Received February 20, 2022, accepted March 3, 2022, date of publication March 8, 2022, date of current version March 17, 2022.

Digital Object Identifier 10.1109/ACCESS.2022.3157875

Robust Mixed Performance Control of Uncertain T-S Fuzzy Systems With Interval Time-Varying Delay by Sampled-Data Input

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The work of Chang-Hua Lien was supported by the Ministry of Science and Technology of Taiwan under Grant MOST 109-2221-E-992-054 and Grant MOST 110-2218-E-006-014-MBK.

ABSTRACT In this paper, a sampled-data parallel distributed compensator (PDC) is proposed to guarantee mixed H_2/H_∞ performance of uncertain T-S fuzzy systems with interval time-varying delay and linear fractional perturbations. A full matrix formulation approach is developed to present our main results in LMI conditions. To achieve better results, new inequality and Lyapunov-Krasovskii functional are developed to improve the conservativeness of the proposed results. Finally, some numerical examples are illustrated to show the use of our main results. In this paper, interval time-varying delay and interval sampling are considered instead of constant delay and periodic sampling in published literatures.

INDEX TERMS H_2/H_∞ performance, interval sampling period, interval time-varying delay, T-S fuzzy system, sampled-data control.

I. INTRODUCTION

Time-delay phenomena are often encountered in various practical systems; such as aircraft stabilization, biology and medicine engineering, chemical engineering systems, control of epidemics, distributed networks, inferred grinding model, manual control, mechanical operation; microwave oscillators, models of lasers, neural networks, nuclear reactors, population dynamic models, rolling mills, ship stabilization, and systems with lossless transmission lines. On the other hand, time delay is often the source of instability and generation of oscillation in many physical systems. Hence, the stability issues of T-S fuzzy systems with time delays have been investigated in recent years [1]–[4]. It is interesting to note that the models of practical systems are always containing several nonlinear properties. Hence the Takagi-Sugeno (T-S) fuzzy system models [5], [6] were introduced to approximate these nonlinear elements in many physical examples. T-S fuzzy system is a useful tool to solve the control design problems in many nonlinear practical applications for dynamic systems; such as guidance and mooring control in autonomous surface vehicle; nonfragile control of permanent-magnet synchronous motor; stabilization of inverted pendulum and motor drive control; predictive and dissipative control of neural networks; predictive control for a diesel engine; performance control of truck-trailer model; dissipative control of wind turbine model; sampled-data control of reaction-diffusion neural networks, vehicle suspension systems, and wind energy conversion systems [7]–[18]. This approach provides a connection between the linear control theory and the fuzzy concept. It is also interesting to note that interval time-varying delay in [10]–[12], [19] is more suitable to describe the transportation delay than constant delay in [20]–[22]. In recent years, stability and performance for T-S fuzzy systems with time delays were investigated by Lyapunov theory and LMI (Linear Matrix Inequality) approach in [10]–[23].

Sampled-data state feedback input is a useful approach to implement some complicate control schemes; such as parallel distributed compensator (PDC) in T-S fuzzy system [20]–[23] and switching control in switched system [24]. Parallel distributed compensator and switching control are always

The associate editor coordinating the review of this manuscript and approving it for publication was Laura Celentano.

used to enrich the performance for T-S fuzzy systems and switched systems, but PDC and switching control are difficult to implement by analog devices. Hence sampled-data state feedback control input is an available consideration to implement PDC and switching control for systems under consideration. Suppose the control input is calculated by a digital device (computer or chip), then the feedback value will be remained until the next sampling instant to reflect the sampled value [16]–[18], [20]–[24]. The allowable upper bound for fixed sampling period $T > 0$ will be an important issue to guarantee the performance of systems under consideration. To implement the distant from state feedback control, networked control technology was provided to finish the goal in the recent years. Aperiodic sampling concept in [20] is a more practical application, but only pointwise sampling period can be guaranteed the performance of systems under consideration. Since the congestion for transmission in network or signal processing of sampler, the actual information transmits to actuator will produce in different sampling periods [19], [22], [23]. Hence interval sampling period is more suitable for practical implementation in sampled-data control systems than constant and pointwise sampling periods [13], [14], [16]–[19], [22], [23], [25]. In this paper, we propose a novel inequality and new Lyapunov-Krasovskii functional to guarantee the mixed performance and design the robust PDC state feedback sampled-data control input with interval sampling period.

In the past, the H_∞ performance of systems under consideration was used to minimize the effect of regulated output with respect to disturbance input and guarantee that the closed-loop system is stable [4], [7], [9], [12], [20], [21], [25]–[27]. On the other hand, the H_2 performance of systems was applied to minimize the dynamics with respect to initial condition of system under consideration and zero disturbance. Hence the system with mixed performance has been an interesting research topic in recent years [20], [26], [27]. In this paper, the mixed H_2/H_∞ performance scheme is proposed to minimize upper bound of H_∞ performance with respect to H_2 measure. Linear fractional perturbation is a general presentation about systems with some uncertain elements or nonlinearities [28], [29]. In this paper, we use LMI optimization approach in [30] to guarantee the mixed H_2/H_∞ performace and design the sampled-data PDC. The main contributions of this paper can be highlighted as follows:

- In this paper, the optimal H_∞ performance for uncertain T-S fuzzy system with interval time-varying delay and linear fractional perturbations is achieved by sampled-data PDC. The H_2 measure can be provided to guarantee the upper bound in response for regulated output of system under consideration.
- To overcome the difficulty about the multiplication and combination of matrices, the full matrix formulation approach is developed in this paper. With the proposed approach, our results can be shown in LMI optimization formulation which can be solved by LMI toolbox of Matlab directly. For more complex system under consideration or other inequalities used, our developed approach is also a good tool for further analysis.
- An upper bound about the sampling period can be evaluated instead of pointwise values in our past results [20]. Interval time-varying delay is considered instead of constant delay in [20]–[22] for the uncertain T-S fuzzy system under consideration. The proposed LMI conditions are easier to solve than the proposed ones in [20].
- To improve the conservativeness for proposed conditions in time-varying delay, inequalities in Lemmas 1 and 2 are used simultaneously. Lyapunov-Krasovskii functional including the term $X^T(t)PX(t)$ is proposed to derive the LMI conditions in our main results. The vector $X(t)$ includes possible information of system under consideration to improve the conservativeness of the proposed results. From the illustrated examples, the better disturbance attenuation and more exact evaluation on H_2 measure have been shown in our main results.

The remainder of this paper is organized as follows. The problem formulation and main results are given in Section 2. Section 3 provides some examples to illustrate the main results. Finally, a conclusion is made in Section 4.

Notations: For a matrix A, we denote the transpose by A^T, symmetric positive (negative) definite by $A > 0$ ($A < 0$). $A \leq B$ means that matrix $B - A$ is symmetric positive semi-definite. $Sym(X) = X + X^T$, I and 0 denote the identity matrix and zero matrix with appropriate dimension, respectively, $E_{q,i} = \begin{bmatrix} 0_{n\times(i-1)n} & I & 0_{n\times(q-i)n} \end{bmatrix} \in \Re^{n\times qn}$, $i = 1, 2, \cdots, q$, $q = 2, 3, \cdots, 15$, $L_2(0,\infty) = \left\{ w \in \Re^m \,\middle|\, \int_0^\infty w^T(t)\, w(t)\, dt < \infty \right\}$, and $\underline{m} = \{1, 2, \cdots, m\}$.

II. PROBLEM FORMULATION AND MAIN RESULT

Consider the following T-S fuzzy system with interval time-varying delay and sampled-data PDC:

Rule i:

If $z_1(t)$ is about M_{i1} and $\cdots z_r(t)$ is about M_{ir}, then

$$\begin{aligned} \dot{x}(t) &= \bar{A}_{0i}(t)\,x(t) + \bar{A}_{1i}(t)\,x(t-h(t)) \\ &\quad + \bar{B}_{ui}(t)\,u(t) + \bar{B}_{wi}(t)\,w(t), \quad t \geq 0, \qquad (1a) \\ y(t) &= \bar{A}_{y0i}(t)\,x(t) + \bar{A}_{y1i}(t)\,x(t-h(t)) \\ &\quad + \bar{B}_{yui}(t)\,u(t) + \bar{B}_{ywi}(t)\,w(t), \quad t \geq 0, \qquad (1b) \\ x(t) &= \varphi_i(t), \quad t \in [-h_M, 0], \; i \in \{1, \cdots, m\}, \qquad (1c) \end{aligned}$$

where $z(t) \in \Re^r$ is a premise variable, M_{ij}, $i \in \{1, \cdots, m\}$, $j \in \{1, \cdots, r\}$, $\bar{A}_{0i}(t) = A_{0i} + \Delta A_{0i}(t)$, $\bar{A}_{1i}(t) = A_{1i} + \Delta A_{1i}(t)$, $\bar{B}_{ui}(t) = B_{ui} + \Delta B_{ui}(t)$, $\bar{B}_{wi}(t) = B_{wi} + \Delta B_{wi}(t)$, $\bar{A}_{y0i}(t) = A_{y0i} + \Delta A_{y0i}(t)$, $\bar{A}_{y1i}(t) = A_{y1i} + \Delta A_{y1i}(t)$, $\bar{B}_{yui}(t) = B_{yui} + \Delta B_{yui}(t)$, $\bar{B}_{ywi}(t) = B_{ywi} + \Delta B_{ywi}(t)$, $x(t) \in \Re^n$ is state at time t, $u(t) \in \Re^p$ is a control input, $w(t) \in \Re^m$ is the disturbance input, $y(t) \in \Re^q$ is the regulated output, $h(t) > 0$ is an interval time-varying delay belonging to $0 < h_m \leq h(t) \leq h_M$, $\dot{h}(t) \leq h_D < 1$, h_m, h_M, and h_D are some positive constants which can be estimated and given in advance, and the initial vector $\varphi_i \in C_0$, where

C_0 is the set of continuous functions from $[-h_M, 0]$ to $\Re^n$. Matrices $A_{0i}, A_{1i} \in \Re^{n\times n}$, $B_{ui} \in \Re^{n\times p}$, $B_{wi} \in \Re^{n\times m}$, A_{y0i}, $A_{y1i} \in \Re^{q\times n}$, $B_{yui} \in \Re^{q\times p}$, and $B_{ywi} \in \Re^{q\times m}$, $i \in \underline{m}$, are constant matrices. $\Delta A_{0i}(t)$, $\Delta A_{1i}(t)$, $\Delta B_{ui}(t)$, $\Delta B_{wi}(t)$, $\Delta A_{y0i}(t)$, $\Delta A_{y1i}(t)$, $\Delta B_{yui}(t)$, and $\Delta B_{ywi}(t)$, $i \in \underline{m}$, are some perturbed matrices and satisfy the following linear fractional perturbation conditions:

$$\begin{aligned} &\left[\,\Delta A_{0i}(t)\;\;\Delta A_{1i}(t)\;\;\Delta B_{ui}(t)\;\;\Delta B_{wi}(t)\,\right] \\ &\quad = M_{xi}\cdot\Delta_{xi}(t)\cdot\left[\,N_{x0i}\;N_{x1i}\;N_{xui}\;N_{xwi}\,\right], \quad (2a) \\ &\left[\,\Delta A_{y0i}(t)\;\;\Delta A_{y1i}(t)\;\;\Delta B_{yui}(t)\;\;\Delta B_{ywi}(t)\,\right] \\ &\quad = M_{yi}\cdot\Delta_{yi}(t)\cdot\left[\,N_{y0i}\;N_{y1i}\;N_{yui}\;N_{ywi}\,\right], \quad (2b) \\ &\Delta_{xi}(t) = \left[I-\Gamma_{xi}(t)\,\Xi_{xi}\right]^{-1}\Gamma_{xi}(t), \quad \Xi_{xi}\Xi_{xi}^T < I, \quad (2c) \\ &\Delta_{yi}(t) = \left[I-\Gamma_{yi}(t)\,\Xi_{yi}\right]^{-1}\Gamma_{yi}(t), \quad \Xi_{yi}\Xi_{yi}^T < I, \quad (2d) \end{aligned}$$

where M_{xi}, M_{zi}, N_{x0i}, N_{x1i}, N_{xui}, N_{xwi}, N_{y0i}, N_{y1i}, N_{yui}, N_{ywi}, Ξ_{xi}, and Ξ_{yi}, $i \in \underline{m}$, are some given constant matrices with appropriate dimensions. $\Gamma_{xi}(t)$ and $\Gamma_{yi}(t)$, $\forall i \in \underline{m}$, are unknown matrices representing the parameter perturbations which satisfy

$$\begin{aligned} &\Gamma_{xi}(t)^T\cdot\Gamma_{xi}(t) \le I, \quad \forall i \in \underline{m},\; t \ge 0, \quad (2e) \\ &\Gamma_{yi}(t)^T\cdot\Gamma_{yi}(t) \le I, \quad \forall i \in \underline{m},\; t \ge 0. \quad (2f) \end{aligned}$$

If we use the standard fuzzy inference method [5], [6], the system (2) is inferred as follows:

$$\begin{aligned} \dot{x} &= \sum_{i=1}^{m} v_i(z(t))\cdot\{\bar{A}_{0i}(t)\,x(t) + \bar{A}_{1i}(t)\,x(t-h(t)) \\ &\quad + \bar{B}_{wi}(t)\,w(t) + \bar{B}_{ui}(t)\,u(t)\} \Big/ \sum_{i=1}^{m} v_i(z(t)) \\ &= \sum_{i=1}^{m} \eta_i(z(t))\cdot\{\bar{A}_{0i}(t)\,x(t) + \bar{A}_{1i}(t)\,x(t-h(t)) \\ &\quad + \bar{B}_{wi}(t)\,w(t) + \bar{B}_{ui}(t)\,u(t)\}, \quad t \ge 0, \quad (3a) \\ y(t) &= \sum_{i=1}^{m} \eta_i(z(t))\cdot\{\bar{A}_{y0i}(t)\,x(t) + \bar{A}_{y1i}(t) \\ &\quad \cdot x(t-h(t)) + \bar{B}_{ywi}(t)\,w(t) + \bar{B}_{yui}(t)\,u(t)\}, \\ &\quad t \ge 0, \quad (3b) \\ x(t) &= \varphi(t) = \sum_{i=1}^{m} \eta_i(z(t))\cdot\varphi_i(t), \quad t \in [-h_M, 0], \quad (3c) \end{aligned}$$

where $v_i(z(t)) = \min_j \Omega_{ij}(z_j(t))$, $\eta_i(z(t)) = v_i(z(t)) / \sum_{i=1}^m v_i(z(t))$, $\Omega_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in fuzzy set M_{ij}. The term $\eta_i(z(t))$ is denoted as the ratio weight of each fuzzy rule. In this paper, we assume $v_i(z(t)) \ge 0$, $i \in \{1, \cdots, m\}$, and $\sum_{i=1}^m v_i(z(t)) > 0$. Hence $\eta_i(z(t)) \ge 0$ and $\sum_{i=1}^m \eta_i(z(t)) = 1$, for all $t \ge 0$. The sampled-data state feedback control input is selected as:

Rule i:

If $z_1(t)$ is about M_{i1} and $\cdots z_r(t)$ is about M_{ir}, then

$$u(t) = -K_i x(T_k), \quad t \in [T_k, T_{k+1}). \quad (4a)$$

In the practical condition, the state of the system will be sampled by a sampler. The sampling instants may be defined by $0 = T_0 < T_1 < T_2 < \cdots$. From the sampling instants, the following time-varying function can be defined:

$$\tau(t) = t - T_k, \quad t \in [T_k, T_{k+1}),$$

where the sampling period $\tau_k = T_{k+1} - T_k$, $\tau_k \le \tau_M = max_{k=0}^{k=\infty}\tau_k$, τ_M is a positive constant which can be estimated and given in advance. In this paper, the sampling period τ_k is interval and defined less than $\tau_M > 0$. Then we have $0 \le \tau(t) \le \tau_M$, $t \ge 0$, and $T_k = t - \tau(t)$, $t \in [T_k, T_{k+1})$.

The sampled-data control input can be inferred as follows:

$$\begin{aligned} u(t) &= -\sum_{i=1}^{m} \eta_i(z(t))\cdot K_i x(T_k) \\ &= -\sum_{i=1}^{m} \eta_i(z(t))\cdot K_i x(t-\tau(t)), \quad t \ge 0, \quad (4b) \end{aligned}$$

where $K_i \in \Re^{p\times n}$, $i \in \underline{m}$, is the designed controller gain. From (3) and (4b), we have

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{m}\sum_{j=1}^{m} \eta_i(z(t))\,\eta_j(z(t)) \\ &\quad \cdot\{\bar{A}_{0i}(t)\,x(t) + \bar{A}_{1i}(t)\,x(t-h(t)) + \bar{B}_{wi}(t)\,w(t) \\ &\quad - \bar{B}_{ui}(t)\,K_j x(t-\tau(t))\}, \quad t \ge 0, \quad (5a) \\ y(t) &= \sum_{i=1}^{m}\sum_{j=1}^{m} \eta_i(z(t))\,\eta_j(z(t)) \\ &\quad \cdot\bar{A}_{y0i}(t)\,x(t) + \bar{A}_{y1i}(t)\,x(t-h(t)) \\ &\quad + \{\bar{B}_{ywi}(t)\,w(t) - \bar{B}_{yui}(t)\,K_j x(t-\tau(t))\}, \quad (5b) \\ x(t) &= \sum_{i=1}^{m} \eta_i(z(t))\cdot\varphi_i(t), \quad t \in [-h_M, 0]. \quad (5c) \end{aligned}$$

The following lemmas will be used to derive the main results in this paper.

Lemma 1 [20]: Let $x(t)$ be a differentiable function: $[t-h_2, t-h_1] \to \Re^n$. For a matrix $R > 0$ and matrices $N_1, N_2, N_3 \in \Re^{4n\times n}$, the following inequality holds:

$$-\int_{t-h_2}^{t-h_1} \dot{x}^T(s)\,R\dot{x}(s)\,ds \le \delta^T(t)\,\Omega\delta(t),$$

where

$$\begin{aligned} \delta^T(t) &= [x^T(t-h_1) \quad x^T(t-h_2) \\ &\quad \frac{1}{h}\int_{t-h_2}^{t-h_1} x^T(s)\,ds \quad \frac{2}{h^2}\int_{t-h_2}^{t-h_1}\int_{t-h_2}^{s} x^T(u)\,duds], \\ h &= h_2 - h_1 \ge 0, \\ \Omega &= h\cdot\left[N_1R^{-1}N_1^T + \frac{1}{3}N_2R^{-1}N_2^T + \frac{1}{5}N_3R^{-1}N_3^T\right] \\ &\quad + Sym(N_1\Pi_1 + N_2\Pi_2 + N_3\Pi_3), \\ \Pi_1 &= E_{4,1} - E_{4,2}, \quad \Pi_2 = E_{4,1} + E_{4,2} - 2E_{4,3}, \\ \Pi_3 &= E_{4,1} - E_{4,2} - 6E_{4,3} + 6E_{4,4}. \end{aligned}$$

Lemma 2: For any matrices R > 0 *and* S, *two nonnegative real numbers h_m and h_M with $h_m \le h(t) \le h_M$, a differentiable vector function $x(t) \in \Re^n$, and*

$$\begin{bmatrix} R & S \\ * & R \end{bmatrix} > 0,$$

the following inequality is satisfied:

$$
\begin{aligned}
&-(h_M-h_m)\cdot\int_{t-h_M}^{t-h_m}\dot{x}^T(s)R\dot{x}(s)\,ds\\
&\le-\begin{bmatrix}x(t-h(t))-x(t-h_M)\\ x(t-h_m)-x(t-h(t))\end{bmatrix}^T\begin{bmatrix}R & S\\ * & R\end{bmatrix}\\
&\begin{bmatrix}x(t-h(t))-x(t-h_M)\\ x(t-h_m)-x(t-h(t))\end{bmatrix}\\
&=Z_2^T(t)\Omega_2 Z_2(t),
\end{aligned}
$$

where

$$
Z_2^T(t)=\begin{bmatrix}x^T(t-h(t)) & x^T(t-h_m) & x^T(t-h_M)\end{bmatrix},
$$

$$
\Omega_2=\begin{bmatrix}-2R+S+S^T & R-S & R-S^T\\ * & -R & S^T\\ * & * & -R\end{bmatrix}.
$$

Proof: By Jensen inequality, we have

$$
\begin{aligned}
&-(h_M-h_m)\cdot\int_{t-h_M}^{t-h_m}\dot{x}^T(s)R\dot{x}(s)\,ds\\
&=-(h_M-h_m)\cdot\left[\int_{t-h_M}^{t-h}(t)\,\dot{x}^T(s)R\dot{x}(s)\,ds\right.\\
&\quad\left.+\int_{t-h(t)}^{t-h_m}\dot{x}^T(s)R\dot{x}(s)\,ds\right]\\
&\le-\left[1+\frac{(h(t)-h_m)}{(h_M-h(t))}\right][x(t-h(t))-x(t-h_M)]^T R\\
&\quad\times[x(t-h(t))-x(t-h_M)]\\
&\quad-\left[1+\frac{(h_M-h(t))}{(h(t)-h_m)}\right][x(t-h_m)-x(t-h(t))]^T R\\
&\quad\times[x(t-h_m)-x(t-h(t))]\\
&\le-\begin{bmatrix}x(t-h(t))-x(t-h_M)\\ x(t-h_m)-x(t-h(t))\end{bmatrix}^T\begin{bmatrix}R & S\\ * & R\end{bmatrix}\\
&\quad\times\begin{bmatrix}x(t-h(t))-x(t-h_M)\\ x(t-h_m)-x(t-h(t))\end{bmatrix}\\
&\quad-\begin{bmatrix}\sqrt{\frac{(h(t)-h_m)}{(h_M-h(t))}}(x(t-h(t))-x(t-h_M))\\ -\sqrt{\frac{(h_M-h(t))}{(h(t)-h_m)}}(x(t-h_m)-x(t-h(t)))\end{bmatrix}^T\\
&\quad\times\begin{bmatrix}R & S\\ * & R\end{bmatrix}\\
&\quad\times\begin{bmatrix}\sqrt{\frac{(h(t)-h_m)}{(h_M-h(t))}}(x(t-h(t))-x(t-h_M))\\ -\sqrt{\frac{(h_M-h(t))}{(h(t)-h_m)}}(x(t-h_m)-x(t-h(t)))\end{bmatrix}\\
&\le-\begin{bmatrix}x(t-h(t))-x(t-h_M)\\ x(t-h_m)-x(t-h(t))\end{bmatrix}^T\begin{bmatrix}R & S\\ * & R\end{bmatrix}\\
&\quad\times\begin{bmatrix}x(t-h(t))-x(t-h_M)\\ x(t-h_m)-x(t-h(t))\end{bmatrix}.
\end{aligned}
$$

This completes the proof. □

Lemma 3 ([28], [29]): Suppose that $\Delta_{xi}(t)$ is defined in (2a) and satisfying (2c), then for real matrices U_i, W_i and X_i with $X_i = X_i^T$, the following statements are equivalent:

(I) The inequality is satisfied

$$
X_i+U_i\Delta_{xi}(t)W_i+W_i^T\Delta_{xi}^T(t)U_i^T<0,
$$

(II) There exists a scalar $\varepsilon_i>0$, such that

$$
\begin{bmatrix}X_i & \varepsilon_i\cdot U_i & W_i^T\\ * & -\varepsilon_i\cdot I & \varepsilon_i\cdot\Xi_{xi}^T\\ * & * & -\varepsilon_i\cdot I\end{bmatrix}<0,
$$

where the matrix Ξ_{xi} is defined in (2c).

Definition 1: Consider the system (5) with (2) and the sampled-data PDC in (4b). Assume

(i) With $w(t)=0$, the system (5) with (2) is asymptotically stable by the sampled-data PDC in (4b).

(ii) With zero initial conditions (i.e. $\varphi(t)=0$, $-h_M\le t\le 0$), the signals $w(t)$ and $y(t)$ are bounded by

$$
\int_0^{\ell_1}y^T(t)y(t)\,dt\le\gamma^2\cdot\int_0^{l}w^T(t)w(t)\,dt,\quad\forall w\ne 0,
$$

for all positive constants ℓ_1 and γ. If the parameter ℓ_1 is selected as ∞, the disturbance input w should be constrained in $L_2(0,\infty)$.

(iii) Under zero disturbance $w(t)=0$, an upper bound $\alpha>0$ can be found satsifying the following condition

$$
\int_0^{\ell_2}y^T(t)y(t)\,dt\le\alpha,
$$

for all positive constant ℓ_2.

Then we say that the system (5) with (2) is asymptotically stabilizable by the sampled-data PDC in (4b) with K_i, H_∞ performance γ, and H_2 measure α.

The delay-dependent LMI optimization results are developed to guarantee the asymptotic stability and mixed performance by the design of sampled-data PDC in (4b).

Theorem 1: Suppose there exist some constants η, such that the following LMI optimization problem:

$$
\text{minimize }\bar{\gamma},\tag{6a}
$$

$$
\textit{subject to}\begin{bmatrix}\hat{R}_5 & \hat{S}_1\\ * & \hat{R}_5\end{bmatrix}>0,\tag{6b}
$$

$$
\begin{bmatrix}\hat{R}_6 & \hat{S}_2\\ * & \hat{R}_6\end{bmatrix}>0,\tag{6c}
$$

$$
\hat{\Psi}_{ij}=\begin{bmatrix}\hat{\Psi}_{11ij} & \hat{\Psi}_{12ij} & \hat{\Psi}_{13i} & 0 & \hat{\Psi}_{15ij} & \hat{\Psi}_{16ij} & \hat{\Psi}_{17}\\ * & -I & 0 & \varepsilon_i\cdot M_{yi} & 0 & 0 & 0\\ * & * & -\varepsilon_i\cdot I & 0 & \varepsilon_i\cdot\Xi_{xi} & 0 & 0\\ * & * & * & -\varepsilon_i\cdot I & 0 & \varepsilon_i\cdot\Xi_{yi} & 0\\ * & * & * & * & -\varepsilon_i\cdot I & 0 & 0\\ * & * & * & * & * & -\varepsilon_i\cdot I & 0\\ * & * & * & * & * & * & \hat{\Psi}_{77}\end{bmatrix}
$$

$$
<0,\quad i\in\underline{N},\ j\in\underline{N},\tag{6d}
$$

where

$$\begin{aligned}
\hat{\Psi}_{11ij} &= Sym\left(\Delta_2^T\hat{P}\Delta_1 + \left(\eta\cdot E_{16,1}^T + E_{16,16}^T\right)\hat{\Gamma}_{1ij}\right) + \hat{\Omega}_1 \\
&\quad + \hat{\Omega}_2 + \hat{\Omega}_3, \\
\hat{\Psi}_{12ij} &= \hat{\Gamma}_{2ij}^T, \quad \hat{\Psi}_{13i} = \varepsilon_i\cdot\left(\eta\cdot E_{16,1}^T + E_{16,16}^T\right)M_{xi}, \\
\hat{\Psi}_{15ij}^T &= [N_{x0i}\hat{U}^T\ N_{x1i}\ \hat{U}^T \\
&\quad 0\ 0\ -N_{xui}\hat{K}_j\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ N_{xwi}\ 0], \\
\hat{\Psi}_{16ij}^T &= [N_{y0i}\hat{U}^T\quad N_{y1i}\hat{U}^T \\
&\quad 0\ 0\ -N_{yui}\hat{K}_j\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ N_{ywi}\ 0], \\
\hat{\Psi}_{17} &= [\sqrt{h_m}\cdot\Lambda_1^T\hat{N}_{11}\ \sqrt{h_m}\cdot\Lambda_1^T\hat{N}_{12}\ \sqrt{h_m}\cdot\Lambda_1^T\hat{N}_{13} \\
&\quad \sqrt{h_M}\cdot\Lambda_2^T\hat{N}_{21}\ \sqrt{h_M}\cdot\Lambda_2^T\hat{N}_{22}\ \sqrt{h_M}\cdot\Lambda_2^T\hat{N}_{23} \\
&\quad \sqrt{h_M - h_m}\cdot\Lambda_3^T\hat{N}_{31}\ \sqrt{h_M - h_m}\cdot\Lambda_3^T\hat{N}_{32} \\
&\quad \sqrt{h_M - h_m}\cdot\Lambda_3^T\hat{N}_{33}\ \sqrt{\tau_M}\cdot\Lambda_4^T\hat{N}_{41}\ \sqrt{\tau_M}\cdot\Lambda_4^T\hat{N}_{42} \\
&\quad \sqrt{\tau_M}\cdot\Lambda_4^T\hat{N}_{43}], \\
\hat{\Psi}_{77} &= diag[-\hat{R}_1 - 3\hat{R}_1 - 5\hat{R}_1 - \hat{R}_2 - 3\hat{R}_2 - 5\hat{R}_2 \\
&\quad - \hat{R}_3 - 3\hat{R}_3 - 5\hat{R}_3 - \hat{R}_4 - 3\hat{R}_4 - 5\hat{R}_4], \\
\hat{\Omega}_1 &= \sum_{i=1}^{3} Sym(\Lambda_1^T\hat{N}_{1i}\Pi_i\Lambda_1 + \Lambda_2^T\hat{N}_{2i}\Pi_i\Lambda_2 + \Lambda_3^T\hat{N}_{3i}\Pi_i\Lambda_3 \\
&\quad + \Lambda_4^T\hat{N}_{4i}\Pi_i\Lambda_4), \\
\hat{\Omega}_2 &= \Lambda_5^T\begin{bmatrix} -2\hat{R}_5 + \hat{S}_1 + \hat{S}_1^T & \hat{R}_5 - \hat{S}_1 & \hat{R}_5 - \hat{S}_1^T \\ * & -\hat{R}_5 & \hat{S}_1^T \\ * & * & -\hat{R}_5 \end{bmatrix}\Lambda_5 \\
&\quad + \Lambda_6^T\begin{bmatrix} -2\hat{R}_6 + \hat{S}_2 + \hat{S}_2^T & \hat{R}_6 - \hat{S}_2 & \hat{R}_6 - \hat{S}_2^T \\ * & -\hat{R}_6 & \hat{S}_2^T \\ * & * & -\hat{R}_6 \end{bmatrix}\Lambda_6, \\
\hat{\Omega}_3 &= E_{16,1}^T\hat{Q}E_{16,1} - (1 - h_D)\cdot E_{16,2}^T\hat{Q}_4E_{16,2} \\
&\quad - E_{16,3}^T(\hat{Q}_1 - \hat{Q}_3)E_{16,3} - E_{16,4}^T(\hat{Q}_2 + \hat{Q}_3)E_{16,4} \\
&\quad - E_{16,6}^T\hat{Q}_5E_{16,6} - \bar{A}\cdot E_{16,15}^TE_{16,15} \\
&\quad - \eta\cdot Sym\left(E_{16,1}^T\hat{U}^TE_{16,16}\right) \\
&\quad - E_{16,16}^T\left[\hat{U} + \hat{U}^T - \hat{R}\right]E_{16,16}, \\
\hat{Q} &= \hat{Q}_1 + \hat{Q}_2 + \hat{Q}_4 + \hat{Q}_5, \\
\hat{R} &= h_m\cdot\hat{R}_1 + h_M\cdot\hat{R}_2 + (h_M - h_m)\cdot(\hat{R}_3 + \hat{R}_5) \\
&\quad + \tau_M\cdot(\hat{R}_4 + \hat{R}_6), \\
\Pi_1 &= E_{4,1} - E_{4,2}, \quad \Pi_2 = E_{4,1} + E_{4,2} - 2E_{4,3}, \\
\Pi_3 &= E_{4,1} - E_{4,2} - 6E_{4,3} + 6E_{4,4}, \\
\Delta_1 &= \Big[E_{16,1}^T\quad h_m\cdot E_{16,7}^T\quad h_M\cdot E_{16,8}^T\quad \left(h_M - h_m\right)\cdot E_{16,9}^T \\
&\quad \tau_M\cdot E_{16,10}^T\quad 0.5\cdot h_m^2\cdot E_{16,11}^T\quad 0.5\cdot h_M^2\cdot E_{16,12}^T \\
&\quad 0.5\cdot(h_M - h_m)^2\cdot E_{16,13}^T 0.5\cdot\tau_M^2\cdot E_{16,14}^T\Big]^T, \\
\Delta_2 &= [E_{16,16}^T\quad \left(E_{16,1} - E_{16,3}\right)^T\quad \left(E_{16,1} - E_{16,4}\right)^T \\
&\quad \left(E_{16,3} - E_{16,4}\right)^T\quad \left(E_{16,1} - E_{16,6}\right)^T \\
&\quad h_m\cdot\left(E_{16,7} - E_{16,3}\right)^T\quad h_M\cdot\left(E_{16,8} - E_{16,4}\right)^T \\
&\quad (h_M - h_m)\cdot\left(E_{16,9} - E_{16,4}\right)^T \\
&\quad \tau_M\cdot\left(E_{16,10} - E_{16,6}\right)^T]^T,
\end{aligned}$$

$$\begin{aligned}
\Lambda_1 &= [\,E_{16,1}^T\ \ E_{16,3}^T\ \ E_{16,7}^T\ \ E_{16,11}^T\,]^T, \\
\Lambda_2 &= [\,E_{16,1}^T\ \ E_{16,4}^T\ \ E_{16,8}^T\ \ E_{16,12}^T\,]^T, \\
\Lambda_3 &= [\,E_{16,3}^T\ \ E_{16,4}^T\ \ E_{16,9}^T\ \ E_{16,13}^T\,]^T, \\
\Lambda_4 &= [\,E_{16,1}^T\ \ E_{16,6}^T\ \ E_{16,10}^T\ \ E_{16,14}^T\,]^T, \\
\Lambda_5 &= \left[\,E_{16,2}^T\ \ E_{16,3}^T\ \ E_{16,4}^T\,\right]^T, \\
\Lambda_6 &= \left[\,E_{16,5}^T\ \ E_{16,1}^T\ \ E_{16,6}^T\,\right]^T, \\
\hat{\Gamma}_{1ij} &= [A_{0i}\hat{U}^T\quad A_{1i}\hat{U}^T\quad 0\quad 0\quad -B_{ui}\hat{K}_j \\
&\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad B_{wi}\quad 0], \\
\hat{\Gamma}_{2ij} &= [A_{y0i}\hat{U}^T\ A_{y1i}\hat{U}^T\ 0\ 0\ -B_{yui}\hat{K}_j\ 0\ 0\ 0\ 0\ 0 \\
&\quad 0\ 0\ 0\ 0\ B_{ywi}\ 0],
\end{aligned}$$

has a feasible solution with some constants $\bar{\gamma} > 0$, $\varepsilon_i > 0$, $i \in \underline{N}$, *a nonsingular matrix* $\hat{U} \in \Re^{n\times n}$, *some positive definite symmetric matrices* $\hat{P} \in \Re^{9n\times 9n}$, $\hat{Q}_i$, $\hat{R}_j \in \Re^{n\times n}$, $i \in \underline{5}, j \in \underline{6}$, *and some matrices* $\hat{S}_1, \hat{S}_2 \in \Re^{n\times n}$, $\hat{N}_{ij} \in \Re^{4n\times n}$, $i \in \underline{4}, j \in \underline{3}$. *Then the system (5) with (2) is asymptotically stabilizable by the sampled-data PDC in (4b) with* $K_i = \hat{K}_i\hat{U}^{-T}$, H_∞ *performance* $\gamma = \sqrt{\bar{\gamma}}$, *and* H_2 *measure given by*

$$\begin{aligned}
\alpha &= X^T(0)PX(0) + \int_{-h_m}^{0}\varphi^T(s)Q_1\varphi(s)\,ds \\
&\quad + \int_{-h_M}^{0}\varphi^T(s)Q_2\varphi(s)\,ds + \int_{-h_M}^{-h_m}\varphi^T(s)Q_3\varphi(s)\,ds \\
&\quad + \int_{-h_M}^{0}\varphi^T(s)Q_4\varphi(s)\,ds + \int_{-\tau_M}^{0}\varphi^T(s)Q_5\varphi(s)\,ds \\
&\quad + \int_{-h_m}^{0}\int_{s}^{0}\dot{\varphi}^T(u)R_1\dot{\varphi}(u)duds \\
&\quad + \int_{-h_M}^{0}\int_{s}^{0}\dot{\varphi}^T(u)R_2\dot{\varphi}(u)duds \\
&\quad + \int_{-h_M}^{-h_m}\int_{s}^{0}\dot{\varphi}^T(u)(R_3 + R_5)\dot{\varphi}(u)duds \\
&\quad + \int_{-\tau_M}^{0}\int_{s}^{0}\dot{\varphi}^T(u)(R_4 + R_6)\dot{\varphi}(u)duds, \qquad (6e)
\end{aligned}$$

where

$$\begin{aligned}
P &= \hat{\hat{\hat{U}}}\hat{P}\hat{\hat{\hat{U}}}^T > 0, \quad \hat{\hat{\hat{U}}} = diag\,[U\ U\ U\ U\ U\ U\ U\ U\ U], \\
U &= \hat{U}^{-1}, \quad Q_i = U\hat{Q}_iU^T > 0, \quad R_j = U\hat{R}_jU^T > 0, \\
&\quad i \in \underline{5}, \quad j \in \underline{6}, \\
X(0) &= \Big[\varphi^T(0)\quad \int_{-h_m}^{0}\varphi^T(s)\,ds\quad \int_{-h_M}^{0}\varphi^T(s)\,ds \\
&\quad \int_{-h_M}^{-h_m}\varphi^T(s)\,ds\quad \int_{-\tau_M}^{0}\varphi^T(s)\,ds \\
&\quad \int_{-h_m}^{0}\int_{-h_m}^{s}\varphi^T(u)\,duds \\
&\quad \int_{-h_M}^{t}\int_{-h_M}^{s}\varphi^T(u)\,duds\quad \int_{-h_M}^{-h_m}\int_{-h_M}^{s}\varphi^T(u)\,duds \\
&\quad \int_{-\tau_M}^{0}\int_{-\tau_M}^{s}\varphi^T(u)\,duds\Big]^T.
\end{aligned}$$

Proof: Define the Lyapunov-Krasovskii functional

$$\begin{aligned} V(x_t) &= X^T(t) P X(t) + \int_{t-h_m}^{t} x^T(s) Q_1 x(s)\,ds \\ &\quad + \int_{t-h_M}^{t} x^T(s) Q_2 x(s)\,ds + \int_{t-h_M}^{t-h_m} x^T(s) Q_3 \\ &\quad \times x(s)\,ds + \int_{-h_m}^{0}\int_{t+s}^{t} \dot{x}^T(u) R_1 \dot{x}(u)\,duds \\ &\quad + \int_{-h_M}^{0}\int_{t+s}^{t} \dot{x}^T(u) R_2 \dot{x}(u)\,duds \\ &\quad + \int_{-h_M}^{-h_m}\int_{t+s}^{t} \dot{x}^T(u)(R_3+R_5)\dot{x}(u)\,duds \\ &\quad + \int_{-\tau_M}^{0}\int_{t+s}^{t} \dot{x}^T(u)(R_4+R_6)\dot{x}(u)\,duds, \end{aligned} \tag{7}$$

where $P, Q_i, R_j, i \in \underline{5}, j \in \underline{6}$, are defined in (6e), and

$$\begin{aligned} X(t) &= \Big[x^T(t) \ \int_{t-h_m}^{t} x^T(s)\,ds \ \int_{t-h_M}^{t} x^T(s)\,ds \ \int_{t-h_M}^{t-h_m} x^T(s)\,ds \\ &\quad \int_{t-\tau_M}^{t} x^T(s)\,ds \ \ \int_{t-h_m}^{t}\int_{t-h_m}^{s} x^T(u)\,duds \\ &\quad \int_{t-h_M}^{t}\int_{t-h_M}^{s} x^T(u)\,duds \\ &\quad \int_{t-h_M}^{t-h_m}\int_{t-h_M}^{s} x^T(u)\,duds \ \ \int_{t-\tau_M}^{t}\int_{t-\tau_M}^{s} x^T(u)\,duds\Big]^T. \end{aligned}$$

The time derivatives of $V(x_t)$ along the trajectories of system (5) with (2) satisfy

$$\begin{aligned} \dot{V}(x_t) &= \dot{X}^T(t) P X(t) + X^T(t) P \dot{X}(t) + x^T(t) Q x(t) \\ &\quad - x^T(t-h_m)(Q_1 - Q_3) x(t-h_m) \\ &\quad - x^T(t-h_M)(Q_2 + Q_3) x(t-h_M) \\ &\quad - (1-\dot{h}(t)) \cdot x^T(t-h(t)) Q_4 x(t-h(t)) \\ &\quad - x^T(t-\tau_M) Q_5 x(t-\tau_M) + \dot{x}^T(t) R \dot{x}(t) - \nabla(t) \\ &\quad - \int_{t-h_M}^{t-h_m} \dot{x}^T(s) R_5 \dot{x}(s)\,ds - \int_{t-\tau_M}^{t} \dot{x}^T(s) R_6 \dot{x}(s)\,ds, \end{aligned} \tag{8a}$$

where

$$\begin{aligned} Q &= Q_1 + Q_2 + Q_4 + Q_5, \\ R &= h_m \cdot R_1 + h_M \cdot R_2 + (h_M - h_m) \cdot (R_3 + R_5) \\ &\quad + \tau_M \cdot (R_4 + R_6), \\ \nabla(t) &= \int_{t-h_m}^{t} \dot{x}^T(s) R_1 \dot{x}(s)\,ds + \int_{t-h_M}^{t} \dot{x}^T(s) R_2 \dot{x}(s)\,ds \\ &\quad + \int_{t-h_M}^{t-h_m} \dot{x}^T(s) R_3 \dot{x}(s)\,ds + \int_{t-\tau_M}^{t} \dot{x}^T(s) R_4 \dot{x}(s)\,ds. \end{aligned}$$

Define

$$\begin{aligned} Y(t) &= \Big[x^T(t) \ x^T(t-h(t)) \ x^T(t-h_m) \ x^T(t-h_M)) \\ &\quad x^T(t-\tau(t)) \ x^T(t-\tau_M) \ \frac{1}{h_m}\int_{t-h_m}^{t} x^T(s)\,ds \\ &\quad \frac{1}{h_M}\int_{t-h_M}^{t} x^T(s)\,ds \frac{1}{h_M-h_m}\int_{t-h_M}^{t-h_m} x^T(s)\,ds \\ &\quad \frac{1}{\tau_M}\int_{t-\tau_M}^{t} x^T(s)\,ds \ \tfrac{2}{h_m^2}\int_{t-h_m}^{t}\int_{t-h_m}^{s} x^T(u)\,duds \\ &\quad \frac{2}{h_M^2}\int_{t-h_M}^{t}\int_{t-h_M}^{s} x^T(u)\,duds \\ &\quad \frac{2}{(h_M-h_m)^2}\int_{t-h_M}^{t-h_m}\int_{t-h_M}^{s} x^T(u)\,duds \\ &\quad \frac{2}{\tau_M^2}\int_{t-\tau_M}^{t}\int_{t-\tau_M}^{s} x^T(u)\,duds \ w^T(t)\, \dot{x}^T(t)\Big]^T, \\ \Gamma_{1ij}(t) &= [\bar{A}_{0i}(t) \ \bar{A}_{1i}(t) \ 0 \ 0 \ -\bar{B}_{ui}(t) K_j \ 0 \ 0 \\ &\quad 0\,0\,0\,0\,0\,0\,0\, \bar{B}_{wi}(t) \ 0], \\ \Gamma_{2ij}(t) &= [\bar{A}_{y0i}(t) \ \bar{A}_{y1i}(t) \ 0 \ 0 \ -\bar{B}_{yui}(t) \ K_j \ 0 \ 0 \\ &\quad 0\,0\,0\,0\,0\,0\,0\, \bar{B}_{ywi}(t) \ 0], \end{aligned}$$

we have

$$\begin{aligned} X(t) &= \Delta_1 Y(t), \\ \dot{X}(t) &= \Delta_2 Y(t), \end{aligned}$$

where $\Delta_i, i \in \underline{2}$, are defined in (6c). By Lemma 1 with $\nabla(t)$ in (8a), we have

$$-\nabla(t) \le Y^T(t) \Omega_1 Y(t), \tag{8b}$$

where

$$\begin{aligned} \Omega_1 &= h_m \cdot \Lambda_1^T \exists_1 \Lambda_1 + h_M \cdot \Lambda_2^T \exists_2 \Lambda_2 \\ &\quad + (h_M - h_m) \cdot \Lambda_3^T \exists_3 \Lambda_3 + \tau_M \cdot \Lambda_4^T \exists_4 \Lambda_4 + \bar{\Omega}_1, \\ \exists_i &= N_{i1} R_i^{-1} N_{i1}^T + \frac{1}{3} N_{i2} R_i^{-1} N_{i2}^T + \frac{1}{5} N_{i3} R_i^{-1} N_{i3}^T, \quad i \in \underline{4}, \\ \bar{\Omega}_1 &= \sum_{i=1}^{3} Sym(\Lambda_1^T N_{1i} \Pi_i \Lambda_1 + \Lambda_2^T N_{2i} \Pi_i \Lambda_2 \\ &\quad + \Lambda_3^T N_{3i} \Pi_i \Lambda_3 + \Lambda_4^T N_{4i} \Pi_i \Lambda_4), \end{aligned}$$

$N_{ij} \in \Re^{4n \times n}$, $i \in \underline{4}, j \in \underline{3}$, should be selected and Λ_i, Π_j, $i \in \underline{4}, j \in \underline{3}$, are defined in (6c). From Lemma 2, we have

$$\begin{aligned} -\int_{t-h_M}^{t-h_m} \dot{x}^T(s) R_5 \dot{x}(s)\,ds + \int_{t-\tau_M}^{t} \dot{x}^T(s) R_6 \dot{x}(s)\,ds \\ \le Y^T(t) \Omega_2 Y(t), \end{aligned} \tag{8c}$$

where

$$\begin{aligned} \Omega_2 &= \Lambda_5^T \begin{bmatrix} -2R_5 + S_1 + S_1^T & R_5 - S_1 & R_4 - S_1^T \\ * & -R_5 & S_1^T \\ * & * & -R_5 \end{bmatrix} \Lambda_5 \\ &\quad + \Lambda_6^T \begin{bmatrix} -2R_6 + S_2 + S_2^T & R_6 - S_2 & R_5 - S_2^T \\ * & -R_6 & S_2^T \\ * & * & -R_6 \end{bmatrix} \Lambda_6, \end{aligned}$$

Λ_5 and Λ_6 are defined in (6c). Then from system (5), we have

$$\sum_{i=1}^{m}\sum_{j=1}^{m}\eta_i(z(t))\,\eta_j(z(t)) \cdot\left[\left(-\dot{x}(t)+\Gamma_{1ij}(t)\,Y(t)\right)^T U^T\left(\eta x(t)+\dot{x}(t)\right) + \left(\eta x(t)+\dot{x}(t)\right)^T U\left(-\dot{x}(t)+\Gamma_{1ij}(t)\,Y(t)\right)\right]=0. \tag{8d}$$

By (8a)-(8d), we can obtain the following result with $\bar{\gamma}=\gamma^2$:

$$\dot{V}(x_t)+y^T(t)\,y(t)-\gamma^2\cdot w^T(t)\,w(t) \le \sum_{i=1}^{m}\sum_{j=1}^{m}\eta_i(z(t))\,\eta_j(z(t))\cdot Y^T(t)\,\Pi_{ij}(t)\,Y(t), \tag{8e}$$

where

$$\begin{aligned}\Pi_{ij}(t) &= \Delta_2^T P\Delta_1+\Delta_1^T P\Delta_2+\Omega_1+\Omega_2+\Omega_3 \\ &\quad + Sym\left(\left(\eta\cdot E_{16,1}^T+E_{16,16}^T\right)U\Gamma_{1ij}(t)\right) \\ &\quad +\Gamma_{2ij}^T(t)\,\Gamma_{2ij}(t),\\ \Omega_3 &= E_{16,1}^T Q E_{16,1}-(1-h_D)\cdot E_{16,2}^T Q_4 E_{16,2}\\ &\quad - E_{16,3}^T(Q_1-Q_3)E_{16,3}-E_{16,4}^T(Q_2+Q_3)E_{16,4}\\ &\quad - E_{16,6}^T Q_5 E_{16,6}-\gamma^2\cdot E_{16,15}^T E_{16,15}\\ &\quad -\eta\cdot Sym\left(E_{16,1}^T U E_{16,16}\right)\\ &\quad - E_{16,16}^T\left[U+U^T-R\right]E_{16,16}.\end{aligned} \tag{8f}$$

Define the following matrices:

$$\begin{aligned}\Sigma_{ij}(t) &= \begin{bmatrix}\Sigma_{1ij}(t) & \Gamma_{2ij}^T(t)\\ * & -I\end{bmatrix}=\begin{bmatrix}\bar{A}_{1ij} & \bar{A}_{2ij}^T\\ * & -I\end{bmatrix}+\Delta_{xyij}(t),\\ \Sigma_{1ij}(t) &= \Delta_2^T P\Delta_1+\Delta_1^T P\Delta_2+\Omega_1+\Omega_2+\Omega_3\\ &\quad + Sym\left(\left(\eta\cdot E_{16,1}^T+E_{16,16}^T\right)U\Gamma_{1ij}(t)\right),\\ \bar{A}_{1ij} &= \Delta_2^T P\Delta_1+\Delta_1^T P\Delta_2+\Omega_1+\Omega_2+\Omega_3\\ &\quad + Sym\left(\left(\eta\cdot E_{16,1}^T+E_{16,16}^T\right)U\bar{\Gamma}_{1ij}\right),\\ \bar{A}_{1ij} &= [A_{0i}\; A_{1i}\; 0\; 0\; -B_{ui}\,K_j\; 0\; 0\\ &\quad 0\;0\;0\;0\;0\;0\;0\; B_{wi}\; 0],\\ \bar{A}_{2ij} &= [A_{y0i}\; A_{y1i}\; 0\; 0\; -B_{yui}\,K_j\; 0\; 0\\ &\quad 0\;0\;0\;0\;0\;0\;0\; B_{ywi}\; 0],\\ \Delta_{xyij}(t) &= Sym\left(\begin{bmatrix}\left(\eta\cdot E_{16,1}^T+E_{16,16}^T\right)UM_{xi} & 0\\ 0 & M_{yi}\end{bmatrix}\right.\\ &\quad\left.\times\begin{bmatrix}\Delta_{xi}(t) & 0\\ 0 & \Delta_{yi}(t)\end{bmatrix}N_{xyij}\right),\\ N_{xyij} &= \begin{bmatrix} N_{x0i}\; N_{x1i}\; 0\; 0\; -N_{xui}K_j \\ 0\;0\;0\;0\;0\;0\;0\;0\;0\; N_{xwi}\; 0\\ N_{y0i}\; N_{y1i}\; 0\; 0\; -N_{yui}K_j\; 0\\ 0\;0\;0\;0\;0\;0\;0\;0\; N_{ywi}\; 0\end{bmatrix}.\end{aligned} \tag{9a}$$

Then we can define the following enlarged matrices

$$\Psi_{ij}=\begin{bmatrix}\bar{\Sigma}_{1i} & \bar{\Gamma}_{2ij}^T & \Psi_{13i} & 0 & \Psi_{15ij} & \Psi_{16ij}\\ * & -I & 0 & \varepsilon_i\cdot M_{yi} & 0 & 0\\ * & * & -\varepsilon_i\cdot I & 0 & \varepsilon_i\cdot\Xi_{xi} & 0\\ * & * & * & -\varepsilon_i\cdot I & 0 & \varepsilon_i\cdot\Xi_{yi}\\ * & * & * & * & -\varepsilon_i\cdot I & 0\\ * & * & * & * & * & -\varepsilon_i\cdot I\end{bmatrix}, \tag{9b}$$

where

$$\begin{aligned}\Psi_{13i} &= \varepsilon_i\cdot\left(\eta\cdot E_{16,1}^T+E_{16,16}^T\right)UM_{xi},\\ \Psi_{15ij}^T &= \left[N_{x0i}\; N_{x1i}\; 0\; 0\; -N_{xui}\,K_j\; 0\;0\;0\;0\;0\;0\;0\;0\;0\; N_{xwi}\; 0\right],\\ \Psi_{16ij}^T &= \left[N_{y0i}\; N_{y1i}\; 0\; 0\; -N_{yui}\,K_j\; 0\;0\;0\;0\;0\;0\;0\;0\;0\; N_{ywi}\; 0\right].\end{aligned}$$

By the conditions in (6c) and with the following results

$$\begin{aligned}\hat{\hat{U}} &= diag[U\;U\;U\;U\;U\;U\;U\;U\;U\;U\;U\;U\;U\;I\;U\\ &\quad I\;I\;I\;I],\\ \hat{U} &= U^{-1},\quad \hat{\hat{\hat{U}}}=diag\left[U\;U\;U\;U\;U\;U\;U\;U\right],\\ \hat{P} &= \hat{\hat{U}}^{-1}P\hat{\hat{U}}^{-T}>0,\quad \hat{S}_i=U^{-1}S_iU^{-T},\\ \hat{Q}_i &= U^{-1}Q_iU^{-T}>0,\quad \hat{R}_i=U^{-1}R_iU^{-T}>0,\end{aligned}$$

we have

$$\hat{\hat{U}}^{-1}\Psi_{ij}\hat{\hat{U}}^{-T}<0.$$

This implies $\Psi_{ij}<0$ in (9b). By Lemma 3 and Schur complement of [22] with (9b), we have the following conditions in (9a) and (8f), respectively:

$$\Sigma_{ij}(t)<0,\quad \Pi_{ij}(t)<0.$$

With $w(t)=0$ and from (8e) with $\Pi_{ij}(t)<0$, we have

$$\dot{V}(x_t)<0,\quad \forall x(t)\neq 0,\ \text{for all } t\in[T_k,T_{k+1}).$$

The system (5) with (2) is asymptotically stable by the sampled-data PDC in (4b). From (8e) with $\Pi_{ij}(t)<0$ in (8f), we can integrate the equation in (8e) from 0 to $\ell>0$ to yield

$$V(x_\ell)-V(\varphi)+\int_0^\ell y^T(t)\,y(t)\,dt -\gamma^2\cdot\int_0^\ell w^T(t)\,w(t)\,dt\le 0. \tag{10}$$

With the zero initial condition (i.e. $\varphi(t)=0,\ -h_M\le t\le 0$), $\ell=\ell_1$, and (10), we have

$$V(\varphi)=0,\quad V\left(x_{\ell_1}\right)\ge 0.$$

From the above derivations, the following condition can be guaranteed

$$\int_0^{\ell_1} y^T(t)\,y(t)\,dt\le\gamma^2\cdot\int_0^{\ell_1} w^T(t)\,w(t)\,dt,\quad \forall w\neq 0,$$

for all positive constants ℓ_1 and γ. From the condition in (10) with $\ell = \ell_2$ and $w(t) = 0$, we have

$$V\left(x_{\ell_2}\right) - V(\varphi) + \int_0^{\ell_2} y^T(t)\, y(t)\, dt \le 0.$$

From (6d) and (7) with $V\left(x_{\ell_2}\right) \ge 0$ and, we have

$$\int_0^{\ell_2} y^T(t)\, y(t)\, dt \le V(\varphi) \le \alpha.$$

This completes the proof. □

Remark 1: In this paper, the sampling period $\tau_k = T_{k+1} - T_k \le \tau_M$ can be allowed in an interval. It is more efficient than periodic sampling and easy to implement in the real world. The delay $0 < h_m \le h(t) \le h_M$ is also assumed varying in a given interval. It is more practical than constant delay. This term $X^T(t)PX(t)$ used in Lyapunov-Krasovskii functional (7) will be a flexible choice. The possible dynamics of system can be included into the time derivative of the functional in (7). Since the high dimensional matrix operations, a full matrix formulation approach is developed to present the multiplication and combination of matrices ($E_{i,j}$). The inequalities in Lemmas 1 and 2 are simultaneous applied in derivations for positive definitive matrices R_3 and R_5 for interval time-varying delay $h(t)$, R_4 and R_6 for sampling period τ_k, respectively. It is interesting to note that the inequality in Lemma 1 is less conservative than Wirtinger-based one [20]. The major advantage of Lemma 2 is that the delayed state terms $x(t - h(t))$ and $x(t - \tau(t))$ can be included into the derivation for the main LMI conditions. More efficient results can be proposed for interval time-varying delay and sampling period.

Remark 2: If the upper bound of variation of interval time-varying h_D is larger than 1 or unknown, the proposed results in Theorem 1 of this paper are also valid by selecting the matrix $\hat{Q}_4 = 0$.

Remark 3: In our past results in [20], some given pointwise sampling intervals will be proposed to improve the fixed sampling. In this paper, the proposed approach will allow that the sampling period belongs to an estimated interval $[0, \tau_M]$. The interval time-varying delay is considered in this paper instead of constant delay in [20]. It will be more practical and flexible than the our published results in [20].

Remark 4: The larger values for sampling intervalτ_M, lower bound h_m, upper bound h_M, interval $h_M - h_m$ of interval time-varying delay are better to provide flexibility and less conservativeness for our proposed results. The smaller value for γ will provide better at disturbance attenuation for system under consideration.

Remark 5: In Theorem 1, we use LMI conditions in (6a)-(6c) to find the feasible solution for a known positive parameter $\bar{\gamma} = \gamma^2$. The H_2 measure of the system under consideration can be calculated from our proposed result in (6d). Smaller values of γ and α will imply better disturbance attenuation and H_2 measure, respectively.

III. ILLUSTRATIVE EXAMPLES

This section includes three examples to demonstrate the use and main contribution of the proposed results. Some comparisons have been made in Examples 1-2. Example 3 is a practical nonlinear system which has been designed by our developed approach to achieve mixed performance for its corresponding T-S fuzzy system. Some simulation diagrams have been provided to show the efficiency of proposed results.

Example 1: Consider the T-S fuzzy system (5) with (2) and the following parameters [20]:

$$\begin{aligned}
&m = 2, \quad A_{01} = \begin{bmatrix} -2.1 & 0.1 \\ 1 & -2 \end{bmatrix}, \quad A_{02} = \begin{bmatrix} -1.9 & 0 \\ -0.2 & -1.1 \end{bmatrix}, \\
&A_{11} = \begin{bmatrix} -0.5 & 0.05 \\ -0.4 & -0.45 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} -0.45 & 0 \\ -0.55 & -0.6 \end{bmatrix}, \\
&B_{w1} = B_{w2} = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, \quad B_{u1} = B_{u2} = \begin{bmatrix} 1 \\ -0.2 \end{bmatrix}, \\
&A_{y01} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad A_{y02} = \begin{bmatrix} 0.5 & -0.6 \end{bmatrix}, \\
&A_{y11} = \begin{bmatrix} -0.8 & 0.6 \end{bmatrix}, \quad A_{y12} = \begin{bmatrix} -0.2 & 1 \end{bmatrix}, \\
&B_{yw1} = B_{yw2} = -0.6, \quad x(t) = \begin{bmatrix} 1 & -1 \end{bmatrix}^T, \\
&\qquad t \in [-h_M, 0], \\
&B_{yui} = M_{xi} = M_{yi} = N_{xji} = N_{xwi} = N_{xui} = N_{yji} = N_{ywi} \\
&\qquad = \Xi_{xi} = \Xi_{yi} = 0, \quad i = 1, 2, \quad j = 0, 1.
\end{aligned} \tag{11}$$

With the sampling interval $0 < \tau_i \le 0.25 = \tau_M$, $h_m = 0$, $h_M = 2$, and from the statements in Remark 5, LMI conditions in (6b)-(6d) of Theorem 1 with $\eta = 1$ and $\gamma = 0.95$ have a feasible solution with

$$\begin{aligned}
\hat{K}_1 &= [0.0016 \quad -0.0005], \\
\hat{K}_2 &= [0.0016 \quad 0.0001], \\
\hat{U} &= \begin{bmatrix} 0.3441 & 0.1178 \\ 0.1471 & 0.3352 \end{bmatrix}.
\end{aligned}$$

Then the system (5) with (2) and (11) is asymptotically stabilizable by the sampled-data PDC in (4b) with

$$K_1 = \hat{K}_1 \hat{U}^{-T} = [0.006 \quad -0.0041], \tag{12a}$$

$$K_2 = \hat{K}_2 \hat{U}^{-T} = [0.0055 \quad -0.0022], \tag{12b}$$

H_∞ performance $\gamma = 0.95$, and H_2 measure $\alpha = 258.3151$. In Theorem 1, we can minimize the disturbance attenuation to $\gamma = 0.942$ with $\alpha = 929.0876$. Some comparisons are made in Table 1 to show that the proposed results in this paper are more flexible and practical than [20] and [21]. In [20], the optimal results are $\gamma = 0.9504$ and $\alpha = 16520$.

Example 2: Consider the uncertain T-S fuzzy system (5) with (2) and the following parameters:

$$\begin{aligned}
&m = 2, \quad A_{01} = \begin{bmatrix} -1.1 & 0.1 \\ 0.2 & -1 \end{bmatrix}, \quad A_{02} = \begin{bmatrix} -1.2 & 0 \\ -0.1 & -1.1 \end{bmatrix}, \\
&A_{11} = \begin{bmatrix} -0.9 & 0.05 \\ -0.2 & -1 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} -1 & 0.1 \\ -0.25 & -1 \end{bmatrix}, \\
&B_{w1} = \begin{bmatrix} 0.2 \\ 0.01 \end{bmatrix}, \quad B_{w2} = \begin{bmatrix} 0.2 \\ 0.02 \end{bmatrix}, \quad B_{u1} = \begin{bmatrix} 1 \\ -0.1 \end{bmatrix},
\end{aligned}$$

TABLE 1. Some comparisons of the sampled-data T-S fuzzy time-delay system (5) with (2) and (11).

Some comparisons about the sampled-data T-S fuzzy time-delay system (5) with (2) and (11).			
Results	Time delay $h(t)$	Sampling period	H_∞ performance γ H_2 measure α
Results of [21]	$h(t)=2$ Constant delay	$\tau_i = 0.2$ Fixed sampling	$\gamma = 1.0279$
Theorem 1 of [20]		$\tau_i = 0.2$ $\tau_j = 0.25$ for some i and j, pointwise sampling	γ=4 $\alpha = 17793$
Theorem 2 of [20]			γ=0.98 $\alpha = 1370.8$
Results of Theorem 1 in this paper	$0 \le h(t) \le 2$ $h(t)$ unknown	$0 < \tau_i \le 0.25 = \tau_M$	$\gamma = 0.942$ $\alpha = 929.0876$

$$B_{u2} = \begin{bmatrix} 1 \\ 0.1 \end{bmatrix}, \quad A_{y01} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad A_{y02} = \begin{bmatrix} 0.8 & -0.1 \end{bmatrix},$$
$$A_{y11} = \begin{bmatrix} -0.8 & 0.6 \end{bmatrix}, \quad A_{y12} = \begin{bmatrix} -0.2 & 1 \end{bmatrix}, \quad B_{yw1} = 0.3,$$
$$B_{yw2} = -0.5, \quad M_{xi} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.02 \end{bmatrix}, \quad M_{yi} = 0.01,$$
$$N_{x0i} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \end{bmatrix}, \quad N_{x1i} = \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$N_{xui} = N_{xwi} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$
$$N_{y0i} = N_{y1i} = \begin{bmatrix} 0.1 & 0 \end{bmatrix}, \quad N_{ywi} = N_{yui} = 0,$$
$$\Xi_{xi} = \Xi_{yi} = 0.1, \quad i = 1, 2,$$
$$x(t) = \begin{bmatrix} 1 & -1 \end{bmatrix}^T, \quad t \in [-h_M, 0]. \tag{13}$$

With the sampling interval $0 < \tau_i \le 0.11 = \tau_M$, $h_m = 0$, $h_M = 0.2$, LMI optimization problem in (6a)-(6d) of Theorem 1 with $\eta = 1$ has a feasible solution with

$$\bar{\gamma} = 0.2674, \quad \hat{K}_1 = [0.7259 \quad 0.7714] \cdot 10^{-3},$$
$$\hat{K}_2 = [0.0009 \quad 0.001], \quad \hat{U} = \begin{bmatrix} 2.0829 & 0.3245 \\ 0.3471 & 0.1278 \end{bmatrix}.$$

Then the system (5) with (2) and (13) is asymptotically stabilizable by the sampled-data PDC in (4b) with

$$K_1 = \hat{K}_1 \hat{U}^{-T} = [-0.001 \quad 0.0088], \tag{14a}$$
$$K_2 = \hat{K}_2 \hat{U}^{-T} = [-0.0015 \quad 0.0121], \tag{14b}$$

H_∞ performance $\gamma = \sqrt{A} = 0.5171$, and H_2 measure $\alpha = 32.8703$. Some comparisons are made in Table 2. In the proposed approach in 20], the optimal results are $\gamma = 0.561$ and $\alpha = 81.3265$.

In the mixed performance of system under consideration, smaller values of γ and α will imply better attenuation on disturbance and exact estimation on H_2 measure, respectively. From Tables 1-2, better attenuation effect of T-S fuzzy system with parameters in (11) or (13) can be achieved by our proposed sampled-data PDC (4) with feedback gains (12) or (14), respectively. The more exact estimation on H_2 measure can be gotten from our proposed approach in this paper. The periodic and pointwise sampling period can be extended to interval sampling one. Interval time-varying delay has been investigated instead of constant delay.

TABLE 2. Some comparisons of the sampled-data T-S fuzzy time-delay system (5) with (2) and (13).

Some comparisons about the sampled-data T-S fuzzy time-delay system (5) with (2) and (12).			
Results	Time delay $h(t)$	Sampling period	H_∞ performance γ H_2 measure α
Theorem 1 of [20]	$h(t) = 0.2$ Constant delay	$\tau_i = 0.1$ $\tau_j = 0.11$ for some i and j, pointwise sampling	γ=0.668 $\alpha = 1039.6$
Theorem 2 of [20]			γ=0.558 $\alpha = 132.5842$
Results of Theorem 1 in this paper	$0 \le h(t) \le 0.2$ $h(t)$ is time-varying and unknown	$0 < \tau_i \le 0.11 = \tau_M$	$\gamma = 0.5171$ $\alpha = 32.8703$

Example 3: Consider a nonlinear mass-spring-damper system [31]:

$$M\ddot{s}(t) + g(s(t), \dot{s}(t)) + f(s(t)) + \varphi_1(s(t))\,w(t) = \varphi_2(s(t))\,u(t), \tag{15}$$

where $s(t)$ is the displacement, M is the mass, u is the input force, w is the disturbance input, $g(s, \dot{s})$, $f(s)$, $\varphi_1(s)$, and $\varphi_2(s)$ are nonlinear terms with respect to damper, spring, w, and u, respectively.

By the same assumption in [31] with $M = 1$, $g(s(t), \dot{s}(t)) = -0.75\dot{s}(t)$, $f(s) = 0.67s(t)^3 - 0.05s(t)$, $\varphi_1(s(t)) = -0.5 - 0.1s(t)^2$, and $\varphi_2(s) = 1 - 0.1s(t)^2$. Define $x_1(t) = s(t)$, $x_2(t) = \dot{s}(t)$, nonlinear system (15) can rewritten as

$$\dot{x}_1(t) = x_2(t), \tag{16a}$$
$$\dot{x}_2(t) = -0.67x_1^3(t) + 0.05x_1(t) + 0.75x_2(t) + \left(0.5 + 0.1x_1^2(t)\right)w(t) + \left(1 - 0.1x_1^2(t)\right)u(t), \tag{16b}$$
$$y(t) = x_2(t) + 0.5u(t), \tag{16c}$$

where $x(0) = \begin{bmatrix} 1 & -1 \end{bmatrix}^T$, $|x_1(t)| \le 1.5$, and $|x_2(t)| \le 2.5$. Since the displacement variable $s(t)$ can be measured easily, we can choose the premise variable as $z(t) = s(t) = x_1(t) \in \Re$ and represent the system (16) by the following T-S fuzzy model with normalized membership function $\eta_1(x_1(t)) = 1 - x_1^2/(1.5)^2 = 1 - x_1^2/2.25$ and $\eta_2(x_1(t)) = 1 - \eta_1(x_1(t))$:

Rule i:
If $z(t)$ *is about* 0, *then*

$$\dot{x}(t) = A_{01}x(t) + B_{u1}u(t) + B_{w1}w(t), \quad t \geq 0, \quad (17a)$$
$$y(t) = A_{y01}x(t) + B_{yu1}u(t), \quad t \geq 0. \quad (17b)$$

If $z(t)$ *is about* ± 1.5, *then*

$$\dot{x}(t) = A_{02}x(t) + B_{u2}u(t) + B_{w2}w(t), \quad t \geq 0, \quad (17c)$$
$$y(t) = A_{y02}x(t) + B_{yu2}u(t), \quad t \geq 0, \quad (17d)$$

where

$$A_{01} = \begin{bmatrix} 0 & 1 \\ -0.67 \times 0^2 + 0.05 & 0.75 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0.05 & 0.75 \end{bmatrix},$$
$$A_{02} = \begin{bmatrix} 0 & 1 \\ -0.67 \times (\pm 1.5)^2 + 0.05 & 0.75 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1.4575 & 0.75 \end{bmatrix},$$
$$B_{u1} = \begin{bmatrix} 0 \\ 1 - 0.1 \times 0^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$B_{u2} = \begin{bmatrix} 0 \\ 1 - 0.1 \times (\pm 1.5)^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.775 \end{bmatrix},$$
$$B_{w1} = \begin{bmatrix} 0 \\ 0.5 + 0.1 \times 0^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix},$$
$$B_{w2} = \begin{bmatrix} 0 \\ 0.5 + 0.1 \times (\pm 1.5)^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.775 \end{bmatrix},$$
$$A_{y01} = A_{y02} = \begin{bmatrix} 0 & 1 \end{bmatrix},$$
$$B_{yu1} = B_{yu2} = 0.5.$$

With the sampling interval $0 < \tau_i \leq 0.16 = \tau_M$, *LMI conditions in (6a)-(6c) of Theorem 1 with* $\eta = 1$ *have a feasible solution with*

$$\bar{\gamma} = 0.8482, \quad \hat{K}_1 = [-2.4332 \quad 5.4494],$$
$$\hat{K}_2 = [-2.4329 \quad 5.4493], \quad \hat{U} = \begin{bmatrix} 0.7648 & -1.1738 \\ 0.1328 & 2.137 \end{bmatrix}.$$

Then the system (17) is asymptotically stabilizable by the sampled-data PDC in (4b) with

$$K_1 = \hat{K}_1\hat{U}^{-T} = [0.6683 \quad 2.5085], \quad (18a)$$
$$K_2 = \hat{K}_2\hat{U}^{-T} = [0.6686 \quad 2.5084], \quad (18b)$$

H_∞ *performance* $\gamma = \sqrt{\bar{A}} = 0.921$, *and* H_2 *measure* $\alpha = 8$. *With disturbance* $w(t) = 0.2e^{-0.2t}\sin(20t)$ *shown in Figure 1, the output* $y(t) \in \Re$ *of nonlinear system (16) without control input is shown in Figure 2. From the simulation, the displacement overtakes the constraints in system (16). The device of system will be destroyed in a short time. By using the sampled-data PDC in (4b) with* K_1 *and* K_2 *in (18), the output* $y(t) \in \Re$ *and sampled-data PDC* $u(t) = u(T_k) \in \Re, \forall t \in [T_k, T_{k+1})$ *with zero initial*

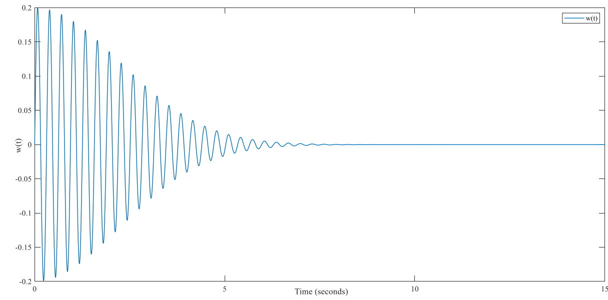

FIGURE 1. The disturbance input.

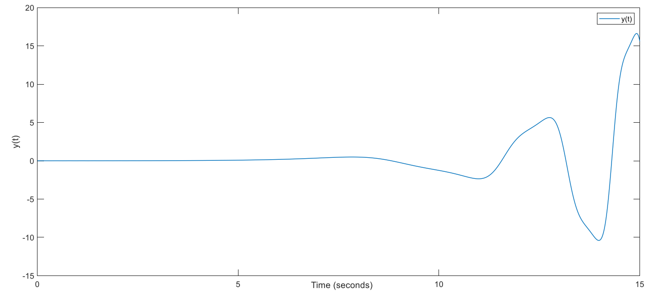

FIGURE 2. The output vector $y(t) \in \Re$ without control input.

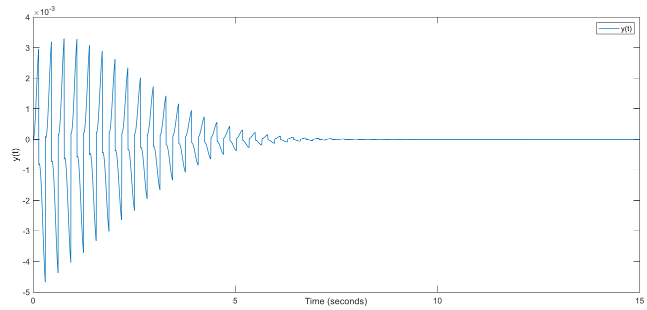

FIGURE 3. The output $y(t) \in \Re$ with zero initial state for the proposed compensator.

state of nonlinear system (16) are shown in Figures 3 and 4, respectively. From Figure 3, the H_∞ *performance* $\gamma = 0.921$ *can be observed. Under the proposed compensator and initial state, the output* $y(t) \in \Re$, *state trajectories* $x(t) \in \Re^2$, *and sampled-data control* $u(t) = u(T_k) \in \Re, \forall t \in [T_k, T_{k+1})$ *with zero disturbance are shown in Figures 5-7, respectively. From the simulation results, the proposed sampled-data PDC in (4b) with (18) is effective for disturbance rejection and* H_2 *measure.*

In this simulation, the selection of fuzzy rule is setting on center and boundary points of displacement variable $s(t)$. *The number of fuzzy rule is 2. If we would like to make better approximation for original nonlinear system in (15)-(16). The following T-S fuzzy model can be given by:*

Rule i:
If $z(t)$ is about 0, *then*

$$\dot{x}(t) = A_{01}x(t) + B_{u1}u(t) + B_{w1}w(t), \quad t \geq 0, \quad (19a)$$
$$y(t) = A_{y01}x(t) + B_{yu1}u(t), \quad t \geq 0. \quad (19b)$$

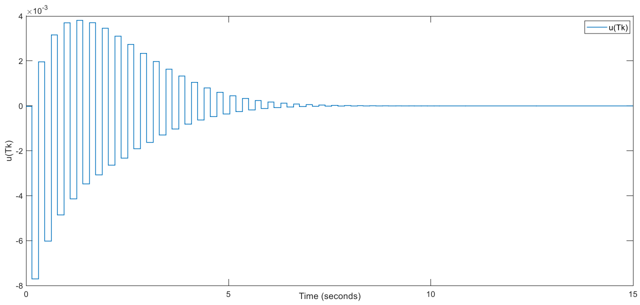

FIGURE 4. The sampled-data input $u(T_k) \in \Re$ with zero initial state.

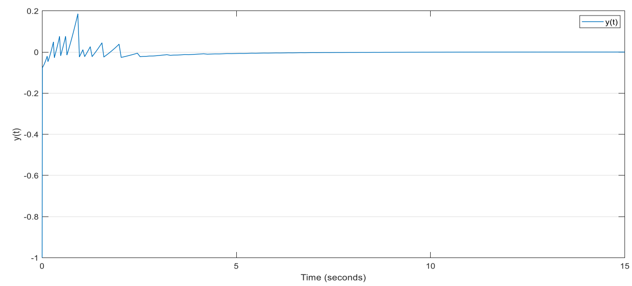

FIGURE 5. The output $y(t) \in \Re$ with zero disturbance for the proposed sampled-data input.

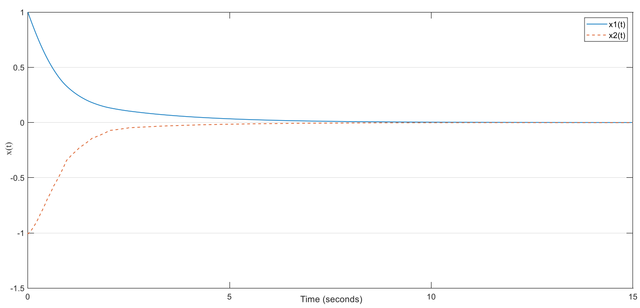

FIGURE 6. The state trajectories $x(t) \in R^2$ with zero disturbance for the proposed sampled-data input.

If $z(t)$ is about ± 1.5, *then*

$$\dot{x}(t) = A_{02}x(t) + B_{u2}u(t) + B_{w2}w(t), \quad t \geq 0, \quad (19c)$$

$$y(t) = A_{y02}x(t) + B_{yu2}u(t), \quad t \geq 0. \quad (19d)$$

If $z(t)$ is about ± 0.75, *then*

$$\dot{x}(t) = A_{03}x(t) + B_{u3}u(t) + B_{w3}w(t), \quad t \geq 0, \quad (19e)$$

$$y(t) = A_{y03}x(t) + B_{yu3}u(t), \quad t \geq 0, \quad (19f)$$

where $A_{0i}, B_{ui}, B_{wi}, A_{y0i}, A_{yui}, i = 1, 2$, *are given in (17), and*

$$A_{03} = \begin{bmatrix} 0 & 1 \\ -0.67 \times (\pm 0.75)^2 + 0.05 & 0.75 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.326875 & 0.75 \end{bmatrix},$$

$$B_{u3} = \begin{bmatrix} 0 \\ 1 - 0.1 \times (\pm 0.75)^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.94375 \end{bmatrix},$$

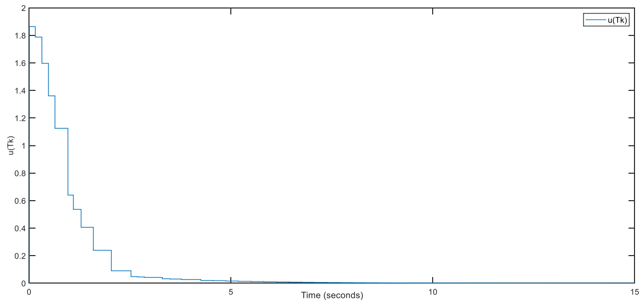

FIGURE 7. The sampled-data input $u(T_k) \in \Re$ with zero disturbance.

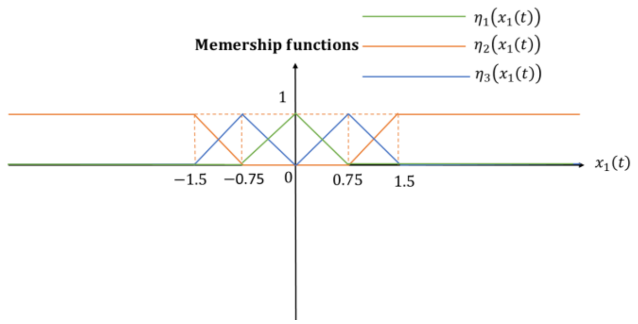

FIGURE 8. The corresponding membership functions for T-S fuzzy system.

$$B_{w3} = \begin{bmatrix} 0 \\ 0.5 + 0.1 \times (\pm 0.75)^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.55625 \end{bmatrix},$$

$$A_{y03} = A_{y01} = \begin{bmatrix} 0 & 1 \end{bmatrix},$$

$$B_{yu3} = B_{yu1} = 0.5.$$

The normalized membership function of T-S fuzzy system in (19) is illustrated in Figure 8. In general, larger number of fuzzy rule will make more approximate to original nonlinear system under consideration. But it may cause that LMI optimization problem of design scheme for sampled-data PDC is infeasible.

IV. CONCLUSION

In this paper, a sampled-data PDC has been proposed to guarantee mixed H_2/H_∞ performance of uncertain T-S fuzzy systems with interval time-varying delay and linear fractional perturbations. Full matrix formulation approach has been investigated to improve the conservativeness of proposed results. New inequality and Lyapunov-Krasovskii functional have been developed to guarantee the efficiency of the proposed results in this paper. Finally, some numerical examples have been illustrated to show the main results. In this paper, we consider interval time-varying delay and interval sampling period instead of constant delay and pointwise sampling period in [20], respectively. Furthermore, some interesting research topics for T-S fuzzy systems can be investigated, e.g., asynchronous non-PDC controller and quantizer [32], [33], dissipativity and dissipative control [15], [16], event-triggered control [33], [34], finite-time control [27]–[32], nonfragile control [14], passivity and passive control [25]. All these would constitute our future research work.

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