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Two-Dimensional Extended Warranty Strategy for an Economic Dependence Multi-Component Series System With Grid Search Algorithm

ENZHI DONG[1,2], ZHONGHUA CHENG[1], AND RONGCAI WANG[1]
[1]Army Engineering University of PLA at Shijiazhuang, Shijiazhuang 050003, China
[2]Hebei Key Laboratory of Condition Monitoring and Assessment of Mechanical Equipment, Baoding 071003, China

Corresponding author: Zhonghua Cheng (a15032073178@sina.com)

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ABSTRACT Extended warranty is an optional contract that often provides additional protection for users after the basic warranty expires. Considering the high warranty cost borne by manufacturers and the lack of system availability guarantee, the present study proposes a two-dimensional extended warranty approach for a multi-component series system considering economic dependence. Based on the imperfect preventive maintenance model and the combination of free warranty and pro-rata warranty policies, the manufacturer's and user's warranty cost model and availability model for a single component are established. Considering the economic dependence between multi-components, the preventive maintenance of each component is combined using the grouping maintenance approach. In the case study, we consider the reconnaissance equipment as the research object and use the grid search method to obtain the two-dimensional extended warranty scheme of reconnaissance equipment considering economic dependence. Comparative and sensitivity analyses verify that the approach can effectively reduce the extended warranty cost of the manufacturer and improve the availability of the multi-component system. Additionally, the approach can provide support for warranty cost calculation and system availability estimation under different extended warranty periods.

INDEX TERMS Maintenance management, grouping maintenance, imperfect preventive maintenance, two-dimensional extended warranty, combined warranty, multi-component series system.

I. INTRODUCTION

The extra warranty period supplied by the manufacturer for its equipment at the end of the basic warranty, which is an extension of the basic warranty, is referred to the extended warranty [1]. Extended warranty is an optional contract that often provides additional protection for users after the basic warranty expires and must be purchased separately at an additional fee [2]. Extended warranties have been a lucrative source of revenue for manufacturers. Also, many users choose for extended warranty services to alleviate the financial burden of equipment failures [3]. However, the lack of standard decision-making methods and the support of a scientific auxiliary decision-making model substantially raise the warranty cost of the manufacturer during the extended warranty period, and the equipment availability cannot be guaranteed. Therefore, formulating a reasonable extended warranty strategy, reducing the warranty cost, and improving the availability of the system are the major challenges that remain to be overcome.

The advancement of technology and improvement of modern industrial sectors have led to the development of various new complex equipment, which have been used widely in many fields. New complex equipment such as ships, automobiles, and aeroengines are highly systematical and have a complex structure. Moreover, they consist of many multi-component series systems, and the life of the multi-component series system is mostly affected by calendar time and usage, so two-dimensional warranty is usually adopted for them [4]. In addition, these systems consist of multiple components in a series, and the failure of a single component results in system failure. Simultaneously, downtime of any component can result in system downtime, providing opportunities for maintenance of other components [5]. Combining the maintenance work of each component can effectively reduce downtime and warranty cost [6]. Economic

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dependence means that the combination of maintenance of system components renders the warranty cost of the system higher or lower than the sum cost of the respective maintenance of system components [7]. Grouping maintenance is a maintenance strategy usually adopted when considering economic dependence. For the multi-component series system, the grouping maintenance strategy considering economic dependence can effectively reduce the warranty cost and improve the system availability [8].

Based on the actual situation of the multi-component series system, this study examines the two-dimensional extended warranty strategy for the multi-component series system. A regular preventive maintenance strategy is adopted, and preventive maintenance is imperfect maintenance. The effect of imperfect preventive maintenance is described by the improvement factor. In this study, imperfect preventive maintenance strategy and corrective maintenance strategy are effectively integrated, and the manufacturer's and user's warranty cost model and availability model for a single component are established basing on it. Considering the economic dependence between multi components in a series, the preventive maintenance benchmark interval is introduced; the preventive maintenance time of each component is adjusted to an integral multiple of the benchmark interval. As a result, the preventive maintenance work of multi components is combined to reduce the manufacturer's warranty cost and improve the availability of the multi-component system.

Considering the design utilisation rate as the limit, the combination of free warranty and pro-rata warranty policies is adopted, implying that when the actual utilisation rate is less than the design utilisation rate, all corrective maintenance and preventive maintenance costs during the warranty period shall be borne by the manufacturer. When the actual utilisation rate is greater than the design utilisation rate, the maintenance costs during the warranty period shall be separately borne by the manufacturer and the user in a certain proportion according to the time or usage of equipment at the time of maintenance activities. This approach reduces the manufacturer's warranty cost and improves the user's awareness of the rational use of equipment. Most studies have considered the lowest warranty cost as the decision-making goal. The present study considers the availability constraint, which implies that the availability should meet the minimal requirements of users to satisfy the demands of manufacturers and users.

A. PAPER ORGANIZATION

The remainder of this article is organised into four sections. Section 2 provides a literature review of related studies. Section 3 presents the description of the model and provides a description of notations based on reasonable assumptions. Based on the failures of single-component systems, the imperfect preventive maintenance and minimal maintenance strategies are adopted to establish the two-dimensional extended warranty cost and availability models. In the cost model, a combination of free warranty and pro-rata warranty policies is adopted. Finally, the combination strategy of preventive maintenance is introduced. Section 4 presents a real case scenario of a reconnaissance equipment manufacturer of China to illustrate the applicability of our model. Finally, Section 5 presents the conclusions.

II. LITERATURE REVIEW

A. TWO-DIMENSIONAL EXTENDED WARRANTY

Recently, scholars have increasingly focused on the two-dimensional extended warranty strategy. At present, the research on two-dimensional extended warranty is increasingly highlighting the heterogeneity of users, that is, the warranty scheme is tailored for different users [9]–[11]. More and more attention is paid to the establishment of warranty policy to strike a balance between manufacturers and users, that is, to make both manufacturers and users acceptable [11], [12]. However, since the formulation of extended warranty strategy is in the hands of the manufacturer, most of the current researches are from the manufacturer's point, aiming at the lowest cost or maximum profit of extended warranty, with little consideration of users' requirements for equipment availability [13]–[16]. Furthermore, most studies have focused on a single-component system, and the economic dependence between multi-components is ignored.

B. ECONOMIC DEPENDENCE

In the research on warranty strategies for multi-component systems, researchers have increasingly focused on the economic dependence between multi-components and have considered combination strategies of maintenance work from varied perspectives. Reference [17] proposed a maintenance optimisation method for a series-parallel system with multi-state components considering economic dependence and state-dependent inspection intervals. Reference [18] proposed a condition-based maintenance strategy model for two-component systems with stochastic and economic dependence. Dao *et al.* [19] explored the formulation of a selective maintenance strategy for a multi-state series parallel system related to economic dependence: decision makers can select components for maintenance according to maintenance objectives, resource availability, and maintenance time and cost of each component to reduce maintenance cost. Su and Chen [20] considered the economic dependence between wind turbine components. Furthermore, the author established a long-term average cost rate model under the condition-based maintenance strategy considering the renewal process theory and analysed the optimal detection cycle of a multi-component system.

C. PREVENTIVE MAINTENANCE

There are many definitions of system availability. This paper mainly studies the operation availability of the system, which represents the ratio of the available time length of the system in a specific time interval to the length of the whole interval. The availability of a system depends to some extent on effective maintenance and inspection, in addition to the structural design, quality and reliability of the components that make up it. Preventive maintenance refers to a series of maintenance on the premise of no system failure or damage, which can effectively avoid the occurrence of system failure with loss characteristics. References [21]–[24] introduced preventive maintenance strategy earlier in the study of warranty policy. In the later study, scholars considered more influential factors, making the formulation of warranty policies based on preventive maintenance strategy more consistent with the reality. For example, Huang *et al.* [25] adopted the bivariate method, considered the time and usage of repairable products, and used regular preventive maintenance to formulate a two-dimensional warranty strategy for repairable products. Wang *et al.* [26] analysed the effects of customers' non-punctuality on the optimisation of preventive maintenance strategy and the resulting warranty costs. Dai *et al.* [27] jointly optimised the proportion of preventive maintenance cost, the number of preventive maintenance activities, and the corresponding preventive maintenance level. Currently, studies examining the two-dimensional extended warranty for a multi-component system are inadequate. Preventive maintenance can effectively increase availability and reduce warranty cost for complex products whose failure rate increases with time [28]. Therefore, it is necessary to add preventive maintenance strategy in the formulation of two-dimensional extended warranty strategy.

D. THE COMBINED WARRANTY STRATEGY

The failure rate of the multi-component system in the extended warranty stage is significantly higher than that in the basic warranty stage, thereby causing immense pressure on the manufacturer with regard to the warranty cost. A reasonable warranty cost sharing mechanism can reduce the manufacturer's warranty cost, expand the profit space, and improve the manufacturer's warranty enthusiasm. Under the one-dimensional warranty mode, the proportional compensation usually determines the cost proportion borne by users according to the time of failure. However, research on the two-dimensional proportional compensation remains scarce. The usual warranty service strategy is free warranty, where the manufacturer bears the entire warranty cost. Liang [29] proposed a combined warranty strategy adopted during the two-dimensional warranty period. The warranty region is divided into multiple sub-regions according to the use intensity. Furthermore, different strategies of proportional warranty, including free warranty and pro-rata warranty, are adopted according to the region of warranty activity. The combined warranty strategy reduces warranty costs, standardises users' product use behavior.

In summary, although the results of various research directions are relatively mature, no research has focused on the formulation of two-dimensional extended warranty strategy for multi-component systems with economic dependence. Based on this, this paper proposes a new two-dimensional extended warranty method, which covers grouping maintenance, periodic imperfect preventive maintenance, and combination of free warranty and pro-rata warranty policies. It can play a significant role in reducing the manufacturer's two-dimensional extended warranty cost, expanding its profit space and improving equipment availability.

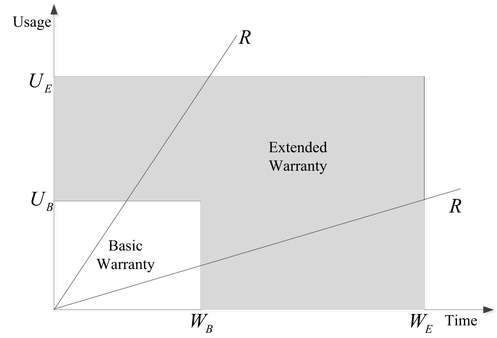

FIGURE 1. Basic warranty and extended warranty regions.

III. MODEL FORMULATION

A. MODEL DESCRIPTION

We assume that users prefer to purchase extended warranty service for equipment and that the warranty is for multi-component series systems. W_B and U_B denote the age and usage limits of the basic warranty region. W_E and U_E denote the age and usage limits of the extended warranty region, as illustrated in Fig. 1. R denotes the system utilisation rate. $R = U(t)/T(t)$, where $U(t)$ denotes the usage of systems at time t, and $T(t)$ denotes the length of service at time t, with $T(t) = t$. During the basic warranty period, corrective maintenance, which is the minimal maintenance, is adopted for all component failures. During the extended warranty period, the multi-component series system adopts regular preventive maintenance, which is imperfect maintenance. Furthermore, the minimal maintenance approach is adopted for failures occurring between preventive maintenance intervals. Considering that the failure rate of multi-component series system is significantly higher in the extended warranty period than that in the basic warranty period, the adoption of free warranty policy will increase the pressure on manufacturers with regard to warranty cost. Thus, the combined strategy of free warranty and pro-rata warranty is adopted in the extended warranty period. Fig. 2 illustrates that when the actual utilisation rate r_l is less than or equal to the design utilisation rate r_1, the warranty cost shall be borne by the manufacturer. When the actual utilisation rate r_l is greater than the design utilisation rate r_1, the manufacturer and the user shall bear part of the warranty cost in proportion according to the location of the two-dimensional extended warranty plane area where the maintenance activities are undertaken. The optimal preventive maintenance interval and improvement factor should be decided to ensure that the manufacturer's two-dimensional extended warranty cost is the lowest, and the availability of systems meets the minimal availability acceptable to users.

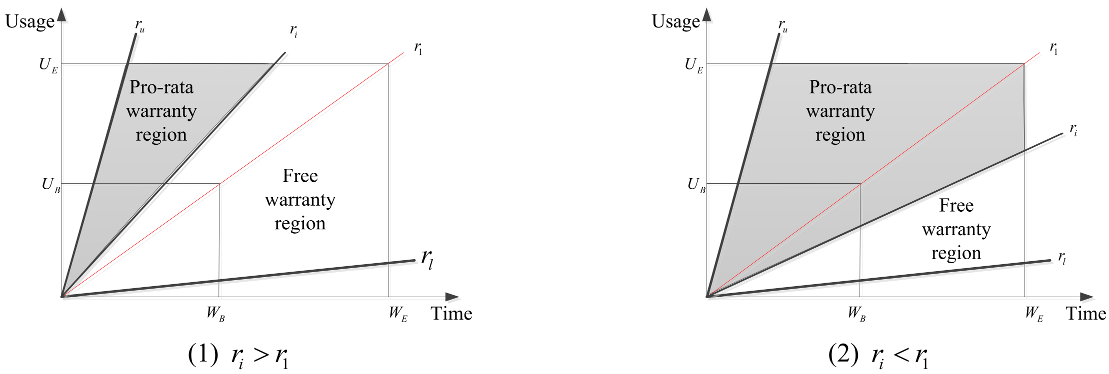

FIGURE 2. Combined strategy of free warranty and pro-rata warranty.

B. ASSUMPTIONS AND NOTATIONS

1) ASSUMPTIONS

We make the following assumptions to develop the mathematical model:

1) After the minimal maintenance, the failure rate of the components remains unchanged. After the imperfect preventive maintenance, the repair degree of the components is between bad as old and good as new.

2) To simplify the analysis steps, we assume that the shape parameters of the basic warranty and the extended warranty regions are the same, that is, $r_B = \frac{W_B}{U_B} = r_E = \frac{W_E}{U_E}$.

3) For the same batch of equipment, the utilisation rate of the same user is the same for the basic warranty and extended warranty periods, and the utilisation rate of different users is a random variable subject to a certain distribution.

4) Only consider the purchase of extended warranty services during the purchase of equipment, and do not consider the purchase of extended warranty services after the end of the basic warranty.

5) The components in the multi-component system are in series.

2) NOTATIONS

$\lambda_i(t\|r)$	: Failure rate function of the i-th component.
(W_B, U_B)	: Two-dimensional basic warranty period.
(W_E, U_E)	: Two-dimensional extended warranty period.
T_i	: The i-th component preventive maintenance interval during the extended warranty period.
θ_i	: The i-th component preventive maintenance improvement factor.
C_{pi}	: The cost of one preventive maintenance by the manufacturer for component i.
C_{Di}	: The preparation costs for preventive maintenance by the manufacturer for component i.
C_{fi}	: The cost of one corrective maintenance by the manufacturer for component i.
T_{pi}	: The time of one preventive maintenance by the manufacturer for component i.
T_{fi}	: The time of one corrective maintenance by the manufacturer for component i.
r_1	: Basic warranty and extended warranty region shape parameters.
r_i	: Design utilisation rate of the i-th component.
I_{t_i}	: Start time of the extended warranty for component i.
O_{t_i}	: End time of the extended warranty for component i.
n_i	: The number of preventive maintenance activities of the i-th component during extended warranty period.
$EC_x(T, I_{t_i}, O_{t_i})$	: The expected warranty cost during the extended warranty period of the i-th component.
$ED_x(T, I_{t_i}, O_{t_i})$	: The expected downtime during the extended warranty period of the i-th component.
EA_i	: The operational availability of the i-th component during the extended warranty period.
m_s	: Number of series components in multi-component systems.
$G_i(r)$	: Cumulative distribution function of the utilisation rate of the i-th component.
$g_i(r)$	: Probability density function of the utilisation rate of the i-th component.
r_{l_i}	: Lower limit of utilisation rate of the i-th component.
r_{u_i}	: Upper limit of utilisation rate of the i-th component.

C. SINGLE COMPONENT FAILURES

Currently, univariate [30], bivariate [31], [32], and composite scale methods [33] are used to construct the two-dimensional failure rate function. We use the univariate method to construct the failure rate function model of a single-component system. In the univariate method, a linear relation is assumed between the usage u of components and time t, that is, $u = rt$. r denotes the utilisation rate of components. The utilisation rate r of components by a single user is constant, and the distribution $G_i(r)$ of r can be obtained by statistics. For a given utilisation rate r, the failure rate function of component i is expressed as $\lambda_i(t|r)$. This is a non-decreasing function of t and r. We refer the study by Yun and Kang [34]:

$$\lambda_i(t|r) = \theta_{0i} + \theta_{1i}r + \theta_{2i}t^2 + \theta_{3i}rt^2 \tag{1}$$

D. IMPERFECT PREVENTIVE MAINTENANCE ACTIONS

Preventive maintenance is imperfect preventive maintenance. After preventive maintenance, the components are between 'good as new' and 'bad as old'. We use the virtual age method [35] to model the effect of imperfect preventive maintenance. Its basic idea is that preventive maintenance reduces the failure rate of components or systems and enhances the performance of systems or components, just as the age of components or systems is reduced for a period of time. The improvement factor (represented by θ_i) is introduced to describe the effect of imperfect preventive maintenance. Assuming that the improvement factor of the component is affected by the actual service life t_i of the component, imperfect preventive maintenance interval T_i, and single imperfect preventive warranty cost C_{pi}, the improvement factor can be expressed as follows:

$$\theta_i = \left(\frac{C_{pi}^0}{C_{pi}^0 - \pi C_{pi}}\right)^{\frac{T_i - t_i}{T_i}} \tag{2}$$

where, C_{pi}^0 denotes the preventive replacement cost of component i; π is the adjustment coefficient that changes with the change of component type, nature, and condition and is usually set according to experience. The improvement factor θ_i is temporarily assumed to be constant for research and calculation.

After the k-th imperfect preventive maintenance, the failure rate of the component or system decreases to the failure rate $k\theta_i T_i$ before the maintenance. Therefore, the failure rate of the component or system is as follows:

$$\begin{cases} \lambda_{0i}(t|r) = \lambda(t|r) \\ \lambda_{1i}(t|r) = \lambda(t - \theta_i T_i|r) \\ \vdots \\ \lambda_{ki}(t|r) = \lambda(t - k\theta_i T_i|r) \end{cases} \tag{3}$$

where $\lambda_{ki}(t|r)$ denotes the failure rate function between the k-th and $(k+1)$-th imperfect preventive maintenance of component i under the utilisation rate r.

E. TWO-DIMENSIONAL EXTENDED WARRANTY COST

The failure and preventive maintenance of two-dimensional warranty equipment usually occur at a point in the rectangular region. The equipment in the extended warranty period have experienced the basic warranty period. So in the extended warranty period, with the passage of time, the failure rate gradually increases. If the manufacturer completely bears the warranty cost at this stage, it will bring great pressure to the manufacturer, thereby resulting in the manufacturer's low enthusiasm for warranty. Therefore, the combined strategy of free warranty and pro-rata warranty is adopted.

Under pro-rata warranty, the warranty cost shall be borne by the user and the manufacturer in a certain proportion. The user bearing ratio is determined according to the rectangular region surrounded by the warranty claim point and the origin. Let t denotes the calendar time when the warranty claim is generated, and l denotes the usage when the warranty claim is generated. Assuming that the usage is linearly related to the calendar time, the correlation can be expressed as follows:

$$l = tr \tag{4}$$

Let C denote the current warranty claim, in which the part to be borne by the user and the part to be borne by the manufacturer can be expressed as follows [29]:

$$Y(t,l) = \begin{cases} \dfrac{t^2 r}{W \cdot U} \cdot C & 0 \le t \le W \text{ and } 0 \le r \le \dfrac{U}{t} \\ C & t \ge W \text{ and } r > \dfrac{U}{t} \end{cases} \tag{5}$$

$$X(t,l) = \begin{cases} \dfrac{W \cdot U - t^2 r}{W \cdot U} \cdot C & 0 \le t \le W \text{ and } 0 \le r \le \dfrac{U}{t} \\ 0 & t \ge W \text{ and } r > \dfrac{U}{t} \end{cases} \tag{6}$$

Since the utilisation rate is a random variable and different users have different utilisation rates, the start and end times of extended warranty period under different utilisation rates are different. In practice, r_1 and r have different quantitative relationships. The extended warranty period and the number of preventive maintenance activities of components differ according to the different quantitative relationships between the two [36].

When $r_l \le r \le r_1$, the start time of the extended warranty for components $I_{t_i} = W_B$, and the end time of the extended warranty $O_{t_i} = W_E$. So the number of preventive maintenance n_i is:

$$n_i = \begin{cases} \dfrac{W_E - W_B}{T_i + T_i(\theta_i)} - 1 & \dfrac{W_E - W_B}{T_i + T_i(\theta_i)} \in N^* \\ \left\lfloor \dfrac{W_E - W_B}{T_i + T_i(\theta_i)} \right\rfloor & \dfrac{W_E - W_B}{T_i + T_i(\theta_i)} \notin N^* \end{cases} \tag{7}$$

In a similar way, When $r_u \ge r > r_1$, the start time of the extended warranty for components I_{t_i} is:

$$I_{t_i} = \frac{U_B}{r} \tag{8}$$

And the end time of the extended warranty O_{t_i} is:

$$O_{t_i} = \frac{U_E}{r} \tag{9}$$

So the number of preventive maintenance n_i is:

$$n_i = \begin{cases} \frac{O_{t_i} - I_{t_i}}{T_i + T_i(\theta_i)} - 1 & \frac{O_{t_i} - I_{t_i}}{T_i + T_i(\theta_i)} \in N^* \\ \left\lfloor \frac{O_{t_i} - I_{t_i}}{T_i + T_i(\theta_i)} \right\rfloor & \frac{O_{t_i} - I_{t_i}}{T_i + T_i(\theta_i)} \notin N^* \end{cases} \tag{10}$$

According to the imperfect preventive maintenance model in 2.4, for component i, after the k-th preventive maintenance, the failure rate function of the component is as follows:

$$\lambda_i(t|r) = \lambda_i(t - k\theta_i T_i|r) \; I_{t_i} + kT_i \le t < I_{t_i} + (k+1)T_i \quad 0 \le k \le n_i \tag{11}$$

Then, during the interval from the k-th preventive maintenance to the $k+1$-th preventive maintenance, the number of minimal maintenance activities M_1 is as follows:

$$\begin{aligned} M_1 &= \int_{I_{t_i}+k(T_i+T_i(\theta_i))}^{I_{t_i}+k(T_i+T_i(\theta_i))+T_i} \lambda_i(t|r)dt \\ &= \int_{I_{t_i}+kT_i}^{I_{t_i}+(k+1)T_i} \lambda_i(t - k\theta_i T_i|r)dt \\ & 0 \le k \le n_i - 1 \end{aligned} \tag{12}$$

During the period from the n_i-th preventive maintenance to the end of the extended warranty period, the number of minimal maintenance activities M_2 is as follows:

$$M_2 = \int_{I_{t_i}+n_i(T_i+T_i(\theta_i))}^{O_{t_i}} \lambda_i(t|r)dt = \int_{I_{t_i}+nT_i}^{O_{t_i}} \lambda_i(t - n_i\theta_i T_i|r)dt \tag{13}$$

In the extended warranty period, the expected warranty cost comprises preventive maintenance and minimal maintenance costs. Indeed, an unequal quantitative relationship exists between r_1 and r_i; thus, different situations should be discussed according to the quantitative relationship between r_1 and r_i when calculating the extended warranty cost. Fig. 3 shows the two-dimensional extended warranty period under different utilization rates.

Case 1 When $r_i > r_1$:

1) $r_l \le r < r_1$

The warranty cost borne by the manufacturer during the extended warranty period of the component is as follows:

$$EC_{x1}(T_i, I_{t_i}, O_{t_i}) = \int_{r_l}^{r_1} \{n_i[C_i(\theta_i) + C_{Di}] + \sum_{k=0}^{n_i-1} C_{fi}M_1 + C_{fi}M_2\}dG_i(r) \tag{14}$$

As indicated in the Fig. 3(1), since the actual utilisation rate does not exceed the design utilisation rate, users do not need to bear the warranty cost during the two-dimensional extended warranty period of the component.

2) $r_1 \le r < r_i$

The warranty cost borne by the manufacturer during the two-dimensional extended warranty period of components is as follows:

$$EC_{x2}(T_i, I_{t_i}, O_{t_i}) = \int_{r_1}^{r_i} \{n_i[C_i(\theta_i) + C_{Di}] + \sum_{k=0}^{n_i-1} C_{fi}M_1 + C_{fi}M_2\}dG_i(r) \tag{15}$$

As illustrated in the Fig. 3(2), similar to case (1), the user does not need to bear the warranty cost during the two-dimensional extended warranty period of the component.

3) $r_i \le r < r_u$

As indicated in the Fig. 3(3), since the actual utilisation rate of components has exceeded the design utilisation rate, the manufacturer and the user shall bear a certain proportion of the warranty cost during the two-dimensional extended warranty period. The warranty cost borne by the manufacturer during the two-dimensional extended warranty period of components is as follows:

$$\begin{aligned} &EC_{x3}(T_i, I_{t_i}, O_{t_i}) \\ &= \int_{r_i}^{r_u} \int_{\frac{U_B}{r}}^{\frac{U_E}{r}} \frac{W_E \cdot U_E - t_1 \cdot t_1 \cdot r}{W_E \cdot U_E} \{n_i[C_i(\theta_i) + C_{Di}] \\ &+ \sum_{k=0}^{n_i-1} C_{fi}M_1 + C_{fi}M_2\} \, dt_1 dG_i(r) \end{aligned} \tag{16}$$

The warranty cost borne by the user during the two-dimensional extended warranty period of components is as follows:

$$\begin{aligned} &EC_{x4}(T_i, I_{t_i}, O_{t_i}) \\ &= \int_{r_i}^{r_u} \int_{\frac{U_B}{r}}^{\frac{U_E}{r}} \frac{t_1 \cdot t_1 \cdot r}{W_E \cdot U_E} \{n_i[C_i(\theta_i) + C_{Di}] \\ &+ \sum_{k=0}^{n_i-1} C_{fi}M_1 + C_{fi}M_2\} \, dt_1 dG_i(r) \end{aligned} \tag{17}$$

Case 2 When $r_i < r_1$:

4) $r_l \le r < r_i$

The warranty cost borne by the manufacturer during the two-dimensional extended warranty period of components is as follows:

$$EC_{x5}(T_i, I_{t_i}, O_{t_i}) = \int_{r_l}^{r_i} \{n_i[C_i(\theta_i) + C_{Di}] + \sum_{k=0}^{n_i-1} C_{fi}M_1 + C_{fi}M_2\}dG_i(r) \tag{18}$$

As illustrated in the Fig. 3(4), since the actual utilisation rate does not exceed the design utilisation rate, users do not need to bear the warranty cost during the two-dimensional extended warranty period of components.

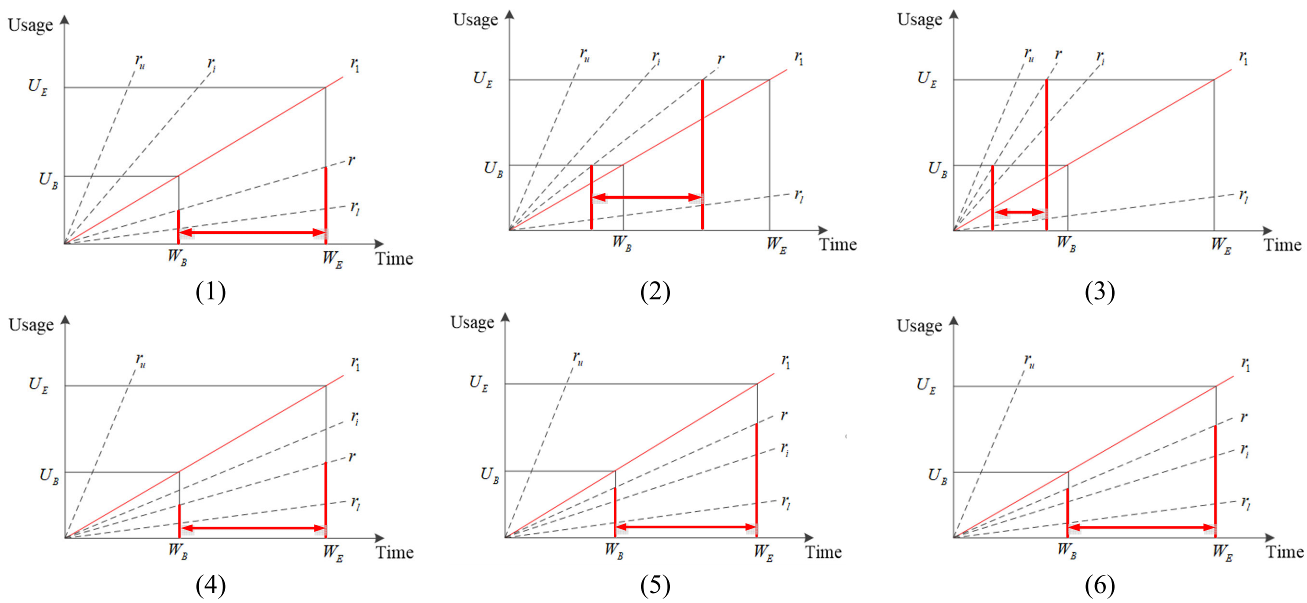

FIGURE 3. Schematic diagram of two-dimensional extended warranty under different utilization rates.

5) $r_i \leq r < r_1$

As indicated in the Fig. 3(5), since the actual utilisation rate of components has exceeded the design utilisation rate, the manufacturer and the user shall bear a certain proportion of the warranty cost during the two-dimensional extended warranty period. The warranty cost borne by the manufacturer during the two-dimensional extended warranty period of components is as follows:

$$\begin{aligned} &EC_{x6}(T_i, I_{t_i}, O_{t_i}) \\ &\quad = \int_{r_i}^{r_1} \int_{W_B}^{W_E} \frac{W_E \cdot U_E - t_1 \cdot t_1 \cdot r}{W_E \cdot U_E} \{n_i[C_i(\theta_i) + C_{Di}] \\ &\qquad + \sum_{k=0}^{n_i-1} C_{fi}M_1 + C_{fi}M_2\} dt_1 dG_i(r) \end{aligned} \tag{19}$$

The warranty cost borne by the user during the two-dimensional extended warranty period of components is as follows:

$$\begin{aligned} &EC_{x7}(T_i, I_{t_i}, O_{t_i}) \\ &\quad = \int_{r_i}^{r_1} \int_{W_B}^{W_E} \frac{t_1 \cdot t_1 \cdot r}{W_E \cdot U_E} \{n_i[C_i(\theta_i) + C_{Di}] \\ &\qquad + \sum_{k=0}^{n_i-1} C_{fi}M_1 + C_{fi}M_2\} dt_1 dG_i(r) \end{aligned} \tag{20}$$

6) $r_1 \leq r < r_u$

As indicated in the Fig. 3(6), similar to case (2), since the actual utilisation rate of components has exceeded the design utilisation rate, the manufacturer and the user shall bear a certain proportion of the warranty cost during the two-dimensional extended warranty period. The warranty cost borne by the manufacturer during the two-dimensional extended warranty period of components is as follows:

$$\begin{aligned} &EC_{x8}(T_i, I_{t_i}, O_{t_i}) \\ &\quad = \int_{r_1}^{r_u} \int_{\frac{U_B}{r}}^{\frac{U_E}{r}} \frac{W_E \cdot U_E - t_1 \cdot t_1 \cdot r}{W_E \cdot U_E} \{n_i[C_i(\theta_i) + C_{Di}] \\ &\qquad + \sum_{k=0}^{n_i-1} C_{fi}M_1 + C_{fi}M_2\} dt_1 dG_i(r) \end{aligned} \tag{21}$$

The warranty cost borne by the user during the two-dimensional extended warranty period of components is as follows:

$$\begin{aligned} &EC_{x9}(T_i, I_{t_i}, O_{t_i}) \\ &\quad = \int_{r_1}^{r_u} \int_{\frac{U_B}{r}}^{\frac{U_E}{r}} \frac{t_1 \cdot t_1 \cdot r}{W_E \cdot U_E} \{n_i[C_i(\theta_i) + C_{Di}] \\ &\qquad + \sum_{k=0}^{n_i-1} C_{fi}M_1 + C_{fi}M_2\} dt_1 dG_i(r) \end{aligned} \tag{22}$$

To sum up, the total expected cost borne by the user during the warranty period is as follows:

$$EC_{1i} = \begin{cases} EC_{x4}(T_i, I_{t_i}, O_{t_i}) r_i > r_1 \\ EC_{x7}(T_i, I_{t_i}, O_{t_i}) + EC_{x9}(T_i, I_{t_i}, O_{t_i}) r_i \leq r_1 \end{cases} \tag{23}$$

The total expected cost borne by the manufacturer during the two-dimensional extended warranty period of the component is as follows:

$$EC_{2i} = \begin{cases} EC_{x1}(T_i, I_{t_i}, O_{t_i}) + EC_{x2}(T_i, I_{t_i}, O_{t_i}) \\ \quad + EC_{x3}(T_i, I_{t_i}, O_{t_i}) \quad r_i > r_1 \\ EC_{x5}(T_i, I_{t_i}, O_{t_i}) + EC_{x6}(T_i, I_{t_i}, O_{t_i}) \\ \quad + EC_{x8}(T_i, I_{t_i}, O_{t_i}) \quad r_i \leq r_1 \end{cases} \tag{24}$$

TABLE 1. The downtime of the component when $r_i > r_1$.

	Value range of r	Downtime	Number
$r_i > r_1$	$r_l \le r < r_1$	$\int_{r_l}^{r_1} [n_i T_i(\theta_i) + \sum_{k=0}^{n_i-1} T_{fi} M_1 + T_{fi} M_2] dG_i(r)$	(1)
	$r_1 \le r < r_i$	$\int_{r_1}^{r_i} [n_i T_i(\theta_i) + \sum_{k=0}^{n_i-1} T_{fi} M_1 + T_{fi} M_2] dG_i(r)$	(2)
	$r_i \le r < r_u$	$\int_{r_1}^{r_i} [n_i T_i(\theta_i) + \sum_{k=0}^{n_i-1} T_{fi} M_1 + T_{fi} M_2] dG_i(r)$	(3)
$r_i < r_1$	$r_l \le r < r_i$	$\int_{r_l}^{r_1} [n_i T_i(\theta_i) + \sum_{k=0}^{n_i-1} T_{fi} M_1 + T_{fi} M_2] dG_i(r)$	(4)
	$r_i \le r < r_1$	$\int_{r_1}^{r_i} [n_i T_i(\theta_i) + \sum_{k=0}^{n_i-1} T_{fi} M_1 + T_{fi} M_2] dG_i(r)$	(5)
	$r_1 \le r < r_u$	$\int_{r_1}^{r_i} [n_i T_i(\theta_i) + \sum_{k=0}^{n_i-1} T_{fi} M_1 + T_{fi} M_2] dG_i(r)$	(6)

F. TWO-DIMENSIONAL EXTENDED WARRANTY AVAILABILITY

During the two-dimensional extended warranty period, the downtime of components mainly constitutes corrective maintenance downtime and preventive maintenance downtime. Availability is the ratio of trouble-free time to the total service time within the extended warranty period of components, and it can be expressed as the following formula [37]:

$$EA = \frac{\text{Extended Warranty period} - \text{Expected downtime}}{\text{Extended Warranty period}} \tag{25}$$

Therefore, the key to calculating the availability of components during the two-dimensional extended warranty period is to calculate the downtime of components. We can refer to the calculation method of warranty cost in the two-dimensional extended warranty period to calculate the downtime of components in the two-dimensional extended warranty period. We only need to replace the corrective maintenance cost and preventive maintenance cost in some formulas with the corrective maintenance time and the preventive maintenance time. The downtime of components during the two-dimensional extended warranty period is as indicated in the following Table 1.

The total downtime of the component during the two-dimensional extended warranty period is as follows:

$$ED = \begin{cases} (1) + (2) + (3) & r_i > r_1 \\ (4) + (5) + (6) & r_i \le r_1 \end{cases} \tag{26}$$

The availability of components during the two-dimensional extended warranty period is as follows:

$$EA_i = \frac{W_E - W_B - ED}{W_E - W_B} \tag{27}$$

The decision-making objective of this model is to minimise the warranty cost borne by the manufacturer of each component in the two-dimensional extended warranty period. The constraint is that the availability of components in the two-dimensional extended warranty period should be greater than or equal to a set minimum value, and this minimum value of availability should be acceptable to users. Accordingly, the two-dimensional extended warranty decision model of a single component can be expressed as follows [38]:

$$\begin{cases} min\ EC_{2i} \\ s.t. \\ EA_i \ge A_{0i} \end{cases} \tag{28}$$

where A_{0i} denotes the lowest value of component availability acceptable to the user.

The total warranty cost of multi-component series system during the two-dimensional extended warranty period is as follows:

$$EC_{total2} = \sum_{i=1}^{m_s} EC_{2i} \tag{29}$$

As any component shutdown will cause system shutdown, the availability of the multi-component series system in the extended warranty period is 1 minus the sum of the unavailability of each component. The availability of the multi-component series system in the two-dimensional extended warranty period is as follows:

$$EA = 1 - \sum_{i=1}^{m_s} (1 - EA_i) \tag{30}$$

G. COMBINATION STRATEGY OF PREVENTIVE MAINTENANCE

By solving formula (28), the optimal preventive maintenance interval of each single component can be obtained. Before grouping maintenance, the preventive maintenance time array

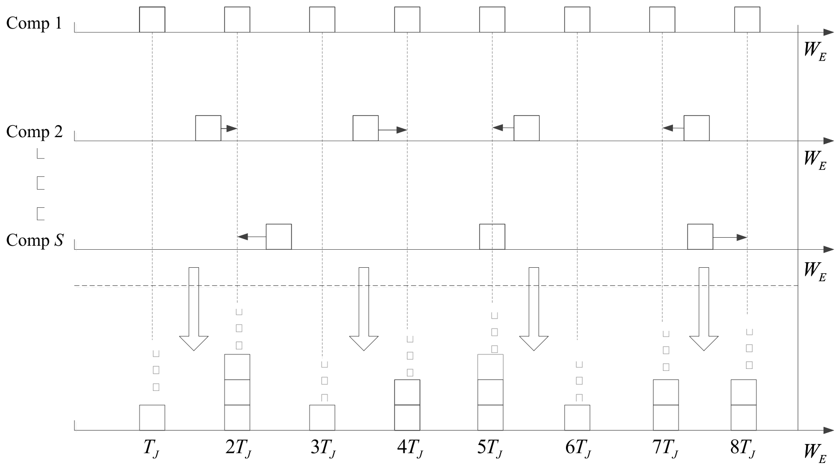

FIGURE 4. Adjustment of preventive maintenance time for components.

of each component is as follows:

$$\mathbf{T_{iA}} = (T_i^*, 2T_i^*, 3T_i^*, \ldots, kT_i^*)\ 1 \leq i \leq m_s;\ 1 \leq k \leq n_i \tag{31}$$

Due to the different failure rules of each component, the optimal preventive maintenance interval is different. If preventive maintenance is undertaken according to their optimal preventive maintenance interval, it is bound to increase the shutdown times and downtime of multi-component system as well as increase the expenditure of preventive maintenance preparation cost. Reasonable preventive maintenance combination strategy can adjust multiple preventive maintenance works of multiple components to the same time, effectively reduce the warranty cost of the system in the two-dimensional extended warranty period, and enhance the availability level of the system.

The preventive maintenance benchmark interval T_J of the multi-component system is introduced to adjust the preventive maintenance time of each component to an integral multiple of the preventive maintenance benchmark interval. The preventive maintenance time array of each component after adjustment is as follows:

$$\mathbf{T^*_{iA}} = (T^*_{i1}, T^*_{i2}, T^*_{i3}, \ldots, T^*_{ik}) T^*_{ik} = \left\lceil \frac{kT_i^*}{T_J} \right\rceil \cdot T_J\ ;\ 1 \leq i \leq m_s \tag{32}$$

If the last preventive maintenance time after the grouping maintenance exceeds the extended warranty period, the preventive maintenance shall be adjusted to the maximum integer multiple of the preventive maintenance benchmark interval to ensure that the number of preventive maintenance activities before and after the grouping maintenance of each component remains unchanged. Considering the preventive maintenance time of each component after adjustment as the input, the two-dimensional extended warranty cost and availability of the system are obtained using the models in 3.5 and 3.6. The grouping maintenance model is indicated in the Fig. 4.

IV. CASE STUDY

A. PROBLEM DESCRIPTION AND CASE PREPARATION

A reconnaissance equipment is mainly used for reconnaissance and surveillance of ground targets. It mainly comprises laser rangefinder, thermal imager, and radar. The normal operation of reconnaissance equipment requires the close cooperation of three components, that is, the three components work at the same time to ensure the predetermined function of reconnaissance equipment.

The three components of this reconnaissance equipment are independent warranty units and have different failure modes and failure laws; However, they all show degradation in two dimensions of time (Calendar time, unit: years) and usage (Work time, unit: Kilohour). Therefore, using the two-dimensional failure rate function to describe the failure law of each component is essential. As the three components are technology-intensive products, users can only do simple maintenance work such as wiping, drying, and powering on their surfaces. Works such as condition detection, failure prevention, and corrective maintenance need to be completed by the manufacturer. The basic warranty period of reconnaissance equipment is (2years, 2Kh), that is, $W = 2$ years, $U = 2$Kh. The extended warranty period is (3years, 3Kh), that is, $W_E = 3$years, $U_E = 3$Kh. The user signs a two-dimensional extended warranty contract with the manufacturer while purchasing the equipment. Determining the optimal preventive maintenance interval and improvement factor of imperfect preventive maintenance is crucial so that the availability of reconnaissance equipment in the two-dimensional extended warranty period can reach the minimum acceptable to users, reduce the warranty cost, and expand the profit margins of manufacturers.

One year is approximately regarded as 360 days. The parameters of each part within the extended warranty period are indicated in the Table 2.

As shown in Table 2, the preventive maintenance time and preventive maintenance cost of components are linearly related to the improvement factor of imperfect preventive

TABLE 2. Parameter setting.

	r_i (Kh/y)	r_l (Kh/y)	r_u (Kh/y)	C_{fi} /CNY	T_{fi} /Years	C_{Di} /CNY	A_{0i}	$T_i(\theta_i)$ /Years	$C_i(\theta_i)$ /CNY
Laser rangefinder	3.5	0.5	4	2500	6/360	100	0.90	$\frac{3}{360}(1+\theta_i)$	$1000(1+\theta_i)$
Thermal imager	3.8	0.5	4	1200	8/360	100	0.90	$\frac{4}{360}(1+\theta_i)$	$800(1+\theta_i)$
Radar	3.7	0.5	4	2900	6/360	100	0.90	$\frac{4}{360}(1+\theta_i)$	$1300(1+\theta_i)$

TABLE 3. Algorithm basic steps.

Algorithm -Basic steps of GS

Input $T_i = 0.1{:}0.1{:}2.9$
Input $\theta_i = 0.1{:}0.1{:}0.9$
while b≤261 do
b=1;c=1
for $i = 1{:}2.9$
for $j = 1{:}0.9$
Calculate the EC_{1i}^{c}, EC_{2i}^{c} and EA_i^c corresponding to $(T_i(i), \theta_i(j))$
Store $(T_i, \theta_i, EC_{2i}^{c})_c$, $(T_i, \theta_i, EC_{1i}^{c})_c$, and $(T_i, \theta_i, EA_i^{c})_c$
c=c+1
end for
end for
for c=1:261
if $EA_i^c < A_{0i}$ **then**
Remove EC_{1i}^{c}, EC_{2i}^{c} and EA_i^c corresponding to group $(T_i(i), \theta_i(j))$
else
Retain EC_{1i}^{c}, EC_{2i}^{c} and EA_i^c corresponding to group $(T_i(i), \theta_i(j))$
end if
end for
b=b+1
end while
Output the minimum EC_{2i} and the corresponding EC_{1i} and EA_i of the group (T_i, θ_i)

maintenance, that is, with the improvement factor, the corresponding time and cost expenditure also increase.

For different users, the component utilisation follows the two-parameter Weibull distribution, and each component follows the same distribution. The probability density function expression of two-parameter Weibull distribution is as follows:

$$g(r) = \frac{\delta}{\eta}\left(\frac{r}{\eta}\right)^{\delta-1} e^{-(\frac{r}{\eta})^{\delta}} \quad (0.5Kh/year < r < 4Kh/year)$$

In this case, δ is 2 and η is 2.5 years.

Each of the three components has a major failure mode. The main failure mode of laser rangefinder is that the laser cannot emit laser. The main failure mode of thermal imager is that it cannot image normally. The main failure mode of radar is that the system has no response or false alarm of target position. The method in appendix is used to fit the two-dimensional failure rate function, and the failure rate function of the main failure modes of the three components is as follows:

Laser rangefinder:

$$\lambda_1(t|r) = 0.2 + 0.015r + 0.01t^2 + 0.014rt^2$$

Thermal imager:

$$\lambda_2(t|r) = 0.1 + 0.06r + 0.003t^2 + 0.001rt^2$$

Radar:

$$\lambda_3(t|r) = 0.3 + 0.02r + 0.01t^2 + 0.02rt^2$$

B. SOLUTION OF OPTIMAL EXTENDED WARRANTY SCHEME FOR SINGLE COMPONENT

According to the single-component two-dimensional extended warranty decision model in 2.6, the grid search method is used for optimisation. Grid search method is an exhaustive search algorithm. Given all the possible values of decision variables, the algorithm can find the optimal decision variables by calculating all corresponding objective function values and then provide the best scheme for decision makers.

Let T_i take value in $[0.1years, 2.9years]$ in steps of 0.1 years and θ_i take value in [0.1, 0.9] in steps of 0.1, respectively. Next, calculate the expected warranty cost EC_{2i} borne by the manufacturer; the expected warranty cost EC_{1i} borne by the user; and the expected availability EA_i of the components corresponding to each group of (T_i, θ_i) and store them. Finally, 261 groups (T_i, θ_i, EC_{2i}), (T_i, θ_i, EC_{1i}), and (T_i, θ_i, EA_i) are obtained. In combination with the availability constraints of the model, the unqualified groups are removed. Then we compare the remaining EC_{2i}, and find the minimum value. At the same time, the corresponding preventive maintenance interval and improvement factor are output, that is, the optimal two-dimensional extended warranty scheme for a single component is obtained.

In the programming process, when the actual utilization rate $r < r_1$, as I_{t_i} and O_{t_i} are fixed, the preventive maintenance times n_i can be determined after the preventive maintenance interval T_i is set, which is independent of the actual utilization rate r. At this point, obtaining the extended warranty cost and availability is easy. When $r > r_1$, I_{t_i} and O_{t_i} are constantly changing with the actual utilization rate r; thus, it is difficult to use the algorithm for programming and problem solving. The approximate algorithm that converts continuous to discrete is adopted to solve the problem of extended warranty cost and availability. The specific process is as follows:

Step 1. We divide $[r_1, r_u]$ into 20 sub-intervals on average. These sub-intervals are respectively expressed as $[r_{a-1}, r_a), a = 1, 2, \ldots, 20$, and r_a can be expressed as follows:

$$r_a = r_1 + a \times \frac{r_u - r_1}{20}, \quad a = 1, 2, \ldots, 20$$

Step 2. We determine the average utilization rate corresponding to each sub-interval, which can be expressed as follows:

$$\overline{r_a} = \int_{r_{a-1}}^{r_a} \frac{sg(s)}{G(r_a) - G(r_{a-1})} ds$$

Step 3. We determine the probability corresponding to each sub-interval, which can be expressed as follows:

$$P_a = G(r_a) - G(r_{a-1})$$

Step 4. Within the utilization rate range, the average utilization rate $\overline{r_a}$ can be used for approximate calculation to obtain $I_{t_i} = \frac{U_B}{\overline{r_a}}$ and $O_{t_i} = \frac{U_E}{\overline{r_a}}$. On this basis, the extended warranty cost C_a and availability A_a can be obtained. Combined with the probability P_a corresponding to the sub-interval, the expected extended warranty cost and availability of the sub-interval are obtained.

$$EC_a = C_a P_a, \quad EA_a = A_a P_a$$

Step 5. We calculate the expected extended warranty cost and availability of the system at $r > r_1$.

$$EC_{r>r_1} = \sum_{a=1}^{20} EC_a, \quad EA_{r>r_1} = \sum_{a=1}^{20} EA_a$$

Fig. 5 illustrates the algorithm flow, and the algorithm basic steps are presented in Table 3.

Using these data of output, three-dimensional images of EC_{2i}, EC_{1i}, and EA_i can be drawn. The radar component is taken as an example to illustrate the image trend.

Fig. 6 presents the variation trend of extended warranty cost and system availability with preventive maintenance interval and improvement factors of radar, in which (1) and (2) represent the manufacturer's extended warranty cost EC_{2i} and user's extended warranty cost EC_{1i}, respectively. As shown in Fig. 6 (1), when the improvement factor and preventive maintenance interval are small, the manufacturer needs to bear more warranty cost during the extended warranty period. Furthermore, when the improvement factor and preventive maintenance interval are large, the expected warranty cost borne by the manufacturer increases further. According to the image trend, an optimal preventive maintenance interval and improvement factor exist to minimise the expected warranty cost borne by manufacturers. Fig. 6 (2) shows the same trend as Fig. 6(1), that is, the user's warranty cost of each component presents the same trend as the manufacturer's warranty cost. However, from the perspective of magnitude of warranty cost, the user's warranty cost is much smaller than the manufacturer's warranty cost.

Fig. 6(3) presents the change trend of radar availability with preventive maintenance interval and improvement factor. Optimal preventive maintenance interval and improvement factor maximise the expected availability of radar.

In the model established in this study, the manufacturer's warranty cost and multi-component system availability are related to the preventive maintenance interval and improvement factor, and their values will have varying degrees of impact on the two indicators. To observe these impacts more intuitively, dimension reduction analysis is performed in Fig. 7.

Fig. 7 (1) represents the change trend of the manufacturer's extended warranty cost of radar with improvement factor after a fixed preventive maintenance interval. Further, Fig. 7 (2) represents the change trend of extended warranty availability of radar with improvement factor after a fixed preventive maintenance interval. The image clearly indicates that an extremely large or small preventive maintenance interval is not conducive to cost saving and improving the component

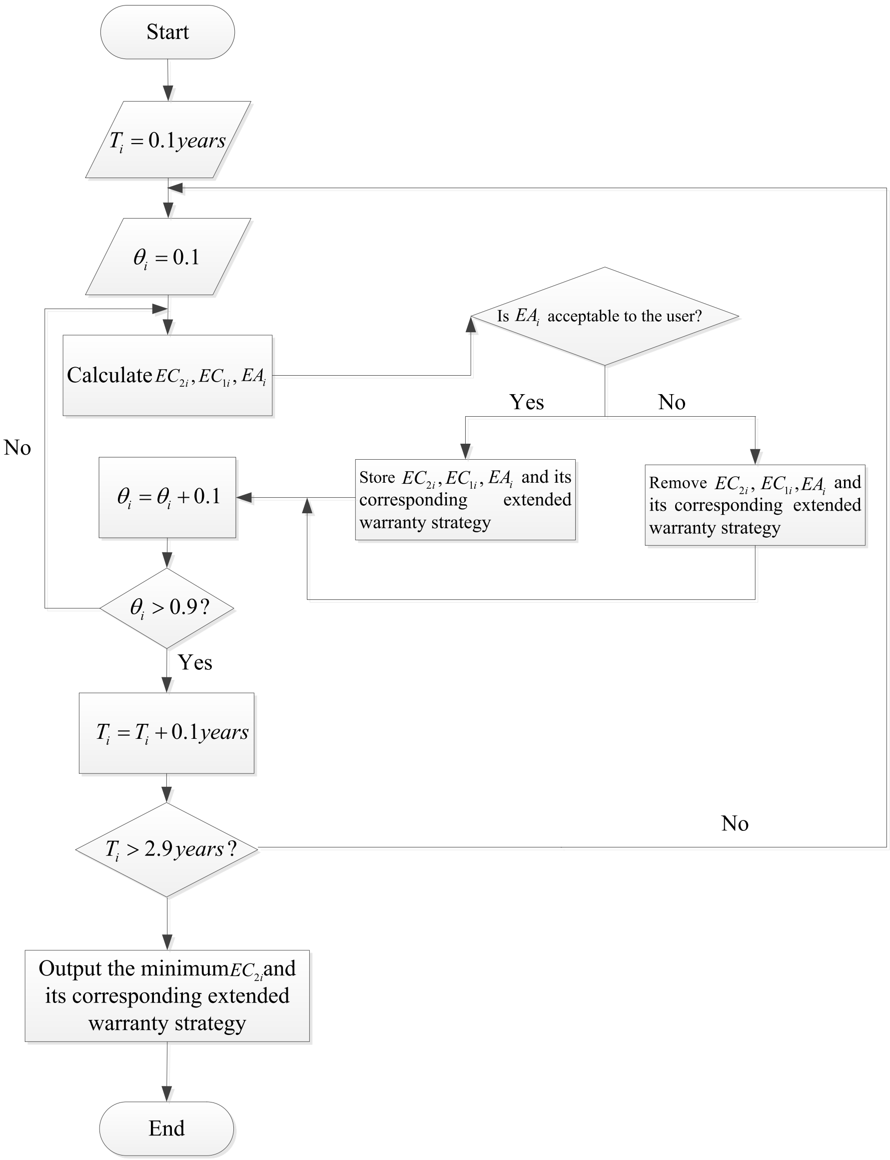

FIGURE 5. Algorithm flow chart.

availability level by manufacturers. An optimal preventive maintenance interval minimises the manufacturer's warranty cost or maximises the component availability within the extended warranty period. Therefore, determining the preventive maintenance interval by using a quantitative analysis model is essential. The empirical method may result in warranty cost waste or reduce availability.

Fig. 7 (3) represents the change trend of the manufacturer's extended warranty cost of radar with the preventive maintenance interval. Further, Fig. 7(4) represents the change trend of extended warranty availability of radar with the preventive maintenance interval. With an increase in the improvement factor, the manufacturer's warranty cost decreases and the extended warranty availability of the multi-component system increases. This is mainly because the higher the improvement factor of imperfect preventive maintenance, the lower is the failure rate during the preventive maintenance interval, and the lower is the cost of minimum maintenance after multi-component system failure in addition to less downtime. In general, manufacturers should continuously improve the technical level of preventive maintenance and strengthen technological innovation and maintenance personnel training to seek greater improvement factors of imperfect preventive maintenance, reduce warranty costs, and improve the availability of multi-component systems. This approach can also improve user's satisfaction and enhance marketing competitiveness of the manufacturer.

As we focus on the manufacturer's minimum warranty cost, the optimal preventive maintenance interval and improvement factor can be obtained by searching the manufacturer's minimum expected warranty cost of the radar in Fig. 6 (1). On this basis, combined with

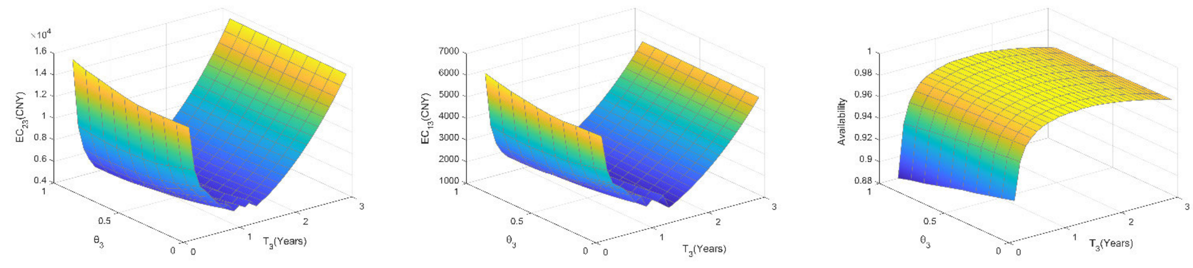

FIGURE 6. Three-dimensional images of the Radar two-dimensional extended warranty scheme.

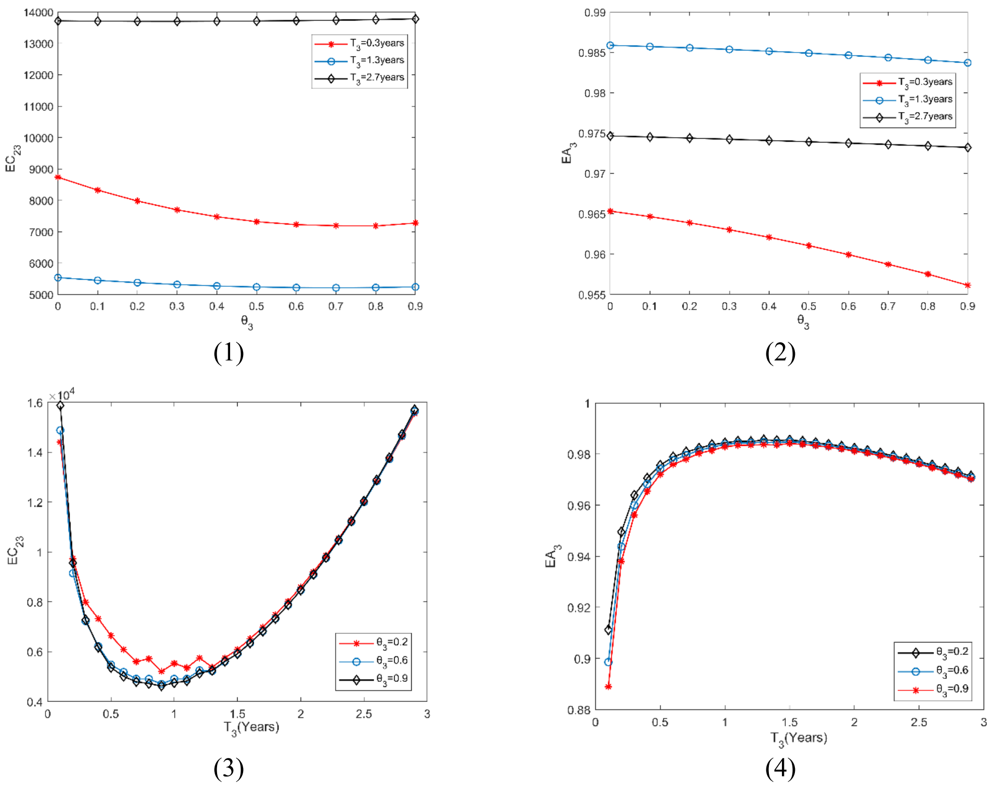

FIGURE 7. Dimension reduction analysis.

TABLE 4. Single component optimal extended warranty scheme.

Component number	Optimal preventive maintenance interval/Years	Optimal improvement factor	Manufacturer's expected warranty cost/CNY	user's expected warranty cost/CNY	Expected availability of components	Accept or not
1	1.6	0.9	2440.084	456.842	0.993	Yes
2	2	0.9	1116.493	117.817	0.992	Yes
3	0.9	0.8	4623.587	1786.2	0.982	Yes
Total			8180.16	2360.86	0.966	

Fig. 6(2) and Fig. 6(3), we calculate the user's expected warranty cost and component expected availability and judge whether the optimal scheme can be accepted by the user according to the constraints. Laser Rangefinder and Thermal

Imager images show the same pattern as radar, and the optimal scheme is obtained in the same way as radar. The optimal scheme of single component is presented in the Table 4.

Table 2 indicates that the minimum availability acceptable to users of all components is 0.9, and the expected availability of components corresponding to the manufacturer's lowest expected warranty cost is higher than this value. Therefore, the optimal preventive maintenance interval and improvement factor with the manufacturer's lowest expected warranty cost are the optimal extended warranty scheme of components. The optimal preventive maintenance intervals of laser rangefinder, thermal imager, and radar are 1.6, 2, and 0.9 years, respectively, and the optimal improvement factors of the three components are 0.9, 0.9, and 0.8, respectively.

Before grouping maintenance, each component shall carry out preventive maintenance according to the optimal preventive maintenance interval separately. As each preventive maintenance requires a certain preventive maintenance preparation cost and each preventive maintenance will cause equipment shutdown, additional warranty costs will be increased and the availability of equipment will be reduced. We use the grouping maintenance method proposed in 2.7 to combine the preventive maintenance work of each component. Through this method, multiple adjacent preventive maintenance works are combined. Multiple preventive maintenance activities only require one preventive maintenance preparation cost to reduce the expenditure of equipment preventive maintenance preparation cost. Multiple preventive maintenance works are carried out at the same time, and the preventive maintenance downtime takes the maximum of all preventive maintenance working hours to reduce the equipment preventive maintenance downtime and improve equipment availability. Finding the preventive maintenance benchmark interval that minimises the manufacturer's warranty cost is crucial. The specific method is to make T_J take a value between $[0.7years, 2.7years]$ and calculate the sum of the manufacturer's warranty costs EC_{total2} of the three components corresponding to any T_J to obtain the change trend diagram of EC_{total2} with T_J.

Fig. 8 indicates that with the increase of preventive maintenance benchmark interval, the manufacturer's warranty cost first decreases and then increases. The optimal preventive maintenance benchmark interval is 0.9 years. At this time, the manufacturer's extended warranty cost is 7074.4 CNY, which is 13.5% lower than 8180.6 CNY before grouping maintenance.

Fig. 9 depicts the adjustment of preventive maintenance of components after grouping maintenance. After grouping maintenance, the extended warranty times of reconnaissance equipment are reduced from 5 to 3. The preventive maintenance of laser rangefinder and thermal imager are simultaneously undertaken with the second preventive maintenance of radar. The comparison of availability for components before and after adjustment is presented in Table 5.

After grouping maintenance, the availability of reconnaissance equipment will be slightly improved due to the reduction of preventive maintenance times and downtime of reconnaissance equipment within the extended warranty period. Grouping maintenance reduces the manufacturer's warranty cost and improves equipment availability. Therefore, it is a win–win strategy for manufacturers and users to adopt grouping maintenance in the extended warranty period.

C. COMPARATIVE ANALYSIS

1) COMPARISON BETWEEN WITH AND WITHOUT COMBINED WARRANTY

The introduction of the warranty cost sharing mechanism in which both the manufacturer and the user bear the warranty cost in proportion leads the user to bear a certain warranty cost during the two-dimensional extended warranty period of the equipment. This part of the cost does not constitute a high proportion of the total warranty cost but is paid by the user to the manufacturer on the basis of the extended warranty price; therefore, it can expand the manufacturer's profit margin to a certain extent. Assuming that the profit demand of the manufacturer for providing extended warranty service for each part of reconnaissance equipment is 800 CNY per year, the expansion proportion of the manufacturer's profit margin can be obtained as shown in Table 6.

Introduction of the mechanism of combined strategy of free warranty and pro-rata warranty has significantly expanded the manufacturer's profit margin. Therefore, extended warranty has become an essential source of revenue for manufacturers. Of course, users can also regulate their own use behavior to avoid excessive equipment utilisation, which is conducive to reducing the risk of equipment failure and avoiding more warranty expenses. The enthusiasm of the manufacturer to participate in the extended warranty of equipment is aroused, and the maintenance activities carried out according to the optimal preventive maintenance plan can effectively ensure the availability of components to meet the requirements of the user.

2) COMPARISON BETWEEN BEFORE AND AFTER TAKING PREVENTIVE MAINTENANCE

During the extended warranty period, regular preventive maintenance shall be undertaken for all components of reconnaissance equipment. If preventive maintenance measures are not taken for each component and only corrective maintenance strategy is used, that is, minimum maintenance strategy after failure, the manufacturer's warranty cost and availability of each component within the extended warranty period can be calculated accordingly. The obtained value is compared with the value when preventive maintenance measures are taken, and Table 7 can be obtained.

The comparison indicates that during the extended warranty period, after adopting preventive maintenance measures for reconnaissance equipment, the manufacturer's extended warranty cost can be saved by 32.4% and the availability of equipment can be increased by 25.1%. This is mainly because preventive maintenance helps reduce the failure rate

TABLE 5. Comparison of availability of components before and after combined maintenance.

Component number	Expected availability of components before adjustment	Expected availability of components after adjustment	Accept or not	Change proportion
1	0.993	0.992	Yes	-0.1007%
2	0.992	0.9935	Yes	0.151%
3	0.982	0.982	Yes	0
Total	0.966	0.968		0.207%

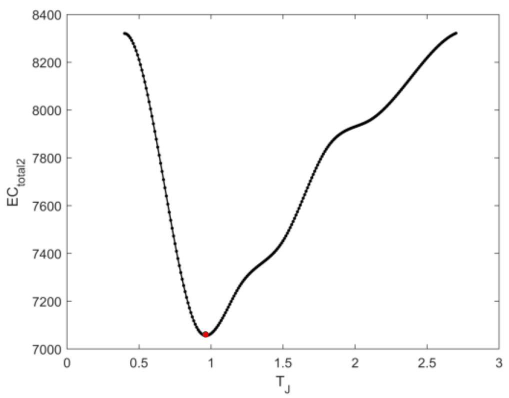

FIGURE 8. Change trend of manufacturer's warranty cost.

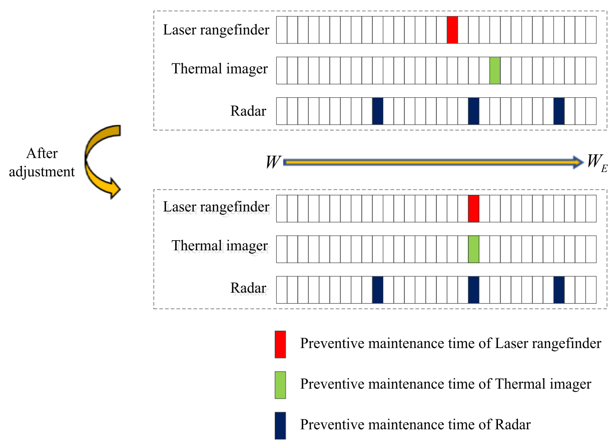

FIGURE 9. Preventive maintenance adjustment of each component.

of the multi-component system and the occurrence of failures. Therefore, preventive maintenance strategy plays an important role in reducing manufacturer's warranty cost and improving equipment availability.

D. SENSITIVITY ANALYSIS

The aforementioned case is based on the extended warranty period of 3 years (i.e. 1,080 days). It is worth discussing whether the optimal extended warranty scheme obtained with changes in the extended warranty period is applicable. Therefore, the extended warranty period is changed, and the preventive maintenance benchmark interval and corresponding extended warranty cost and availability of the system are calculated using the same method as the case, as shown in Table 8.

With the change in the extended warranty period, the basic interval of preventive maintenance will have irregular changes (Table 8). Moreover, the system extended warranty cost increases and the system availability decreases. The extended warranty period and the system warranty cost exhibit an obvious positive correlation, whereas the extended warranty period and the system availability exhibit an obvious negative correlation.

The data of extended warranty period and system warranty cost are fitted using MATLAB Curve Fitting Toolbox, and the fitting method is polynomial, with the degree being two. The regression function can be expressed as follows:

$$EC = 0.004881W_E^2 - 2.447W_E + 4173$$

The fitting effect is shown in Fig. 10. The adjust R-square is 0.9954, indicating that the fitting effect of the model is good.

The extended warranty period and system availability are fitted in the same manner. The regression function can be expressed as follows:

$$EA = 6.836\exp(-9)W_E^2 - 4.149\exp(-5)W_E + 1.004$$

The fitting effect is shown in Fig. 11. The adjust R-square is 0.9571, indicating that the fitting effect of the model is good.

In practical application, the fitting results can estimate the optimal extended warranty cost and system availability in different extended warranty periods, which can provide a reasonable basis for determining the extended warranty period. Users and manufacturers can choose the extended warranty period according to the actual situation to meet the demands of both parties.

However, it is undeniable that there is an inherent contradiction between the extended warranty cost and the availability of warranty objects, that is, maintaining and improving the availability of multi-component systems often leads to the increase of extended warranty cost. Therefore, how to balance this pair of contradictions and formulate an extended warranty scheme that satisfies both manufacturers and users is

TABLE 6. Expansion proportion of manufacturer's warranty cost.

Component number	Manufacturer's expected warranty cost after combined warranty (CNY)	Manufacturer's expected warranty cost before combined warranty (CNY)	Manufacturer's profit demand (CNY)	The expansion proportion of profit margin
1	2440.084	2,896.92	2400	19%
2	1116.493	1,234.32	2400	4.9%
3	4623.587	6,409.787	2400	49.4%
Total	8180.16	10,541.027	7200	33%

TABLE 7. Comparison of preventive maintenance and corrective maintenance.

Component number	Take corrective maintenance measures		Take preventive maintenance measures			Manufacturer cost change rate	Availability change rate
	EC_{2i} /CNY	EA_i	T_i /Years	EC_{2i} /CNY	EA_i		
1	3425.41	0.951	1.5	2440.08	0.993	-28.8%	4.42%
2	1853.72	0.924	2	1116.49	0.992	-39.8%	7.36%
3	6823.45	0.897	0.9	4623.59	0.982	-32.2%	9.48%
EC_{total2}	12102.58			8180.16		-32.4%	
EA		0.772			0.966		25.1%

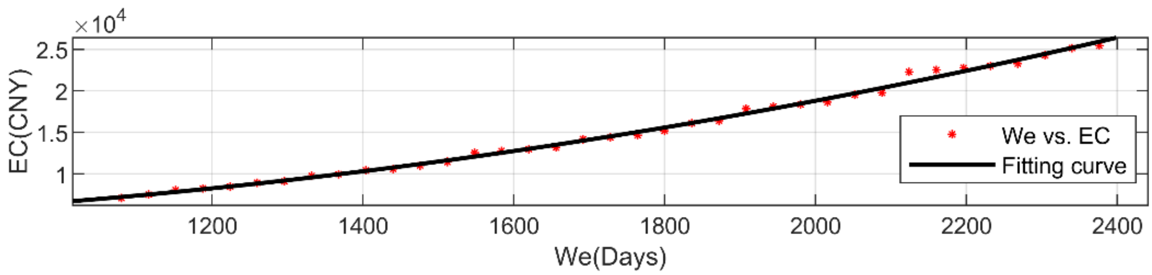

FIGURE 10. Fitting curve of We and EC.

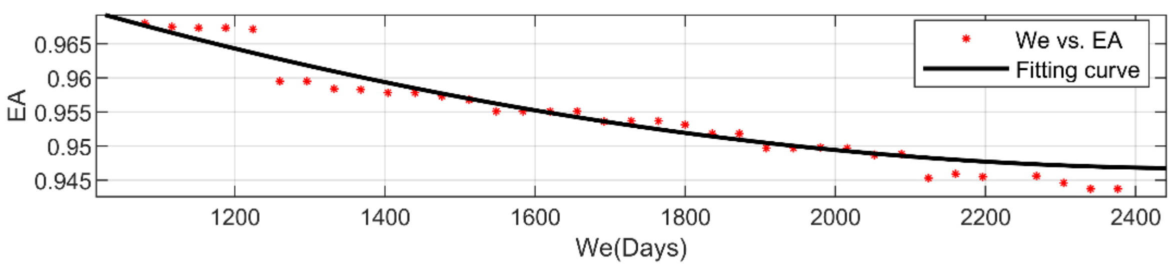

FIGURE 11. Fitting curve of We and EA.

an important issue in the current research.. Cost-effectiveness ratio refers to the ratio of input cost to output benefit. If there is less input and more output, the cost-effectiveness ratio is low; More input and less output result in higher

TABLE 8. Extended warranty scheme.

Scheme	W_E / Days	T_J / Days	EC/CNY	EA	Cost-effectiveness ratio
1	1080	324	7074.4	0.9680	6.77
2	1188	432	8256.6	0.9673	6.16
3	1296	360	9087.4	0.9595	6.06
4	**1404**	**324**	**10413.7**	**0.9578**	**5.99**
5	1512	288	11480.3	0.9568	6.36
6	1620	360	12962.5	0.9551	7.09
7	1728	324	14406.9	0.9537	7.76
8	1836	468	16117.9	0.9518	9.22
9	1944	360	18093.2	0.9497	9.8
10	2052	288	19536.5	0.9487	10.04
11	2160	252	22520.5	0.9459	11.02
12	2268	252	23277.5	0.9456	10.85
13	2376	288	25458.6	0.9437	11.35

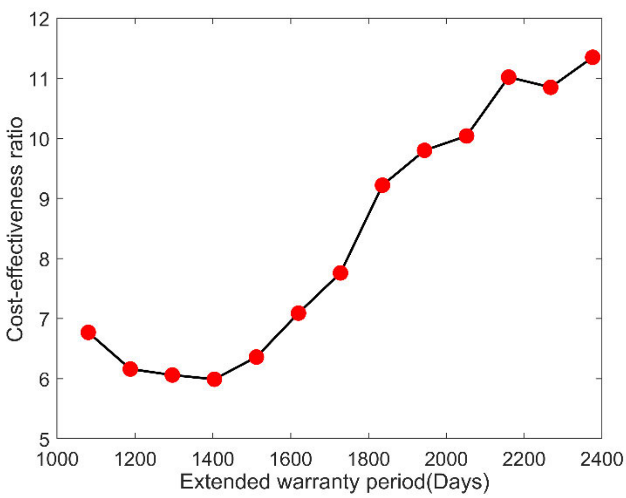

FIGURE 12. Schematic diagram of cost-efficiency ratio change.

cost-effectiveness ratio. In the extended warranty of the warranty object, the warranty cost can be regarded as the input cost, and the availability can be regarded as the output benefit. Therefore, lower cost-effectiveness ratio is the common goal pursued by users and manufacturers. Table 8 shows the cost-effectiveness ratio under different extended warranty schemes. The change of cost-effectiveness ratio with the extended warranty period is shown in the Figure 12.

As can be seen from Fig. 12, with the increase of extended warranty period, the cost-effectiveness ratio first decreases and then increases. The cost-effectiveness ratio is the smallest, and the two-dimensional extended warranty period is (3.9years, 3.9Kh). At this time, the optimal preventive maintenance interval is 324 days. It can be seen that the method proposed in this paper can provide a quantitative analysis method for the determination of the optimal two-dimensional extended warranty period.

V. CONCLUSION

This study proposes a two-dimensional extended warranty approach for a multi-component series system considering economic dependence. We consider imperfect preventive maintenance, minimum maintenance, and grouping maintenance strategies and adopt a combination of free warranty and pro-rata warranty policies. Based on the failure rate function of the two-dimensional approach for a single component, the manufacturer's and user's warranty cost and availability models of single component are established. In the case analysis, the reconnaissance equipment is considered as the research object. Furthermore, the study uses the grid search method to acquire the two-dimensional extended warranty scheme of reconnaissance equipment considering economic dependence. Through comparative and sensitivity analyses, the following conclusions can be derived:

(1) Imperfect preventive maintenance and grouping maintenance strategies can effectively reduce the manufacturer's extended warranty cost and enhance system availability. Thus, these strategies are win–win strategies for both manufacturers and users.

(2) The combination of free warranty and pro-rata warranty policies can effectively expand the manufacturer's profit

margin and solve the problem of high warranty cost during the extended warranty period.

(3) The sensitivity analysis results indicate that this method can provide support for warranty cost calculation and system availability estimation under different extended warranty periods.

Nevertheless, some aspects are worth studying in the future. First, in addition to economic dependence, failure dependence and structure dependence exist among multi components. Developing a two-dimensional extended warranty scheme considering failure dependence and structure dependence between multi components is challenging. Second, grouping maintenance is the only means to combine preventive maintenance for components, which can also be achieved through opportunity maintenance. Opportunity maintenance is a condition-based maintenance method that considers the state of the components, which may be more advantageous in saving warranty costs and enhancing system availability. Therefore, examining the two-dimensional extended warranty strategy based on opportunity maintenance would be interesting.

APPENDIX A

Fitting of two dimensional failure rate function (1)

The model contains unknown parameters. It is necessary to determine the specific expressions of $\lambda_i(t|r)$ and $G_i(r)$ by counting the failure time of samples in a period of time within the warranty period and the usage in case of failure. The general steps are as follows:

Step 1: Select components sample size L (the number of observable components is L). n_i represents the number of failures of the i-th component during the warranty period. $t_{i,j}$ and $u_{i,j}$ respectively represent the time and usage of the j-th failure of the i-th component. Calculate the value of r_i, $r_i = u_{i,j}/t_{i,j}$, $1 \le i \le L$; $1 \le j \le n_i$.

Step 2: Group all r_i into a total of M groups, the m-th interval is represented by $[h_{m-1}, h_m)$. Draw the histogram of r_i, then fit probability density function $g_i(r)$ of utilization rate.

Step 3: $\overline{h_m}$ is the median of interval $[h_{m-1}, h_m)$, and set I_m represents the set of all failure times of components whose utilization rate falls in this interval. The failure rate function $\lambda_i(t|\overline{h_m})$ when the component utilization rate is $\overline{h_m}$ is fitted through the failure time data, as shown in Fig. A.1.

Step 4: Put all the failure rate curves fitted under the specific utilization rate into the three-dimensional space, and fit the three-dimensional surface through these curves to obtain the two-dimensional failure rate function $\lambda_i(t|r)$ about t and r.

APPENDIX B

Fitting of probability density function of utilization (33) of $g(r)$.

Set X as the overall of the utilization rate data. X_1, X_2, X_3, …, X_n are samples from population X, and x_1, x_2, x_3, …, x_n are sample values corresponding to samples. Then the probability that X_1, X_2, X_3, …, X_n equals

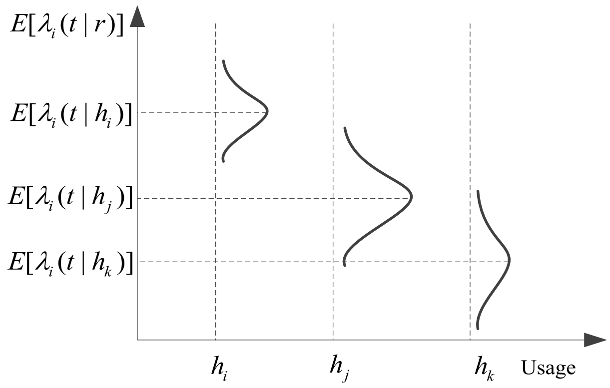

FIGURE 13. Failure rate function of components under different utilization rates.

x_1, x_2, x_3, …, x_n is:

$$P(\delta, \eta) = P(x_1, x_2, x_3, \ldots, x_n; \delta, \eta) = \prod_{i=1}^{n} g(x_i; \delta, \eta)$$

$P(\delta, \eta)$ is the likelihood function of $x_1, x_2, x_3, \ldots, x_n$. Then the sample value $x_1, x_2, x_3, \ldots, x_n$ are fixed. the maximum value of $P(\delta, \eta)$ is found within the value range of δ, η, and take this value $\widehat{\delta}, \widehat{\eta}$ as the estimated value of δ, η. Generally, it is considered that $\widehat{\delta}, \widehat{\eta}$ can be obtained by partial differentiation of likelihood function $P(\delta, \eta)$. Because both likelihood function and log likelihood function can take extreme values at the same value. Then the maximum likelihood estimation of $\widehat{\delta}, \widehat{\eta}$ can also be obtained by partial differentiation of the log likelihood function, that is:

$$\begin{cases} \dfrac{\partial \ln P(\delta, \eta)}{\partial \delta} = 0 \\ \dfrac{\partial \ln P(\delta, \eta)}{\partial \eta} = 0 \end{cases}$$

$\widehat{\delta}, \widehat{\eta}$ is obtained by solving the equations.

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ENZHI DONG received the Bachelor of Arts degree from the Renmin University of China, Beijing, in 2020. He is currently pursuing the M.S. degree in management science and engineering with Army Engineering University, Shijiazhuang. His research interests include equipment support theory, and application and maintenance engineering.

ZHONGHUA CHENG received the M.S. and Ph.D. degrees in weapon system and application engineering from the Ordnance Engineering College, Shijiazhuang, in 2005. He is currently a Professor with Army Engineering University, Shijiazhuang. His research interests include management science and engineering, maintenance engineering, and reliability engineering.

RONGCAI WANG received the B.S. degree in equipment support from Army Engineering University, Shijiazhuang, in 2019, where he is currently pursuing the M.S. degree in management science and engineering. His research interests include management science and engineering, and maintenance engineering.

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