IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received January 30, 2022, accepted February 18, 2022, date of publication March 3, 2022, date of current version March 14, 2022.

Digital Object Identifier 10.1109/ACCESS.2022.3156643

Observer-Based Design of Motion Control Systems in Sliding Mode Control Framework

ASIF ŠABANOVIĆ[1,2], (Life Senior Member, IEEE), AND TARIK UZUNOVIĆ[3], (Senior Member, IEEE)

[1]Faculty of Engineering and Natural Sciences, International University of Sarajevo, 71210 Sarajevo, Bosnia and Herzegovina
[2]Academy of Sciences and Arts of Bosnia and Herzegovina, 71000 Sarajevo, Bosnia and Herzegovina
[3]Department of Automatic Control and Electronics, Faculty of Electrical Engineering, University of Sarajevo, 71000 Sarajevo, Bosnia and Herzegovina

Corresponding author: Tarik Uzunović (tuzunovic@etf.unsa.ba)

This work was supported in part by the Ministry of Civil Affairs of Bosnia and Herzegovina, and in part by the Ministry for Science, Higher Education and Youth of Canton Sarajevo.

ABSTRACT This paper presents a comprehensive treatment of the complex motion control systems in the Sliding Mode Control (SMC) framework. The single and multi degrees of freedom (DOF) plants and applications to haptics and functionally related systems are discussed. The proposed algorithms are based on the application of the equivalent control observer and the convergence term that guaranty stability of the closed loop in a Lyapunov sense and enforces the sliding mode on selected manifolds. Presented SMC design leads to a solution that easily could be modified to include majority of the algorithms presented in the literature.

INDEX TERMS Sliding mode, motion control systems, robot control, motion observers, actuators.

I. INTRODUCTION

The Sliding Mode Control (SMC) is still attracting a significant interest among application engineers and researchers due to the simplicity of design and implementation. The analysis and design of the systems with sliding modes are well presented in several books [1]–[4]. The status overview, survey and tutorial papers [5], [6] have been published covering many aspects of the theory. Among these works, [6] stands aside as an attempt, to give a comprehensive guide for engineers on the design solutions for real-life engineering applications in which authors discuss some analytical problems present in a sliding mode application along with methods to eliminate chattering. In the discrete-time implementation, the sliding mode motion appears for a continuous (in a sense of the discrete-time systems) control input [7], [8]. Applications of the so-called higher order sliding modes [3], [9], the sliding mode observers [10] in the realization of the sliding mode control are attracting attention. Uses of SMC for networked control systems are discussed in [11].

The application of the sliding modes to mechanical systems has a long history. Early attempts sought the direct application of discontinuous control to motion systems with force as control input [12], [13]. This led to claims of the chattering phenomena and numerous attempts to alleviate the effects of chattering induced by switching control input [1], [4], [6], [14]. The direct application of the SMC to motion control systems with force (torque) as a control input is still very popular despite the fact that discontinuous control causes chattering. Discrete time implementation of SMC control does not require discontinuous control [7] but it requires calculation of the equivalent control.

In this paper the aim is to present a SMC based design procedure in motion control tasks, which leads to a controller that could be easily applied and would guaranty the behavior expected from a robust control with sliding modes. The focus is on (i) the implementable design solutions for motion control tasks that could be easily applied for linear as well as nonlinear systems affine in control, (ii) discussion of the generic algorithms while avoiding presentation of many solutions that in essence do not have very significant differences, (iii) show implementation of proposed algorithms to common motion control problems – trajectory tracking, force control, real-world haptic systems, the haptics motion reconstruction, parallel position/force control tasks, to name some. The details are kept to the minimum but still provided to the level that would make easier to follow the design. The interested or novice control engineer is directed to [1]–[4] for the fundamentals of sliding mode control theory and application.

The associate editor coordinating the review of this manuscript and approving it for publication was Alfeu J. Sguarezi Filho.

In this paper, we present a control design methodology that eliminates the need for equivalent control calculation, and thus full information on state, disturbances and plant parameters, or the need for high amplitude of the convergence control term (which is discontinuous in conventional SMC) by applying equivalent control observer. Therefore, a control designer needs to select only the convergence term that guaranties stability of the closed loop. The parameters of this term define the convergence in the closed loop. Therefore, the presented approach is simple for application and tuning. Moreover, it is shown that design approach from this paper is applicable to most of the algorithms presented in the literature.

This paper is organized as follows. In Section 2, preliminaries about sliding modes, control input selection along with the models for different tasks of multi-body systems in configuration and operational space are discussed. Section 3 presents the design of sliding mode controller for different problems in motion control including new formulation of hybrid position-force control. Possible applications are discussed in Section 4. Section 5 concludes the paper.

II. PRELIMINARIES

In most modern discussions on the motion control systems, the actuators with their power supply are treated as a force source so the design of the motion controller takes forces as input and mechanism position (or some function of) as output. The numerous issues – control problems – are discussed within motion control. These problems include position, force and parallel position/force control in robotics [15]–[18], the control of systems that need to establish a certain way of cooperation [19], real-world haptics [20] and the reconstruction of the haptics motion on only master or slave system [21] are some that are mostly discussed in the literature. In this section the properties and design of the control systems with sliding modes along with the mathematical models used in the motion control tasks will be discussed in some details. The aim is to set a background results for the more detailed discussion of the sliding application in motion control systems.

A. SLIDING MODES IN DYNAMIC SYSTEMS

The main ideas of the control enforcing the sliding mode motion will be shown for systems described by

$$\mathbf{M}\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{B}\mathbf{u} + \mathbf{h}(\mathbf{x},t) \tag{1}$$

where $\mathbf{M} \in \Re^{n\times n}$ is a full rank positive definite matrix; $\mathbf{x} \in \Re^{n\times 1}$ is the state vector; $\mathbf{B} \in \Re^{n\times m}$ is the full column rank matrix; $\mathbf{u} \in \Re^{m\times 1}$ is the control input vector; $\mathbf{f}(\mathbf{x}) \in \Re^{n\times 1}$ is a vector function; $\mathbf{h}(\mathbf{x},t) \in \Re^{n\times 1}$ is the exogenous disturbance. All parameters, variables, and disturbances are assumed bounded with known upper bounds consistent with operational requirements. If the vector $\mathbf{f}(\mathbf{x}) \in \Re^{n\times 1}$ could be expressed as a linear function $\mathbf{f}(\mathbf{x}) = \mathbf{A}\mathbf{x}$ then system describes a linear dynamics with pair $(\mathbf{A}, \mathbf{B})$ assumed controllable. As will be shown later in this section the velocity dynamics in motion control systems could be expressed in a similar way as in (1).

In the sliding mode approach, the control input should be selected to constrain the system's motion in the manifold $\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{0} \in \Re^{m\times 1}$. The components of the vector $\boldsymbol{\sigma}(\mathbf{x})$ are assumed continuous with $\mathbf{G} = [\partial\sigma/\partial\mathbf{x}] \in \Re^{m\times n}$ a full row rank matrix.

It is said that system (1) exhibits the sliding mode motion in manifold $\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{0}$ if (i) manifold is reached in a finite time $t = t_0$, and (ii) if for $t \geq t_0$ the system state is constrained to the manifold.

The projection of the system dynamics into the sliding mode manifold is described by

$$\begin{cases} \dot{\boldsymbol{\sigma}} = \left(\mathbf{G}\mathbf{M}^{-1}\mathbf{B}\right)\left(\mathbf{u} - \mathbf{u}^{eq}\right);\ \det\left(\mathbf{G}\mathbf{M}^{-1}\mathbf{B}\right) \neq 0 \\ \mathbf{u}^{eq} = -\left(\mathbf{G}\mathbf{M}^{-1}\mathbf{B}\right)^{-1}\mathbf{G}\mathbf{M}^{-1}\left(\mathbf{f}(\mathbf{x}) + \mathbf{h}(\mathbf{x},t)\right) \end{cases} \tag{2}$$

The $\mathbf{u}^{eq}(\mathbf{x},t)$ stands for so-called equivalent control [3], [4] which enforces zero rate of change of the sliding mode function $\dot{\boldsymbol{\sigma}}(\mathbf{u} = \mathbf{u}^{eq}) = \mathbf{0}$. For the manifold consistent initial conditions, $\boldsymbol{\sigma}(\mathbf{x}(\mathbf{0})) = \mathbf{0}$, the equivalent control (2) applied to the system (1) enforces the $(n - m)$ order sliding mode dynamics

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{M}^{-1}\mathbf{P}\left(\mathbf{f}(\mathbf{x}) + \mathbf{h}(\mathbf{x},t)\right);\ \boldsymbol{\sigma}(\mathbf{x}) = \mathbf{0}; \\ \mathbf{P} = \left[\mathbf{I} - \mathbf{B}\left(\mathbf{G}\mathbf{M}^{-1}\mathbf{B}\right)^{-1}\mathbf{G}\mathbf{M}^{-1}\right] \end{cases} \tag{3}$$

The sliding mode projection operator $\mathbf{P}$ satisfies two very important relationships $\mathbf{P}\mathbf{B} = \mathbf{0}$ and $\mathbf{G}\mathbf{M}^{-1}\mathbf{P} = \mathbf{0}$. In the sliding mode the system is free to move in the tangential plane of the manifold $\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{0}$. Note that if the dimensions of the vector $\mathbf{x}$ and $\mathbf{u}$ are the same,i.e., $\dim(\mathbf{x}) = \dim(\mathbf{u})$, then motion in sliding mode is described by $\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{0}$. If system dynamics and the disturbance satisfy $(\mathbf{f}(\mathbf{x}) + \mathbf{h}(\mathbf{x},t)) = \boldsymbol{\zeta}(\mathbf{x},t) + \mathbf{B}\boldsymbol{\eta}$ then dynamics (3) reduces to $\dot{\mathbf{x}} = \mathbf{M}^{-1}\mathbf{P}\boldsymbol{\zeta}(\mathbf{x},t);\ \boldsymbol{\sigma}(\mathbf{x}) = \mathbf{0}$. The motion in sliding mode does not depend on the vector $\boldsymbol{\eta}$.

B. CONTROL INPUT SELECTION

The control input should be designed to enforce (i) the finite time convergence to and (ii) the stability of the motion in sliding manifold. The specific structure of the control action is not part of the specification. If $\mathbf{u} = \mathbf{u}^{eq}$ is applied as the control input then motion is constrained to a manifold in which $\boldsymbol{\sigma}$ is constant, $\boldsymbol{\sigma} = const$, thus the additional component should be added to ensure the convergence to and stability of the sliding mode manifold. Let control enforcing sliding mode in manifold is selected as

$$\mathbf{u} = \mathbf{u}^{eq} - \rho\boldsymbol{\Omega}^{-1}\boldsymbol{\Psi}(\boldsymbol{\sigma}) \tag{4}$$

where $\boldsymbol{\Omega} = \left(\mathbf{G}\mathbf{M}^{-1}\mathbf{B}\right)$. The control (4) would enforce sliding mode in manifold if the vector valued function $\boldsymbol{\Psi}(\boldsymbol{\sigma})$ is selected so the derivative $\dot{V}\big|_{\sigma\neq 0} = \boldsymbol{\sigma}^{\mathrm{T}}\dot{\boldsymbol{\sigma}}$ of the Lyapunov function candidate $V = 0.5\boldsymbol{\sigma}^{\mathrm{T}}\boldsymbol{\sigma}$, satisfies

$\dot{V}\big|_{\sigma\neq 0} = \boldsymbol{\sigma}^{\mathrm{T}}\dot{\boldsymbol{\sigma}} = -\rho\boldsymbol{\sigma}^{\mathrm{T}}\boldsymbol{\Psi}(\boldsymbol{\sigma}) < 0$. Then for $\boldsymbol{\sigma} \neq \mathbf{0}$ the dynamics of sliding mode function becomes

$$\boldsymbol{\sigma}^{\mathrm{T}}\left(\dot{\boldsymbol{\sigma}} + \rho\boldsymbol{\Psi}(\boldsymbol{\sigma})\right) = \mathbf{0} \tag{5}$$

In sliding mode $\dot{V}\big|_{\boldsymbol{\sigma}=\mathbf{0}} = 0$ and $\mathbf{u} = \mathbf{u}^{eq}$ so the term $\rho\boldsymbol{\Omega}^{-1}\boldsymbol{\Psi}(\boldsymbol{\sigma})$ should not contribute to motion in manifold. If this term is discontinuous then in manifold $\boldsymbol{\sigma} = \mathbf{0}$ its average value must be zero – thus switching will occur. These conditions do not specify if control is continuous or discontinuous nor the nature of the convergence process. In continuous time design, the finite time convergence to the manifold could be achieved if the control is either discontinuous or is continuous non-Lipschitz function [1]–[4]. In discrete time design the sliding mode could be achieved with continuous control input [4], [7], [8]. A variety of algorithms could be generated by using different methods for equivalent control estimation and/or for different selection of the convergence term. Our goal is not listing many of them here, but rather to show generic structures that are easy to implement.

The need for the equivalent control calculation, and thus full information on state, disturbances and plant parameters, could be avoided by estimation of the equivalent control from dynamics (2). If pair $(\mathbf{u}, \boldsymbol{\sigma})$ is measured and the change of the equivalent control is slow in comparison with observer dynamics, the equivalent control could be estimated by

$$\left.\begin{aligned}\dot{\boldsymbol{\xi}} &= -\mathbf{L}\boldsymbol{\xi} + \mathbf{L}\left(\mathbf{u} + \mathbf{L}\boldsymbol{\Omega}^{-1}\sigma\right)\\ \boldsymbol{\xi} &= \hat{\mathbf{u}}^{eq} + \mathbf{L}\boldsymbol{\Omega}^{-1}\boldsymbol{\sigma}\end{aligned}\right\} \Rightarrow \dot{\hat{\mathbf{u}}}^{eq} + \mathbf{L}\hat{\mathbf{u}}^{eq} = \mathbf{L}\mathbf{u}^{eq} \tag{6}$$

Here $\boldsymbol{\xi}$ is an auxiliary variable, $diag(\mathbf{L}) = l_{ii} > 0$ is a design parameter. Insertion of the estimated equivalent control into (4) the control input becomes

$$\mathbf{u} = \hat{\mathbf{u}}^{eq} - \rho\boldsymbol{\Omega}^{-1}\boldsymbol{\Psi}(\boldsymbol{\sigma}), \quad \boldsymbol{\Omega} = \mathbf{G}\mathbf{M}^{-1}\mathbf{B} \tag{7}$$

If (7) is applied as the control input by selecting observer gain $\mathbf{L}$ such that the dynamic separation conditions met for the augmented system (2), (6), the stability of the solution $\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{0}$ is guaranteed and the sliding mode dynamics is then described as in (3).

Before starting the discussion on the sliding modes in motion control let us first look at dynamics of the systems and the tasks.

C. MODELS

1) CONFIGURATION SPACE

dynamics of a rigid fully actuated multi-body n-DOF system is given by [4]

$$\mathbf{A}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{b}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) + \boldsymbol{\tau}^{ext} = \boldsymbol{\tau}. \tag{8}$$

Here $\mathbf{q}$ stands for the system configuration vector; $\mathbf{A}(\mathbf{q})$ is a positive definite matrix; $\mathbf{b}(\mathbf{q}, \dot{\mathbf{q}})$ stands for Coriolis forces, viscous friction and centripetal forces; $\mathbf{g}(\mathbf{q})$ stands for gravity terms; $\boldsymbol{\tau} \in \Re^{nx1}$ stands for generalized joint forces and $\boldsymbol{\tau}^{ext}$ stands for the external forces projection to configuration space. The system variables and parameters are bounded with known lower and upper bounds consistent with the operational domain D_0. Expressing $\mathbf{A}(\mathbf{q}) = \mathbf{A}_n(\mathbf{q}) + \Delta\mathbf{A}(\mathbf{q})$, with a nominal value $\mathbf{A}_n(\mathbf{q})$ and a bounded uncertainty $\Delta\mathbf{A}(\mathbf{q})$, model (8) could be rewritten as

$$\begin{cases}\ddot{\mathbf{q}} = \mathbf{A}_n^{-1}\left(\boldsymbol{\tau} - \boldsymbol{\tau}^{dis}\right)\\ \boldsymbol{\tau}^{dis} = \mathbf{b}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) + \Delta\mathbf{A}(\mathbf{q})\ddot{\mathbf{q}} + \boldsymbol{\tau}^{ext}\end{cases} \tag{9}$$

2) THE PROJECTION

$\ddot{\boldsymbol{\varphi}}(\mathbf{q}) = \boldsymbol{\Phi}_q\ddot{\mathbf{q}} + \dot{\boldsymbol{\Phi}}_q\dot{\mathbf{q}}$ of the dynamics (8) to **constraint manifold** $\boldsymbol{\varphi}(\mathbf{q}) = \mathbf{0} \in \Re^{p\times 1}$, $p < n$, can be written

$$\begin{cases}\ddot{\boldsymbol{\varphi}} = \mathbf{M}_\varphi^{-1}\left(\mathbf{f}_\varphi - \mathbf{f}_\varphi^{dis}\right); \mathbf{M}_\varphi^{-1} = \left(\boldsymbol{\Phi}_q\mathbf{A}_n^{-1}\boldsymbol{\Phi}_q^{\mathrm{T}}\right)\\ \mathbf{f}_x^{dis} = \boldsymbol{\Phi}_q\mathbf{A}_n^{-1}\boldsymbol{\tau}^{dis} - \boldsymbol{\Phi}_q\mathbf{A}_n^{-1}\boldsymbol{\Gamma}_\varphi^{\mathrm{T}}\boldsymbol{\tau}_0 - \mathbf{M}_\varphi\dot{\boldsymbol{\Phi}}_q\dot{\mathbf{q}}\\ \boldsymbol{\tau} = \boldsymbol{\Phi}_q^{\mathrm{T}}\mathbf{f}_\varphi + \boldsymbol{\Gamma}_\varphi^{\mathrm{T}}\boldsymbol{\tau}_0.\end{cases} \tag{10}$$

Here $\boldsymbol{\Phi}_q = \left(\partial\boldsymbol{\varphi}/\partial\mathbf{q}\right)$ is constraint Jacobian, $\mathbf{f}_\varphi, \mathbf{f}_\varphi^{dis}$ are constraint space control and disturbance force respectively, $\boldsymbol{\tau}^{dis}$ is given in (9), $\boldsymbol{\tau}^{ext} = \boldsymbol{\Phi}_q^{\mathrm{T}}\boldsymbol{\lambda}$ is the projection of interaction force $\boldsymbol{\lambda}$ into configuration space. Due to the redundancy of the constraint $p < n$ mapping the constraint control $\mathbf{f}_\varphi$ and $\boldsymbol{\tau}_0 \in \Re^{n\times 1}$ - the arbitrary configuration space generalized force - into the configuration space is given by $\boldsymbol{\tau} = \boldsymbol{\Phi}_q^{\mathrm{T}}\mathbf{f}_\varphi + \boldsymbol{\Gamma}_\varphi^{\mathrm{T}}\boldsymbol{\tau}_0$. Selection of matrix $\boldsymbol{\Gamma}_\varphi^{\mathrm{T}}$ will be discussed later in the text.

3) THE DYNAMICS OF A REDUNDANT TASK

$\mathbf{x}(\mathbf{q}) \in \Re^{m\times 1}$, $m < n$, governed by $\ddot{\mathbf{x}} = \mathbf{J}\ddot{\mathbf{q}} + \dot{\mathbf{J}}\dot{\mathbf{q}}$ with Jacobian $\mathbf{J} = (\partial\mathbf{x}/\partial\mathbf{q})$ can be, for system (8), expressed as

$$\begin{cases}\ddot{\mathbf{x}} = \mathbf{M}_n^{-1}\left(\mathbf{f}_x - \mathbf{f}_x^{dis}\right);\ \mathbf{M}_n^{-1} = \left(\mathbf{J}\mathbf{A}_n^{-1}\mathbf{J}^{\mathrm{T}}\right)\\ \mathbf{f}_x^{dis} = \mathbf{M}_n\mathbf{J}\mathbf{A}_n^{-1}\boldsymbol{\tau}^{dis} - \mathbf{M}_n\mathbf{J}\mathbf{A}_n^{-1}\boldsymbol{\Gamma}_x^{\mathrm{T}}\boldsymbol{\tau}_0 - \mathbf{M}_n\dot{\mathbf{J}}\dot{\mathbf{q}}\\ \boldsymbol{\tau} = \mathbf{J}^{\mathrm{T}}\mathbf{f}_x + \boldsymbol{\Gamma}_x^{\mathrm{T}}\boldsymbol{\tau}_0\end{cases} \tag{11}$$

with the operational space control force $\mathbf{f}_x$, disturbance $\mathbf{f}_x^{dis}$, respectively. The disturbance $\boldsymbol{\tau}^{dis}$ is given in (9). Mapping operational space control force to configuration space is given by $\boldsymbol{\tau} = \mathbf{J}^{\mathrm{T}}\mathbf{f}_x + \boldsymbol{\Gamma}_x^{\mathrm{T}}\boldsymbol{\tau}_0$ where $\boldsymbol{\tau}_0 \in \Re^{n\times 1}$ is an arbitrary configuration space force vector. Selection of matrix $\boldsymbol{\Gamma}_x^{\mathrm{T}}$ would be discussed later.

4) THE FORCE DUE TO INTERACTION

with environment at $\boldsymbol{\varphi}_e(\mathbf{x}) = \mathbf{0} \in \Re^{r\times 1}$, $r < m < n$ in operational space is often modeled as a spring-damper $\boldsymbol{\lambda}(\mathbf{x}) = D_e\dot{\mathbf{e}}_\varphi + K_e\mathbf{e}_\varphi$ system with parameters $D_e, K_e > 0$ and $\mathbf{e}_\varphi = \boldsymbol{\varphi}(\mathbf{x}) - \boldsymbol{\varphi}_e(\mathbf{x})$. The motion in the normal direction to surface $\boldsymbol{\varphi}_e(\mathbf{x}) = \mathbf{0}$ would result in interaction force $\boldsymbol{\lambda}$ while the system is free to move in the tangential plane. The dynamics of the interaction force modeled as spring-damper, could be expressed as

$$\begin{cases}\dot{\boldsymbol{\lambda}}^*(\mathbf{x}) = \mathbf{M}_\lambda^{-1}\left(\mathbf{f}_\lambda - \mathbf{f}_\lambda^{dis}\right); \mathbf{M}_\lambda^{-1} = \left(\boldsymbol{\Phi}_x\mathbf{M}_n^{-1}\boldsymbol{\Phi}_x^{\mathrm{T}}\right)\\ \mathbf{f}_\lambda^{dis} = \mathbf{M}_\lambda\boldsymbol{\Phi}_x\mathbf{M}_n^{-1}\mathbf{f}_x^{dis} + \mathbf{M}_\lambda(\dot{\boldsymbol{\varphi}}_e - \boldsymbol{\Phi}_x\mathbf{M}_n^{-1}\boldsymbol{\Gamma}_{\lambda x}^{\mathrm{T}}\mathbf{f}_{xo}\\ \quad - D_e^{-1}K_e\dot{\mathbf{e}}_\varphi - \dot{\boldsymbol{\Phi}}_x\dot{\mathbf{x}})\\ \mathbf{f}_x = \boldsymbol{\Phi}_x^{\mathrm{T}}\mathbf{f}_\lambda + \boldsymbol{\Gamma}_{\lambda x}^{\mathrm{T}}\mathbf{f}_{xo}\end{cases} \tag{12}$$

Here $\dot{\boldsymbol{\lambda}}^* = D_e^{-1}\dot{\boldsymbol{\lambda}}(\mathbf{x})$ is scaled interaction force; $\boldsymbol{\Phi}_x = (\partial\boldsymbol{\varphi}/\partial\mathbf{x})$ is the constraint Jacobian; $\mathbf{f}_\lambda$, $\mathbf{f}_\lambda^{dis}$ are control and disturbance forces in constraint space, respectively; and $\mathbf{f}_x^{dis}$ is given in (11). Mapping of the constraint control forces into the operational space is given by $\mathbf{f}_x = \boldsymbol{\Phi}_x^{\mathrm{T}}\mathbf{f}_\lambda + \boldsymbol{\Gamma}_{\lambda x}^{\mathrm{T}}\mathbf{f}_{xo}$, where $\mathbf{f}_{xo}$ is an arbitrary operational space force.

A specific situation appears if a set of systems needs to establish some functional relationship. As example let two single DOF systems are required to maintain relationship defined by a vector valued function $\boldsymbol{\xi}^{\mathrm{T}}(q_1, q_2) = [\xi_1\ \xi_2]$ and that equation $\boldsymbol{\xi}(q_1, q_2) = \mathbf{0}$ has unique solution for $q_1\&q_2$. The dynamics $\ddot{\boldsymbol{\xi}} = \boldsymbol{\Psi}\ddot{\mathbf{q}} + \dot{\boldsymbol{\Psi}}\dot{\mathbf{q}}$, $\Psi = \partial\boldsymbol{\xi}/\partial\mathbf{q}$ could be written as

$$\begin{cases} \ddot{\boldsymbol{\xi}} = \boldsymbol{\Psi}\mathbf{A}_n^{-1}\Psi^{\mathrm{T}}\left(\boldsymbol{\tau}_\xi - \boldsymbol{\tau}_\xi^{dis}\right); \mathbf{A}_n = diag\,(a_{n1}, a_{n2}) \\ \boldsymbol{\tau}_\xi^{dis} = \left(\Psi\mathbf{A}_n^{-1}\Psi^{\mathrm{T}}\right)^{-1}\Psi\mathbf{A}_n^{-1}\boldsymbol{\tau}_{1,2}^{dis} - \left(\Psi\mathbf{A}_n^{-1}\Psi^{\mathrm{T}}\right)^{-1}\dot{\boldsymbol{\Psi}}\dot{\mathbf{q}} \\ \boldsymbol{\tau}_\xi = \left[\tau_{\xi 1}\ \tau_{\xi 2}\right], \boldsymbol{\tau}_{1,2}^{dis} = \left[\tau_1^{dis}\ \tau_2^{dis}\right] \end{cases} \tag{13}$$

Models (9)-(13) stand for different projections of the dynamics (8) and all are presented as structure $\ddot{\mathbf{R}} = \boldsymbol{\Omega}\left(\mathbf{f} - \mathbf{f}^{dis}\right)$ where $\boldsymbol{\Omega}$ is a full rank matrix of appropriate dimension, $\mathbf{f}^{dis}$ stands for generalized matched disturbance, and $\mathbf{f}$ is the control force. If the control tasks could be defined in a suitable way, structurally the same controller could be applied to all problems. Note here that generalized disturbance for systems (9)-(13) could be estimated as an unknown input by applying simple disturbance observer [22].

Models (9), (10), (11) and (13) could be written in the form

$$\left.\begin{matrix} \ddot{\mathbf{z}} = \boldsymbol{\Omega}_z\left(\mathbf{f}_z - \mathbf{f}_z^{dis}\right) \\ \Downarrow \\ \dot{\mathbf{z}} = \mathbf{v}_z \\ \dot{\mathbf{v}}_z = \boldsymbol{\Omega}_z\left(\mathbf{f}_z - \mathbf{f}_z^{dis}\right) \end{matrix}\right\} \begin{matrix} \mathbf{z} = \mathbf{q}\ or\ \boldsymbol{\varphi}\ or\ \mathbf{x}\ or\ \boldsymbol{\xi} \\ \mathbf{f}_z = \boldsymbol{\tau}\ or\ \mathbf{f}_\varphi\ or\ \mathbf{f}_x\ or\ \mathbf{f}_\xi \\ \mathbf{f}_z^{dis} = \boldsymbol{\tau}^{dis}\ or\ \mathbf{f}_\varphi^{dis}\ or\ \mathbf{f}_x^{dis}\ or\ \mathbf{f}_\xi^{dis} \\ \boldsymbol{\Omega}_z = \mathbf{A}_\varphi^{-1}\ or\ \mathbf{M}_\varphi^{-1}\ or\ \mathbf{M}_n^{-1}\ or\ \mathbf{M}_\xi^{-1} \end{matrix} \tag{14}$$

where $\dim(\mathbf{z}) = \dim(\mathbf{f}_z)$ and $\boldsymbol{\Omega}_z$ is a square full rank matrix. The force dynamics $\dot{\boldsymbol{\lambda}}^* = \boldsymbol{\Omega}_\lambda\left(\mathbf{f}_\lambda - \mathbf{f}_\lambda^{dis}\right)$ is already in the same form as the $\dot{\mathbf{v}}_z = \boldsymbol{\Omega}_z\left(\mathbf{f}_z - \mathbf{f}_z^{dis}\right)$.

D. FORMULATION OF CONTROL TASKS

The motion control issues are related to the problems of tracking desired trajectory and modifying motion to obtain desired interaction with environment. In literature, these issues are usually treated as separate [12], [13], [15], [16] but in real, to obtain natural human-like behavior the motion without interaction and motion in contact with environment should be combined and a seamless transition between the two is needed [21].

From dynamics (14) the tracking error $\mathbf{e}_z = \mathbf{z} - \mathbf{z}^{ref}$ would converge to zero if $\mathbf{v}_z = \dot{\mathbf{z}}^{ref} - \mathbf{C}\mathbf{e}_z$ and control is designed to enforce sliding mode on manifold $\boldsymbol{\sigma}_z = \dot{\mathbf{e}}_z + \mathbf{C}\mathbf{e}_z = \mathbf{0}$. Here $\mathbf{C}$ is a square full rank positive definite (usually a diagonal) matrix.

When enforcement of sliding mode in selected manifold is performed, then in sliding mode dynamics $\boldsymbol{\sigma}_z = \dot{\mathbf{e}}_z + \mathbf{C}\mathbf{e}_z = \mathbf{0}$ governs the control error and asymptotic convergence is guaranteed. If sliding mode is established for $\boldsymbol{\sigma}_z = \boldsymbol{\eta} \neq \mathbf{0}$ then tracking error converges to $\mathbf{e}_z \to \mathbf{C}^{-1}\boldsymbol{\eta}$. This is an interesting feature that allows modification of the system behavior in sliding mode. As an example tracking of interaction force $\boldsymbol{\lambda} = D_e\dot{\mathbf{e}}_\varphi + K_e\mathbf{e}_\varphi$ with $\mathbf{e}_\varphi = \boldsymbol{\varphi} - \boldsymbol{\varphi}_e$, position of obstacle $\boldsymbol{\varphi}_e$ and reference force $\boldsymbol{\lambda}^{ref}$ yields dynamics $\dot{\mathbf{e}}_\varphi + D_e^{-1}K_e\mathbf{e}_\varphi = D_e^{-1}\boldsymbol{\lambda}^{ref}$ thus the environment tracking error depend on the force reference.

As discussed above the sliding mode manifold for trajectory tracking could be formulated as $(m \le n)$

$$S_z = \left\{\mathbf{q}, \dot{\mathbf{q}} \in \Re^n : \boldsymbol{\sigma}_z = \dot{\mathbf{e}}_z + \mathbf{C}\mathbf{e}_z = \mathbf{0}, \boldsymbol{\sigma}_z \in \Re^{m\times 1}\right\} \tag{15}$$

The sliding mode manifold for force control could be defined as $(p \le m \le n)$

$$S_\lambda = \left\{\mathbf{q}, \dot{\mathbf{q}} \in \Re^n : \boldsymbol{\sigma}_\lambda = \boldsymbol{\lambda} - \boldsymbol{\lambda}^{ref} = \mathbf{0}, \boldsymbol{\sigma}_\lambda \in \Re^{p\times 1}\right\} \tag{16}$$

These formulations could be applied for redundant as well as non-redundant tasks. Motion in manifold (15) or (16) guarantees tracking of the position or force but still does not secure the interaction between the two modes of motion. Establishing a functional relationship between motion and force control tasks to mimic a natural human-like motion in which the motion along desired trajectory is modified when controlled system interacts with obstacle requires the selection of the sliding mode function as a function of both motion and interaction force tracking, as in

$$S_{z\lambda} = \left\{\mathbf{q}, \dot{\mathbf{q}} \in \Re^n : \boldsymbol{\sigma}_z + \mathbf{H}\boldsymbol{\lambda} = \mathbf{0}, \boldsymbol{\sigma}_z \in \Re^m, \boldsymbol{\lambda} \in \Re^p\right\} \tag{17}$$

where matrix $\mathbf{H}$ defines the distribution of the interaction force to the components of the generalized error - a mapping between the force and generalized error spaces. Implementation of the control enforcing sliding mode in manifold (17) yields $\boldsymbol{\sigma}_z = -\mathbf{H}\boldsymbol{\lambda}$. It results in a pure trajectory tracking if $\boldsymbol{\lambda} = \mathbf{0}$. If $\boldsymbol{\lambda} \neq \mathbf{0}$ then tracking, in the axes determined by the projection matrix $\mathbf{H}$, will be modified and balance between the tracking error $\mathbf{e}_z$ and the interaction force $\boldsymbol{\lambda}$ will be established. In order to maintain desired interaction force the trajectory tracking sliding mode functions should be bounded to satisfy the relationship $\boldsymbol{\sigma}_z^* = -\mathbf{H}\boldsymbol{\lambda}^{ref}$. The control task specification as in (17) includes both, the trajectory tracking and the force control and could be treated as general motion control task formulation.

III. CONTROL SYSTEM DESIGN

The dynamics of sliding mode function (15), (16) and (17) for generalized plant dynamics (14) can be expressed as

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{\Omega}\left(\mathbf{f} - \mathbf{f}^{eq}\right) \tag{18}$$

where $\boldsymbol{\sigma} = \boldsymbol{\sigma}_z$ or $\boldsymbol{\sigma}_\lambda$ or $\boldsymbol{\sigma}_{z\lambda}$ stands sliding mode function defined in (15), (16) or (17); $\mathbf{f}^{eq}$ stands for a continuous vector valued function – so-called equivalent control. The control input $\mathbf{f} = \mathbf{f}_z$ and control distribution matrix $\boldsymbol{\Omega} = \boldsymbol{\Omega}_z$ are defined as in (14).

As shown in Section 2 control input $\mathbf{f} = \mathbf{f}^{eq} - \rho\mathbf{\Omega}^{-1}\mathbf{\Psi}(\boldsymbol{\sigma})$ with properly selected vector valued function $\mathbf{\Psi}(\boldsymbol{\sigma})$ would enforce sliding mode in manifold $\boldsymbol{\sigma} = \mathbf{0}$. For $\boldsymbol{\sigma} \neq \mathbf{0}$ the dynamics of sliding mode function becomes

$$\dot{\boldsymbol{\sigma}} + \rho\mathbf{\Psi}(\boldsymbol{\sigma}) = \mathbf{0} \tag{19}$$

If pair $(\boldsymbol{\sigma}, f)$ is measured, use of the observer (6) allows estimation of $\hat{\mathbf{f}}^{eq}$ and control input implementation as

$$f = \hat{\mathbf{f}}^{eq} - \rho\mathbf{\Omega}^{-1}\mathbf{\Psi}(\boldsymbol{\sigma}) \tag{20}$$

Control (20) yields the dynamics of sliding mode function $\dot{\boldsymbol{\sigma}} + \rho\mathbf{\Psi}(\boldsymbol{\sigma}) = \mathbf{\Omega}^{-1}\left(\hat{\mathbf{f}}^{eq} - \mathbf{f}^{eq}\right)$ and consequently, with proper separation of the observer and sliding mode function dynamics the $\hat{\mathbf{f}}^{eq} \rightarrow \mathbf{f}^{eq}$ and the sliding mode is enforced in manifold $\boldsymbol{\sigma} = \mathbf{0}$.

The selection of the convergence term is a design parameter and could be discontinuous or continuous. Some of examples of the implementation of algorithm (20) that enforces sliding mode in manifold $\boldsymbol{\sigma} = \mathbf{0}$ are shown below but many more could be derived taking $\mathbf{\Psi}(\boldsymbol{\sigma})$ as discontinuous or non-Lipschitz function:

- $\mathbf{f} = -M sign(\boldsymbol{\sigma}), M > 0$ and $\mathbf{f} = -M\boldsymbol{\sigma}/\|\boldsymbol{\sigma}\|$ are often used in early works on robotic control with sliding modes. It may cause chattering and is not recommended in the motion control systems;
- $\mathbf{f} = sat\left(\hat{\mathbf{f}}^{eq} - \rho\mathbf{\Omega}^{-1}\boldsymbol{\sigma} - \rho_1\mathbf{\Omega}^{-1}sign(\boldsymbol{\sigma})\right)$ - a combination of estimation of equivalent control and discontinuity term. Selection of small $\rho_1 > 0$ may minimize chattering;
- Selection of $\mathbf{\Psi}(\boldsymbol{\sigma})$ as non-Lipchitz function, for example $\mathbf{f} = sat\left(\hat{\mathbf{f}}^{eq} - \rho\mathbf{\Omega}^{-1}\|\boldsymbol{\sigma}\|^{\eta} sign(\boldsymbol{\sigma})\right)$, $0 < \eta < 1$ - would lead to finite time convergence [3]. Selection of small η may cause chattering;
- control $\mathbf{f} = sat\left(\hat{\mathbf{f}}^{eq} - \rho\mathbf{\Omega}^{-1}\boldsymbol{\sigma}\right)$ in continuous time would guaranty asymptotic convergence and strictly speaking it would not enforce sliding mode motion. In the discrete-time, with sampling interval T, application of $\mathbf{f}(k+1) = sat\left(\hat{\mathbf{f}}^{eq}(k) - \rho\mathbf{\Omega}^{-1}(k)\boldsymbol{\sigma}(k)\right)$ yields sliding mode function dynamics $\boldsymbol{\sigma}(k+1) = (I - T\rho)\boldsymbol{\sigma}(k)$ and sliding mode could be attained [4], [7], [8].

A variety of algorithms could be generated by using different methods for equivalent control estimation or for different selection of the convergence term. Our goal is not listing many of them here, but rather to show generic structures that are easy to implement. The solutions which include the equivalent control observer are in general easier for application because: (i) the observer design uses only the control error, does not require all system parameters and nonlinear terms (as shown in equation (6); (ii) the convergence term in the control input is tuned for a system in which estimated equivalent control essentially compensates the disturbances thus the tuning is made for the nominal system $\dot{\boldsymbol{\sigma}} = -\rho\mathbf{\Psi}(\boldsymbol{\sigma})$; (iii) the chattering in ideal case is fully eliminated due to the fact that convergence term is zero when motion reaches sliding mode manifold (as follows from $\dot{\boldsymbol{\sigma}} = -\rho\mathbf{\Psi}(\boldsymbol{\sigma})$ and for $\boldsymbol{\sigma} = \mathbf{0} \Rightarrow \dot{\boldsymbol{\sigma}} = \mathbf{0} \Rightarrow \rho\mathbf{\Psi}(\boldsymbol{\sigma}) = \mathbf{0}$ as shown in the stability analysis. In real systems, the chattering depends on the measurement noise propagation in control error and in equivalent control estimation channels (usually observer possesses the low pass filter features so noise is reduced).

IV. APPLICATIONS

In this section the application of algorithm (20) for different tasks in the configuration and operation space will be discussed. The presentation will be concentrated in selection of the sliding mode manifold, the determination of the structure of $\mathbf{f}^{eq}$ and the derivation of the system dynamics with sliding mode in selected manifold. For redundant tasks the configuration space control force will be expressed as $\boldsymbol{\tau} = \mathbf{\Lambda}^{\mathrm{T}}\mathbf{f} + \mathbf{\Gamma}^{\mathrm{T}}\boldsymbol{\tau}_0$ with $\mathbf{\Lambda}$ being the appropriate Jacobian matrix. The structure of the matrix $\mathbf{\Gamma}$ will be determined in each of the tasks.

A. CONFIGURATION SPACE CONTROL

1) THE CONFIGURATION SPACE TRAJECTORY TRACKING

error $\mathbf{e}_q = \mathbf{q} - \mathbf{q}^{ref}$, where $\mathbf{q}^{ref}$ is a reference, will converge to zero if system (9) is forced to exhibit sliding mode motion in manifold $\boldsymbol{\sigma}_q = \dot{\mathbf{e}}_q + \mathbf{C}_q\mathbf{e}_q = \mathbf{0}$, $\boldsymbol{\sigma}_q \in \Re^{n\times 1}$. $\mathbf{C}_q \in \Re^{n\times n}$ is positive definite full rank matrix (usually selected diagonal). The configuration space dynamics (9) yields the dynamics of the sliding mode function $\boldsymbol{\sigma}_q$ as

$$\begin{cases} \dot{\boldsymbol{\sigma}}_q = \mathbf{A}_n^{-1}\left(\boldsymbol{\tau} - \boldsymbol{\tau}^{eq}\right) \\ \boldsymbol{\tau}^{eq} = \boldsymbol{\tau}^{dis} + \mathbf{A}_n\left(\ddot{\mathbf{q}}^{ref} - \mathbf{C}_q\dot{\mathbf{e}}_q\right) \end{cases} \tag{21}$$

where $\boldsymbol{\tau}^{eq}$ stands for equivalent control, $\boldsymbol{\tau}^{dis}$ is defined in (9). Insertion $\boldsymbol{\tau} = \hat{\boldsymbol{\tau}}^{eq} - \rho\mathbf{A}_n\mathbf{\Psi}\left(\boldsymbol{\sigma}_q\right)$, into configuration space dynamics (8) yields the closed loop sliding mode dynamics $\dot{\boldsymbol{\sigma}}_q + \rho\mathbf{\Psi}\left(\boldsymbol{\sigma}_q\right) = \mathbf{A}_n^{-1}\left(\hat{\boldsymbol{\tau}}^{eq} - \boldsymbol{\tau}^{eq}\right)$. The equivalent control could be estimated by an observer as in (6). Consequently if $\hat{\boldsymbol{\tau}}^{eq} \rightarrow \boldsymbol{\tau}^{eq}$ then sliding mode closed loop dynamics is governed by $\dot{\mathbf{e}}_q + \mathbf{C}_q\mathbf{e}_q = \mathbf{0}$ and $\mathbf{e}_q \rightarrow \mathbf{0}$.

2) THE CONFIGURATION SPACE STATE CONSTRAINT

$\boldsymbol{\varphi}(\mathbf{q}) = \mathbf{0} \in \Re^{p\times 1}$, $p < n$, with $\mathbf{\Phi}_q = (\partial\boldsymbol{\varphi}/\partial\mathbf{q}) \in \Re^{p\times n}$ as a full row rank constraint Jacobian, could be maintained by input force if sliding mode is enforced on sliding mode manifold $\boldsymbol{\sigma}_\varphi = \dot{\boldsymbol{\varphi}} + \mathbf{C}_\varphi\boldsymbol{\varphi} = \mathbf{0} \in \Re^{p\times 1}$ where $\mathbf{C}_\varphi \in \Re^{p\times p}$ is a full rank matrix. The dynamics of sliding mode function with constraint dynamics as in (10) becomes

$$\begin{cases} \dot{\boldsymbol{\sigma}}_\varphi = \mathbf{M}_\varphi^{-1}\left(\mathbf{f}_\varphi - \mathbf{f}_\varphi^{eq}\right) \\ \mathbf{f}_\varphi^{eq} = \mathbf{f}_\varphi^{dis} - \mathbf{M}_\varphi\mathbf{C}_\varphi\dot{\boldsymbol{\varphi}} \end{cases} \tag{22}$$

$\mathbf{f}_\varphi^{dis}$ and $\mathbf{M}_\varphi$ are defined in (10) and $\boldsymbol{\tau}_0 \in \Re^{n\times 1}$ is arbitrary force vector in configuration space. Control $\mathbf{f}_\varphi = \hat{\mathbf{f}}_\varphi^{eq} - \rho\mathbf{M}_\varphi\mathbf{\Psi}\left(\boldsymbol{\sigma}_\varphi\right)$ enforces sliding mode motion in manifold $\boldsymbol{\sigma}_\varphi = \mathbf{0}$ and from $\dot{\boldsymbol{\varphi}} + \mathbf{C}_\varphi\boldsymbol{\varphi} = \mathbf{0}$ the convergence $\boldsymbol{\varphi}(\mathbf{q}) \rightarrow \mathbf{0}$ is guaranteed. Insertion $\boldsymbol{\tau} = \mathbf{\Phi}_q^{\mathrm{T}}\mathbf{f}_\varphi^{eq} + \mathbf{\Gamma}_\varphi^{\mathrm{T}}\boldsymbol{\tau}_0$ into (8) yields system dynamics in sliding mode as

$$\begin{cases} \mathbf{\Gamma}_{\varphi}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{A}_n^{-1}\mathbf{\Gamma}_{\varphi}^{\mathrm{T}}\boldsymbol{\tau}^{dis} = \mathbf{A}_n^{-1}\mathbf{\Gamma}_{\varphi}^{\mathrm{T}}\boldsymbol{\tau}_0 \\ \boldsymbol{\sigma}_{\varphi}(\mathbf{q}) = \mathbf{\Phi}_q\dot{\mathbf{q}} + \mathbf{C}_{\varphi}\varphi(\mathbf{q}) = \mathbf{0} \\ \mathbf{\Gamma}_{\varphi} = \mathbf{I} - \mathbf{A}_n^{-1}\mathbf{\Phi}_q^{\mathrm{T}}\left(\mathbf{\Phi}_q\mathbf{A}_n^{-1}\mathbf{\Phi}_q^{\mathrm{T}}\right)^{-1}\mathbf{\Phi}_q \end{cases} \tag{23}$$

where $\boldsymbol{\tau}_0$ is an arbitrary configuration space force vector. Projection matrix $\mathbf{\Gamma}_{\varphi}$ satisfies $\mathbf{\Phi}_q\mathbf{\Gamma}_{\varphi} = \mathbf{0}$ and $\mathbf{\Gamma}_{\varphi}\mathbf{A}_n^{-1}\mathbf{\Phi}_q^{\mathrm{T}} = \mathbf{0}$. The $\mathbf{\Phi}_q^{\#} = \mathbf{A}_n^{-1}\mathbf{\Phi}_q^{\mathrm{T}}\left(\mathbf{\Phi}_q\mathbf{A}_n^{-1}\mathbf{\Phi}_q^{\mathrm{T}}\right)^{-1}$ is the generalized pseudo-inverse of constraint Jacobian and matrix $\mathbf{\Gamma}_{\varphi} = \mathbf{I} - \mathbf{\Phi}_q^{\#}\mathbf{\Phi}_q$ is its null-space projection matrix. In sliding mode on manifold $\boldsymbol{\sigma}_{\varphi}(\mathbf{q}) = \mathbf{0}$ motion described by (23) is constrained the null-space of constraint Jacobian and is dynamically decoupled from the motion in constrained direction. The (23) reflects the configuration space dynamics not observable at the measured output - constraint - and could be used to enhance system performance, control a separate task or the posture of the system.

B. OPERATIONAL SPACE CONTROL

1) TASK CONTROL

task $\mathbf{x}(\mathbf{q}) \in \Re^{m\times 1}$ tracking its $\mathbf{x}^{ref}$ reference could be formulated as enforcement of sliding mode in manifold $\boldsymbol{\sigma}_x = \dot{\mathbf{e}}_x + \mathbf{C}_x\mathbf{e}_x = \mathbf{0}$, $\mathbf{C}_x > 0$, $\mathbf{e}_x = \mathbf{x} - \mathbf{x}^{ref}$. With task dynamics as in (11) the dynamics of sliding mode function can be expressed as

$$\begin{cases} \dot{\boldsymbol{\sigma}}_x = \mathbf{M}_n^{-1}\left(\mathbf{f}_x - \mathbf{f}_x^{eq}\right); \mathbf{M}_n^{-1} = \left(\mathbf{J}\mathbf{A}_n^{-1}\mathbf{J}^{\mathrm{T}}\right) \\ \mathbf{f}_x^{eq} = \mathbf{f}_x^{dis} + \mathbf{M}_n\left(\ddot{\mathbf{x}}^{ref} - \mathbf{C}_x\dot{\mathbf{e}}_x\right) \end{cases} \tag{24}$$

where the disturbance $\mathbf{f}_x^{dis}$ is expressed as in (11). Control $\mathbf{f}_x = \hat{\mathbf{f}}_x^{eq} - \rho\mathbf{M}_n\mathbf{\Psi}(\boldsymbol{\sigma}_x)$ would enforce sliding mode in manifold $\boldsymbol{\sigma}_x = \mathbf{0}$ with reaching dynamics $\dot{\boldsymbol{\sigma}}_x + \rho\mathbf{\Psi}(\boldsymbol{\sigma}_x) = \mathbf{0}$. The task control force mapping into configuration space is given by $\boldsymbol{\tau} = \mathbf{J}^{\mathrm{T}}\mathbf{f}_x + \mathbf{\Gamma}_x^{\mathrm{T}}\boldsymbol{\tau}_0$, where $\boldsymbol{\tau}_0$ is an arbitrary configuration space force vector. Insertion into configuration space dynamics (8) yields the sliding mode equations of motion

$$\begin{cases} \mathbf{\Gamma}_x\ddot{\mathbf{q}} + \mathbf{A}_n^{-1}\mathbf{\Gamma}_x^{\mathrm{T}}\boldsymbol{\tau}^{dis} = \mathbf{A}_n^{-1}\mathbf{\Gamma}_x^{\mathrm{T}}\boldsymbol{\tau}_0 \\ \boldsymbol{\sigma}_x\left(\mathbf{x}, \mathbf{x}^{ref}\right) = \mathbf{0} \\ \mathbf{\Gamma}_x = \left(\mathbf{I} - \mathbf{A}_n^{-1}\mathbf{J}^{\mathrm{T}}\left(\mathbf{J}\mathbf{A}_n^{-1}\mathbf{J}^{\mathrm{T}}\right)^{-1}\mathbf{J}\right) \end{cases} \tag{25}$$

The structure is the same as in the constraint control – motion is constrained to sliding mode manifold and $\mathbf{\Gamma}_x$ satisfies $\mathbf{J}\mathbf{\Gamma}_x = \mathbf{0}$ and $\mathbf{J}\mathbf{A}_n^{-1}\mathbf{\Gamma}_x^{\mathrm{T}} = \mathbf{0}$. Motion in manifold $\boldsymbol{\sigma}_x = \mathbf{0}$ allows the usage of the $(n-m)$ degrees to enforce some other task or relationship in null-space of the task Jacobian. This shows the similarities between task and constraint and could be effectively used for the decoupling of the task and posture control.

2) FORCE CONTROL

The dynamics of the operational space force error $\boldsymbol{\sigma}_{\lambda} = \boldsymbol{\lambda}(\mathbf{x}) - \boldsymbol{\lambda}^{ref}$, with force reference $\boldsymbol{\lambda}^{ref}$ and $\boldsymbol{\lambda}(\mathbf{x}) \in \Re^{r\times 1}$ measured force with dynamics as in (12) can be expressed as

$$\begin{cases} \dot{\boldsymbol{\sigma}}_{\lambda}^{*} = D_e^{-1}\dot{\boldsymbol{\sigma}}_{\lambda} = \mathbf{M}_{\lambda}^{-1}\left(\mathbf{f}_{\lambda} - \mathbf{f}_{\lambda}^{eq}\right) \\ \mathbf{f}_{\lambda}^{eq} = \mathbf{f}_{\lambda}^{dis} + D_e^{-1}\mathbf{M}_{\lambda}\dot{\boldsymbol{\lambda}}^{ref} \end{cases} \tag{26}$$

Here $\mathbf{f}_{\lambda} \in \Re^{r\times 1}$ is control force, $\mathbf{\Phi}_{\lambda} \in \Re^{r\times m}$ is constraint Jacobian in operation space, $\mathbf{f}_{\lambda}^{dis} \in \Re^{m\times 1}$, $\mathbf{M}_{\lambda} \in \Re^{m\times m}$ and spring-damper model of the force are as in (12). The dynamics (26) has the same structure as constraint (22) or task (24) dynamics thus the structure of control could be the same. Control $\mathbf{f}_{\lambda} = \mathbf{f}_{\lambda}^{eq} - \rho\mathbf{M}_{\lambda}\mathbf{\Psi}(\boldsymbol{\sigma}_{\lambda})$ enforces sliding mode in manifold $\boldsymbol{\sigma}_{\lambda} = \mathbf{0}$. Insertion $\mathbf{f}_x = \mathbf{\Phi}_{\lambda}^{\mathrm{T}}\mathbf{f}_{\lambda}^{eq} + \mathbf{\Gamma}_{\lambda}^{\mathrm{T}}\mathbf{f}_0$, where $\mathbf{f}_0 \in \Re^{m\times 1}$ is an arbitrary operational space force, into task dynamics (11) yields sliding mode dynamics in operational space

$$\begin{cases} \mathbf{\Gamma}_{\lambda}\ddot{\mathbf{x}} + \mathbf{M}_{\lambda}^{-1}\mathbf{\Gamma}_{\lambda}^{\mathrm{T}}\mathbf{f}_x^{dis} = \mathbf{M}_{\lambda}^{-1}\mathbf{\Gamma}_{\lambda}^{\mathrm{T}}\mathbf{f}_0 \;\&\; \boldsymbol{\sigma}_{\lambda}\left(\boldsymbol{\lambda}, \boldsymbol{\lambda}^{ref}\right) = \mathbf{0} \\ \mathbf{\Gamma}_{\lambda} = \left(\mathbf{I} - \mathbf{M}_{\lambda}^{-1}\mathbf{\Phi}_{\lambda}^{\mathrm{T}}\left(\mathbf{\Phi}_{\lambda}\mathbf{M}_{\lambda}^{-1}\mathbf{\Phi}_{\lambda}^{\mathrm{T}}\right)^{-1}\mathbf{\Phi}_{\lambda}\right) \end{cases} \tag{27}$$

where $\mathbf{f}_x^{dis}$ is expressed as in (11). Dynamics (27) describes the $(m-r)$ dimensional operation space dynamics constrained to the tangential plane in the interaction point. It is easy to verify that its projection in the force direction is equal to zero. This shows the relationship between force and task control – the axes not involved in the force control could be used to generate motion. In the proposed solution the information on the constraint Jacobian is needed and it is applicable when system is in interaction with environment.

C. HYBRID POSITION-FORCE CONTROL

In interaction with environment motion should be modified and should excerpt desired force on the obstacle while moving in the tangential plane at the point of interaction. Such a situation is treated in well established hybrid position-force control framework in which operational space is partitioned onto the direction of the free motion and the direction of constraint in which the force will be controlled. In such a way the position and force control are analyzed independently to take advantage of well-known control techniques for each and are combined at level of the configuration force [16], [20]. The hybrid position-force control is applicable when system is in interaction with environment.

Let operation space position vector $\mathbf{x} \in \Re^{m\times 1}$ is portioned onto the $\mathbf{x}_x = \mathbf{S}\mathbf{x} \in \Re^{m\times 1}$ in the direction of the free motion and $\mathbf{x}_f = \mathbf{S}_{\perp}\mathbf{x} \in \Re^{m\times 1}$ in the direction of constraint. Here $\mathbf{S} \in \Re^{m\times m}$ is diagonal matrix with ones in every direction in which position is controlled and zeroes in directions force is controlled and $\mathbf{S}_{\perp} = \mathbf{I} - \mathbf{S}$. Let interaction is modeled as a spring-damper yielding $D_e^{-1}\boldsymbol{\lambda} = \dot{\mathbf{e}}_{\lambda} + D_e^{-1}K_e\mathbf{e}_{\lambda}$ with parameters $D_e, K_e > 0$ and $\mathbf{e}_{\lambda} = \mathbf{x}_f - \mathbf{x}_f^e$, $\mathbf{x}_f^e$ is position of obstacle. With such a partition, the sliding mode function and force dynamics could be represented as

$$\left.\begin{array}{l}\boldsymbol{\sigma}_x = \mathbf{S}\boldsymbol{\sigma} \\ \boldsymbol{\sigma}_f = \mathbf{S}_\perp\boldsymbol{\sigma} \\ \boldsymbol{\lambda} = D_e\dot{\mathbf{e}}_\lambda + K_e\mathbf{e}_\lambda\end{array}\right\} \begin{array}{l}\dot{\boldsymbol{\sigma}}_x = \mathbf{S}\left(\ddot{\mathbf{x}} - \ddot{\mathbf{x}}^{ref} + \mathbf{C}\dot{\mathbf{e}}\right) \\ \dot{\boldsymbol{\sigma}}_f = \mathbf{S}_\perp\left(\ddot{\mathbf{x}} - \ddot{\mathbf{x}}^{ref} + \mathbf{C}\dot{\mathbf{e}}\right) \\ D_e^{-1}\dot{\boldsymbol{\lambda}} = \mathbf{S}_\perp\left(\ddot{\mathbf{x}} - \ddot{x}_f^e + K_e\dot{\mathbf{e}}_\lambda\right)\end{array} \tag{28}$$

From (28) follows that $\boldsymbol{\sigma}_f$ and $\boldsymbol{\lambda}$ have the same acceleration distribution matrix so they could be combined into a single variable $\boldsymbol{\sigma}_{\lambda f} = \boldsymbol{\sigma}_f + h\boldsymbol{\lambda}$, $h = \eta D_e^{-1}$, $h > 0$ and the dynamics of the $\boldsymbol{\sigma}_{\lambda f}$ could be expressed as

$$\begin{aligned}\dot{\boldsymbol{\sigma}}_{\lambda f} &= \dot{\boldsymbol{\sigma}}_f + h\dot{\boldsymbol{\lambda}} = \mathbf{S}_\perp\left(\ddot{\mathbf{x}} - \ddot{\mathbf{x}}^{ref} + \mathbf{C}\dot{\mathbf{e}}\right) + h\dot{\boldsymbol{\lambda}} \\ \boldsymbol{\sigma}_{\lambda f} &= \boldsymbol{\sigma}_f + h\boldsymbol{\lambda} = \mathbf{0} \Rightarrow \boldsymbol{\sigma}_f = -h\boldsymbol{\lambda}\end{aligned} \tag{29}$$

If sliding mode is established in manifold $\boldsymbol{\sigma}_{\lambda f} = \mathbf{0}$ the interaction force $\boldsymbol{\lambda}$ is balanced by the error in tracking the $\mathbf{x}_f^{ref} = \mathbf{S}_\perp\mathbf{x}^{ref}$ and if interaction force is zero the sliding mode is established in manifold $\boldsymbol{\sigma}_f = \mathbf{S}_\perp\boldsymbol{\sigma}$. It is obvious if force needs to be controlled to track its reference then $\boldsymbol{\sigma}_f = \mathbf{S}_\perp\boldsymbol{\sigma}$ should be bounded by the force reference. By enforcing sliding mode in $\boldsymbol{\sigma}_x = \mathbf{S}\boldsymbol{\sigma} = \mathbf{0}$ & $\boldsymbol{\sigma}_{\lambda f} = \mathbf{S}_\perp\boldsymbol{\sigma} + h\boldsymbol{\lambda} = \mathbf{0}$ the trajectory tracking in interaction-free motion ($\boldsymbol{\lambda} = \mathbf{0}$) and interaction force control combined with motion in interaction-free direction is established. The dynamics of the sliding mode functions $\boldsymbol{\sigma}_{\lambda f} = \mathbf{S}_\perp\boldsymbol{\sigma} + h\boldsymbol{\lambda}$ and $\boldsymbol{\sigma}_x = \mathbf{S}\sigma$ could be written as

$$\begin{aligned}\dot{\boldsymbol{\sigma}}_x &= (\mathbf{SJ})\,\mathbf{A}_n^{-1}(\mathbf{SJ})^{\mathrm{T}}\left(\mathbf{f}_x - \mathbf{f}_x^{eq}\right) \\ \dot{\boldsymbol{\sigma}}_{\lambda f} &= (\mathbf{S}_\perp\mathbf{J})\,\mathbf{A}_n^{-1}(\mathbf{S}_\perp\mathbf{J})^{\mathrm{T}}\left(\mathbf{f}_\lambda - \mathbf{f}_\lambda^{eq}\right)\end{aligned} \tag{30}$$

The control inputs are

$$\left\{\begin{array}{l}\mathbf{f}_x = \hat{\mathbf{f}}_x^{eq} - \rho\mathbf{M}_x\boldsymbol{\Psi}\left(\boldsymbol{\sigma}_x\right);\ \mathbf{M}_x^{-1} = (\mathbf{SJ})\,\mathbf{A}_n^{-1}(\mathbf{SJ})^{\mathrm{T}} \\ \mathbf{f}_\lambda = \hat{\mathbf{f}}_\lambda^{eq} - \rho\mathbf{M}_\lambda\boldsymbol{\Psi}\left(\boldsymbol{\sigma}_{\lambda f}\right);\ \mathbf{M}_\lambda^{-1} = (\mathbf{S}_\perp\mathbf{J})\,\mathbf{A}_n^{-1}(\mathbf{S}_\perp\mathbf{J})^{\mathrm{T}} \\ \boldsymbol{\tau} = (\mathbf{SJ})^{\mathrm{T}}\mathbf{f}_x + (\mathbf{S}_\perp\mathbf{J})^{\mathrm{T}}\mathbf{f}_\lambda\end{array}\right. \tag{31}$$

A natural fusion of the motion and force control lies in a modification of the movement if an interaction appears and maintaining the motion control if interaction is removed. During interaction, the motion is performed in the tangential plane in the contact point and motion in the constraint direction determines the force. Such a behavior could be realized if sliding mode function is selected as a function of both motion tracking and interaction force as in (17) $\boldsymbol{\sigma}_{x\lambda} = \dot{\mathbf{e}}_x + \mathbf{C}\mathbf{e}_x + \mathbf{H}\boldsymbol{\lambda}$, $\boldsymbol{\sigma}_{x\lambda} \in \Re^{m\times 1}$, $\mathbf{C} > 0$, the force $\boldsymbol{\lambda} \in \Re^{p\times 1}$, $\mathbf{H} \in \Re^{m\times p}$ defines the distribution of the interaction force to the components of the generalized tracking error. With task dynamics defined as in (11) the dynamics of sliding mode function (17) could be expressed as

$$\left\{\begin{array}{l}\dot{\boldsymbol{\sigma}}_{x\lambda} = \mathbf{M}_n^{-1}\left(\mathbf{f}_x - \mathbf{f}_{x\lambda}^{eq}\right);\ \mathbf{M}_n^{-1} = \left(\mathbf{J}\mathbf{A}_n^{-1}\mathbf{J}^{\mathrm{T}}\right); \\ \mathbf{f}_{x\lambda}^{eq} = \mathbf{f}_x^{dis} + \mathbf{M}_n\left(\ddot{\mathbf{x}}^{ref} - \mathbf{C}_x\dot{\mathbf{e}}_x - \mathbf{H}\dot{\boldsymbol{\lambda}}\right)\end{array}\right. \tag{32}$$

Here $\mathbf{f}_x^{dis}$ is defined in (11). The control $\mathbf{f}_x = \mathbf{f}_{x\lambda}^{eq} - \rho\mathbf{M}_n\boldsymbol{\Psi}\left(\boldsymbol{\sigma}_{x\lambda}\right)$ enforces sliding mode in manifold $\sigma_{x\lambda} = \mathbf{0}$ and dynamics in constrained direction is described by $\dot{\mathbf{e}}_x + \mathbf{C}\mathbf{e}_x = -\mathbf{H}\boldsymbol{\lambda}$. In order to keep interaction force within desired limits the $\boldsymbol{\sigma}_x = \dot{\mathbf{e}}_x + \mathbf{C}\mathbf{e}_x$ should be limited by $-\mathbf{H}\boldsymbol{\lambda}^{\max} \le \boldsymbol{\sigma}_x \le -\mathbf{H}\boldsymbol{\lambda}^{\min}$. The limits may be considered as references for the interaction force that we want to track.

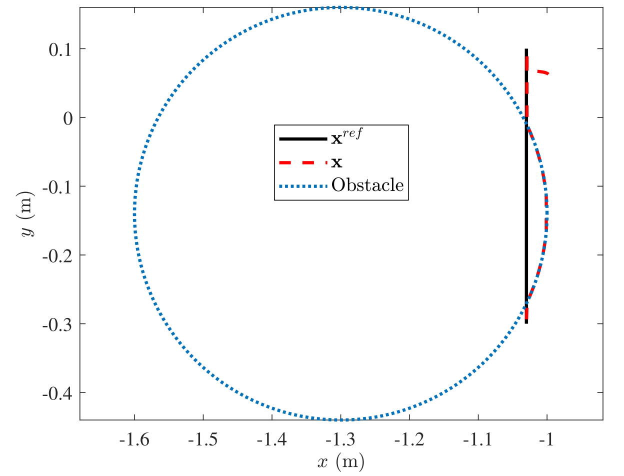


FIGURE 1. **Robot's trajectory, reference trajectory, and obstacle for observer-based control.**

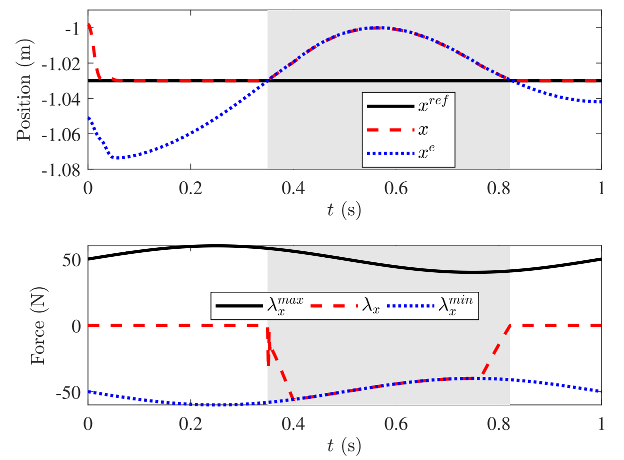


FIGURE 2. ***x*-direction position and interaction force for observer-based control.**

In order to illustrate the concept presented here, it will be assumed that hybrid position-force control is applied to a planar manipulator, discussed in detail in [23]. Two different control algorithms were applied: (i) observer-based control $\mathbf{f}_x = \hat{\mathbf{f}}_{x\lambda}^{eq} - \rho\mathbf{M}_n\boldsymbol{\sigma}_{x\lambda} - \rho_1\mathbf{M}_n\mathrm{sign}\left(\boldsymbol{\sigma}_{x\lambda}\right)$, (ii) classical discontinuous control $\mathbf{f}_x = -M\mathrm{sign}\left(\boldsymbol{\sigma}_{x\lambda}\right)$.

The results for the observer-based control are depicted in Figures 1-3. The shaded parts of the graphs in Figures 2 and 3 correspond to the time interval in which the contact between the manipulator and the circular obstacle (its edge is given in blue color) exists. The x^e and y^e denote the coordinates of the point belonging to an obstacle that is the closest to the manipulator, or the position of the contact point belonging to an obstacle. During the free motion, the manipulator is able to track the reference trajectory (see Figure 1). During the contact, the manipulators is tracking the reference trajectory

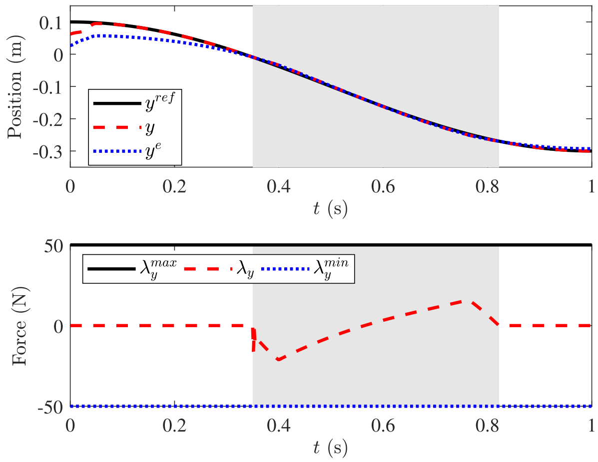


FIGURE 3. ***y*-direction position and interaction force for observer-based control.**

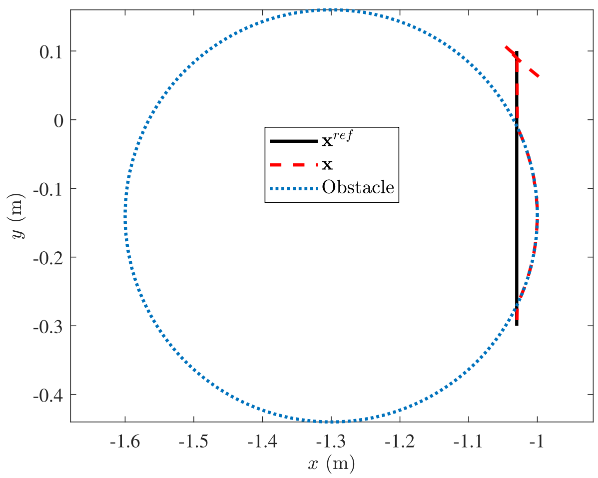


FIGURE 4. **Robot's trajectory, reference trajectory, and obstacle for classical discontinuous control.**

in the y-direction (see Figure 3). However, this is not the case for the x-direction motion, the tracking is not possible, and force is controlled in this direction. Therefore, the reference interaction force in the x-direction is tracked (see Figure 2).

The results for the discontinuous control are depicted in Figures 4-6. The shaded parts of the graphs in Figures 5 and 5 correspond to the time intervals in which the contact between the manipulator and the circular obstacle (its edge is given in blue color) exists. It can be observed that oscillatory motion exists when the manipulator establishes/loses the contact with environment (look at many shaded intervals Figures 5 and 5). It is so-called woodpecker phenomenon. The force in the x-direction is not controlled anymore, and the discontinuous control is obviously inferior to here proposed observer-based control strategy.

D. HAPTIC SYSTEMS

Recently, haptics is becoming important field in motion control. In the core of the problem lie the control structures enabling a human operator to interact with the remote environment, while having a realistic feeling of the interaction

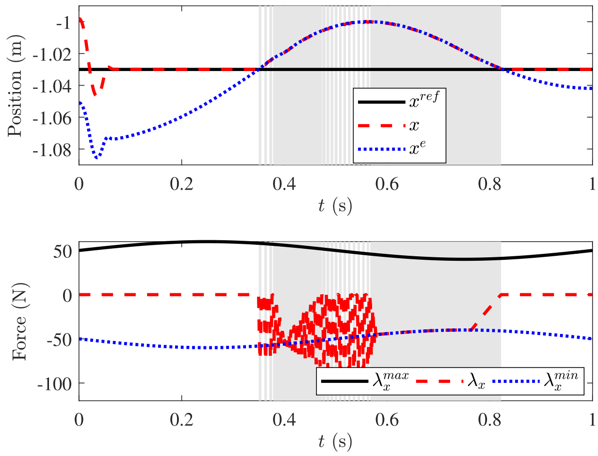


FIGURE 5. ***x*-direction position and interaction force for classical discontinuous control.**

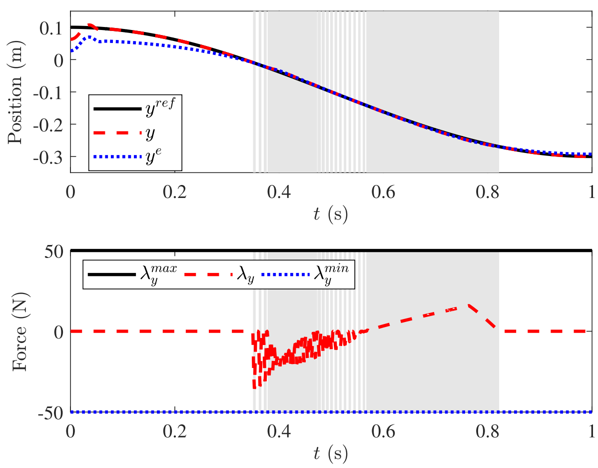


FIGURE 6. ***y*-direction position and interaction force for classical discontinuous control.**

force and transparent interaction. The operation of the master-slave system can be formulated as the concurrent trajectory and force tracking, with human operator setting the position reference (x_m) to the slave side (x_s) while the slave side is executing the motion task and transfers the interaction force back to the master side. The operator sets the level of the interaction force thus the force that master-side device is exerting to the operator (f_h) is equal (or proportional) to the force (f_e) slave device exerts to the environment. Such operation arrangements could be mathematically formulated as a requirement to establish sliding mode motion such that the position tracking $e_x = x_s - x_m$ and force tracking errors $e_{bf} = f_h + f_e$ are concurrently equal to zero [20]. Colloquially, we may view this arrangement as *"extending the hands of the operator by a long stick with a tool on its far end"*.

For simplicity, let master device dynamics and slave devices dynamic are given as in

$$\left.\begin{array}{l} a_m\ddot{q}_m + b_m + g_m = \tau_m - f_h + \tau_{op} \\ a_s\ddot{q}_s + b_s + g_s = \tau_s - f_e \\ f_h = d_h\dot{e}_{mo} + k_h e_{mo};\ e_{mo} = x_m - x_{op} \\ f_e = d_e\dot{e}_{se} + k_e e_{se};\ e_{se} = x_s - x_e \end{array}\right\} \mathbf{A}\ddot{\mathbf{q}} + \mathbf{b} + \mathbf{g} + \mathbf{f} = \boldsymbol{\tau}_b \tag{33}$$

with $\mathbf{A} = diag\,[a_m, a_s]$, $\mathbf{q}^{\mathrm{T}} = [q_m\ q_s]$, $\boldsymbol{\tau}_b^T = [\tau_m\ \tau_s]$, $\mathbf{b}^{\mathrm{T}} = [b_m\ b_s]$, $\mathbf{g}^{\mathrm{T}} = [g_m\ g_s]$, $\mathbf{f}^{\mathrm{T}} = \left[\left(f_h - \tau_{op}\right) f_e\right]$.

The τ_{op} is the operator input force, x_{op}, x_e are operator and environment position respectively, (d_h, k_h), (d_e, k_e) are spring-damper parameters on the operator and slave side respectively, τ_m, τ_s are master and slave control inputs respectively. Let interaction with the operator and with the environment are modeled as a spring-damper system (the same procedure could be applied if the interaction forces are modeled as simple spring model) [18].

Now, the haptic (bilateral) operation could be formulated as a requirement to enforce sliding mode in the manifold $\boldsymbol{\sigma}_b = \mathbf{0}$ where vector valued sliding mode function $\boldsymbol{\sigma}_b^{\mathrm{T}} = \left[\sigma_{bx}\ \sigma_{bf}\right]$ is

$$\boldsymbol{\sigma}_b = \begin{bmatrix} \sigma_{bx} \\ \sigma_{bf} \end{bmatrix} = \begin{bmatrix} \dot{e}_x + c_{bx} e_x \\ k_f \left(f_h + f_e\right) \end{bmatrix}; \quad c_{bx}, \ k_f > 0$$

$$\underbrace{\begin{bmatrix} \sigma_{bx} \\ \sigma_{bf} \end{bmatrix}}_{\boldsymbol{\sigma}_b} = \underbrace{\begin{bmatrix} -1 & 1 \\ d_{op} & d_e \end{bmatrix}}_{\mathbf{J}} \begin{bmatrix} \dot{x}_m \\ \dot{x}_s \end{bmatrix} + \underbrace{\begin{bmatrix} -c_{bx} & c_{bx} \\ k_{op} & k_e \end{bmatrix}}_{\mathbf{C}} \begin{bmatrix} x_m \\ x_s \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \xi\left(x_{op}, x_e\right) \end{bmatrix}}_{\boldsymbol{\xi}\left(x_{op}, x_e\right)}$$

$$\xi\left(x_{op}, x_e\right) = -d_{op}\dot{x}_{op} - k_{op}x_{op} - d_e\dot{x}_e - k_e x_e$$

$$\boldsymbol{\sigma}_b = \mathbf{J}\dot{\mathbf{q}} + \mathbf{C}\mathbf{q} + \boldsymbol{\xi}\left(x_{op}, x_e\right) \tag{34}$$

The dynamics of sliding mode function could be expressed as

$$\begin{aligned} \dot{\boldsymbol{\sigma}}_b &= \mathbf{J}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \dot{\boldsymbol{\xi}}\left(x_{op}, x_e\right) \\ \dot{\boldsymbol{\sigma}}_b &= \mathbf{J}\mathbf{A}^{-1}\mathbf{J}^{\mathrm{T}}\left(\boldsymbol{\tau}_b - \boldsymbol{\tau}_b^{eq}\right) \\ \boldsymbol{\tau}_b^{eq} &= \left(\mathbf{J}\mathbf{A}^{-1}\mathbf{J}^{\mathrm{T}}\right)^{-1} \mathbf{J}\mathbf{A}^{-1}\left(\mathbf{b} + \mathbf{g} + \mathbf{f}\right) - \mathbf{C}\dot{\mathbf{q}} - \dot{\xi}\left(x_{op}, x_e\right) \end{aligned} \tag{35}$$

The (34) stands for the task dynamics of the two physically separated systems which are required to establish desired functional relationship. It has the same structure as in all cases discussed above, thus the control that would guarantee sliding mode in manifold $\boldsymbol{\sigma}_b = \mathbf{0}$ could be selected as $\boldsymbol{\tau}_b = \hat{\boldsymbol{\tau}}_b^{eq} - \rho\left(\mathbf{J}\mathbf{A}^{-1}\mathbf{J}^{\mathrm{T}}\right)^{-1}\boldsymbol{\Psi}\left(\boldsymbol{\sigma}_b\right)$, and observer (6) could be used to estimate $\hat{\boldsymbol{\tau}}_b^{eq}$. By changing force τ_{op} the operator sets the equilibrium in the bilateral control system. Bilateral control with scaling position and force (for example in micromanipulation systems) can be solved in the same way. The haptic systems are part of a wider class of functionally related systems in which the systems are forced to maintain a certain functional relationship [19], [22].

E. REPRODUCTION OF HAPTIC MOTION

The reproduction of haptic interaction requires accurate replay of the position and the interaction force without human operator (just as a replay of music or video is done without person or unit that recorded sound or image).

Let motion generalized error $\sigma_{bx}^{rec} = c_{bx}x^{rec} + \dot{x}^{rec}$; $c_{bx} > 0$ and the interaction force f_e^{rec} be recorded during the bilateral operation on a master-slave system. Assume that during reproduction the motion generalized error $\sigma_{bx}^{rep} = c_{bx}x^{rep} + \dot{x}^{rep}$ and force f_e^{rep} are measured. Let the generalized motion-reproduction error is defined as $\sigma_x = \sigma_{bx}^{rep} - \sigma_{bx}^{rec}$. If control enforces the convergence to and stability of equilibrium $\sigma_x = 0$ the reproduced trajectory x^{rep} will follow the recorded path x^{rec}. The position-force reproduction generalized error and its dynamics could be expressed as

$$\begin{cases} \sigma_{bxf}^{rep} = \left(\sigma_{bx}^{rep} - \sigma_{bx}^{rec}\right) + k\left(hf_e^{rep} - hf_e^{rec}\right) \\ \dot{\sigma}_{bx\varphi}^{rep} = K^{rep}\left(f_x^{rep} - f_{eq}^{rep}\right) \end{cases} \tag{36}$$

Here force control gain $k > 0$ is a design parameter, $K^{rep} > 0$ and f_x^{rep} stand for the gain and input force of the system reproducing haptic motion (slave or master), the f_{eq}^{rep} is the control enforcing zero rate of change of position-force reproduction error (36). In the equilibrium, the motion generalized error σ_x is balanced by the force error $k\left(hf_\varphi^{rep} - hf_\varphi^{rec}\right)$ and if interaction force is $f_e = D_e\left(\dot{x} - \dot{x}_e\right) + K_e\left(x - x_e\right)$, where K_e, D_e are strictly positive constants, x_e is the position of the environment, and the initial position and parameters of the environment in the recording and reproduction phases are the same. Then in sliding mode

$$\begin{aligned} &\left(1 + hD_e\right)\dot{e}_x + \left(C_x + hK_e\right)e_x = khD_e\dot{e}_{xe} + khK_e e_{xe} \\ &e_{xe} = x_e^{rep} - x_e^{rec}; \quad K_e^{rep} = K_e^{rec} = K_e; \\ &D_e^{rep} = D_e^{rec} = D_e \end{aligned} \tag{37}$$

The proposed solution offers accurate force reproduction even in the case $x_e^{rep} \neq x_e^{rec}$, as a key advantage in comparison with other methods presented in [24].

V. CONCLUSION

This paper showed that majority motion control tasks can be discussed in a single framework, since the dynamics of all tasks can be written in the same form. Thus, the same control structure is applicable. The paper concentrated to structures based on SMC. However, it gives enough information to apply numerous other control algorithms to all of the tasks.

REFERENCES

[1] V. Utkin, A. Poznyak, Y. V. Orlov, and A. Polyakov, *Road Map for Sliding Mode Control Design*. Cham, Switzerland: Springer, 2020.

[2] C. Edwards and S. Spurgeon, *Sliding Mode Control: Theory and Applications*. Boca Raton, FL, USA: CRC Press, 1998.

[3] Y. Shtessel, C. Edwards, L. Fridman, and A. Levant, *Sliding Mode Control and Observation*. New York, NY, USA: Springer, 2014.

[4] V. Utkin, J. Guldner, and M. Shijun, *Sliding Mode Control in Electromechanical Systems*, vol. 34. Boca Raton, FL, USA: CRC Press, 1999.

[5] F. Piltan and N. B. Sulaiman, "Review of sliding mode control of robotic manipulator," *World Appl. Sci. J.*, vol. 18, no. 12, pp. 1855–1869, 2012.

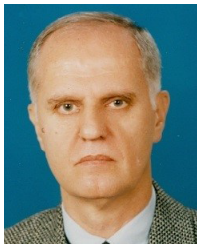
[6] K. D. Young, V. I. Utkin, and U. Ozguner, "A control engineer's guide to sliding mode control," *IEEE Trans. Control Syst. Technol.*, vol. 7, no. 3, pp. 328–342, May 1999.

[7] S. V. Drakunov and V. Utkin, "On discrete-time sliding modes," in *Nonlinear Control Systems Design 1989*. Amsterdam, The Netherlands: Elsevier, 1990, pp. 273–278.

[8] G. Golo and Č. Milosavljević, "Robust discrete-time chattering free sliding mode control," *Syst. Control Lett.*, vol. 41, no. 1, pp. 19–28, 2000.

[9] V. Utkin, A. Poznyak, Y. Orlov, and A. Polyakov, "Conventional and high order sliding mode control," *J. Franklin Inst.*, vol. 357, no. 15, pp. 10244–10261, Oct. 2020.

[10] S. Drakunov and V. Utkin, ''Sliding mode observers. Tutorial,'' in *Proc. 34th IEEE Conf. Decis. Control*, Dec. 1995, pp. 3376–3378.
[11] J. Hu, H. Zhang, H. Liu, and X. Yu, ''A survey on sliding mode control for networked control systems,'' *Int. J. Syst. Sci.*, vol. 52, no. 6, pp. 1129–1147, Apr. 2021.
[12] K. K. D. Young, ''Controller design for a manipulator using theory of variable structure systems,'' *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-8, no. 2, pp. 101–109, Feb. 1978.
[13] J. J. Slotine and S. S. Sastry, ''Tracking control of non-linear systems using sliding surfaces, with application to robot manipulators,'' *Int. J. Control*, vol. 38, no. 2, pp. 465–492, Aug. 1983.
[14] V. Utkin and H. Lee, ''Chattering problem in sliding mode control systems,'' in *Proc. Int. Workshop Variable Struct. Syst. (VSS)*, 2006, pp. 346–350.
[15] O. Khatib, ''A unified approach for motion and force control of robot manipulators: The operational space formulation,'' *IEEE J. Robot. Autom.*, vol. RA-3, no. 1, pp. 43–53, Feb. 1987.
[16] W. D. Fisher and M. S. Mujtaba, ''Hybrid position/force control: A correct formulation,'' *Int. J. Robot. Res.*, vol. 11, no. 4, pp. 299–311, Aug. 1992.
[17] K. Erbatur, M. O. Kaynak, and A. Sabanovic, ''A study on robustness property of sliding-mode controllers: A novel design and experimental investigations,'' *IEEE Trans. Ind. Electron.*, vol. 46, no. 5, pp. 1012–1018, Oct. 1999.
[18] K. D. Young, ''Sliding mode for constrained robot motion control,'' in *Variable Structure Control for Robotics and Aerospace Application*. Amsterdam, The Netherlands: Elsevier, 1993, pp. 157–172.
[19] T. Uzunović and A. Šabanović, *Motion Control of Functionally Related Systems*. Boca Raton, FL, USA: CRC Press, 2020.
[20] K. Ohnishi, S. Katsura, and T. Shimono, ''Motion control for real-world haptics,'' *IEEE Ind. Electron. Mag.*, vol. 4, no. 2, pp. 16–19, Jun. 2010.
[21] T. Uzunovic, A. Sabanovic, M. Yokoyama, and T. Shimono, ''Novel algorithm for effective position/force control,'' *IEEJ J. Ind. Appl.*, vol. 8, no. 6, pp. 960–966, 2019.
[22] A. Sabanovic and K. Ohnishi, *Motion Control Systems*. Hoboken, NJ, USA: Wiley, 2011.
[23] T. Uzunovic, A. Sabanovic, M. Yokoyama, and T. Shimono, ''Novel algorithm for position/force control of multi-DOF robotic systems,'' in *Proc. IEEE 16th Int. Workshop Adv. Motion Control (AMC)*, Sep. 2020, pp. 265–270.
[24] T. Okano, R. Oboe, K. Ohnishi, and T. Murakami, ''Comparative study of soft motion for motion copying system with environmental variations,'' in *Proc. IEEE/ASME Int. Conf. Adv. Intell. Mechatronics (AIM)*, Jul. 2018, pp. 646–651.



ASIF ŠABANOVIĆ (Life Senior Member, IEEE) received the B.S., M.S., and Dr.Eng. degrees in electrical engineering from the University of Sarajevo, Bosnia and Herzegovina.

After graduation, he joined the Energoinvest-IRCA Sarajevo. He was a Visiting Researcher at IPU, Moscow; a Visiting Professor at Caltech, Pasadena, CA, USA; a Visiting Professor at Keio University, Yokohama, Japan; a Full Professor at Yamaguchi University, Ube, Japan; the Head of the Robotics Department, Tubitak-MAM, Istanbul. He is currently an Emeritus Professor and a member of the Academy of Sciences and Arts of Bosnia and Herzegovina. His research interests include control systems, motion control systems, robotics, mechatronics, and power electronics.

TARIK UZUNOVIĆ (Senior Member, IEEE) received the B.Eng. and M.Eng. degrees in electrical engineering from the University of Sarajevo, Sarajevo, Bosnia and Herzegovina, in 2008 and 2010, respectively, and the Ph.D. degree in mechatronics from Sabanci University, Istanbul, Turkey, in 2015.

He is currently an Associate Professor with the Department of Automatic Control and Electronics, Faculty of Electrical Engineering, University of Sarajevo. His research interests include control theory, motion control, robotics, and mechatronics.