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LDPC Coded Multi-User Massive MIMO Systems With Low-Complexity Detection

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ABSTRACT We design a low-density parity-check (LDPC) coded multi-user (MU) massive multiple-input multiple-output (MIMO) system with an iterative joint detection and decoding (JDD) algorithm. As a low-complexity MU detection scheme, we consider a factor graph based belief propagation detection with Gaussian approximation of interference, called a FG-GAI BP detection. We introduce a factor graph representation of LDPC coded MU massive MIMO system and define the message updating rule in the JDD process. We devise a design tool for analyzing extrinsic information transfer (EXIT) characteristics of messages exchanged in JDD, based on which degree distribution of LDPC codes and a JDD strategy are efficiently designed for coded MU massive MIMO systems. A JDD strategy and LDPC codes are designed such that a fast convergence of JDD and a low bit error probability are attained. It is observed that the coded MU massive MIMO system equipped with LDPC codes and the JDD strategy designed by the proposed method shows a lower bit error rate than conventional ones with a given number of iterations.

INDEX TERMS Massive MIMO, multi-user, LDPC codes, joint detection and decoding, low complexity, density evolution.

I. INTRODUCTION

Along with the continuous increase of demand for wireless and mobile services, performance requirements for wireless communication system including data rate, spectral efficiency and energy efficiency are getting strengthened [1], [2]. As one of promising solutions to meet such tight performance requirements, a multi-input and multi-output (MIMO) technology was proposed and tremendous amount of research works have been conducted in various aspects of MIMO systems [3], [4]. Recently, it was found that the use of massive number of antennas at transmitter and receiver can improve the spectral efficiency and save energy significantly in wireless communication systems [5], [6]. As a result, the massive MIMO technology has been under active study in both academia and engineering fields, and it was chosen as one of key technologies for the next generation cellular networks, known as the fifth generation (5G) systems [7]. Among various forms of massive MIMO schemes, a multi-user (MU) massive MIMO system, in which

a base station (BS) is equipped with a large number of antennas to serve many user equipments (UE) simultaneously over the same time-frequency resource, has been actively studied to be practically adopted in 5G systems [8]–[10].

Since the signal detection in massive MIMO systems requires high amount of computations, the complexity reduction of signal detection algorithm has been a great concern to implement a massive MIMO technique in practical systems [11]–[20]. Suboptimal linear detection algorithms have been intensively studied for the purpose of complexity reduction, where zero forcing (ZF) detection [11], [12] and minimum mean squared error (MMSE) detection [13]–[16] are well known examples. When implementing these linear detectors, various alternatives of matrix inversion are applied [14]–[16]. Low-complexity detection algorithms based on belief propagation (BP) over factor graph (FG) have also been proposed [19], [20]. Among those, the FG-based BP detection with Gaussian approximation of interference (GAI), called FG-GAI BP detection, is known as one of promising solutions for complexity reduction in the receiver of massive MIMO systems [20]. Despite using low amount of computational complexity, the FG-GAI BP

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detection shows sufficiently low probability of error through iterative message passing on factor graphs [20].

Error control coding is widely used in a variety of communication systems to improve the communication reliability or reduce the transmit power at the cost of bandwidth expansion. Low-density parity-check (LDPC) codes have recently been adopted in various communication standards thanks to the powerful error correction capability [21]–[23]. LDPC codes can be efficiently and systematically designed by using the density evolution algorithm [24], whose well-known example is the extrinsic information transfer (EXIT) chart [25]. LDPC codes have also been applied to MIMO systems to enhance the transmission reliability. Huge amount of research works have been performed regarding the design of LDPC coded MIMO systems and the analysis on the iterative detection and decoding algorithm [26]–[29].

It is necessary to improve further the transmission reliability of massive MIMO systems because the performance requirement for data rate, spectral efficiency and energy efficiency is obviously keep getting tighter. Thus, it is a natural approach to apply LDPC codes having a powerful error-correction capability to the massive MIMO systems, by which LDPC coded massive MIMO system is devised. To obtain a maximum performance gain, a joint detection and decoding (JDD) scheme needs to be implemented at the receiver. Since decoding process requires additional computational complexity, a low-complexity detection algorithm had better be applied to JDD. In [30], non-binary LDPC codes are designed for coded massive MIMO systems considering modified MMSE and matched filter (MF) soft-output detectors. In [31]–[33], binary and non-binary LDPC codes for massive MIMO systems using the FG-GAI BP detection are designed. Protograph LDPC codes for massive MIMO systems [34] and LDPC coded space shift keying (SPK) for massive MIMO system with a low-complexity detector [35] are also proposed. In [17], [18], iterative soft-input soft-output (SISO) MMSE detectors are proposed to enhance the signal detection of massive MIMO systems.

In JDD mechanism, one JDD iteration is composed of a detection phase followed by a decoding phase, where each phase may consist of multiple local iterations. JDD had better converge by a smaller number of local iterations, i.e., with a faster convergence speed, to meet the latency requirement. Thus, we need to design LDPC coded MU massive MIMO system to achieve lower BER with faster convergence speed. In designing LDPC codes, the threshold, meaning the value of E_b/N_0 over which BER improves abruptly, is used as an indicator for evaluating BER. The threshold is determined by the structure of factor graph representing the overall system. The convergence speed is controlled by the ratio of local iteration numbers in detection phase and decoding phase composing one JDD iteration, which will be called a JDD strategy. There exist various techniques to predict the threshold of LDPC coded systems. However, there exist few works considering JDD convergence speed when designing LDPC codes in the system. Thus, there is a strong need for

an efficient design tool by which we can check if messages flowing in JDD evolve in the way of satisfying two design goals: lower threshold and faster JDD convergence.

As a solution to meet this demand, we propose an EXIT analysis tool which is useful for designing JDD strategy and accordingly designing LDPC codes for MU massive MIMO system, where the FG-GAI BP detection is considered as a low-complexity detection algorithm. The proposed tool can be used to analyze the density evolution of messages and predict the convergence behavior of JDD. We represent the coded MU massive MIMO system by a factor graph composed of observation nodes, symbol nodes, bit nodes and check nodes connected through edges, and we define updating rules of messages flowing in JDD. We formulate the bit-level EXIT characteristics of input and output messages of FG-BP GAI detector based on the factor graph representation, and combine this with the decoder to form an overall EXIT characteristic function representing the whole JDD process.

By using the EXIT characteristic function, we optimize the JDD strategy to result in a fast JDD convergence and degree distributions of factor graph representing LDPC codes to result in a low threshold. In the optimization of degree distributions, we include an additional constraint regarding the placement of edges between bit nodes and check nodes in a practical point of view. Then, the parity check matrix of LDPC codes is constructed by using the progressive edge growth (PEG) algorithm [36]. It is observed from simulations that the LDPC coded MU massive MIMO system designed by the proposed method shows improved performance in view of BER and JDD convergence speed over systems designed by conventional methods.

This paper is organized as follows. In Sec. II, we present a graph model for coded MU massive MIMO system. In Sec. III, we introduce how JDD operates and messages are updated. We propose an EXIT analysis tool to analyze the behavior of JDD of the LDPC coded MU massive MIMO system in Sec. IV. In Sec. V, we design LDPC codes and a JDD strategy by using the proposed EXIT analysis tool. In Sec. VI, we present BER performances of the proposed LDPC coded MU massive MIMO system in various points of view and compare those with conventional ones. Finally, we conclude this paper in Sec. VII.

II. SYSTEM MODEL AND GRAPH REPRESENTATION

We consider uplink communications of coded MU massive MIMO system, in which n_U UEs transmit data to a BS as depicted in Fig. 1. We suppose each UE has a single antenna and a BS is equipped with n_R multiple antennas. Each UE encodes K information bits to a N -bit codeword with the code rate $R = K/N$ and modulates it as M_o -ary QAM symbols. Then, $L = N/\log_2 M_o$ symbols generated at each UE are transmitted to BS over L channel uses. We suppose that all UEs transmit codeword symbols to BS in a synchronous manner.

Let $x_k^{(l)}$ denote the l -th symbol generated at the UE k , where $k = 1, \dots, n_U$ and $l = 1, \dots, L$. We let

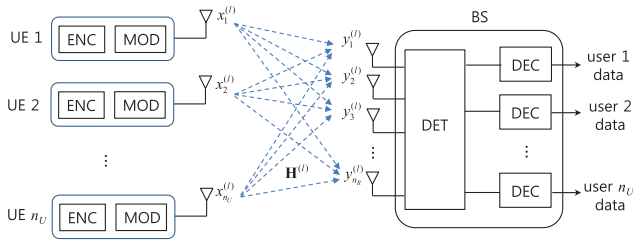


FIGURE 1. Coded MU massive MIMO system.

$\mathbf{x}^{(l)} = [x_1^{(l)} \dots x_{n_U}^{(l)}]^T \in \mathbb{C}^{n_U \times 1}$ denote a symbol vector transmitted from n_U UEs to BS at the l -th channel use. We also let $\mathbf{y}^{(l)} = [y_1^{(l)} \dots y_{n_R}^{(l)}]^T \in \mathbb{C}^{n_R \times 1}$, $\mathbf{w}^{(l)} = [w_1^{(l)} \dots w_{n_R}^{(l)}]^T \in \mathbb{C}^{n_R \times 1}$ and $\mathbf{H}^{(l)} \in \mathbb{C}^{n_R \times n_U}$ denote the received signal vector, the additive noise vector and the channel gain matrix, respectively, all at the l -th channel use. Let $h_{ij}^{(l)}$ denote the (i, j) -th entry of $\mathbf{H}^{(l)}$. Suppose that all $w_i^{(l)}$ are independent and identically distributed (i.i.d.) zero-mean circular symmetric complex white Gaussian noise with variance of σ^2 , and all $h_{ij}^{(l)}$ are i.i.d. complex Gaussian with zero mean and unit variance. The input-output relation of the MU massive MIMO system at the l -th channel use is expressed as

$$\mathbf{y}^{(l)} = \mathbf{H}^{(l)}\mathbf{x}^{(l)} + \mathbf{w}^{(l)} \quad (1)$$

and its real-valued representation is written by

$$\bar{\mathbf{y}}^{(l)} = \bar{\mathbf{H}}^{(l)}\bar{\mathbf{x}}^{(l)} + \bar{\mathbf{w}}^{(l)}, \quad (2)$$

where

$$\begin{aligned} \bar{\mathbf{x}}^{(l)} &= [\Re\{x_1^{(l)}\} \Im\{x_1^{(l)}\} \dots \Re\{x_{n_U}^{(l)}\} \Im\{x_{n_U}^{(l)}\}]^T \in \mathbb{A}^{2n_U \times 1} \\ \bar{\mathbf{y}}^{(l)} &= [\Re\{y_1^{(l)}\} \Im\{y_1^{(l)}\} \dots \Re\{y_{n_R}^{(l)}\} \Im\{y_{n_R}^{(l)}\}]^T \in \mathbb{R}^{2n_R \times 1} \\ \bar{\mathbf{w}}^{(l)} &= [\Re\{w_1^{(l)}\} \Im\{w_1^{(l)}\} \dots \Re\{w_{n_R}^{(l)}\} \Im\{w_{n_R}^{(l)}\}]^T \in \mathbb{R}^{2n_R \times 1} \\ \bar{\mathbf{H}}^{(l)} &= \begin{bmatrix} \bar{\mathbf{H}}_{11}^{(l)} & \bar{\mathbf{H}}_{12}^{(l)} & \dots & \bar{\mathbf{H}}_{1n_U}^{(l)} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{\mathbf{H}}_{n_R1}^{(l)} & \bar{\mathbf{H}}_{n_R2}^{(l)} & \dots & \bar{\mathbf{H}}_{n_Rn_U}^{(l)} \end{bmatrix} \in \mathbb{R}^{2n_R \times 2n_U} \end{aligned}$$

with

$$\bar{\mathbf{H}}_{ij}^{(l)} = \begin{bmatrix} \Re\{h_{ij}^{(l)}\} & -\Im\{h_{ij}^{(l)}\} \\ \Im\{h_{ij}^{(l)}\} & \Re\{h_{ij}^{(l)}\} \end{bmatrix} \in \mathbb{R}^{2 \times 2}.$$

Note that $\Re\{\cdot\}$ and $\Im\{\cdot\}$ represent real and imaginary part of a complex variable, respectively, and \mathbb{A} denotes the set of values for real-valued transmit symbols.

The receiver of coded MU massive MIMO system with the real-valued representation given in (2) can be expressed by a bipartite graph shown in Fig. 2. The receiver consists of L detectors and n_U decoders, where each detector corresponds to each channel use and each decoder corresponds to each UE. Each detector consists of $2n_R$ observation nodes and $2n_U$ symbol nodes connected through edges. Each decoder is composed of N variable nodes (or bit nodes) and $N - K$

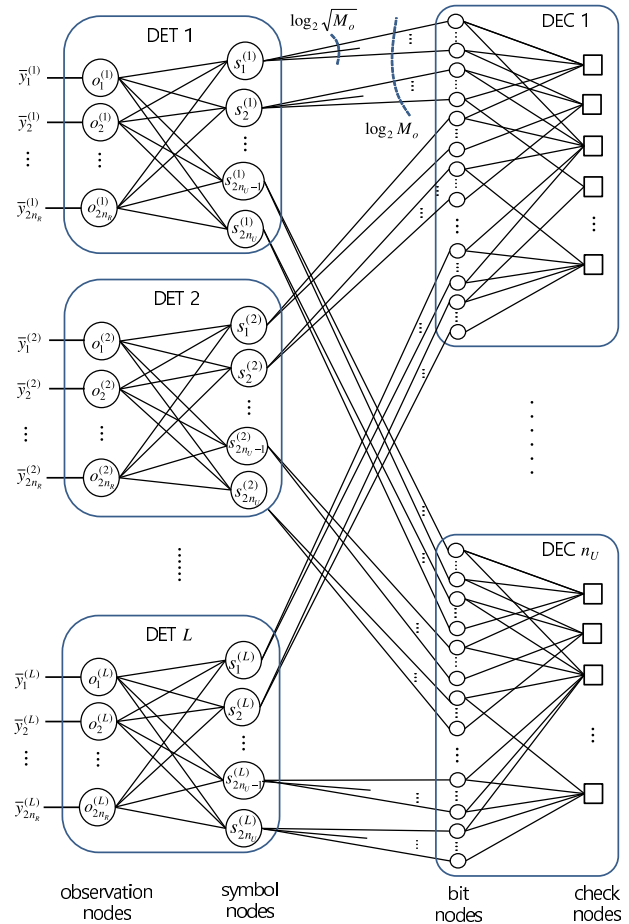


FIGURE 2. Graph representation for the receiver of LDPC coded MU massive MIMO system.

check nodes. Let $o_i^{(l)}$ denote an observation node belonging to the l -th detector, whose input is $\bar{y}_i^{(l)}$, $1 \leq i \leq 2n_R$, $1 \leq l \leq L$. We also let $s_{2k-1}^{(l)}$ and $s_{2k}^{(l)}$, $1 \leq k \leq n_U$, denote symbol nodes belonging to the l -th detector, which correspond to $\Re\{x_k^{(l)}\}$ and $\Im\{x_k^{(l)}\}$, respectively. Note that $s_{2k-1}^{(l)}$ and $s_{2k}^{(l)}$ is connected to the first half and the second half of the l -th group of $\log_2 M_o$ bit nodes in the k -th decoder, respectively.

III. JOINT DETECTION AND DECODING

For given received signals over L channel uses, i.e., $\bar{\mathbf{y}}^{(1)}, \bar{\mathbf{y}}^{(2)}, \dots, \bar{\mathbf{y}}^{(L)}$, the receiver performs a JDD process in an iterative manner over the factor graph presented in Fig. 2. Detectors perform N_{det} iterations and pass resulting messages to decoders, then decoders perform N_{dec} iterations and pass computed messages to detectors. This cycle is called one JDD iteration or a global iteration. Note that each detection iteration and each decoding iteration will be called a local iteration. We perform N_g global iterations for a whole JDD process.

We consider a low-complexity FG-GAI BP-based detection algorithm [33] and a sum-product decoding algorithm in a JDD process. We focus on the l -th detector, in which $\bar{y}_i^{(l)}$, $\bar{x}_i^{(l)}$ and $\bar{w}_i^{(l)}$ denote the i -th entry of $\bar{\mathbf{y}}^{(l)}$, $\bar{\mathbf{x}}^{(l)}$ and $\bar{\mathbf{w}}^{(l)}$,

respectively, and $\bar{h}_{ij}^{(l)}$ denotes the (i, j) -th entry of $\bar{\mathbf{H}}^{(l)}$. We rewrite (2) as [31]

$$\bar{y}_i^{(l)} = \bar{h}_{ik}^{(l)} \bar{x}_k^{(l)} + z_{ik}^{(l)}, \quad i = 1, \dots, 2n_R, \quad (3)$$

where $z_{ik}^{(l)} \triangleq \sum_{j=1, j \neq k}^{2n_U} \bar{h}_{ij}^{(l)} \bar{x}_j^{(l)} + \bar{w}_i^{(l)}$ is an interference plus noise in case of detecting symbol $\bar{x}_k^{(l)}$, $k = 1, \dots, 2n_U$, from the received signal $\bar{y}_i^{(l)}$. If n_U is large enough, $z_{ik}^{(l)}$ can be approximated as a Gaussian random variable [33] with a mean $\mu_{z_{ik}}^{(l)}$ and a variance $\sigma_{z_{ik}}^2$, where

$$\mu_{z_{ik}}^{(l)} = \sum_{j=1, j \neq k}^{2n_U} \bar{h}_{ij}^{(l)} \sum_{s \in \mathbb{A}} s \cdot \Pr^{(i)}\{\bar{x}_j^{(l)} = s\} \quad (4)$$

and

$$\sigma_{z_{ik}}^2 = \sum_{j=1, j \neq k}^{2n_U} (\bar{h}_{ij}^{(l)})^2 \left\{ \sum_{s \in \mathbb{A}} s^2 \cdot \Pr^{(i)}\{\bar{x}_j^{(l)} = s\} - \left(\sum_{s \in \mathbb{A}} s \cdot \Pr^{(i)}\{\bar{x}_j^{(l)} = s\} \right)^2 \right\} + \frac{\sigma^2}{2}. \quad (5)$$

Note that $\Pr^{(i)}\{\bar{x}_j^{(l)} = s\}$ denotes the *a priori* probability of $\bar{x}_j^{(l)}$ from the viewpoint of the observation node $o_i^{(l)}$.

Let us define $\alpha_{ik}^{(l)}(s) \triangleq \Pr\{\bar{y}_i^{(l)} | \bar{\mathbf{H}}^{(l)}, \bar{x}_k^{(l)} = s\}$ and $\beta_{ki}^{(l)}(s) \triangleq \Pr\{\bar{x}_k^{(l)} = s | \bar{\mathbf{H}}^{(l)}, \bar{\mathbf{y}}_i^{(l)}\}$ representing the likelihood and the extrinsic probability, respectively, of $\bar{x}_k^{(l)} = s$ evaluated at the observation node $o_i^{(l)}$ for a given $\bar{\mathbf{H}}^{(l)}$, where $\bar{\mathbf{y}}_i^{(l)}$ denotes a received signal vector excluding an entry $\bar{y}_i^{(l)}$. We let $\gamma_k^{(l)}(s) \triangleq \Pr\{\bar{x}_k^{(l)} = s\}$ represent the *a priori* probability of $\bar{x}_k^{(l)} = s$. Note that $\alpha_{ik}^{(l)}(s)$ and $\beta_{ki}^{(l)}(s)$ are obtained by [31], [33]

$$\alpha_{ik}^{(l)}(s) \approx \frac{1}{\sqrt{2\pi\sigma_{z_{ik}}^2}} \exp\left(-\frac{(\bar{y}_i^{(l)} - \bar{h}_{ik}^{(l)}s - \mu_{z_{ik}}^{(l)})^2}{2\sigma_{z_{ik}}^2}\right) \quad (6)$$

and

$$\beta_{ki}^{(l)}(s) = \kappa \prod_{j=1, j \neq i}^{2n_R} \alpha_{jk}^{(l)}(s) \cdot \gamma_k^{(l)}(s), \quad (7)$$

where κ is a constant. By the iterative detection principle, the extrinsic probability $\beta_{ji}^{(l)}(s)$ plays the role of the *a priori* probability $\Pr^{(i)}\{\bar{x}_j^{(l)} = s\}$ in (4) and (5) during iterations. So, for a given $\gamma_k^{(l)}(s)$, the message $\beta_{ki}^{(l)}(s)$ is used for updating $\mu_{z_{ik}}^{(l)}$ and $\sigma_{z_{ik}}^2$ by (4) and (5), and consequently updating $\alpha_{ik}^{(l)}(s)$ by (6). Of course, $\beta_{ki}^{(l)}(s)$ is updated by $\alpha_{ik}^{(l)}(s)$ as in (7). Consequently, $\alpha_{ik}^{(l)}(s)$ and $\beta_{ki}^{(l)}(s)$ are updated in a recursive manner via detection iterations. The FG-GAI BP detection algorithm is summarized in Algorithm 1.

After N_{det} detection iterations, each symbol node $s_k^{(l)}$ computes the log-likelihood ratios (LLR) of code bits composing the symbol $\bar{x}_k^{(l)}$ and delivers these to corresponding bit nodes in decoders. Let $a[t]$ denote the t -th bit in the bit-stream

Algorithm 1 FG-GAI BP Detection

```

1 for m = 1 to Ndet do
2   for i = 1 to 2nR do
3     for j = 1 to 2nU do
4       ξμij ← h̄ij(l) · ∑s∈A s · βji(l)(s)
5       ξσij2 ← (h̄ij(l))2 ∑s∈A s2 · βji(l)(s) - ξμij2
6       ξμi ← ∑j=12nU ξμij
7       ξσi2 ← ∑j=12nU ξσij2 + σ2/2
8       for k = 1 to 2nU do
9         μzik ← ξμi - ξμik
10        σzik2 ← ξσi2 - ξσik2
11        αik(l)(s) ← 1 / √(2πσzik2) exp(-((ȳi(l) - h̄ik(l)s - μzik)2 / (2σzik2)),
12        ∀s ∈ A.
13   for k = 1 to 2nU do
14     βk(s) ← κ ∏j=12nR αjk(l)(s) · γk(l)(s), ∀s ∈ A
15     for i = 1 to 2nR do
16       βki(l)(s) ← βk(s) / αik(l)(s), ∀s ∈ A
    
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composing a symbol a . The LLR of $\bar{x}_k^{(l)}[t]$ is defined by

$$L(\bar{x}_k^{(l)}[t]) \triangleq \log \frac{\Pr\{\bar{x}_k^{(l)}[t]=0\}}{\Pr\{\bar{x}_k^{(l)}[t]=1\}} \text{ and obtained as [33]}$$

$$L(\bar{x}_k^{(l)}[t]) = \log \frac{\sum_{s \in \mathbb{S}_t^0} \Pr\{\bar{x}_k^{(l)} = s | \bar{\mathbf{H}}^{(l)}, \bar{\mathbf{y}}^{(l)}\}}{\sum_{s \in \mathbb{S}_t^1} \Pr\{\bar{x}_k^{(l)} = s | \bar{\mathbf{H}}^{(l)}, \bar{\mathbf{y}}^{(l)}\}} = \log \frac{\sum_{s \in \mathbb{S}_t^0} \prod_{i=1}^{2n_R} \alpha_{ik}^{(l)}(s)}{\sum_{s \in \mathbb{S}_t^1} \prod_{i=1}^{2n_R} \alpha_{ik}^{(l)}(s)}, \quad (8)$$

where $\mathbb{S}_t^0 = \{s|s[t] = 0\}$ and $\mathbb{S}_t^1 = \{s|s[t] = 1\}$, and the last equality comes from $\Pr\{\bar{x}_k^{(l)} = s | \bar{\mathbf{H}}^{(l)}, \bar{\mathbf{y}}^{(l)}\} \propto \prod_{i=1}^{2n_R} \Pr\{\bar{y}_i^{(l)} | \bar{\mathbf{H}}^{(l)}, \bar{x}_k^{(l)} = s\}$. If we let $r = (2(l-1) + \text{mod}(k+1, 2)) \cdot \log_2 \sqrt{M_o} + t$, the message $L(\bar{x}_k^{(l)}[t])$ is delivered to the r -th bit node in the k -th decoder, denoted by v_r^k .

By using $L(\bar{x}_k^{(l)}[t])$ obtained in (8) as the channel LLR of the corresponding bit node $v = v_r^k$, i.e., $L_v = L(\bar{x}_k^{(l)}[t])$ for $v = v_r^k$, the sum-product decoding is performed in an iterative manner. Let L_{vc} and L_{cv} denote the message sent from the bit node v to the check node c , and the message sent from the check node c to the bit node v , respectively. Then, these messages are updated [24] as

$$L_{vc} = L_v + \sum_{c' \in C_v \setminus c} L_{c'v}, \quad (9)$$

and

$$L_{cv} = \prod_{v' \in V_c \setminus v} \text{sign}(L_{v'c}) \cdot \phi\left(\sum_{v' \in V_c \setminus v} \phi(|L_{v'c}|)\right), \quad (10)$$

where $\phi(x) = \log \frac{\exp(x)+1}{\exp(x)-1}$. Note that $C_v \setminus c$ denotes the set of check nodes except c connected to the bit node v and $V_c \setminus v$

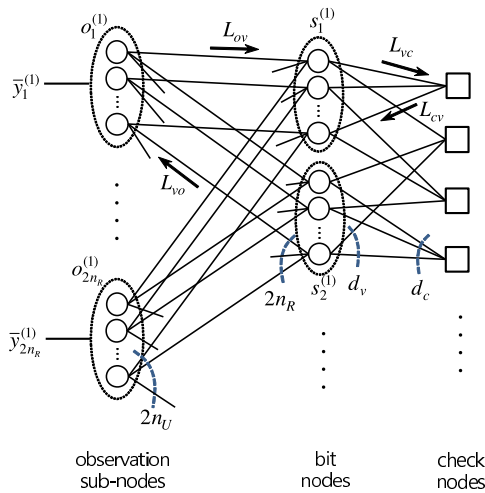


FIGURE 3. Factor graph of LDPC coded MU massive MIMO system for the bit-level EXIT analysis of JDD.

denotes the set of bit nodes except v connected to the check node c .

After N_{dec} decoding iterations, the LLR of $\bar{x}_k^{(l)}[t]$ is computed as $L'(\bar{x}_k^{(l)}[t]) = \sum_{c \in C_{v_r}^k} L_{cv_r}^k$ and delivered to the symbol node $s_k^{(l)}$ in the detector l . Then, the *a priori* probability $\gamma_k^{(l)}(s)$ is computed as

$$\gamma_k^{(l)}(s) = \prod_{t=1}^{\log_2 \sqrt{M_o}} \frac{\exp((1-s[t]) \cdot L'(\bar{x}_k^{(l)}[t]))}{1 + \exp(L'(\bar{x}_k^{(l)}[t]))} \quad (11)$$

and used in (7) at the next global iteration.

After N_g global iterations, we make decision on each code bit such that $\bar{x}_k^{(l)}[t]$ is estimated as 1 if $L(\bar{x}_k^{(l)}[t]) + L'(\bar{x}_k^{(l)}[t]) < 0$ and as 0 otherwise. The overall procedure of JDD is presented in Algorithm 2.

Algorithm 2 Joint Detection and Decoding (JDD)

- 1 Initialize : $\beta_{ki}^{(l)}(s) = \frac{1}{\sqrt{M_o}}, \forall l, i, k, s,$
 $\Pr\{\bar{x}_k^{(l)} = s\} = \frac{1}{\sqrt{M_o}}, \forall l, k, s,$ and $L_{cv} = 0, \forall c, v.$
- 2 **for** $l' = 1$ to N_g **do**
- 3 **for** $l = 1$ to L **do**
- 4 Run Algorithm 1 in the l -th detector.
- 5 Compute $L_v, \forall v,$ by (8).
- 6 **for** $l'' = 1$ to N_{dec} **do**
- 7 Update L_{vc} and $L_{cv}, \forall v, c,$ by (9) and (10), respectively.
- 8 Compute $\gamma_k^{(l)}(s), \forall l, k, s,$ by (11).
- 9 Determine the value of code bit corresponding to $v, \forall v$

IV. EXIT ANALYSIS OF JOINT DETECTION AND DECODING

We propose an analysis tool for studying the behavior of JDD in the coded MU massive MIMO system in terms of extrinsic

information transfer (EXIT) characteristics of component units. We investigate the mutual information between code bits and corresponding soft-valued messages traveling in JDD. For this purpose, we construct a bit-level factor graph by decomposing observation nodes and symbol nodes in Fig. 2 as sub-nodes, where each observation node and symbol node is decomposed into $\log_2 \sqrt{M_o}$ sub-nodes. Each symbol sub-node is connected to the corresponding bit node in an one-to-one manner, so that symbol sub-nodes can be simply merged by bit nodes. As a result, we obtain a bit-level factor graph as shown in Fig. 3, where only a part of graph associated with observation nodes $o_1^{(1)}, \dots, o_{2n_R}^{(1)}$ and symbol nodes $s_1^{(1)}, s_2^{(1)}$ is depicted.

Let L_{vo} and L_{ov} denote bit LLRs sent from a bit node to an observation sub-node and from an observation sub-node to a bit node, respectively. For an observation node $o = o_i^{(l)}$ and a bit node v corresponding to $\bar{x}_k^{(l)}[t]$, the bit LLRs are obtained by $L_{vo} = \log \frac{\sum_{s \in \mathbb{S}_0^l} \beta_{ki}^{(l)}(s)}{\sum_{s \in \mathbb{S}_1^l} \beta_{ki}^{(l)}(s)}$ and $L_{ov} = \log \frac{\sum_{s \in \mathbb{S}_0^l} \alpha_{ik}^{(l)}(s)}{\sum_{s \in \mathbb{S}_1^l} \alpha_{ik}^{(l)}(s)}$. By letting U denote a code bit, we define $I_{VO} = I(U; L_{vo})$ and $I_{OV} = I(U; L_{ov})$, where $I(U; X)$ is the mutual information between U and X . Note that I_{VO} and I_{OV} are input and output, respectively, of observation sub-nodes in terms of bit-level EXIT characteristics. We also define $I_{CV} = I(U; L_{cv})$ and $I_{VC} = I(U; L_{vc})$, where I_{CV} and I_{VC} are input and output, respectively, of bit nodes as well as output and input, respectively, of check nodes in terms of EXIT characteristics.

Allowing slight abuse of notation, we represent I_{CV} associated with degree- d_c check nodes as $I_{CV}(d_c)$. We also represent I_{VC} associated with degree- d_v bit nodes as $I_{VC}(d_v)$. In the similar manner, we use $I_{VO}(d_v)$ and $I_{OV}(d_v)$ to denote I_{VO} and I_{OV} , respectively, associated with degree- d_v bit nodes. Then, we define averages of these as $\bar{I}_{CV} = \sum_{d_c=2}^{d_{c,max}} \rho_{d_c} I_{CV}(d_c), \bar{I}_{VC} = \sum_{d_v=2}^{d_{v,max}} \lambda_{d_v} I_{VC}(d_v)$ and $\bar{I}_{OV} = \sum_{d_v=2}^{d_{v,max}} \lambda_{d_v} I_{OV}(d_v)$, where λ_{d_v} and ρ_{d_c} denote the fractions of edges that are connected to degree- d_v bit nodes and degree- d_c check nodes, respectively, and $d_{v,max}$ and $d_{c,max}$ are maxima of d_v and d_c , respectively.

By defining $J(\sigma_X)$ as [27]

$$J(\sigma_X) = 1 - \int_{-\infty}^{\infty} \frac{e^{-(\xi - \sigma_X^2/2)^2/2\sigma_X^2}}{\sqrt{2\pi\sigma_X^2}} \cdot \log_2[1 + e^{-\xi}] d\xi, \quad (12)$$

we obtain $I(U; X) = J(\sigma_X)$, where σ_X^2 is the variance of a normally distributed random variable X . At bit nodes, the message going to a target node is generated by summing up all incoming messages except one from a target node, so L_{vo} is obtained by summing up $2n_R - 1$ copies of L_{ov} and d_v copies of L_{cv} . Then, $I_{VO}(d_v)$ is obtained as

$$I_{VO}(d_v) = J\left(\sqrt{(2n_R - 1) \cdot [J^{-1}(I_{OV}(d_v))]^2 + d_v \cdot [J^{-1}(\bar{I}_{CV})]^2}\right), \quad (13)$$

Algorithm 3 Density Evolution in Terms of EXIT Characteristics

```

1 Initialize:  $\bar{I}_{CV} = 0$  and  $I_{OV}(d_v) = 0, \forall d_v$ 
2 for  $l' = 1$  to  $N_g$  do
3   for  $d_v = 2$  to  $d_{v,\max}$  do
4     for  $m = 1$  to  $N_{det}$  do
5        $I_{OV}(d_v) \leftarrow f_O \left( J \left( \sqrt{(2n_R - 1) \cdot [J^{-1}(I_{OV}(d_v))]^2 + d_v \cdot [J^{-1}(\bar{I}_{CV})]^2} \right), \frac{E_b}{N_0} \right)$ 
6    $\bar{I}_{OV} \leftarrow \sum_{d_v} \lambda_{d_v} I_{OV}(d_v)$ 
7   for  $l'' = 1$  to  $N_{dec}$  do
8      $\bar{I}_{VC} \leftarrow \sum_{d_v=1}^{d_{v,\max}} \lambda_{d_v} \cdot J \left( \sqrt{2n_R \cdot [J^{-1}(I_{OV}(d_v))]^2 + (d_v - 1) \cdot [J^{-1}(\bar{I}_{CV})]^2} \right)$ 
9      $\bar{I}_{CV} \leftarrow \sum_{d_c=1}^{d_{c,\max}} \rho_{d_c} \cdot (1 - J(\sqrt{d_c - 1} \cdot J^{-1}(1 - \bar{I}_{VC})))$ 

```

where $I_{OV}(d_v)$ is defined as a function of $I_{VO}(d_v)$ as

$$I_{OV}(d_v) = f_O \left(I_{VO}(d_v), \frac{E_b}{N_0} \right). \quad (14)$$

The EXIT function of observation sub-node, $f_O(\cdot)$, is obtained in a polynomial form by using Monte Carlo simulation [27] and a curve fitting technique. In the same manner, the LLR message L_{VC} is obtained by summing up $2n_R$ copies of L_{ov} and $d_v - 1$ copies of L_{cv} . Thus,

$$I_{VC}(d_v) = J \left(\sqrt{2n_R \cdot [J^{-1}(I_{OV}(d_v))]^2 + (d_v - 1) \cdot [J^{-1}(\bar{I}_{CV})]^2} \right), \quad (15)$$

where the EXIT function of check node is defined as [27]

$$I_{CV}(d_c) \approx 1 - J \left(\sqrt{d_c - 1} \cdot J^{-1}(1 - \bar{I}_{VC}) \right). \quad (16)$$

The density evolution of soft messages exchanged in the JDD process in terms of EXIT characteristics is summarized in Algorithm 3.

By running Algorithm 3, we trace the value of the average mutual information \bar{I}_{VC} of output message at the bit node updated through iterations. If the value of \bar{I}_{VC} reaches 1 through a sufficient number of iterations at a certain E_b/N_0 , this implies that the JDD converges and the decoding succeeds at this E_b/N_0 . The minimum value of E_b/N_0 at which \bar{I}_{VC} converges to 1 is defined as the threshold.

Let us consider 3-D EXIT chart composed of two EXIT surfaces, each of which represents EXIT characteristics of bit node and check node, respectively. The space between two EXIT surfaces may or may not form a tunnel with an entry and an exit. If a tunnel is formed, JDD trajectory penetrates through the tunnel by 3-D zigzag movements and reaches the point of $\bar{I}_{VC} = 1$. This situation corresponds to the convergence of JDD and a low BER is obtained. On the other hand, if a tunnel is not formed, JDD trajectory gets stuck at a point of $\bar{I}_{VC} < 1$ resulting in high BER. We can obtain JDD trajectory by connecting points of \bar{I}_{VC} , \bar{I}_{CV} and \bar{I}_{OV} , which are obtained by running Algorithm 3,

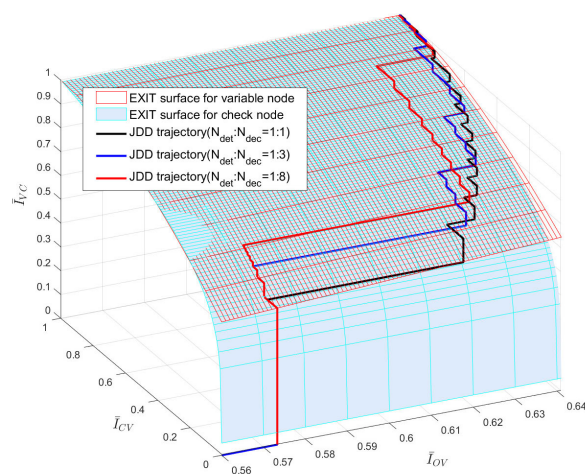


FIGURE 4. 3-D EXIT chart and trajectories of JDD for (3, 6)-regular LDPC coded MU massive MIMO system with $n_U = 10$ and $n_R = 64$ at $E_b/N_0 = -6$ [dB], where 4-QAM is used for modulation. Note that \bar{I}_{OV} denotes the mutual information per observation node.

with straight lines. By a detection iteration, the trajectory experiences a straight movement in the direction of \bar{I}_{OV} -axis. By a decoding iteration, the trajectory experiences a pair of orthogonal movements on the \bar{I}_{VC} - \bar{I}_{CV} plane. Fig. 4 shows an example of 3-D EXIT chart and some JDD trajectories obtained with different JDD strategies, which means the value of $N_{det} : N_{dec}$ composing one global iteration. It is observed that using different JDD strategies result in distinct JDD trajectories. A JDD trajectory reaching the point of $\bar{I}_{VC} = 1$ with a small number of zigzag movements implies a fast convergence of JDD. Since a tunnel between EXIT surfaces is uneven or bumpy, we need to design an efficient strategy for trajectory movement.

Thus, for the purpose of designing LDPC coded MU massive MIMO system to attain low BER with a fast convergence speed, we need to judge if a tunnel exists between two EXIT surfaces and investigate by how many zigzag movements JDD trajectory reaches the point of $\bar{I}_{VC} = 1$. As an efficient tool for conducting these judgement

and investigation, we propose an EXIT analysis tool, which traces the value of \bar{I}_{VC} by running Algorithm 3. The details of the proposed EXIT analysis tool is presented in Sec. V.

V. DESIGN OF LDPC CODES AND JDD STRATEGY

We have mainly two design parameters for building LDPC coded MU massive MIMO system. The first one is the degree distribution of factor graph representing LDPC codes. The second one is the ratio of $N_{det} : N_{dec}$ composing one global iteration of JDD process, which is called a JDD strategy. Note that the first design parameter determines the existence of a tunnel for a given E_b/N_0 , while the second design parameter determines the convergence speed of JDD.

To design JDD strategy resulting in a fast convergence, we trace the evolution of \bar{I}_{VC} with respect to the total number of local iterations by running Algorithm 3 as shown in Fig. 5. Recall that a local iteration means each detection iteration and decoding iteration. Note that N_g global iterations conducted with the ratio $N_{det} : N_{dec}$ result in total $N_g(N_{det} + N_{dec})$ local iterations. A smaller number of local iterations required for \bar{I}_{VC} to reach 1 implies a faster convergence speed of JDD. It is clear that different JDD strategies result in distinct convergence speeds.

We design LDPC codes for the MU massive MIMO system by using the proposed EXIT analysis tool so that the lowest threshold and the fastest convergence of JDD are achieved. In general, LDPC codes are designed through two steps, which are the degree distribution optimization and the edge placement between bit nodes and check nodes, or a parity-check matrix construction. Degree distributions of bit nodes and check nodes from the edge perspective are represented in the form of polynomials as [23]

$$\lambda(x) = \sum_{d_v=2}^{d_v,max} \lambda_{d_v} x^{d_v-1} \text{ and } \rho(x) = \sum_{d_c=2}^{d_c,max} \rho_{d_c} x^{d_c-1}, \quad (17)$$

respectively. Then, the code rate R is given by [23]

$$R(\lambda, \rho) = 1 - \frac{\sum_{d_c=2}^{d_c,max} \rho_{d_c}/d_c}{\sum_{d_v=2}^{d_v,max} \lambda_{d_v}/d_v}, \quad (18)$$

where $\lambda = \{\lambda_2, \dots, \lambda_{d_v,max}\}$ and $\rho = \{\rho_2, \dots, \rho_{d_c,max}\}$.

We determine degree distributions λ and ρ to maximize the code rate $R(\lambda, \rho)$ guaranteeing the convergence of JDD at a given E_b/N_0 by using the EXIT analysis introduced in Sec. IV. For a given target code rate, we perform the EXIT analysis with various values of E_b/N_0 and find the smallest E_b/N_0 resulting in the maximum $R(\lambda, \rho)$ exceeding the target code rate. Such E_b/N_0 is called the threshold and will be denoted by $(E_b/N_0)^*$. The corresponding degree distributions are considered optimal and will be denoted by (λ^*, ρ^*) .

Next, we place edges between bit nodes and check nodes in a factor graph based on (λ^*, ρ^*) to satisfy the following criteria [23]:

- (a) Avoid short cycles involving only degree-2 bit nodes.

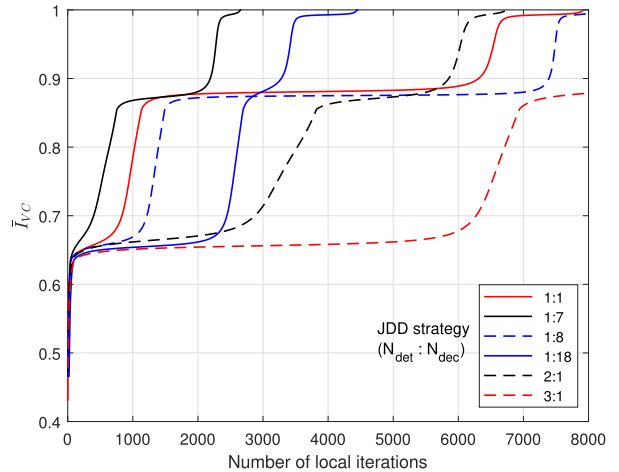


FIGURE 5. Evolution of \bar{I}_{VC} with respect to the total number of local iterations for some JDD strategies, where $R = 1/2$, $n_U = 6$, $n_R = 16$.

- (b) Length-4 cycles need to be avoided.
- (c) All degree-2 bit nodes need to represent only non-systematic bits.

The criterion (c) is satisfied if the following condition

$$\lambda_2 \leq 2 \sum_{d_c=2}^{d_c,max} \rho_{d_c}/d_c \quad (19)$$

is met [31]. Furthermore, the condition (19) can be incorporated in the degree distribution optimization process as a constraint [31]. The criteria (a) and (b) can be satisfied by placing edges between nodes based on the progressive edge growth (PEG) algorithm [36]. Then, the degree distribution for a given E_b/N_0 is determined as

$$\begin{aligned} & \max_{\lambda, \rho} R(\lambda, \rho) \\ & \text{s.t. } \bar{I}_{VC} = 1 \text{ after running Algorithm 3,} \\ & \lambda_2 \leq 2 \sum_{d_c=2}^{d_c,max} \rho_{d_c}/d_c, \\ & \sum_{d_c=2}^{d_c,max} \rho_{d_c} = \sum_{d_v=2}^{d_v,max} \lambda_{d_v} = 1 \text{ with } \rho_{d_c}, \lambda_{d_v} \geq 0, \end{aligned} \quad (20)$$

where the first constraint guarantees the convergence of JDD and the second constraint is used to satisfy the criterion (c) introduced earlier. The lowest E_b/N_0 , at which the resultant maximum $R(\lambda, \rho)$ exceeds the target rate, is considered the threshold $(E_b/N_0)^*$ and the corresponding degree distributions are defined as (λ^*, ρ^*) .

In addition, we take into consideration the convergence speed of JDD when designing LDPC codes. The fastest convergence of JDD algorithm can be achieved by selecting the optimal JDD strategy. The convergence speed is predicted by observing the evolution of \bar{I}_{VC} as shown in Fig. 5. The degree distribution achieving $(E_b/N_0)^*$ as well as the fastest JDD convergence will be considered the overall optimal and will be denoted as $(\lambda^\dagger, \rho^\dagger)$. Consequently, LDPC codes for

TABLE 1. Optimal degree distributions (λ^* , ρ^*) of rate-1/2 LDPC codes for MU massive MIMO system with some candidate JDD strategies, where $(n_U \times n_R) = (6 \times 16)$ and (6×64) are considered, and $d_{v,max} = 24$ is used.

$n_U \times n_R$	6×16						6×64						
	$N_{det} : N_{dec}$	1 : 1	1 : 7	1 : 8	1 : 18	2 : 1	3 : 1	1 : 1	1 : 2	1 : 6	1 : 7	1 : 8	1 : 10
λ_2		0.25766	0.25765	0.25757	0.25685	0.2566	0.25655	0.23112	0.23019	0.23319	0.231	0.23196	0.22944
λ_3		0.2013	0.20204	0.1999	0.20808	0.19995	0.2086	0.18313	0.16131	0.19338	0.16081	0.18372	0.17621
λ_4		0.02609	0.00878	0.01077	0.00219	0.00784	0.00435		0.00732	0.04823			0.00524
λ_5			0.00124					0.09537	0.17804	0.06896	0.0211	0.09714	0.02665
λ_6		0.05778	0.1703	0.17942	0.15828	0.22033	0.10628				0.11916		0.22411
λ_7		0.1795	0.0523	0.04517	0.0122	0.00247	0.13497	0.16213		0.18653	0.10778	0.16518	
λ_8			0.03416		0.09028			0.02464	0.10835	0.02645		0.01946	
λ_9		0.00771		0.03759	0.00978	0.04268	0.02393	0.00465		0.00466	0.0067	0.00451	0.01232
λ_{10}			0.01267					0.00399				0.00398	0.00152
λ_{11}				0.00183	0.00168		0.00709		0.00535	0.0051			
λ_{12}						0.01388			0.00359	0.00119		0.00604	0.01366
λ_{13}		0.01002	0.00119	0.00949	0.00499				0.00183				
λ_{14}		0.00161			0.00416	0.00745	0.00025		0.00161			0.00071	
λ_{15}		0.01579	0.00181	0.00348		0.00363	0.00934			0.00288			
λ_{16}		0.00637	0.01071					0.0025					
λ_{17}			0.00128	0.00915		0.00363	0.00424						0.00909
λ_{18}		0.00919	0.01374		0.01053		0.00411						
λ_{19}					0.0107								
λ_{20}						0.00224					0.00232		
λ_{21}				0.03951					0.0055	0.01999		0.0026	0.00782
λ_{22}								0.0014	0.00317		0.00196		
λ_{23}									0.04048				
λ_{24}		0.22699	0.23212	0.20612	0.23027	0.2393	0.24028	0.29107	0.25326	0.25766	0.30095	0.2847	0.29396
ρ_7		0.01947	0.03564	0.01598	0.00479	0.02296	0.00003						
ρ_8		0.98053	0.96436	0.98402	0.99521	0.97704	0.99997	0.456	0.39566	0.54993	0.42131	0.48963	0.38846
ρ_9								0.544	0.60434	0.45007	0.57869	0.51037	0.61154
Capacity		-3.63 [dB]						-9.99 [dB]					
$(E_b/N_0)^*$		-3.53 [dB]						-9.77 [dB]					

TABLE 2. Optimal degree distributions (λ^* , ρ^*) of rate-1/2 LDPC codes for MU massive MIMO system with some candidate JDD strategies, where $(n_U \times n_R) = (10 \times 16)$ and (10×64) are considered, and $d_{v,max} = 24$ is used.

$n_U \times n_R$	10×16						10×64						
	$N_{det} : N_{dec}$	1 : 1	1 : 6	1 : 7	1 : 10	1 : 18	3 : 1	1 : 1	1 : 6	1 : 7	1 : 10	2 : 1	3 : 1
λ_2		0.2709	0.26778	0.26801	0.26972	0.26751	0.26698	0.23752	0.23444	0.23545	0.23687	0.2387	0.23685
λ_3		0.18545	0.19357	0.21132	0.1982	0.19252	0.19074	0.18626	0.17452	0.17362	0.1406	0.1853	0.18946
λ_4		0.04003	0.01868		0.00493		0.01594	0.00825	0.00254	0.00776	0.09976	0.02727	0.0022
λ_5		0.04319	0.04781		0.03902	0.09724	0.05435	0.00782	0.08786	0.05923	0.02338	0.0059	0.00924
λ_6		0.04879	0.08192	0.19122	0.13976	0.0517	0.07636	0.1947	0.13674	0.18362	0.06668	0.17099	0.18466
λ_7		0.09536	0.05975		0.02508	0.07298	0.08029	0.03045	0.00286		0.0763		0.07437
λ_8		0.02205	0.03393	0.03733	0.02658		0.01597	0.04218	0.00712	0.01101	0.0545		0.00642
λ_9				0.0116		0.00056		0.00606	0.04328			0.09997	
λ_{10}						0.01438				0.01748			
λ_{11}			0.00284	0.00438				0.00081	0.00043				0.00231
λ_{12}						0.00069	0.0003	0.00052	0.01073	0.01443			
λ_{13}		0.0194		0.00037	0.00417							0.00194	
λ_{14}		0.00965	0.00507		0.00533	0.00932	0.00379			0.00331	0.00263	0.00033	
λ_{15}		0.00071											0.00894
λ_{16}			0.00349			0.00406					0.00102		
λ_{17}		0.0006			0.00146		0.00048		0.00729	0.00096		0.00015	
λ_{18}					0.00162	0.01984							
λ_{19}			0.01192		0.04071		0.00965			0.00239		0.00121	0.00506
λ_{20}									0.00055				
λ_{21}													0.00386
λ_{22}				0.00774				0.00279		0.01036	0.01544		0.01487
λ_{23}				0.04673				0.023			0.00009	0.02661	
λ_{24}		0.26387	0.27324	0.22131	0.24341	0.26922	0.28515	0.25965	0.29164	0.28039	0.28272	0.24163	0.26177
ρ_7		0.10198	0.04967	0.12017	0.08971	0.03138	0.0152						
ρ_8		0.89802	0.95033	0.87983	0.91029	0.96862	0.9848	0.60457	0.49241	0.5244	0.51169	0.64375	0.59523
ρ_9								0.39543	0.50759	0.4756	0.48831	0.35625	0.40477
Capacity		-1.16 [dB]						-7.67 [dB]					
$(E_b/N_0)^*$		-1.09 [dB]						-7.61 [dB]					

TABLE 3. Optimal degree distributions (λ^* , ρ^*) of rate-3/4 LDPC codes for MU massive MIMO system with some candidate JDD strategies, where $(n_U \times n_R) = (6 \times 16)$ and (6×64) are considered, and $d_{v,max} = 20$ is used.

$n_U \times n_R$	6×16						6×64						
	$N_{det} : N_{dec}$	1 : 1	1 : 2	1 : 6	1 : 8	1 : 10	2 : 1	1 : 1	1 : 6	1 : 7	1 : 8	1 : 18	3 : 1
λ_2		0.25153	0.25086	0.24979	0.25131	0.25126	0.25067	0.22195	0.22112	0.22072	0.22114	0.22115	0.22142
λ_3		0.24695	0.25841	0.27338	0.25148	0.25333	0.26273	0.22886	0.2405	0.2413	0.24071	0.2369	0.23387
λ_4		0.05149	0.02339	0.00034	0.04076	0.03551	0.01418					0.01362	
λ_5								0.09901				0.01389	0.08153
λ_6									0.12908	0.10564	0.12005		
λ_7			0.10298			0.0579	0.12295		0.02053	0.0768	0.05256	0.23249	
λ_8	0.12851			0.22142	0.20515			0.20539	0.11484	0.10959	0.03423		0.21368
λ_9	0.03275	0.20663	0.12047	0.01264	0.26284	0.14833	0.00426				0.12453	0.01927	0.02975
λ_{10}	0.20208					0.03968	0.06396		0.06723	0.00916			
λ_{11}		0.06785			0.18716	0.01832		0.03067			0.03487	0.08481	0.01279
λ_{12}			0.07289		0.01299		0.07982		0.08308	0.04628			
λ_{13}	0.05706							0.0973				0.09032	0.13317
λ_{14}										0.02632	0.09923		
λ_{15}		0.05111						0.07881		0.15996	0.01277	0.01471	
λ_{16}			0.03335						0.07306				0.03233
λ_{17}	0.00267	0.02488				0.07574	0.04426						
λ_{18}					0.00508			0.02562	0.04878			0.05029	0.00058
λ_{19}			0.01056										
λ_{20}	0.02695	0.01389	0.0178	0.03343	0.00541	0.0131		0.00812	0.00178	0.00423	0.05991	0.02254	0.04086
ρ_{14}	0.00079	0.00074	0.00015	0.00101	0.00059	0.00054							
ρ_{15}	0.99921	0.99926	0.99985	0.99899	0.99941	0.99946		0.00525	0.00031	0.00251	0.0018	0.00923	0.00046
ρ_{16}								0.99475	0.99969	0.99749	0.9982	0.99077	0.99954
Capacity				-2.10 [dB]							-8.50 [dB]		
$(E_b/N_0)^*$				-2.00 [dB]							-8.35 [dB]		

TABLE 4. Optimal degree distributions (λ^* , ρ^*) of rate-3/4 LDPC codes for MU massive MIMO system with some candidate JDD strategies, where $(n_U \times n_R) = (10 \times 16)$ and (10×64) are considered, and $d_{v,max} = 20$ is used.

$n_U \times n_R$	10×16						10×64						
	$N_{det} : N_{dec}$	1 : 1	1 : 2	1 : 8	1 : 10	2 : 1	3 : 1	1 : 1	1 : 2	1 : 6	1 : 7	1 : 8	1 : 18
λ_2		0.2638	0.26397	0.26392	0.26427	0.26471	0.26482	0.22379	0.22471	0.22413	0.2245	0.22363	0.22479
λ_3		0.26233	0.26143	0.26119	0.26147	0.251	0.24719	0.24426	0.22361	0.2377	0.233	0.24104	0.22962
λ_4		0.01095	0.00065	0.01539	0.00131	0.02064	0.03047	0.00426	0.04376	0.00533	0.0085	0	0.01776
λ_5		0.01276	0.02095			0.03983	0.04445	0.01578					0.0108
λ_6				0.00997	0.08914				0.059	0.1501	0.17402	0.15169	0.09403
λ_7			0.15616		0.00082			0.1592	0.07237				0.10437
λ_8	0.22554			0.24725	0.11206	0.05857	0.00336						
λ_9	0.00609	0.0365				0.17618	0.15552	0.0864	0.12129	0.09314	0.01348	0.13377	
λ_{10}				0.00108				0.12778	0.05605	0.12671	0.13009	0.0411	
λ_{11}									0.08064	0.0004	0.06125	0.04783	0.07553
λ_{12}	0.00071				0.14117	0.01316	0.14174	0.00546					
λ_{13}		0.15231								0.00979			0.15306
λ_{14}												0.06048	
λ_{15}	0.15666			0.08371							0.06892		0.08022
λ_{16}						0.08967				0.12144			
λ_{17}							0.06059	0.08642			0.06922		
λ_{18}		0.03663	0.0331	0.06569								0.09011	
λ_{19}									0.06504				
λ_{20}	0.06116	0.0714	0.08439	0.06408	0.08624	0.05187		0.04665	0.05354	0.03125	0.01702	0.01034	0.00983
ρ_{14}	0.00053	0.00664	0.00491	0.00733	0.00043	0.0009							
ρ_{15}	0.99947	0.99336	0.99509	0.99267	0.99957	0.9991		0.00186	0.0013	0.00318	0.00596	0.00363	0.00549
ρ_{16}								0.99814	0.9987	0.99682	0.99404	0.99637	0.99451
Capacity				0.33 [dB]							-6.25 [dB]		
$(E_b/N_0)^*$				0.39 [dB]							-6.12 [dB]		

MU massive MIMO system are constructed by the following procedure:

- (i) perform the optimization process (20) for various E_b/N_0 and candidate JDD strategies.
- (ii) determine $(E_b/N_0)^*$ and the optimal JDD strategy, and find the corresponding degree distribution $(\lambda^\dagger, \rho^\dagger)$.

- (iii) construct the parity-check matrix of LDPC codes from $(\lambda^\dagger, \rho^\dagger)$ by using the PEG algorithm.

By following the above procedure, we can efficiently construct LDPC codes for MU massive MIMO system showing the lower threshold $(E_b/N_0)^*$, or equivalently the better error correcting capability, and the faster convergence of JDD.

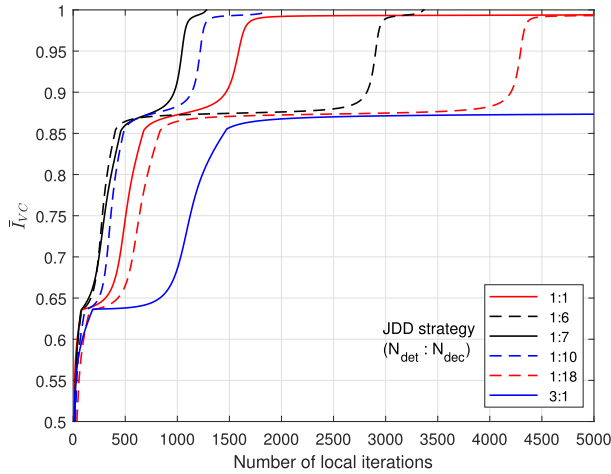


FIGURE 6. Evolution of \bar{I}_{VC} with respect to the total number of local iterations for some JDD strategies, where $R = 1/2$, $n_U = 10$, $n_R = 16$.

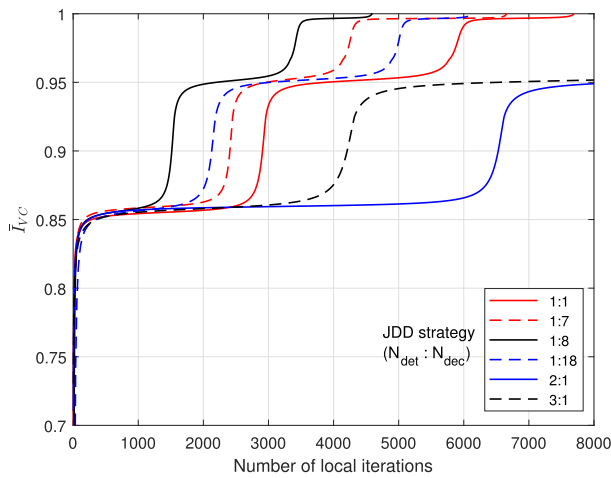


FIGURE 7. Evolution of \bar{I}_{VC} with respect to the total number of local iterations for some JDD strategies, where $R = 3/4$, $n_U = 6$, $n_R = 16$.

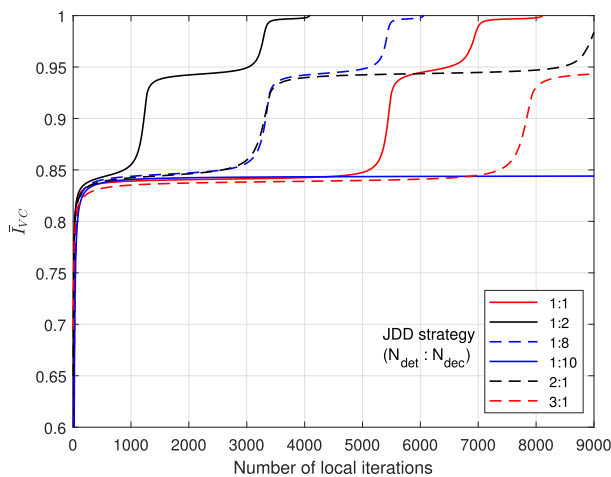


FIGURE 8. Evolution of \bar{I}_{VC} with respect to the total number of local iterations for some JDD strategies, where $R = 3/4$, $n_U = 10$, $n_R = 16$.

VI. NUMERICAL RESULTS

We consider LDPC coded MU massive MIMO systems with $n_U = 6$ or 10 and $n_R = 16$ or 64 . We call the MU massive

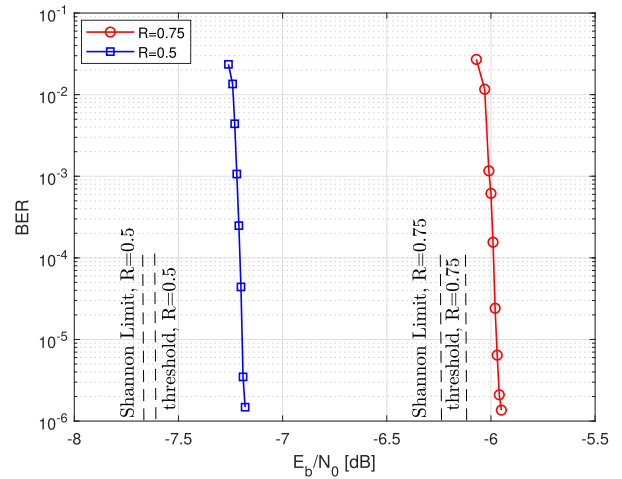


FIGURE 9. BER of LDPC coded MU massive MIMO system with $N = 64000$, $n_U = 10$ and $n_R = 64$, where the total number of local iterations is high enough to result in the convergence of JDD.

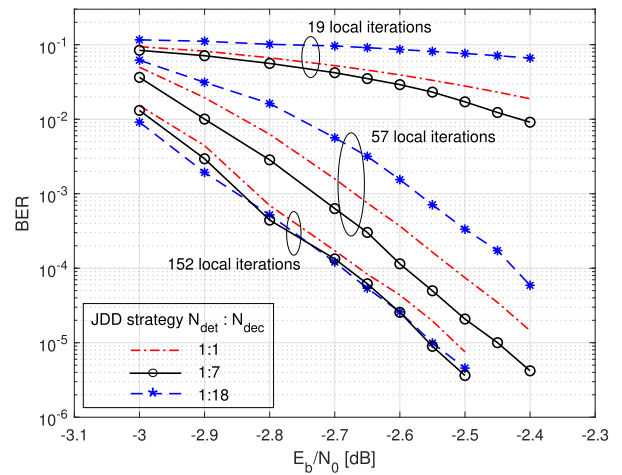


FIGURE 10. BER of rate-1/2 LDPC coded MU massive MIMO system with $N = 2304$ over 6×16 channel obtained with different JDD strategies and different total numbers of local iterations.

MIMO system with n_U UEs and n_R BS antennas as $n_U \times n_R$ channel. We suppose the channel gain of each pair of UE and BS antenna is i.i.d. complex Gaussian with zero mean and unit variance, while additive noises at BS antennas are assumed to be i.i.d. zero-mean circular symmetric complex white Gaussian. Each UE encodes information bits to rate-1/2 or 3/4 LDPC codes and maps code bits to 4-QAM symbols by Gray-mapping.

We solve the optimization problem (20) for each E_b/N_0 and JDD strategy by using the differential evolution algorithm [37]. In degree distribution optimization, we use the concentrated check node degree distribution [24], i.e., $\rho(x) = \rho_{dc}x^{d_c} + (1 - \rho_{dc})x^{d_c+1}$. For each massive MIMO channel and code rate under consideration, we find degree distributions (λ^*, ρ^*) achieving the threshold $(E_b/N_0)^*$ by distinct JDD strategies. Among (λ^*, ρ^*) found for various JDD strategies, we determine the overall optimal distribution $(\lambda^\dagger, \rho^\dagger)$ resulting in the fastest JDD convergence.

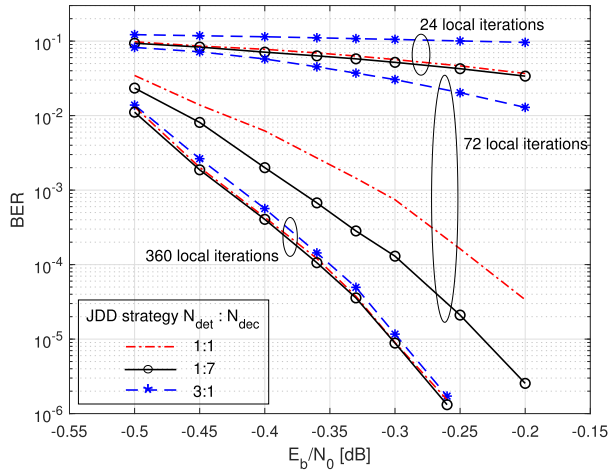


FIGURE 11. BER of rate-1/2 LDPC coded MU massive MIMO system with $N = 2304$ over 10×16 channel obtained with different JDD strategies and different total numbers of local iterations.

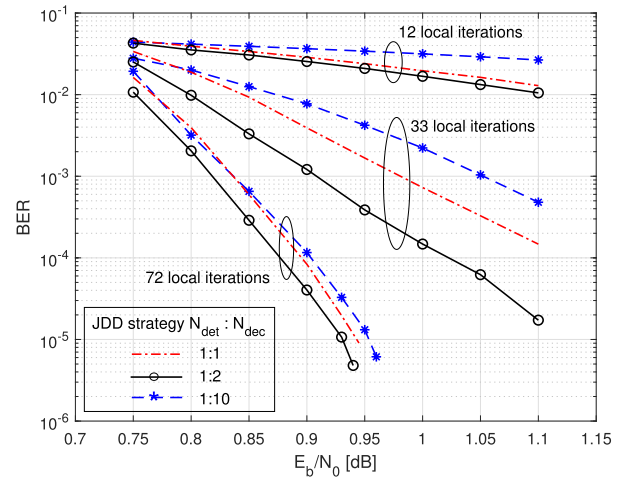


FIGURE 13. BER of rate-3/4 LDPC coded MU massive MIMO system with $N = 4608$ over 10×16 channel obtained with different JDD strategies and different total numbers of local iterations.

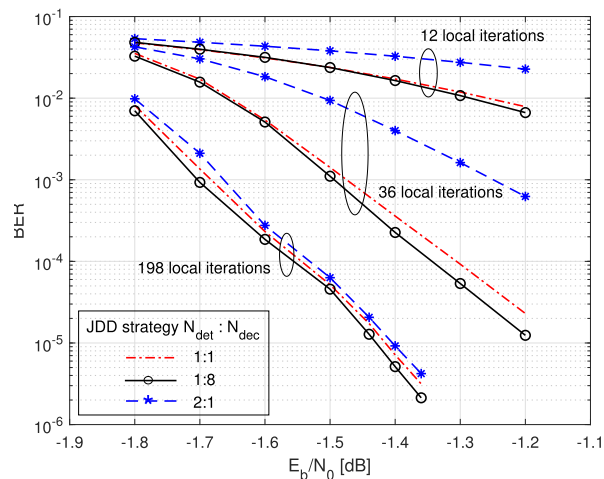


FIGURE 12. BER of rate-3/4 LDPC coded MU massive MIMO system with $N = 4608$ over 6×16 channel obtained with different JDD strategies and different total numbers of local iterations.

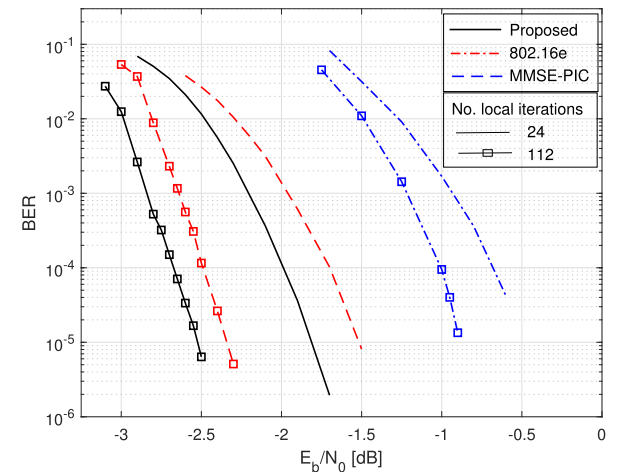


FIGURE 14. BER of rate-1/2 LDPC coded MU massive MIMO system designed by the proposed scheme and conventional schemes, where $N = 2304$, $n_U = 6$ and $n_R = 16$.

In Table 1 - Table 4, we list degree distributions (λ^*, ρ^*) of LDPC codes for MU massive MIMO systems achieving the threshold with some candidate JDD strategies over $n_U \times n_R$ channels and code rates under consideration. By a practical reason, λ_{dv} and ρ_{dc} having negligible values are enforced to be null. In each table, the channel capacity and the threshold value obtained by EXIT analysis are also listed. It is found that for a given $n_U \times n_R$ channel, the same threshold is obtained irrespective of the choice of JDD strategy. This implies that equivalent error correcting capabilities are obtained by different JDD strategies if an infinite number of iterations are allowed, while different convergence speeds are obtained with distinct JDD strategies.

Fig. 5 - Fig. 8 show the evolutions of \bar{I}_{VC} with respect to the total number of local iterations obtained by various JDD strategies for given massive MIMO channel and code rate. Note that evolution curves are obtained by EXIT analysis

for MU massive MIMO system equipped with LDPC codes whose degree distributions are listed in Table 1 - Table 4. It is obvious that distinct JDD strategies may result in different convergence speeds. We use boldface fonts to list values of $(\lambda^\dagger, \rho^\dagger)$ resulting in the threshold with the fastest JDD convergence speed for each massive MIMO channel and code rate in Table 1 - Table 4.

We construct parity-check matrices of LDPC codes with specific blocklengths N for MU massive MIMO system with some candidate JDD strategies by applying the PEG algorithm [36] to the obtained degree distributions (λ^*, ρ^*) . In Fig. 9, we plot the BER of MU massive MIMO system employing optimized LDPC codes with sufficiently large blocklength, i.e., $N = 64000$. It is observed that the threshold prediction based on EXIT analysis agrees quite well with the result of BER simulation. This observation verifies the practical effectiveness of the proposed EXIT analysis in predicting the performance of coded MU massive MIMO systems.

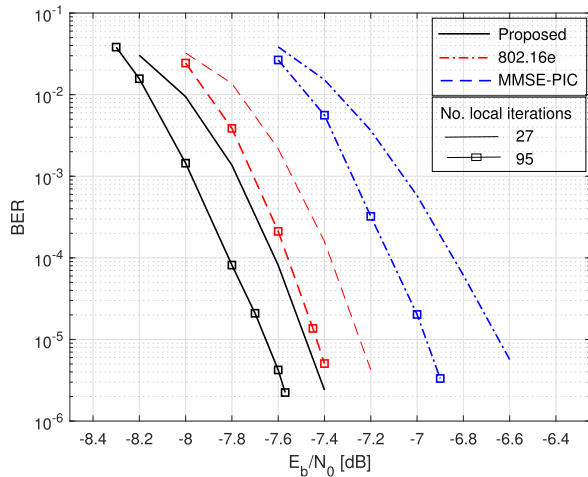


FIGURE 15. BER of rate-3/4 LDPC coded MU massive MIMO system designed by the proposed scheme and conventional schemes, where $N = 4608$, $n_U = 6$ and $n_R = 64$.

In Fig. 10 - Fig. 13, we plot BER performances of coded MU massive MIMO system with short to medium blocklength whose LDPC codes are constructed by using (λ^*, ρ^*) listed in Table 1 - Table 4. In each figure, we plot BER curves obtained by three different total numbers of local iterations to show different JDD convergence speeds attained by distinct JDD strategies. It is observed that the convergence speed of JDD strategies can be predicted well by investigating the evolution of \bar{I}_{VC} obtained by the proposed EXIT analysis as depicted in Fig. 5 - Fig. 8. Using the optimal JDD strategy and overall optimal degree distribution $(\lambda^\dagger, \rho^\dagger)$ results in the fastest convergence of JDD for each MIMO channel and code rate. The lower BER can be obtained for a given total number of local iterations if the optimal JDD strategy and the corresponding degree distributions are used in the construction of LDPC codes.

In Fig. 14 and Fig. 15, we compare BER performances of the LDPC coded MU massive MIMO systems designed by the proposed scheme and conventional schemes. As conventional schemes, we consider the MU massive MIMO system employing 802.16e LDPC codes [38] and JDD strategy $N_{det} : N_{dec} = 1 : 1$ as well as the MU massive MIMO system using MMSE-PIC detection [18] with JDD strategy $N_{det} : N_{dec} = 1 : 1$ and correspondingly designed optimal LDPC codes. It is observed that the MU massive MIMO system designed as proposed outperforms those designed in conventional manners. The coding gain of the proposed system at BER of 10^{-5} after convergence with $N = 2304$, $R = 1/2$ over 6×16 MIMO channel is about 0.2 dB over 802.16e LDPC based system and about 1.6 dB over MMSE-PIC based system. The coding gain of the proposed system at BER of 10^{-5} after convergence with $N = 4608$, $R = 3/4$ over 6×64 MIMO channel is about 0.3 dB over 802.16e LDPC based system and about 0.8 dB over MMSE-PIC based system.

VII. CONCLUSION

In this paper, we designed LDPC coded MU massive MIMO system equipped with an iterative JDD algorithm using the

low-complexity FG-GAI BP detection. We defined a factor graph representation of the LDPC coded MU massive MIMO system and defined updating rules for messages flowing in the JDD. We proposed an EXIT analysis tool to investigate the behavior of iterative JDD algorithm of coded massive MIMO receiver. Based on the EXIT analysis, we designed jointly irregular LDPC codes through the optimization of degree distributions and the JDD strategy to achieve the lowest BER and the fastest JDD convergence. The coded MU massive MIMO system equipped with the proposed LDPC codes and the proposed JDD strategy results in the improved BER performance over conventional schemes.

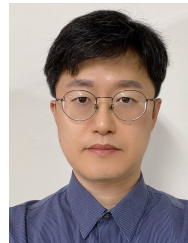
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