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Intuitionistic Fuzzy c-Ordered Means Clustering Algorithm

MEENAKSHI KAUSHAL, (Student Member, IEEE),
AND Q. M. DANISH LOHANI, (Senior Member, IEEE)
Department of Mathematics, South Asian University, New Delhi 110021, India

Corresponding author: Q. M. Danish Lohani (danishlohani@cs.sau.ac.in)

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ABSTRACT Atanassov intuitionistic fuzzy set (AIFS) has the capability to deal with various uncertain situations, so its popularity among researchers is quite high. It has been observed that Euclidean distance measure based AIFS clustering algorithms perform well on imprecise datasets. The performance of Euclidean measure based clustering algorithms deteriorates due to the presence of outliers/noise within a dataset. In the paper, an extension of the algorithm given by Leski, Jacek M. [Fuzzy Sets and Systems, 286 (2016): 114-133] is proposed as intuitionistic fuzzy *c*-ordered means clustering algorithm. This paper analyses the functionality of clustering algorithms over outliers/noises based datasets. In IFCOM, an alternate of Euclidean distance known as Loss function is used. Moreover, IFCOM uses intuitionistic fuzzy OWA to combat the ill effects of the noises and outliers. The proposed algorithm exploits a typicality function based weighing ordering approach. The approach assigins lower weights to the outliers. Hence, the catastrophic behavior is not observed in IFCOM while dealing outliers possessing synthetic and UCI machine learning datasets.

INDEX TERMS Fuzzy clustering, intuitionistic fuzzy sets, IFCM, loss functions.

I. INTRODUCTION

Clustering is a machine learning technique which identifies true patterns within datasets and each element is assigned a specific group, known as cluster. The similar elements are kept in one cluster, so they get separated from the dissimilar elements. In the literature, the clustering domain has been revolutionized due to the introduction of fuzzy set based clustering algorithms (see [1]–[4] and [5]). A well known generalization of fuzzy set called Atanassov intuitionistic fuzzy set (AIFS) is considered for clustering due to flexibility of the set. The conventional fuzzy c means algorithm [6] uses fuzzy Euclidean metric, which has been generalized to intuitionistic fuzzy c means in [7]. Mainly, intuitionistic fuzzy clustering algorithms help in solving the problems of medical image segmentation ([8], [9]), brain image segmentation [10] and MRI segmentation [11]. In general, the evolution of well separated compact clusters over lower dimensional datasets declares the AIFS based clustering algorithm successful. Moreover, an AIFS based clustering algorithm improvises with the variations in distance measures. Kumar *et al.* computed distance between cluster center and pixel for medical image segmentation with the help of Hausdorff metric instead of Euclidean distance (see [12]). Kernel function helps in the identification of overlapping clusters (see [13]) and such results are difficult to obtain using the Euclidean distance (see [14]). A detailed study of intuitionistic fuzzy metric and norm has been done in the literature (see [15]. [16]–[18]).

Clustering becomes a data mining technique when it clusters the noisy/outliers possessing datasets. Usually, the performance of clustering algorithm degrades over noisy/outliers based datasets [19]. To combat this issue, there are proposals in the literature where the datasets containing noises and heavy tailed outliers are satisfactorily clustered (see [19], [20]). The sensitivity aspect of a fuzzy clustering algorithm is minimized by Leski (see [21], [22]). The Euclidean distance measure is replaced by Huber loss function in [21] and [22] to obtain better clustering results. Vapnik introduced ϵ-insensitive loss function to

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minimize the tolerance limit of errors during clustering [23]. The overfitting of the clustering algorithm on a training dataset is also controlled by ϵ-insensitive loss function. The catastrophic behavior unfolded under the influence of outliers/noises in clustering algorithm is put to hold by Huber loss function [24]. The results shown in [24] explain how a small deviation in number of noises/outliers leads to unexpected fluctuations in clustering. Generally, in AIFS based clustering algorithms, the components of distance measures are assigned optimal weights for better clustering. The negative impact of non-informative elements such as noises/outliers in datasets are reduced with assignment of lower weights to them. Moreover, the informative elements are allocated optimal weights with respect to centroids position within clusters. Yager's ordered averaging operator (OWA) [25] and typicality function together decides the importance of data-items to reduce the noises/outliers during clustering.

Now, we discuss the motivation of the paper as follows:

- Euclidean distance measure (EDM) based clustering algorithm delivers compromised performance over outliers/noises possessing datasets. The sensitivity of EDM is reflected while clustering outliers/noise based datasets.
- Fuzzy c ordered means clustering algorithm (FCOM) works well over the datasets which contain outliers/noises. So, the study of FCOM is being extended to AIFS environment.
- Here clustering of outliers/noises possessing datasets is carried out. So, the intention of the paper is to study the clustering as a data mining technique.
- The experimental study of proposed clustering algorithm over both UCI and synthetic datasets is being done.

Further, we discuss the contribution of the paper:

- The sensitivity analysis of Euclidean distance measure (EDM) and Loss functions in the clustering algorithms has been performed. We obtained that Essential loss function is less sensitive in comparison to EDM.
- In order to deal with AIFS environment, we propose intuitionistic fuzzy c ordered means clustering algorithm (IFCOM).
- The role of data-items is measured in terms of informative and non-informative elements during clustering. The concept of typicality function is utilized to allocate least weights to outliers.
- The proposed IFCOM is compared with three algorithms, namely IFCM [7], IFWCOM [26], and GA-IFWCOM [26].

The rest of the paper is organized as follows: Section II outlines the basic background knowledge about AIFS and AIFS based clustering algorithms. The mathematics of the proposed intuitionistic fuzzy c ordered means clustering algorithm (IFCOM) has been given in section III. Meanwhile, section IV is dedicated for the experimental analysis. Finally, conclusion is stated in section V.

II. A BRIEF DISCUSSION ABOUT AIFS AND IFCM ALGORITHM

In the section, we discuss AIFS and a traditional AIFS based clustering algorithm, namely IFCM, IFWCOM and GA-IFWCOM.

A. SOME IMPORTANT DETAILS OF AIFS

An Atanassov intuitionistic fuzzy set A in the universe of discourse $U = \{x_1, x_2, \ldots, x_n\}$ is defined as,

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \text{ in } U\}. \tag{1}$$

The functions μ_A: $U \rightarrow$ [0,1] and ν_A: $U \rightarrow$ [0,1] assign membership value and non-membership value to each element x in U respectively, if

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1. \tag{2}$$

AIFS contains intuitionistic fuzzy index called hesitancy $\pi_A(x)$, such that:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x). \tag{3}$$

Atanassov intuitionistic fuzzy set [27] is proposed as a generalization of Zadeh's fuzzy set. The estimation of uncertainty in imprecise and vague datasets by the help of AIFS yields good results. The intuitionistic fuzzy index assigns a scalar value to each data-item which is equal to complement of sum value of membership and the non-membership. The hesitancy component of AIFS is helpful in the efficient modeling of uncertainty. The equality relationship used in fuzzy set has been generalized to inequality (2) in AIFS, which satisfy sum of membership and non-membership values is less than equal to one. The hesitancy has contributed to several applications in the field of clustering (see [28], [29]), decision making ([30], [31]), medical image segmentation [32], pattern and face recognition [33].

B. A BRIEF REVIEW OF IFCM ALGORITHM

Intuitionistic fuzzy c-means (IFCM) clustering algorithm was introduced by Xu and Wu [7]. The IFCM algorithm clubs the theory of AIFS and fuzzy c-means algorithm. A good partitioning is observed between clusters due to the exploitation of hesitancy in the algorithm. IFCM is an iterative algorithm with fixed numbers of initial clusters. There is a random selection of initial cluster centroids. Mathematically, the clustering problem is proposed as follows:

Problem I: Let universe of discourse $U = \{x_i, 1 \leq i \leq P\}$ has P data-items. The ith data-item, $x_i = (x_{id})_{d=1}^{D}$ is a D dimensional feature vector with x_{id} as the dth feature value. We have to cluster U into c classes under the constraints $u_{li} \geq 0$ for all $1 \leq l \leq c, 1 \leq i \leq P$; $0 \leq \sum_{i=1}^{P} u_{li} \leq P$ for all $1 \leq l \leq c$; and $\sum_{l=1}^{c} u_{li} = 1$ for all $1 \leq i \leq P$. Here, $M = [u_{li}]_{c\times P}$ is the initial membership matrix.

Solution: First of all, the AIFS counterpart $\Omega = \{\tilde{x}_{id} = (\mu_{id}, \nu_{id}, \pi_{id}), 1 \leq i \leq P, 1 \leq d \leq D\}$ of U is deduced with the usage of three functions based AIFS generation. Then, an arbitrary selection of c numbers of cluster centroids $\{\tilde{v}_l = (\mu_{ld}, \nu_{ld}, \pi_{ld})_{d=1}^{D}, 1 \leq l \leq c\}$ is done. Here, $D_1(\tilde{x}_i, \tilde{v}_l)$ assigns initial membership value u_{li} to $\tilde{x}_i$ in $\tilde{v}_l$, and hence $M = [u_{li}]_{c\times P}$ is obtained. The membership value of ith data item in lth cluster and cluster centroid gets updated after each iteration as per (4) and (5), respectively:

$$u_{li} = \frac{1}{\sum_{l=1}^{c}\left(\frac{D_1(\tilde{x}_i,\tilde{v}_l)}{D_1(\tilde{x}_i,\tilde{v}_l)}\right)^{\frac{2}{m-1}}}, \tag{4}$$

$$\bar{\mu}_{ld} = \frac{\sum_{i=1}^{P} u_{li}^m \mu_{id}}{\sum_{i=1}^{P} u_{li}^m}, \quad \bar{\nu}_{ld} = \frac{\sum_{i=1}^{P} u_{li}^m \nu_{id}}{\sum_{i=1}^{P} u_{li}^m}, \quad \bar{\pi}_{ld} = \frac{\sum_{i=1}^{P} u_{li}^m \pi_{id}}{\sum_{i=1}^{P} u_{li}^m}. \tag{5}$$

The equations (4) and (5) are derived as a solution to the optimization problem. The mathematical formulation of problem is:

$$J = \min \sum_{l=1}^{c}\sum_{i=1}^{P} u_{li}^m D_1^2(\tilde{x}_i, \tilde{v}_l), \tag{6}$$

$$D_1^2(\tilde{x}_i, \tilde{v}_l) = \frac{1}{2n}\sum_{d=1}^{D}((\mu_{id} - \bar{\mu}_{ld})^2 + (\nu_{id} - \bar{\nu}_{ld})^2 + (\pi_{id} - \bar{\pi}_{ld})^2),$$

under the constraints,

$$\sum_{l=1}^{c} u_{li} = 1 \quad \text{for all } 1 \leq i \leq P, \tag{7}$$

$$u_{li} \geq 0 \quad \text{for all } 1 \leq l \leq c,\ 1 \leq i \leq P, \tag{8}$$

$$0 \leq \sum_{i=1}^{p} u_{li} \leq P \quad \text{for all } 1 \leq l \leq c. \tag{9}$$

Here, m is the fuzzy index. This is the brief mathematical introduction of IFCM algorithm.

Though IFCM gives better clustering results while handling the uncertainty that arises due to imprecise information present in the dataset, it is unable to distinguish between informative and non-informative data-items (outliers and noises) accurately. The criterion function of IFCM is comprised of AIFS based Euclidean distance measure and fuzzy index m. Solely on the basis of Euclidean distance measure, it is difficult to handle outliers within a real dataset. Therefore, the real valued datasets containing outliers/noises are not accurately clustered by the IFCM algorithm.

C. A BRIEF REVIEW OF FEATURE WEIGHTED ALGORITHMS

Kuo *et al.* [26] introduced an algorithm called IFWCOM to overcome the influence of noise and outliers in the dataset. It incorporates role of hesitation degree and a feature weighted technique in FCOM [22]. The criterion function of IFWCOM is defined as follows:

$$J(M, V) = \sum_{l=1}^{c}\sum_{i=1}^{P} \beta_{li} u_{li}^m D_2^w(x_i, v_l) \tag{10}$$

where

$$D_2^w(x_i, v_l) = \sum_{d=1}^{D}[L(x_{id} - v_{ld}) \times w_d^{\theta} \tag{11}$$

Here, Logarithmic loss function is used as a similarity measure in (11) (see [22]). The iterative process for membership matrix M, weight matrix W and centroid matrix V uses following mathematical formulations:

$$u_{si} = \frac{f_i[D_2^w(x_i, v_l)]^{\frac{1}{1-m}}}{\left[\sum_{l=1}^{c} \beta_{li} \times D_2^w(x_i, v_l))^{\frac{1}{1-m}}\right]} \tag{12}$$

$$w_d = \frac{(\sum_{i=1}^{P}\sum_{l=1}^{c} u_{li}^m L(x_{id}, v_{ld})]^{\frac{1}{1-\theta}}}{\sum_{a=1}^{D}(\sum_{i=1}^{P}\sum_{l=1}^{c} u_{li}^m L(x_{ia}, v_{la}))^{\frac{1}{1-\theta}}} \tag{13}$$

The global typicality function f_i and local typicality function β_{li} are defined in detail in next section. The typicality function controls the impact of outliers and noise in the dataset under the fuzzy environment. Lastly, the mathematical formula for cluster centroids is given as follows:

$$v_{ld} = \frac{\sum_{i=1}^{P} \beta_{si}(u_{il}^*)^m h_{lid}(x_{id})}{\sum_{i=1}^{P} \beta_{si}(u_{il}^*)^m h_{lid}} \tag{14}$$

Here, the function h is a loss function based real valued mapping. The membership value u^* is updated using equation $u_{li}^* = u_{li} + \pi_{li}$ and (12). IFWCOM is further generalized to GA-IFWCOM using genetic algorithm to obtain a global optimal solution. In the genetic algorithm technique, the famous Roulette wheel selection is used for initialization of cluster centroids.

III. PROPOSED INTUITIONISTIC FUZZY C ORDERED MEANS CLUSTERING ALGORITHM

Here, the loss function $\tilde{L}$ and typicality β_{li} replace $D_1^2(\tilde{x}_i, v_l)$ in the criterion J. It results a new optimization problem that leads to IFCOM. Mathematically, we have,

$$\begin{cases} J(M, V) = \min \sum_{l=1}^{c}\sum_{i=1}^{P} \beta_{li} u_{li}^m \sum_{d=1}^{D} \tilde{L}(\tilde{x}_{id} - \tilde{v}_{ld}), \\ \quad \text{where,} \\ \tilde{L}(\tilde{x}_{id} - \tilde{v}_{ld}) = h(e_{lid})\big[(\mu_{id} - \bar{\mu}_{ld})^2 + (\nu_{id} - \bar{\nu}_{ld})^2 + \\ (\pi_{id} - \bar{\pi}_{ld})^2\big]. \end{cases} \tag{15}$$

The constraints for the problem are (8) and (9) along with,

$$\sum_{l=1}^{c} \beta_{li} u_{li} = f_i, \quad \text{for all } 1 \leq i \leq P. \tag{16}$$

The function h is defined as mapping $h : \mathbb{R} \rightarrow \mathbb{R}$, such that,

$$h(e_{lid}) = \begin{cases} 0, & e_{lid} = 0 \\ \frac{L(e_{lid})}{e_{lid}^2}, & e_{lid} \neq 0 \end{cases}$$

With respect to dth dimension, the residue of ith data-item in the lth cluster is calculated using $e_{lid} = (\tilde{x}_{id} - v_{ld})$. Loss function [22] defines a mapping h. Typicality is a weighted t-norm that operates on $\hat{\beta}_{lid}$. Here, $\hat{\beta}_{lid}$ is the value being assigned to dth dimension of ith data-item with respect to lth cluster. The collection $\{\hat{\beta}_{lid}\}$ is dealt by defining a function β_{li} and it calculates the local typicality of ith data-item with respect to the lth cluster. In other words, the function β_{li} computes weight for ith data-item with respect to lth cluster based on dimension-wise typicality values $\hat{\beta}_{lid}$. The local typicality of ith data-item for lth cluster is derived using (17) and (18):

$$\beta_{li} = \prod_{d=1}^{D} \hat{\beta}_{lid}, \tag{17}$$

and,

$$\hat{\beta}_{lid} = \frac{1}{1 + exp\left[\frac{2.944}{p_a P}(\chi_{lid} - p_c p)\right]}. \tag{18}$$

The piece-wise linearly weighted OWA function has been used in (18). The residual value e_{lid} measures the difference between a data-item and centroid. The ranking of dth dimension of ith data-item within lth cluster is done by the help of function χ_{lid}. The data-item nearest to lth cluster is given rank 1, and farthest data-item is ranked P. The global typicality of the ith data-item is estimated using a function f_i. The global typicality gives an idea about overall behavior of ith data-item in the lth cluster. We have,

$$f_i = \max\{\beta_{1i}, \beta_{2i}, \ldots, \beta_{ci}\}. \tag{19}$$

The solution of the optimization problem (15) is obtained in the form of an iterative scheme. This scheme is utilized to propose the IFCOM algorithm. The section III-A provides a mathematical solution to the optimization problem (15).

A. MAIN RESULTS

In the section, we have proved two theorems to solve the optimization problem. We are using membership matrix M and cluster center matrix V.

Theorem 1: Let $\theta : \mathbb{M}_{lk} \times \mathbb{V}_{ld} \rightarrow \mathbb{R}$, such that $\theta(M, V) = J(M, V)$, where $V \in \mathbb{V}_{ld}$ is fixed. Then M^* is a strict local minima if M^* is deduced from (25). Here, $\mathbb{M}_{lk}$ and $\mathbb{V}_{ld}$ are the collections that contains membership matrices and cluster center matrices, respectively.

Proof: The Lagrangian $G(M, V)$ of criterion function (15) and constraint (16) is defined in (20) by the help of Lagrange's multipliers $\lambda_i(1 \leq i \leq n)$. The parameter $m \in (0, 1) \cup (1, \infty)$ is a weighting exponent for memberships. Now,

$$\begin{aligned} &G(M, V) \\ &= \sum_{i=1}^{P}\left(\sum_{l=1}^{c} \beta_{li} u_{li}^m \sum_{d=1}^{D}(h(e_{lid})((\mu_{id} - \tilde{\mu}_{ld})^2\right. \\ &\quad \left. + (\nu_{id} - \tilde{\nu}_{ld})^2 + (\pi_{id} - \tilde{\pi}_{ld})^2) - \sum_{i=1}^{p}\lambda_i\left[\sum_{l=1}^{c}\beta_{li}u_{li} - f_i\right]\right) \end{aligned} \tag{20}$$

The derivatives of the Lagrangian condition (20) is set equal to zero with respect to lagrange multiplier λ_i as follows:

$$\frac{\partial G(M, V)}{\partial \lambda_i} = -\left[\sum_{l=1}^{c}\beta_{li}u_{li} - f_i\right] = 0, \quad \forall 1 \leq i \leq P \tag{21}$$

Similarly, derivatives of the Lagrangian condition (20) is set equal to zero with respect to membership parameter u_{si},where $1 \leq i \leq P$,$1 \leq s \leq c$ as follows:

$$\begin{aligned} \frac{\partial G(M, V)}{\partial u_{si}} &= \beta_{si} m u_{si}^{m-1} \sum_{d=1}^{D} h(e_{lid})(((\mu_{ia} - \tilde{\mu}_{sa})^2 \\ &\quad + (\nu_{ia} - \tilde{\nu}_{sa})^2 + (\pi_{ia} - \tilde{\pi}_{sa})^2)) - \lambda_i\beta_{si} = 0 \end{aligned} \tag{22}$$

Solving (22), we get:

$$\begin{aligned} u_{si} = \left(\frac{\lambda_i}{m}\right)^{\frac{1}{m-1}} \sum_{d=1}^{D} h(e_{lid})(((\mu_{id} - \tilde{\mu}_{sd})^2 + (\nu_{id} - \tilde{\nu}_{sd})^2 \\ + (\pi_{id} - \tilde{\pi}_{sd})^2))^{\frac{1}{1-m}} \end{aligned} \tag{23}$$

On combining (21) and (23), we have,

$$\begin{aligned} &\sum_{l=1}^{c}\beta_{li}\left(\frac{\lambda_i}{m}\right)^{\frac{1}{m-1}}\left(\sum_{d=1}^{D} h(e_{lid})((\mu_{ia} - \tilde{\mu}_{sd})^2 + (\nu_{id} - \tilde{\nu}_{sd})^2\right. \\ &\quad \left. + (\pi_{id} - \tilde{\pi}_{sd})^2)\right)^{\frac{1}{1-m}} = f_i \\ &\left(\frac{\lambda_i}{m}\right)^{\frac{1}{m-1}}\sum_{l=1}^{c}\beta_{li}\left(\sum_{d=1}^{D} h(e_{lid})((\mu_{ia} - \tilde{\mu}_{sd})^2 + (\nu_{id} - \tilde{\nu}_{sd})^2\right. \\ &\quad \left. + (\pi_{id} - \tilde{\pi}_{sd})^2)\right)^{\frac{1}{1-m}} = f_i \end{aligned} \tag{24}$$

The division of (23) by (24) results an iterative formula for membership value u_{si} as follows:

$$u_{si} = \frac{\left\{ f_i \sum_{d=1}^{D} h(e_{lid})\left((\mu_{id} - \tilde{\mu}_{sd})^2 + (\nu_{id} - \tilde{\nu}_{sd})^2 + (\pi_{id} - \tilde{\pi}_{sd})^2\right)^{\frac{1}{1-m}} \right\}}{\left\{ \sum_{l=1}^{c} \beta_{ci}\left(\sum_{d=1}^{D} h(e_{lid})((\mu_{ia} - \tilde{\mu}_{sd})^2 + (\nu_{id} - \tilde{\nu}_{sd})^2 + (\pi_{id} - \tilde{\pi}_{sd})^2)\right)^{\frac{1}{1-m}} \right\}} \tag{25}$$

Theorem 2: The optimal minima of the problem which has been defined in (15), (16) is obtained at a point $V = V^*$. Here, the point $V = V^*$ is derived on the basis of (31), (32) and (33).

Proof: The Lagrangian $G(M, V)$ of criterion function (15) and constraint (16) is defined in (26) by the help of Lagrange's multipliers $\lambda_i (1 \leq i \leq n)$. The parameter $m \in (0, 1) \cup (1, \infty)$ is a weighting exponent for memberships. Now,

$$G(M, V) = \sum_{i=1}^{P} \left(\sum_{l=1}^{c} \beta_{li} u_{li}^{m} \sum_{d=1}^{D} (h(e_{lid})((\mu_{id} - \tilde{\mu}_{ld})^2 + (\nu_{id} - \tilde{\nu}_{ld})^2 + (\pi_{id} - \tilde{\pi}_{ld})^2) - \sum_{i=1}^{p} \lambda_i \left[\sum_{l=1}^{c} \beta_{li} u_{li} - f_i \right] \right) \tag{26}$$

Setting the derivative (27) equal to 0, we get:

$$\frac{\partial \tilde{G}(M, V)}{\partial \tilde{\mu}_{sa}} = 0 \tag{27}$$

$$\frac{\partial \tilde{G}(M, V)}{\partial \tilde{\nu}_{sa}} = 0 \tag{28}$$

$$\frac{\partial \tilde{G}(M, V)}{\partial \tilde{\pi}_{sa}} = 0 \tag{29}$$

Solving (27), we get,

$$\frac{\partial G(M, V)}{\partial \tilde{\mu}_{sa}} = \sum_{i=1}^{P} \left(\beta_{si} u_{si}^{m} h(e_{sia}) [-2(\mu_{ia} - \tilde{\mu}_{sa})] \right) = 0 \tag{30}$$

$$\tilde{\mu}_{sa} = \frac{\sum_{i=1}^{P} \beta_{si} u_{si}^{m} h(e_{sia}) \mu_{id}}{\sum_{i=1}^{P} \beta_{si} u_{si}^{m} h(e_{sia})} \tag{31}$$

Similarly, from (28) and (29), we get:

$$\tilde{\nu}_{sa} = \frac{\sum_{i=1}^{P} \beta_{si} u_{si}^{m} h(e_{sia}) \nu_{id}}{\sum_{i=1}^{P} \beta_{si} u_{si}^{m} h(e_{sia})} \tag{32}$$

$$\tilde{\pi}_{sa} = \frac{\sum_{i=1}^{P} \beta_{si} u_{si}^{m} h(e_{sia}) \pi_{id}}{\sum_{i=1}^{P} \beta_{si} u_{si}^{m} h(e_{sia})} \tag{33}$$

Corollary 1: If $\beta_{li} = \beta$ for all l, i and $h(e_{lia})$ is an Essential loss function [22], the mathematical formulation of IFCOM is reduced to IFCM. In other words, IFCOM is a generalization of IFCM.

B. PROPOSAL OF IFCOM CLUSTERING ALGORITHM

In Table 1, the mathematical symbols used in IFCOM are given.

The initially given $U = \{x_1, x_2, \ldots, x_p\}$ is a real valued dataset. Each data-point x_i contains D criteria values, so, $x_i = (a_{ij}^{\circ})_{j=1}^{j=D}$. We compute the AIFS counterpart of U by three functions based generalized Yager's intuitionistic fuzzy generator [34]. Hence, the AIFSs derived against x_i is $\tilde{x}_i = (\mu_{ij}, \nu_{ij}, \pi_{ij})_{j=1}^{D}$, such that,

$$\mu_{ij} = (\phi_{ij})^{\gamma}, \quad \gamma > 0, \tag{34}$$

$$\nu_{ij} = (1 - (\phi_{ij})^{\alpha\gamma})^{\frac{1}{\alpha}}, \quad 0 < \alpha \leq 1, \tag{35}$$

$$\pi_{ij} = 1 - \mu_{ij} - \nu_{ij}. \tag{36}$$

TABLE 1. Mathematical symbols used.

Symbol	Description of the Symbol
α	Tuning parameter in Yager'complement function
β_{ck}	Typicality of kth data point in cth cluster
γ	Tuning parameter in Yager'complement function
D	Number of attributes
χ	Data item's ordinal number
m	Fuzziness index
c	Number of clusters
P	Number of data points
J	Criterion function
u_{li}	Membership degree of ith data point in lth cluster
x_k	$(a_{ij}^{\circ})_{j=1}^{j=D}$, kth data point, where a_{ij}° is a non-negative value
$\tilde{x}_k$	Atanassov intuitionistic fuzzy data-item, $(\mu_{id}, \nu_{id}, \pi_{id})$
v_l	Centroid of lth cluster
f_k	Global typicality of kth data point
$h(e)$	Loss function

Here, the element a_{ij}° is assigned a membership value μ_{ij}, non-membership value ν_{ij}, and hesitancy value π_{ij}. The proposed algorithm clusters U with the help of an iterative procedure involving membership matrix $M = [u_{li}]_{P \times c}$ and cluster centroids matrix $V = [\tilde{v}_1, \tilde{v}_2, \cdots, \tilde{v}_c]^T$. The AIFSs $\tilde{v}_l = (\mu_{lj}, \nu_{lj}, \pi_{lj})_{j=1}^{j=D}$, $1 \leq l \leq c$ correspond to cluster centroids. The flow chart of IFCOM clustering algorithm is illustrated in Figure 1.

IV. EXPERIMENTAL ANALYSIS

A. CLUSTERING OVER OUTLIERS POSSESSING SYNTHETIC DATASET

Generation of a synthetic dataset [22]: In the experiment, a three cluster based 2D dataset is generated with the help of Gaussian distribution function, $\sum_{j=1}^{3} \theta_j N(b_j, \sigma_j)$ with $\theta_1 = \theta_2 = \theta_3 = 5$, $b_1 = (-25, -25)^T$, $b_2 = (25, -25)^T$, $b_3 = (25, 25)^T$ and $\sigma_1 = \sigma_2 = \sigma_3 = (1, 0; 0, 1)$. The dataset consists of 75 data-items. Here three well separated clusters are constructed, such that each contains 25 data-items (see Figure 2(a)). The focus of the experiment is to study certain intuitionistic fuzzy set based clustering algorithms, namely IFCM ([36], [37]), proposed IFCOM, IFWCOM [26], GA-IFWCOM [26]. The given dataset is transformed into intuitionistic fuzzy dataset using step A of algorithm I. Then, each data-item is assigned a membership value while exploiting `norm` function in MATLAB software 50 times. Further, the non-membership values and hesitancy values are deduced using (35) and (36). Furthermore, the numbers of outliers located at point $(0, 1)^T$ are varying in step size of 5 units from 0 to 30. So, one by one six numbers of outliers ($\{5, 10, 15, 20, 25, 30\}$) are handled. The cluster centroids are depicted in the form of triangles (see Figure 2). The initial centroids $(0.7, 0.7)^T$, $(0.7, 0.3)^T$ and $(0.3, 0.3)^T$ are

Algorithm 1 A procedure to Implement IFCOM

A. *Dataset pre-processing*:
The normalizaton function $\phi : X \rightarrow [0, 1]$ is considered for AIFS generation, and a positively valued parameter γ tunes ϕ. The algorithm is randomly initialized 50 times for the computation of optimal ϕ. We have P datapoints in AIFS counterpart $\Omega = \{\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_P\}$ of U.

B. *Initialization:*
In IFCOM pre-fixing of initial number of clusters, say c is required. The value that delivers mostly good clustering results is $m = 2$ (see [35]). The global typicality function f_i, membership matrix M and centroid matrix V are initialized as follows:

Step 1: Local β_{li} and global f_i typicalities initializations: Here each data-item $\tilde{x}_i$, $1 \leq i \leq P$, is initially allocated value equal to 1 in all c clusters. The algorithm computes β_{li} using (17). Finally, global typicality f_i is calculated by (19).

Step 2. Derivation of membership matrix, M: The *rand* function (of the MATLAB) arbitrarily assigns values to data-items in each cluster. These values are normalized using (16), and then we use them as initial membership values of data-items. The initial membership matrix M contains initial membership values of data-items $\tilde{x}_i$, $1 \leq i \leq P$ with respect to clusters $\tilde{v}_l$, where $1 \leq l \leq c$.

Step 3. Computation of cluster centroids, V: The equations (31), (32) and (33) process the values of M, and hence, we get a set of c initial clusters $V = [\tilde{v}_1, \tilde{v}_2, \cdots, \tilde{v}_c]^T$, $\tilde{v}_l = (\mu_{lj}, \nu_{lj}, \pi_{lj})_{j=1}^{j=D}$.

D. *Updation of Membership matrix, M*: The initial membership values of data points with respect to clusters are contained in M. The membership values describe the connection between data-items and clusters. Here, the membership matrix $M = [u_{il}]$ has order $P \times c$, where P is the number of data points and c is the number of clusters. The membership matrix M is updated by the help of (25).

E. *Updation of typicality, f_i*: We update local typicality β_{li} with the help of (17) and (18). The updation of typicality reduces the ill impact of outliers/noises in dataset. The global typicality is calculated by the help of (19).

F. *Updation of centroid matrix, V:* The updation of typicality β_{li} updates centroid collection V (see (31), (32), (33)).

G. *Convergence criterion*: The convergence of the IFCOM algorithm is decided by the error which is a positive number, ϵ equal to 10^{-5}. The convergence is met if the termination criterion $\sum_{l=1}^{c} \frac{d_2(\hat{v}_l(t), \hat{v}_l(t+1))}{c} < \epsilon$ is reached else the algorithm is repeated from Step D.

marked with diamond shape symbols for the three clusters, respectively.

1) COMPUTATIONAL COST REDUCTION

The exploration of fuzzy index value $m = 2$ in algorithm often delivers suitable clustering results (see [35]). So, we have experimented upon IFCOM and IFCM algorithms exploring $m = 2$ and its certain neighbouring points. The sensitivity of generalized Yager's generation function is found low at $\alpha = 0.5$ (see [37]), so same value has been used throughout the experiment. The selection of (m, α) equals to (2,0.5) has lowered the computational cost of experiment. Further, a parameter called error rate E_v is used to compare the performances of algorithms. Mathematically, $E_v = ||v_e - v_l||$, where the exact centroid v_e is calculated by arithmetic mean and approximate centroid v_l is obtained via clustering algorithm.

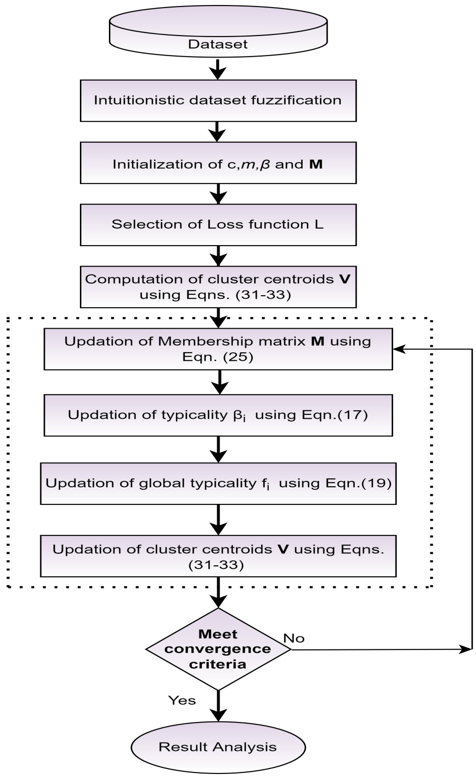

FIGURE 1. Proposed architecture of IFCOM clustering algorithm.

2) PERFORMANCE ANALYSIS OF IFCM CLUSTERING ALGORITHM

First, the study of IFCM is carried out over a dataset which has no outliers. Here $E_v = 0.0063$ confirms that experimentally derived centroids converge to exact centroids (see Figure 2(a)). Now, let us dope the dataset with 5 outliers, and then 10 outliers. After doping, the experimentally derived centroids are converging slightly away from exact centroids (see Figures 2(b), 2(c)). The points of convergence of experimentally derived centroids catastrophically changes from exact centroids, if the dataset contains 20, 25 or 30 outliers (see Figures 2(e), 2(f), and we have E_v equal to 0.3599 and 0.8622, respectively). The performance of IFCM deteriorates as numbers of outliers increases, or in otherwords, algorithm has sensitivity towards outliers.

There is a spike in E_v (see Figure 3), therefore it is not suitable to deal outliers possessing datasets using IFCM.

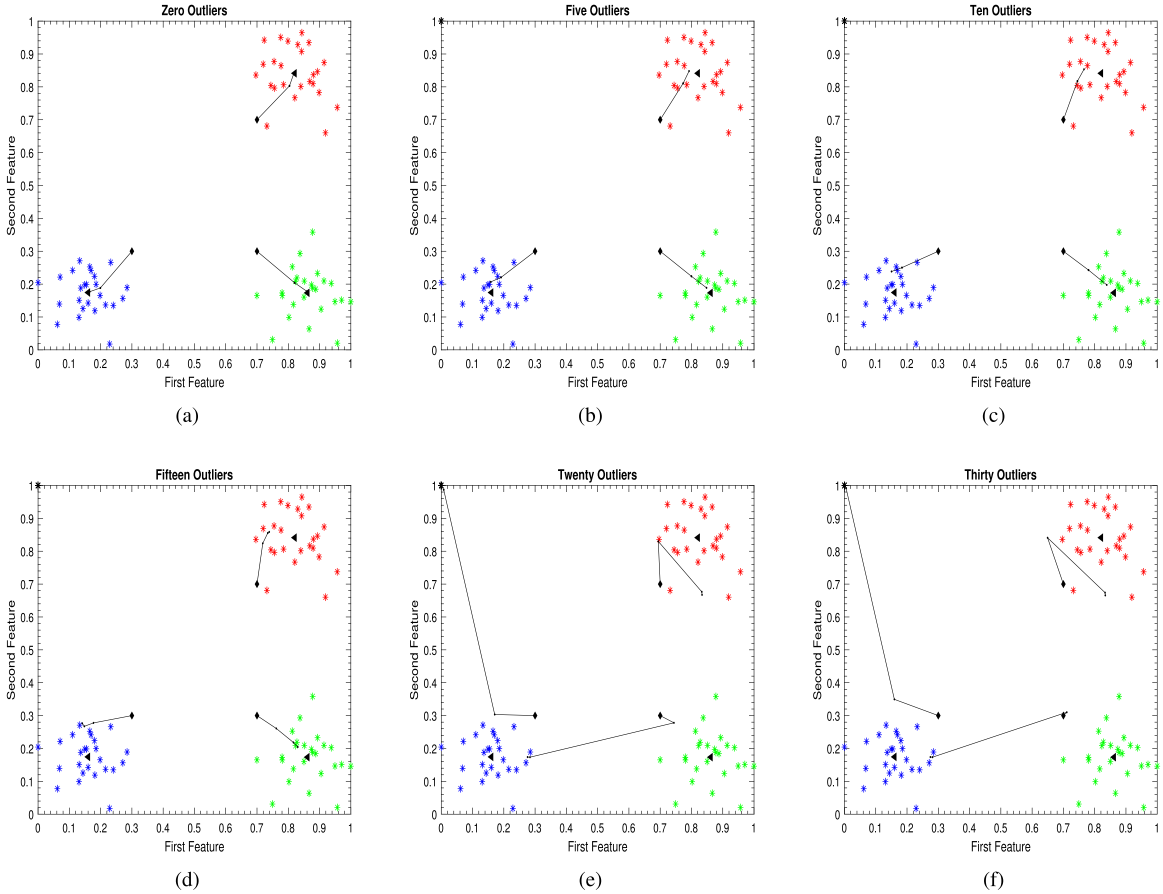

FIGURE 2. Study of cluster centers using IFCM clustering algorithm in presence of outliers in the dataset.

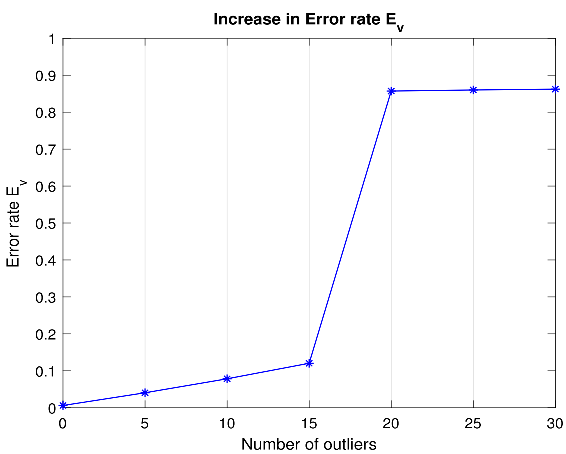

FIGURE 3. Increase in the Error rate with increase in number of outliers in the dataset.

More the numbers of outliers in the dataset, larger is the deviation of experimental centroids from exact centroids. We have not observed improvement in the performance of IFCM even after selecting initial centroids near to exact centroids. IFCM is a fast converging algorithm [28], but the requirement of the algorithm is to address its sensitivity issue. We have incorporated typicality function based ordering approach in the proposal of IFCOM algorithm for the redressal of sensitivity issue. The typicality function allocates high weights to informative data-items and low weights to non-informative data-items called outliers.

3) PERFORMANCE ANALYSIS OF IFCOM CLUSTERING ALGORITHM

The experimental results obtained by using Essential loss function are discussed. The ill effects of outliers are well in control under IFCOM, so catastrophical changes in the positions of centroids are not observed (see Figure 4). Here, typicality function properly handles the varying numbers of outliers present in the dataset. The typicality function based value allocated to each data-item is graphically demonstrated by the help of histogram (see Figure 5). In Figure 5(a), exactly eight data-items of the dataset are allocated lower typicality values that includes five outliers and three data-items. Further, nineteen data-items are allocated lower typicality values, where fifteen are outliers (see Figure 5(c)). Furthermore in Figure 5(f), thirty two data-items are given lower weights/ typicality values out of which thirty are the numbers of outliers. A sudden decline in the error rate of IFCOM algorithm is observed while dealing large numbers of outliers (see Figure 6). As the numbers of outliers (non-informative data-items) increases, the error rate decreases.

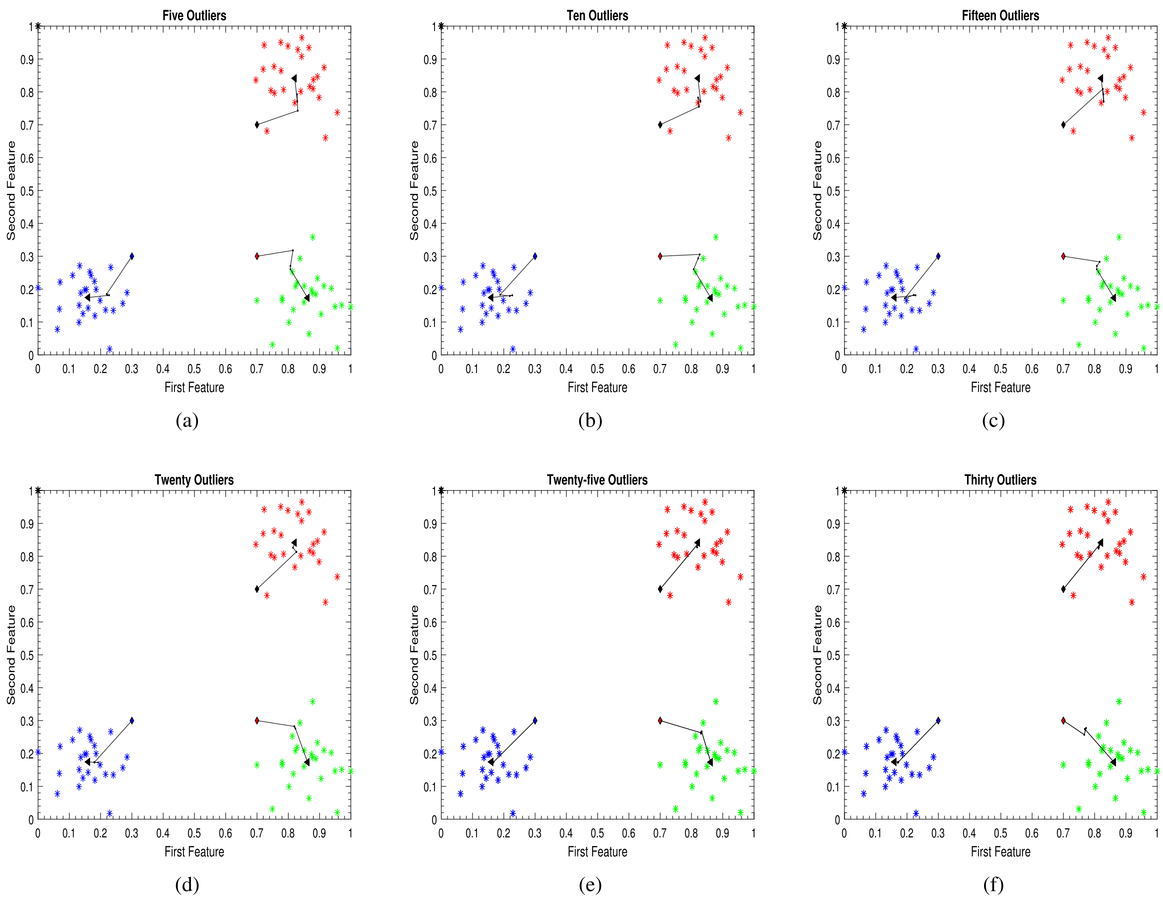

FIGURE 4. Study of centroids using proposed IFCOM clustering algorithm in presence of outliers in the dataset.

It shows that IFCOM has successfully reduced the ill effects of outliers during clustering. Moreover, the sudden change in position of three final cluster centroids is rectified in IFCOM.

Finally, the comparison between IFCOM algorithm and IFCM is done as follows:

1. The sensitivity of IFCOM is less in comparison to IFCM (see Figure 2).

2. The IFCM catastrophically changes behaviour in the presence of outliers. The inclusion of typicality concept in IFCM proposes IFCOM, and hence experimental centroids converge to exact centroids (see Figure 4).

3. A thorough study regarding the role of typicality function in IFCOM has been discussed (see Figure 5). Here, least weights are allocated to the non-informative data-items (called outliers), which results in the enhancement of efficacy of IFCOM in comparison to IFCM.

4. Finally, the decrease in error rate of the proposed algorithm with increase in number of outliers depicts the out-performance of IFCOM over IFCM clustering algorithm.

B. CLUSTERING UCI DATASETS

The datasets utilized for the comparison of IFCOM, FCOM, IFWCOM and GA-IFWCOM algorithms are taken from UCI machine learning repository. We have considered six UCI datasets for experimentation and their details are provided in Table 2.

TABLE 2. Summary of UCI datasets.

Dataset	Instances	Features	Classes	Outliers
Iris	150	4	3	0
Wine	178	13	3	10
WBCD	569	30	2	0
Glass	214	9	62	9
Seeds	210	7	3	0
Wifi	2000	7	4	0

[1]source:https://archive.ics.uci.edu/ml/index.php

Clustering accuracy and rand index are the two benchmark measuring indexes used for comparing FCOM, IFWCOM, GA-IFWCOM and IFCOM clustering algorithms. The clustering accuracy (CA) is defined as the number of correctly classified data points p_c divided by total number of data-points p, i.e., $\frac{p_c}{p} \times 100\%$. CA is useful for labeled dataset. Rand index (R_I) is another index that works for labeled dataset. Here, true set of clusters of the dataset is denoted by $\mathbb{C}$ and $\mathbb{C}'$ is the cluster set derived by the help of algorithm. There are four categories SD, DS, SS or DD (here S stands for same cluster and D stands for different cluster) of data-points. R_I calculates the fraction of the total number of pairs that are

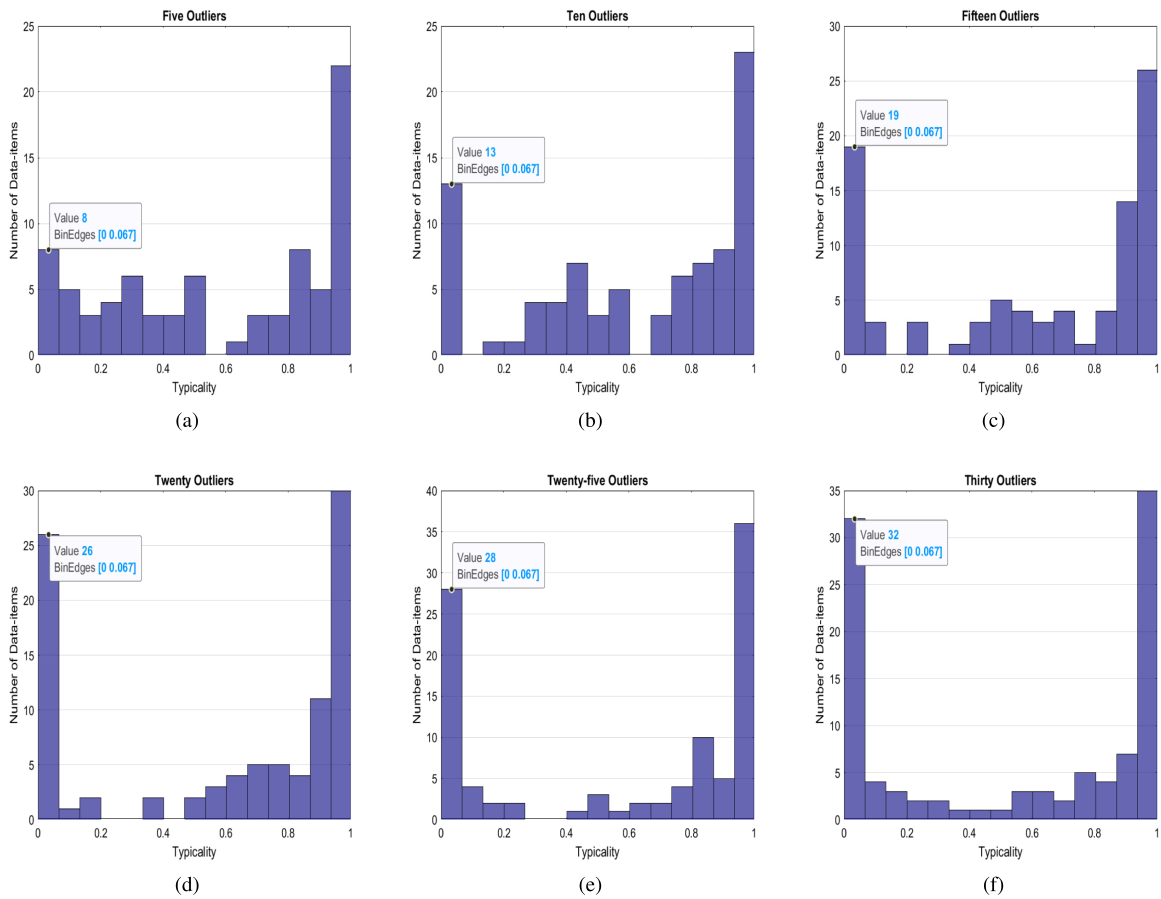

FIGURE 5. Distribution of typicality values between informative data-items and non-informative data-items (outliers).

TABLE 3. Clustering results of IFCOM algorithm on UCI datasets.

Dataset	CA	R_I	(m, α, γ)	Loss function
Iris	**92**	**0.87**	**(2.0,0.5,0.5)**	**Essential**
	70	0.78	(2.0,0.5,2.0)	Huber
	73	0.74	(1.5,0.5,2.0)	Logarithmic
	85	0.88	(2.0,0.5,2.0)	Logarithmic-Linear
Wine	**97.5**	**0.89**	**(2.5,0.5,2.5)**	**Essential**
	88.5	0.89	(2.0,0.5,2.0)	Huber
	82	0.83	(2.5,0.5,2.0)	Logarithmic
	83	0.889	(2.5,0.5,2.0)	Logarithmic-Linear
WBCD	78	0.775	(2.0,0.5,2.0)	Essential
	87	**0.865**	**(2.5,0.5,2.5)**	**Huber**
	74	0.743	(2.0,0.5,2.0)	Logarithmic
	79	0.78	(2.0,0.5,2.5)	Logarithmic-Linear
Glass	58	0.54	(2.5,0.5,2.5)	Essential
	57.5	0.565	(2.5,0.5,2.0)	Huber
	45	0.47	(2.0,0.5,2.5)	Logarithmic
	58	**0.585**	**(2.5,0.5,2.0)**	**Logarithmic-Linear**
Seeds	**92**	**0.91**	**(2.0,0.5,2.5)**	**Essential**
	84.5	0.85	(2.5,0.5,2.5)	Huber
	82	0.83	(2.0,0.5,2.5)	Logarithmic
	90.5	0.88	(2.0,0.5,2.5)	Logarithmic-Linear
Wifi	78	0.77	(2.5,0.5,2.0)	Essential
	89.5	**0.895**	**(2.0,0.5,2.5)**	**Huber**
	64	0.63	(2.0,0.5,2.5)	Logarithmic
	70.5	0.76	(2.5,0.5,2.5)	Logarithmic-Linear

[2]Best clustering accuracy, CA and rand index, R_I are marked as bold values.

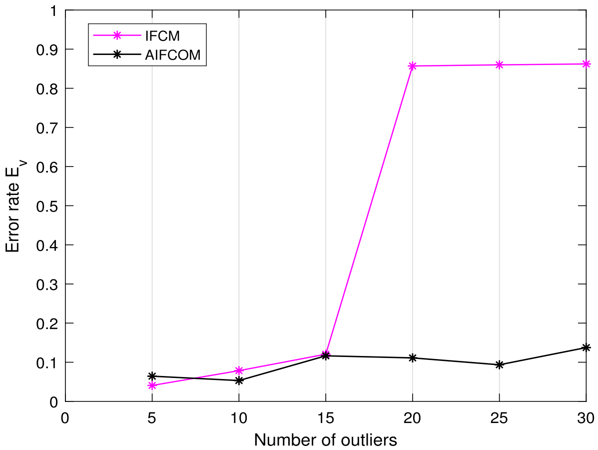

FIGURE 6. Reduction of Error rate in AIFS based clustering algorithm using typicality function.

TABLE 4. Clustering results on UCI datasets by various algorithms.

Dataset	Clustering Algorithms	CA	R_I	Running time (in sec.) (for CA)
Iris	FCOM	90	0.84	0.02
	IFWCOM	95.02	0.89	0.02
	GA-IFWCOM	96	0.91	12.39
	IFCOM	92	0.87	0.03
Wine	FCOM	96.61	0.91	0.08
	IFWCOM	96.65	0.90	0.09
	GA-IFWCOM	97	0.91	48.61
	IFCOM	97.5	89	0.07
WBCD	FCOM	93.85	0.87	0.57
	IFWCOM	93.85	0.87	0.70
	GA-IFWCOM	93.85	0.87	256.36
	IFCOM	87	0.865	0.60
Glass	FCOM	53.66	0.41	0.39
	IFWCOM	61.70	0.57	0.50
	GA-IFWCOM	62.62	0.58	104.79
	IFCOM	58	0.58	0.44
Seeds	FCOM	90.48	0.86	0.03
	IFWCOM	91.41	0.88	0.05
	GA-IFWCOM	91.43	0.88	31.18
	IFCOM	92	0.91	0.05
Wifi	FCOM	93.70	0.83	12.34
	IFWCOM	96.15	0.86	17.59
	GA-IFWCOM	96.42	0.89	1527.92
	IFCOM	89.5	0.895	16.16

[3]For results of FCOM, IFWCOM and GA-IFWCOM algorithms, please see [26]

either in SS or DD. The mathematical formula is given as:

$$R_I = \frac{t_1 + t_4}{R} \quad (37)$$

where, t_1 = number of SS, t_2 = number of DS, t_3 = number of SD and t_4 = number of DD with $R = t_1 + t_2 + t_3 + t_4$. The value of index lies in the interval [0,1].

In the proposed algorithm, we set three parameters to initialize it, namely Yager complement α, fuzzy index m and parameter $\gamma \in (0, 5]$ for fuzzification of given dataset (see [34], [37]). The Yager complement parameter is set to value $\alpha = 0.5$ to increase the role of hesitant factor during fuzzification. The fuzzy index, m is chosen from the interval [1.5,2]. Here, we set the stopping criteria as 10^{-5} which has been taken for FCOM (see [26]). We have fixed the maximum number of iterations for the experiment as 100. The loss functions used in the experiment are Essential loss function, Huber loss function, Logarithmic loss function and Logarithmic-Linear loss function (see [22]). In Table 3, the clustering results obtained using IFCOM algorithm over the six UCI datasets are given. The performance of IFCOM has been compared with other three algorithms in Table 4. The average values of clustering accuracy (CA) and rand index (R_I) obtained for Iris dataset are recorded as 92 percent and 0.87 which shows its better performance than FCOM. Similarly, IFCOM works well for wine dataset with clustering accuracy of 97.5 and rand index equals to 0.89 for Essential loss function. The best clustering accuracy CA obtained by IFCOM for datasets WBCD, Glass, Seeds and Wifi are 87, 58, 92 and 89.5 respectively. The best rand index R_I obtained by IFCOM for datasets WBCD, Glass, Seeds and Wifi are 0.865, 0.585, 0.91 and 0.895, respectively. We have marked the optimal parameters that resulted in best values obtained by IFCOM. In Table 4, we can clearly see that IFCOM performs well in comparison to FCOM for four datasets namely Iris, Wine, Glass and Seeds. The running time for IFCOM seems to be very less in comparison to IFWCOM and GA-IFWCOM clustering algorithms. But unfortunately, a clear cut performance edge of IFCOM over IFWCOM and GA-IFWCOM does not appear in the results.

V. FINAL REMARKS

To begin with, the paper has incorporated typicality function based weighing ordering approach in IFCM, and thus sensitivity of algorithm towards non-informative data-items (outliers) is put to control. Moreover, the proposed IFCOM has assigned lower weights to non-informative data-items (such as outliers) and higher weights to informative data-items. Here, in the criterion function, the distance is replaced with the loss function. We found that Essential loss function has mostly delivered better results in comparison to other loss functions. In other words, an improvement in the results of IFCOM clustering algorithm is observed due to Essential loss function. In addition, our future work is directed towards the further improvement of proposed clustering algorithm. The allocation of lower weights to outliers can be further refined using the concept of dimensional weights over data-items. It may be a line of action to improve IFCOM for image segmentation. Some of the well known algorithms for image segmentation are proposed in [38] and [28].

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(Meenakshi Kaushal and Q. M. Danish Lohani contributed equally to this work.)

CONFLICT OF INTEREST

On behalf of both the authors, the corresponding author states that there is no conflict of interest.

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MEENAKSHI KAUSHAL (Student Member, IEEE) received the master's degree in mathematics from the Department of Mathematics, Maharishi Dayanand University, India, in 2013. She is currently pursuing the Ph.D. degree in mathematics with South Asian University, India. She has been funded a Junior Research Fellowship (JRF) and a Senior Research Fellowship (SRF) from the University Grant Commission, Government of India. She made an international visit to Rio de Generio, Brazil, to present her paper in FUZZ IEEE 2018 which was funded by South Asian University. She has published seven papers in reputed journals/conferences. Her current research interests include the field of machine learning, optimization, data envelopment analysis, intuitionistic fuzzy sets, type-2 fuzzy sets, and operator theory.

Q. M. DANISH LOHANI (Senior Member, IEEE) received the Ph.D. degree from the Department of Mathematics, Aligarh Muslim University, India, in 2009. He is currently working as an Associate Professor with the Department of Mathematics, South Asian University, India. He has published more than 50 scientific articles in well-known journals, including the IEEE Transactions on Fuzzy Systems, *Applied Soft Computing*, and *Chaos, Solitons and Fractals*, and reputed conferences, such as Fuzz-IEEE, IEEE-CEC, and IJCNN. His current research interests include study of both theoretical and application aspects of fuzzy, intuitionistic fuzzy, type-2 fuzzy sets, operators, and summability theory. He is a member of the IEEE Computational Intelligence Society and the IEEE Computer Society.