

Received January 17, 2022, accepted February 4, 2022, date of publication February 25, 2022, date of current version March 7, 2022.

Digital Object Identifier 10.1109/ACCESS.2022.3154779

# Adaptive Fuzzy Backstepping Control Based on Dynamic Surface Control for Uncertain Robotic Manipulator

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This work was supported in part by the Doctor Foundation of Heze University under Grant XY19BS14, in part by the Engineering Laboratory of Heze City, in part by the Key Laboratory of Heze City, and in part by the Engineering Laboratory of Shandong Province.

**ABSTRACT** In the actual operation site, the dynamics of robotic manipulators is affected by two uncertainties, i.e., external disturbances and modeling errors. In this paper, an adaptive fuzzy backstepping controller based on dynamic surface control is proposed to track and control the robotic manipulator while considering both uncertainties and also making a distinction. Firstly, a feedback control technique is used to convert the robotic manipulator dynamics model into two first-order systems, and the control inputs to be designed are introduced. Secondly, the uncertain modeling errors are approximated using two fuzzy networks, and the external disturbances are assumed to be less than some upper limit. Thirdly, in order to weaken the traditional problem of “explosion of complexity” in the design of the adaptive backstepping controller, a dynamic surface control technique is used in this paper. Then, the stability of the designed controller is demonstrated using Lyapunov theory. Finally, simulations are performed with a two-linked robotic manipulator to show the effectiveness of the designed controller, and then, to show the superiority of the controller, simulation results are compared with the results obtained by other control algorithms.

**INDEX TERMS** Adaptive fuzzy control, backstepping control, dynamic surface control, manipulator.

## I. INTRODUCTION

The robotic manipulator, as a typical dynamics system, has been commonly used in industrial fields. However, in the industrial fields, most uncertainties affect control performance. With the improvement of demand for control performance, these uncertainties should be estimated by some techniques, such as disturbance observer [1], neural network [2] and fuzzy logic [3], [4]. So, many advanced control theories based on these techniques are widely used in robotic manipulators to improve tracking accuracy, shorten stabilization time, enhance robustness and so on considering the uncertainties of modelling errors and external disturbances. One fuzzy system-fuzzy neural network-backstepping controller is proposed in [5] to guarantee the accurate, stable and efficient control of complex robotic manipulator system with uncertainties and disturbances. In order to improve disturbance rejection of a 3-degrees-of-freedom overhead

transmission line de-icing robotic manipulator, an observer-based backstepping terminal sliding mode controller is studied in [6]. In order to handle the communication delay, various nonlinearities and uncertainties in teleoperation manipulator system, a globally stable adaptive fuzzy backstepping controller is designed in [7]. To tackle the tracking control problem of uncertain electrically driven robotic manipulators, an adaptive fuzzy voltage-based backstepping controller is developed in [8]. A finite time adaptive backstepping fault tolerant controller based on fractional-order theory is proposed in [9] for robotic manipulators in the presence of uncertainties, unknown external load disturbances and actuator faults to achieve fast response and high-precision tracking performance. Reference [10] investigates a new controller based on the combination of sliding mode control and backstepping control strategy for the tracking control problem of the uncertain welding robot.

However, the backstepping method used in the above-mentioned controller design has a strong limitation, that is “explosion of complexity” caused by the repeated

The associate editor coordinating the review of this manuscript and approving it for publication was Dazhong Ma<sup>ID</sup>.

differentiation of the designed virtual control signal [11]. With pursuing progress in cybernetics, a pioneering technique called dynamic surface control (DSC) is put forward in [12] and [13] in order to design more appropriate controllers for uncertain and mismatched nonlinear systems in the strict feedback form by adopting first-order filters through the procedures of recursive design. Inspired by the results of this study, many successful cases of DSC-based diversification techniques have sprung up. To overcome the “explosion of complexity” problem during the construction of the adaptive backstepping controller, the DSC technique is adopted in [14] for nonlinear input-delay systems. Four examples show that the DSC-based composite controller designed in [15] can not only track the desired trajectory well, but also present a better approximation capability when dealing with the control problem of complex unknown systems with disturbances. In [16], by adding the DSC technique into adaptive fuzzy backstepping control (AFBC), the developed controller can not only overcome the general problem but also prevent the control singularity problem completely. In [17], threefold merits are shown in dealing the stability of switched uncertain nonlinear systems by integrating the adaptive observer method and the DSC technique.

With the development of the DSC technique, this novel technique has started to be applied in many engineering fields. In [18] and [19], DSC-based adaptive controllers are successfully proposed and proved vaultful for uncertain permanent magnet synchronous motors. Dandan Lei and his co-authors successfully developed composite controllers based on DSC technique for micro-gyroscope [20], [21] to improve the timeliness and effectiveness of tracking and other performances in the presence of model uncertainties and external disturbances. In [22], the effectiveness of the new application of DSC on uncertain robot manipulators is verified effectiveness by using Lyapunov theory and simulating results. Both [23] and [24] adopt DSC technique to propose advanced tracking controllers for induction motor servo drive and DC/DC boost converter respectively.

Motivated by the successful application of DSC techniques in engineering fields, this paper proposes an adaptive fuzzy backstepping dynamic surface control (AFBDSC) for uncertain robotic manipulators to improve control performances. In the presence of uncertainties, to reduce the number of fuzzy rules, two fuzzy networks are adopted to approximate the uncertain modeling parameters. To compensate for these approximation errors, the backstepping control is brought in designing the controller. The DSC technique is incorporated to eliminate the general problem of “explosion of complexity” produced by using the backstepping control. The successful combination of these control theories improves control performances of robotic manipulator systems considering the modeling uncertainties and external disturbances. The main contributions of this paper can be summarized as follows.

First: Two kind of uncertainties, i.e., the external disturbances and the modeling errors are considered in order to

reflect the actual operation of robotic manipulator. For distinguishing the interferential effects of the two kind uncertainties, the uncertain modeling parameters are approximated using fuzzy networks, whereas the external disturbances are assumed to be less than some upper limit.

Second: A feedback control technique is used to transform the n-link robotic manipulator dynamics based on the Lagrange equation into two first-order subsystems. Then, the DSC is integrated into the adaptive fuzzy control scheme by using the backstepping design framework, where two fuzzy networks are used to estimate the modeling error in order to reduce the number of fuzzy rules, and the DSC technique is used to overcome the general phenomenon of “explosion of complexity” inherent in the backstepping design method.

Third: Most of the present research results, when applying DSC technology to control problems, target single-input, single-output (SISO) systems, while the robot manipulator system is a multi-input, multi-output system, so the successful application of DSC technology in this paper can make a small contribution to promote the application of this technology.

The rest of this paper is organized as follows. Section 2 carries out the robotic manipulator system and fuzzy logic system. Section 3 develops the control algorithm of the AFBDC for the uncertain robotic manipulator. Section 4 carries out the simulation results of a two-link robotic manipulator and a simulated comparison. Section 5 summarizes the full text.

## II. SYSTEMS DESCRIPTION

### A. ROBOTIC MANIPULATOR SYSTEM

Ignoring the detailed derivation based on the Lagrange equation, the dynamics equation for a rigid n-link robotic manipulator can be simply shown as [25], [26]:

$$M(q)\ddot{q} + h(q, \dot{q}) = \tau \quad (1)$$

where  $q, \dot{q}, \ddot{q} \in R^n$  are vectors of the joint displacement, velocity and acceleration, respectively.  $M(q) \in R^{n \times n}$  is an inertial matrix of the robotic manipulator,  $h(q, \dot{q}) \in R^n$  couples the Coriolis, centrifugal forces  $C(q, \dot{q})\dot{q} \in R^n$  and gravitational forces  $G(q) \in R^n$  used in other studies,  $\tau \in R^n$  is the generalized control torque.

As authors did in [25] and [26], the influence of uncertainties on the manipulator system has been increasingly considered in studies. In this paper, these uncertainties are also considered, so dynamics (1) can be rewritten as:

$$M_0(q)\ddot{q} + h_0(q, \dot{q}) = \tau + \rho + f \quad (2)$$

where  $M_0(q)$  and  $h_0(q, \dot{q})$  are nominal models,  $\rho = -\Delta M(q)\ddot{q} - \Delta h(q, \dot{q})$  denotes the uncertain modeling parameters and  $\Delta M(q)$ ,  $\Delta h(q, \dot{q})$  are unknown items of models.  $f$  denotes the onefold external disturbances, and it is bounded in the following.

Almost all the related literatures introduce the following property about the inertial matrix  $M_0(q)$  before developing controller design.

*Property 1:*  $M_0(q)$  is a positive-definite and symmetric matrix, satisfying the following inequality for all  $q$ :

$$\lambda_m \|q\|^2 \leq q^T M_0(q) q \leq \lambda_M \|q\|^2$$

where  $\lambda_m < \lambda_M$  are positive numbers.

in order to use the backstepping theory in designing the controller, external control input  $u$  is first introduced into the generalized control torque [27], [28]:

$$\tau = h_0(q, \dot{q}) + M_0(q)u \quad (3)$$

Then after the simple calculation of (2) and (3), the following more concise second-order system is obtained:

$$\ddot{q} = \phi(q, \dot{q}, \ddot{q}) + u + d \quad (4)$$

Finally, it is decomposed into two first-order subsystems, described as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \phi(q, \dot{q}, \ddot{q}) + u + d \end{cases} \quad (5)$$

where  $x_1 = q, x_2 = \dot{q}, \phi(q, \dot{q}, \ddot{q}) = M_0^{-1}(q)\rho$  is on behalf of the modeling uncertainties and the bounded  $d = M_0^{-1}(q)f$  is on behalf of the external uncertainties, which satisfies:

$$|d_i| \leq d_0 \quad (6)$$

where  $d_i$  is the  $i^{\text{th}}$  element of the vectors  $d$  and  $d_0$  is a small positive constant.

To give an ideal development, the fuzzy logic systems are used in the following to approximate the modeling uncertainties.

### B. FUZZY LOGIC SYSTEM

Fuzzy control, as an intelligent control technique, is widely used in uncertain nonlinear (dynamic) systems due to its super nonlinear approximation ability. The fuzzy logic system has been discussed extensively in these studies, so this paper provides a brief introduction to the fuzzy logic system.

Since uncertainty is approximated by the fuzzy logic system, without loss of generality, the uncertainty system and the output of its approximation can be recorded as  $f(\omega) \in R$  and  $y$  respectively with input vector  $\omega = [\omega_1, \dots, \omega_n]^T \in R^n$ . Generally speaking, a fuzzy logic system contains four parts [28], i.e. the fuzzifier, the fuzzy inference engine, the defuzzifier and the knowledge base composed by rulebase and database. When the first three parts are chosen as the singleton fuzzifier, the product inference engine and the center-average defuzzifier respectively, and the rulebase is chosen as the If-Then rules, then it creates a mapping from the input  $\omega$  to the output  $y$  shown as:

$$y = \frac{\sum_{i=1}^N \tilde{y}^i \prod_{j=1}^n \mu_j^i(\omega_j)}{\sum_{i=1}^N \left( \prod_{j=1}^n \mu_j^i(\omega_j) \right)} = \theta^T \psi(\omega) \quad (7)$$

where  $N$  is the total number of fuzzy rules,  $\theta = [\tilde{y}^1, \dots, \tilde{y}^N]^T$  is the weight vector and each element of which

is an adjustable value where the fuzzy membership function  $\mu_{B^i}(\tilde{y}^i)$  shows the maximum value, choosing  $\mu_{B^i}(\tilde{y}^i) = 1$  usually.  $\psi(\omega) = [\psi_1, \dots, \psi_N]^T$  is called the fuzzy basis function vector, whose element is of the form:

$$\psi_i = \frac{\prod_{j=1}^n \mu_j^i(\omega_j)}{\sum_{i=1}^N \left( \prod_{j=1}^n \mu_j^i(\omega_j) \right)}, \quad i = 1, \dots, N \quad (8)$$

### III. AFBSDC DESIGN

In this section, the AFBSDC is designed for the uncertain robotic manipulator.

#### A. TWO LEMMAS

*Lemma 1:* For any given two vectors  $A, B \in R^n$ , there is:

$$(A^T B)^2 \leq A^T A B^T B$$

*Lemma 2:* For any given two positive numbers  $a$  and  $b$ , there is:

$$2\sqrt{ab} \leq (ab + 1)$$

*Remark 1:* Lemma 1 is actually the Cauchy-Schwarz inequality in Euclidean space.

#### B. FUZZY APPROXIMATION

For the sake of reducing the total number of fuzzy rules in designing the controller, firstly the modeling uncertainties  $\phi(q, \dot{q}, \ddot{q})$  can be divided into the following two parts:

$$\phi(q, \dot{q}, \ddot{q}) = \phi^1(q, \dot{q}) + \phi^2(q, \ddot{q}) \quad (9)$$

with:

$$\begin{cases} \phi^1(q, \dot{q}) = -M_0^{-1}(q)\Delta h(q, \dot{q}) \\ \phi^2(q, \ddot{q}) = -M_0^{-1}(q)\Delta M(q)\ddot{q} \end{cases} \quad (10)$$

Then each part is approximated by using the fuzzy logic system mentioned above, and two outputs are obtained as:

$$\begin{cases} \hat{\phi}^1(q, \dot{q}|\theta^1) = [\theta_1^{1T} \psi^1(q, \dot{q}), \dots, \theta_n^{1T} \psi^1(q, \dot{q})]^T \\ \quad = \theta^{1T} \psi^1(q, \dot{q}) \\ \hat{\phi}^2(q, \ddot{q}|\theta^2) = [\theta_1^{2T} \psi^2(q, \ddot{q}), \dots, \theta_n^{2T} \psi^2(q, \ddot{q})]^T \\ \quad = \theta^{2T} \psi^2(q, \ddot{q}) \end{cases} \quad (11)$$

So, the lumped approximation of the  $\phi(q, \dot{q}, \ddot{q})$  is:

$$\hat{\phi}(q, \dot{q}, \ddot{q}) = \theta^{1T} \psi^1(q, \dot{q}) + \theta^{2T} \psi^2(q, \ddot{q}) \quad (12)$$

Letting  $\theta^{1*}$  and  $\theta^{2*}$  be the the optimal weight matrices of each fuzzy logic system, the minimum approximation output errors is defined as follows:

$$\begin{cases} \varepsilon^1 = \phi^1(q, \dot{q}) - \theta^{1*T} \psi^1(q, \dot{q}) \\ \varepsilon^2 = \phi^2(q, \ddot{q}) - \theta^{2*T} \psi^2(q, \ddot{q}) \end{cases} \quad (13)$$

These errors are bounded on the basis of the approximation principle of the fuzzy system, that is to say there are very small positive constants  $\sigma^1$  and  $\sigma^2$  satisfying:

$$\begin{cases} |\varepsilon_i^1| \leq \sigma^1 \\ |\varepsilon_i^2| \leq \sigma^2 \end{cases} \quad (14)$$

with  $\varepsilon_i^1$  and  $\varepsilon_i^2$  being the  $i^{\text{th}}$  element of the vectors  $\varepsilon^1$  and  $\varepsilon^2$  respectively.

Lastly, the approximate weight matrix errors are defined as:

$$\begin{cases} \tilde{\theta}^1 = \theta^{1*} - \theta^1 \\ \tilde{\theta}^2 = \theta^{2*} - \theta^2 \end{cases} \quad (15)$$

### C. CONTROLLER DESIGNS

In this subsection, the control input  $u$  in (3)-(5) is designed based on the AFBDC technique.

Let the given joint displacement, velocity and acceleration be  $q_d, \dot{q}_d, \ddot{q}_d$  respectively. The tracking error can be stated as follows:

$$e = x_1 - q_d \quad (16)$$

And the time derivative of (16) is:

$$\dot{e} = \dot{x}_1 - \dot{q}_d = x_2 - \dot{q}_d \quad (17)$$

After choosing the first Lyapunov function:

$$V_1 = \frac{1}{2}e^T e \quad (18)$$

and differentiating it with respect to time, there is:

$$\dot{V}_1 = e^T \dot{e} = e^T (x_2 - \dot{q}_d) \quad (19)$$

For seeking  $\dot{V}_1 \leq 0$ , as other articles did, a virtual control  $\bar{x}_2$  is also defined here by the following expression:

$$\bar{x}_2 = -k_1 e + \dot{q}_d \quad (20)$$

where  $k_1$  is a positive constant.

On account of the drawbacks of the multiple surface sliding control [29], with a time constant  $\tau$ , let  $\bar{x}_2$  pass through the following first-order filter to obtain another filter virtual control vector  $\alpha$ :

$$\tau \dot{\alpha} + \alpha = \bar{x}_2, \alpha(0) = \bar{x}_2(0) \quad (21)$$

The boundary layer error and the surface error are respectively defined as follows:

$$y = \alpha - \bar{x}_2 \quad (22)$$

$$s = x_2 - \alpha \quad (23)$$

After differentiating  $s$  with respect to time, one obtains:

$$\dot{s} = \dot{x}_2 - \dot{\alpha} = \phi(q, \dot{q}, \ddot{q}) + u + d - \dot{\alpha} \quad (24)$$

Lastly the control input is designed as follows:

$$u = -\hat{\phi}^1(q, \dot{q}|\theta^1) - \hat{\phi}^2(q, \ddot{q}|\theta^2) - (d_0 + \sigma^1 + \sigma^2)I_n \text{sgn}(s) + \dot{\alpha} - e - k_2 s \quad (25)$$

where  $\text{sgn}(s)$  is a sign function vector. One chooses the weight updating algorithms as follows:

$$\begin{cases} \dot{\theta}_i^1 = -\Gamma_{1i}^{-1} s_i \psi^1(q, \dot{q}) \\ \dot{\theta}_i^2 = -\Gamma_{2i}^{-1} s_i \psi^2(q, \ddot{q}) \end{cases} \quad (26)$$

where  $k_2$  and  $\Gamma_{ji}(j = 1, 2)$  are known as positive constants.

*Remark 2:* It should be noted that, like being described in the existing research results, the meaning of the dynamic surface in this paper can be shown by the following two points. On one hand, a new dynamic equation Eq. (21) is obtained by introducing the first-order filter. On the other hand, the Eq. (23) defines the dynamic surface error.

### D. STABILITY ANALYSIS

In this subsection, the stability of the closed-loop system is studied by using the Lyapunov theory to analyze the following theorem.

*Theorem 1:* Considering the closed-loop system consisting of Eq. (5) and Eq. (25), if the weight updating algorithm is chosen as Eq. (26), then the convergences of the tracking error and all the system parameters can be guaranteed and converge to zero.

*Proof:* Based on the tracking error, considering the boundary layer error (22), the surface error (23) and the weight matrix errors (15), another Lyapunov function is chosen as follows:

$$V = V_1 + \frac{1}{2}y^T y + \frac{1}{2}s^T s + \frac{1}{2} \left( \sum_{i=1}^n \tilde{\theta}_i^{1T} \Gamma_{1i} \tilde{\theta}_i^1 + \sum_{i=1}^n \tilde{\theta}_i^{2T} \Gamma_{2i} \tilde{\theta}_i^2 \right) \quad (27)$$

The time derivative of (27) is:

$$\dot{V} = \dot{V}_1 + y^T \dot{y} + s^T \dot{s} + \sum_{i=1}^n \tilde{\theta}_i^{1T} \Gamma_{1i} \dot{\tilde{\theta}}_i^1 + \sum_{i=1}^n \tilde{\theta}_i^{2T} \Gamma_{2i} \dot{\tilde{\theta}}_i^2 \quad (28)$$

Considering the above given equations, because of:

$$\begin{aligned} \dot{V}_1 &= e^T \dot{e} = e^T (x_2 - \dot{q}_d) = e^T (s + \alpha - \dot{q}_d) \\ &= e^T (s + y + \bar{x}_2 - \dot{q}_d) \\ &= e^T (s + y + -k_1 e + \dot{q}_d - \dot{q}_d) \\ &= -k_1 e^T e + e^T s + e^T y \end{aligned} \quad (29)$$

$$\begin{aligned} y^T \dot{y} &= y^T (\dot{\alpha} - \dot{\bar{x}}_2) = y^T \left( \frac{\bar{x}_2 - \alpha}{\tau} - \dot{\bar{x}}_2 \right) = y^T \left( -\frac{y}{\tau} - \dot{\bar{x}}_2 \right) \\ &= -\frac{1}{\tau} y^T y + y^T (-\dot{\bar{x}}_2) = -\frac{1}{\tau} y^T y + y^T (k_1 \dot{e} - \dot{q}_d) \end{aligned} \quad (30)$$

$$\begin{aligned} s^T \dot{s} &= s^T (\dot{x}_2 - \dot{\alpha}) = s^T (\phi(q, \dot{q}, \ddot{q}) + u + d - \dot{\alpha}) \\ &= s^T \left( \phi(q, \dot{q}, \ddot{q}) - \hat{\phi}^1(q, \dot{q}|\theta^1) - \hat{\phi}^2(q, \ddot{q}|\theta^2) - (d_0 + \sigma^1 + \sigma^2)I_n \text{sgn}(s) + \dot{\alpha} - e - k_2 s + d - \dot{\alpha} \right) \\ &= -s^T e + s^T \left( \varepsilon^1 + \tilde{\theta}^1 \psi^1(q, \dot{q}) + \varepsilon^2 + \tilde{\theta}^2 \psi^2(q, \ddot{q}) - (d_0 + \sigma^1 + \sigma^2)I_n \text{sgn}(s) - k_2 s + d \right) \end{aligned}$$

$$(31)$$

and:

$$\begin{aligned} & \sum_{i=1}^n \tilde{\theta}_i^{1T} \Gamma_{1i} \dot{\tilde{\theta}}_i^1 + \sum_{i=1}^n \tilde{\theta}_i^{2T} \Gamma_{2i} \dot{\tilde{\theta}}_i^2 \\ & = s^T \left( -\tilde{\theta}^1 \psi^1(q, \dot{q}) - \tilde{\theta}^2 \psi^2(q, \dot{q}) \right) \end{aligned} \quad (32)$$

Then, there is:

$$\begin{aligned} \dot{V} & = -e^T K_1 e + e^T y - \frac{1}{\tau} y^T y + y^T (K_1 \dot{e} - \ddot{q}_d) \\ & \quad + s^T \left( \varepsilon^1 + \varepsilon^2 - (d_0 + \sigma^1 + \sigma^2) I_n \operatorname{sgn}(s) - k_2 s + d \right) \\ & = -e^T K_1 e - \frac{1}{\tau} y^T y - k_2 s^T s + y^T (e + k_1 \dot{e} - \ddot{q}_d) \\ & \quad + s^T \left( \varepsilon^1 + \varepsilon^2 - (d_0 + \sigma^1 + \sigma^2) I_n \operatorname{sgn}(s) + d \right) \end{aligned} \quad (33)$$

Considering Eq. (6) and Eq. (14), no matter  $s_i \leq 0$  or  $s_i > 0$ , there is always:

$$\dot{V} \leq -k_1 e^T e - \frac{1}{\tau} y^T y - k_2 s^T s + y^T (e + k_1 \dot{e} - \ddot{q}_d) \quad (34)$$

where  $s_i$  is the  $i^{\text{th}}$  element of the surface error  $s$ .

*Remark 3:* Let  $S = s^T (\varepsilon^1 + \varepsilon^2 - (d_0 + \sigma^1 + \sigma^2) I_n \operatorname{sgn}(s) + d)$  in Eq. (33), when  $s_i \leq 0$ , there is:

$$\begin{aligned} S & = s^T \left( \varepsilon^1 + \varepsilon^2 + (d_0 + \sigma^1 + \sigma^2) + d \right) \\ & = s^T \left( \varepsilon^1 + \sigma^1 + \varepsilon^2 + \sigma^2 + d + d_0 \right) \leq 0 \end{aligned}$$

and when  $s_i > 0$ , there also is:

$$\begin{aligned} S & = s^T \left( \varepsilon^1 + \varepsilon^2 - (d_0 + \sigma^1 + \sigma^2) + d \right) \\ & = s^T \left( \varepsilon^1 - \sigma^1 + \varepsilon^2 - \sigma^2 d - d_0 \right) \leq 0 \end{aligned}$$

Defining  $A = (e + k_1 \dot{e} - \ddot{q}_d)$  and letting the upper limit be  $B$ , naturally, we come to:

$$A^T A \leq B^T B \quad (35)$$

Taking the Lemma 1 and the Lemma 2 into consideration, Eq. (34) becomes:

$$\begin{aligned} \dot{V} & \leq -k_1 e^T e - \frac{1}{\tau} y^T y - k_2 s^T s + y^T A \\ & \leq -k_1 e^T e - \frac{1}{\tau} y^T y - k_2 s^T s + \sqrt{(y^T A)(y^T A)} \\ & \leq -k_1 e^T e - \frac{1}{\tau} y^T y - k_2 s^T s + \sqrt{A^T A y^T y} \\ & \leq -k_1 e^T e - \frac{1}{\tau} y^T y - k_2 s^T s + \frac{1}{2} (A^T A y^T y + 1) \\ & = -k_1 e^T e - k_2 s^T s + \left( \frac{1}{2} A^T A - \frac{1}{\tau} \right) y^T y + \frac{1}{2} \end{aligned} \quad (36)$$

Choosing  $k_1 \geq 1 + r$ ,  $r > 0$ ,  $k_2 \geq (1/2) + r$ ,  $(1/\tau) \geq (1/2) B^T B + r$ , then Eq. (36) becomes:

$$\dot{V} \leq (1 - k_1) e^T e + \left( \frac{1}{2} - k_2 \right) s^T s$$

$$\begin{aligned} & + \left( \frac{1}{2} A^T A - \frac{1}{2} B^T B - r \right) y^T y + \frac{1}{2} \\ & \leq -r e^T e - r s^T s - r y^T y + \frac{1}{2} \\ & = -2r \left( \frac{1}{2} e^T e + \frac{1}{2} s^T s + \frac{1}{2} y^T y \right) + \frac{1}{2} \end{aligned} \quad (37)$$

Let  $C = (1/2)(e^T e + s^T s + y^T y)$  and when  $r \geq (1/4C)$ , Eq. (37) becomes:

$$\dot{V} \leq -2 \frac{1}{4C} C + \frac{1}{2} = 0$$

Therefore, according to Lyapunov theory, the tracking error  $e$  can achieve as small as the desired range and all the system parameters are bounded. So, the stability is assured.

The control structure of the proposed controller is shown detailedly in Figure 1. The control structure contains two closed-loops. The closed-loop I gives the tracking errors  $e$ , and the closed-loop II gives the proposed controller  $u$ .

#### IV. SIMULATION RESULTS

In this section, we will illustrate the effectiveness of the proposed controller using a two-link robot manipulator shown in Figure 2 as an example. The two-link robot manipulator is one important simulation object, which is widely adopted in relevant literatures [30]–[32] to show the validity of the proposed method.

The inertial matrix  $M_0(q)$  and the coupling term  $h_0(q, \dot{q})$  in Eq. (2) are:

$$M_0(q) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, h_0(q, \dot{q}) = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

where:

$$\begin{cases} M_{11} = (m_1 + m_2)l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 \cos(q_2) \\ M_{12} = M_{21} = m_2 l_2^2 + m_2 l_1 l_2 \cos(q_2) \\ M_{22} = m_2 l_2^2 \\ h_1 = -m_2 l_1 l_2 \sin(q_2) \dot{q}_2 (\dot{q}_1 + \dot{q}_2) + (m_1 + m_2) \cos(q_2) g \\ h_2 = m_2 l_1 l_2 \sin(q_2) \dot{q}_1^2 \end{cases}$$

Without loss of generality, these parameter values are simply shown in the following Table 1.

TABLE 1. The system model parameters.

Parameters	Values	Units
The length of first link ( $l_1$ )	1	$m$
The length of second link ( $l_2$ )	0.5	$m$
The mass of first link ( $m_1$ )	1	$kg$
The mass of second link ( $m_2$ )	0.5	$kg$

Assume that external disturbances and modeling errors are described by:

$$\begin{cases} \Delta M(q) = 0.2 M_0(q) \\ \Delta h(q, \dot{q}) = 0.2 h_0(q, \dot{q}) \\ d = 0.5 [\sin(2t), \sin(2t)]^T \end{cases}$$



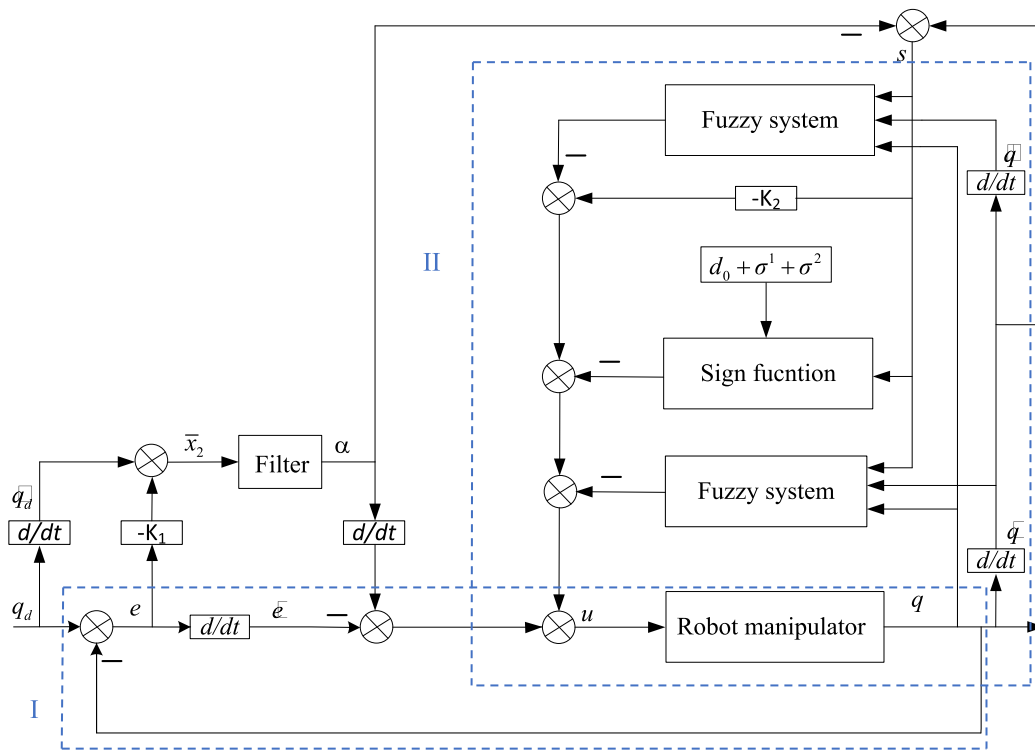


FIGURE 1. The control structure.

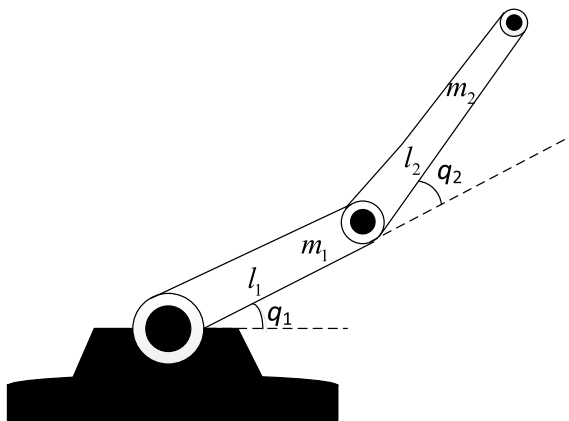


FIGURE 2. Two-link robot manipulator.

Other parameters are respectively chosen as:  $d_0 = 1q_d = [0.3\sin(t), 0.5\sin(t)]^T$ ,  $\Gamma_{11} = \Gamma_{12} = \Gamma_{21} = \Gamma_{22} = 0.01$ ,  $\sigma_1 = \sigma_2 = 2$ ,  $k_1 = 100$ ,  $k_2 = 25$  and  $\tau = 0.04$ .

In simulations, five fuzzy levels, i.e., NB, NS, ZO, PS, PB [33] are selected on the universe of each input variable and the membership is chosen as the following Gaussian function:

$$\mu_{A_i^l}(x_i) = \exp \left[ - \left( \frac{x_i - \bar{x}_i^l}{\pi/24} \right)^2 \right]$$

where  $\bar{x}_i^l$  are  $-\pi/6, -\pi/12, 0, \pi/12, \text{ and } \pi/6$  respectively.

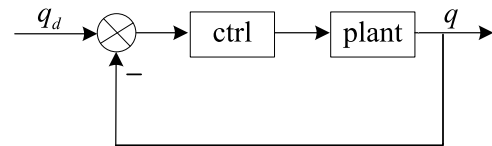


FIGURE 3. The simulation model.

The simulation process is built in the Matlab/Simulink soft, and the simulation model can be simplify shown in Figure 3. In Figure 3, both “ctrl” module and “plant” module are S-Function blocks, which are be written in Matlab to describe the controller Eq. (25) and the two-link robot manipulator Eq. (2) respectively.

The corresponding simulation results are shown in Figure 4, where Figure 4 (a) is trajectory tracking and Figure 4 (b) is control input.

In order to investigate the superiority of the developed controllers adopting the DSC technique, a simulated comparison of the backstepping adaptive fuzzy controller (BAFC) [34] is shown in Figure 5, where Figure 5 (a) is trajectory tracking and Figure 5 (b) is also control input. Here in the simulation results of BAFC, the same parameters are chosen.

From the comparison results, we know that the proposed controller can guarantee that the actual trajectory tracks the desired trajectory more exactly and the vibration of control input mitigates much more.

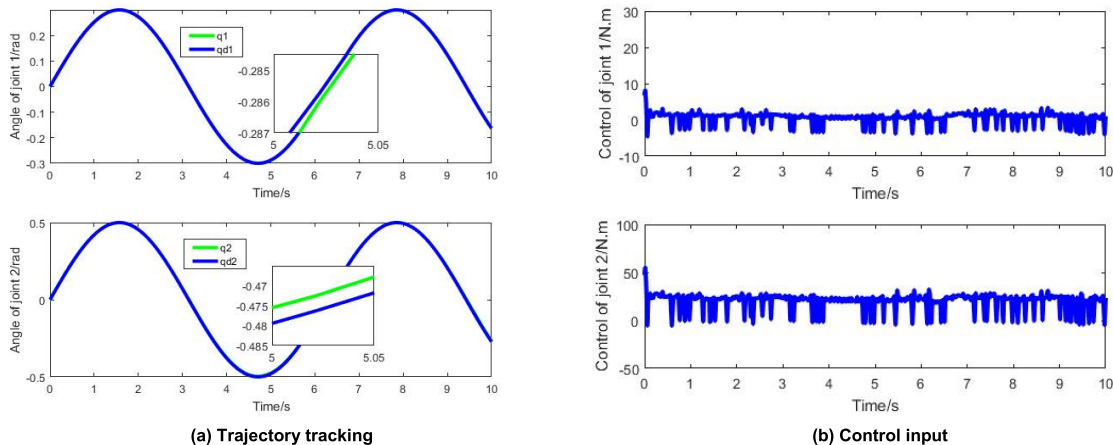


FIGURE 4. Simulation results with proposed controller.

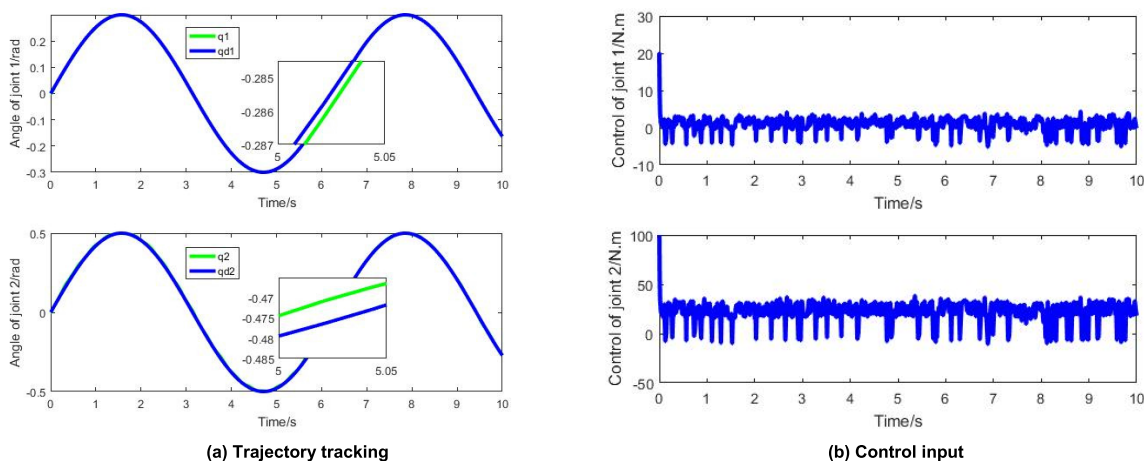


FIGURE 5. Simulation results with BAFC.

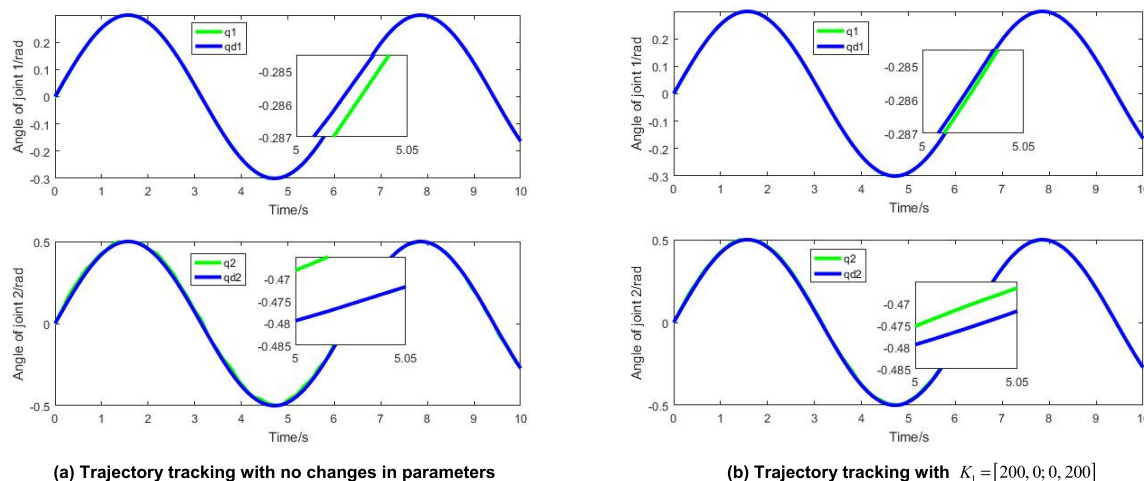


FIGURE 6. Simulation results with  $\Delta M(q) = 0.4M_0(q)$  and  $\Delta h(q, \dot{q}) = 0.4h_0(q, \dot{q})$ .

Remark 4: These parameter values of the two-link are chosen with a bit simplification, but they are more representative. Other parameters of the proposed controller are chosen after

some simulation experiments. When the control plant, the modeling errors and the external disturbances change, the parameters of the controller have to be given a regulation

to get a better control effectiveness. For example, when the modeling errors increase to 40%, the simulation results are shown in Figure 6. Figure 6 (a) is obtained with no changes in parameters of the proposed controller. Now, the trajectory tracking is larger. After adjusting  $K_1 = [200, 0; 0, 200]$ , the trajectory tracking is shown in Figure 6(b), which presents better control performance.

## V. CONCLUSION

This paper presents a hybrid combination of the AFBC and the DSC for the tracking control of robotic manipulators. The developed controller eliminates the corporate influence of the external disturbance and the modeling uncertainties. Firstly, in the process of design, the dynamics of the n-link uncertain robotic manipulator is transformed into two first-order subsystems. Secondly, the modeling uncertainties are approximated using fuzzy logic system. Thirdly, the DSC is introduced to overcome the general phenomenon of “explosion of complexity” when use the backstepping control scheme. Then, the convergence and the stability are assured using the Lyapunov theory. Lastly, a simulation example of a two-link manipulator is given to show its effectiveness.

It should be noted that, although the DSC technique has a lot of advantages, it is better to combine other control techniques besides the adaptive fuzzy control-based on in this paper, such as T-S fuzzy control, neural network control, disturbance observer-based control, sliding mode control, and so on to achieve more superior control performances. In the future work, on one hand, it is challenging to study more control techniques which can be combined with the DSC technique. On the other hand, it is meaningful to apply the proposed controller to the specific robotic manipulator.

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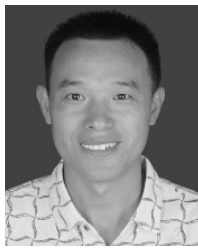
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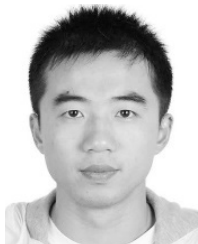
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