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Adaptive Cooperative Tracking Control of Multiple Trains With Uncertain Dynamics and External Disturbances Based on Adjacent Information

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ABSTRACT Automatic train operation is an important part of the train control system. As train operating intervals continue to shorten and train speeds continue to increase, multiple train cooperative control is currently an important technology to further improve the efficiency of train operation and line passing capacity. However, considering various factors such as the nonlinearity and uncertainty of the train dynamics model and the complexity of the line conditions, this creates even greater demands on the design of the controller. In this study, we propose an adaptive cooperative tracking control method for multiple trains using adjacent information. For the multiple-train coordinated tracking control in the presence of model uncertainties, unknown parameters, and external disturbances, a distributed cooperative control scheme for multiple trains is designed using the displacement, velocity, and acceleration information of adjacent trains, combined with radial basis function neural networks and adaptive methods. A fast high-order sliding mode observer is used to estimate the train velocity and acceleration information. Stability and convergence are proved for single and multiple trains utilizing Lyapunov stability analysis. Simulation examples demonstrate the effectiveness of the proposed controller.

INDEX TERMS Adaptive methods, cooperative control, multiple trains, radial basis function networks, stability analysis, trajectory tracking, uncertainty.

I. INTRODUCTION

With its safety, efficiency and low carbon footprint, urban railway transportation is becoming an indispensable mode of transport in the development of major cities [1]. The development of contemporary railway systems has been considerably aided by communication-based train control systems (CBTC) [2]. For signaling systems equipped with CBTC, which operate in moving block or virtual block forms, real-time information about the trains in front of them, vehicle characteristics and other information is used to calculate the moving authority of trains through advanced communication, positioning and control technologies, thereby reducing the interval between trains and improving the operational efficiency of railway lines, and has been widely used in urban rail transit systems [3]. Automatic train operation (ATO) is a very important part of the train control system, acting in all phases of train operation, and one of its main tasks is

speed regulation. As train operating intervals continue to shorten and train speeds continue to increase, it is necessary to explore control strategies that use adaptive and intelligent control methods.

In recent decades, a great deal of research has been carried out by researchers on methods for single train automatic operation control. In [4], a model-free adaptive controller combining neural networks (NN) and proportional-integral-derivative (PID) algorithms is proposed in order to be able to adjust the PID gain adaptively and efficiently. The proposed NNPID controller has better tracking and is more energy saving. For high-speed trains with time-varying resistance coefficients, an adaptive model predictive control (MPC) technique was proposed in [5]. An adaptive update rule for estimated parameters and a multiply restricted MPC for the estimated system are combined to create the adaptive MPC. In [6], a robust adaptive nonsingular terminal sliding mode control technique was developed for solving the automatic train operation system's position and velocity tracking control problem in the presence of unknown parameters, model

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uncertainty, and external disturbances. In [7], intelligent train operation algorithms based on expert systems and reinforcement learning are proposed without the use of accurate train models and offline optimized speed profiles, respectively, and the superiority of the algorithms is verified by means of a simulation platform.

Trains are not isolated in the line network and the states between trains are coupled. In addition, with the development of advanced communication technologies, train-to-train communication in real time has become possible [8]–[10]. Multi-train cooperative control is currently an important technology to further improve the efficiency of train operation and line passing capacity. In [11], the implementation of one-parameter adaptive cooperative control of multiple trains in predecessor following and bidirectional architecture modes is proposed. To solve the problem of “explosion of complexity” in backstepping method design, a dynamic surface control mechanism is added into the algorithm. A multi-agent system is used to model the motion of high-speed trains in [12]. The new coordinated cruise control strategy for multiple trains based on neighbouring trains’ information was designed to ensure that each train can track the intended speed and that high-speed trains run safely and efficiently, using potential fields and the LaSalle’s invariance principle. Based on the multi-agent system model above, [13] investigates the coordinated control of multiple high-speed trains in the presence of actuator saturation. In [14], the cooperative control for multiple high-speed trains was addressed to achieve the speed and position of high-speed trains that are assured to be constrained to certain speed limits and authorized distances certified by automatic train protection and movement authority respectively. In [15], a novel distributed optimal control algorithm based on a distributed message passing mechanism was developed using an alternating direction method of multipliers and a model predictive control approach. The controller is updated based on train information with common global variables rather than exchanging any information with other trains. In [16], considering train controller output constraints and safe train following distance, a multi-train cooperative control model was proposed to adjust train following headway based on cooperative adaptive cruise control. The simulation results were analysed to illustrate the effectiveness of the proposed method.

In the problem of multi-train cooperative control, the design of this controller poses higher requirements, considering various factors such as the non-linearity and uncertainty of the train dynamics model and the complexity of the line conditions. The combination of non-linear and intelligent control methods can effectively solve these problems. A neural networks-based auto-tune PID-like controller is suggested in [17] for underwater vehicles. The neural network’s purpose is to automatically estimate the best set of PID gains for system stability. In [18], a full-regulated neural network with a double hidden layer recurrent neural network structure is presented for a class of dynamic systems and an adaptive

global sliding-mode controller is based on this. When compared to a general neural network with a single hidden layer, the novel proposed network may improve network accuracy and generalization, reduce network weights, and speed up network training. In [19], the backstepping method was used to develop a direct one-parameter adaptive neural network control scheme for a class of nonlinear stochastic strict-feedback systems with unknown time delays. In [20], neural networks are used to design adaptive motion controllers for underactuated wheeled inverted pendulum model. In [21], based on the characteristics of fuzzy neural network without prior knowledge of uncertainty and sufficient observed data, the online approximation of the uncertain dynamics of the robot is carried out, and the adaptive fuzzy neural network control is proposed. In [22], a fuzzy gain-scheduling sliding mode control method is proposed by combining sliding mode control with fuzzy logic, thus solving the attitude regulation problem of unmanned quadcopters with parameter uncertainty and external disturbances.

Motivated by these above observations, this study proposes an adaptive cooperative tracking control method for multiple trains using adjacent information. Specifically, the contributions of this study are presented as follows: (1) For the multiple train coordinated tracking control in the presence of model uncertainties, unknown parameters and external disturbances, a distributed cooperative control scheme for multiple trains is designed using the displacement, velocity, and acceleration information of adjacent trains, combined with radial basis function neural networks and adaptive methods. Neural networks with online adjustment characteristics and adaptive methods are used to estimate uncertain dynamics models and external disturbances, and the stability and convergence of single and multiple trains are guaranteed by Lyapunov methods. (2) Based on the above design, an observer-based multi-train cooperative tracking controller is designed to reduce the burden on sensors while maintaining good tracking performance, with only the position information of neighboring trains provided.

The remaining of this paper is organized as follows. Section 2 introduces the problem formulation and preliminaries. Section 3 gives the design of coordinated tracking control of multiple trains. The stability analysis of the designed controllers is given in Section 4. Simulation results are given in Section 5 to illustrate the effectiveness of the proposed methods. Conclusions are presented in Section 6.

II. PROBLEM FORMULATION

Considering only the longitudinal motion of the train, the dynamics of the train i can be described as follows [23]:

$$\begin{cases} \frac{dx_i(t)}{dt} = v_i(t) \\ \frac{dv_i(t)}{dt} = a_i(t) \\ M_i a_i(t) = F_i(t) - f_{ib}(v_i) - f_{ie}(x_i, v_i, t) \\ \quad - d_{ie}(x_i, v_i, t) \\ f_{ie} = f_{ir} + f_{ic} + f_{it} \end{cases} \quad (1)$$

where t represents a continuous time indicator; M_i denotes the total mass of the train, including the weights of the train, passengers and baggage; $x_i(t)$, $v_i(t)$, $a_i(t)$ are the real-time displacement, velocity, and acceleration, respectively; $F_i(t)$ is implemented force, that is, the traction or braking force of the train; $f_{ib}(v_i)$ is the specific basic resistance of the train operation.; $f_{ie}(x_i, v_i, t)$ is the combination of the additional resistance, which includes ramp resistance f_{ir} , line curve resistance f_{ic} and tunnel resistance f_{it} ; $d_{ie}(x_i, v_i, t)$ represents unmodeled dynamics, such as time-varying external disturbance.

The basic resistance consisting mainly of mechanical and air resistance can be expressed as the following Davis equation [24]:

$$f_{ib}(v_i) = c_{i0}(t) + c_{i1}(t)v_i + c_{i2}(t)v_i^2 \quad (2)$$

where $c_{i0}(t)$, $c_{i1}(t)$ and $c_{i2}(t)$ are the time-varying empirical positive coefficients. Generally these coefficients are obtained through numerous experiments.

Therefore, the train dynamics model adopted in this paper is rewritten as follows:

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t) - f_i(v_i) - d_i(x_i, v_i, t) \end{cases} \quad (3)$$

where $d_i(x_i, v_i, t) = (f_{ie}(x_i, v_i, t) + d_{ie}(x_i, v_i, t))/M$, $f_i(v_i) = f_{ib}(v_i)/M$ and $u_i(t) = F_i(t)/M$ is the acceleration and deceleration of train i , which is chosen to be designed as the control input later. To design the controller, there is a need to introduce the following assumptions.

Assumption 1: The pre-specified trajectory x_d for the first train is known and bounded. The first and second order derivatives of x_d are also known and bounded.

Assumption 2: The Davis equation coefficients $c_{i0}(t)$, $c_{i1}(t)$, $c_{i2}(t)$ and the combined disturbance $d_i(x_i, v_i, t)$ are all unknown but continuously bounded, that is, $|c_{i0}(t)| \leq c_{i0}^+$, $|c_{i1}(t)| \leq c_{i1}^+$, $|c_{i2}(t)| \leq c_{i2}^+$, $|d_i(x_i, v_i, t)| \leq d_i^+$.

Assumption 3: The ideal neural network weights W^* are bounded, i.e., $\|W^*\| \leq \|W_N^*\|$, where $\|W_N^*\|$ is the boundary of $\|W^*\|$.

For the train dynamics model (3), considering that the basic resistance and combined disturbances of the trains are unknown, the multiple train cooperative control law is designed with the following control objectives: to ensure that the displacement and velocity of the first train can follow the desired trajectory, that the displacement of the following train can maintain a pre-specified distance from the preceding train while the velocity remains consistent, and that the closed-loop signals for all trains are guaranteed to be uniformly ultimately bounded.

III. CONTROL DESIGN

A. COOPERATIVE CONTROL DESIGN

To begin with the cooperative tracking control design, define the following displacement tracking errors

$$\begin{cases} e_1 = x_d - x_1 \\ e_i = x_{i-1} - x_i + L_s, i = 2, \dots, n \end{cases} \quad (4)$$

where x_i is the real-time position of train i , x_d and L_s denote the pre-specified position trajectory versus time for the first train and separation distance for the following train, respectively. The derivative of e_i is obtained as

$$\begin{cases} \dot{e}_1 = v_d - v_1 \\ \dot{e}_i = v_{i-1} - v_i, i = 2, \dots, n \end{cases} \quad (5)$$

The filtered error is designed as follows

$$s_i = \lambda_i e_i + \dot{e}_i, \quad i = 1, \dots, n \quad (6)$$

where λ_i is a positive design parameter for train i . It can be known from (6) that the filtered error variables only take into account the information of train i . To introduce information about the adjacent trains, the coupling error variable is defined as follows

$$\begin{cases} E_i = \mu_i s_i - s_{i+1}, i = 1, \dots, n-1 \\ E_n = \mu_n s_n \end{cases} \quad (7)$$

where μ_i is a positive constant design parameter. It can be concluded that the boundedness or asymptotic convergence of s_i and E_i are equivalent, which is briefly explained as follows.

Let $\mathcal{M}_1 = [s_1 s_2 \dots s_n]^T$ and $\mathcal{M}_2 = [E_1 E_2 \dots E_n]^T$. The relationship of \mathcal{M}_1 and \mathcal{M}_2 is governed by $\mathcal{M}_2 = \mathcal{A}\mathcal{M}_1$, where

$$\mathcal{A} = \begin{bmatrix} \mu_1 & -1 & & & & \\ & \mu_2 & -1 & & & \\ & & \ddots & \ddots & & \\ & & & \ddots & -1 & \\ & & & & & \mu_n \end{bmatrix}.$$

Because μ_i for all i are positive, \mathcal{A} is a non-singular matrix. This guarantees the above perspective.

Define J_i as

$$\begin{cases} J_i = \mu_i \lambda_i \dot{e}_i + \mu_i \dot{v}_{i-1} - \lambda_{i+1} \dot{e}_{i+1} + \dot{v}_{i+1}, \\ \quad i = 1, \dots, n-1 \\ J_n = \mu_n \lambda_n \dot{e}_n + \mu_n \dot{v}_{n-1}. \end{cases} \quad (8)$$

The time derivative of (7) can be calculated as

$$\begin{aligned} \dot{E}_i &= \mu_i \dot{s}_i - \dot{s}_{i+1} \\ &= \mu_i (\lambda_i \dot{e}_i + \dot{v}_{i-1} - \dot{v}_i) - (\lambda_{i+1} \dot{e}_{i+1} + \dot{v}_i - \dot{v}_{i+1}) \\ &= -(\mu_i + 1) \dot{v}_i + J_i. \end{aligned} \quad (9)$$

In this study, considering its excellent universal approximation property [25], the radial basis function (RBF) neural network will be used to approximate $f_i(v_i)$ as follows

$$\begin{cases} f_i = W^{*T} h(Z) + \epsilon \\ \hat{f}_i = \hat{W}^T h(Z) \end{cases} \quad (10)$$

where Z is the input vector of the neural network; W^* is the neural network ideal weights; ϵ is the neural network's approximation error and ϵ_N is the boundary value of ϵ ; $h(Z)$

is the basis function vector, and the basis function is chosen as

$$h_j = \exp\left(\frac{\|Z - n_{cj}\|^2}{2n_{bj}^2}\right) \quad (11)$$

where j, n_{cj}, n_{bj} represents the j th node of the hidden layer of the neural network, the central value of the Gaussian function and the width of the Gaussian function, respectively.

The cooperative tracking control law is designed as

$$\begin{cases} u_i = \frac{k_i + 0.5(\mu_i + 1)}{\mu_i + 1} E_i + \frac{1}{\mu_i + 1} J_i + \hat{f}_i + \tanh(\omega_i E_i) \hat{d}_i, \\ i = 1, \dots, n-1 \\ u_n = \frac{k_n + 0.5\mu_n}{\mu_n} E_n + \frac{1}{\mu_n} J_n + \hat{f}_n + \tanh(\omega_n E_n) \hat{d}_n \end{cases} \quad (12)$$

where k_i is positive design constant; \hat{f}_i is the estimation of the unknown basic operating resistance $f_i(v_i)$; \hat{d}_i is updated by the following adaptive laws:

$$\begin{cases} \dot{\hat{d}}_i = \delta_i((\mu_i + 1) E_i \tanh(\omega_i E_i) - \sigma_i \hat{d}_i) \quad i = 1, \dots, n-1 \\ \dot{\hat{d}}_n = \delta_n(\mu_n E_n \tanh(\omega_n E_n) - \sigma_n \hat{d}_n) \end{cases} \quad (13)$$

where δ_i, σ_i are positive design parameters. The RBF neural network weights are updated by

$$\begin{cases} \dot{\hat{W}}_i = \gamma_i(\mu_i + 1) E_i h(Z) - \gamma_i \eta_i \hat{W}_i \quad i = 1, \dots, n-1 \\ \dot{\hat{W}}_n = \gamma_n \mu_n E_n h(Z) - \gamma_n \eta_n \hat{W}_n \end{cases} \quad (14)$$

with $\gamma_i > 0$ and $\eta_i > 0$ being design parameters.

B. COOPERATIVE CONTROL DESIGN USING OBSERVER

As can be noted from the design above, the controller requires the displacement, velocity, and acceleration of neighbouring trains. For the consideration of reducing the amount of information to be collected, this section designs a multi-train cooperative controller based on a fast high-order sliding mode differentiator, with the advantage that only position information is required, while velocity and acceleration information is obtained through the observer. First, it is assumed that the following inequalities hold:

$$\begin{cases} |\tilde{v}_i| = |\hat{v}_i - v_i| \leq \kappa_{ai} \\ |\tilde{a}_i| = |\hat{a}_i - a_i| \leq \kappa_{bi} \end{cases} \quad (15)$$

where κ_{ai} and κ_{bi} are positive constants indicating the boundary value of the observation error; $\tilde{v}_i = \hat{v}_i - v_i$ and $\tilde{a}_i = \hat{a}_i - a_i$ are the observation errors. It is noted that velocity and acceleration information is not available in this section, so the variables are redefined as follows

$$\begin{cases} \hat{s}_i = \lambda_i e_i + \dot{e}_i, \quad i = 1, \dots, n \\ \dot{E}_i = \mu_i \hat{s}_i - \hat{s}_{i+1}, \quad i = 1, \dots, n-1 \\ \dot{E}_n = \mu_n \hat{s}_n \\ \hat{J}_i = \mu_i \lambda_i \dot{e}_i + \mu_i \hat{v}_{i-1} - \lambda_{i+1} \dot{e}_{i+1} + \dot{v}_{i+1}, \\ i = 1, \dots, n-1 \\ \hat{J}_n = \mu_n \lambda_n \dot{e}_n + \mu_n \hat{v}_{n-1}. \end{cases} \quad (16)$$

Similar to the design procedures in the above section, the cooperative control using observer is proposed as follows

$$\begin{cases} u_i = \frac{k_i + 0.5}{\mu_i + 1} \hat{E}_i + \frac{1}{\mu_i + 1} \hat{J}_i + \hat{f}_i + \tanh(\omega_i \hat{E}_i) \hat{d}_i, \\ i = 1, \dots, n-1 \\ u_n = \frac{k_n + 0.5}{\mu_n} \hat{E}_n + \frac{1}{\mu_n} \hat{J}_n + \hat{f}_n + \tanh(\omega_n \hat{E}_n) \hat{d}_n, \end{cases} \quad (17)$$

where \hat{d}_i is updated by

$$\begin{cases} \dot{\hat{d}}_i = \delta_i((\mu_i + 1) \hat{E}_i \tanh(\omega_i \hat{E}_i) - \sigma_i |\hat{E}_i| \hat{d}_i), \\ i = 1, \dots, n-1 \\ \dot{\hat{d}}_n = \delta_n(\mu_n \hat{E}_n \tanh(\omega_n \hat{E}_n) - \sigma_n |\hat{E}_n| \hat{d}_n); \end{cases} \quad (18)$$

\hat{W}_i is updated by

$$\begin{cases} \dot{\hat{W}}_i = \gamma_i(\mu_i + 1) \hat{E}_i h(Z) - \gamma_i \eta_i |\hat{E}_i| \hat{W}_i, \\ i = 1, \dots, n-1 \\ \dot{\hat{W}}_n = \gamma_n \mu_n \hat{E}_n h(Z) - \gamma_n \eta_n |\hat{E}_n| \hat{W}_n. \end{cases} \quad (19)$$

IV. STABILITY ANALYSIS

Lemma 1. It is said that a group of multiple trains is platoon stable if

$$\text{given any } \tau > 0, \|e_i(0)\|_\infty < \xi \Rightarrow \sup \|e_i(\cdot)\|_\infty < \tau$$

where ξ is a constant [11].

Theorem 1: Under Assumptions 1-3, considering the train dynamics model (3), the proposed controller (12) and adaptation laws (13) and (14) guarantee that all the signals in the closed-loop system are uniformly ultimately bounded.

Proof of Theorem 1: Choose the following Lyapunov candidate as

$$V_{ai} = \frac{1}{2} E_i^2 + \frac{1}{2\gamma_i} \tilde{W}_i^T \tilde{W}_i + \frac{1}{2\delta_i} \tilde{d}_i^2 \quad (20)$$

and its time derivative can be calculated as

$$\dot{V}_{ai} = E_i \dot{E}_i + \frac{1}{\gamma_i} \tilde{W}_i^T \dot{\tilde{W}}_i + \frac{1}{\delta_i} \tilde{d}_i \dot{\tilde{d}}_i. \quad (21)$$

Select $\tilde{W}_i := \hat{W}_i - W^*$ and $\tilde{d}_i := \hat{d}_i - d_i$ as the estimation errors. Substituting (12) into (9), we have

$$\begin{aligned} \dot{E}_i = & -(k_i + 0.5(\mu_i + 1)) E_i - (\mu_i + 1) \left(\tilde{W}_i^T h(Z) - \epsilon_i \right) \\ & - (\mu_i + 1) \left(\tanh(\omega_i E_i) \hat{d}_i - d_i \right). \end{aligned} \quad (22)$$

Therefore, the Lyapunov function can be written as

$$\begin{aligned} \dot{V}_{ai} = & \frac{1}{\delta_i} \tilde{d}_i \dot{\tilde{d}}_i - E_i(\mu_i + 1) \left(\tanh(\omega_i E_i) \hat{d}_i - d_i \right) \\ & + \frac{1}{\gamma_i} \tilde{W}_i^T \dot{\tilde{W}}_i - (k_i + 0.5(\mu_i + 1)) E_i^2 \\ & - E_i(\mu_i + 1) \left(\tilde{W}_i^T h(Z) - \epsilon_i \right). \end{aligned} \quad (23)$$

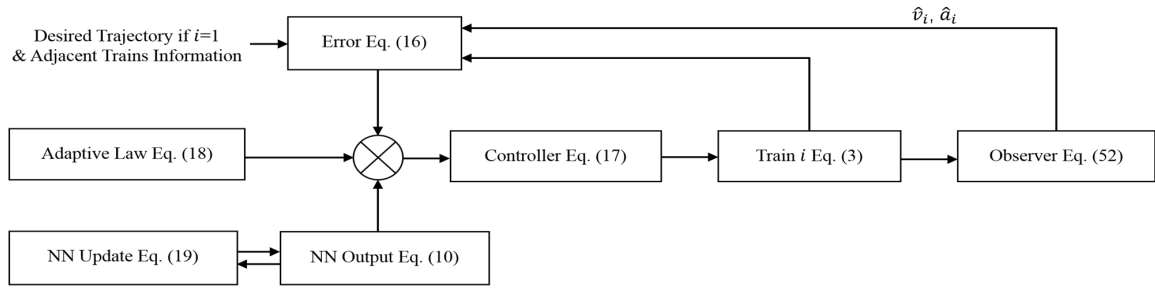


FIGURE 1. Block diagram of the adaptive cooperative tracking control system using the observer.

Incorporating the adaptive laws (13) and (14), one has

$$\begin{aligned} \dot{V}_{ai} &= -\sigma_i \tilde{d}_i \hat{d}_i + (\mu_i + 1) E_i (d_i - \tanh(\omega_i E_i) d_i) \\ &\quad - k_i E_i^2 - \eta_i \tilde{W}_i^T \hat{W}_i - 0.5 (\mu_i + 1) E_i^2 + \epsilon_i (\mu_i + 1) E_i. \end{aligned} \quad (24)$$

According to [26], [27], we obtain some inequalities as follows

$$\begin{cases} (\mu_i + 1) d_i^+ (|E_i| - E_i \tanh(\omega_i E_i)) \leq 0.2785 \frac{(\mu_i + 1) d_i^+}{\omega_i} \\ -\sigma_i \tilde{d}_i \hat{d}_i \leq \sigma_i \frac{d_i^2}{2} - \sigma_i \frac{\tilde{d}_i^2}{2} \\ -\eta_i \tilde{W}_i^T \hat{W}_i \leq \eta_i \frac{\|W_i^*\|^2}{2} - \eta_i \frac{\|\tilde{W}_i\|^2}{2} \\ (\mu_i + 1) \epsilon_i E_i \leq (\mu_i + 1) \frac{\epsilon_i^2}{2} + (\mu_i + 1) \frac{E_i^2}{2}. \end{cases} \quad (25)$$

Equation (25) becomes

$$\begin{aligned} \dot{V}_{ai} &\leq -k_i E_i^2 - \eta_i \frac{\tilde{W}_i^T \tilde{W}_i}{2} - \sigma_i \frac{\tilde{d}_i^2}{2} \\ &\quad + \eta_i \frac{\|W_N^*\|^2}{2} + (\mu_i + 1) \frac{\epsilon_N^2}{2} + \sigma_i \frac{d_i^2}{2} + 0.2785 \frac{\mu_i + 1}{\omega_i} d_i^+. \end{aligned} \quad (26)$$

Let

$$\begin{cases} \alpha_i := \frac{\min\{k_i, \frac{\eta_i}{2}, \frac{\sigma_i}{2}\}}{\max\{\frac{1}{2}, \frac{1}{2\gamma_i}, \frac{1}{2\sigma_i}\}} \\ \beta_i := \eta_i \frac{\|W_N^*\|^2}{2} + (\mu_i + 1) \frac{\epsilon_N^2}{2} + \sigma_i \frac{d_i^2}{2} \\ \quad + 0.2785 \frac{\mu_i + 1}{\omega_i} d_i^+. \end{cases} \quad (27)$$

It gives

$$\dot{V}_{ai} \leq -\alpha_i V_{ai} + \beta_i. \quad (28)$$

Multiplying (28) by $e^{\alpha_i t}$ and integrating from 0 to t , yields

$$0 \leq V_{ai}(t) \leq e^{-\alpha_i t} \left(V_{ai}(0) - \frac{\beta_i}{\alpha_i} \right) + \frac{\beta_i}{\alpha_i}, \quad \forall t > 0. \quad (29)$$

Equation (29) shows that $V_{ai}(t)$ is bounded. And it can be concluded that

$$\begin{cases} |E_i| \leq \sqrt{\frac{2\beta_i}{\alpha_i}} \\ \|\tilde{W}_i\| \leq \sqrt{\frac{2\gamma_i \beta_i}{\alpha_i}} \\ |\tilde{d}_i| \leq \sqrt{\frac{2\delta_i \beta_i}{\alpha_i}}. \end{cases} \quad (30)$$

The error variables can be adjusted to be arbitrarily small by choosing appropriate parameters. The proof is completed.

Theorem 2: Under the Assumptions 1-3, considering the train dynamics model (3), the proposed controller (12) and adaptation laws (13) and (14) guarantee that a group of multiple trains is platoon stable.

Proof of Theorem 2: Choose the following Lyapunov function for multiple trains:

$$V_b = \sum_{i=1}^n V_{ai}. \quad (31)$$

The derivative of (31) can be calculated as

$$\dot{V}_b \leq -\sum_{i=1}^n (\alpha_i V_{ai}) + \sum_{i=1}^n \beta_i \leq -\mathbb{A} V_b + \sum_{i=1}^n \beta_i \quad (32)$$

where $\mathbb{A} = \min\{\alpha_1, \dots, \alpha_n\}$. From (32), it can be obtained that

$$0 \leq V_b(t) \leq e^{-\mathbb{A}t} \left(V_b(0) - \frac{\sum_{i=1}^n \beta_i}{\mathbb{A}} \right) + \frac{\sum_{i=1}^n \beta_i}{\mathbb{A}}, \quad (33)$$

which indicates that there exists a time moment T such that for any $t > T$, the coupling error E_i , estimation error \tilde{W}_i and \tilde{d}_i is ultimately bounded by

$$\begin{cases} |E_i| \leq \sqrt{2 \frac{\sum_{i=1}^n \beta_i}{\mathbb{A}}} \\ \|\tilde{W}_i\| \leq \sqrt{2\gamma_i \frac{\sum_{i=1}^n \beta_i}{\mathbb{A}}} \\ |\tilde{d}_i| \leq \sqrt{2\delta_i \frac{\sum_{i=1}^n \beta_i}{\mathbb{A}}}. \end{cases} \quad (34)$$

Platoon stability is guaranteed for multiple trains. The proof is completed.

Theorem 3: The adopted train dynamics model is given by (3). Under Assumptions 1-3, if the control law is designed as (17) and adaptive law (18) and (19) are selected, then the closed-loop system is uniformly ultimately bounded.

Proof of Theorem 3: The Lyapunov function is considered as follows

$$V_{ci} = \frac{1}{2\gamma_i} \hat{W}_i^T \hat{W}_i. \quad (35)$$

The derivative of V_{ci} is calculated as

$$\dot{V}_{ci} = \frac{1}{\gamma_i} \hat{W}_i^T \dot{\hat{W}}_i. \quad (36)$$

Incorporating (19), one has

$$\begin{aligned} \dot{V}_{ci} &= \hat{W}_i^T \left((\mu_i + 1) \hat{E}_i h(Z) - \eta_i \hat{W}_i \left| \hat{E}_i \right| \right) \\ &\leq - \left\| \hat{W}_i \right\| \left| \hat{E}_i \right| \left(\eta_i \left\| \hat{W}_i \right\| - (\mu_i + 1) h^+ \right) \end{aligned} \quad (37)$$

where $\|h(Z)\| \leq h^+$. Noticing that $\dot{V}_{ci} \leq 0$ if $\left\| \hat{W}_i \right\| > (1/\eta_i)(\mu_i + 1) h^+$, according to the Lyapunov theorem, \hat{W}_i is ultimately bounded by

$$\left\| \hat{W}_i \right\| \leq \frac{1}{\eta_i} (\mu_i + 1) h^+. \quad (38)$$

The Lyapunov function is considered as follows

$$V_{di} = \frac{1}{2\delta_i} \hat{d}_i^2. \quad (39)$$

Similarly, in view of the adaptive law (18), the time derivative of (39) can be calculated as

$$\begin{aligned} \dot{V}_{di} &= \frac{1}{\delta_i} \hat{d}_i \dot{\hat{d}}_i = \hat{d}_i \left((\mu_i + 1) \hat{E}_i \tanh(\omega_i \hat{E}_i) - \sigma_i \left| \hat{E}_i \right| \hat{d}_i \right) \\ &\leq - \left| \hat{d}_i \right| \left| \hat{E}_i \right| \left(\sigma_i \left| \hat{d}_i \right| - (\mu_i + 1) \right). \end{aligned} \quad (40)$$

Noticing that $\dot{V}_{di} \leq 0$ if $\left| \hat{d}_i \right| > (1/\sigma_i)(\mu_i + 1)$, \hat{d}_i is ultimately bounded by

$$\left| \hat{d}_i \right| \leq \frac{1}{\sigma_i} (\mu_i + 1). \quad (41)$$

Consider the Lyapunov function

$$V_{ei} = \frac{1}{2} E_i^2. \quad (42)$$

Substitute (17) into (9), it yields

$$\begin{aligned} \dot{E}_i &= -\tilde{J}_i - (k_i + 0.5) \hat{E}_i - (\mu_i + 1) \left(\tilde{W}_i^T h(Z) - \epsilon_i \right) \\ &\quad - (\mu_i + 1) \left(\tanh(\omega_i \hat{E}_i) \hat{d}_i - d_i \right). \end{aligned} \quad (43)$$

We have the derivative of (42) as follows:

$$\begin{aligned} \dot{V}_{ei} &= -\tilde{J}_i E_i - (k_i + 0.5) \hat{E}_i E_i - (\mu_i + 1) E_i \left(\tilde{W}_i^T h(Z) - \epsilon_i \right) \\ &\quad - (\mu_i + 1) E_i \left(\tanh(\omega_i \hat{E}_i) \hat{d}_i - d_i \right) \\ &= - (k_i + 0.5) E_i^2 - E_i \left((k_i + 0.5) \hat{E}_i + \tilde{J}_i \right) \\ &\quad - (\mu_i + 1) E_i \left(\tilde{W}_i^T h(Z) - \epsilon_i \right) \\ &\quad - (\mu_i + 1) E_i \left(\tanh(\omega_i \hat{E}_i) \hat{d}_i - d_i \right). \end{aligned} \quad (44)$$

It is worth noticing the above fact and the following inequalities

$$\begin{cases} \tilde{E}_i \leq \mu_i \kappa_{ai} + \kappa_{a(i+1)} \\ \tilde{J}_i \leq \kappa_{b(i+1)} + \lambda_i (\kappa_{ai} + \kappa_{a(i+1)}) \\ \quad + \mu_i (\kappa_{b(i-1)} + \lambda_i \kappa_{a(i-1)} + \lambda_i \kappa_{ai}). \end{cases} \quad (45)$$

Equation (44) becomes

$$\begin{aligned} \dot{V}_{ei} &\leq - (k + 0.5) E_i^2 + |E_i| \bar{\beta}_i \\ &\leq -k_1 E_i^2 + \frac{\bar{\beta}_i^2}{2} \\ &= -\bar{\alpha}_i V_{ei} + \frac{1}{2} \bar{\beta}_i^2. \end{aligned} \quad (46)$$

Multiplying (28) by $e^{\bar{\alpha}_i t}$ and integrating from 0 to t , it follows

$$0 \leq V_{ei}(t) \leq e^{-\bar{\alpha}_i t} \left(V_{ei}(0) - \frac{\bar{\beta}_i^2}{2\bar{\alpha}_i} \right) + \frac{\bar{\beta}_i^2}{2\bar{\alpha}_i}, \quad \forall t > 0. \quad (47)$$

Therefore, $V_{ei}(t)$ is bounded. And it can be concluded that

$$|E_i| \leq \sqrt{\frac{\bar{\beta}_i^2}{\bar{\alpha}_i}}. \quad (48)$$

The proof is completed.

Theorem 4: The adopted train dynamics model is given by (3). Under Assumptions 1-3, if the control law is designed as (17) and adaptive law (18) and (19) are selected, then a group of multiple trains is platoon stable.

Proof of Theorem 4: The Lyapunov function is considered as follows

$$V_f = \sum_{i=1}^n V_{ei} \quad (49)$$

and take derivative V_f of along time, yields

$$\begin{aligned} \dot{V}_f &\leq - \sum_{i=1}^n (\bar{\alpha}_i V_{ei}) + \sum_{i=1}^n \left(\frac{1}{2} \bar{\beta}_i^2 \right) \\ &\leq -\mathbb{B} V_f + \sum_{i=1}^n \left(\frac{1}{2} \bar{\beta}_i^2 \right) \end{aligned} \quad (50)$$

with $\mathbb{B} = \min\{\bar{\alpha}_1, \dots, \bar{\alpha}_n\}$. Thus, by following along the similar lines with the proof of Theorem 2, we can conclude that

$$|E_i| \leq \sqrt{\frac{\sum_{i=1}^n (\bar{\beta}_i^2)}{\mathbb{B}}}. \quad (51)$$

Platoon stability is guaranteed for multiple trains. The proof is completed.

V. SIMULATION RESULTS

Simulation examples for tracking control are conducted to verify the effectiveness of the proposed control method. The number of trains is 4 in the simulation. The pre-specified trajectory for the first train and the disturbance for all trains are given in Figures 2 and 3 respectively. All trains have an initial speed of 0 and initial positions of 0 m, 400 m, 800 m and 1200 m respectively. The distance between the two trains is set at 400 m. The basic resistance

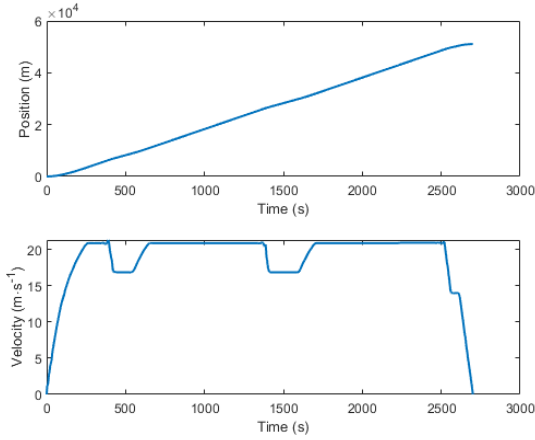


FIGURE 2. The pre-specified position and velocity trajectory for train 1.

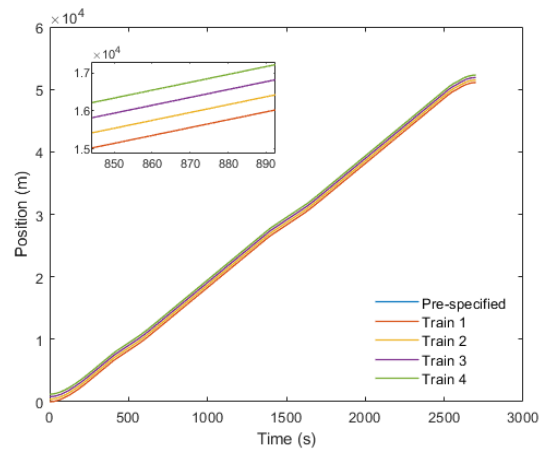


FIGURE 4. Position values of the four trains versus time using Theorem 1-2.

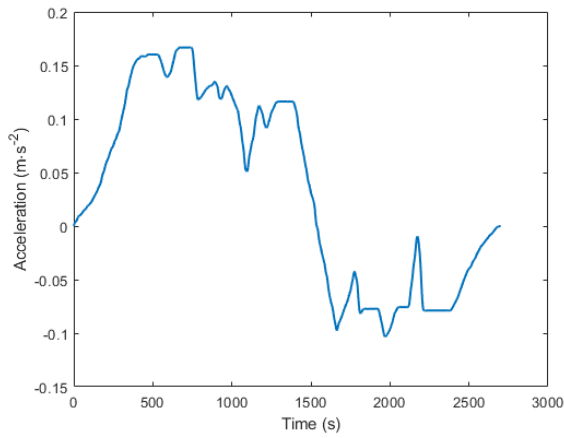


FIGURE 3. The disturbance for all trains.

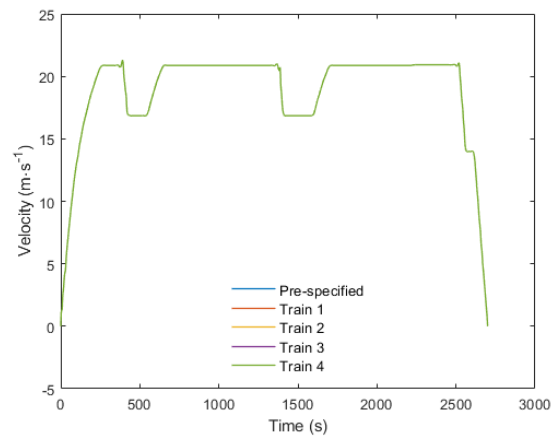


FIGURE 5. Velocity values of the four trains versus time using Theorem 1-2.

forces of trains are simulated by $0.9 + 0.002v_1 + 0.00015v_1^2$, $1.0 + 0.004v_2 + 0.0002v_2^2$, $0.7 + 0.001v_3 + 0.0004v_3^2$, and $0.8 + 0.002v_4 + 0.0003v_4^2$, respectively.

The simulation will use the following fast high-order sliding mode observer, whose stability is demonstrated in [28].

$$\begin{cases} \dot{\hat{x}}_i = -\rho_{i1} |\hat{x}_i - x_i|^{\frac{2}{3}} \operatorname{sgn}(\hat{x}_i - x_i) - \rho_{i1} (\hat{x}_i - x_i) + \hat{v}_i \\ \dot{\hat{v}}_i = -\rho_{i2} |\hat{v}_i - \dot{\hat{x}}_i|^{\frac{1}{2}} \operatorname{sgn}(\hat{v}_i - \dot{\hat{x}}_i) - \rho_{i2} (\hat{v}_i - \dot{\hat{x}}_i) + \hat{a}_i \\ \dot{\hat{a}}_i = -\rho_{i3} |\hat{a}_i - \dot{\hat{v}}_i|^{\frac{q}{p}} \operatorname{sgn}(\hat{a}_i - \dot{\hat{v}}_i) - \rho_{i3} (\hat{a}_i - \dot{\hat{v}}_i) \end{cases} \quad (52)$$

The design parameters applying Theorem 1-2 are chosen as: $\lambda_i = 4, \mu_i = 10, \omega_i = 0.15, k_i = 20, \sigma_i = 0.001, \delta_i = 1e^{-7}, \gamma_i = 1, \eta_i = 0.001$. The design parameters applying Theorem 3-4 are chosen as: $\lambda_i = 5, \mu_i = 1, \omega_i = 0.15, k_i = 25, \sigma_i = 0.001, \delta_i = 1e^{-7}, \gamma_i = 1, \eta_i = 0.001, \rho_{11} = 8, \rho_{12} = 4, \rho_{13} = 2, \rho_{i1} = 6, \rho_{i2} = 3, \rho_{i3} = 1, q = 7, p = 9, i = 1, \dots, 4$. The initial values of \hat{a}_i are

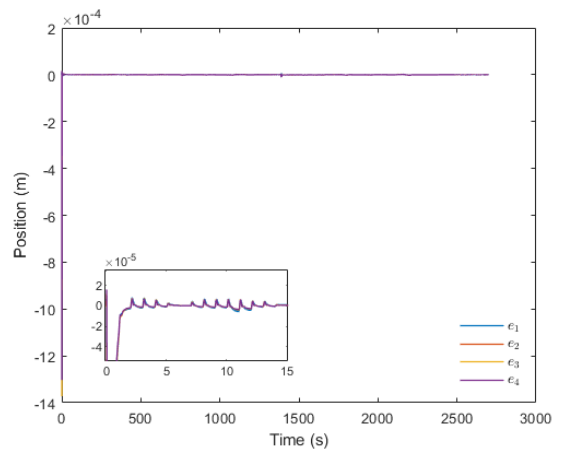


FIGURE 6. Position tracking errors of the four trains using Theorem 1-2.

set as 0.0001. The input vectors of the RBF neural networks are set to $Z = [e_i, \dot{e}_i]^T$. The widths of the Gaussian basis function of the neural networks containing 11 nodes are all

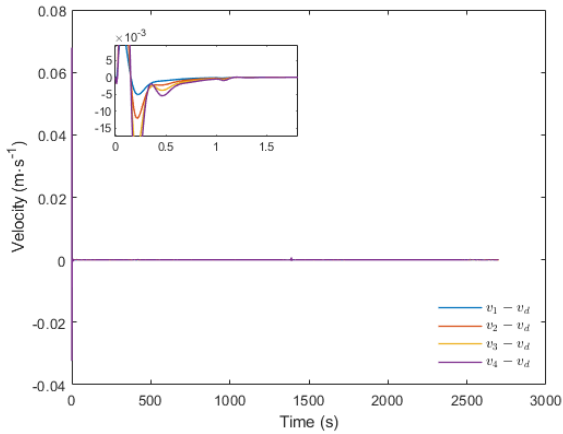


FIGURE 7. Velocity tracking errors of the four trains using Theorem 1-2.

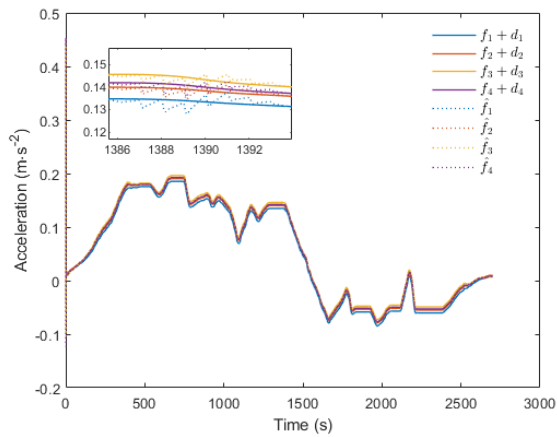


FIGURE 8. Neural networks estimation using Theorem 1-2.

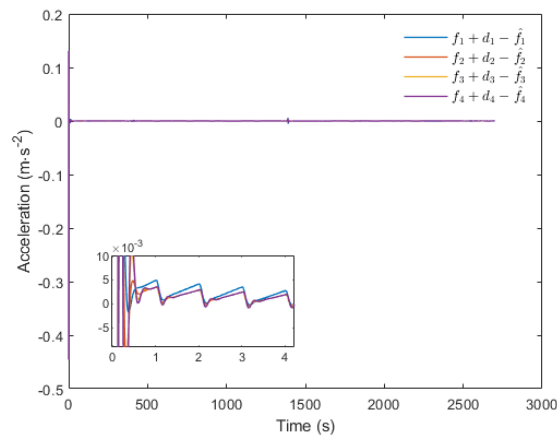


FIGURE 9. Neural networks estimation error using Theorem 1-2.

10, with the following central values:

$$\begin{bmatrix} -20 & -16 & -12 & -8 & -4 & 0 & 4 & 8 & 12 & 16 & 20 \\ -20 & -16 & -12 & -8 & -4 & 0 & 4 & 8 & 12 & 16 & 20 \end{bmatrix}. \quad (53)$$

The initial weights of the neural networks are all 0.1.

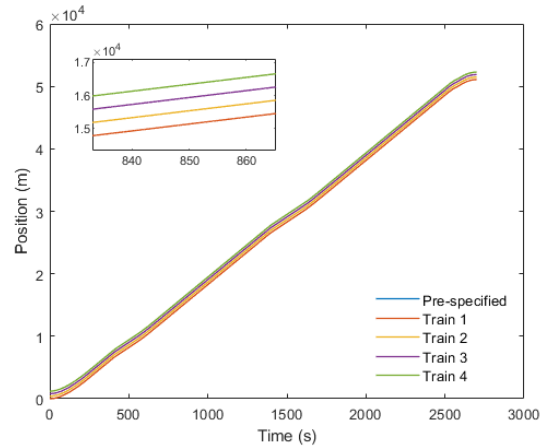


FIGURE 10. Position values of the four trains versus time using Theorem 3-4.

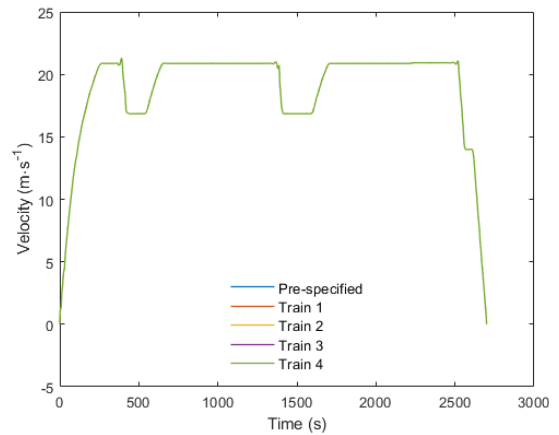


FIGURE 11. Velocity values of the four trains versus time using Theorem 3-4.

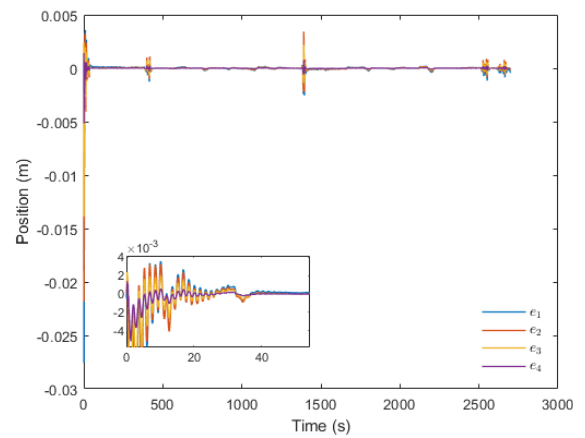


FIGURE 12. Position tracking errors of the four trains using Theorem 3-4.

The simulation results for Theorems 1-2 are shown in Figures 4-9. The first train tracks the given trajectory, while the following trains maintain the set distance while the speed

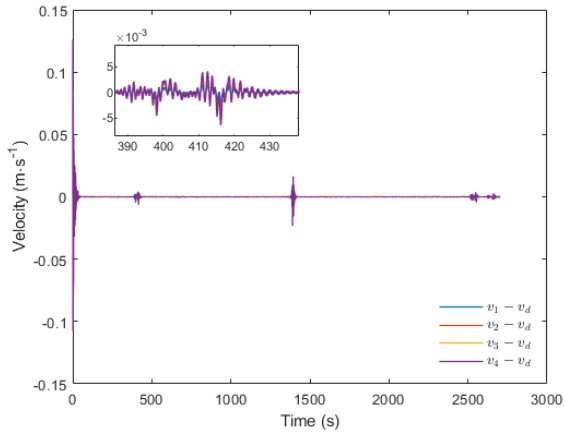


FIGURE 13. Velocity tracking errors of the four trains using Theorem 3-4.

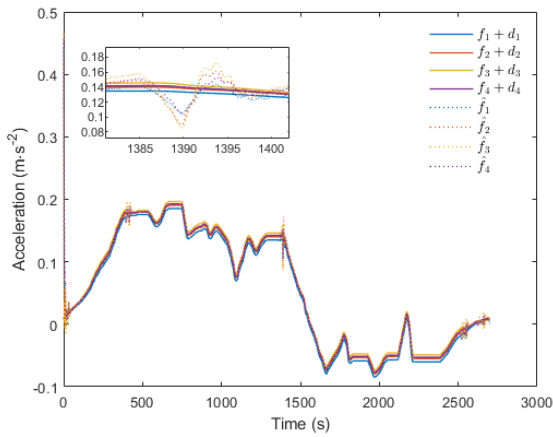


FIGURE 14. Neural networks estimation using Theorem 3-4.

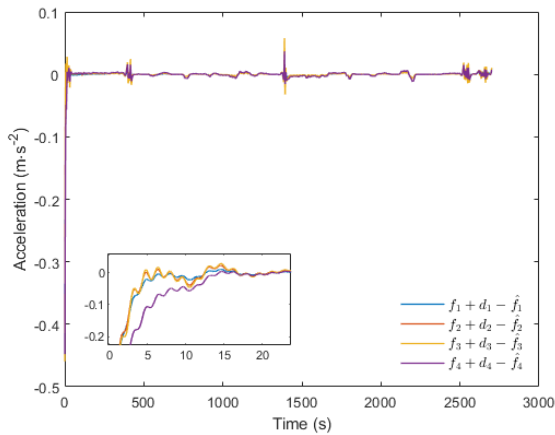


FIGURE 15. Neural networks estimation error using Theorem 3-4.

converges. From Figures 6-7 it can be seen that the tracking errors for displacement and speed are small. The simulation results for Theorems 3-4 are shown in Figures 10-17. The tracking performance for position and velocity is given in Figures 10-13. According to Figures 16-17, the observer used in the simulation is able to observe the speed and acceleration of the trains very well. It is observed that the

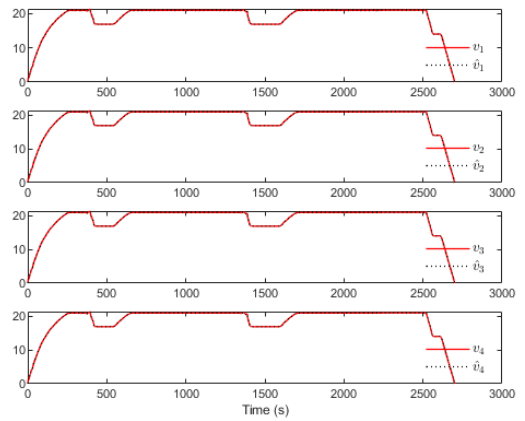


FIGURE 16. Velocity observations for the four trains.

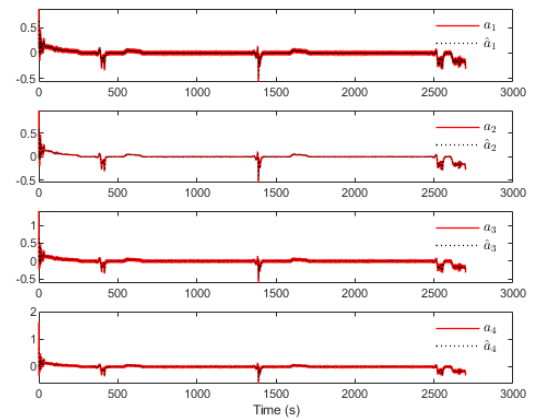


FIGURE 17. Acceleration observations for the four trains.

cooperative controllers of Theorems 3-4 have a slight loss of tracking accuracy but significantly reduce the burden of sensor acquisition due to multi-state information through the observer technique. It is apparent that the designed control schemes can effectively ensure that the closed-loop system signals are bounded and the platoon stability is achieved.

VI. CONCLUSION

This paper designs multiple train distributed cooperative tracking control schemes. A combination of radial basis function neural networks and adaptive methods is used to cope with model uncertainties, unknown parameters and external disturbances in the train operating environment. A fast high-order sliding mode observer is used to estimate the train velocity and acceleration information. Stability and convergence are proved for single and multiple trains by means of Lyapunov stability analysis. Simulation examples demonstrate the effectiveness of the proposed controller. While uncertain dynamics and external disturbances have been considered in this paper, due to the complexity of the actual train operating environment, there are still several problems with multiple train control that have yet to be investigated, such as sensor delays and actuator saturation, which will be explored in future work.

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