An Extended Fuzzy CPT-TODIM Model Based on Possibility Theory and Its Application to Air Target Dynamic Threat Assessment

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ABSTRACT Dynamic target threat assessment can be regarded as a classical multiple attribute group decision-making (MAGDM) issue. Triangular fuzzy numbers (TFNs) can fully describe imprecise and fuzzy information for target threat assessment. Thus, to consider the changing trend of target detection information in a certain period and the influence of commanders' psychological factors on target threat assessment results, this study proposes an extended CPT-TODIM method based on possibility theory (PT-CPT-TODIM) to solve target threat assessment. Of note, the CRITIC method is used to determine the attribute weights of the probability mean matrix and the probability standard deviation matrix, which can make the target threat assessment result more rational. A numerical example and comparison analysis demonstrate the efficiency and feasibility of the newly proposed CPT-TODIM method.

INDEX TERMS CPT-TODIM, possibility theory, triangular fuzzy number, CRITIC method, dynamic air target threat assessment.

I. INTRODUCTION

The rapid development of weapons and equipment information technology and the wide application of new aviation equipment have exacerbated the antagonists and complexity of modern air combat [1]. How to effectively use battlefield target detection information to assist commanders in scientifically assisting decision-making has become a research hotspot. Then, in the face of a complex air combat situation, a reasonable threat assessment of enemy targets is essential for improving survivability and winning the initiative on the battlefield.

Target threat assessment aims to analyze the potential combat capabilities of enemy targets through the target's attribute information and target combat intentions to obtain a quantitative description of the target's threat level [2]. Due to the complexity of the combat environment and the limitation of battlefield commanders' cognition, it is difficult for commanders to give accurate target threat assessment information in actual battlefield decision-making.

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To express the target detection information, some target threat assessment methods have been proposed. An intuitive fuzzy number is used to represent target detection information, and the threat degree of different targets is compared and selected according to [3]. Interval intuitionistic fuzzy numbers (IVIFs) are used to represent target detection information in [4], [5]. Reference [6] studied the target threat assessment based on generalized intuitionistic fuzzy soft set (GIFSS). However, the fuzzy interval is usually difficult to define using intuitionistic fuzzy number to represent target detection information, which leads to the distortion and deviation of target threat assessment information. In addition, the assembly process of IVIFs and GIFSS is more complicated, which makes the calculation complexity of target threat assessment higher. In contrast, the triangular fuzzy number can not only effectively represent the value range of the uncertain information in the target detection information but also highlight the center point where the target threat is most likely. Therefore, it can compensate for the lack of accuracy of intuitionistic fuzzy numbers. Moreover, triangular fuzzy numbers are easier to assemble; thus, they can reduce the computational complexity in the target threat assessment.

Currently, target threat assessments utilizing triangular fuzzy numbers have been rarely reported. Therefore, it is an interesting research topic to apply triangular fuzzy numbers for target threat assessment.

The threat degree of the target depends on various factors, such as the target's combat capability and the target's combat situation. It is difficult to analyze these attributes, which should be combined with the knowledge and experiences of military experts. Threat assessment of targets can be regarded as a decision-making problem [4]. Many MADM methods have been proposed to solve target threat assessment problems, such as VIKOR [3], neural networks [7], extended gray correlation [8], and Bayesian networks [9]. Although these methods can solve threat assessment problems under the corresponding battlefield environments, these methods are not suitable for considering the influence of the battlefield commanders' psychological state on the target threat assessment. Fortunately, TODIM (an acronym in Portuguese for interactive multicriteria decision-making) can capture the psychological behavior of decision-makers, and it is widely used in various areas, such as multicriteria rental evaluation [10] and green supplier selection [11]. To further solve the MADM problem based on the TODIM method, Tian *et al.* [12] developed an extended TODIM method for green supplier selection based on cumulative prospect theory. Zhao *et al.* [13] proposed an extended CPT-TODIM method for urban ecological risk assessment. Zhao *et al.* [14] designed a CPT-TODIM method for multiple attribute group decision-making with 2-tuple linguistic neutrosophic sets. Zhang *et al.* [15] studied a CPT-TODIM method for multiple attribute group decision-making under a 2-tuple linguistic Pythagorean fuzzy environment. Zhao *et al.* [16] proposed a Pythagorean fuzzy TODIM method based on CPT for MAGDM. Liao *et al.* [17] proposed the CPT-TODIM method under a probabilistic hesitant fuzzy setting. Zhao *et al.* [18] combined the CPT-TODIM method with an interval– Pythagorean fuzzy set and established a green supplier selection model.

Thus, this study improves TODIM, which is based on CPT, and investigates its use for target threat assessment under a triangular fuzzy environment. In contrast to previous studies on the extensions of the CPT-TODIM method under a fuzzy battlefield environment, in this study, we introduce the concept of Markowitz's portfolio mean–variance methodology into the CPT-TODIM method with triangular fuzzy numbers, and the study aims to develop an extended fuzzy CPT-TODIM method based on possibility theory for the threat assessment of air targets. To be exact, in the extended fuzzy CPT-TODIM method, the battlefield commander will select an ideal target with a high possibilistic mean value and high possibilistic standard deviation. The possibilistic mean value reflects the overall change trend of information and can reflect the centralized change trend of target attribute information in target threat assessment. The possibilistic standard deviation is an indicator to measure the volatility of uncertain information. The greater is the

volatility of the target attribute information, the greater is the risk of the target's threat. Then, in the extended CPT-TODIM method, the possibilistic mean values matrix and the possibilistic standard deviation are constructed to compute the integrated superiority degree of each target. In addition, the importance weights of difference features are assessed based on the CRITIC method for each corresponding feature. Then, the threat degree of all targets can be ranked according to the overall superiority degree of each target. Moreover, we also compare the results of our newly proposed method and existing methods through an example.

The main contributions are as follows. First, the characteristics of air target detection information affecting target threat assessment are considered; the air target threat indicator set is determined, and the uncertainty of detection information in the process of target threat assessment is described by triangular fuzzy numbers. Second, a developed CPT-TODIM method based on possibility theory is introduced to represent the psychological state of the battlefield commander. Third, the proposed method combs the target threat assessment process, which contributes to further assisting commanders in making scientific decisions. This combination has application prospects in corresponding cases, which can boost and replenish the research.

The rest of this article is structured in the following way to achieve the abovementioned purposes. Section 2 presents the basic knowledge of this article. Section 3 provides the proposed model. In Section 4, the applicability of the proposed model is demonstrated through target threat assessment. Subsequently, conclusions are provided in Section 5.

II. RELATED KOWLEDGE

In this section, the essential knowledge about possibility theory [19], [20], TODIM method based on CPT [12], [21] and CRITIC method [22] are reviewed.

A. POSSIBILITY THEORY

Definition 1: Let $A \in x$ be a fuzzy number with $A^\lambda = [a_1(\lambda), a_2(\lambda)]$, $\lambda \in [0, 1]$. The possibilistic mean values of fuzzy number A are defined as $M(A) = \int_0^1 \lambda(a_1(\lambda) + a_2(\lambda))d\lambda$. In addition, the possibilistic variance is as follows: $\text{Var(A)} = \frac{1}{2} \int_0^1 \lambda(a_1(\lambda) - a_2(\lambda))^2 d\lambda$.

Definition 2: Let A be a triangular fuzzy number, $x \in X$ and the membership degree is as follows:

$$U_{A(x)} = \begin{cases} 0 & x \leq a_1 \\ \dfrac{x - a_1}{a_q - a_1} & a_1 \leq x \leq a_q \\ \dfrac{a_r - x}{a_r - a_q} & a_q \leq x \leq a_r \\ 0 & x \geq a_r \end{cases} \quad (1)$$

where $a_1 \leq a_q \leq a_r$. Another equivalent form of A is $A = (a - \alpha, a, a + \beta)$, satisfying $a = a_q$, $\alpha = a_q - a_1$ and $\beta = a_r - a_q$. According to Definition 1, the possibilistic

mean value of triangular fuzzy number A is described as

$$M(A) = \int_0^1 \lambda((a - (1 - \lambda)\alpha) + (a + (1 - \lambda)\beta))d\lambda$$

$$= a + \frac{1}{6}(\beta - \alpha) \qquad (2)$$

The possibilistic variance of triangular fuzzy number A can be written as

$$\mathrm{Var}(A) = \frac{1}{2}\int_0^1 \lambda((a - (1 - \lambda)\alpha) - (a + (1 - \lambda)\beta))^2 d\lambda$$

$$= \frac{1}{24}(\beta + \alpha)^2 \qquad (3)$$

B. CPT-TODIM METHOD

The calculation process of the CPT-TODIM method is reviewed. A decision matrix obtained from DMs is as follows, in which the schemes and attributes are provided by decision-makers.

$$D = \begin{array}{c|cccccc} & c_1 & c_2 & \cdots & c_g & \cdots & c_s \\ x_1 & w_{11} & w_{12} & \cdots & w_{1g} & \cdots & w_{1s} \\ x_2 & w_{21} & w_{22} & \cdots & w_{2g} & \cdots & w_{2s} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ x_u & w_{u1} & w_{u2} & \cdots & w_{ug} & \cdots & w_{us} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ x_m & w_{m1} & w_{m2} & \cdots & w_{mg} & \cdots & w_{ms} \end{array}$$

The weighting vector of attributes is represented by $\theta = (\theta_1, \theta_2, \ldots, \theta_s)$, which satisfies $\sum_{g=1}^{s} \theta_g = 1$.

Step 1: When $w_{ug} - w_{kg} \geq 0$, the converted probability weight is computed by (4):

$$\phi_{ukg}^+(\theta_g) = \theta_g^\delta \Big/ (\theta_g^\delta + (1 - \theta_g)^\delta)^{\frac{1}{\delta}} \qquad (4)$$

Otherwise, the converted probability weight is computed by (5):

$$\phi_{ukg}^-(\theta_g) = \theta_g^\varepsilon \Big/ (\theta_g^\varepsilon + (1 - \theta_g)^\varepsilon)^{\frac{1}{\varepsilon}} \qquad (5)$$

Step 2: Determine the relative weight $\phi_{ukg}^*(\theta_g)$ of

$$\phi_{ukg}^*(\theta_g) = \phi_{ukg}(\theta_r) \Big/ \max\left\{\phi_{ukg}(\theta_o) \,|o \in s\right\} \ g \in s, \ \forall(u, k) \qquad (6)$$

Step 3: Equation (7) is applied to determine the relative prospect superiority of scheme I_u to I_k in attribute T_g.

$$\kappa_g(I_u, I_k) = \begin{cases} \dfrac{\phi_{ukg}^*(\theta_g) \cdot (w_{ug} - w_{kg})^\varsigma}{\sum\limits_{g=1}^{s} \phi_{ukg}^*(\theta_g)} & w_{ug} > w_{kg} \\[4ex] 0, & w_{ug} = w_{kg} \\[2ex] -\tau \cdot \dfrac{\sum\limits_{g=1}^{s} \phi_{ukg}^*(\theta_g) \cdot (w_{ug} - w_{kg})^\omega}{\phi_{ukg}^*(\theta_g)} & w_{ug} < w_{kg} \end{cases} \qquad (7)$$

where ς, w, τ are the parameters.

Step 4: The superiority degree of the scheme of scheme I_u over other schemes is calculated as Eq. (8).

$$\Im(I_u) = \sum_{k=1}^{m} \sum_{g=1}^{s} \kappa_g(I_u, I_k) \quad u = 1, 2, \ldots, m \qquad (8)$$

Step 5: Acquire the overall superiority degree $\Omega(I_u)$ of scheme I_u.

$$\Omega(I_u) = \frac{\Im(I_u) - \min\{\Im(I_u)\}}{\max_u\{\Im(I_u)\} - \min_u\{\Im(I_u)\}} \quad u = 1, 2, \ldots, m \qquad (9)$$

Step 6: Rank the overall superiority degree.

Rank the overall superiority degree $\Omega(I_u)$ to obtain the best scheme that has the largest $\Omega(I_u)$ value.

C. CRITIC METHOD

The CRITIC method comprehensively measures the objective weighting method of the index weight through the contrast intensity and conflict between the indices.

(1) Calculate the correlation coefficient among attributes:

$$\bar{a}_j = \frac{1}{m}\sum_{i=1}^{m} a_{ij}; \quad j = 1, 2, \cdots, n \qquad (10)$$

$$\rho_{jk} = \sum_{i=1}^{m} (a_{ij} - \bar{a}_j)(a_{ik} - \bar{a}_k)$$

$$\Big/ \sqrt{\sum_{i=1}^{m} (a_{ij} - \bar{a}_j)^2 \sum_{i=1}^{m}(a_{ik} - \bar{a}_k)^2} \qquad (11)$$

(2) Calculate the standard deviation and the index for all attributes:

$$\sigma_j = \sqrt{\frac{1}{m-1}\sum_{i=1}^{m}(a_{ij} - \bar{a}_j)^2}; \quad j = 1, 2, \cdots, n \qquad (12)$$

$$C_j = \sigma_j \sum_{k=1}^{n}(1 - \rho_{jk}); \quad j = 1, 2, \cdots, n \qquad (13)$$

(3) Calculate the weight of attributes.

$$w_j = \frac{C_j}{\sum\limits_{j=1}^{n} C_j}; \quad j = 1, 2, \cdots, n \qquad (14)$$

The weights of the attributes are sorted in descending order.

III. THE PROPOSED EXTENDED CPT-TODIM BASED ON THE POSSIBILITY THEORY

Based on the abovementioned information, in this section, a new model is established to answer the issue of target threat assessment with triangular fuzzy number information.

A. THREAT ASSESSMENT INDEX SYSTEM

The scientific evaluation index set is the basis of target threat evaluation, and the key to establishing the evaluation index set is to determine the evaluation index. Due to the complexity and variability of air target operations, it is

necessary to construct a target evaluation system from different perspectives that reflect the representative factors of the target's threat level. Combining the actual situation of the battlefield, this article mainly considers the target's threat assessment from two major aspects: the target's combat capabilities and the target's combat situation. Among them, the target combat capabilities include two aspects: target type, target maneuvering ability; target velocity, target height, target distance and target arrival time reflect the combat situation. The target evaluation set is shown in Figure 1.
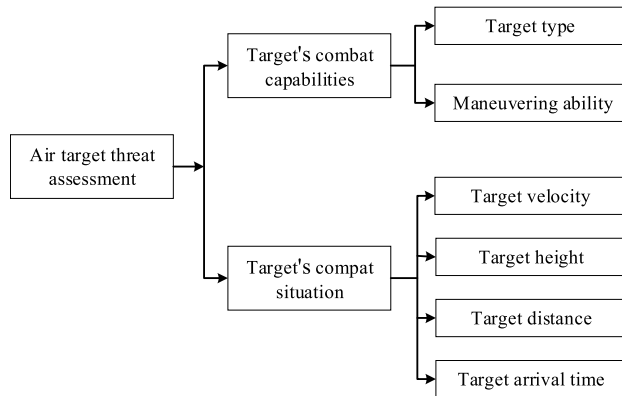

FIGURE 1. Air target threat assessment index system.

B. THE PROPOSED NOVEL TARGET THREAT ASSESSMENT MODEL

Considering the advantage of CPT-TODIM and triangular fuzzy number, this study proposes extended CPT-TODIM based on possibility theory to solve dynamic target assessment. Figure 2 shows the flowchart of the method and the detailed steps.

In a real battlefield environment, it is difficult for commanders to obtain exact information about all battlefield maneuver targets. Therefore, commanders often utilize fuzzy theory to evaluate battlefield maneuver targets. In this section, we propose a new and novel fuzzy CPT-TODIM method based on possibility theory to support target threats.

Step 1: The initial decision matrix of the target evaluation index at different moments is determined.

We assume $X = \{X_i | i = 1, 2, \cdots, m\}$ as a finite set of possible targets and $C = \{C_j | j = 1, 2, \cdots, n\}$ as a finite set of attributes of targets. Because the information of targets is fuzzy and uncertain during target threat assessment, the battlefield commander utilizes a triangular fuzzy number. For
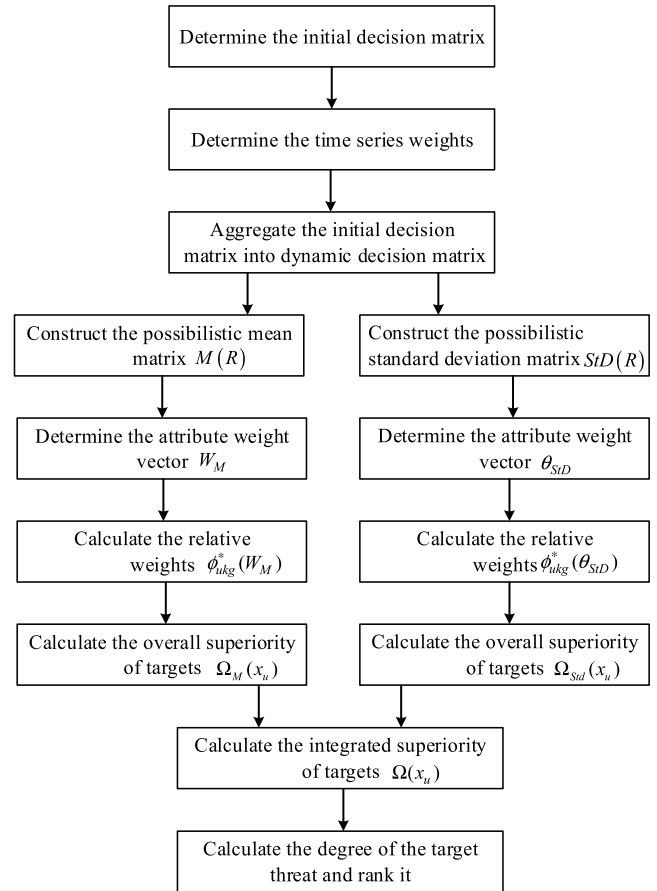

FIGURE 2. Algorithm flow chart of the proposed air target threat assessment method.

example, an expert confirms that the threat degree of a target under a certain attribute will not exceed 0.8, will not be less than 0.6, and that its most likely threat degree is 0.75; then, the evaluation value of this target can be expressed by a triangular fuzzy number as (0.6, 0.75, 0.8). At moment t_k, the target threat assessment problem with triangular fuzzy numbers can be expressed in matrix format D_k, as shown at the bottom of the page.

Step 2: Time series weights are obtained by the inverse Poisson distribution method.

As time changes, battlefield information also keeps changing; thus, the degree of threat caused by the target decision matrix information at different moments is different. In the actual combat process, the decision-making information at the current moment is important for the threat assessment of

$$D_k = \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_m \end{array} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \\ (a_{11} - \alpha_{11}, a_{11}, a_{11} + \beta_{11}) & (a_{12} - \alpha_{12}, a_{12}, a_{12} + \beta_{12}) & \cdots & (a_{1n} - \alpha_{1n}, a_{1n}, a_{1n} + \beta_{1n}) \\ (a_{21} - \alpha_{21}, a_{21}, a_{21} + \beta_{21}) & (a_{22} - \alpha_{22}, a_{22}, a_{22} + \beta_{22}) & \cdots & (a_{2n} - \alpha_{2n}, a_{2n}, a_{2n} + \beta_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (a_{m1} - \alpha_{m1}, a_{m1}, a_{m1} + \beta_{m1}) & (a_{m2} - \alpha_{m2}, a_{m2}, a_{m2} + \beta_{m2}) & \cdots & (a_{mn} - \alpha_{mn}, a_{mn}, a_{mn} + \beta_{mn}) \end{array}$$

aerial targets. The weight of the time series is calculated by the inverse Poisson distribution method.

$$\eta_k = \frac{k!}{\phi^k} \times (\sum_{k=1}^{l} \frac{k!}{\phi^k})^{-1} \qquad (15)$$

where $\eta_k \geq 0$ and $\sum_{k=1}^{l} \eta_k = 1, 0 \leq \phi \leq 2$.

Step 3: The comprehensive decision matrix R is obtained by fusing multiple time evaluation information.

$$R = (r_{m1}, r_{m2}, \cdots, r_{mn}) \; R = \sum_{k=1}^{L} \eta_k D_k \; r_{mn} = \sum_{k=1}^{L} \eta_k d_{k,mn}$$

Step 4: The possibilistic mean value of the triangular fuzzy number is obtained by Formula 2, and according to Formula 3, the possibilistic variance of the triangular fuzzy number is obtained.

$$M(R) = \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_m \end{array} \begin{array}{cccc} c_1 & c_1 & \cdots & c_n \\ M(r_{11}) & M(r_{12}) & \cdots & M(r_{11}) \\ M(r_{21}) & M(r_{221}) & \cdots & M(r_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ M(r_{m1}) & M(r_{m2}) & \cdots & M(r_{mn}) \end{array}$$

$$StD(R) = \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_m \end{array} \begin{array}{cccc} c_1 & c_1 & \cdots & c_n \\ Std(r_{11}) & Std(r_{12}) & \cdots & Std(r_{11}) \\ Std(r_{21}) & Std(r_{221}) & \cdots & Std(r_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ Std(r_{m1}) & Std(r_{m2}) & \cdots & Std(r_{mn}) \end{array}$$

Step 5: Determining the index weight is the key to air combat threat assessment. The weights of the probability mean matrix and the probability standard deviation matrix are calculated by the CRITIC method.

$$W_M = \begin{bmatrix} w_1 & w_2 & \cdots & w_n \end{bmatrix}$$
$$\theta_{StD} = \begin{bmatrix} \theta_1 & \theta_2 & \cdots & \theta_n \end{bmatrix}$$

Step 6: Determine the relative weights $\phi^*_{ukg}(W_M)$ and $\phi^*_{ukg}(\theta_{StD})$.

When $M(r)_{ug} > M(r)_{kg}$, the converted probability weight is as follows:

$$\phi^+_{ukg}(W_M) = w_g^\delta \Big/ (w_g^\delta + (1 - w_g)^\delta)^{\frac{1}{\delta}} \qquad (16)$$

Otherwise, the converted probability weight is computed by (17):

$$\phi^-_{ukg}(W_M) = w_g^\varepsilon \Big/ (w_g^\varepsilon + (1 - w_g)^\varepsilon)^{\frac{1}{\varepsilon}} \qquad (17)$$

When $Std(r)_{ug} > Std(r)_{kg}$, the converted probability weight is as follows:

$$\phi^+_{ukg}(\theta_{StD}) = \theta_g^\delta \Big/ (\theta_g^\delta + (1 - \theta_g)^\delta)^{\frac{1}{\delta}} \qquad (18)$$

Otherwise, the converted probability weight is computed by (19):

$$\phi^-_{ukg}(\theta_{StD}) = \theta_g^\varepsilon \Big/ (\theta_g^\varepsilon + (1 - \theta_g)^\varepsilon)^{\frac{1}{\varepsilon}} \qquad (19)$$

Step 7: Determine the superiority degree $\Im_M(x_u)$ and $\Im_{StD}(x_u)$ of the targets X_u over the others, which is calculated by Eqs. (20) and (21).

$$\Im_M(x_u) = \sum_{k=1}^{m} \sum_{g=1}^{s} MK_g(x_u, x_g) \qquad (20)$$

$$\Im_{StD}(x_u) = \sum_{k=1}^{m} \sum_{g=1}^{s} StDK_g(x_u, x_g) \qquad (21)$$

where $M\kappa_g(I_u, I_k)$ and $StD\kappa_g(I_u, I_k)$, as shown at the bottom of the page.

Step 8: Acquire the overall superiority degree $\Omega_M(x_u)$ and $\Omega_{StD}(x_u)$.

$$\Omega_M(x_u) = \frac{\Im_M(x_u) - \min_u \{\Im_M(x_u)\}}{\max_u \{\Im_M(x_u)\} - \min_u \{\Im_M(x_u)\}}$$
$$u = 1, 2, \ldots, m \qquad (22)$$

$$\Omega_{StD}(x_u) = \frac{\Im_{StD}(x_u) - \min_u \{\Im_{StD}(x_u)\}}{\max_u \{\Im_{StD}(x_u)\} - \min_u \{\Im_{StD}(x_u)\}}$$
$$u = 1, 2, \ldots, m \qquad (23)$$

$$M\kappa_g(I_u, I_k) = \begin{cases} \dfrac{\phi^*_{ukg}(W_M) \cdot (M(r)_{ug} - M(r)_{kg})^\varsigma}{\sum\limits_{g=1}^{s} \phi^*_{ukg}(W_M)} & M(r)_{ug} > M(r)_{kg} \\ 0, & M(r)_{ug} = M(r)w_{kg} \\ -\tau \cdot \dfrac{\sum\limits_{g=1}^{s} \phi^*_{ukg}(W_M) \cdot (M(r)_{ug} - M(r)_{kg})^\omega}{\phi^*_{ukg}(W_M)} & M(r)_{ug} < M(r)_{kg} \end{cases}$$

$$StD\kappa_g(I_u, I_k) = \begin{cases} \dfrac{\phi^*_{ukg}(\theta_{StD}) \cdot (StD(r)_{ug} - StD(r)_{kg})^\varsigma}{\sum\limits_{g=1}^{s} \phi^*_{ukg}(\theta_{StD})} & Std(r)_{ug} > Std(r)_{kg} \\ 0, & Std(r)_{ug} = Std(r)w_{kg} \\ -\tau \cdot \dfrac{\sum\limits_{g=1}^{s} \phi^*_{ukg}(\theta_{StD}) \cdot (StD(r)_{ug} - StD(r)_{kg})^\omega}{\phi^*_{ukg}(\theta_{StD})} & Std(r)_{ug} < Std(r)_{kg} \end{cases}$$

TABLE 1. Assessment information on targets $x_1 - x_5$ at time t_1.

target	c_1	c_2	c_3	c_4	c_5	c_6
x_1	(0.70 0.72 0.75)	(0.78 0.80 0.82)	(0.60 0.65 0.70)	(0.70 0.73 0.76)	(0.72 0.75 0.78)	(0.70 0.75 0.79)
x_2	(0.68 0.70 0.73)	(0.67 0.68 0.70)	(0.60 0.62 0.65)	(0.70 0.72 0.80)	(0.80 0.82 0.85)	(0.75 0.78 0.82)
x_3	(0.55 0.57 0.63)	(0.46 0.50 0.52)	(0.45 0.50 0.52)	(0.35 0.37 0.41)	(0.60 0.62 0.65)	(0.63 0.64 0.65)
x_4	(0.40 0.42 0.45)	(0.35 0.37 0.40)	(0.58 0.60 0.64)	(0.22 0.25 0.28)	(0.30 0.32 0.35)	(0.12 0.15 0.18)
x_5	(0.45 0.46 0.48)	(0.30 0.32 0.33)	(0.50 0.52 0.54)	(0.12 0.16 0.17)	(0.50 0.52 0.55)	(0.22 0.25 0.28)

Step 9: The joint superiority degree of each target is calculated by the following formula.

$$\Omega(x_u) = ((\Omega_M(x_u)) \wedge \alpha + (\Omega_{StD}(x_u) \wedge \beta)/2 \quad (24)$$

where $\alpha + \beta = 1$, α and β represent the importance of $\Omega_M(x_u)$ and $\Omega_{StD}(x_u)$ respectively, and the threat level of each target is ranked according to $\Omega(x_u)$.

IV. NUMERICAL INSTANCE AND DISCUSSION

A. NUMERICAL INSTANCE

In the following, we will solve the problem of dynamic air target threat assessment using the PT-CPT-TODIM method described in this paper.

Step 1: The initial decision matrix is determined as follows.

Our side encounters five air targets through detection equipment. The air target set $X = \{x_1, x_2 \cdots x_5\}$ corresponds to {bomber, fighter, missile, early warning aircraft, armed helicopter}. The attribute set $C = \{c_1, c_2 \cdots c_6\}$ consists of six attributes, which correspond to {target type, maneuvering ability, target arrival time, target velocity, target height, target distance}. The information on the air targets at three time points t_1, t_2, and t_3 is shown in Tables 1-3.

Step 2: Taking into account the dynamic time-varying nature of air combat situation information, a Poisson distribution-based time series weight calculation model is established to process multitime air combat situation information. The time series weight is obtained when $\phi = 1.5$ as follows:

$$\eta = \{0.2000, \ 0.2667, \ 0.5333\}$$

Step 3: The dynamic decision matrix is shown in Table 4.

Step 4: Construct the possibilistic mean value matrix using Formula (2) and the possibilistic standard variance matrix using Formula (3) $M(R)$ and $StD(R)$, as shown at the bottom of the page.

Step 5: The CRITIC method is used to calculate the attribute weights of the probability mean and the probability standard deviation matrix.

The weight of the possibilistic mean value matrix:

$$W_M = \begin{bmatrix} 0.0788 & 0.1354 & 0.2101 & 0.1554 & 0.1763 & 0.2439 \end{bmatrix}$$

The weight of the possibilistic standard variance matrix:

$$\theta_{StD} = \begin{bmatrix} 0.1820 & 0.1138 & 0.1700 & 0.1655 & 0.1474 & 0.2213 \end{bmatrix}$$

Step 6: Determine the relative weights $\phi_{ukg}^*(W_M)$ and the results are shown in Tables 5,6,7,8, and 9.

Step 7: To determine the relative prospect superiority $M\kappa_g(I_u, I_k)$ of the possibilistic mean value matrix, the results are shown in Tables 10, 11, 12, 13, 14 ($\varsigma = 0.88$, $\omega = 0.88$, $\tau = 2.25$, based on the experiment [23]).

Step 8: Determine the relative weights $\phi_{ukg}^*(\theta_{StD})$ and the results are shown in Tables 15, 16, 17, 18, and 19.

Step 9: The relative prospect superiority $StD\kappa_g(I_u, I_k)$ of the possibilistic standard deviation matrix is shown in Tables 20, 21, 22, 23, and 24.

Step 10: Acquire the superiority degree $\Omega_M(x_u)$, $\Omega_{StD}(x_u)$ and $\Omega(x_u)$ ($\alpha = 0.5$, $\beta = 0.5$) of the target ($i = 1, 2, 3, 4, 5$), and the results are shown in Table 25. According to $\Omega_2 > \Omega_1 > \Omega_3 > \Omega_4 > \Omega_5$, the ranking result of the

$$M(R) = \begin{bmatrix} 0.7217 & 0.7707 & 0.7322 & 0.8127 & 0.8104 & 0.6901 \\ 0.7017 & 0.7292 & 0.6688 & 0.7930 & 0.8488 & 0.8217 \\ 0.5767 & 0.5367 & 0.4906 & 0.4853 & 0.6292 & 0.5764 \\ 0.4217 & 0.4259 & 0.5673 & 0.2478 & 0.3630 & 0.2051 \\ 0.4617 & 0.3486 & 0.5667 & 0.2439 & 0.5612 & 0.3056 \end{bmatrix}$$

$$StD(R) = \begin{bmatrix} 0.0102 & 0.0103 & 0.0155 & 0.0155 & 0.0090 & 0.0211 \\ 0.0102 & 0.0088 & 0.0233 & 0.0184 & 0.0129 & 0.0143 \\ 0.0163 & 0.0133 & 0.0143 & 0.0122 & 0.0075 & 0.0112 \\ 0.0102 & 0.0080 & 0.0106 & 0.0128 & 0.0108 & 0.0122 \\ 0.0061 & 0.0083 & 0.0131 & 0.0113 & 0.0129 & 0.0106 \end{bmatrix}$$

TABLE 2. Assessment information on targets $x_1 - x_5$ at time t_2.

target	c_1	c_2	c_3	c_4	c_5	c_6
x_1	(0.70 0.72 0.75)	(0.75 0.77 0.79)	(0.75 0.78 0.80)	(0.78 0.80 0.82)	(0.75 0.78 0.79)	(0.68 0.72 0.76)
x_2	(0.68 0.70 0.73)	(0.70 0.73 0.74)	(0.63 0.65 0.70)	(0.70 0.75 0.80)	(0.82 0.85 0.86)	(0.78 0.80 0.83)
x_3	(0.55 0.57 0.63)	(0.50 0.53 0.56)	(0.48 0.52 0.55)	(0.55 0.58 0.61)	(0.63 0.66 0.67)	(0.55 0.58 0.62)
x_4	(0.40 0.42 0.45)	(0.38 0.41 0.45)	(0.55 0.58 0.60)	(0.25 0.28 0.32)	(0.32 0.34 0.36)	(0.16 0.19 0.22)
x_5	(0.45 0.46 0.48)	(0.32 0.35 0.37)	(0.50 0.53 0.55)	(0.15 0.20 0.22)	(0.52 0.55 0.58)	(0.26 0.32 0.33)

TABLE 3. Assessment information on targets $x_1 - x_5$ at time t_3.

target	c_1	c_2	c_3	c_4	c_5	c_6
x_1	(0.70 0.72 0.75)	(0.73 0.76 0.79)	(0.72 0.73 0.80)	(0.85 0.85 0.90)	(0.83 0.85 0.87)	(0.60 0.65 0.72)
x_2	(0.68 0.70 0.73)	(0.72 0.75 0.77)	(0.60 0.70 0.76)	(0.80 0.85 0.90)	(0.82 0.86 0.90)	(0.80 0.85 0.88)
x_3	(0.55 0.57 0.63)	(0.51 0.56 0.58)	(0.45 0.47 0.52)	(0.45 0.48 0.51)	(0.60 0.62 0.63)	(0.52 0.55 0.58)
x_4	(0.40 0.42 0.45)	(0.45 0.45 0.47)	(0.52 0.55 0.57)	(0.20 0.23 0.26)	(0.36 0.39 0.42)	(0.19 0.24 0.25)
x_5	(0.45 0.46 0.48)	(0.34 0.36 0.38)	(0.57 0.60 0.65)	(0.28 0.30 0.33)	(0.55 0.58 0.62)	(0.31 0.32 0.35)

TABLE 4. Dynamic decision matrix R.

target	c_1	c_2	c_3	c_4	c_5	c_6
x_1	(0.70 0.72 0.75)	(0.75 0.77 0.80)	(0.70 0.73 0.78)	(0.77 0.81 0.85)	(0.79 0.81 0.83)	(0.64 0.69 0.74)
x_2	(0.68 0.70 0.73)	(0.70 0.73 0.75)	(0.61 0.67 0.72)	(0.75 0.80 0.84)	(0.82 0.85 0.88)	(0.78 0.82 0.85)
x_3	(0.55 0.57 0.63)	(0.50 0.54 0.56)	(0.46 0.49 0.53)	(0.46 0.48 0.52)	(0.61 0.63 0.65)	(0.55 0.58 0.60)
x_4	(0.40 0.42 0.45)	(0.41 0.42 0.45)	(0.54 0.57 0.59)	(0.22 0.25 0.28)	(0.34 0.36 0.39)	(0.17 0.20 0.23)
x_5	(0.45 0.46 0.48)	(0.32 0.35 0.37)	(0.54 0.57 0.60)	(0.21 0.25 0.27)	(0.53 0.56 0.60)	(0.29 0.30 0.33)

TABLE 5. Relative weights $\phi^*_{1kg}(W_M)$.

	c_1	c_2	c_3	c_4	c_5	c_6
ϕ^*_{12}	0.5728	0.7467	0.9234	0.7982	0.8247	1
ϕ^*_{13}	0.5728	0.7467	0.9234	0.7982	0.8483	0.9931
ϕ^*_{14}	0.5728	0.7467	0.9234	0.7982	0.8483	0.9931
ϕ^*_{15}	0.5728	0.7467	0.9234	0.7982	0.8483	0.9931

target threat assessment is obtained: $x_2 > x_1 > x_3 > x_4 > x_5$. According to the initial decision matrix data shown in Table 1-3, the target threat assessment results are consistent with reality.

In addition, different α and β can have a corresponding impact on the target threat assessment result. According

to the commander's judgment on the battlefield situation, dynamically adjust the values of α and β to obtain the corresponding target threat ranking results as shown in Figure 3. The target threat ranking results of sub-figures (a), (f), (g), (h) and (i) are $x_2 > x_1 > x_3 > x_4 > x_5$; the results of subpicture (j) are $x_1 > x_2 > x_3 > x_5 > x_4$, and the results of

TABLE 6. Relative weights $\phi_{2kg}^*(W_M)$.

	c_1	c_2	c_3	c_4	c_5	c_6
ϕ_{21}^*	0.5131	0.7100	0.9214	0.7707	0.8541	1
ϕ_{23}^*	0.5768	0.7518	0.9298	0.8037	0.8541	1
ϕ_{24}^*	0.5768	0.7518	0.9298	0.8037	0.8541	1
ϕ_{25}^*	0.5768	0.7518	0.9298	0.8037	0.8541	1

TABLE 7. Relative weights $\phi_{3kg}^*(W_M)$.

	c_1	c_2	c_3	c_4	c_5	c_6
ϕ_{31}^*	0.5096	0.7052	0.9151	0.7654	0.8247	1
ϕ_{32}^*	0.5096	0.7052	0.9151	0.7654	0.8247	1
ϕ_{34}^*	0.5728	0.7467	0.9151	0.7982	0.8483	0.9931
ϕ_{35}^*	0.5728	0.7467	0.9151	0.7982	0.8483	0.9931

TABLE 8. Relative weights $\phi_{4kg}^*(W_M)$.

	c_1	c_2	c_3	c_4	c_5	c_6
ϕ_{41}^*	0.5096	0.7052	0.9151	0.7654	0.8247	1
ϕ_{42}^*	0.5096	0.7052	0.9151	0.7654	0.8247	1
ϕ_{43}^*	0.5096	0.7052	0.9234	0.7654	0.8247	1
ϕ_{45}^*	0.5096	0.7467	0.9234	0.7982	0.8247	1

TABLE 9. Relative weights $\phi_{5kg}^*(W_M)$.

	c_1	c_2	c_3	c_4	c_5	c_6
ϕ_{51}^*	0.5096	0.7052	0.9151	0.7654	0.8247	1
ϕ_{52}^*	0.5096	0.7052	0.9151	0.7654	0.8247	1
ϕ_{53}^*	0.5096	0.7052	0.9234	0.7654	0.8247	1
ϕ_{54}^*	0.5728	0.7052	0.9151	0.7654	0.8483	0.9931

TABLE 10. Relative prospect superiority of the targets x_1 to others.

	c_1	c_2	c_3	c_4	c_5	c_6
κ_{12}	0.0038	0.0093	0.0168	0.0047	-0.7527	-1.8371
κ_{13}	0.0214	0.0426	0.0542	0.0619	0.0386	0.0300
κ_{14}	0.0407	0.0599	0.0387	0.0996	0.0856	0.1076
κ_{15}	0.0359	0.0716	0.0389	0.1002	0.0512	0.0877

TABLE 11. Relative prospect superiority of the targets x_2 to others.

	c_1	c_2	c_3	c_4	c_5	c_6
κ_{21}	-0.6688	-0.9178	-1.0288	-0.4015	0.0102	0.0352
κ_{23}	0.0188	0.0359	0.0415	0.0590	0.0458	0.0590
κ_{24}	0.0383	0.0535	0.0252	0.0968	0.0920	0.1329
κ_{25}	0.0334	0.0654	0.0254	0.0974	0.0580	0.1137

TABLE 12. Relative prospect superiority of the targets x_3 to others.

	c_1	c_2	c_3	c_4	c_5	c_6
κ_{31}	-3.8096	-4.1950	-3.3258	-5.2551	-2.8645	-1.5670
κ_{32}	-3.3432	-3.5337	-2.5439	-5.0065	-3.3914	-3.0826
κ_{34}	0.0228	0.0221	-1.2521	0.0462	0.0543	0.0852
κ_{35}	0.0175	0.0352	-1.2425	0.0469	0.0163	0.0646

TABLE 13. Relative prospect superiority of the targets x_4 to others.

	c_1	c_2	c_3	c_4	c_5	c_6
κ_{41}	-7.2234	-5.9000	-2.3757	-6.3457	-5.3457	-5.6179
κ_{42}	-6.7979	-5.2712	-1.5493	-6.8217	-6.8217	-6.9389
κ_{43}	-4.0470	-2.1762	0.0204	-4.0255	-4.0255	-4.4492
κ_{45}	-1.2481	0.0163	0.0003	-3.1541	-3.1541	-1.4301

TABLE 14. Relative prospect superiority of the targets x_5 to others.

	c_1	c_2	c_3	c_4	c_5	c_6
κ_{51}	-6.3687	-7.0501	-2.3842	-8.5029	-3.7916	-4.5802
κ_{52}	-5.9356	-6.4372	-1.5583	-8.2704	-4.3004	-5.9337
κ_{53}	-3.1122	-3.4679	0.0202	-3.9802	-1.2112	-3.3709
κ_{54}	0.0070	-1.6102	-0.0190	-0.1069	0.0425	0.0274

TABLE 15. Relative weights $\phi^{*}_{1kg}(\theta_{StD})$.

	c_1	c_2	c_3	c_4	c_5	c_6
ϕ^{*}_{12}	0.9095	0.7247	0.8524	0.8387	0.7833	1
ϕ^{*}_{13}	0.8873	0.6714	0.8802	0.8387	0.8216	1
ϕ^{*}_{14}	0.9095	0.7247	0.8802	0.8387	0.7833	1
ϕ^{*}_{15}	0.9095	0.7247	0.8802	0.8387	0.7833	1

TABLE 16. Relative weights $\phi_{2kg}^*(\theta_{StD})$.

	c_1	c_2	c_3	c_4	c_5	c_6
ϕ_{21}^*	0.9095	0.6714	0.8802	0.8687	0.8216	0.9965
ϕ_{23}^*	0.8873	0.6714	0.8802	0.8687	0.8216	1
ϕ_{24}^*	0.9095	0.7247	0.8802	0.8687	0.8216	1
ϕ_{25}^*	0.9095	0.7247	0.8802	0.8687	0.8216	1

TABLE 17. Relative weights $\phi_{3kg}^*(\theta_{StD})$.

	c_1	c_2	c_3	c_4	c_5	c_6
ϕ_{31}^*	0.9095	0.7247	0.8524	0.8387	0.7833	0.9965
ϕ_{32}^*	0.9095	0.7247	0.8524	0.8387	0.7833	0.9965
ϕ_{34}^*	0.9095	0.7247	0.8802	0.8387	0.7833	0.9965
ϕ_{35}^*	0.9095	0.7247	0.8802	0.8687	0.7833	1

TABLE 18. Relative weights $\phi_{4kg}^*(\theta_{StD})$.

	c_1	c_2	c_3	c_4	c_5	c_6
ϕ_{41}^*	0.9095	0.6714	0.8524	0.8687	0.8216	0.9965
ϕ_{42}^*	0.9095	0.6714	0.8524	0.8387	0.7833	0.9965
ϕ_{43}^*	0.8873	0.6714	0.8524	0.8687	0.8216	1
ϕ_{45}^*	0.9095	0.6714	0.8524	0.8687	0.7833	1

TABLE 19. Relative weights $\phi_{5kg}^*(\theta_{StD})$.

	c_1	c_2	c_3	c_4	c_5	c_6
ϕ_{51}^*	0.8904	0.6738	0.8554	0.8717	0.8245	1
ϕ_{52}^*	0.8904	0.6738	0.8554	0.8416	0.8245	1
ϕ_{53}^*	0.8904	0.6738	0.8554	0.8416	0.8245	1
ϕ_{54}^*	0.8904	0.7273	0.8833	0.8416	0.8245	1

TABLE 20. Relative prospect superiority of the targets x_1 to others.

	c_1	c_2	c_3	c_4	c_5	c_6
κ_{12}	0	0.0004	-0.1874	-0.2453	-0.1125	0.0024
κ_{13}	-0.1459	-0.1028	0.0004	-0.0622	0.0005	0.0034
κ_{14}	0	0.0006	0.0016	-0.0762	-0.0558	0.0030
κ_{15}	0.0014	0.0006	0.0008	-0.0377	-0.1131	0.0035

TABLE 21. Relative prospect superiority of the targets x_2 to others.

	c_1	c_2	c_3	c_4	c_5	c_6
κ_{21}	0	-0.0564	0.0024	0.0030	0.0012	-0.1439
κ_{23}	-0.1468	-0.1477	0.0027	0.0025	0.0016	0.0012
κ_{24}	0	0.0002	0.0036	0.0023	0.0007	0.0008
κ_{25}	0.0014	0.0001	0.0030	0.0027	0	0.0014

TABLE 22. Relative prospect superiority of the targets x_3 to others.

	c_1	c_2	c_3	c_4	c_5	c_6
κ_{31}	0.0020	0.0008	-0.0371	0.0007	-0.0482	-0.2003
κ_{32}	0.0020	0.0012	-0.2130	-0.1991	-0.1492	-0.0721
κ_{34}	0.0020	0.0014	0.0012	-0.0185	-0.0957	-0.0286
κ_{35}	0.0031	0.0013	0.0004	0.0003	-0.1510	0.0002

TABLE 23. Relative prospect superiority of the targets x_4 to others.

	c_1	c_2	c_3	c_4	c_5	c_6
κ_{41}	0	-0.0822	-0.1253	0.0009	0.0006	-0.1803
κ_{42}	0	-0.0324	-0.2815	-0.1854	-0.0659	-0.0409
κ_{43}	-0.1460	-0.1701	-0.0970	0.0002	0.0010	0.0004
κ_{45}	0.0014	-0.0124	-0.0676	0.0005	-0.0664	0.0006

TABLE 24. Relative prospect superiority of the targets x_5 to others.

	c_1	c_2	c_3	c_4	c_5	c_6
κ_{51}	-0.1021	-0.0733	-0.0678	0.0004	0.0012	-0.2084
κ_{52}	-0.1015	-0.0228	-0.2367	-0.2178	0	-0.0824
κ_{53}	-0.2274	-0.1614	-0.0366	-0.0298	0.0016	-0.0153
κ_{54}	-0.1031	0.0001	0.0008	-0.0451	0.0007	-0.0410

TABLE 25. Degrees of the target threats based on the proposed method.

Target	$\Omega_{M}(x_n)$	$\Omega_{SD}(x_n)$	$\Omega(x_n)$
x_1	1	0.4967	0.8524
x_2	0.9959	1	0.9990
x_3	0.5498	0.4386	0.7019
x_4	0	0.1598	0.1999
x_5	0.1486	0	0.1927

sub-pictures (b), (c), (d), and (e) are $x_2 > x_1 > x_3 > x_5 > x_4$. The abovementioned results show that dynamically adjusting the values of α and β can provide commanders with flexible auxiliary decision-making information.
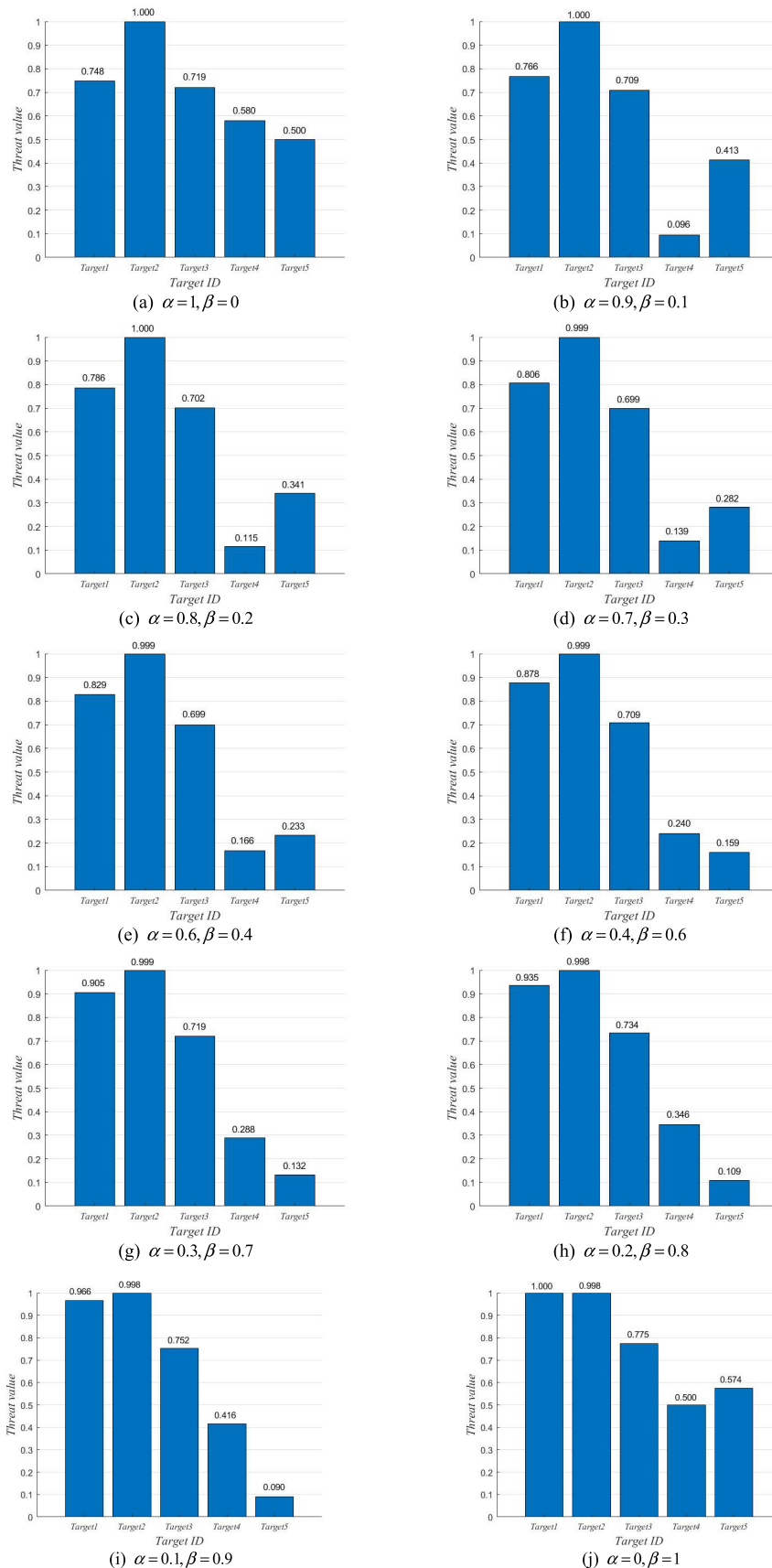
FIGURE 3. Target threat ranking results with different values of α and β.

TABLE 26. Comparison analysis.

	Target	$R_M(x_u)$	ranking	$R_{StD}(x_u)$	ranking	$R(x_u)$	ranking
PT-Grey-Correlation	x_1	0.8181	1	0.5395	2	0.6643	2
	x_2	0.8155	2	0.6069	1	0.7035	1
	x_3	0.4552	3	0.4588	3	0.4570	3
	x_4	0.1871	5	0.2884	4	0.2323	5
	x_5	0.2377	4	0.2643	5	0.2506	4
PT-TOPSIS	x_1	0.8772	2	0.5570	2	0.6990	2
	x_2	0.9349	1	0.5695	1	0.7297	1
	x_3	0.5033	3	0.4615	3	0.4820	3
	x_4	0.0908	5	0.2481	4	0.1501	5
	x_5	0.1901	4	0.2202	5	0.2046	4
PT-VIKOR	x_1	0	1	0.3658	2	0.3024	2
	x_2	0.0156	2	0	1	0.0625	1
	x_3	0.7233	3	0.7173	3	0.8487	3
	x_4	1	5	0.7402	4	0.9302	4
	x_5	0.8396	4	1	5	0.9581	5
PT-MABAC	x_1	0.1953	2	0.0008	2	0.0981	2
	x_2	0.2109	1	0.0028	1	0.1068	1
	x_3	-0.0098	3	0.00002	3	-0.0049	3
	x_4	-0.1953	5	-0.0015	4	-0.0984	5
	x_5	-0.1440	4	-0.0021	5	-0.0730	4
PT-CPT-TODIM	x_1	1	1	0.4967	2	0.8524	2
	x_2	0.9959	2	1	1	0.9990	1
	x_3	0.5498	3	0.4386	3	0.7019	3
	x_4	0	5	0.1598	4	0.1999	4
	x_5	0.1486	4	0	5	0.1927	5

B. COMPARATIVE ANALYSIS

It is necessary to use other methods to verify the PT-CPT-TODIM method. In this part, we select the extended grey correlation model based on the possibility theory (PT-grey-correlation) [8] and the extended TOPSIS model based on the possibility theory (PT-TOPSIS) [24]. We also extended VIKOR [3] and MABAC [25] using the possibility theory, and simulation experiments are conducted using the data in this study. The outcomes of the above five methods are calculated in Table 26. In Table 26, the results calculated by the five methods using the probability mean matrix are denoted by $R_M(x_u)$; the result calculated using the probability standard deviation matrix is denoted by $R_{StD}(x_u)$, and the final calculation result is denoted by $R(x_u)$. As shown in Table 26, the ranking results of x_2, x_1 and x_3 are the same for each method. The ranking results of x_4 and x_5 are not in the same order. Further analysis is as follows: the threat levels of targets 1 and 2 obtained by the five methods using the probability mean matrix are almost the same, indicating that the concentration trends of target 1 and target 2 are very close. When using the probability standard deviation to measure the fluctuation of targets 1 and 2, we find that target 2 has a larger fluctuation change than target 1, indicating that target 2 is a greater threat than target 1. The threat ranking results of targets 4 and 5 from the probability mean and probability standard deviation of the five methods are the same, and the result obtained from the probability mean is that target 5 is a greater threat than target 4. However, from the perspective of the probability standard deviation, target 4 is more threatening than target arrival. This shows that the maneuver change of target 5 is larger than that of target 4. From the perspective of the final ranking results, the five methods give auxiliary decision-making results from different perspectives; thus, in the actual battlefield environment, the commander

needs to further clarify the threat level of targets 4 and 5 in combination with the battlefield environment.

Combined with the initial decision matrix of the target evaluation index, compared with the other four methods, the method proposed in this study considers the influence of the battlefield commanders' psychological state on target threat assessment, which is very important for target threat assessment. It is observed that the target threat assessment result obtained by the proposed method is more reasonable; then, it is concluded that the target threat assessment method based on the extended CPT-TODIM method based on the possibility theory is effective.

V. CONCLUSION

In conclusion, this study proposes a novel target threat assessment method based on the CPT-TODIM method and possibility theory. Compared with existing researchers, this study has the following advantages: (1) the combination of the CPT-TODIM method and possibility theory overcomes the deficiency of the traditional scheme, e.g., the changing trend of target detection information in a certain period have been considered on target threat assessment results. (2) The triangular fuzzy number fully describes the uncertainty, making target detection information more informative. (3) The proposed method studies the influence of the battlefield commanders' psychological state on target threat assessment and realizes the dynamic threat assessment of aerial targets.

Due to the advantage of the proposed method, it is reasonable to believe that the extended PT-CPT-TODIM approach for target threat assessment will be more widely applied in other uncertainty fields, such as risk assessment, multiattribute case retrieval, green supplier selection, and low-speed wind farm site selection [26]–[29].

The application of probabilistic hesitant fuzzy sets makes them more practical in a complex battlefield environment. Therefore, more effort should be put into probabilistic hesitant fuzzy sets and the combination of possibility theory in the future. In addition, this newly proposed method can also integrate other multiple criteria decision-making models [30]–[32] to better solve battlefield situation assessments such as multiground-target threats and target recognition [33], [34].

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