

Dynamic H_∞ Consensus of Higher-Order Nonlinear Multi-Agent Systems With General Directed Topology

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ABSTRACT This paper investigates the leaderless H_∞ consensus problem of multi-agent systems with higher-order Lipschitz dynamics and external disturbance. The topology is assumed to be directed. Distributed controllers using only relative outputs information of the adjacent agents are constructed. In order to deal with the non-symmetric property of the associated Laplacian matrix of directed topology, some properties of Laplacian matrix are introduced. By introducing an appropriate state transformation, the H_∞ consensus problem is converted to a low-dimensional system stability problem which is solved using Lyapunov stability analysis method. The effectiveness of the proposed control design is demonstrated through simulation examples.

INDEX TERMS Multi-agent systems, H_∞ consensus, directed topology, external disturbance.

I. INTRODUCTION

The consensus of multi-agent systems have received compelling attention due to its wide-spread applications in satellites formation [1], unmanned systems [2], and distributed reconfigurable sensor networks [3], [4]. Since that agents are coupled together through a communication network, the consensus achievement is also affected by the system dynamics and network topology [5]. Leaderless consensus is one of the important research directions. Under various topology assumptions, many research results have been achieved including both integrator types [6]–[9] and general dynamics [10], [11].

Considering the fact that some system are usually described with nonlinear dynamics, researchers have studied the consensus problem of higher-order nonlinear multi-agent systems. In [12], the leaderless global H_∞ consensus of Lipschitz multi-agent systems was studied with strongly connected graphs. Then the results were extended to the case that the topology containing a directed spanning tree [13]. In [14], the leader-following consensus was studied with switching directed topologies using multiple Lyapunov functions method. It was proved that the consensus could be achieved if each topology contains a spanning tree and the

dwelling time was larger than a positive threshold. In [15], the leaderless consensus problem with general directed fixed and switching topologies was investigated. Several topology depending Lyapunov functions were constructed. The leaderless consensus of higher-order Lipschitz multi-agent systems was studied in [16] with jointly connected topologies, where common Lyapunov function method and Cauchy convergence criterion were used. Consensus controllers using relative state feedback were introduced for one-sided Lipschitz nonlinear multi-agent systems with directed strongly connected topology [17] and undirected topology [18], separately. In [19], leader-following exponential consensus was investigated using sampled-data information. When the union graph having a directed spanning tree rooted at the leader, [20] considered the performance consensus tracking problem of singular Lipschitz multi-agent systems. Both the leaderless and leader-following guaranteed-performance consensus problems were investigated in [21] with strongly connected and balanced topologies.

It is worth mentioning that the results in [12], [14]–[21] are obtained with the assumption that the relative state information of neighboring agents is available. However, in some real applications, only output information is available and there may be external disturbances. Inspired by this, output based consensus controllers are constructed with different topology conditions. Under fixed directed topology, [22] investigated

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the leader-following consensus using the output information. [23] addressed the leader-following consensus problem with a directed topology using event-triggered schemes. Under undirected topology, [24] proposed the fully distributed leaderless consensus controller using output feedback, in which the consensus controllers had adaptive coupling weights. Under the assumption that the interaction topology contained a directed spanning tree with the leader agent being the root and its subgraph associated with all following agents were undirected, the leader-following consensus problem for nonlinear multi-agent systems was considered using distributed adaptive consensus protocols [25]. In [26] the leaderless H_∞ consensus problem for Lipschitz nonlinear system was solved with relative output feedback. However, the topology was constrained with undirected case. Under undirected topology, [27] proposed reduced-order observer-based leaderless consensus controller.

It can be observed that, as for directed topology, the most of the existing conclusions of output based consensus controller for Lipschitz nonlinear multi-agent systems focus on the leader-following consensus problem [22], [23], [25]. In contrast, conclusions of the leaderless consensus problem are limited to some special topologies, such as undirected topology [24], [26], [27]. A general conclusion is that, due to the asymmetric nature of directed topology, it is more challenging to construct Lyapunov functions for the directed topology than undirected topology. Due to the network coupling in the multi-agent systems, the topology affects group behaviour. It is of great significance to design the consensus controller under general topology condition. Actually, the combination of higher-order Lipschitz dynamics, directed topology, and external disturbance make the H_∞ leaderless consensus problem much challenging.

Motivated by the above discussion, this paper aims to solve the leaderless H_∞ consensus problem of Lipschitz multi-agent systems with external disturbance using output feedback under quite general communication topology conditions. To the best of our knowledge, dynamic leaderless consensus protocols using output feedback are proposed herein for the first time for higher-order Lipschitz nonlinear systems both with general directed topology and external disturbance.

The main contributions of this paper are as follows.

First, the network among the agents is modelled by a general directed topology containing a spanning tree which can include the undirected topologies [24], [26], [27] as special cases. Actually, in practical applications, the communication between agents is usually not bidirectional due to many factors such as obstructions and communication capabilities. Only in highly constrained circumstance would such a communication network results in an undirected graph. Thus, it is more practical and reasonable to consider the general directed topology. Thus the topology constraint is greatly relaxed.

Second, the leaderless consensus problem is solved for the Lipschitz nonlinear multi-agent systems with output-feedback based consensus controller without using the states information. The results is suitable for situations where the

system states cannot be obtained directly. In this regard, it is more practical than the results in which the states feedback based consensus controller.

Third, this paper considers the H_∞ leaderless consensus problem with external disturbances. It is worth noting that from the perspective of analytical difficulty, the leaderless consensus problem is more difficult than the leader-following consensus problem, because that there is no specified leader in the leaderless consensus problem to construct the consensus error system with directed topology.

The rest of this work is organized as follows. Section 2 introduces some useful definitions and notations. In section 3, output based consensus controllers are introduced. In section 4, two simulation examples are given. Section 5 is the conclusion.

II. PRELIMINARIES

In this paper, \otimes denotes the Kronecker product. For $\mu \in \mathbb{C}$, $Re(\mu)$ denotes the real part of μ . $\lambda(A)$ denotes the eigenvalues of matrix A . Matrix $A > 0$ expresses that A is positive definite.

The information flow among the agents is depicted by a general directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with $\mathcal{V} = \{1, 2, \dots, N\}$ being the vertex set, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ being the directed edges set. $\mathcal{A} = [a_{ij}]_{N \times N}$ represents the adjacency matrix with $a_{ij} > 0$ if $i \times j \in \mathcal{E}$ and $a_{ij} = 0$ if $i \times j \notin \mathcal{E}$. The Laplacian matrix of the graph \mathcal{G} is defined as $\mathcal{L} = [\mathcal{L}_{ij}]_N$, with $\mathcal{L}_{ii} = \sum_{j \neq i} a_{ij}$ and $\mathcal{L}_{ij} = -a_{ij}$. A directed graph is said to have a spanning tree if there is a directed path from one vertex to every other node. The communication topology in multi-agent systems is expressed via a general directed graph containing a directed spanning tree.

The following conclusion will be used in the following sections.

Lemma 1 [28]: Assume that D and S are real matrices with compatible dimensions. For any given $x, y \in \mathbb{R}^n$, and matrices $P > 0$, the following inequality holds

$$2x^T D S y \leq x^T D P D^T x + y^T S^T P^{-1} S y.$$

III. PROBLEM FORMULATION AND MAIN RESULTS

A. PROBLEM FORMULATION

Let there is a group of N identical agents. The dynamics of the i -th agent are specified as

$$\begin{aligned} \dot{x}_i &= A x_i + B u_i + D_1 f(x_i) + D_2 \omega_i \\ y_i &= C x_i \quad i = 1, 2, \dots, N, \end{aligned} \quad (1)$$

where $x_i \in \mathbb{R}^n$ represents the state of the i -th agent, $u_i \in \mathbb{R}^p$ is the control input to be designed, $y_i \in \mathbb{R}^q$ and $\omega_i \in \mathcal{L}_2[0, \infty)$ are the output and the external disturbance, respectively. A, B, C, D_1 , and D_2 denotes the constant real matrices. $f(\cdot)$ is the nonlinear function and satisfies Lipschitz condition, i.e.,

$$\|f(x, t) - f(y, t)\| \leq \rho \|x - y\|, \quad \forall x, y \in \mathbb{R}^n, t \geq 0,$$

where $\rho > 0$ is the Lipschitz constant.

The consensus of system (1) is said to be achieved if $\lim_{t \rightarrow \infty} \|x_i(t) - x_j(t)\| = 0, \forall i, j \in N$. However, when there is external disturbance, the consensus may cannot be achieved accurately. Thus, the H_∞ consensus is introduced as follows, in which the system can reach consensus and meanwhile maintains a desirable disturbance attenuation performance. The consensus performance variables are defined as

$$z_i = \frac{1}{N} \sum_{j=1}^N C(x_i - x_j), \quad i = 1, 2, \dots, N,$$

where $C \in \mathbb{R}^{q_1 \times n}$ is a constant matrix.

Definition 1: The system (1) can realize H_∞ consensus with a prescribed disturbance attenuation level $\gamma > 0$ under the zero-initial conditions, if

$$J = \int_0^\infty (z^T(t)z(t) - \gamma^2 \omega^T(t)\omega(t))dt < 0.$$

B. MAIN RESULTS

Here, by using only relative output information of neighboring agents, the following dynamic consensus controllers are introduced:

$$\begin{aligned} \dot{\hat{x}}_i &= (A + BF)\hat{x}_i + cH \sum_{j=1}^N a_{ij}(C(\hat{x}_j - \hat{x}_i) - (y_j - y_i)) \\ u_i &= F\hat{x}_i, \quad i = 1, 2, \dots, N, \end{aligned} \quad (2)$$

where $\hat{x}_i \in \mathbb{R}^n$ is the intermediate variable, F and H are the feedback gain matrices to be designed, c is the coupling strength to be selected, a_{ij} is the element of the adjacency matrix \mathcal{A} of the graph \mathcal{G} .

Let $e_i = x_i - \hat{x}_i$, then from (1) and (2), we can obtain the closed-loop dynamics

$$\begin{aligned} \dot{x} &= (I_N \otimes (A + BF))x - (I_N \otimes BF)e \\ &\quad + (I_N \otimes D_1)F(x) + (I_N \otimes D_2)\omega \\ \dot{e} &= (I_N \otimes A - c\mathcal{L} \otimes HC)e + (I_N \otimes D_1)F(x) \\ &\quad + (I_N \otimes D_2)\omega \\ z &= (U \otimes C)x, \end{aligned} \quad (3)$$

where $x = [x_1^T, \dots, x_N^T]^T$, $e = [e_1^T, \dots, e_N^T]^T$, $\omega = [\omega_1^T, \dots, \omega_N^T]^T$, $F(x) = [f(x_1)^T, \dots, f(x_N)^T]^T$, $\mathcal{L} \in \mathbb{R}^{N \times N}$ is the Laplacian matrix of the graph \mathcal{G} , and $U = I_N - \frac{1}{N}\mathbf{1}_N\mathbf{1}_N^T$. We can see that the row sum of U is zero and U is a special Laplacian matrix of a completed graph.

Before moving forward, the following conclusions obtained in our earlier works are introduced to obtain the results.

Lemma 2 [13], [29]: For a matrix R whose row sum is zero and a full row rank matrix $E \in \mathbb{R}^{(N-1) \times N}$ defined as

$$E = \begin{bmatrix} 1 & -1 & 0 & \dots & 0 \\ 0 & 1 & -1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}, \quad (4)$$

one can find a matrix $M \in \mathbb{R}^{N \times (N-1)}$ such that $R = ME$. Furthermore, if R is a Laplacian matrix of a graph containing a directed spanning tree, one has that $\text{Re}(\lambda(EM)) > 0$.

Lemma 3 [13], [29]: For a matrix $Q > 0$ with the eigenvalues having positive real parts, there exist a matrix $Q > 0$ and a scalar $\alpha > 0$ such that

$$R^T Q + QR > \alpha Q, \quad (5)$$

where $0 < \alpha < 2 \min\{\text{Re}(\lambda(R))\}$.

In light of the property of matrix U in (3) and Lemma 2, we set $\xi = (E \otimes I_n)x$ and $\zeta = (E \otimes I_n)e$. Then, (3) can be rewritten as

$$\begin{aligned} \dot{\xi} &= (I_{N-1} \otimes (A + BF))\xi - (I_{N-1} \otimes BF)\zeta \\ &\quad + (I_{N-1} \otimes D_1)\Phi(x) + (E \otimes D_2)\omega \\ \dot{\zeta} &= (I_{N-1} \otimes A - cEM \otimes HC)\zeta \\ &\quad + (I_{N-1} \otimes D_1)\Phi(x) + (E \otimes D_2)\omega \\ z &= (W \otimes C)\xi, \end{aligned} \quad (6)$$

where $\Phi(x) = (E \otimes I_n)F(x)$. M and W are the matrices such that $\mathcal{L} = ME$ and $U = WE$.

The definition of ξ ensures that $\xi=0$, if and only if $\lim_{t \rightarrow \infty} \|x_i(t) - x_j(t)\| = 0$, therefore consensus is achieved.

Theorem 1: For the multi-agent systems in (1), the relative output feedback based controllers in (2) ensure H_∞ consensus of system (1) if there exist constants $c > 0$, $\mu > 0$, and matrices $P_1 > 0$, $P_2 > 0$ such that the following inequalities are satisfied:

$$\begin{bmatrix} \tilde{A}_1 & P_1 D_1 & I_n & C^T & P_1 D_2 \\ D_1^T P_1 & -\frac{\eta_0}{\varphi_1} I_n & 0 & 0 & 0 \\ I_n & 0 & -\frac{\varphi_2}{2\rho^2 \eta_0} I_n & 0 & 0 \\ C & 0 & 0 & -\frac{\varphi_2}{\beta} I_n & 0 \\ D_2^T P_1 & 0 & 0 & 0 & -\frac{\gamma^2}{2\varphi_1 \lambda_0} I_n \end{bmatrix} < 0, \quad (7)$$

$$\begin{bmatrix} \tilde{A}_2 & P_2 D_1 & P_2 D_2 \\ D_1^T P_2 & -\frac{\eta_0}{\varphi_1} I_n & 0 \\ D_2^T P_2 & 0 & -\frac{\gamma^2}{2\varphi_1 \lambda_0} I_n \end{bmatrix} < 0, \quad (8)$$

where $\tilde{A}_1 = A^T P_1 + P_1 A - \mu P_1 B B^T P_1$, $\lambda_0 = \max\{\lambda(EE^T)\}$, $\tilde{A}_2 = A^T P_2 + P_2 A - c\alpha C^T C + \mu P_1 B B^T P_1$, $\varphi_1 = \max\{\lambda(Q)\}$, $\varphi_2 = \min\{\lambda(Q)\}$, $\beta = \max\{\lambda(W^T W)\}$, η_0 is a positive constant, $0 < \alpha < 2 \min\{\text{Re}(\lambda(EM))\}$, and Q is a feasible solution of (5) with $R = EM$. The feedback matrices are designed as $F = -\mu B^T P_1$ and $H = P_2^{-1} C^T$.

Proof: Consider the following Lyapunov candidate

$$V = V_1 + V_2 = \xi^T (Q \otimes P_1) \xi + \zeta^T (Q \otimes P_2) \zeta, \quad (9)$$

where $V_1 = \xi^T (Q \otimes P_1) \xi$ and $V_2 = \zeta^T (Q \otimes P_2) \zeta$.

The derivation of V_1 along the trajectory of system (6) is

$$\begin{aligned} \dot{V}_1 = & \xi^T \left(Q \otimes \left((A + BF)^T P_1 + P_1 (A + BF) \right) \right) \xi \\ & - 2\zeta^T \left(Q \otimes (BF)^T P_1 \right) \xi + 2\Phi(x)^T \left(Q \otimes D_1^T P_1 \right) \xi \\ & + 2\omega^T \left(E^T Q \otimes D_2^T P_1 \right) \xi. \end{aligned} \quad (10)$$

Substituting $F = -\mu B^T P_1$ into (10)(9) yields

$$\begin{aligned} \dot{V}_1 \leq & \xi^T \left(Q \otimes \left(A^T P_1 + P_1 A - 2\mu P_1 B B^T P_1 \right) \right) \xi \\ & + 2\zeta^T \left(Q \otimes \mu P_1 B B^T P_1 \right) \xi + 2\Phi(x)^T \left(Q \otimes D_1^T P_1 \right) \xi \\ & + 2\omega^T \left(E^T Q \otimes D_2^T P_1 \right) \xi. \end{aligned} \quad (11)$$

According to the Lipschitz condition and Lemma 1, we have

$$\begin{aligned} & 2\Phi(x)^T \left(Q \otimes D_1^T P_1 \right) \xi \\ & \leq \frac{1}{\eta_0} \Phi(x)^T \Phi(x) + \eta_0 \xi^T \left(Q^2 \otimes P_1 D_1 D_1^T P_1 \right) \xi \\ & \leq \rho^2 \eta_0 \xi^T \xi + \frac{1}{\eta_0} \xi^T \left(Q^2 \otimes P_1 D_1 D_1^T P_1 \right) \xi, \end{aligned} \quad (12)$$

where η_0 is a positive constant.

Then (11) can be rewritten as

$$\begin{aligned} \dot{V}_1 \leq & \xi^T \left(Q \otimes \left(A^T P_1 + P_1 A - 2\mu P_1 B B^T P_1 \right) \right) \xi \\ & + 2\zeta^T \left(Q \otimes \mu P_1 B B^T P_1 \right) \xi + \rho^2 \eta_0 \xi^T \xi \\ & + \xi^T \left(Q^2 \otimes \frac{1}{\eta_0} P_1 D_1 D_1^T P_1 \right) \xi \\ & + 2\omega^T \left(E^T Q \otimes D_2^T P_1 \right) \xi. \end{aligned} \quad (13)$$

Since $Q \leq \varphi_1 I_{N-1}$ and $Q \geq \varphi_2 I_{N-1}$, where $\varphi_1 = \max \{\lambda(Q)\}$, $\varphi_2 = \min \{\lambda(Q)\}$, we can obtain that

$$\begin{aligned} \dot{V}_1 \leq & \xi^T \left(Q \otimes \left(A^T P_1 + P_1 A - 2\mu P_1 B B^T P_1 \right) \right) \xi \\ & + \varphi_1 \frac{1}{\eta_0} P_1 D_1 D_1^T P_1 + \frac{\rho^2 \eta_0}{\varphi_2} I_n \\ & + 2\zeta^T \left(Q \otimes \mu P_1 B B^T P_1 \right) \xi \\ & + 2\omega^T \left(E^T Q \otimes D_2^T P_1 \right) \xi. \end{aligned} \quad (14)$$

The derivation of V_2 along the trajectory of system (6) is

$$\begin{aligned} \dot{V}_2 = & \zeta^T \left(Q \otimes \left(A^T P_2 + P_2 A \right) \right) \zeta \\ & - 2c\zeta^T \left((EM)^T Q \otimes (HC)^T P_2 \right) \zeta \\ & + 2\Phi(x)^T \left(Q \otimes D_1^T P_2 \right) \zeta \\ & + 2\omega^T \left(E^T Q \otimes D_2^T P_2 \right) \zeta. \end{aligned} \quad (15)$$

Substituting $H = P_2^{-1} C^T$ into (15)(13) yields

$$\begin{aligned} \dot{V}_2 = & \zeta^T \left(Q \otimes \left(A^T P_2 + P_2 A \right) \right) \zeta \\ & - c\zeta^T \left(\left((EM)^T Q + QEM \right) \otimes C^T C \right) \zeta \end{aligned}$$

$$+ 2\Phi(x)^T \left(Q \otimes D_1^T P_2 \right) \zeta + 2\omega^T \left(E^T Q \otimes D_2^T P_2 \right) \zeta. \quad (16)$$

According to Lemma 2 and Lemma 3, we can obtain the following conclusion

$$(EM)^T Q + QEM > \alpha Q, \quad (17)$$

where $0 < \alpha < 2 \min \{\text{Re}(\lambda(EM))\}$. It follows from (16) that

$$\begin{aligned} \dot{V}_2 = & \zeta^T \left(Q \otimes \left(A^T P_2 + P_2 A - c\alpha C^T C \right) \right) \zeta \\ & + 2\Phi(x)^T \left(Q \otimes D_1^T P_2 \right) \zeta + 2\omega^T \left(E^T Q \otimes D_2^T P_2 \right) \zeta. \end{aligned} \quad (18)$$

Similarly, using the Lipschitz condition, the following conclusion can be obtained

$$\begin{aligned} \dot{V}_2 \leq & \zeta^T \left(Q \otimes \left(A^T P_2 + P_2 A - c\alpha C^T C \right) \right) \zeta + \rho^2 \eta_0 \xi^T \xi \\ & + \zeta^T \left(Q^2 \otimes \frac{1}{\eta_0} P_2 D_1 D_1^T P_2 \right) \zeta \\ & + 2\omega^T \left(E^T Q \otimes D_2^T P_2 \right) \zeta \\ & \leq \zeta^T \left(Q \otimes \left(A^T P_2 + P_2 A - c\alpha C^T C \right) \right) \zeta \\ & + \xi^T \left(Q \otimes \frac{\rho^2 \eta_0}{\varphi_2} I_n \right) \xi + \zeta^T \left(Q \otimes \frac{\varphi_1}{\eta_0} P_2 D_1 D_1^T P_2 \right) \zeta \\ & + 2\omega^T \left(E^T Q \otimes D_2^T P_2 \right) \zeta \\ & = \zeta^T \left(Q \otimes \left(A^T P_2 + P_2 A - c\alpha C^T C \right) \right) \zeta \\ & + \xi^T \left(Q \otimes \frac{\rho^2 \eta_0}{\varphi_2} I_n \right) \xi + 2\omega^T \left(E^T Q \otimes D_2^T P_2 \right) \zeta. \end{aligned} \quad (19)$$

Using (14) and (19), we have

$$\begin{aligned} \dot{V} = & \dot{V}_1 + \dot{V}_2 \\ \leq & \xi^T \left(Q \otimes \left(A^T P_1 + P_1 A - 2\mu P_1 B B^T P_1 \right) \right) \xi \\ & + \frac{\varphi_1}{\eta_0} P_1 D_1 D_1^T P_1 + \frac{2\rho^2 \eta_0}{\varphi_2} I_n \\ & + 2\zeta^T \left(Q \otimes \mu P_1 B B^T P_1 \right) \xi + 2\omega^T \left(E^T Q \otimes D_2^T P_1 \right) \xi \\ & + \zeta^T \left(Q \otimes \left(A^T P_2 + P_2 A - c\alpha C^T C \right) \right) \zeta \\ & + \frac{\varphi_1}{\eta_0} P_2 D_1 D_1^T P_2 \\ & + 2\omega^T \left(E^T Q \otimes D_2^T P_2 \right) \zeta. \end{aligned} \quad (20)$$

For the case of $\omega_i(t) = 0$, it follows from the above analysis that

$$\begin{aligned} \dot{V} \leq & \xi^T \left(Q \otimes \left(A^T P_1 + P_1 A - 2\mu P_1 B B^T P_1 \right) \right) \xi \\ & + \frac{\varphi_1}{\eta_0} P_1 D_1 D_1^T P_1 + \frac{2\rho^2 \eta_0}{\varphi_2} I_n \\ & + 2\zeta^T \left(Q \otimes \mu P_1 B B^T P_1 \right) \xi \end{aligned}$$

$$\begin{aligned}
 & +\zeta^T \left(Q \otimes \left(\begin{array}{c} A^T P_2 + P_2 A - c\alpha C^T C \\ + \frac{\varphi_1}{\eta_0} P_2 D_1 D_1^T P_2 \end{array} \right) \right) \zeta \\
 \leq & \xi^T \left(Q \otimes \left(\begin{array}{c} A^T P_1 + P_1 A - \mu P_1 B B^T P_1 \\ + \frac{\varphi_1}{\eta_0} P_1 D_1 D_1^T P_1 + \frac{2\rho^2 \eta_0}{\varphi_2} I_n \end{array} \right) \right) \xi \\
 & +\zeta^T \left(Q \otimes \left(\begin{array}{c} A^T P_2 + P_2 A - c\alpha C^T C \\ + \frac{\varphi_1}{\eta_0} P_2 D_1 D_1^T P_2 + \mu P_1 B B^T P_1 \end{array} \right) \right) \zeta.
 \end{aligned} \tag{21}$$

where the following inequality is used

$$\begin{aligned}
 & 2\zeta^T \left(Q \otimes \mu P_1 B B^T P_1 \right) \xi \\
 & \leq \mu \zeta^T \left(Q \otimes P_1 B B^T P_1 \right) \zeta + \mu \xi^T \left(Q \otimes P_1 B B^T P_1 \right) \xi.
 \end{aligned}$$

Since $\omega_i(t) = 0$, the following conclusions can be obtained from (7) and (8) using the Schur complement lemma

$$A^T P_1 + P_1 A - \mu P_1 B B^T P_1 + \frac{\varphi_1}{\eta_0} P_1 D_1 D_1^T P_1 + \frac{2\rho^2 \eta_0}{\varphi_2} I_n < 0, \tag{22}$$

$$A^T P_2 + P_2 A - c\alpha C^T C + \frac{\varphi_1}{\eta_0} P_2 D_1 D_1^T P_2 + \mu P_1 B B^T P_1 < 0. \tag{23}$$

Then, it follows from (21) that $\dot{V} < 0$. Thus the consensus can be achieved when there is no external disturbance.

Next, we will consider the case when $\omega_i(t) \neq 0$ and the H_∞ performance of the system. Let $\beta = \max \{ \lambda (W^T W) \}$ and $\varphi_2 = \min \{ \lambda (Q) \}$. Then, we have

$$z^T z < \frac{\beta}{\varphi_2} \xi^T \left(Q \otimes C^T C \right) \xi, \tag{24}$$

$$\gamma^2 \omega^T (I_{N-1} \otimes I_n) \omega = \gamma^2 \omega^T \omega. \tag{25}$$

Using (24) and (25), we can obtain from (20) that

$$\begin{aligned}
 \dot{V} \leq & \xi^T \left(Q \otimes \left(\begin{array}{c} A^T P_1 + P_1 A - \mu P_1 B B^T P_1 \\ + \frac{\varphi_1}{\eta_0} P_1 D_1 D_1^T P_1 + \frac{2\rho^2 \eta_0}{\varphi_2} I_n + \frac{\beta}{\varphi_2} C^T C \end{array} \right) \right) \xi \\
 & +\zeta^T \left(Q \otimes \left(\begin{array}{c} A^T P_2 + P_2 A - c\alpha C^T C \\ + \frac{\varphi_1}{\eta_0} P_2 D_1 D_1^T P_2 + \mu P_1 B B^T P_1 \end{array} \right) \right) \zeta \\
 & +2\omega^T \left(E^T Q \otimes D_2^T P_2 \right) \zeta + 2\omega^T \left(E^T Q \otimes D_2^T P_1 \right) \xi \\
 & -z^T z + \gamma^2 \omega^T \omega - 2\omega^T \left(I_{N-1} \otimes \frac{\gamma^2}{2} I_n \right) \omega \\
 = & \begin{bmatrix} \xi \\ \omega \end{bmatrix}^T \Theta_1 \begin{bmatrix} \xi \\ \omega \end{bmatrix} + \begin{bmatrix} \zeta \\ \omega \end{bmatrix}^T \Theta_2 \begin{bmatrix} \zeta \\ \omega \end{bmatrix} - z^T z + \gamma^2 \omega^T \omega,
 \end{aligned} \tag{26}$$

where

$$\begin{aligned}
 \Theta_1 & = \begin{pmatrix} Q \otimes \Xi_1 & QE \otimes P_1 D_2 \\ E^T Q \otimes D_2^T P_1 & -I_{N-1} \otimes \frac{\gamma^2}{2} I_n \end{pmatrix}, \\
 \Theta_2 & = \begin{pmatrix} Q \otimes \Xi_2 & QE \otimes P_2 D_2 \\ E^T Q \otimes D_2^T P_2 & -I_{N-1} \otimes \frac{\gamma^2}{2} I_n \end{pmatrix},
 \end{aligned}$$

$$\begin{aligned}
 \Xi_1 & = A^T P_1 + P_1 A - \mu P_1 B B^T P_1 + \frac{\varphi_1}{\eta_0} P_1 D_1 D_1^T P_1 \\
 & + \frac{2\rho^2 \eta_0}{\varphi_2} I_n + \frac{\beta}{\varphi_2} C^T C \\
 \Xi_2 & = A^T P_2 + P_2 A - c\alpha C^T C + \frac{\varphi_1}{\eta_0} P_2 D_1 D_1^T P_2 \\
 & + \mu P_1 B B^T P_1.
 \end{aligned}$$

Noting that $\Theta_1 < 0$ and $\Theta_2 < 0$ if and only if the following inequalities hold

$$-Q \otimes \frac{\gamma^2}{2\varphi_1} I_n < 0, \tag{27}$$

$$Q \otimes \Xi_1 + \frac{2}{\gamma^2} QEE^T Q \otimes P_1 D_2 D_2^T P_1 < 0, \tag{28}$$

$$Q \otimes \Xi_2 + \frac{2}{\gamma^2} QEE^T Q \otimes P_2 D_2 D_2^T P_2 < 0. \tag{29}$$

Since that $\lambda_0 = \max \{ \lambda (EE^T) \}$ and $\varphi_1 = \max \{ \lambda (Q) \}$, one has

$$\begin{aligned}
 & Q \otimes \Xi_1 + \frac{2}{\gamma^2} QEE^T Q \otimes P_1 D_2 D_2^T P_1 \\
 & < Q \otimes \left(\Xi_1 + \frac{2\varphi_1 \lambda_0}{\gamma^2} P_1 D_2 D_2^T P_1 \right),
 \end{aligned} \tag{30}$$

$$\begin{aligned}
 & Q \otimes \Xi_2 + \frac{2}{\gamma^2} QEE^T Q \otimes P_2 D_2 D_2^T P_2 \\
 & < Q \otimes \left(\Xi_2 + \frac{2\varphi_1 \lambda_0}{\gamma^2} P_2 D_2 D_2^T P_2 \right),
 \end{aligned} \tag{31}$$

where $\lambda_0 = \max \{ \lambda (EE^T) \}$ and $\varphi_1 = \max \{ \lambda (Q) \}$.

By the Schur complement lemma, inequalities (7) and (8) imply that

$$\Xi_1 + \frac{2\varphi_1 \lambda_0}{\gamma^2} P_1 D_2 D_2^T P_1 < 0, \tag{32}$$

$$\Xi_2 + \frac{2\varphi_1 \lambda_0}{\gamma^2} P_2 D_2 D_2^T P_2 < 0. \tag{33}$$

Considering (30), (31), (32), and (33) at the same time, we can conclude that inequalities (28) and (29) hold. That further implies that $\Theta_1 < 0$ and $\Theta_2 < 0$. It follows from (26) that

$$\dot{V} + z^T z - \gamma^2 \omega^T \omega < 0. \tag{34}$$

Since that $V(0) = 0$, the following inequality is satisfied

$$\begin{aligned}
 J & = \int_0^\infty \left(z^T(t) z(t) - \gamma^2 \omega^T(t) \omega(t) \right) dt \\
 & = \int_0^\infty \left(z^T(t) z(t) - \gamma^2 \omega^T(t) \omega(t) + \dot{V} \right) dt \\
 & \quad - V(\infty) + V(0) \\
 & < \int_0^\infty \left(z^T(t) z(t) - \gamma^2 \omega^T(t) \omega(t) + \dot{V} \right) dt.
 \end{aligned} \tag{35}$$

That means that system (1)(1) achieves the H_∞ consensus. The proof is completed.

Remark 1: Leaderless consensus problems of Lipschitz multi-agent systems using output information were studied in [24], [26], [27]. However, the results were constrained

with some special topologies, such as undirected or strongly connected topology. It is worth noting that the topology used here can include undirected or strongly connected topology as special cases. Comparing with [24], [26], [27], the topology constraints are relaxed to a more general situation. In practical applications, the communication between agents is usually not bidirectional due to many factors such as obstructions and communication capabilities. Thus, the results obtained in this paper are less conservative.

Remark 2: In [14], [22], [23], the leader-following output based consensus controllers for Lipschitz multi-agent systems are constructed under directed topologies. There is a specific leader in the system, which provides a great convenience for constructing the consensus error system. In contrast with the results in [14], [22], [23], leaderless consensus problem is investigated here. By using the properties of the Laplacian matrix obtained in our earlier works [13], [29], the leaderless H_∞ consensus problem is very succinctly transformed into a H_∞ control problem of a low-dimensional system. It is worth pointing out that, in the homogeneous multi-agent systems, leaderless consensus problem can include leader-following consensus problem as a special case. Thus, the conclusions obtained here can also solve the leader-following consensus problem. A simulation example will be provided to verify the applicability of this conclusion.

Remark 3: It should be pointed out that although this article assumes that the directed topology graph is fixed, based on the processing method of this article, the H_∞ consensus problem can also be dealt with using multiply Lyapunov function method with directed switching topologies.

Remark 4: The traditional method of analyzing the H_∞ consensus problem is to use the diagonalization method to decompose the system into the H_∞ control problem of N-1 subsystems corresponding to the N-1 non-zero eigenvalues of the Laplacian matrix. However, when the topology is directional and the system has non-linear terms at the same time, the traditional analysis method is very difficult. Here, a brief analysis method is used. The leaderless H_∞ consensus problem is very succinctly transformed into a low-dimensional system H_∞ control problem which is solved using Lyapunov function method. The solution of the problem also benefits from the properties of the Laplacian matrix given in Lemma 2.

Remark 5: It should be pointed out that the global information of the topology structure is needed in the proposed approach. Specifically, the structural information of the topology is required to calculate the parameter α using the smallest nonzero eigenvalue of the graph Laplacian matrix in Lemma 3. The research results have certain limitations. However, this shortcoming can be overcome if we can know the structure of the topology in advance.

Remark 6: It can be seen that the performance index γ affects the feasibility of the inequality (7). Actually, if inequality (7) holds, the following inequality is a necessary

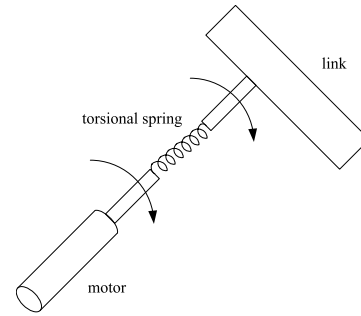


FIGURE 1. Robotic system with revolute joint.

condition.

$$A^T P_1 + P_1 A - \mu P_1 B B^T P_1 < 0$$

That means that (A, B) should be controllable. The feasibility of (7) and (8) can be easily checked from standard LMI routines. Note that a possible solution P_1 of (7) can be calculated first with a positive constant η_0 . Then, the possible solution P_2 of (8) with the fixed P_1 can be obtained.

If it is assumed that the system is linear and there is no external interference, the following corollary can be obtained.

Corollary 1: For the multi-agent systems in (1) with $D_1 = 0$ and $D_2 = 0$, the relative output feedback based controllers in (2) ensure consensus if there exist constants $c > 0$, $\mu > 0$, and matrices $P_1 > 0$, $P_2 > 0$ such that the following inequality is satisfied:

$$A^T P_1 + P_1 A - 2\mu P_1 B B^T P_1 < 0$$

$$A^T P_2 + P_2 A - c\alpha C^T C < 0,$$

where $0 < \alpha < 2 \min \{Re(\lambda(EM))\}$, The feedback matrices are designed as $F = -\mu B^T P_1$ and $H = P_2^{-1} C^T$.

IV. SIMULATION

In this section, two simulation examples are provided to verify the applicability of consensus controller proposed above with two different topology conditions. Consider a group of one-link manipulators with revolute joints actuated by a DC, as shown in Figure 1.

Let $N = 4$ in (1) and

$$x_i = \begin{bmatrix} x_{i1} \\ x_{i2} \\ x_{i3} \\ x_{i4} \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.26 & 48.6 & 0 \\ 0 & 0 & 0 & 10 \\ 1.95 & 0 & -1.95 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix},$$

$$D_1 = 0.1 * I_4, \quad D_2 = [0.1 \quad 0 \quad 0 \quad 0.05]^T,$$

$$C = [1 \quad 0 \quad 0 \quad 0],$$

$$f(x_i) = [0 \quad 0 \quad 0 \quad -0.333 \sin(x_{i3})]^T.$$

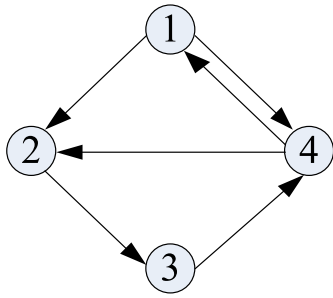


FIGURE 2. Communication topology in example 1.

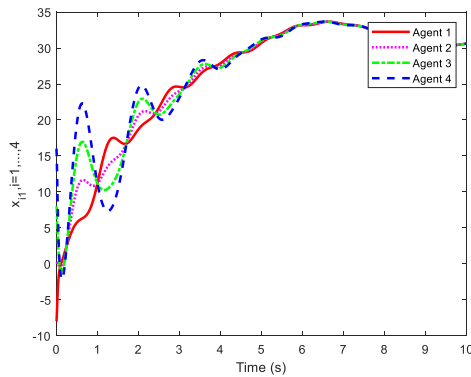


FIGURE 3. State trajectories of $x_{1i} i = 1, \dots, 4$.

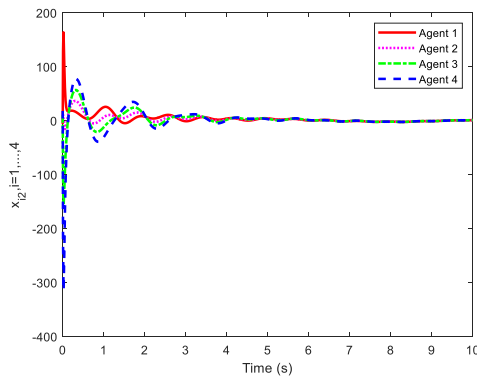


FIGURE 4. State trajectories of $x_{2i} i = 1, \dots, 4$.

We can obtain that $\rho = 0.333$. The external disturbance is defined as $\omega(t) = [1\bar{\omega}(t) 0.8\bar{\omega}(t) 2\bar{\omega}(t) 0.5\bar{\omega}(t)]^T$, where

$$\bar{\omega}(t) = \begin{cases} 2, & 0 \leq t \leq 6 \\ 0, & t > 6. \end{cases}$$

Example 1: In this example, the leaderless H_∞ consensus problems is investigated with the topology shown in Figure 2. It is shown that the topology contains a directed spanning tree.

The Laplacian matrix of this graph is

$$\mathcal{L} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 2 & 0 & -1 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & -1 & 2 \end{bmatrix}.$$

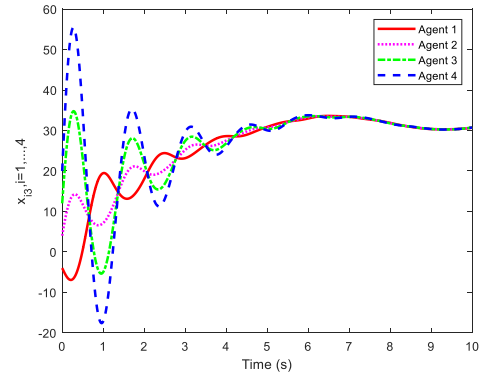


FIGURE 5. State trajectories of $x_{3i} i = 1, \dots, 4$.

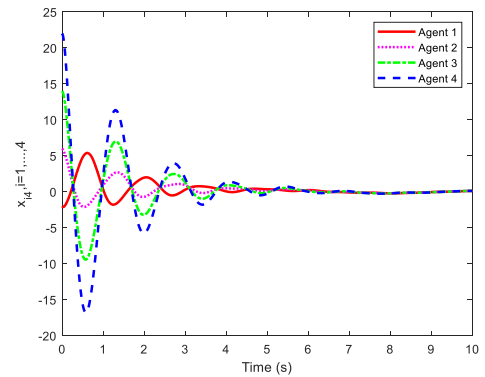


FIGURE 6. State trajectories of $x_{4i} i = 1, \dots, 4$.

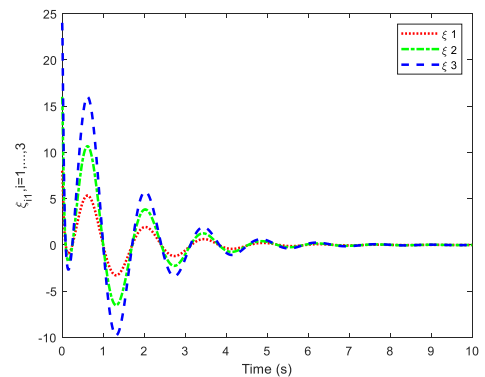


FIGURE 7. State trajectories of $\xi_{1i} i = 1, \dots, 3$.

Then one has that $\min \{Re(\lambda(EM))\} = 2$. Thus, we chose $\alpha = 0.1$. Then it follows from (5) that

$$Q = \begin{bmatrix} 15.8748 & 3.9377 & -5.2133 \\ 3.9377 & 11.6496 & -3.2365 \\ -5.2133 & -3.2365 & 13.0430 \end{bmatrix}.$$

Then one can obtain that $\varphi_1 = 22.2984$, $\varphi_2 = 8.9549$. Choose the H_∞ performance index $\gamma = 2$. Set $\eta_0 = 8.6$, $\mu = 1000$, and $c = 2000$. Solving the inequality (7) and (8) using the LMI toolbox of Matlab, we get the following feasible

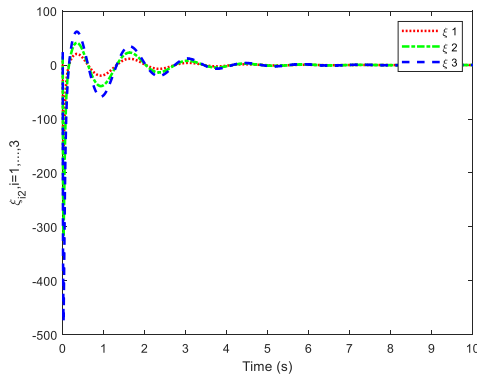


FIGURE 8. State trajectories of $\xi_{i2} i = 1, \dots, 3$.

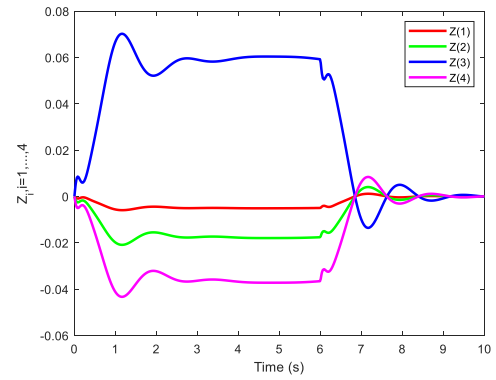


FIGURE 11. Trajectories of $Z_i, i = 1, 2, 3, 4$.

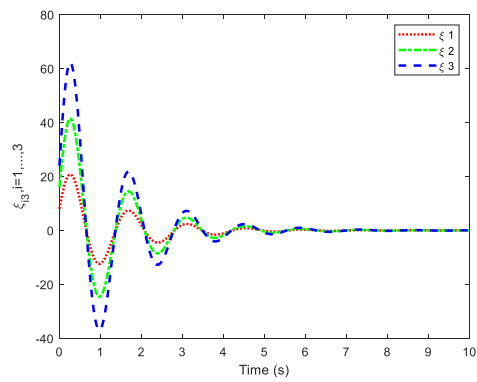


FIGURE 9. State trajectories of $\xi_{i3} i = 1, \dots, 3$.

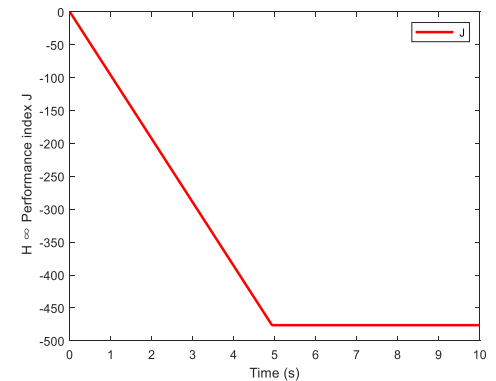


FIGURE 12. Trajectories of performance index J .

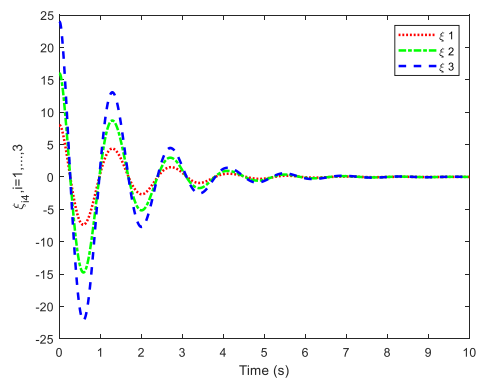


FIGURE 10. State trajectories of $\xi_{i4} i = 1, \dots, 3$.

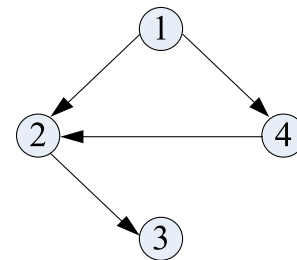


FIGURE 13. Communication topology in example 2.

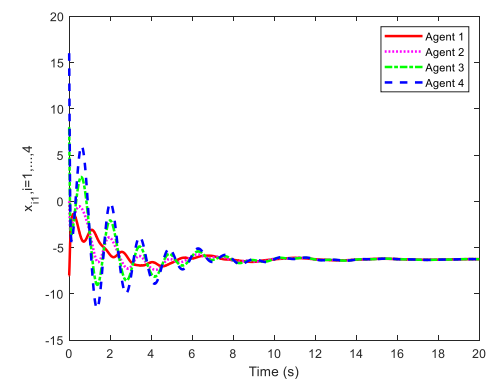


FIGURE 14. State trajectories of $x_{i1} i = 1, \dots, 4$.

solutions

$$P_1 = \begin{bmatrix} 32 & -293 & 6 & -8.2 \\ -293 & 9330.4 & -16.6 & -1 \\ 6 & -16.6 & 57.7 & -3 \\ -8.2 & -1 & -3 & 11.3 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 2.9125 & -0.0415 & 0.0048 & -0.0166 \\ -0.0415 & 0.0079 & -0.0434 & 0.0175 \\ 0.0048 & -0.0434 & 0.3103 & -0.2252 \\ -0.0166 & 0.0175 & -0.2252 & 0.3315 \end{bmatrix}.$$

The feedback gain matrices in the dynamic consensus controller (2) can be chosen as

$$F = [-41.1981 \ -3.6191 \ 1.7128 \ -29.9585],$$

$$H = [19.1 \ 1300.9 \ 261.9 \ 110.4]^T.$$

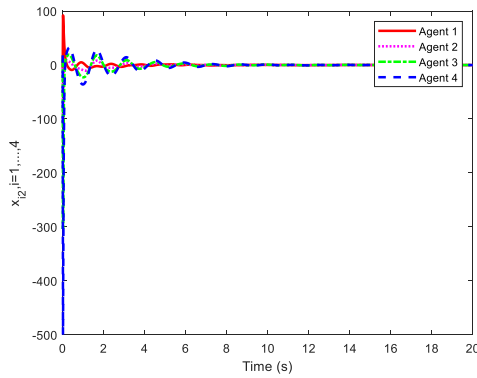


FIGURE 15. State trajectories of $x_{i2} i = 1, \dots, 4$.

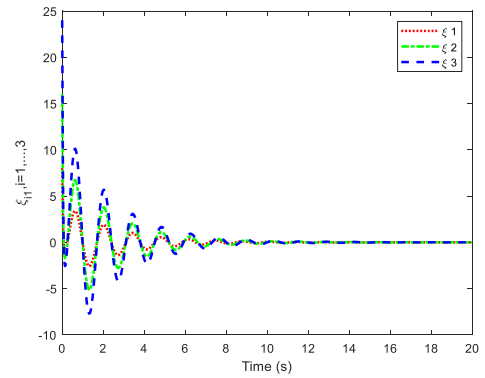


FIGURE 18. State trajectories of $\xi_{i1} i = 1, \dots, 3$.

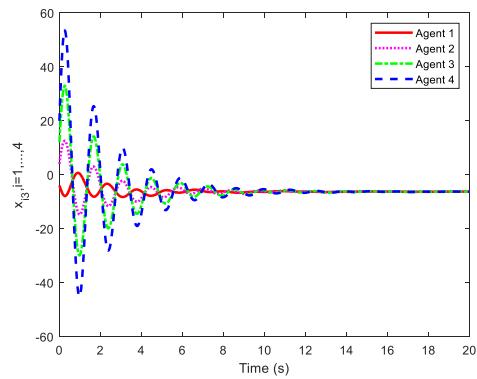


FIGURE 16. State trajectories of $x_{i3} i = 1, \dots, 4$.

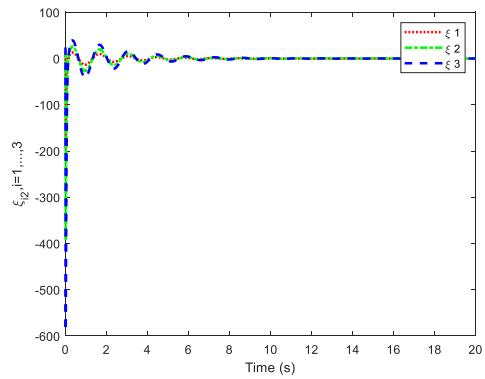


FIGURE 19. State trajectories of $\xi_{i2} i = 1, \dots, 3$.

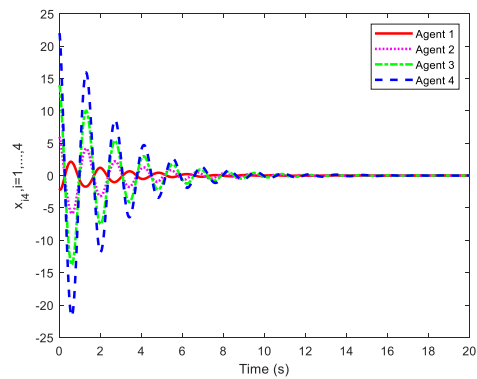


FIGURE 17. State trajectories of $x_{i4} i = 1, \dots, 4$.

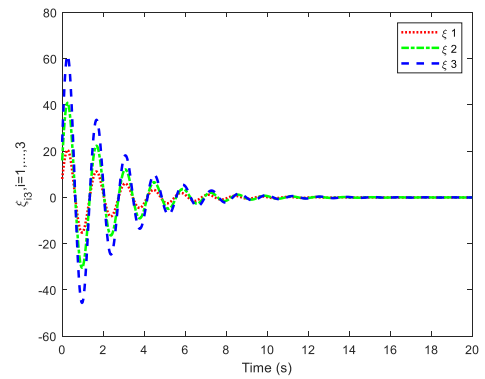


FIGURE 20. State trajectories of $\xi_{i3} i = 1, \dots, 3$.

For the case without disturbances, Figures. 2-5 depict the trajectory graph of the states and Figures. 6-9 show the trajectory graph of the consensus error ξ . It is shown that the consensus error ξ converges to zero, which means that the system can achieve consensus under the proposed output-feedback based consensus controller.

Figure 10 shows the trajectories of the performance variables z_i with zero-initial condition and external disturbances. The performance index J is shown in Figure 11. It is shown that J fulfills the conditions in definition 1, which means that the H_∞ consensus is achieved.

Example 2: In this example, the leader-following consensus problem without external disturbance is investigated with the topology shown in Figure 13. It is shown that the topology contains a directed spanning tree with agent 1 being the leader.

The Laplacian matrix of this graph is

$$\mathcal{L} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & -1 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}.$$

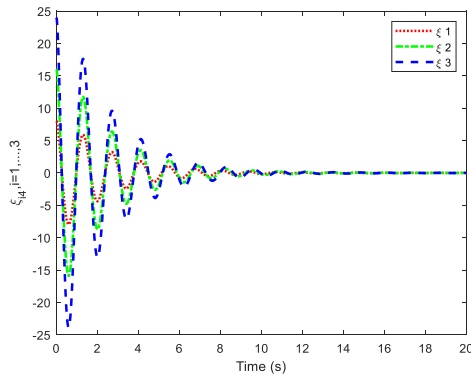


FIGURE 21. State trajectories of $\xi_{i,j} \ i = 1, \dots, 3$.

Then one can obtain that $\varphi_1 = 1.2778$ and $\varphi_2 = 0.2501$. Set $\mu = 100$ and $c = 500$, solving the inequalities (7) and (8), we get the following feasible solutions

$$P_1 = \begin{bmatrix} 6.0604 & 0.0533 & -1.2274 & 4.5442 \\ 0.0533 & 0.0062 & -0.0108 & 0.0388 \\ -1.2274 & -0.0108 & 1.6808 & -0.1346 \\ 4.5442 & 0.0388 & -0.1346 & 11.9906 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 34.8833 & -2.2935 & -12.5398 & -2.4343 \\ -2.2935 & 1.6693 & -8.9003 & 12.2477 \\ -12.5398 & -8.9003 & 74.6456 & -92.0940 \\ -2.4343 & 12.2477 & -92.0940 & 225.5706 \end{bmatrix}.$$

The feedback gain matrices in the dynamic consensus controller (2) can be chosen as

$$F = [-115.2230 \quad -13.3506 \quad 23.2852 \quad -83.9076],$$

$$H = [0.1211 \quad 0.7316 \quad 0.1213 \quad 0.0111]^T.$$

For the case without disturbances, Figures. 14-17 depict the trajectory graph of the states and Figures. 18-21 show the trajectory graph of the consensus error ξ . It is shown that the consensus error ξ converges to zero and the states of followers can track the states of the leader. That means that the system can also achieve leader-following consensus under the proposed output-feedback based consensus controller.

V. CONCLUSION

This paper has investigated the leaderless H_∞ consensus of higher-order Lipschitz nonlinear multi-agent systems with external disturbance under general directed topology. The only constraint on the topology structure is having a directed spanning tree, which is a quite general condition. By using relative outputs information, dynamic consensus controllers have been proposed. Sufficient conditions have been obtained, under which the Lipschitz multi-agent systems can achieve leaderless H_∞ consensus with a guaranteed H_∞ performance for a group of nonlinear agents subject to external disturbances. The key technology used here is the properties of Laplacian matrix in our earlier work.

However, the controller constructed in this article requires continuous communication. This brings a lot of communi-

cation burden. An interesting topic is to consider the H_∞ consensus problem with discontinuous communication.

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