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Finite Time Annular Domain Robust Stability Analysis and Controller Design for T-S Fuzzy Interval Positive Systems

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ABSTRACT This paper is concerned with the finite time annular domain robust stability (FTADRS) analysis and controller design for T-S fuzzy positive systems with interval uncertainties. The concept of finite time annular domain stability is first introduced for positive systems. Based on this and using the copositive Lyapunov function approach, some sufficient conditions for FTADRS are derived. Subsequently, the finite time annular domain robust controller is designed via the linear programming technique. Finally, two numerical examples and an application example are employed to show the effectiveness of our results.

INDEX TERMS Finite-time annular domain robust stability, T-S fuzzy positive systems, interval uncertainties, copositive Lyapunov function, linear programming.

I. INTRODUCTION

Positive systems, whose states always remain in the nonnegative orthant if its initial states and inputs are nonnegative, find wide applications in real-world life such as the pest natural enemy system based on the Lotka-Volterra model [1], the DC-DC buck converter [2], and the immune-tumor system [3]. As a result, positive systems have attracted considerable attentions in recent years, and many significant results have been reported in the literatures. To mention a few, stability problems were discussed in [4]–[7], the controller design issues were considered in [8]–[10], and the filtering problem was addressed in [11], [12]. It should be noted that the existing researches on positive systems are mainly concerned with the case of linear systems. Actually, most of positive systems possess more or less some nonlinear characteristics. For example, the energy converter system [2] and the tumor-immune system [3] do belong to the scope of nonlinear positive systems. Besides, in the real world, most of positive systems contain uncertainties, hence robustness issues should be concerned in the system analysis and synthesis.

In order to guarantee that system states do not exceed a specific upper limit in finite time, the concept of finite time stability (FTS) is proposed, and many important results have been reported [13]–[17]. However, in some special cases, it requires that system states do not only exceed a specific upper bound, but also do not fall below a specific lower bound in finite time. For example, it is always needed to take multiple medical measures for diabetes patients to keep their blood sugar levels within a safe range(i.e., 70-180 mg/dL) [18], [19]. If the blood glucose concentration is not in this range, it will lead to a series of serious complications and even death. To characterize the above dynamic behavior, a new FTS concept is proposed by Yan [20], which requires the states to be maintained in a given region with both upper and lower bounds in finite time. Such the new FTS concept is also referred to as the finite time annular domain stability (FTADS). Unlike the traditional FTS, the lower bound of system states is also constrained to a specific level in the definition of FTADS. Hence, new constraints need to be added to ensure that the system states do not fall below the lower bound, which often leads to difficulty in FTADS analysis and control synthesis. The concept of FTADS was first proposed for stochastic systems, and now it has been extended to several types of dynamic systems, such as stochastic Markov jump systems [21], [22], stochastic systems with Wiener and Poisson noises [23], [24], impulsive switched systems [25] and networked switched systems [26]. In fact, FTADS is

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also of great research significance for positive systems. For example, in the above-mentioned glucose-regulatory system, the blood sugar levels of diabetes patients are positive and must be maintained between 70-180 mg/dL [18], [19], and in the blood circulation system, the systolic blood pressure of a normal person is positive and must be maintained between 90-130mmHg. However, to the best of our knowledge, there is no result on the FTADS analysis and control synthesis of positive systems. So, it is desirable to further extend the concept of FTADS to positive systems. It should be noted that in the design of controllers, in order to ensure the positivity of closed-loop systems, new constraints need to be added. This will make it more difficult to deal with the nonlinear terms. Besides, nonlinear and uncertainty factors are also considered in system model, which make the traditional FTADS analysis approaches cannot be directly applied. Hence, developing new methods on the FTADS analysis and control synthesis of nonlinear positive uncertain systems is imperative, which motivates us to conduct this work.

Note that the T-S fuzzy model is a powerful tool to approximate complex nonlinear systems, and T-S fuzzy model-based methods have been widely used to analyze kinds of nonlinear systems, see, e.g., [27]–[30] and the references therein. As a result, the T-S model has also been employed to investigate nonlinear positive systems, and some significant results have been obtained, see for example, [31]–[40]. Among them, asymptotic/exponential stability/stabilization issues are the main concerns, e.g., [31]–[38], and only a few works considered the FTS issue, e.g., [39], [40]. Therefore, numerous successful applications of T-S fuzzy model-based methodology motivate us to employ the T-S model as a vehicle to analyze the FTADS of nonlinear positive systems.

In this paper, a new concept of FTADS is proposed for positive systems, and the FTADRS analysis and controller design for T-S fuzzy positive systems with interval uncertainties are respectively considered. The main contributions of the paper are highlighted as follows: 1) Different from the previous works on FTADS [20]–[26], the positivity of the system is considered, i.e., system states are always constrained in the nonnegative orthant. Besides, model uncertainty and nonlinear characteristics are also considered. Hence, the system model in this paper is more complex than [20]–[26]. 2) For positive systems, a new definition of FTADS is proposed. Compared with the traditional FTS definition of positive systems in [15], the lower bound of system states is fully considered in the new definition. Hence, our definition in this paper is more comprehensive. 3) By using copositive Lyapunov functions, the FTADS condition is established for T-S fuzzy positive systems with interval uncertainties. Moreover, the stabilization condition by state feedback controller is obtained. When the lower bound is 0 ($i.e.$, $\xi_1 = \xi_3 = 0$), the obtained conditions reduce to the traditional FTS form. Hence, our method is more general than those in [15].

The rest of this paper is organized as follows: Section II introduces some definitions and preliminaries. A sufficient condition of FTADRS is given in Section III. Section IV is devoted to state feedback controllers design. Two numerical examples and an application example are given in Section V. Section VI is the conclusion of this paper.

Notations: $\mathbf{1}$ denotes a column vector with all 1 elements. $A \in \mathcal{M}$: all elements of matrix A are nonnegative except its diagonal elements. $A \succeq 0$ ($A \succ 0$): all elements of matrix or vector A are nonnegative (positive). $b = \lceil a \rceil$: b is an integer and satisfies that $a \leq b < a + 1$. The "w.r.t." is an abbreviation of "with respect to".

II. PRELIMINARIES AND PROBLEM FORMULATION

Consider the following T-S fuzzy system with interval uncertainties:

Rule i : IF $\vartheta_1(t)$ is M_{i1}, $\vartheta_2(t)$ is M_{i2} and $\cdots$ and $\vartheta_l(t)$ is M_{il}, THEN

$$\dot{x}(t) = A_i x(t) + B_i u(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state of the system; $u(t) \in \mathbb{R}^s$ is the input of the system; $\vartheta_i(t)$ are the known premise variables and M_{ij} ($i = 1, 2, \cdots, r$, r is the number of model rules, $j = 1, 2, \cdots, l$, l is the number of premise variables) is fuzzy set, respectively. $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times s}$ are uncertain system matrices, but with known bounds, i.e.,

$$\underline{A}_i \preceq A_i \preceq \overline{A}_i, \tag{2}$$

$$\underline{B}_i \preceq B_i \preceq \overline{B}_i, \tag{3}$$

where $\overline{A}_i$, $\overline{B}_i$ and $\underline{A}_i$, $\underline{B}_i$ are the upper and lower bound matrices of A_i, B_i. By fuzzy blending, we can obtain the following overall T-S fuzzy model:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(\vartheta(t))(A_i x(t) + B_i u(t)), \tag{4}$$

where $\vartheta^T(t) = [\vartheta_1(t), \vartheta_2(t), \ldots, \vartheta_p(t)]$; $h_i(\vartheta(t))$ are the membership functions with satisfying

$$h_i(\vartheta(t)) \geq 0, \sum_{i=1}^{r} h_i(\vartheta(t)) = 1. \tag{5}$$

Definition 1 [41]: System (4) *is positive if, for any input $u(t) \succeq 0$ and $x_0 \succeq 0$, $x(t) \succeq 0$ holds for all $t > 0$.*

Lemma 1 [42]: System (4) *is said to be positive, if A_i are $\mathcal{M}$ matrices and $B_i \succeq 0$.*

Lemma 2: System (4) *is said to be positive, if $\underline{A}_i$ are $\mathcal{M}$ matrices and $\underline{B}_i \succeq 0$.*

proof: Since $\underline{A}_i$ are $\mathcal{M}$ matrices, its off-diagonal elements are all nonnegative. In view of $\underline{A}_i \preceq A_i$, it can be check that all off-diagonal elements of A_i are nonnegative, i.e., A_i are $\mathcal{M}$ matrices. Inequality (3) implies that $B_i \succeq \underline{B}_i \succeq 0$. Hence, by Lemma 1, system (4) is positive. □

Lemma 3 [43]: If there is a constant $\varpi > 0$ and a matrix A satisfying $A + \varpi I \succ 0$, then $A \in \mathcal{M}$.

Definition 2: For given positive scalars $\xi_1, \xi_2, \xi_3, \xi_4, T$ with $\xi_2 > \xi_4 > \xi_3 > \xi_1 \geq 0$, and a vector $\mathcal{R} \succ 0$, if

$$\xi_3 \leq x_0^T \mathcal{R} \leq \xi_4 \Rightarrow \xi_1 < x^T(t)\mathcal{R} < \xi_2, \ \forall t \in [0, T],$$

then positive system (4) *is FTADS w.r.t.* $(\xi_1, \xi_2, \xi_3, \xi_4, T, \mathcal{R})$.

Remark 1: The FTADS concept is originally developed for linear stochastic systems by Yan et al. in [20]. Definition 2 is a natural extension of the original FTADS and implies that the trajectory of $x^T(t)\mathcal{R}$ starting from $[\xi_3, \xi_4]$ cannot escape $[\xi_1, \xi_2]$. Different from previous definition of FTADS, the nonnegative characteristics of states are fully taken into account in Definition 2 and the selection of $\mathcal{R}$ changes from matrix to vector. Based on this new definition, the copositive Lyapunov function can be selected and the conditions of FTADS analysis and controller design in the form of linear programming are obtained, which can be solved by the linear programming technique.

Remark 2: It should be noted that Definition 2 reduces to the traditional FTS form associated with positive systems when $\xi_1 = \xi_3 = 0$, see, e.g., [15]. Hence, our Definition 2 is more flexible than the traditional FTS, and thus can potentially derive more general results.

Lemma 4 [20]: If there exist a nonnegative function $\mathfrak{g}(t)$ and nonnegative constants $\mathfrak{a}$, $\mathfrak{b}$ such that

$$\mathfrak{g}(t) \le \mathfrak{a} + \mathfrak{b}\int_0^t \mathfrak{g}(s)ds, \ \forall t \in [0, T]$$

holds, then

$$\mathfrak{g}(t) \le \mathfrak{a}e^{\mathfrak{b}t}, \ \forall t \in [0, T].$$

Lemma 5 [20]: If there exist a nonnegative function $\mathfrak{g}(t)$ and nonnegative constants $\mathfrak{a}$, $\mathfrak{b}$ such that

$$\mathfrak{g}(t) \ge \mathfrak{a} + \mathfrak{b}\int_0^t \mathfrak{g}(s)ds, \ \forall t \in [0, T]$$

holds, then

$$\mathfrak{g}(t) \ge \mathfrak{a}e^{\mathfrak{b}t}, \ \forall t \in [0, T].$$

III. FINITE-TIME ANNULAR DOMAIN ROBUST STABILITY ANALYSIS

In this section, a sufficient condition is given to test the FTADRS of positive system (4) when $u(t) = 0$.

Theorem 1: For constants $\alpha > 0$, $\beta \ge 0$, $\xi_2 > \xi_4 > \xi_3 \ge \xi_1 \ge 0$, and a vector $\mathcal{R} \succ 0$ with $\xi_4 \ge x_0^T\mathcal{R} \ge \xi_3$, if there exist a vector $\mathcal{P} \succ 0$ and positive constants ω_1, ω_2 such that

$$\overline{A}_i^T\mathcal{P} - \alpha\mathcal{P} \prec 0, \tag{6}$$

$$\beta\mathcal{P} - \underline{A}_i^T\mathcal{P} \prec 0, \tag{7}$$

$$\omega_1\mathcal{R} - \mathcal{P} \prec 0, \tag{8}$$

$$\mathcal{P} - \omega_2\mathcal{R} \prec 0, \tag{9}$$

$$\xi_4\omega_2 - \xi_2\omega_1 e^{-\alpha T} < 0, \tag{10}$$

$$\xi_1\omega 2 - \xi_3\omega_1 < 0, \tag{11}$$

hold, then system (4) is FTADRS w.r.t. $(\xi_1, \xi_2, \xi_3, \xi_4, T, \mathcal{R})$.

Proof: We use two steps to prove Theorem 1.

Step 1: $x_0^T\mathcal{R} \le \xi_4 \Rightarrow x^T(t)\mathcal{R} < \xi_2$.

Consider the following copositive Lyapunov function:

$$V(x(t)) = x^T(t)\mathcal{P}, \qquad \mathcal{P} \succ 0. \tag{12}$$

By (6), we can obtain

$$\begin{aligned}\dot{V}(x(t)) - \alpha V(x(t)) &= x^T(t)\sum_{i=1}^r h_i(\vartheta(t))A_i^T\mathcal{P} \\ &\quad - \alpha x^T(t)\mathcal{P} \\ &= x^T(t)\sum_{i=1}^r h_i(\vartheta(t))(A_i^T\mathcal{P} - \alpha\mathcal{P}) \\ &\le x^T(t)\sum_{i=1}^r h_i(\vartheta(t))(\overline{A}_i^T\mathcal{P} - \alpha\mathcal{P}) \\ &< 0. \end{aligned} \tag{13}$$

Multiplying both sides of (13) by $e^{-\alpha t}$ and integrating from 0 to t, one has

$$V(x(t)) < V(x(0)) + \alpha\int_0^t V(x(s))ds$$

By Lemma 4, we have:

$$V(x(t)) < V(x(0))e^{\alpha t}, \ \forall t \in [0, T]. \tag{14}$$

Considering (8) and (9), one has

$$\omega_1 x^T(t)\mathcal{R} < x^T(t)\mathcal{P} < \omega_2 x^T(t)\mathcal{R}, \ \forall t \in [0, T]. \tag{15}$$

By (12) and (15), the following inequality holds:

$$V(x(0))e^{\alpha t} = x_0^T\mathcal{P}e^{\alpha t} < \omega_2 x_0^T\mathcal{R}e^{\alpha T} \tag{16}$$

Considering (14)-(16), we have

$$x^T(t)\mathcal{R} < \frac{\omega_2}{\omega_1}x_0^T\mathcal{R}e^{\alpha T}$$

and

$$\frac{\omega_2}{\omega_1}\xi_4 e^{\alpha T} < \xi_2, \ \forall t \in [0, T]. \tag{17}$$

Then, by the fact of $x_0^T\mathcal{R} \le \xi_4$, one has $x^T(t)\mathcal{R} < \xi_2$, $\forall t \in [0, T]$.

Step 2: $\xi_3 \le x_0^T\mathcal{R} \Rightarrow \xi_1 < x^T(t)\mathcal{R}$.

By (7), we have

$$\begin{aligned}\dot{V}(x(t)) - \beta V(x(t)) &= x^T(t)\sum_{i=1}^r h_i(\vartheta(t))A_i^T\mathcal{P} \\ &\quad - \beta x^T(t)\mathcal{P} \\ &= x^T(t)\sum_{i=1}^r h_i(\vartheta(t))(A_i^T\mathcal{P} - \beta\mathcal{P}) \\ &\ge x^T(t)\sum_{i=1}^r h_i(\vartheta(t))(\underline{A}_i^T\mathcal{P} - \beta\mathcal{P}) \\ &> 0. \end{aligned} \tag{18}$$

Multiplying both sides of (18) by $e^{-\beta t}$ and integrating from 0 to t, one has

$$V(x(t)) > V(x(0)) + \beta\int_0^t V(x(s))ds. \tag{19}$$

According to Lemma 5, we can obtain:

$$V(x(t)) > V(x(0))e^{\beta t}. \tag{20}$$

By (12) and (15), one has

$$V(x(0))e^{\beta t} = x_0^T \mathcal{P} e^{\beta t} > \omega_1 x_0^T \mathcal{R}. \tag{21}$$

Considering (11) (15) (20) and (21), we have

$$\xi_1 < \xi_3 \frac{\omega_1}{\omega_2}$$

and

$$x_0^T \mathcal{R} \frac{\omega_1}{\omega_2} < x^T(t)\mathcal{R}, \ \forall t \in [0, T]. \tag{22}$$

Then, by the fact of $x_0^T \mathcal{R} \geq \xi_3$, one has $x^T(t)\mathcal{R} > \xi_1$, $\forall t \in [0, T]$.

Hence, combining the above two steps, we obtain that

$$\xi_3 \leq x_0^T \mathcal{R} \leq \xi_4 \Rightarrow \xi_1 < x^T(t)\mathcal{R} < \xi_2, \ \forall t \in [0, T].$$

By Definition 2, system (4) is FTADRS w.r.t. $(\xi_1, \xi_2, \xi_3, \xi_4, T, \mathcal{R})$. □

Remark 3: It should be noted that the values of α and β may have an effect on the feasibility of the FTADRS criterion (i.e., Theorem 1). Therefore, the values of α and β must be appropriately selected. Considering (2) *and* (6)-(7)*, we have $\beta\mathcal{P} \prec \underline{A}_i^T \mathcal{P} \prec \overline{A}_i^T \mathcal{P} \prec \alpha\mathcal{P}$. Then, by* (10) *and* (11)*, $0 \leq \beta < \alpha < \frac{1}{T} ln \frac{\xi_2\xi_3}{\xi_1\xi_4}$ is obtain. In other words, the values of α and β are always limited in $[0, \frac{1}{T} ln \frac{\xi_2\xi_3}{\xi_1\xi_4})$.*

Remark 4: To calculate a more accurate feasible range of α and β, a grid search algorithm is designed as shown in Figure 1. In Figure 1, α_1 and β_1 are the initial values of α and β, and they can be set as zeros. l_1 and l_2 are the given step sizes. n and m are the maximum numbers of iterations respectively for α and β, and they can be set as $n = \lceil \frac{ln \frac{\xi_2\xi_3}{\xi_1\xi_4}}{Tl_1} \rceil$ and $m = \lceil \frac{ln \frac{\xi_2\xi_3}{\xi_1\xi_4}}{Tl_2} \rceil$ according to Remark 3.

When A_i and B_i are known matrices (i.e., without considering uncertainties), Theorem 1 reduces to the following corollary.

Corollary 1: For constants $\alpha > 0, \beta \geq 0, \beta \geq 0, \xi_2 > \xi_4 > \xi_3 \geq \xi_1 \geq 0$, and a vector $\mathcal{R} \succ 0$ with $\xi_4 \geq x_0^T \mathcal{R} \geq \xi_3$, if there exist a vector $\mathcal{P} \succ 0$ and positive constants ω_1, ω_2 such that (8)-(11) *and the following inequalities hold:*

$$A_i^T \mathcal{P} - \alpha\mathcal{P} \prec 0, \tag{23}$$

$$\beta\mathcal{P} - A_i^T \mathcal{P} \prec 0, \tag{24}$$

then system (4) *is FTADS w.r.t.* $(\xi_1, \xi_2, \xi_3, \xi_4, T, \mathcal{R})$.

Proof: Letting $\overline{A}_i = \underline{A}_i$, inequalities (6) and (7) immediately reduce to (23) and (24). □

Especially in the absence of uncertainties and $\xi_1 = \xi_3 = 0$, Theorem 1 reduces to the traditional FTS criterion, i.e., the following Corollary 2.

Corollary 2: For positive constants $\alpha, \xi_2 > \xi_4 > 0$, and a vector $\mathcal{R} \succ 0$ with $\xi_4 \geq x_0^T \mathcal{R}$, if there exist a vector $\mathcal{P} \succ 0$ and positive constants ω_1, ω_2 such that (8)-(10) *and* (23) *hold, then system* (4) *is FTS w.r.t.* $(\xi_2, \xi_4, T, \mathcal{R})$.

Proof: Letting $\overline{A}_i = \underline{A}_i$ and referring to the proof process of Step 1 in Theorem 1, we get that

$$x_0^T \mathcal{R} \leq \xi_4 \Rightarrow x^T(t)\mathcal{R} < \xi_2, \ \forall t \in [0, T].$$

Then, positive system (4) is FTS w.r.t. $(\xi_2, \xi_4, T, \mathcal{R})$ □

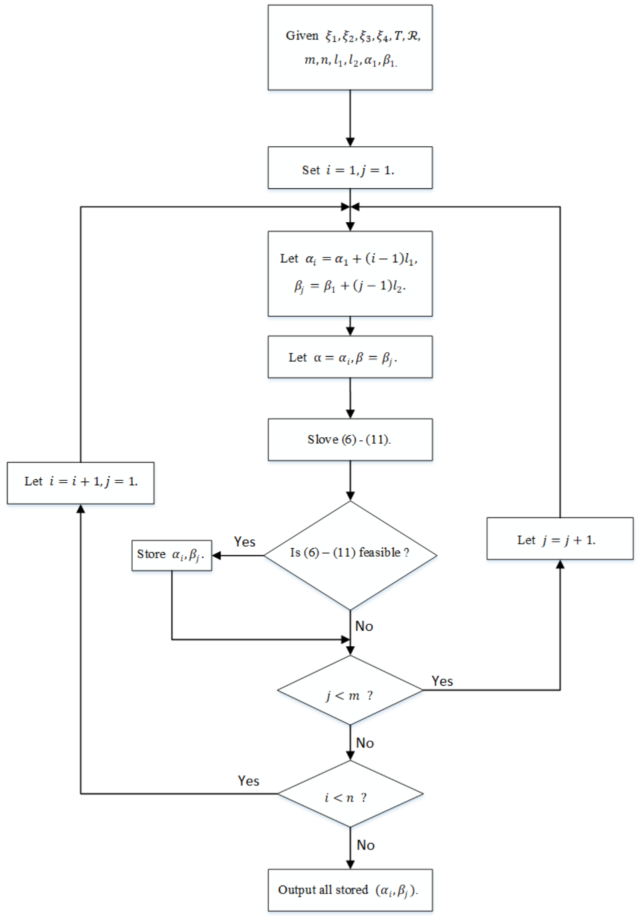

FIGURE 1. An algorithm to obtain the feasible region of α and β.

IV. CONTROLLER DESIGN

In this section, based on the above stability result, a state feedback controller is designed for positive system (4).

Consider the following state feedback controller:

$$u(t) = \sum_{j=1}^{r} h_j(\vartheta(t)) K_j x(t),$$

where $K_j \in \mathbb{R}^{s \times n}$. Then, the closed-loop system is obtained as follow:

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\vartheta(t)) h_j(\vartheta(t))(A_i + B_i K_j)x(t), \tag{25}$$

The controller gain matrices K_j can be obtained by the following theorem.

Theorem 2: For constants $\alpha > 0, \beta \geq 0, \xi_2 > \xi_4 > \xi_3 \geq \xi_1 \geq 0$, and a vector $\mathcal{R} \succ 0$ with $\xi_4 \geq x_0^T \mathcal{R} \geq \xi_3$, if there exist positive constants ω_1, ω_2, h, vectors $\mathcal{P} \succ 0$, O_{ij}, N_{ij} and matrices $Q_j \preceq 0$ such that (8)-(11) *and the following inequalities hold:*

$$\overline{A}_i^T \mathcal{P} - \alpha\mathcal{P} + O_{ij} \prec 0, \tag{26}$$

$$\beta\mathcal{P} - \underline{A}_i^T \mathcal{P} - N_{ij} \prec 0, \tag{27}$$

$$Q_j^T \underline{B}_i^T - O_{ij} 1^T \underline{B}_j^T \preceq 0, \tag{28}$$
$$N_{ij} 1^T \underline{B}_j^T - Q_j^T \overline{B}_i^T \preceq 0, \tag{29}$$
$$(1^T \underline{B}_j^T \mathcal{P})\underline{A}_i + \overline{B}_i Q_j + hI \succeq 0, \tag{30}$$

then closed-loop system (25) *is positive and FTADRS w.r.t.* $(\xi_1, \xi_2, \xi_3, \xi_4, T, \mathcal{R})$.

In this case, the controller gain matrices can be obtained as:

$$K_j = \frac{Q_j}{1^T \underline{B}_j^T \mathcal{P}}. \tag{31}$$

Proof: Considering (30), we have

$$\underline{A}_i + \overline{B}_i K_j + \frac{h}{1^T \underline{B}_j^T \mathcal{P}} I \succeq 0. \tag{32}$$

Then, by Lemma 3, $\underline{A}_i + \overline{B}_i K_j$ are $\mathcal{M}$ matrices. With the fact that $Q_j \preceq 0$, we obtain

$$\underline{A}_i + \overline{B}_i K_j \preceq A_i + B_i K_j.$$

Hence, by Lemma 2, it is easy to prove that system (25) is positive.

It follows from (28) and (29) that

$$K_j^T \underline{B}_i^T \mathcal{P} \preceq O_{ij} \tag{33}$$

and

$$K_j^T \overline{B}_i^T \mathcal{P} \succeq N_{ij}. \tag{34}$$

By (33) and (34), one has

$$N_{ij} \preceq K_j^T B_i^T \mathcal{P} \preceq O_{ij}. \tag{35}$$

Inequalities (26) (27) and (35) imply that

$$(A_i + B_i K_j)^T \mathcal{P} - \alpha \mathcal{P} \preceq \overline{A}_i^T \mathcal{P} - \alpha \mathcal{P} + O_{ij} \prec 0 \tag{36}$$

and

$$\beta \mathcal{P} - (A_i + B_i K_j)^T \mathcal{P} \preceq \beta \mathcal{P} - \underline{A}_i^T \mathcal{P} - N_{ij} \prec 0. \tag{37}$$

By (36) and (37), one has $\dot{V}(x(t)) - \alpha V(x(t)) < 0$ and $\dot{V}(x(t)) - \beta V(x(t)) > 0$. Then, according to the proof process of (13) - (17) and (18) - (22) in Theorem 1, we can obtain that

$$\xi_3 \le x_0^T \mathcal{R} \le \xi_4 \Rightarrow \xi_1 < x^T(t)\mathcal{R} < \xi_2, \ \forall t \in [0, T].$$

By Definition 2, the closed-loop system (25) is FTADRS w.r.t. $(\xi_1, \xi_2, \xi_3, \xi_4, T, \mathcal{R})$. □

Remark 5: In Theorem 2, we assume that $Q_j \preceq 0$, which leads to the designed controller $K_j \preceq 0$. If this assumption is not satisfied, inequalities (30)-(35) *are not necessarily true, which will bring great difficulties to the controller design for the closed-loop system* (25)*. It should be noted that if the uncertainties are not considered, this assumption can be removed, see e.g., Corollary 3. The controller design problem of symbolic indefinite Q_j has not been solved and needs further research.*

Especially in the absence of uncertainties, Theorem 2 reduces to the following form.

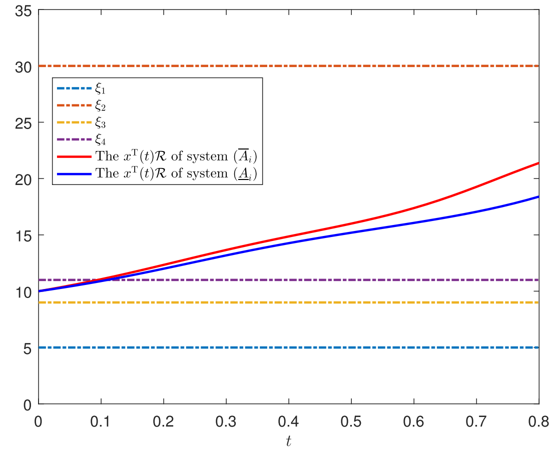

FIGURE 2. The $x^T(t)\mathcal{R}$ of open loop system in Example 1.

Corollary 3: For constants $\alpha > 0$, $\beta \ge 0$, $\xi_2 > \xi_4 > \xi_3 \ge \xi_1 \ge 0$, and a vector $\mathcal{R} \succ 0$ with $\xi_4 \ge x_0^T \mathcal{R} \ge \xi_3$, if there exist positive constants ω_1, ω_2, h, vectors $\mathcal{P} \succ 0$, O_{ij}, N_{ij} and matrices Q_j such that (8)-(11) *and the following inequalities hold:*

$$A_i^T \mathcal{P} - \alpha \mathcal{P} + O_{ij} \prec 0, \tag{38}$$
$$\beta \mathcal{P} - A_i^T \mathcal{P} - N_{ij} \prec 0, \tag{39}$$
$$Q_j^T B_i^T - O_{ij} 1^T B_j^T \preceq 0, \tag{40}$$
$$N_{ij} 1^T B_j^T - Q_j^T B_i^T \preceq 0, \tag{41}$$
$$(1^T B_j^T \mathcal{P}) A_i + B_i Q_j + hI \succeq 0, \tag{42}$$

then closed-loop system (25) *is positive and FTADS w.r.t.* $(\xi_1, \xi_2, \xi_3, \xi_4, T, \mathcal{R})$.

In this case, controller gain matrices can be obtained as:

$$K_j = \frac{Q_j}{1^T B_j^T \mathcal{P}}. \tag{43}$$

Proof; Letting $\overline{A}_i = \underline{A}_i$ and $\overline{B}_i = \underline{B}_i$, inequalities (26)-(30) reduce to (38)-(42). Then, Corollary 3 can be obtained by Theorem 2. □

In particular, in the absence of uncertainties and $\xi_1 = \xi_3 = 0$, Theorem 2 reduces to the following corollary.

Corollary 4: For positive constants α, $\xi_2 > \xi_4 > 0$, and a vector $\mathcal{R} \succ 0$ with $\xi_4 \ge x_0^T \mathcal{R}$, if there exist positive constants ω_1, ω_2, h, vectors $\mathcal{P} \succ 0$, O_{ij} and matrices Q_j such that (8)-(10) (38) (40) *and* (42) *hold, then closed-loop system* (25) *is positive and FTS w.r.t.* $(\xi_2, \xi_4, T, \mathcal{R})$.

In this case, K_j can also be obtained by (43).

Proof: Letting $\overline{A}_i = \underline{A}_i$, $\overline{B}_i = \underline{B}_i$ and referring to (32), we obtain that the closed-loop system (25) is positive.

By (33) and (36), we have $\dot{V}(x(t)) - \alpha V(x(t)) < 0$. Then, similar to the proof process of Step 1 in Theorem 1, the closed-loop system (25) is FTS w.r.t. $(\xi_2, \xi_4, T, \mathcal{R})$. □

V. EXAMPLES

In this section, we use two numerical examples and an application example to illustrate the above results.

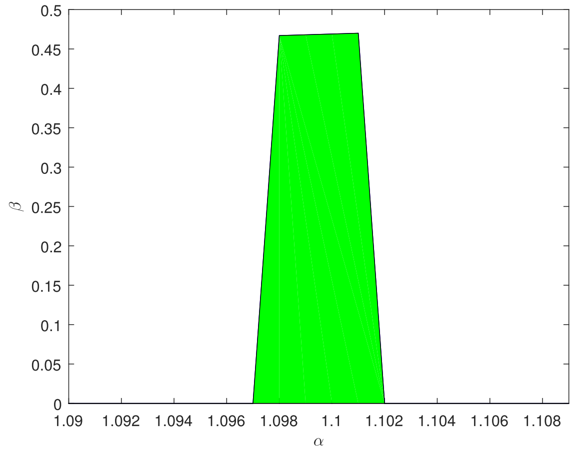

FIGURE 3. A feasible region of α and β in Example 1.

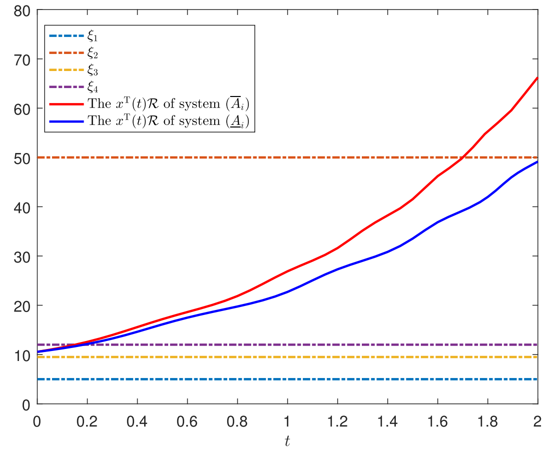

FIGURE 4. The $x^T(t)\mathcal{R}$ of open loop system in Example 2.

A. EXAMPLE 1

Consider system (4) with

$$\underline{A}_1 = \begin{bmatrix} -1.2 & 2.8 \\ 1.3 & -0.7 \end{bmatrix}, \quad \underline{A}_2 = \begin{bmatrix} -1.3 & 2.5 \\ 1 & -0.8 \end{bmatrix},$$
$$\overline{A}_1 = \begin{bmatrix} -1.2 & 3 \\ 1.3 & -0.6 \end{bmatrix}, \quad \overline{A}_2 = \begin{bmatrix} -1 & 2.5 \\ 1 & -0.7 \end{bmatrix},$$
$$h_1(\vartheta(t)) = 1 - cos^2 x_1(t), h_2(\vartheta(t)) = cos^2 x_1(t). \tag{44}$$

By Lemma 2, it is easy to prove that system (44) is positive. Given $\xi_1 = 5, \xi_2 = 30, \xi_3 = 9, \xi_4 = 11, T = 0.8, \mathcal{R} = [1,\ 2]^T$ and $x_0 = [4,\ 3]^T$, then we obtain the trajectory of $x^T(t)\mathcal{R}$, as shown in Figure 2. From Figure 2, we can find that the $x^T(t)\mathcal{R}$ of system $(\underline{A}_i)$ is not lower than ξ_1 and the $x^T(t)\mathcal{R}$ of system $(\overline{A}_i)$ is not higher than ξ_2. Hence, positive system (44) is FTADRS w.r.t. $(5, 30, 9, 11, 0.8, [1,\ 2]^T)$.

By using the algorithm in Figure 1, we obtain a feasible region of α and β, as shown in Figure 3. According to the region, selecting $\alpha = 1.1$, $\beta = 0.2$ and solving inequalities (6)-(11), we obtain $\omega_1 = 82.0003 > 0$, $\omega_2 = 92.7252 > 0$ and $\mathcal{P} = [92.7185,\ 164.0176]^T \succ 0$. This coincides with the simulation result.

B. EXAMPLE 2

Consider system (4) with

$$\underline{A}_1 = \begin{bmatrix} -1.2 & 2.9 \\ 1.3 & -0.7 \end{bmatrix}, \quad \underline{A}_2 = \begin{bmatrix} -1.2 & 2.4 \\ 1 & -0.8 \end{bmatrix},$$
$$\overline{A}_1 = \begin{bmatrix} -1.2 & 3 \\ 1.3 & -0.6 \end{bmatrix}, \quad \overline{A}_2 = \begin{bmatrix} -1 & 2.5 \\ 1 & -0.7 \end{bmatrix},$$
$$\underline{B}_1 = \begin{bmatrix} 1.4 \\ 0.8 \end{bmatrix}, \quad \underline{B}_2 = \begin{bmatrix} 0.85 \\ 0.3 \end{bmatrix},$$
$$\overline{B}_1 = \begin{bmatrix} 1.5 \\ 0.8 \end{bmatrix}, \quad \overline{B}_2 = \begin{bmatrix} 1 \\ 0.4 \end{bmatrix},$$
$$h_1(\vartheta(t)) = 1 - cos^2 x_1(t), h_2(\vartheta(t)) = cos^2 x_1(t). \tag{45}$$

By Lemma 2, it is easy to prove that system (45) is positive. Let $\xi_1 = 5, \xi_2 = 50, \xi_3 = 9.5, \xi_4 = 12, T = 2, \mathcal{R} = [1.3,\ 2.4]^T$ and $x_0 = [3.5,\ 2.5]^T$. The trajectory of $x^T(t)\mathcal{R}$ for system (45) with open loop is shown in Figure 4. From Figure 4, we can see that system (45) is not FTADRS w.r.t. $(5, 50, 9.5, 12, , 2, [1.3,\ 2.4]^T)$. Next, we design a FTADRS controller for system (45).

By the algorithm in Figure 1, we obtain a feasible region of α and β, as shown in Figure 5. Choosing $\alpha = 0.6$, $\beta = 0.1$ and solving (8)-(11) and (26)-(30), we get

$$\mathcal{P} = \begin{bmatrix} 94.3293 \\ 166.8417 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} -56.9199 & -100.2082 \end{bmatrix},$$
$$Q_2 = \begin{bmatrix} -31.3406 - 51.3405 \end{bmatrix}, \quad \omega_1 = 65.2718,$$
$$\omega_2 = 78.1349.$$

Then, the controller gain matrices are obtained as:

$$K_1 = \frac{Q_1}{1^T \underline{B}_1^T \mathcal{P}} = \begin{bmatrix} -0.2144 & -0.3774 \end{bmatrix},$$
$$K_2 = \frac{Q_2}{1^T \underline{B}_2^T \mathcal{P}} = \begin{bmatrix} -0.2407 & -0.3942 \end{bmatrix}. \tag{46}$$

By Lemma 2, it can be easily verified that the closed-loop system is positive. Figure 6 shows the trajectory of $x^T(t)\mathcal{R}$ for (45) under the designed controller (46), from which we can see that the closed-loop system is FTADRS w.r.t. $(5, 50, 9.5, 12, 2, [2.3,\ 2.4]^T)$.

C. EXAMPLE 3

Consider the following pest-natural enemy system based on the Lotka-Volterra model [1]:

$$\begin{cases} \dot{x}_1(t) = x_1(t)(\rho_1 - \rho_2 x_2(t)) + u(t), \\ \dot{x}_2(t) = x_2(t)(\rho_3 x_1(t) - \rho_4), \end{cases} \tag{47}$$

where $x_1(t)$ and $x_2(t)$ are the population densities of insect pest and natural enemy. ρ_1, ρ_2, ρ_3 and ρ_4 are all positive parameters, and one may refer to [1] for more details.

For the convenience of stability analysis and controller design, we establish a T-S fuzzy model for system (47). Let premise variable $\vartheta(t) \triangleq x_2(t)$, and define $x^T(t) \triangleq [x_1(t), x_2(t)]$, $\vartheta_{max} \triangleq max\{x_2(t)\}$ and $\vartheta_{min} \triangleq min\{x_2(t)\}$.

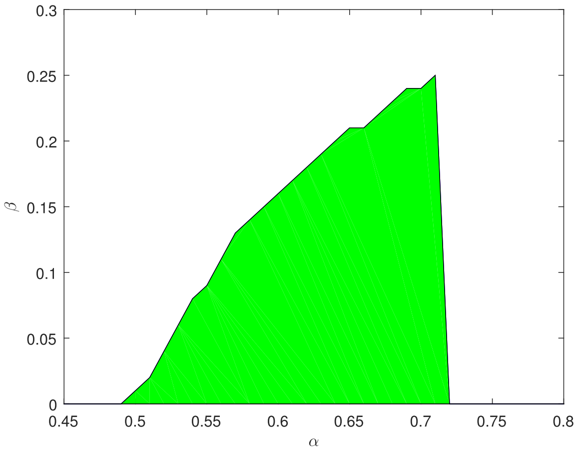

FIGURE 5. The feasible region of α and β in Example 2.

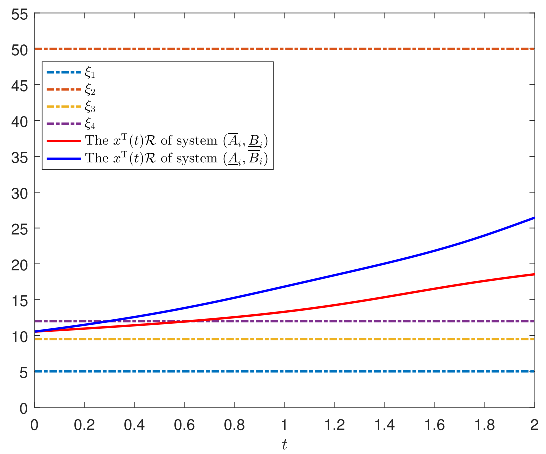

FIGURE 6. The $x^T(t)\mathcal{R}$ of closed-loop system under state feedback controller in Example 2.

Then, system (47) can be approximated by a T-S model with three rules:

IF $\vartheta(t)$ is ϑ_{min}, THEN

$$\dot{x}(t) = A_1x(t) + B_1u(t).$$

IF $\vartheta(t)$ is $\frac{\vartheta_{max}+\vartheta_{min}}{2}$, THEN

$$\dot{x}(t) = A_2x(t) + B_2u(t),$$

IF $\vartheta(t)$ is ϑ_{max}, THEN

$$\dot{x}(t) = A_3x(t) + B_3u(t),$$

where

$$A_1 = \begin{bmatrix} \rho_1 - \rho_2\vartheta_{min} & 0 \\ \rho_3\vartheta_{min} & -\rho_4 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} \rho_1 - \rho_2\frac{\vartheta_{max}+\vartheta_{min}}{2} & 0 \\ \rho_3\frac{\vartheta_{max}+\vartheta_{min}}{2} & -\rho_4 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} \rho_1 - \rho_2\vartheta_{max} & 0 \\ \rho_3\vartheta_{max} & -\rho_4 \end{bmatrix},$$
$$B_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad i = 1, 2, 3.$$

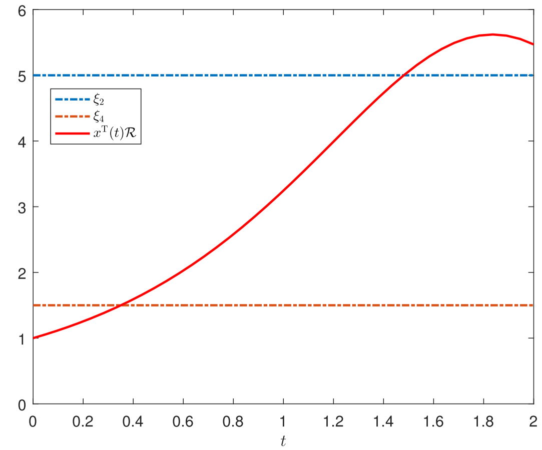

FIGURE 7. The $x^T(t)\mathcal{R}$ of open loop system (47) in Example 3.

Selecting the following membership functions: $h_1(\vartheta(t)) = max\{1-\frac{2}{\vartheta_{max}+\vartheta_{min}}x_2(t), 0\}$, $h_3(\vartheta(t)) = max\{\frac{2}{\vartheta_{max}+\vartheta_{min}}x_2(t)-1, 0\}$, $h_2(\vartheta(t)) = 1-h_1(\vartheta(t))-h_3(\vartheta(t))$, the overall T-S fuzzy model can be written as:

$$\dot{x}(t) = \sum_{i=1}^{3} h_i(\vartheta(t))(A_ix(t) + B_iu(t)). \tag{48}$$

Let $\rho_1 = 4.9$, $\rho_2 = 1.9$, $\rho_3 = 0.11$, $\rho_4 = 0.21$, $\vartheta_{min} = 0$, $\vartheta_{max} = 10$ and $x_0 = [1, \ 2]^T$. By Lemma 1, it is easy to prove that system (48) is positive. We assume that the desired population density of insect pest is $x_1(t) < 5, \forall t \in [0, 2]$. For the convenience of simulation, let $\xi_2 = 5$, $\xi_4 = 1.5$, $T = 2$ and $\mathcal{R} = [1, \tau]^T$, where τ is a positive number and small enough. Hence, the population density of insect pest can be denoted by $x^T(t)\mathcal{R}$ since $x^T(t)\mathcal{R} \approx x_1(t)$. Given $\tau = 0.000001$, the trajectory of $x^T(t)\mathcal{R}$ for (48) can be obtained, as shown in Figure 7. From Figure 7, we can find that the population density of insect pest is not within the desired range. Next, we design a FTS controller for system (48) to control the population density of insect pest to satisfy that $x_1(t) < 5, \forall t \in [0, 2]$. Selecting $\alpha = 0.4$ and solving Corollary 4, we get

$$\mathcal{P} = \begin{bmatrix} 6.7261 \\ 0.00001 \end{bmatrix}, \quad \mathcal{Q}_1 = \mathcal{Q}_2 = \mathcal{Q}_3 = \begin{bmatrix} -147.4021 & 0 \end{bmatrix},$$
$$\omega_1 = 6.6882, \quad \omega_2 = 9.9649.$$

Then, the controller gain matrices are obtained as:

$$K_1 = K_2 = K_3 = \frac{\mathcal{Q}_1}{1^TB_1^T\mathcal{P}} = \begin{bmatrix} -21.9151 & 0 \end{bmatrix}.$$

By Lemma 1, it is easy to prove that the closed-loop system is positive. The evolution of insect pest population density under the designed controller is shown in Figure 8. From Figure 8, we can find that the population density of insect pest is effectively controlled within the desired range by the designed controller.

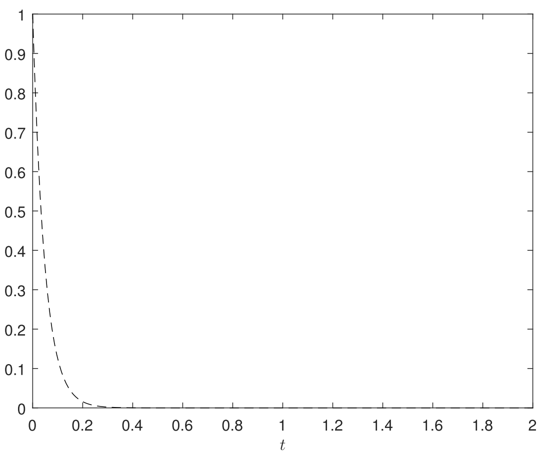

FIGURE 8. **The population density curve of insect pest under the designed controller in Example 3.**

VI. CONCLUSION

In this study, the problems of FTADRS analysis and controller design for T-S fuzzy positive systems with interval uncertainties have been investigated. The concept of FTADS has been introduced for positive systems for the first time. Based on this, the FTADRS analysis for uncertain T-S fuzzy positive systems has been addressed by using the copositive Lyapunov function method. Moreover, the state feedback controller has been designed, which can guarantee the closed-loop systems to be positive and FTADRS. At the end, several examples have illustrated the effectiveness of the proposed results. In the future work, FTADRS analysis for positive systems with time delays and external disturbances will be considered.

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