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Non-Fragile Fault-Tolerant Control Design for Fractional-Order Nonlinear Systems With Distributed Delays and Fractional Parametric Uncertainties

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ABSTRACT This work discuss the stabilization issue for a class of fractional-order nonlinear systems together with time delay, parametric uncertainties and actuator faults. Precisely, the considered system comprises of two delays namely distributed delay and time-varying delay. Moreover, the occurrence of the actuator faults and fractional parametric uncertainties may induce poor performance of the systems. To overcome these issue, a non-fragile fault-tolerant controller is designed which makes the system asymptotically stable with the specified mixed H_∞ and passive performance index. A fractional Razumikhin theorem is applied to handle the distributed delay term in the stabilization analysis. With the aid of suitable Lyapunov-Krasovskii functional, the sufficient conditions are established in terms of linear matrix inequalities together with Razumikhin stability theorem for getting the required results. By virtue of this, the controller gain matrix is obtained by solving the obtained LMIs and the graphical results are simulated using FOMCON toolbox. Later, the potency of the developed results are validated by virtue of three numerical examples including a rocket motor chamber.

INDEX TERMS Fractional-order nonlinear systems, distributed delay, reliable controller, gain perturbation, fractional uncertainties, mixed H_∞, passive performance.

I. INTRODUCTION

The concept of fractional calculus has been demanding considerable attention in recent years. This is primarily due to the long-range memory property and the historical dependence. Specifically, the fractional-order (FO) dynamical systems are more accurate than the integer-order systems, hence the stability and stabilization analysis of FO systems are more intricate than the systems of integer-order. Despite their complexity, fractional-order systems have acquired significant attention due to their remarkable applications in the field of control engineering and several works have been presented in literature [1]–[5]. For instance, the robust stabilization problem for FO systems under interval uncertainties is discussed in [1] and [2]. An adaptive observer-based design for a nonlinear FO system is given in [3]. The authors in [4] and [5] studied the robust dissipativity-based control problem for uncertain nonlinear FO systems via output feedback and state feedback, respectively. Generally, most of the practical applications might not be linear in nature. Therefore, it is of utmost importance to consider the nonlinearity in the fractional-order control systems upon studying their stability and stabilization analysis [6]–[10]. In [6], a nonlinear FO system with time-delays is considered and the stability is achieved via LMI-based design through observer-based control problem. The authors in [9] have studied the stability analysis by developing sufficient conditions for FO nonlinear systems with time-varying delay.

Moreover, time-delays are ineluctable during the application of the FO nonlinear control systems in real-world processes. In general, there exist constant and time-varying delays that represent the time-lag during transmission,

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processing and so on. Such presence of delays may influence the qualitative properties of the system and may affect the stability of the control systems. Hence, the study on fractional-order nonlinear control systems with delays is significant. In addition, the propagation delays are distributed over a period of time and may influence the control system to oscillate more. Hence, it is mandatory to investigate the stability and stabilizability of control systems with time-dependent and distributed delay. The fractioning and partitioning nature of distributed delays has been attracting the researchers and a great number of noteworthy results on integer-order systems with distributed-delayed have been obtained [11], [12]. However, in fractional-order systems, the presence of an integral term in the system poses a difficulty in calculating the fractional derivative of the Lyapunov function. Recently, this problem demands considerable attention among the researchers and hence few works on stability and stabilization of fractional-order distributed delay system is discussed in [14]–[18]. However, in this work, the integral term in the system state will be dealt by virtue of LMIs with the help of Cauchy matrix inequality and Razumikhin stability theorem.

In addition to the time-delays, another concerning issue regarding the stabilization of FO control systems is actuator failure. The control systems may undergo unexpected changes due to the failure of the actuators during the transmission of signals that may cause instability or degrade the system performance. Hence, it is important and necessary to construct a suitable controller that could be tolerant to faults. As a result, there are so many works that exhibit the prominence of fault-tolerant controller [19]–[21]. In [19], the problem of fault occurrence in T-S fuzzy fractional-order systems is handled by considering a robust H_∞ adaptive sliding mode fault-tolerant controller. On another research front, the fragility of the designed controller may be caused by certain variations in its gain parameters which may deteriorate the controller performance. Some attempts have been made to tackle this variation by considering a non-fragile controller in [22]–[24]. In [23], a resilient controller is devised to overcome the effect of gain perturbations for a class of FO linear delayed systems. The authors in [24] discussed the issue of perturbation in gain matrices by designing a non-fragile passification controller for a class of FO nonlinear systems in conjunction with time-dependent uncertainties with the bounded norm.

At the same time, the system may face difficulties in the form of uncertainties due to inaccurate modelling and approximations etc, hence it is necessary to deal with them during modelling of the system. In particular, the fractional parametric uncertainties are more general than the norm bounded uncertainties and are useful in modelling the changes caused by the environment, identification errors and so on [25]–[27]. The authors in [25] obtained the non-fragile fault detection filter design for delayed singular Markovian jump systems with the presence of linear fractional parameter uncertainties. Over the past few decades, H_∞ control is considered to be an effective approach for disturbance attenuation. Additionally, the passivity theory has also been implemented for some complex control systems for disturbance attenuation. From the practical point of view, some system requires either one of these performances at a particular point of time. To facilitate this problem, a unified mixed H_∞ and passive performance is considered and is widely applied in several works of literature for dealing with the disturbances [28]–[30].

On the other hand, the Lyapunov functional-based approach is applied for the study on the stability of the system under both integer and fractional-order [23], [27]. Among which, the construction of Lyapunov functional and finding its derivative plays a major role in the stability and stabilization of control systems. Nevertheless, the product rule and Leibnitz rule will not hold similar to the case of integer-order ones for fractional derivatives. Thus, in order to examine the stability and stabilization of control systems, the derivative of Lyapunov functions in the fractional sense is intricate. Motivated by the aforementioned facts, it is interesting to develop a non-fragile fault-tolerant (NFT) controller for a class of fractional-order nonlinear systems with time-varying delay, distributed delay, fractional parametric uncertainties and disturbances. Hence, we develop the NFT controller for the above-said system with mixed H_∞ and passive performance index $\hat{\gamma}$. Furthermore, the contribution of this work is listed as:

- Stabilization problem of FO nonlinear systems with time-varying delay, distributed delay, fractional parametric uncertainties and exogenous disturbances is formulated.
- On the account of actuator faults and gain perturbations, an NFT controller is devised.
- The distributed delay is considered in the form of an integral term in the system state, which will be solved in the way of linear matrix inequalities along with Cauchy matrix inequality and Razumikhin stability theorem.
- Specifically with Lyapunov stability theory, the LMIs are obtained to attain the asymptotical stability with mixed H_∞ and passive disturbance attenuation.
- Finally, the theoretical results are verified by presenting three numerical examples including the practical application of a system of rocket motor chambers. The last example validates the proposed work in comparison with the conventional state feedback control method.

II. PRELIMINARIES AND SYSTEM DESCRIPTION

In this section, let us address the stabilization problem of the FO nonlinear control system in terms of Caputo fractional derivative. The prime motive for considering the fractional derivatives is that it gathers the entire information of the function, unlike the integer-order derivatives which focus only on the adjacent points. On this account, we present the Caputo fractional derivative definition for a function $f(t)$ [2] with fractional-order α as

$$ {}^{C}D_{t_0}^{\alpha} f(t) = \mathcal{I}_{t_0}^{q-\alpha} \frac{d^q}{dt^q} f(t) $$

$$= \frac{1}{\Gamma(q-\alpha)} \int_{t_0}^{t} (t-s)^{q-\alpha-1} f^{(q)}(s)\, ds, \quad (1)$$

where q is a positive integer with the constraint $0 \leq q - 1 \leq \alpha < q$; $\mathcal{I}_{t_0}^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t-s)^{\alpha-1} f(s)\, ds$ and $\Gamma(.)$ is a gamma function. From now, we adopt D^{α} to notate the Caputo derivative for convenience.

Now, let us analyse the FO uncertain nonlinear control system in accordance with time-varying and distributed delays as follows

$$\begin{cases} D^{\alpha} \mathrm{x}(t) = \bar{\mathrm{A}}\mathrm{x}(t) + \bar{\mathrm{A}}_{\tau} \mathrm{x}(t-\tau(t)) + \bar{\mathrm{A}}_{d} \int_{t-d}^{t} \mathrm{x}(s) ds \\ \qquad\qquad + h(\mathrm{x}(t)) + \bar{\mathrm{B}} u^{F}(t) + \mathrm{B}_{w} \mathfrak{d}(t), \\ \mathrm{y}(t) = \mathrm{C}\mathrm{x}(t), \\ x(t) = \varphi(t), \quad t \in [-\tau, 0], \end{cases} \quad (2)$$

where $\mathrm{x}(t) \in \mathbb{R}^{n}$, $\mathrm{y}(t) \in \mathbb{R}^{m}$ and $u^{F}(t) \in \mathbb{R}^{p}$ implies the vectors of state, output and faulty control input respectively, $h(\mathrm{x}(t))$ is a nonlinear function with respect to the current state $\mathrm{x}(t)$, $\mathfrak{d}(t) \in L_2[0, \infty)$ denotes the disturbance function and $\varphi(t)$ is a function that is continuously differentiable representing the initial condition on $t \in [-\tau, 0]$. $\tau(t)$ is the time-dependent delay function sufficed by the constraint $0 \leq \tau(t) \leq \tau$ and $\dot{\tau}(t) \leq \bar{\tau}$ and $d \in \mathbb{R}^{+}$ represents the distributed delay. $\bar{\mathrm{A}}, \bar{\mathrm{A}}_{\tau}, \bar{\mathrm{A}}_{d}$ are time invariant matrices with fractional uncertainties described as

$$\begin{cases} \begin{bmatrix} \bar{\mathrm{A}} & \bar{\mathrm{A}}_{\tau} & \bar{\mathrm{A}}_{d} \end{bmatrix} \\ \quad = \begin{bmatrix} \mathrm{A} & \mathrm{A}_{\tau} & \mathrm{A}_{d} \end{bmatrix} + H\Theta(t) \begin{bmatrix} F_1 & F_2 & F_3 \end{bmatrix}, \\ \bar{\mathrm{B}} = \mathrm{B} + H\Theta(t) F_4 \\ \Theta(t) = U(t)(I - VU(t))^{-1}, \quad I - V^{T}V > 0, \end{cases} \quad (3)$$

where $U(t)$ is a known matrix function satisfying $U^{T}(t)U(t) \leq I$ and the matrices $\mathrm{A}, \mathrm{A}_{\tau}, \mathrm{A}_{d}, \mathrm{B}, \mathrm{B}_{w}$ and C are compatibly dimensioned known constant matrices.

It should be mentioned that there may be undesirable changes in the control systems owing to the occurrence of actuator faults. As a consequence, it is vital to consider a fault-tolerant controller that can automatically retrieves the system performance even in the presence of faults. Accordingly, the control input which is resistant to faults is designed as follows

$$u^{F}(t) = Ru(t), \quad (4)$$

where $R = diag\{r_1, r_2 \ldots r_p\}$ is the actuator fault matrix satisfying $0 \leq r_l^{min} \leq r_l \leq r_l^{max} \leq 1$, $l \in \{1, 2, \ldots, p\}$. Moreover, if r_l takes the value 1, then the actuator works normal, else if it takes the value 0, then the actuator fails to work. On this note, the actuator undergo partial failure if it takes the value $(0, 1)$. Consequently, we define, $R^{max} = \mathrm{diag}\{r_1^{max}, r_2^{max}, \ldots, r_l^{max}\}$, $R^{min} = \mathrm{diag}\{r_1^{min}, r_2^{min}, \ldots, r_l^{min}\}$, $R_0 = \frac{r_l^{max}+r_l^{max}}{2}$, $R_1 = \frac{r_l^{max}-r_l^{min}}{2}$, then the matrix R can be rewritten as

$$R = R_0 + R_1\Sigma, \quad \Sigma = \mathrm{diag}\{e_1, e_2, \ldots, e_l\}, e_l \in [-1, 1]. \quad (5)$$

Further, the non-fragile state-feedback controller $u(t)$ can be taken in the following form

$$u(t) = \bar{\mathrm{K}}\mathrm{x}(t), \quad (6)$$

where $\bar{\mathrm{K}} = \mathrm{K} + \Delta\mathrm{K}$, wherein K is the controller gain matrix that is to be evaluated, here $\Delta\mathrm{K}$ is the controller gain variation that takes the form $LG(t)S$, where L and S are appropriately dimensioned constant matrices; Also, $G(t)$ is any function in matrix form fulfilling the condition $G^{T}(t)G(t) \leq I$.

Therefore, with the aid of equations from (2)-(6), the consequent closed-loop fractional-order nonlinear system can be obtained as

$$\begin{cases} D^{\alpha} \mathrm{x}(t) = (\bar{\mathrm{A}} + \bar{\mathrm{B}}R\bar{\mathrm{K}})\mathrm{x}(t) + \bar{\mathrm{A}}_{\tau}\mathrm{x}(t-\tau(t)) \\ \qquad\qquad + \bar{\mathrm{A}}_{d} \int_{t-d}^{t} \mathrm{x}(s) ds + h(\mathrm{x}(t)) + \mathrm{B}_{w}\mathfrak{d}(t), \\ \mathrm{y}(t) = \mathrm{C}\mathrm{x}(t). \end{cases} \quad (7)$$

On this note, some of the prerequisite lemmas and definitions which play a crucial part in the derivation analysis is recalled here.

Assumption 1: For given a matrix N of suitable dimension, the nonlinear vector function $h(\mathrm{x}(t))$ is said to satisfy the global Lipschitz condition $h^{T}(\mathrm{x}(t))h(\mathrm{x}(t)) \leq \mathrm{x}^{T}(t)N^{T}N\mathrm{x}(t)$.

Definition 1 [29]: For given scalar θ and $\mathfrak{d}(t) \in L_2[0, \infty)$, if the inequality $\int_0^t (-\hat{\gamma}^{-1}\theta \mathrm{y}^{T}(v)\mathrm{y}(v) + 2(1-\theta)\mathrm{y}^{T}(v)\mathfrak{d}(v))dv \geq -\hat{\gamma} \int_0^t \mathfrak{d}^{T}(v)\mathfrak{d}(v)dv\}$, $\forall t > 0$ holds, then the system (7) is said to be asymptotically stable under zero initial conditions with satisfied mixed H_{∞} and passive performance index $\hat{\gamma} > 0$.

Lemma 1 [4]: Given a vector of continuous and differentiable function $\chi(t) \in \mathbb{R}^{n}$ and a positive definite matrix $\mathfrak{P} \in \mathbb{R}^{n \times n}$, we have

$$D^{\alpha}(\chi^{T}(t)\mathfrak{P}\chi(t)) \leq (\chi^{T}(t)\mathfrak{P})D^{\alpha}\chi(t) + (D^{\alpha}\chi(t))^{T}\mathfrak{P}\chi(t), \forall \alpha \in (0, 1], t \geq t_0.$$

Lemma 2 (Razumikhin-Type Stability[2]): For some continuous and non-decreasing functions $\beta_1, \beta_2, \beta_3$ with β_2 strictly increasing and $\beta_1(0) = \beta_2(0) = \beta_3(0) = 0$, suppose that a number $p > 1$ and a continuous differentiable function $\mathcal{V} : \mathbb{R} \times \mathbb{R}^{n} \to \mathbb{R}$ exists so that:

1) $\beta_1(||\chi||) \leq \mathcal{V}(t, \chi(t)) \leq \beta_2(||\chi||)$, for $t \in \mathbb{R}$, $\chi \in \mathbb{R}^{n}$
2) $D^{\alpha}\mathcal{V}(t, \chi(t)) \leq -\beta_3(||\chi||)$ if $\mathcal{V}(t+s, \chi(t+s)) < p\mathcal{V}(t, \chi(t))$, $\forall s \in [-d, 0]$, $t \geq 0$, then the zero solution of the system (2) is asymptotically stable.

III. MAIN RESULTS

This section summarizes the stability analysis of the closed-loop system (7) by deriving adequate conditions. More precisely, by selecting a relevant Lyapunov-Krasovskii functional and making use of Razumikhin theorem, the stabilization of the addressed FO nonlinear control system (2) is assured by the non-fragile fault-tolerant control law along with prescribed disturbance attenuation level. In particular, the properties of Caputo fractional derivative and fractional integral are adopted to prove the mixed H_∞ and passive disturbance attenuation performance. In the following theorem, the desired conditions supporting the stabilization analysis are presented.

Theorem 1: For given scalar $\alpha \in (0,1)$, positive scalars $d, \bar{\tau}, \epsilon, \hat{\gamma}$ and known matrices R_0, R_1, the considered system (2) is asymptotically stabilized by the NFT controller (4) and meets the mixed H_∞ and passive performance, suppose there exists positive-definite matrices P, Q, scalars $\mu_j, j = 1,2,3$ and matrix Y of compatible dimension thereby satisfying the subsequent LMI:

$$\Upsilon_{[13\times 13]} < 0, \tag{8}$$

where the terms of Υ are furnished as $\Upsilon_{1,1} = \mathrm{A}X + \mathrm{B}R_0Y + X^T\mathrm{A}^T + (\mathrm{B}R_0Y)^T + \bar{Q} + d\epsilon X$, $\Upsilon_{1,2} = \mathrm{A}_\tau X$, $\Upsilon_{1,3} = \mathrm{B}_w - (1-\theta)X\mathrm{C}^T$, $\Upsilon_{1,4} = I$, $\Upsilon_{1,5} = d\mathrm{A}_dX$, $\Upsilon_{1,6} = XN^T$, $\Upsilon_{1,7} = \sqrt{\theta}X\mathrm{C}^T$, $\Upsilon_{1,8} = \mu_1H$, $\Upsilon_{1,9} = XF_1^T + Y^TR_0^TF_4^T$, $\Upsilon_{1,10} = \mu_2\mathrm{B}R_0L$, $\Upsilon_{1,11} = XS^T$, $\Upsilon_{1,12} = \mu_3\mathrm{B}R_1$, $\Upsilon_{1,13} = Y^T$, $\Upsilon_{2,2} = -(1-\bar{\tau})\bar{Q}$, $\Upsilon_{2,9} = XF_2^T$, $\Upsilon_{3,3} = -\hat{\gamma}$, $\Upsilon_{4,4} = -I$, $\Upsilon_{5,5} = -dX$, $\Upsilon_{5,9} = XF_3^T$, $\Upsilon_{6,6} = -I$, $\Upsilon_{7,7} = -\hat{\gamma}$, $\Upsilon_{8,8} = \Upsilon_{9,9} = -\mu_1$, $\Upsilon_{8,9} = V^T$, $\Upsilon_{9,10} = \mu_2F_4R_0L$, $\Upsilon_{9,12} = \mu_3F_4R_1$, $\Upsilon_{10,10} = \Upsilon_{11,11} = -\mu_2$, $\Upsilon_{10,13} = R^T$, $\Upsilon_{12,12} = \Upsilon_{13,13} = -\mu_3$. Furthermore, the controller gain matrix K is determined by $\mathrm{K} = YX^{-1}$.

Proof: Predominantly, we aim to stabilize the considered system (2) by using the designed NFT controller (4) which will be proved by selecting a suitable Lyapunov-Krasovskii functional in the subsequent manner

$$\mathcal{V}(t) = \mathcal{V}_1(t) + \mathcal{V}_2(t), \tag{9}$$

here $\mathcal{V}_1(t) = \mathrm{x}^T(t)P\mathrm{x}(t)$, $\mathcal{V}_2(t) = D^{(-\alpha+1)}\int_{t-\tau(t)}^{t}\mathrm{x}^T(s)Q\,\mathrm{x}(s)\,ds$. Then, with the support of Lemma 1 and the linearity nature of the Caputo fractional-order derivative, the fractional derivative along the trajectories of the system state is acquired as follows

$$\begin{aligned}
D^\alpha\mathcal{V}(t) \le\ & \mathrm{x}^T(t)PD^\alpha\mathrm{x}(t) + (D^\alpha\mathrm{x}(t))^TP\mathrm{x}(t) + \mathrm{x}^T(t)Q\mathrm{x}(t)\\
&-(1-\bar{\tau})\mathrm{x}^T(t-\tau(t))Q\mathrm{x}(t-\tau(t)),\\
=\ & \mathrm{x}^T(t)P\Big[(\bar{\mathrm{A}}+\bar{\mathrm{B}}R\bar{\mathrm{K}})\mathrm{x}(t) + \bar{\mathrm{A}}_\tau\mathrm{x}(t-\tau(t))\\
&+\bar{\mathrm{A}}_d\int_{t-d}^{t}\mathrm{x}(s)ds + h(\mathrm{x}(t)) + \mathrm{B}_w\mathfrak{d}(t)\Big]\\
&+\Big[(\bar{\mathrm{A}}+\bar{\mathrm{B}}R\bar{\mathrm{K}})\mathrm{x}(t) + \bar{\mathrm{A}}_\tau\mathrm{x}(t-\tau(t))\\
&+\bar{\mathrm{A}}_d\int_{t-d}^{t}\mathrm{x}(s)ds + h(x(t)) + \mathrm{B}_w\mathfrak{d}(t)\Big]^TP\mathrm{x}(t)\\
&+\mathrm{x}^T(t)Q\mathrm{x}(t)\\
&-(1-\bar{\tau})\mathrm{x}^T(t-\tau(t))Q\mathrm{x}(t-\tau(t)),\\
\le\ & \mathrm{x}^T(t)[P(\bar{\mathrm{A}}+\bar{\mathrm{B}}R\bar{\mathrm{K}}) + (\bar{\mathrm{A}}+\bar{\mathrm{B}}R\bar{\mathrm{K}})^TP + Q]\mathrm{x}(t)\\
&-(1-\bar{\tau})\mathrm{x}^T(t-\tau(t))Q\mathrm{x}(t-\tau(t))\\
&+2\mathrm{x}^T(t)P\bar{\mathrm{A}}_\tau\mathrm{x}(t-\tau(t)) + 2\mathrm{x}^T(t)Ph(\mathrm{x}(t))\\
&+2\mathrm{x}^T(t)P\bar{\mathrm{A}}_d\int_{t-d}^{t}\mathrm{x}(s)ds + 2\mathrm{x}^T(t)P\mathrm{B}_w\mathfrak{d}(t).
\end{aligned} \tag{10}$$

Further, the term $2\mathrm{x}^T(t)P\bar{\mathrm{A}}_d\int_{t-d}^{t}\mathrm{x}(s)ds$ is computed in view of the procedure as in [17] and by employing Lemma 2.1 in [17], Lemma 2 and Cauchy matrix inequality, we have

$$\begin{aligned}
&2\mathrm{x}^T(t)P\bar{\mathrm{A}}_d\int_{t-d}^{t}\mathrm{x}(s)ds\\
&\le d\mathrm{x}^T(t)P\bar{\mathrm{A}}_dP^{-1}\bar{\mathrm{A}}_d^TP\mathrm{x}(t)\\
&\quad+\frac{1}{d}\left(\int_{t-d}^{t}\mathrm{x}(s)ds\right)^TP\left(\int_{t-d}^{t}\mathrm{x}(s)ds\right),\\
&\le d\mathrm{x}^T(t)P\bar{\mathrm{A}}_dP^{-1}\bar{\mathrm{A}}_d^TP\mathrm{x}(t) + \int_{t-d}^{t}\mathrm{x}(s)^TP\mathrm{x}(s)ds\\
&\le d\mathrm{x}^T(t)P\bar{\mathrm{A}}_dP^{-1}\bar{\mathrm{A}}_d^TP\mathrm{x}(t) + \int_{-d}^{0}\mathrm{x}(t+s)^TP\mathrm{x}(t+s)ds\\
&\le d\mathrm{x}^T(t)P\bar{\mathrm{A}}_dP^{-1}\bar{\mathrm{A}}_d^TP\mathrm{x}(t) + \epsilon\int_{-d}^{0}\mathrm{x}(t)^TP\mathrm{x}(t)ds\\
&\quad(\text{ since } \mathcal{V}(t+s,\mathrm{x}(t+s)) < \epsilon\mathcal{V}(t,\mathrm{x}(t)))\\
&\Longrightarrow 2\mathrm{x}^T(t)P\bar{\mathrm{A}}_d\int_{t-d}^{t}\mathrm{x}(s)ds \le d\mathrm{x}^T(t)P\bar{\mathrm{A}}_dP^{-1}\bar{\mathrm{A}}_d^TP\mathrm{x}(t)\\
&\quad+d\epsilon\mathrm{x}(t)^TP\mathrm{x}(t).
\end{aligned} \tag{11}$$

Subsequently, with the aid of Assumption 1, the nonlinear term in (10) can be computed as

$$\begin{aligned}
2\mathrm{x}^T(t)Ph(\mathrm{x}(t)) &\le \mathrm{x}^T(t)P^TP\mathrm{x}(t) + h^T(\mathrm{x}(t))h(\mathrm{x}(t))\\
&\le \mathrm{x}^T(t)P^TP\mathrm{x}(t) + \mathrm{x}^T(t)N^TN\mathrm{x}(t).
\end{aligned} \tag{12}$$

Then, substituting the obtained inequalities (11) and (12) in (10), we get the following inequality according to mixed H_∞ and passivity condition

$$\begin{aligned}
D^\alpha\mathcal{V}(t) + \hat{\gamma}^{-1}\theta\mathrm{y}^T(t)\mathrm{y}(t) - 2(1-\theta)\mathrm{y}^T(t)\mathfrak{d}(t)\\
-\hat{\gamma}\mathfrak{d}^T(t)\mathfrak{d}(t) \le \eta^T(t)[\Lambda]_{3\times 3}\eta(t),
\end{aligned}$$

where $\eta(t) = \left[\mathrm{x}^T(t)\ \mathrm{x}^T(t-\tau(t))\ \mathfrak{d}^T(t)\right]^T$ and $\Lambda_{1,1} = P\bar{\mathrm{A}} + P\bar{\mathrm{B}}R\bar{\mathrm{K}} + \bar{\mathrm{A}}^TP + (\bar{\mathrm{B}}R\bar{\mathrm{K}})^TP + Q + dP\bar{\mathrm{A}}_dP^{-1}\bar{\mathrm{A}}_d^TP + d\epsilon P + P^TP + N^TN + \hat{\gamma}^{-1}\theta\mathrm{C}^T\mathrm{C}$, $\Lambda_{1,2} = P\bar{\mathrm{A}}_\tau$, $\Lambda_{1,3} = P\mathrm{B}_w - (1-\theta)\mathrm{C}^T$, $\Lambda_{2,2} = -(1-\bar{\tau})Q$, $\Lambda_{3,3} = -\hat{\gamma}$.

In light of fractional uncertainties, gain fluctuations and fault, by applying the S-procedure and Schur complement lemma to the above matrix inequality, the matrix Λ can be reformulated as $[\hat{\Lambda}]_{13\times 13}$. The elements of $\hat{\Lambda}$ are categorized as follows: $\hat{\Lambda}_{1,1} = P\mathrm{A} + P\mathrm{B}R_0\mathrm{K} + \mathrm{A}^TP + (\mathrm{B}R_0\mathrm{K})^TP + Q + d\epsilon P$, $\hat{\Lambda}_{1,2} = P\mathrm{A}_\tau$, $\hat{\Lambda}_{1,3} = P\mathrm{B}_w - (1-\theta)\mathrm{C}^T$, $\hat{\Lambda}_{1,4} = P^T$, $\hat{\Lambda}_{1,5} = dP\mathrm{A}_d$, $\hat{\Lambda}_{1,6} = N^T$, $\hat{\Lambda}_{1,7} = \sqrt{\theta}\mathrm{C}^T$, $\hat{\Lambda}_{1,8} = \mu_1PH$, $\hat{\Lambda}_{1,9} = F_1^T + \mathrm{K}^TR_0^TF_4^T$, $\hat{\Lambda}_{1,10} = \mu_2P\mathrm{B}R_0L$, $\hat{\Lambda}_{1,11} = S^T$, $\hat{\Lambda}_{1,12} = \mu_3P\mathrm{B}R_1$, $\hat{\Lambda}_{1,13} = K^T$, $\hat{\Lambda}_{2,2} = -(1-\bar{\tau})Q$, $\hat{\Lambda}_{2,9} = F_2^T$, $\hat{\Lambda}_{3,3} = -\hat{\gamma}$, $\hat{\Lambda}_{4,4} = -I$, $\hat{\Lambda}_{5,5} = -dP$, $\hat{\Lambda}_{5,9} = F_3^T$, $\hat{\Lambda}_{6,6} = -I$, $\hat{\Lambda}_{7,7} = -\hat{\gamma}$, $\hat{\Lambda}_{8,8} = -\mu_1$, $\hat{\Lambda}_{8,9} = V^T$, $\hat{\Lambda}_{9,9} = -\mu_1$, $\hat{\Lambda}_{9,10} = \mu_2F_4R_0L$, $\hat{\Lambda}_{9,12} = \mu_3F_4R_1$, $\hat{\Lambda}_{10,10} = -\mu_2$, $\hat{\Lambda}_{10,13} = R^T$, $\hat{\Lambda}_{11,11} = -\mu_2$, $\hat{\Lambda}_{12,12} = -\mu_3$, $\hat{\Lambda}_{13,13} = -\mu_3$.

The matrix $\hat{\Lambda}$ can be linearized by applying the congruence transformation on both sides with the diagonal matrix $diag\{X, X, I, I, X, I, I, I, I, I, I, I, I\}$. Let us denote $P^{-1} = X$, $XQX = \bar{Q}$, $KX = Y$, then we readily retrieve the linear matrix inequality Υ in (8). Now, if the LMI condition (8) holds, then we can acquire

$$D^{\alpha}\mathcal{V}(t)+\hat{\gamma}^{-1}\theta \mathrm{y}^{T}(t)\mathrm{y}(t)-2(1-\theta)\mathrm{y}^{T}(t)\mathfrak{d}(t) -\hat{\gamma}\mathfrak{d}^{T}(t)\mathfrak{d}(t)\leq 0. \quad (13)$$

Further, the preceding inequality (13) is integrated with respect to t by the limits from 0 to T_f, we attain

$$\mathcal{I}^{1}D^{\alpha}\mathcal{V}(T_f)+\int_{0}^{T_f}\hat{\gamma}^{-1}\theta \mathrm{y}^{T}(t)\mathrm{y}(t) -2(1-\theta)\mathrm{y}^{T}(t)\mathfrak{d}(t)-\hat{\gamma}\mathfrak{d}^{T}(t)\mathfrak{d}(t)dt\leq 0. \quad (14)$$

By incorporating the Property 1 and 2 from [4], $\mathcal{I}^{1}D^{\alpha}\mathcal{V}(T_f)$ can be rewritten as

$$\begin{aligned}\mathcal{I}^{1}D^{\alpha}\mathcal{V}(T_f)&=\mathcal{I}^{1-\alpha}\mathcal{I}^{\alpha}D^{\alpha}\mathcal{V}(T_f),\\&=\mathcal{I}^{1-\alpha}(\mathcal{V}(T_f)-V(0))\\&=\mathcal{I}^{1-\alpha}\mathcal{V}(T_f)-\mathcal{I}^{1-\alpha}V(0),\end{aligned} \quad (15)$$

where $\mathcal{I}^{1-\alpha}\mathcal{V}(0)=\frac{1}{\Gamma(1-\alpha)}\int_0^{T_f}(T_f-t)^{-\alpha}V(0)\,dt=0\ \forall\ T_f\geq 0$ and $\mathcal{I}^{1-\alpha}\mathcal{V}(T_f)=\frac{1}{\Gamma(1-\alpha)}\int_0^{T_f}(T_f-t)^{-\alpha}V(t)\,dt\geq 0$. Hence, from the inequality (15), we get

$$\mathcal{I}^{1}D^{\alpha}\mathcal{V}(T_f)\geq 0.$$

Substituting the above inequality in (14), we easily get

$$\begin{aligned}&\int_{0}^{T_f}\hat{\gamma}^{-1}\theta \mathrm{y}^{T}(t)\mathrm{y}(t)-2(1-\theta)\mathrm{y}^{T}(t)\mathfrak{d}(t)\\&\quad-\hat{\gamma}\mathfrak{d}^{T}(t)\mathfrak{d}(t)dt\leq 0,\\&\Longrightarrow \int_{0}^{T_f}-\hat{\gamma}^{-1}\theta \mathrm{y}^{T}(t)\mathrm{y}(t)+2(1-\theta)\mathrm{y}^{T}(t)\mathfrak{d}(t)dt\\&\geq \int_{0}^{T_f}-\hat{\gamma}\mathfrak{d}^{T}(t)\mathfrak{d}(t)dt.\end{aligned} \quad (16)$$

Therefore, it can be concluded from (13) and (16) that the considered FO nonlinear system (2) is stabilized by utilising the non-fragile fault-tolerant controller (4) with the required mixed H_∞ and passive performance index. Thus, the proof is completed. ■

Meanwhile, if the time-varying delays and nonlinearity nature are omitted, then the addressed system in (2) will be remodelled as uncertain fractional-order system with distributed delays in the subsequent manner

$$\begin{cases}D^{\alpha}\mathrm{x}(t)=\bar{\mathrm{A}}\mathrm{x}(t)+\bar{\mathrm{A}}_d\int_{t-d}^{t}\mathrm{x}(s)ds+\mathrm{B}u^{F}(t)+\mathrm{B}_w\mathfrak{d}(t),\\ \mathrm{y}(t)=\mathrm{C}\mathrm{x}(t),\end{cases} \quad (17)$$

Corollary 1: For given scalar $\alpha\in(0,1),\theta\in(0,1)$, positive scalars $d,\epsilon,\hat{\gamma}$ and known matrices R_0, R_1, the considered system (17) is asymptotically stabilized by the NFT controller (4) with the mixed H_∞ and passive performance, suppose there exists a matrix Y of compatible dimension, a positive-definite matrix P and scalars $\mu_z, z=1,2,3$ so that the subsequent LMI satisfies:

$$\Psi_{[10\times 10]}<0, \quad (18)$$

where the terms of Ψ are furnished as $\Psi_{1,1}=\mathrm{A}X+\mathrm{B}R_0Y+X^T\mathrm{A}^T+(\mathrm{B}R_0Y)^T+d\epsilon X$, $\Psi_{1,2}=\mathrm{B}_w-(1-\theta)X\mathrm{C}^T$, $\Psi_{1,3}=d\mathrm{A}_dX$, $\Psi_{1,4}=\sqrt{\theta}X\mathrm{C}^T$, $\Psi_{1,5}=\mu_1H$, $\Psi_{1,6}=XF_1^T+Y^TR_0^TF_4^T$, $\Psi_{1,7}=\mu_2\mathrm{B}R_0L$, $\Psi_{1,8}=XS^T$, $\Psi_{1,9}=\mu_3\mathrm{B}R_1$, $\Psi_{1,10}=Y^T$, $\Psi_{2,2}=-\hat{\gamma}$, $\Psi_{3,3}=-dX$, $\Psi_{3,6}=XF_3^T$, $\Psi_{4,4}=-\hat{\gamma}$, $\Psi_{5,5}=\Psi_{6,6}=-\mu_1$, $\Psi_{5,6}=V^T$, $\Psi_{6,7}=\mu_2F_4R_0L$, $\Psi_{6,9}=\mu_3F_4R_1$, $\Psi_{7,7}=\Psi_{8,8}=-\mu_2$, $\Psi_{7,10}=R^T$, $\Psi_{9,9}=\Psi_{10,10}=-\mu_3$. Further, the controller gain matrix K is determined by $K=YX^{-1}$.

On the other hand, in the absence of distributed delays and nonlinearity, the addressed system in (2) will be remodelled with $\tau(t)=\tau$ as follows

$$\begin{cases}D^{\alpha}\mathrm{x}(t)=\bar{\mathrm{A}}\mathrm{x}(t)++\bar{\mathrm{A}}_\tau\mathrm{x}(t-\tau)+\mathrm{B}u^{F}(t)+\mathrm{B}_w\mathfrak{d}(t),\\ \mathrm{y}(t)=\mathrm{C}\mathrm{x}(t),\end{cases} \quad (19)$$

Corollary 2: For given scalar $\alpha\in(0,1),\theta\in(0,1)$, positive scalars $\tau,\epsilon,\hat{\gamma}$ and known matrices R_0, R_1, the considered system (19) is asymptotically stabilized by the NFT controller (4) with the mixed H_∞ and passive performance, suppose there exists a matrix Y of compatible dimension, a positive-definite matrix P and scalars $\mu_z, z=1,2,3$ so that the subsequent LMI satisfies:

$$\hat{\Psi}_{[10\times 10]}<0, \quad (20)$$

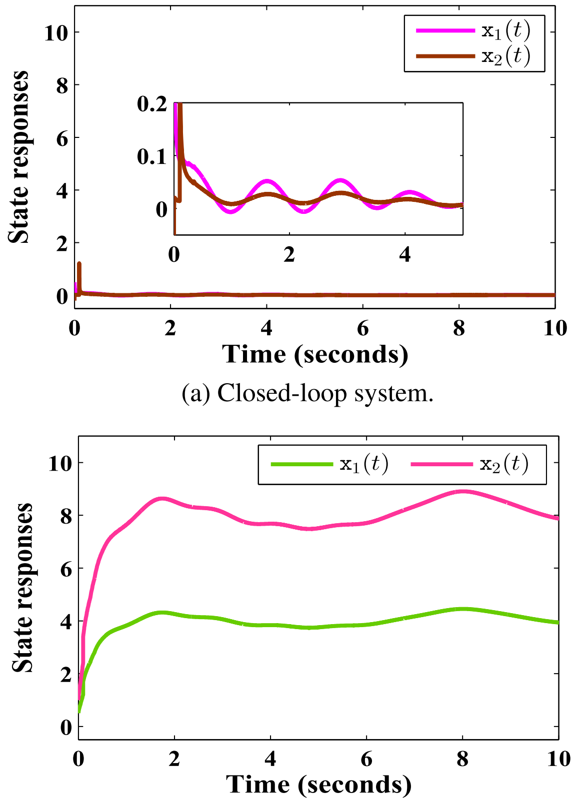
where the terms of $\hat{\Psi}$ are furnished as $\hat{\Psi}_{1,1}=\mathrm{A}X+\mathrm{B}R_0Y+X^T\mathrm{A}^T+(\mathrm{B}R_0Y)^T+d\epsilon X$, $\hat{\Psi}_{1,2}=\mathrm{A}_\tau X$, $\hat{\Psi}_{2,2}=-(1-\bar{\tau})\bar{Q}$, $\hat{\Psi}_{1,3}=\mathrm{B}_w-(1-\theta)X\mathrm{C}^T$, $\hat{\Psi}_{1,4}=\sqrt{\theta}X\mathrm{C}^T$, $\hat{\Psi}_{1,5}=\mu_1H$, $\hat{\Psi}_{1,6}=XF_1^T+Y^TR_0^TF_4^T$, $\hat{\Psi}_{1,7}=\mu_2\mathrm{B}R_0L$, $\hat{\Psi}_{1,8}=XS^T$, $\hat{\Psi}_{1,9}=\mu_3\mathrm{B}R_1$, $\hat{\Psi}_{1,10}=Y^T$, $\hat{\Psi}_{3,3}=-\hat{\gamma}$, $\hat{\Psi}_{3,6}=XF_3^T$, $\hat{\Psi}_{4,4}=-\hat{\gamma}$, $\hat{\Psi}_{5,5}=\hat{\Psi}_{6,6}=-\mu_1$, $\hat{\Psi}_{5,6}=V^T$, $\hat{\Psi}_{6,7}=\mu_2F_4R_0L$, $\hat{\Psi}_{6,9}=\mu_3F_4R_1$, $\hat{\Psi}_{7,7}=\hat{\Psi}_{8,8}=-\mu_2$, $\hat{\Psi}_{7,10}=R^T$, $\hat{\Psi}_{9,9}=\hat{\Psi}_{10,10}=-\mu_3$. Further, the controller gain matrix K is determined by $K=YX^{-1}$.

IV. NUMERICAL EXAMPLES

The applicability of the estimated theoretical results is verified in this section by presenting three numerical examples. Especially, in Example 1, the efficacy of the designed NFT controller is endorsed through a fractional-order nonlinear system with appropriate system matrices. Moreover, the proposed controller is applied to a system of rocket motor chambers and the corresponding results are depicted with simulations in Example 2. A comparative study is made in Example 3 to show the effectiveness of the proposed control strategy.

Example 1: Let us consider the following parameters for a fractional nonlinear system with time-varying delay and distributed delay;

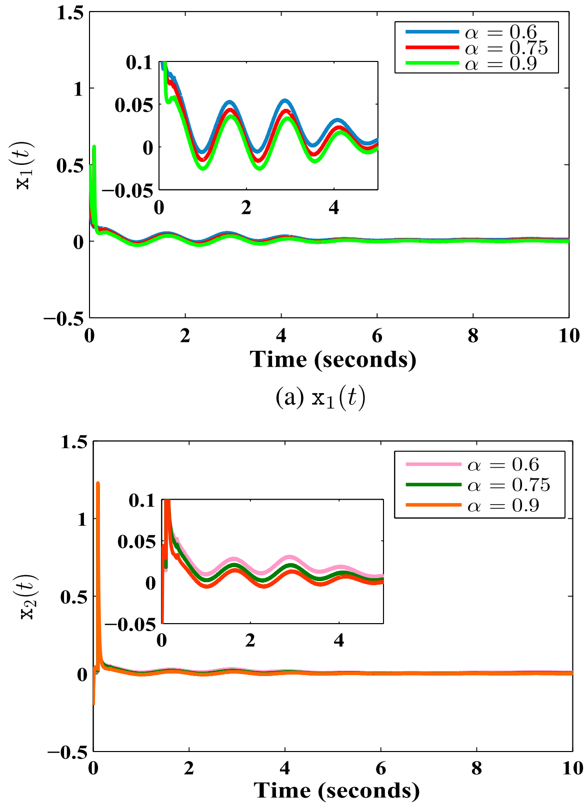
$$\mathrm{A}=\begin{bmatrix}-0.6 & -2.1\\ -1 & -0.3\end{bmatrix},\ \mathrm{A}_\tau=\begin{bmatrix}0.3 & 0.2\\ 0.1 & 0.4\end{bmatrix},$$
$$\mathrm{A}_d=\begin{bmatrix}0.1 & 0.3\\ 0.4 & 0.2\end{bmatrix},\ \mathrm{B}=\begin{bmatrix}0.2 & 0.5\\ 0.4 & 0.2\end{bmatrix},$$
$$\mathrm{C}=\begin{bmatrix}0.2 & 0.4\end{bmatrix},\quad \text{and}\quad \mathrm{B}_w=\begin{bmatrix}0.7\\ 0.2\end{bmatrix}.$$



(a) Closed-loop system.

(b) Open-loop system.

FIGURE 1. State responses of the system.



(a) $x_1(t)$

(b) $x_2(t)$

FIGURE 2. Trajectories of system states under different fractional-orders.

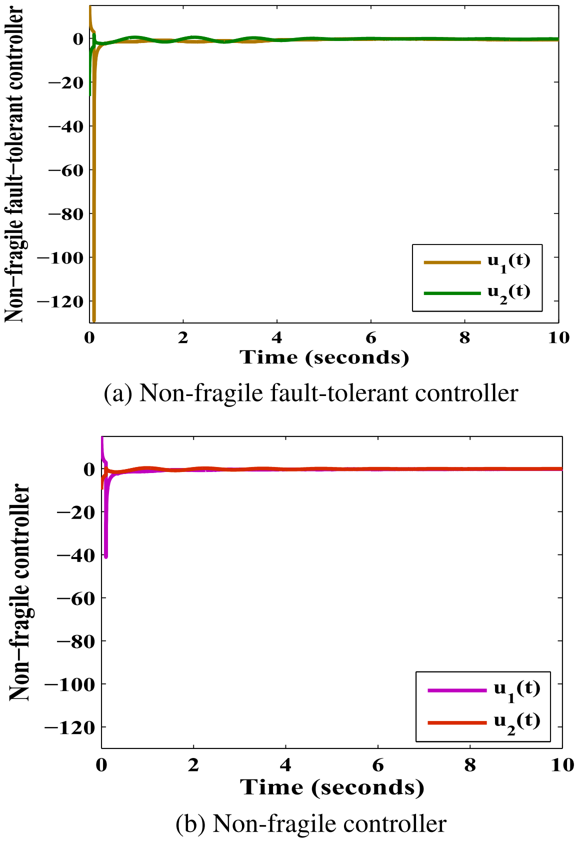
TABLE 1. Obtained gains under different performance index.

$\theta = 0$	$K = \begin{bmatrix} 14.5380 & -97.5672 \\ -45.7609 & -6.3663 \end{bmatrix}$
$\theta = 0.5$	$K = \begin{bmatrix} 52.3869 & -131.5161 \\ -43.5959 & 22.4177 \end{bmatrix}$
$\theta = 1$	$K = \begin{bmatrix} 49.4701 & -123.1365 \\ -46.8482 & 24.1628 \end{bmatrix}$

In light of the fractional parametric uncertainties in the system model and the gain perturbations in the control law, the corresponding matrices are considered as follows

$$H = \begin{bmatrix} 0.4 & 0.7 \end{bmatrix}^T, \quad F_1 = \begin{bmatrix} -0.1 & 0.2 \end{bmatrix},$$
$$F_2 = \begin{bmatrix} 0.2 & 0.13 \end{bmatrix}, \quad F_3 = \begin{bmatrix} 0.4 & 0.7 \end{bmatrix}, F_4 = 0, V = 0.4,$$
$$= \begin{bmatrix} -0.7 & 0.5 \end{bmatrix}^T \quad \text{and} \quad S = \begin{bmatrix} -0.3 & 0.4 \end{bmatrix}.$$

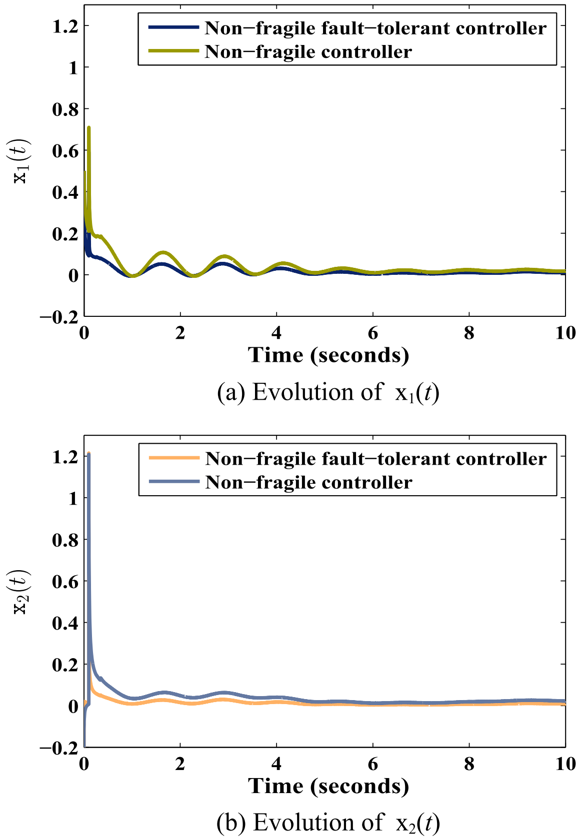
Further, the delay bounds with respect to time-varying delay and distributed delay is adopted as $\bar{\tau} = 0.3$ and $d = 0.3$. The fault function of actuator present in the input signal is fixed in the range $0.3 < R < 0.9$ such that the parameters R_0 and R_1 takes the value 0.6 and 0.3 respectively. Besides, the signal representing the external disturbance $\eth(t)$ for the system is selected as $\eth(t) = \exp(-0.4t)\sin(5t)$. Then, the nonlinear function present in the system takes the form $h(x(t)) = x_1 x_2 - x_1^2$ and other parameters are selected as $N = \begin{bmatrix} 0.2 & 0.4 \\ 0.8 & 0.7 \end{bmatrix}$, $\epsilon = 0.4$. Also, when $\theta = 0.5$, the mixed H_∞ and passive performance is accounted with disturbance attenuation level $\hat{\gamma}$ that is selected as 0.7. The fractional-order α is chosen as 0.6 with initial conditions $x(0) = \begin{bmatrix} 0.5 & 0.2 \end{bmatrix}^T$. With the above set of parameters, the stability of the system is addressed by obtaining the feasible solutions in Theorem 1 via MATLAB LMI toolbox and FOMCON toolbox. Moreover, the controller gain matrix under different performances is provided in Table 1. Upon considering the above parameters and the determined controller gain matrix, the simulated results of the established system under fractional-order $\alpha = 0.6$ with mixed H_∞ and passive performance are presented in the Figs.1-6. The response of the system states in the presence and absence of the controller is presented in Fig.1. From Fig. 1 (a), the prominence of the NFT controller in stabilizing the system is revealed. Also, in Fig. 1 (b), the unavailability of the controller has created a great impact on the system such that it leads the curves to diverge to a greater extent. Additionally, to manifest the identity of the considered FO system, the behaviour of the system under different fractional-order is depicted in Fig. 2. Thus, the NFT controller is competent enough to tackle and stabilize the system under different fractional-orders. The response



(a) Non-fragile fault-tolerant controller

(b) Non-fragile controller

FIGURE 3. Control responses of the system.



(a) Evolution of $x_1(t)$

(b) Evolution of $x_2(t)$

FIGURE 4. State responses using the NFT controller (4) **and non-fragile controller.**

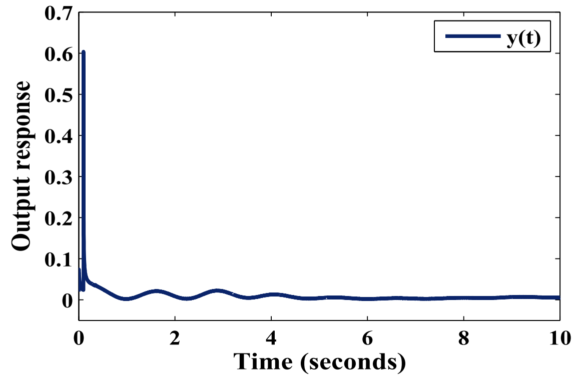


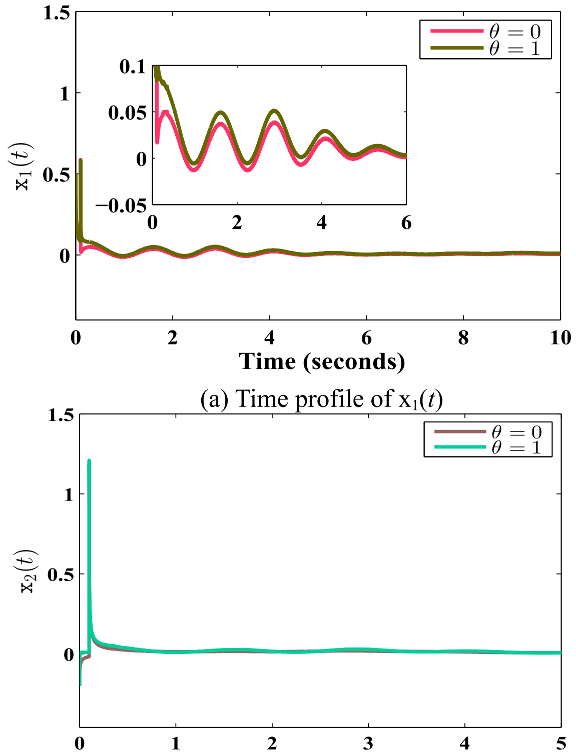
FIGURE 5. Output responses of the system.

of the NFT controller and the non-fragile controller in the absence of actuator faults is presented in Fig. 3. In response to the presented controllers in Fig. 3, the dynamics of the system states under the occurrence of actuator fault is portrayed in Fig. 4. It is evident from this figure that when the actuator becomes faulty, the NFT controller has the potential to provide better performance in achieving stability compared with the non-fragile controller. Consequently, the response of the output curve of the considered fractional-order nonlinear system is displayed in Fig. 5. In addition, various disturbance attenuation performances can be reduced from mixed H_∞ and passive performance. Suppose if θ takes the values 1 or 0, then the disturbance approach will be reduced to H_∞ or passivity performance respectively with disturbance attenuation index $\hat{\gamma}$. On this note, with the obtained gain values in Table 1, the depiction of the system behaviour under different performance is given in Fig. 6. On the whole, it can be concluded that the considered FO nonlinear system with time-varying delay, distributed delay, fractional uncertainties, disturbance and actuator faults achieve the stabilization by virtue of the proposed NFT controller.

Example 2: Let us study the stabilization of liquid mono propellant rocket motor with a pressure feeding system. By assuming the non-steady flow and considering non-uniform lag, the combustion chamber and the pressure feeding system is represented by dynamical model as follows [14]

$$\begin{aligned}
D^\alpha \sigma(t) &= (\upsilon - 1)\sigma(t) \\
&\quad + \int_0^1 [\beta(t-\delta(\varsigma)) - \upsilon\sigma(t-\delta(\varsigma))]d\varsigma, \\
D^\alpha \beta_1(t) &= \frac{1}{\xi J}\left[-\phi(t) + \frac{p_0 - p_1}{2\Delta p}\right], \\
D^\alpha \beta(t) &= \frac{1}{(1-\xi)J}\left[-\beta(t) + \phi(t) - p\sigma(t)\right], \\
D^\alpha \phi(t) &= \frac{1}{E}\left[\beta_1(t) - \beta(t)\right],
\end{aligned}$$

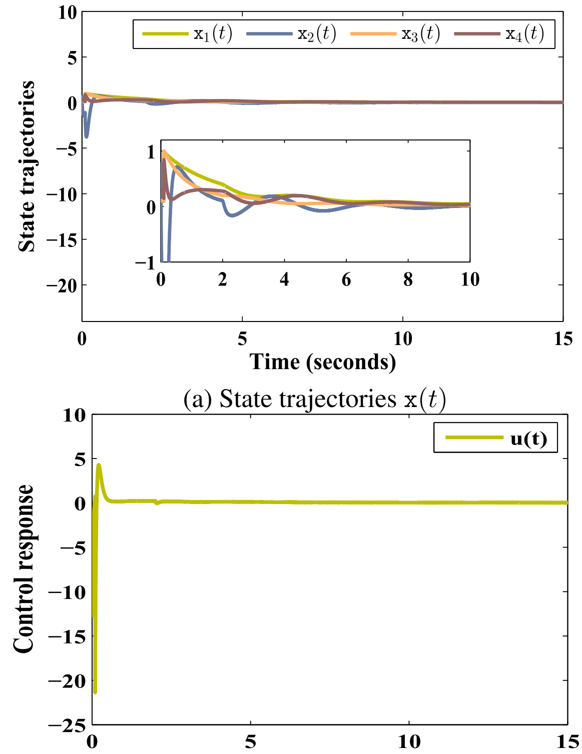
where $\sigma(t) = (p(t) - \tilde{p})/\tilde{p}$ so that $\tilde{p}(t)$ is the instant pressure occuring in combustion chamber with $\tilde{p}$ is its value in steady



(a) Time profile of $x_1(t)$

(b) Time profile of $x_2(t)$

FIGURE 6. State responses under H_∞ ($\theta = 1$) performance and passive ($\theta = 0$) performance.



(a) State trajectories $x(t)$

(b) Control response $u(t)$

FIGURE 7. State responses and control responses.

state; $\beta_1(t) = (\dot{m}_i(t) - \dot{m})/\dot{m}$ such that $\dot{m}(t)$ is the instant mass flow upstream of capacitance and $\dot{m}$ is the value of $\dot{m}_i$ in steady state with $\dot{m}_i(t)$ as the instant mass rate of the liquid propellant; $\beta(t) = (\dot{m}_i - \dot{m})/\dot{m}$; $\phi(t) = (p_1(t) - \tilde{p}_1)/(2\Delta p)$ for $\tilde{p}_1$ takes the value in steady operation of $p_1(t)$ which is the instant pressure in the feeding line at a particular point where the capacitance denoting the elasticity is present; $\Delta p = \tilde{p}_1 - \tilde{p}$ denotes pressure drop in the injector during steady operation; ξ, υ, E, J respectively represents the fractional length for the pressure supply, pressure exponent, elasticity parameter and inertia parameter of the line; t represents the reduced time that is normalized with gas residence time; p_0 is the regulated gas pressure; $\delta = \hat{\tau}/\varsigma_g$ is the reduced time lag with $\varsigma_g, \hat{\tau}$ as the time of gas residence and time lag in steady operation. Let us take the control input as $u = (p_0 - p_1)/(2\Delta p)$ and the values of parameters are chosen as $J = 2, \delta(\varsigma) = d\varsigma, p = 1, \xi = 0.1, E = 1$. Next, the modelled system is altered as follows

$$D^\alpha x(t) = \bar{A}x(t) + \bar{A}_d \int_{t-d}^{t} x(s)ds + Bu^F(t),$$

where $x(t) = \begin{bmatrix}\sigma(t) & \beta_1(t) & \beta(t) & \phi(t)\end{bmatrix}^T$ with the matrices

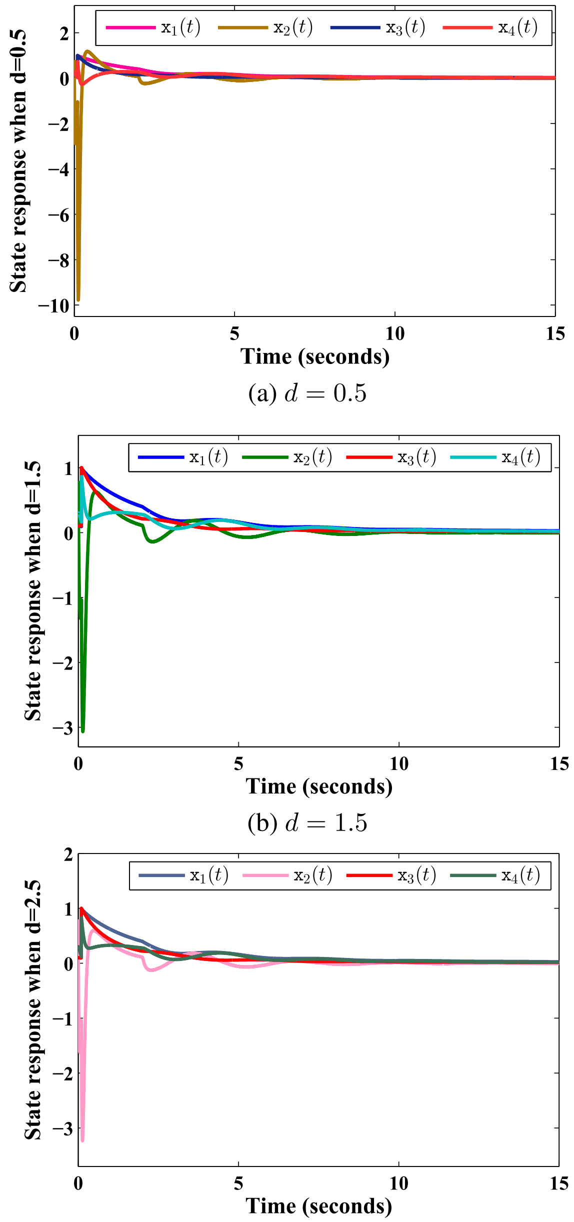
$$A = \begin{bmatrix} \upsilon - 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 \\ -0.5556 & 0 & -0.5556 & 0.5556 \\ 0 & 1 & -1 & 0 \end{bmatrix},$$

TABLE 2. Various disturbance attenuation performance.

Performance index	$\hat{\gamma}$ value
Passivity case($\theta = 0$)	0.000963
Mixed H_∞ and passivity($\theta = 0.5$)	0.000784
H_∞ case($\theta = 1$)	0.000803

$$A_d = \begin{bmatrix} \frac{-\upsilon}{d} & 0 & \frac{1}{d} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ 5 \\ 0 \\ 0 \end{bmatrix}.$$

Based on the above setting, we would also consider the disturbance term additionally with the coefficient matrix $B_w = \begin{bmatrix}0.4 & 0.1 & -0.2 & 0\end{bmatrix}^T$ and the disturbance function $\eth(t) = \exp(-0.4t)\sin(2t)$. Further, the analysis is carried out by choosing the fractional-order $\alpha = 0.9$ and by considering $\upsilon = 0.5, d = 1$. The fault function is assumed to take the value $R \in (0.2, 0.6)$ and the parameter $\epsilon = 0.1$. Then, the matrices pertaining to the uncertainties and gain fluctuations are chosen as $H = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, V = 0.2, F_1 = 0.1A, F_3 = 0.1A_d, F_4 = 0, R = 0.5, S = \begin{bmatrix}-0.5 & 0.7 & 0.6 & 0.4\end{bmatrix}$. By utilizing the designed control algorithm and by making use of



(a) $d = 0.5$

(b) $d = 1.5$

(c) $d = 2.5$

FIGURE 8. Tendency of system states for different time lag.

Corollary 1, the feasibility is computed with performance index $\hat{\gamma} = 0.4$ and the controller gain matrix is obtained as $K = \begin{bmatrix} 91.3705 & -10.4238 & -85.1332 & -63.5359 \end{bmatrix}$.

With the account of all the above parameters, the results on stability analysis for the combustion chamber of the rocket motor are presented in Figs. 7-8. Especially, the dynamics of the system states and trajectory of control input is plotted in Fig. 7 (a) and Fig. 7 (b) respectively. It can be viewed from this figure that the stability of the system is ensured through the proposed NFT controller even in the presence of exogenous disturbance. Also, the control trajectory stabilizes the system state under the fractional-order $\alpha = 0.9$. The considered dynamical system of rocket motor undergoes time-lag that is chosen as d. When this time lag changes, the dynamics of the state trajectories are clearly shown in Fig. 8. From this figure, it can be viewed that the system states are able to attain stability even when the states degrade with respect to different time lags, which reveals the efficiency of the proposed control law. The disturbance attenuation index $\hat{\gamma}$ under different performances is evaluated and tabulated in Table 2. It can be seen that the optimized mixed H_∞ and passivity performance level is better than H_∞ and passivity cases. Hence, it can be concluded that the prescribed mixed H_∞ and passivity performance is effective over external disturbances. It can be concluded that the considered system is stabilized by the non-fragile fault-tolerant controller in spite of different time lag, actuator fault, gain fluctuations and disturbances.

Example 3: Consider the uncertain fractional-order linear system as in Example 2 of [13] with the parameters as follows:

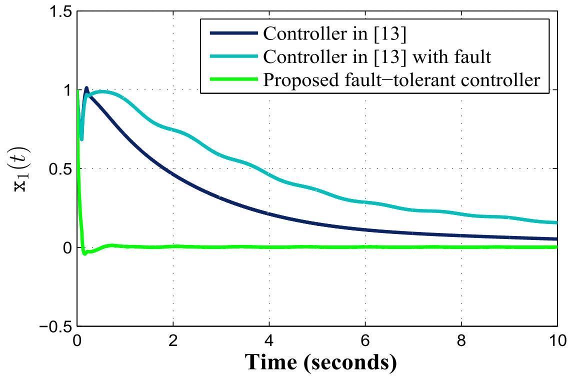
$$A = \begin{bmatrix} -3 & 2 & 1 \\ 2 & -5 & 1 \\ 3 & 1 & -4 \end{bmatrix}, \quad A_\tau = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & -1 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad B_w = \begin{bmatrix} 0.1 & 0.5 & 0.2 \end{bmatrix}^T,$$

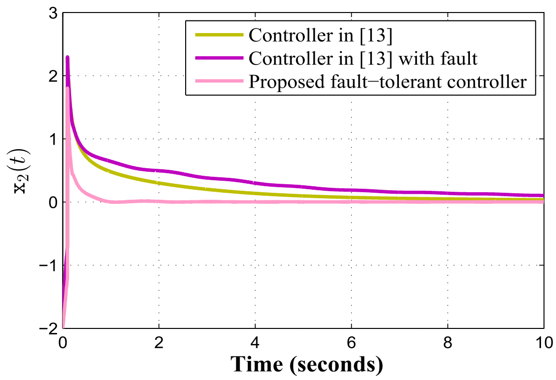
$$H = \begin{bmatrix} 0.1 & 0.1 & 0.2 \end{bmatrix}^T, \quad \tau = 0.1,$$
$$F_1 = \begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix},$$
$$F_4 = 0.1, \quad V = 0.4, \; L = 0.5,$$
$$S = \begin{bmatrix} 0.2 & 0.3 & -0.1 \end{bmatrix} \text{ and } \alpha = 0.9.$$

The fault function is chosen in the range $R \in (0.2, 0.6)$ and disturbance vector is considered as $\eth(t) = \exp(-t)\sin(5t)$. Under initial conditions $x(0) = \begin{bmatrix} 1 & -2 & 3 \end{bmatrix}^T$ and with the aid of Corollary 2, the controller gain matrix is obtained as $K = \begin{bmatrix} -16.9227 & -10.2482 & -7.8420 \end{bmatrix}$. For the comparison purpose, the state trajectories under the control law designed in [13] is plotted in the Fig. 9 along-with the state dynamics of the proposed NFT control scheme. In view of more validation, we also present the state dynamics of [13] under the influence of actuator faults. The corresponding states under controller [13] tend to oscillate more under the presence of faults whereas the proposed NFT controller is efficient enough to tackle the faults which is clearly pictured in Fig. 9. It can also be viewed from this figure that the proposed NFT control law suppresses the effects of actuator faults and makes the system states to converge to zero quickly even under the influence of gain fluctuations and external disturbances. This shows the significance of developed non-fragile fault-tolerant control design.

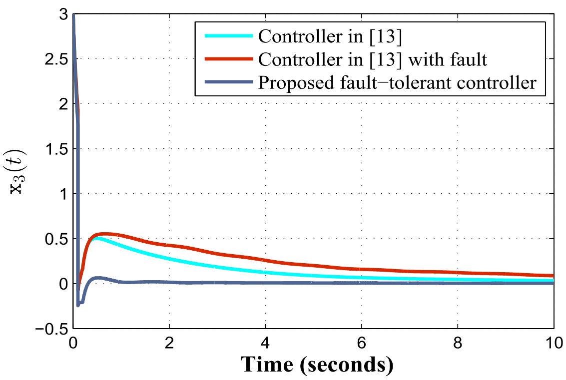
Remark 1: To the best of the authors' knowledge, there are some studies addressing the problem of stability and stabilization of control systems with distributed delays. In particular, the authors in [11] and [12] developed an impulsive and state feedback control for integer-order distributed delay systems. These works have been solved with the help of Lyapunov approach and LMI based criteria. Moreover, there are some works on distributed delay systems [14]–[18]. In [14],



(a) $x_1(t)$



(b) $x_2(t)$



(c) $x_3(t)$

FIGURE 9. Responses of system states under the controller proposed and controller in [13].

the stability of uncertain FO neutral systems with distributed delays and input saturation is reported. Besides, the authors in [17] studied the state feedback control design for polytopic uncertain FO system with distributed delays. However, the problem of non-fragile fault-tolerant control design for non-linear FO system subject to time-varying delay, distributed delay, fractional parametric uncertainties and external disturbance is not yet reported. In particular, the product rule and Leibnitz rule $D^{\alpha}(uv) = (D^{\alpha}u)v + u(D^{\alpha}v)$ does not hold for fractional-order systems. Moticated by the above works, in this paper a novel NFT control protocol is proposed to obatin the required perfromance for the addressed system (2).

V. CONCLUSION

The problem of non-fragile stabilization of fractional-order nonlinear system with distributed delay, time-varying delay and actuator faults is studied. A non-fragile fault-tolerant controller is designed to assure the stabilization of the addressed system even when the actuator faults, gain fluctuations and fractional parametric uncertainties occur. Then, by means of the Lyapunov approach in conjunction with the Razumikhin stability theorem, some adequate constraints are derived by the virtue of linear matrix inequalities which are then solved to acquire controller gain. Finally, three numerical examples including the rocket motor chamber model are manifested to validate the efficacy of the derived results.

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