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High-Order Internal Model Based Barrier Iterative Learning Control for Time-Iteration-Varying Parametric Uncertain Systems With Arbitrary Initial Errors

ZHI YANG [1], WEI WANG [2], YUNTAO ZHANG [1], QIUZHEN YAN [1], AND JIANPING CAI [3]
[1]College of Information Engineering, Zhejiang University of Water Resources and Electric Power, Hangzhou 310018, China
[2]School of Mathematics and Physics, Hebei GEO University, Shijiazhuang 050031, China
[3]College of Electrical Engineering, Zhejiang University of Water Resources and Electric Power, Hangzhou 310018, China

Corresponding author: Qiuzhen Yan (zjhzyqz@gamail.com)

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ABSTRACT In this paper, a high-order internal model based adaptive iterative learning control scheme is proposed to solve the trajectory tracking problem for a class of nonlinear systems with time-iteration-varying parametric uncertainties which are generated from a high-order internal model. A time-varying boundary layer is constructed to remove the nonzero initial error condition in ILC design. An adaptive iterative learning law is designed to deal with the time-iteration-varying parametric uncertainties. For improving the robustness and safety, a barrier Lyapunov function is adopted to controller design, thus making the filtering error constrained during each iteration. Even there exist nonzero initial state errors, the norm of tracking error vector will asymptotically converge to a tunable residual set as the iteration number increases. Simulation results show the effectiveness of the propose high-order internal model based filtering-error constraint adaptive learning scheme.

INDEX TERMS High-order internal model, iterative learning control, iteration-varying parametric uncertainties, barrier Lyapunov function, initial position problem.

I. INTRODUCTION

Iterative learning control (ILC) is an effective control strategy for the systems undertaking repetitive tasks over a finite time interval [1]–[12]. ILC utilizes the system invariance property to improve tracking performance, such that it can be used in many cases where the system modeling is difficult to be carried out. In view of its good application prospects in servo motors [13], traffic flows [14], robot manipulators [15], batch reactors [16], etc, ILC has attracted increasing attention during the past decades.

It is well known that adaptive iterative learning approach is effective in estimating unknown time-invariant constants and time-varying but iteration-independent parameters. Specifically, unknown time-variant constant parameters in ILC systems may be estimated by using differential learning approach [17], which is similar to the parameter estimation strategy in adaptive control. Based on the principle of parameter invariance, time-varying but iteration-independent parameters may be estimated by using difference learning approach [18]. In recent years, the exploration on how to estimate and compensate for more general parametric uncertainties in ILC has never ceased. As a significant progress in this issue, Yin *et al.* [19] proposed two adaptive ILC schemes for uncertain systems whose time-iteration-varying parameters are generated by high-order internal model (HOIM) [20], with zero initial error condition and alignment condition [21] considered, respectively. It should be noticed that resetting the controlled system to zero initial error at each iteration is an impossible job in real applications. Hence, the application

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scope of many ILC algorithms based on zero-initial-error assumptions is very limited. Once applying these ILC algorithms in real systems, system divergence may happen even if the initial error is very slight, which is the so-called the initial position problem of ILC [22]–[24]. On the other hand, a control system under alignment condition means its reference trajectory is smoothly closed, i.e., the initial state of reference trajectory is equal to the final state of reference trajectory. Up to now, for more general situations where neither zero initial errors condition nor alignment condition can be satisfied, how to design iterative learning controller for uncertain systems with time-iteration-varying parameters generated by HOIM is still unclear.

For the purpose of improving the robustness of system and the safety of equipments, there exist the requirements of constraining the system output, the system state, or the output tracking error in some situations. During the past 30 years, many experts and scholars carried out a great amount of theoretical and experimental research at this topic and put forward some theories and approaches, including maximal output admissible set strategy [25], constrained model predictive control [26], reference governor approach [27], convex optimization strategy [28] and barrier Lyaponov function approach [29], [30]. These theories and approaches served as solid references for implementing system constraints in controller design. As far as the system constraint solution in adaptive ILC is concerned, barrier Lyapunov function approach plays an important role for its convenience and effectiveness. The earlier studies have been reported in [31] and [32]. Specifically, [31] discusses the output constraint ILC design for SISO nonlinear systems under alignment condition. Reference [32] proposes an error constraint ILC algorithm for MIMO systems under alignment condition. Later on, barrier Lyapunov function approach for nonparametric systems with nonzero initial errors have been investigated in [33] and [34]. Constrained spatial adaptive iterative learning control are investigated in [35] and [36]. However, none of these works consider the issue of estimating and compensating for time-iteration-varying parameters during operations. How to develop an effective ILC scheme to solve the tracking problem for nonlinear system with time-iteration-varying parameters generated by HOIM, as well as to meet the requirement of arbitrary initial errors and filtering error constraint during operations, has not been addressed yet.

In this work, a HOIM based adaptive ILC scheme is proposed to solve the trajectory tracking problem for a class of time-iteration-varying parametric systems with arbitrary initial errors and filtering error constraint. We adopt a barrier Lyapunov function to address the requirements on system constraint. The technique of time-varying boundary layer is applied for relaxing the zero initial condition in ILC design, such that the reference trajectory is allowed to be any non-repetitive smooth curve with an arbitrary initial value, whether the initial state of reference trajectory is zero or any other bounded nonzero value. Under the proposed adaptive ILC scheme, the norm of tracking error vector will asymptotically converge to a tunable residual set as the iteration number increases, and the constraint to filtering error can be guaranteed. The main contributions are summarized as follows:

1) The initial position problem of ILC for nonlinear systems with iteration-time-varying parameters is considered.
2) The system constraint problem of HOIM based ILC systems is addressed.
3) Iteration-varying trajectory tracking for nonlinear systems with iteration-time-varying parameters is considered.

The paper is organized as follows. Section II introduces the problem formulation. In Section III, we propose a filtering-error constrained adaptive ILC scheme for nonlinear systems with iteration-time-varying parameters under nonzero initial error condition, via using the technique barrier Lyapunov function and time-varying boundary layer. The convergence analysis of closed-loop iteration-time-varying parametric systems is given in Section IV. In Section V, some simulation results are illustrated to verify the effectiveness of the proposed adaptive ILC scheme. Finally, Section VI concludes this work.

II. PROBLEM FORMULATION

Let us consider a class nonlinear dynamic systems operating over time interval $t \in [0, T]$ repetitively as follows:

$$\begin{cases} \dot{x}_{i,k} = x_{i+1,k}, \quad i = 1, 2, \cdots, n-1 \\ \dot{x}_{n,k} = \boldsymbol{\theta}_k^T(t)\boldsymbol{\xi}(\boldsymbol{x}_k) + \boldsymbol{w}^T(t)\boldsymbol{\zeta}(\boldsymbol{x}_k) + g(t)u(v_k), \end{cases} \tag{1}$$

where $k = 0, 1, 2\cdots$ is the iteration index, $\boldsymbol{x}_k = [x_{1,k}, x_{2,k}, \cdots, x_{n,k}]^T \in \mathbb{R}^n$ and v_k is the system vector. The control system is defined over a finite time interval $[0, T]$. $\boldsymbol{w}(t) \in \mathbb{R}^q$ is and iteration-independent unknown bounded parameter vector. $\boldsymbol{\theta}_k = [\theta_{1,k}, \theta_{2,k}, \cdots, \theta_{p,k}]^T \in \mathbb{R}^p$, where $\theta_{j,k}$ is an unknown bounded parameter with respect to both t and k, for $j = 1, 2, 3\cdots, p$. $\theta_{j,k}$ is defined in a bounded closed set Ω,. $\boldsymbol{\zeta}(\boldsymbol{x}_k)$ and $\boldsymbol{\xi}(\boldsymbol{x}_k) = [\xi_1(\boldsymbol{x}_k), \xi_2(\boldsymbol{x}_k), \cdots, \xi_p(\boldsymbol{x}_k)]^T$ is the basis function vector. $u(v_k)$ and v_k are the input and the output of an unknown deadzone nonlinearity. $u(v_k)$ is unavailable for measurement, whose value is determined according to

$$u(v_k) = \begin{cases} m_r(v_k - b_r) & v_k \geq b_r \\ 0 & b_l \leq v_k < b_r \\ m_l(v_k - b_l) & v_k < b_l. \end{cases} \tag{2}$$

Here, $m_r = m_l = m > 0$, $b_r > 0$, $b_l < 0$. m, b_r and b_l are all unknown. Let

$$b_{u,k} = \begin{cases} b_r & v_k \geq b_r \\ u(v_k) & b_l \leq v_k < b_r \\ b_l & v_k < b_l. \end{cases} \tag{3}$$

Based on (2) and (3), (1) can be rewritten as

$$\begin{cases} \dot{x}_{i,k} = x_{i+1,k}, \quad i = 1, 2, \cdots, n-1 \\ \dot{x}_{n,k} = \boldsymbol{\theta}_k^T(t)\boldsymbol{\xi}(\boldsymbol{x}_k) + \boldsymbol{w}^T(t)\boldsymbol{\zeta}(\boldsymbol{x}_k) + gmv_k + gmb_{u,k}. \end{cases} \quad (4)$$

Assumption 1: The time-iteration-varying parameter $\theta_{i,k}$ satisfies the following kth-order internal model in the iteration domain: For $j = 1, 2, \cdots, p$,

$$\theta_{j,k} = h_{j,1}\theta_{j,k-1}(t) + \cdots + h_{j,m_j}\theta_{j,k-m_j}(t), \quad (5)$$

where $h_{j,1}, \cdots, h_{j,m_j}$ are known constant coefficients. $\theta_{j,-1}(t), \cdots, \theta_{j,-m_j}(t)$ are unknown basis functions that are linearly independent.

The control task is to let the system state $\boldsymbol{x}_k(t)$ accurately track the reference signal $\boldsymbol{x}_d(t)$ under both nonzero initial errors and filtering error constraint. For the sake of brevity, the arguments in this paper are sometimes omitted when no confusion is likely to arise.

Remark 1: The deadzone nonlinearity often exists in the actuator of motion control, which has adverse effects on the control performance and even may cause divergence and instability to systems in severe cases. Therefore, for getting better control performance, its is necessary to applying corresponding compensation in the process of controller design. The deadzone input model considered in this work is similar to the one discussed in [37].

III. CONTROL SYSTEM DESIGN

By letting

$$\boldsymbol{\nu}_{j,k} = [\theta_{j,k-m_j+1}, \theta_{j,k-m_j}, \cdots, \theta_{j,k-1}, \theta_{j,k}]^T \quad (6)$$

and

$$H_j = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h_{j,m_j} & h_{j,m_j-1} & h_{j,m_j-2} & \cdots & h_{j,1} \end{pmatrix},$$

we can rewrite (5) as

$$\boldsymbol{\nu}_{j,k} = H_j\boldsymbol{\nu}_{j,k-1} = \cdots = H_j^k\boldsymbol{\nu}_{j,0}. \quad (7)$$

Let $\boldsymbol{\varphi}_{j,k}^T$ denote the last row of matrix H_j^k. From (6) and (7), we have

$$\theta_{j,k} = \boldsymbol{\nu}_{j,0}^T\boldsymbol{\varphi}_{j,k}. \quad (8)$$

Define $\boldsymbol{e}_k = [e_{1,k}, e_{2,k}, \cdots, e_{n,k}]^T = \boldsymbol{x}_k - \boldsymbol{x}_d$ and $s_k = c_1e_{1,k} + c_2e_{2,k} + \cdots + c_{n-1}e_{n-1,k} + e_{n,k}$, where $c_1, c_2, \cdots, c_{n-1}$ are the coefficients of a Hurwitz polynomial $\Delta(D) = D^{n-1} + c_{n-1}D^{n-2} + \cdots + c_1$.

Combining (4) with (7), we get the tracking error dynamics as

$$\begin{cases} \dot{e}_{i,k} = e_{i+1,k}, \quad i = 1, 2, \cdots, n-1 \\ \dot{e}_{n,k} = \sum_{j=1}^{p}(\boldsymbol{\nu}_{j,0}^T\boldsymbol{\varphi}_{j,k}\xi_{j,k}) + \boldsymbol{w}^T(t)\boldsymbol{\zeta}(\boldsymbol{x}_k) - \dot{x}_{n,d} \\ \qquad\quad + gmv_k + gmb_{u,k} \end{cases}$$

According to the definition of s_k, the time derivative of s_k may be obtained as

$$\dot{s}_k = \boldsymbol{c}^T\boldsymbol{e}_k + \sum_{j=1}^{p}(\boldsymbol{\nu}_{j,0}^T\boldsymbol{\varphi}_{j,k}\xi_{j,k}) + \boldsymbol{w}^T(t)\boldsymbol{\zeta}(\boldsymbol{x}_k) + gmv_k + gmb_{u,k} - \dot{x}_{n,d}. \quad (9)$$

Let us choose a candidate barrier Lyapunov function as

$$V_k(t) = \frac{s_{\phi,k}^2}{2(b_s^2 - s_{\phi,k}^2)}, \quad (10)$$

in which

$$s_{\phi,k}(t) = s_k(t) - \phi_k(t)\text{sat}_{-1,1}\left(\frac{s_k(t)}{\phi_k(t)}\right), \quad (11)$$

with

$$\phi_k(t) = |s_k(0)|\mathrm{e}^{-\mu t}, \quad \mu > 0. \quad (12)$$

The saturation function $\text{sat}_{\cdot,\cdot}(\cdot)$ in (11) is defined as follows: For a scalar $\hat{a}$, which is the estimation to a scalar a,

$$\text{sat}_{\underline{a},\bar{a}}(\hat{a}) := \begin{cases} \bar{a}, & \text{if } \hat{a} > \bar{a} \\ \hat{a}, & \text{if } \underline{a} \le \hat{a} \le \bar{a} \\ \underline{a}, & \text{if } \hat{a} < \underline{a}, \end{cases}$$

where $\underline{a}$ and $\bar{a}$ are the lower bound and upper bound of the scalar a, respectively. For a vector $\hat{\boldsymbol{a}} = [\hat{a}_1, \hat{a}_2, \cdots, \hat{a}_m] \in \mathbb{R}^m$, $\text{sat}_{\underline{a},\bar{a}}(\hat{\boldsymbol{a}}) := \left[\text{sat}_{\underline{a},\bar{a}}(\hat{a}_1), \text{sat}_{\underline{a},\bar{a}}(\hat{a}_2), \cdots, \text{sat}_{\underline{a},\bar{a}}(\hat{a}_m)\right]^T$. Note that $\phi_k(0) = s_k(0)$ leads to $s_{\phi,k}(0) = 0$, which is useful to solve the initial position problem of ILC. Since $\phi_k(t)$ converges to zero, it is a reasonable strategy to derive $|s_k(t)| \le \phi_k(t)$ by design iterative learning controller.

By taking the time derivative of V_k along (9), we have

$$\dot{V}_k(t) = \sigma_k s_{\phi,k}\Big[\boldsymbol{c}^T\boldsymbol{e}_k + \sum_{j=1}^{p}(\boldsymbol{\nu}_{j,0}^T\boldsymbol{\varphi}_{j,k}\xi_{j,k}) + \boldsymbol{w}^T(t)\boldsymbol{\zeta}(\boldsymbol{x}_k) + gmv_k + gmb_{u,k} - \dot{x}_{n,d} - \dot{\phi}_k(t)\text{sgn}\big(s_{\phi,k}\big)\Big], \quad (13)$$

where $\sigma_k = \frac{b_s^2}{(b_s^2 - s_{\phi,k}^2)^2}$. By using (11), we have

$$\begin{aligned} s_{\phi,k}\dot{\phi}_k(t)\text{sgn}\big(s_{\phi,k}\big) &= s_{\phi,k}\mu\phi_k(t)\text{sat}_{-1,1}\left(\frac{s_k}{\phi_k}\right) \\ &= \mu s_{\phi,k}(s_k - s_{\phi,k}), \end{aligned} \quad (14)$$

Substituting (14) into (13) yields

$$\begin{aligned} \dot{V}_k(t) &= \sigma_k s_{\phi,k}\Big[\boldsymbol{c}^T\boldsymbol{e}_k + \sum_{j=1}^{p}(\boldsymbol{\nu}_{j,0}^T\boldsymbol{\varphi}_{j,k}\xi_{j,k}) + \boldsymbol{w}^T(t)\boldsymbol{\zeta}(\boldsymbol{x}_k) + gmv_k \\ &\quad + gmb_{u,k} - \dot{x}_{n,d} + \mu(s_{\phi,k} - s_k)\Big] \\ &= \sigma_k s_{\phi,k}gm\Big[(gm)^{-1}\boldsymbol{c}^T\boldsymbol{e}_k + gm^{-1}\sum_{j=1}^{p}(\boldsymbol{\nu}_{j,0}^T\boldsymbol{\varphi}_{j,k}\xi_{j,k}) \\ &\quad + (gm)^{-1}\boldsymbol{w}^T(t)\boldsymbol{\zeta}(\boldsymbol{x}_k) + gmb_{u,k} + v_k + \rho_b\text{sat}_{-1,1}\left(\frac{s_k}{\phi_k}\right) \\ &\quad - (gm)^{-1}\dot{x}_{n,d} + (gm)^{-1}\mu(s_{\phi,k} - s_k)\Big] \end{aligned} \quad (15)$$

Define $\boldsymbol{\beta} = [gm^{-1}, gm^{-1}\mu, (gm)^{-1}\mathbf{w}^T(t)]^T$, $\boldsymbol{\psi}_k = [\mathbf{c}^T\boldsymbol{e}_k - \dot{x}_{n,d}, s_{\phi,k} - s_k, \boldsymbol{\zeta}^T(\boldsymbol{x}_k)]^T$, $\boldsymbol{\eta}_j = gm^{-1}\boldsymbol{\nu}_{j,0}$ and $\rho_b = \sup(|b_{u,k}|)$. Then, (15) can be rewritten as

$$\dot{V}_k(t) = \sigma_k s_{\phi,k} gm\big[\boldsymbol{\beta}^T\boldsymbol{\psi}_k + \sum_{j=1}^{p}(\boldsymbol{\eta}_j^T\boldsymbol{\varphi}_{j,k}\xi_{j,k}) + v_k\big] + \sigma_k|s_{\phi,k}|gm\rho_b. \quad (16)$$

On the basis of (16), we design the control law and learning laws as follows:

$$u_k = -\frac{\gamma_1}{\sigma_k}s_{\phi,k} - \boldsymbol{\beta}_k^T\boldsymbol{\psi}_k - \sum_{j=1}^{p}(\boldsymbol{\eta}_{j,k}^T\boldsymbol{\varphi}_{j,k}\xi_{j,k}) - \rho_{b,k}\mathrm{sat}_{-1,1}\left(\frac{s_k}{\phi_k}\right), \quad (17)$$

$$\boldsymbol{\beta_k} = \mathrm{sat}_{\underline{\beta},\bar{\beta}}(\boldsymbol{\beta}_{k-1}) + \gamma_2\sigma_k s_{\phi,k}\boldsymbol{\psi}_k,\ \boldsymbol{\beta_{-1}} = 0, \quad (18)$$

$$\boldsymbol{\eta}_{j,k} = \mathrm{sat}_{\underline{\eta_j},\bar{\eta_j}}(\boldsymbol{\eta}_{j,k-1}) + \gamma_3\sigma_k s_{\phi,k}\boldsymbol{\varphi}_{j,k}\xi_{j,k}, \quad \boldsymbol{\eta_{j,-1}} = 0, \quad (19)$$

$$\rho_{b,k} = \mathrm{sat}_{0,\bar{\rho}_b}(\rho_{b,k-1}) + \gamma_4\sigma_k|s_{\phi,k}|,\ \rho_{b,-1} = 0, \quad (20)$$

where $\gamma_1 > 0$, $\gamma_2 > 0$, $\gamma_3 > 0$ and $\gamma_4 > 0$. The recommended value intervals for $\gamma_1 - \gamma_4$ are [1, 10], [0.5, 5], [0.5, 5] and [0.01, 1], respectively. In this work, the initial value of $\boldsymbol{\beta}_k, \boldsymbol{\eta}_{j,k}$ and $\rho_{b,k}$ are set as $\boldsymbol{\beta}_{-1}(t) = 0$, $\boldsymbol{\eta}_{j,-1}(t) = 0$ and $\rho_{b,-1}(t) = 0$. In fact, $\boldsymbol{\beta}_{-1}(t)$ and $\boldsymbol{\eta}_{j,-1}(t)$ may be any two real numbers, and setting $\rho_{b,-1}(t)$ to a small positive number or zero is reasonable.

Remark 2: So far, some types of barrier Lyapunov functions have been proposed for state constraint or error constraint in controller design, such as $V = \frac{1}{2}\log\frac{k_b^2}{k_b^2-e^2}$, $V = \frac{k_b^2}{\tan(\frac{\pi e^T e}{2k_b^2})}$, $V = \frac{h}{2}\log\frac{k_b^2}{k_b^2-e^2} + \frac{1-h}{2}\log\frac{k_a^2}{k_a^2-e^2}$, $V = \frac{1}{2}\log\frac{e^2}{k_b^2-e^2}$, etc. These results promote the development of constraint design methods in adaptive control and adaptive ILC.

IV. CONVERGENCE ANALYSIS

Theorem 1: Consider the closed-loop nonlinear system (1) and control law and learning laws (17) -(20). Then, the following facts will hold:

(1) $\lim_{k\to+\infty} s_{\phi,k}(t) = 0$ and $\lim_{k\to+\infty}|s_k(t)| \le |s_k(0)|\mathrm{e}^{-\mu t}$ hold for $t \in [0, T]$.

(2) $|s_{\phi,k}(t)| < b_s$ and $|s_k(t)| \le |s_k(0)|\mathrm{e}^{-\mu t} + |s_{\phi,k}(t)| < |s_k(0)|\mathrm{e}^{-\mu t} + b_s$ are ensured for $k = 0, 1, 2, 3, \cdots, t \in [0, T]$.

(3) Let λ be the positive constant such that is $\Delta(D - \lambda)$ is still a Hurwitz polynomial, then

$$\lim_{k\to+\infty}\|\underline{\boldsymbol{e}}_k(t)\| \le m_\Phi \mathrm{e}^{-\lambda t}\|\underline{\boldsymbol{e}}_k(0)\| + |s_k(0)|\frac{\mathrm{e}^{-\mu t} - \mathrm{e}^{-\lambda t}}{\lambda - \mu} \quad (21)$$

holds for $\lambda \ne \mu$,

$$\lim_{k\to+\infty}\|\underline{\boldsymbol{e}}_k(t)\| \le= m_\Phi \mathrm{e}^{-\lambda t}\|\underline{\boldsymbol{e}}_k(0)\| + |s_k(0)|\mathrm{e}^{-\lambda t}t \quad (22)$$

holds for $\lambda = \mu$; and

$$\lim_{k\to+\infty}|e_{n,k}| \le \sum_{j=1}^{n}|c_j e_{j,k}| + \mathrm{e}^{-\mu t}|s_k(0)| \quad (23)$$

for some positive constant m_Φ and $t \in [0, T]$, where $\underline{\boldsymbol{e}}_k = [e_{1,k}, e_{2,k}, \cdots, e_{n-1,k}]^T$.

(4) All adjustable control parameters $\boldsymbol{\beta}_k(t)$, $\boldsymbol{\vartheta}_k(t)$, $\rho_k(t)$, $\varrho_k(t)$, and internal signals $x_k(t), e_k(t), u_k(t)$ are bounded $\forall t \in [0, T]$ and $\forall k \ge 0$.

Proof: Firstly, let us analyze the difference of barrier Lyapunov functional between the adjacent iterations. Define a barrier Lyapunov functional as follows:

$$L_k = V_k + \frac{gm}{2\gamma_2}\int_0^t \tilde{\boldsymbol{\beta}}_k^T\tilde{\boldsymbol{\beta}}_k d\tau + \sum_{j=1}^{p}\frac{mg}{2\gamma_3}\int_0^t \tilde{\boldsymbol{\eta}}_{j,k}^T\tilde{\boldsymbol{\eta}}_{j,k}d\tau + \frac{gm}{2\gamma_4}\int_0^t \tilde{\rho}_{b,k}^2 d\tau, \quad (24)$$

where $\tilde{\boldsymbol{\beta}}_k = \boldsymbol{\beta} - \boldsymbol{\beta}_k$ and $\tilde{\boldsymbol{\eta}}_{j,k} = \boldsymbol{\eta}_j - \boldsymbol{\eta}_{j,k}$. While $k > 0$, it is obvious that

$$L_k - L_{k-1} = V_k - V_{k-1} + \frac{gm}{2\gamma_2}\int_0^t(\tilde{\boldsymbol{\beta}}_k^T\tilde{\boldsymbol{\beta}}_k - \tilde{\boldsymbol{\beta}}_{k-1}^T\tilde{\boldsymbol{\beta}}_{k-1})d\tau + \sum_{j=1}^{p}\frac{mg}{2\gamma_3}\int_0^t(\tilde{\boldsymbol{\eta}}_{j,k}^T\tilde{\boldsymbol{\eta}}_{j,k} - \tilde{\boldsymbol{\eta}}_{j,k-1}^T\tilde{\boldsymbol{\eta}}_{j,k-1})d\tau + \frac{gm}{2\gamma_4}\int_0^t(\tilde{\rho}_{b,k}^2 - \tilde{\rho}_{b,k}^2)d\tau. \quad (25)$$

Substituting (17) into (16) leads to

$$\dot{V}_k = -\gamma_1 gm s_{\phi,k}^2 + \sigma_k s_{\phi,k} gm\big[\tilde{\boldsymbol{\beta}}_k^T\boldsymbol{\psi}_k + \sum_{j=1}^{p}(\tilde{\boldsymbol{\eta}}_{j,k}^T\boldsymbol{\varphi}_{j,k}\xi_{j,k})\big] + \sigma_k|s_{\phi,k}|gm\tilde{\rho}_{b,k} \quad (26)$$

According to (11), we can see that $s_{\phi,k}(0) = 0$ holds. From (26), we have

$$V_k = -\gamma_1 mg\int_0^t s_{\phi,k}^2 d\tau + \int_0^t \sigma_k s_{\phi,k} gm\tilde{\boldsymbol{\beta}}_k^T\boldsymbol{\psi}_k d\tau + \int_0^t \sigma_k s_{\phi,k} gm\sum_{j=1}^{p}(\tilde{\boldsymbol{\eta}}_{j,k}^T\boldsymbol{\varphi}_{j,k}\xi_{j,k})d\tau + \int_0^t \sigma_k|s_{\phi,k}|gm\tilde{\rho}_{b,k}d\tau. \quad (27)$$

From (18), we obtain

$$\begin{aligned}
&\frac{gm}{2\gamma_2}(\tilde{\boldsymbol{\beta}}_k^T\tilde{\boldsymbol{\beta}}_k - \tilde{\boldsymbol{\beta}}_{k-1}^T\tilde{\boldsymbol{\beta}}_{k-1}) + gm\sigma_k s_{\phi,k}\tilde{\boldsymbol{\beta}}_k^T\boldsymbol{\psi}_k \\
&\le \frac{gm}{2\gamma_2}[(\boldsymbol{\beta} - \boldsymbol{\beta}_k)^T(\boldsymbol{\beta} - \boldsymbol{\beta}_k) - (\boldsymbol{\beta} - \mathrm{sat}_{\underline{\beta},\bar{\beta}}(\boldsymbol{\beta}_{k-1}))^T(\boldsymbol{\beta} - \mathrm{sat}_{\underline{\beta},\bar{\beta}}(\boldsymbol{\beta}_{k-1}))] + gm\sigma_k s_{\phi,k}\tilde{\boldsymbol{\beta}}_k^T\boldsymbol{\psi}_k \\
&\le \frac{gm}{2\gamma_2}(2\boldsymbol{\beta} - \boldsymbol{\beta}_k - \mathrm{sat}_{\underline{\beta},\bar{\beta}}(\boldsymbol{\beta}_{k-1}))^T(\mathrm{sat}_{\underline{\beta},\bar{\beta}}(\boldsymbol{\beta}_{k-1}) - \boldsymbol{\beta}_k) + gm\sigma_k s_{\phi,k}\tilde{\boldsymbol{\beta}}_k^T\boldsymbol{\psi}_k \\
&\le \frac{gm}{\gamma_2}(\boldsymbol{\beta} - \boldsymbol{\beta}_k)^T[\mathrm{sat}_{\underline{\beta},\bar{\beta}}(\boldsymbol{\beta}_{k-1}) - \boldsymbol{\beta}_k + \gamma_2\sigma_k s_{\phi,k}\boldsymbol{\psi}_k] \\
&= 0.
\end{aligned} \quad (28)$$

Substituting (27) and (28) into (25), we have

$$\begin{aligned}
&L_k - L_{k-1} \\
&\leq -V_{k-1} + \int_0^t \sigma_k s_{\phi,k} gm \sum_{j=1}^{p} (\tilde{\boldsymbol{\eta}}_{j,k}^T \boldsymbol{\varphi}_{j,k} \xi_{j,k}) d\tau \\
&\quad + \int_0^t \sigma_k |s_{\phi,k}| gm \tilde{\rho}_{b,k} d\tau + \frac{gm}{2\gamma_4} \int_0^t (\tilde{\rho}_{b,k}^2 - \tilde{\rho}_{b,k}^2) d\tau \\
&\quad + \sum_{j=1}^{p} \frac{mg}{2\gamma_3} \int_0^t (\tilde{\boldsymbol{\eta}}_{j,k}^T \tilde{\boldsymbol{\eta}}_{j,k} - \tilde{\boldsymbol{\eta}}_{j,k-1}^T \tilde{\boldsymbol{\eta}}_{j,k-1}) d\tau \qquad (29)
\end{aligned}$$

From (19), we have

$$\begin{aligned}
&\frac{gm}{2\gamma_3}(\tilde{\boldsymbol{\eta}}_{j,k}^T \tilde{\boldsymbol{\eta}}_{j,k} - \tilde{\boldsymbol{\eta}}_{j,k-1}^T \tilde{\boldsymbol{\eta}}_{j,k-1}) + gm\sigma_k s_{\phi,k} \tilde{\boldsymbol{\eta}}_{j,k}^T \boldsymbol{\varphi}_{j,k} \xi_{j,k} \\
&\leq \frac{gm}{2\gamma_3}[(\boldsymbol{\eta}_j - \boldsymbol{\eta}_{j,k})^T (\boldsymbol{\eta}_j - \boldsymbol{\eta}_{j,k}) - (\boldsymbol{\eta}_j - \mathrm{sat}_{\underline{\eta},\bar{\eta}}(\boldsymbol{\eta}_{j,k-1}))^T (\boldsymbol{\eta}_j \\
&\quad - \mathrm{sat}_{\underline{\eta},\bar{\eta}}(\boldsymbol{\eta}_{j,k-1}))] + gm\sigma_k s_{\phi,k} \tilde{\boldsymbol{\eta}}_{j,k}^T \boldsymbol{\varphi}_{j,k} \xi_{j,k} \\
&\leq \frac{gm}{2\gamma_3}(2\boldsymbol{\eta}_j - \boldsymbol{\eta}_{j,k} - \mathrm{sat}_{\underline{\eta},\bar{\eta}}(\boldsymbol{\eta}_{j,k-1}))^T (\mathrm{sat}_{\underline{\eta},\bar{\eta}}(\boldsymbol{\eta}_{j,k-1}) - \boldsymbol{\eta}_{j,k}) \\
&\quad + gm\sigma_k s_{\phi,k} \tilde{\boldsymbol{\eta}}_{j,k}^T \boldsymbol{\varphi}_{j,k} \xi_{j,k} \\
&\leq \frac{gm}{\gamma_3}(\boldsymbol{\eta}_j - \boldsymbol{\eta}_{j,k})^T [\mathrm{sat}_{\underline{\eta},\bar{\eta}}(\boldsymbol{\eta}_{j,k-1}) - \boldsymbol{\eta}_{j,k} + \gamma_3 \sigma_k s_{\phi,k} \boldsymbol{\varphi}_{j,k} \xi_{j,k}] \\
&= 0. \qquad (30)
\end{aligned}$$

Similarly, from (20), we obtain

$$\begin{aligned}
&\frac{gm}{2\gamma_4}(\tilde{\varrho}_{b,k}^2 - \tilde{\varrho}_{b,k-1}^2) + gm\sigma_k |s_{\phi,k}| \tilde{\varrho}_{b,k} \\
&\leq \frac{gm}{2\gamma_4}(2\varrho_b - \varrho_{b,k} - \mathrm{sat}_{0,\bar{\varrho}}(\varrho_{b,k-1}))(\mathrm{sat}_{0,\bar{\varrho}}(\varrho_{b,k-1}) - \varrho_{b,k}) \\
&\quad + gm\sigma_k |s_{\phi,k}| \tilde{\varrho}_{b,k} \\
&\leq \frac{gm}{\gamma_4}(\varrho_b - \varrho_k)\big[\mathrm{sat}_{0,\bar{\varrho}}(\varrho_{b,k-1}) - \varrho_{b,k} + \gamma_4 gm\sigma_k |s_{\phi,k}| \tilde{\varrho}_{b,k}\big] \\
&= 0. \qquad (31)
\end{aligned}$$

Substituting (30) and (31) into (29), we have

$$L_k - L_{k-1} \leq -V_{k-1} \qquad (32)$$

By using the recursive relation (32) and the definition of V_{k-1}, we can further obtain

$$L_k(t) \leq L_0(t) - \frac{1}{2} \sum_{j=0}^{k-1} \frac{s_{\phi,k}^2}{b_s^2 - s_{\phi,k}^2} \qquad (33)$$

for $k > 0$.

Secondly, we will prove that $b_s^2(t) - s_{\phi,k}^2(t) > 0, \forall k, \forall t$. On the basis of the definition of L_k and (26), we can get the time derivative of L_k as

$$\begin{aligned}
\dot{L}_k &= -\gamma_{1,k} gm s_{\phi,k}^2 + \sigma_k gm s_{\phi,k} (\tilde{\boldsymbol{\beta}}_k^T \boldsymbol{\psi}_k + \sum_{j=1}^{p} \tilde{\boldsymbol{\eta}}_{j,k}^T \boldsymbol{\varphi}_k \xi_k) \\
&\quad + \sigma_k |s_{\phi,k}| gm \tilde{\rho}_{b,k} + \frac{gm}{2\gamma_2} \tilde{\boldsymbol{\beta}}_k^T \tilde{\boldsymbol{\beta}}_k + \sum_{j=1}^{p} \frac{gm}{2\gamma_3} \tilde{\boldsymbol{\eta}}_k^T \tilde{\boldsymbol{\eta}}_k \\
&\quad + \frac{gm}{2\gamma_4} \tilde{\rho}_{b,k}^2. \qquad (34)
\end{aligned}$$

By using (18), we have

$$\begin{aligned}
&\sigma_k s_{\phi,k} gm \tilde{\boldsymbol{\beta}}_k^T \boldsymbol{\psi}_k + \frac{1}{2\gamma_2} gm \tilde{\boldsymbol{\beta}}_k^T \tilde{\boldsymbol{\beta}}_k \\
&= \frac{gm}{2\gamma_2}(\boldsymbol{\beta} - \boldsymbol{\beta}_k)^T (2\boldsymbol{\beta}_k - 2\mathrm{sat}_{\underline{\beta},\bar{\beta}}(\boldsymbol{\beta}_{k-1}) + \boldsymbol{\beta} - \boldsymbol{\beta}_k) \\
&= \frac{gm}{2\gamma_2}[-\boldsymbol{\beta}_k^T \boldsymbol{\beta}_k + \boldsymbol{\beta}^T \boldsymbol{\beta} - 2\boldsymbol{\beta}^T \mathrm{sat}_{\underline{\beta},\bar{\beta}}(\boldsymbol{\beta}_{k-1}) \\
&\quad + 2\boldsymbol{\beta}_k^T \mathrm{sat}_{\underline{\beta},\bar{\beta}}(\boldsymbol{\beta}_{k-1})] \\
&= -\frac{gm}{2\gamma_2}[\boldsymbol{\beta}_k - \mathrm{sat}_{\underline{\beta},\bar{\beta}}(\boldsymbol{\beta}_{k-1})]^T [\boldsymbol{\beta}_k - \mathrm{sat}_{\underline{\beta},\bar{\beta}}(\boldsymbol{\beta}_{k-1})] \\
&\quad + \frac{gm}{2\gamma_2}[\mathrm{sat}_{\underline{\beta},\bar{\beta}}(\boldsymbol{\beta}_{k-1}^T)\mathrm{sat}_{\underline{\beta},\bar{\beta}}(\boldsymbol{\beta}_{k-1}) + \boldsymbol{\beta}^T \boldsymbol{\beta} \\
&\quad - 2\boldsymbol{\beta}^T \mathrm{sat}_{\underline{\beta},\bar{\beta}}(\boldsymbol{\beta}_{k-1})] \\
&\leq \frac{gm}{2\gamma_2}[\mathrm{sat}_{\underline{\beta},\bar{\beta}}(\boldsymbol{\beta}_{k-1}^T)\mathrm{sat}_{\underline{\beta},\bar{\beta}}(\boldsymbol{\beta}_{k-1}) + \boldsymbol{\beta}^T \boldsymbol{\beta} \\
&\quad - 2\boldsymbol{\beta}^T \mathrm{sat}_{\underline{\beta},\bar{\beta}}(\boldsymbol{\beta}_{k-1})] \qquad (35)
\end{aligned}$$

Obviously, each term in $\frac{g}{2\gamma_2}[\mathrm{sat}_{\underline{\beta},\bar{\beta}}(\boldsymbol{\beta}_{k-1}^T)\mathrm{sat}_{\underline{\beta},\bar{\beta}}(\boldsymbol{\beta}_{k-1}) + \boldsymbol{\beta}^T \boldsymbol{\beta} - 2\boldsymbol{\beta}^T \mathrm{sat}_{\underline{\beta},\bar{\beta}}(\boldsymbol{\beta}_{k-1})]$ is bounded. Therefore, there exits a positive number c_β, which satisfies

$$\sigma_k gm s_{\phi,k} \tilde{\boldsymbol{\beta}}_k^T \boldsymbol{\psi}_k + \frac{g}{2\gamma_2} gm \tilde{\boldsymbol{\beta}}_k^T \tilde{\boldsymbol{\beta}}_k \leq c_\beta. \qquad (36)$$

Similarly, by using (19) and (20), there exist positive numbers $c_{\eta,j}$ and c_ρ, which meet

$$\begin{aligned}
&\sigma_k s_{\phi,k} gm \tilde{\boldsymbol{\eta}}_{j,k}^T \boldsymbol{\varphi}_{j,k} \xi_{j,k} + \frac{gm}{2\gamma_j} \tilde{\boldsymbol{\eta}}_{j,k}^T \tilde{\boldsymbol{\eta}}_{j,k} \\
&= \frac{gm}{2\gamma_j}[-\boldsymbol{\eta}_{j,k}^T \boldsymbol{\eta}_{j,k} + \boldsymbol{\eta}_j^T \boldsymbol{\eta}_j - 2\boldsymbol{\eta}_j^T \mathrm{sat}_{\underline{\eta}_j,\bar{\eta}_j}(\boldsymbol{\eta}_{j,k-1}) \\
&\quad + 2\boldsymbol{\eta}_{j,k}^T \mathrm{sat}_{\underline{\eta}_j,\bar{\eta}_j}(\boldsymbol{\eta}_{j,k-1})] \\
&= -\frac{gm}{2\gamma_j}[\boldsymbol{\eta}_{j,k} - \mathrm{sat}_{\underline{\eta}_j,\bar{\eta}_j}(\boldsymbol{\eta}_{j,k-1})]^T [\boldsymbol{\eta}_{j,k} - \mathrm{sat}_{\underline{\eta}_j,\bar{\eta}_j}(\boldsymbol{\eta}_{j,k-1})] \\
&\quad + \frac{gm}{2\gamma_j}[\mathrm{sat}_{\underline{\eta}_j,\bar{\eta}_j}(\boldsymbol{\eta}_{j,k-1}^T)\mathrm{sat}_{\underline{\eta}_j,\bar{\eta}_j}(\boldsymbol{\eta}_{j,k-1}) + \boldsymbol{\eta}_j^T \boldsymbol{\eta}_j \\
&\quad - 2\boldsymbol{\eta}_j^T \mathrm{sat}_{\underline{\eta}_j,\bar{\eta}_j}(\boldsymbol{\eta}_{j,k-1})] \\
&\leq \frac{gm}{2\gamma_j}[\mathrm{sat}_{\underline{\eta}_j,\bar{\eta}_j}(\boldsymbol{\eta}_{j,k-1}^T)\mathrm{sat}_{\underline{\eta}_j,\bar{\eta}_j}(\boldsymbol{\eta}_{j,k-1}) + \boldsymbol{\eta}_j^T \boldsymbol{\eta}_j \\
&\quad - 2\boldsymbol{\eta}_j^T \mathrm{sat}_{\underline{\eta}_j,\bar{\eta}_j}(\boldsymbol{\eta}_{j,k-1})] \\
&\leq c_{\eta,j} \qquad (37)
\end{aligned}$$

and

$$\begin{aligned}
&\sigma_k |s_{\phi,k}| \tilde{\rho}_{b,k} + \frac{1}{2\gamma_5} \tilde{\rho}_k^2 \\
&= \frac{1}{2\gamma_5}[-\rho_k^2 + \rho_b^2 - 2\rho \mathrm{sat}_{\underline{\rho_b},\bar{\rho}_b}(\rho_{b,k-1}) \\
&\quad + 2\rho_k \mathrm{sat}_{\underline{\rho_b},\bar{\rho}_b}(\rho_{b,k-1})] \\
&= \frac{1}{2\gamma_5}[\mathrm{sat}_{\underline{\rho_b},\bar{\rho}_b}(\rho_{b,k-1})\mathrm{sat}_{\underline{\rho_b},\bar{\rho}_b}(\rho_{b,k-1}) + \rho_b^2 \\
&\quad - 2\rho \mathrm{sat}_{\underline{\rho_b},\bar{\rho}_b}(\rho_{b,k-1})] - \frac{1}{2\gamma_5}[\rho_k - \mathrm{sat}_{\underline{\rho_b},\bar{\rho}_b}(\rho_{b,k-1})]^2 \\
&\leq \frac{1}{2\gamma_5}[\mathrm{sat}_{\underline{\rho_b},\bar{\rho}_b}(\rho_{b,k-1})\mathrm{sat}_{\underline{\rho_b},\bar{\rho}_b}(\rho_{b,k-1}) + \rho_b^2 \\
&\quad - 2\rho \mathrm{sat}_{\underline{\rho_b},\bar{\rho}_b}(\rho_{b,k-1})] \\
&\leq c_\rho, \qquad (38)
\end{aligned}$$

respectively. Substituting (35)-(38) into (34) yields

$$\dot{L}_k \leq c_\beta + \sum_{j=1}^{p} c_{\eta,j} + c_\rho. \tag{39}$$

Due to $L_k(0) = 0$, it follows from (39) that

$$L_k(t) \leq t(c_\beta + \sum_{j=1}^{p} c_{\eta,j} + c_\rho) \tag{40}$$

Further, we have

$$V_k(t) = \frac{s_{\phi k}^2(t)}{2(b_s^2 - s_{\phi,k}^2(t))} \leq t(c_\beta + \sum_{j=1}^{p} c_{\eta,j} + c_\rho). \tag{41}$$

Since $s_{\phi,k}^2(0) = 0$ for any k, once $s_{\phi,k}^2(t)$ increases nearly to b_s^2 for any $t \in (0, T]$, which is contrary to the inequality (41). Therefore,

$$s_{\phi,k}^2(t) < b_s^2 \tag{42}$$

holds for $t \in [0, T]$, which is equivalent to the fact that

$$|s_{\phi,k}(t)| < b_s \tag{43}$$

holds for $t \in [0, T]$. Then, according to the definition of $s_{\phi,k}$, we have

$$|s_k(t)| \leq |s_k(0)|\mathrm{e}^{-\mu t} + |s_{\phi,k}(t)| < |s_k(0)|\mathrm{e}^{-\mu t} + b_s. \tag{44}$$

Meanwhile, from (42), we can also see that $V_k(t) \geq 0$ and $L_k(t) \geq 0$ hold. Thus, from (40), we can conclude that both $L_k(t)$ is a negative bounded number. Based on this and (33), we have

$$L_k(t) \leq L_0(t) - \frac{1}{2b_s^2}\sum_{j=0}^{k-1} s_{\phi,k}^2, \tag{45}$$

which leads to

$$\lim_{k\to+\infty} s_{\phi,k}(t) = 0, \tag{46}$$

and

$$\lim_{k\to+\infty} |s_k(t)| \leq |s_k(0)|\mathrm{e}^{-\mu t}. \tag{47}$$

According to the definition of s_k, we have

$$\dot{\boldsymbol{e}}_k = A_c \boldsymbol{e}_k + \boldsymbol{B}_c s_k, \tag{48}$$

where $\boldsymbol{B}_c = [0, 0, \cdots, 0, 1]^T$ and

$$A_c = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -c_1 & -c_2 & -c_3 & \cdots & -c_{n-1} \end{pmatrix}.$$

The solution of (48) in time domain is given by

$$\underline{\boldsymbol{e}}_k(t) = \Phi(t)\underline{\boldsymbol{e}}_k(0) + \int_0^t \Phi(t-\tau)B_c s_k(\tau)d\tau, \tag{49}$$

where the state transition matrix satisfies $\|\Phi(t)\| \leq m_\Phi \mathrm{e}^{-\lambda t}$ for some suitable positive constant m_Φ. Taking norms on (49), we have

$$\|\underline{\boldsymbol{e}}_k(t)\| \leq m_\Phi \mathrm{e}^{-\lambda t}\|\boldsymbol{e}_k(0)\| + \int_0^t \mathrm{e}^{-\lambda(t-\tau)}\|B_c\||s_k(\tau)|d\tau. \tag{50}$$

While $k \to +\infty$, from (47) and (50), if $\lambda \neq \mu$,

$$\begin{aligned} \|\underline{\boldsymbol{e}}_k(t)\| &\leq m_\Phi \mathrm{e}^{-\lambda t}\|\underline{\boldsymbol{e}}_k(0)\| + \int_0^t \mathrm{e}^{-\lambda(t-\tau)}\mathrm{e}^{-\mu t}|s_k(0)|d\tau \\ &\leq m_\Phi \mathrm{e}^{-\lambda t}\|\underline{\boldsymbol{e}}_k(0)\| + |s_k(0)|\frac{\mathrm{e}^{-\mu t} - \mathrm{e}^{-\lambda t}}{\lambda - \mu} \end{aligned} \tag{51}$$

holds; if $\lambda = \mu$,

$$\begin{aligned} \|\underline{\boldsymbol{e}}_k(t)\| &\leq m_\Phi \mathrm{e}^{-\lambda t}\|\underline{\boldsymbol{e}}_k(0)\| + \int_0^t \mathrm{e}^{-\lambda(t-\tau)}\mathrm{e}^{-\mu\tau}|s_k(0)|d\tau \\ &\leq m_\Phi \mathrm{e}^{-\lambda t}\|\underline{\boldsymbol{e}}_k(0)\| + |s_k(0)|\mathrm{e}^{-\lambda t}\int_0^t \mathrm{e}^{(\lambda-\mu)\tau}d\tau \\ &= m_\Phi \mathrm{e}^{-\lambda t}\|\underline{\boldsymbol{e}}_k(0)\| + |s_k(0)|\mathrm{e}^{-\lambda t}t \end{aligned} \tag{52}$$

holds. This concludes (21) and (22). Then, tracking performance shown in (23) can be easily derived by using the definition of s_k.

On the other hand, from (34) and (39), we can see that $s_{\phi,k}, \boldsymbol{\beta}_k, \boldsymbol{\psi}_k, \boldsymbol{\eta}_{j,k}, \boldsymbol{\varphi}_k, \xi_k, \rho_{b,k}$ and $\boldsymbol{\beta}_k$ are bounded. Further, the boundedness of s_k, $\boldsymbol{e}_k$, v_k, u_k and other signals can be ensured. ■

Through constraining $s_{\phi,k}$, we implement the constraint to s_k during each iteration cycle. It should be noted that the reference trajectory is allowed to be iteration-varying in this work. Since $\rho_b = \max(mb_r, m|b_l|)$, a larger $\max(b_r, |b_l|)$ will bring about a worse adverse effect in control performance. To mitigate the damage caused by deadzone nonlinearity, we design the difference learning law (20) to estimate and compensate for ρ_b.

V. NUMERICAL SIMULATION

Consider a one-link robotic manipulator [19]

$$\begin{cases} \dot{x}_{1,k} = x_{2,k}, \\ \dot{x}_{2,k} = \dfrac{M_k g_G l}{J}\sin(x_{1,k}) - \alpha(t)x_{2,k} + \dfrac{1}{J}u(v_k), \end{cases} \tag{53}$$

where $x_{1,k}$ and $x_{2,k}$ are the joint angle and the angular velocity, respectively. J and $\alpha(t)$ are unknown parameters. l and g_G are known parameters. $M_k = m_o - v_0 t + \omega_k$, where $m_o - v_0 t$ are unknown iteration-independent parameter, $\omega_k = -0.2\omega_{k-1} - 0.3\omega_{k-2}$, $\omega_{-1}(t) = \int_0^t 0.08\cos(\frac{\pi\tau}{10})d\tau$, $\omega_0(t) = -\int_0^t (0.96\sin(1.2\tau) + 0.8\cos(2\tau))d\tau$. By letting $\theta_k(t) = \frac{1}{J}\omega_k$, $\xi_k = g_G l\sin(x_{1,k})$, $\boldsymbol{w} = [(m_o - v_0 t)/J, -\alpha(t)]^T$, $\boldsymbol{\zeta}_k = [g_G l\sin(x_{1,k}), x_{2,k}]^T$ and $g(t) = 1/J$, we can transform (53) to the form of (1) with $n = 2, p = 1$.

In the simulation, model parameters are set as $J = 1.667\mathrm{kg\,m^2}$, $l = 0.9\mathrm{m}$, $g_G = 9.8\mathrm{ms^{-2}}$, $\alpha(t) = 0.2/(1 + t/10)$, $m_o = 23\mathrm{kg}$, $v_0 = 4.5\mathrm{kg\cdot s^{-1}}$, $b_r = 0.5$, $b_l = -0.6$, $m_r = 1.2$, $m_l = 1.2$. The reference signal and

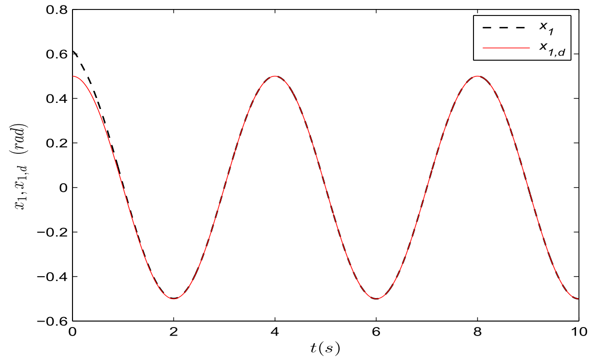

FIGURE 1. x_1 and $x_{1,d}$ (constraint ILC).

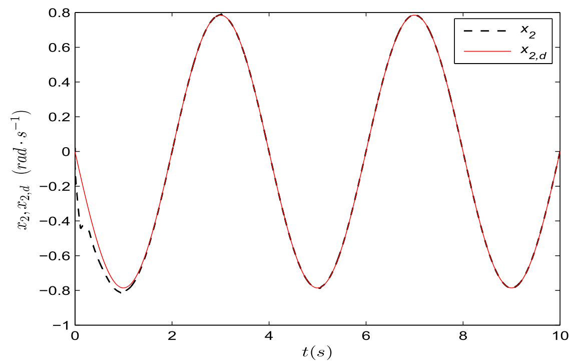

FIGURE 2. x_2 and $x_{2,d}$ (constraint ILC).

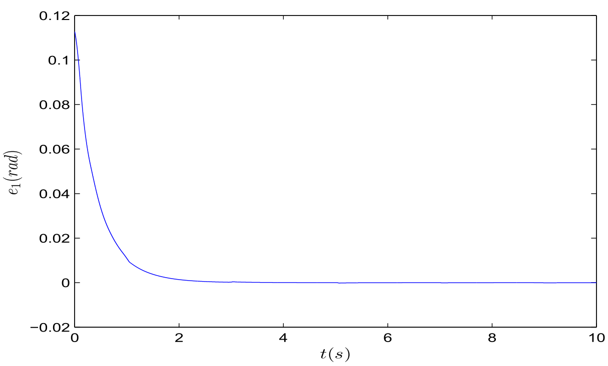

FIGURE 3. The error e_1 (constraint ILC).

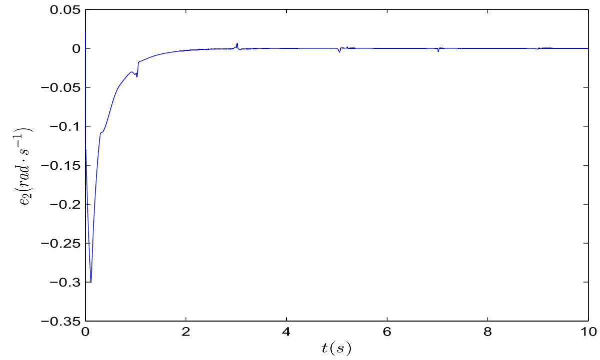

FIGURE 4. The error e_2 (constraint ILC).

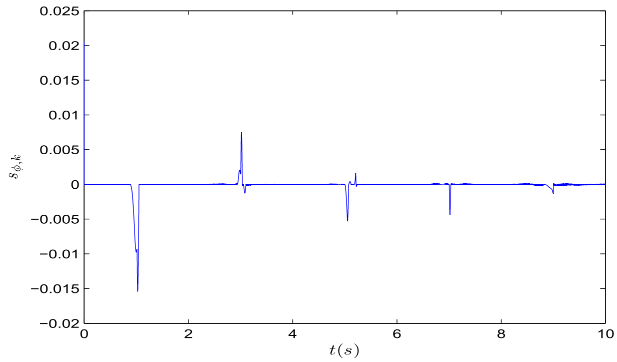

FIGURE 5. $s_{\phi,k}$ during 15th iteration (constraint ILC).

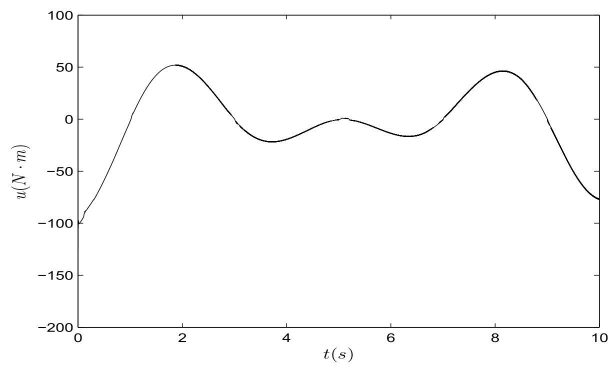

FIGURE 6. Control input (constraint ILC).

initial system state are $\boldsymbol{x}_d(t) = [x_{1,d}(t), x_{2,d}(t)]^T = [(0.05 - 0.1\text{rand}_1(k)) + 0.5\cos(0.5\pi t), -0.25\sin(0.5\pi t)]^T$ and $\boldsymbol{x}_k(0) = [1.2 + 0.1\text{rand}_2(k), 0.05\text{rand}_3(k) - 0.05]^T$, respectively, where $\text{rand}_1(k)$-$\text{rand}_3(k)$ represent random numbers between 0 and 1. The control law and learning laws are constructed according to (17)-(20) is

$$u_k = -\frac{\gamma_1}{\sigma_k} s_{\phi,k} - \boldsymbol{\beta}_k^T \boldsymbol{\psi}_k - \boldsymbol{\eta}_k^T \boldsymbol{\varphi}_k g_G l \sin(x_{1,k}) - \rho_{b,k}\text{sat}_{-1,1}\left(\frac{s_k}{\phi_k}\right), \tag{54}$$

$$\boldsymbol{\beta_k} = \text{sat}_{\underline{\beta},\bar{\beta}}(\boldsymbol{\beta}_{k-1}) + \gamma_2\sigma_k s_{\phi,k}\boldsymbol{\psi}_k, \ \boldsymbol{\beta_{-1}} = 0, \tag{55}$$

$$\boldsymbol{\eta}_k = \text{sat}_{\underline{\eta},\bar{\eta}}(\boldsymbol{\eta}_{k-1}) + \gamma_3\sigma_k s_{\phi,k}\boldsymbol{\varphi}_k g_G l \sin(x_{1,k}), \quad \boldsymbol{\eta}_{-1} = 0, \tag{56}$$

$$\rho_{b,k} = \text{sat}_{0,\bar{\rho}_b}(\rho_{b,k-1}) + \gamma_4\sigma_k|s_{\phi,k}|, \ \rho_{b,-1} = 0, \tag{57}$$

where $s_{\phi,k} = s_k - \phi_k \text{sat}_{-1,1}\left(\frac{s_k}{\phi_k}\right)$, $\phi_k = |s_k(0)|\mathrm{e}^{-5t}$, $s_k = 2e_{1,k} + e_{2,k}$, $\boldsymbol{\psi}_k = [\boldsymbol{c}^T\boldsymbol{e}_k - \dot{x}_{n,d}, s_{\phi,k} - s_k, g_G l\sin(x_{1,k}), x_{2,k}]^T$, $\boldsymbol{\varphi}_k$ is the last row of matrix $\begin{bmatrix} 0 & 1 \\ -0.3 & -0.2 \end{bmatrix}^k$. The control parameters and learning gains in control law (17) and learning laws (18)-(19) are set as follows: $\mu = 5, \gamma_1 = 10, \gamma_2 = 1, \gamma_3 = 1, \mu_4 = 0.05, b_s = 0.4, T = 6, \bar{\eta} = 100, \underline{\eta} = -100, \bar{\rho}_b = 10$.

After 15 iteration cycles, the simulation results are shown in Figs. 1-7. Figs. 1-2 show the profiles of angle position and angular velocity at the 15th learning cycle, respectively.

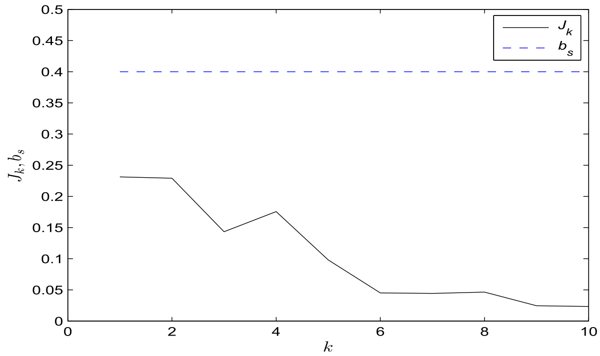

FIGURE 7. $\max_{t\in[0,T]} |s_{\phi,k}(t)|$ along iteraion axis (constraint ILC).

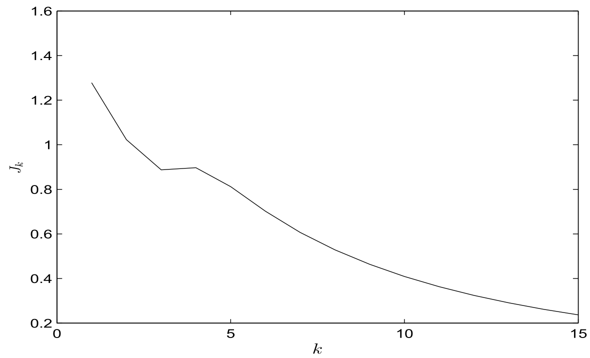

FIGURE 8. $\max_{t\in[0,T]} |s_{\phi,k}(t)|$ along iteraion axis (no-constraint ILC).

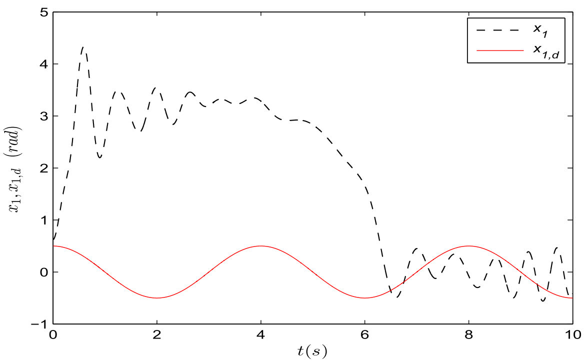

FIGURE 9. x_1 and $x_{1,d}$ (D-type ILC).

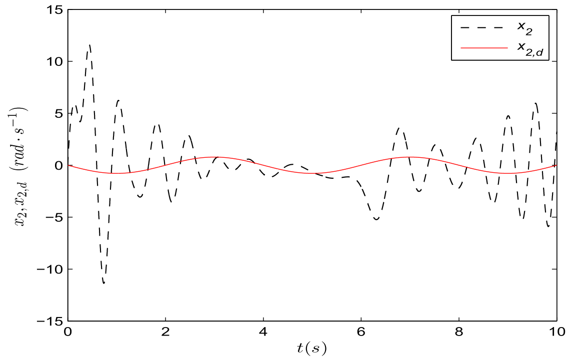

FIGURE 10. x_2 and $x_{2,d}$ (D-type ILC).

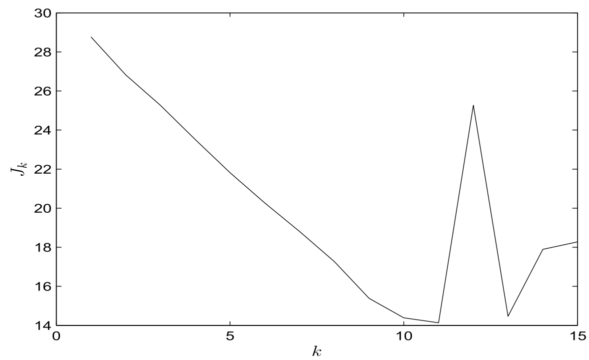

FIGURE 11. $\max_{t\in[0,T]} |s_{\phi,k}(t)|$ along iteraion axis (D-type ILC).

The angle tracking error profile and angular velocity tracking error profile are respectively given in Figs. 3-4. The profile of $s_{\phi,k}$ at the 15th iteration is shown in Fig. 5. From Figs. 2-5, we can see that good asymptotic tracking convergence from $\boldsymbol{x}_k$ to $\boldsymbol{x}_d(t)$ has been obtained as the iteration number increases. The control input at the 15th iteration is shown in Fig. 6. Fig. 7 gives the convergence history of $s_{\phi k}$, where $J_k \triangleq \max_{t\in[0,T]} |s_{\phi,k}(t)|$. From Fig. 7, we can see $|s_{\phi,k}| < b_s$ holds during each learning cycle.

Comparison A: The no-constraint adaptive ILC algorithm is adopted to simulation as follows:

$$u_k = -\gamma_1 s_{\phi,k} - \boldsymbol{\beta}_k^T \boldsymbol{\psi}_k - \boldsymbol{\eta}_k^T \boldsymbol{\varphi}_k g_G l \sin(x_{1,k}) - \rho_{b,k} \mathrm{sat}_{-1,1}\left(\frac{s_k}{\phi_k}\right), \tag{58}$$

$$\boldsymbol{\beta}_k = \mathrm{sat}_{\underline{\beta},\bar{\beta}}(\boldsymbol{\beta}_{k-1}) + \gamma_2 s_{\phi,k} \boldsymbol{\psi}_k, \quad \boldsymbol{\beta}_{-1} = 0, \tag{59}$$

$$\boldsymbol{\eta}_k = \mathrm{sat}_{\underline{\eta},\bar{\eta}}(\boldsymbol{\eta}_{k-1}) + \gamma_3 s_{\phi,k} \boldsymbol{\varphi}_k g_G l \sin(x_{1,k}), \quad \boldsymbol{\eta}_{-1} = 0, \tag{60}$$

$$\rho_{b,k} = \mathrm{sat}_{0,\bar{\rho}_b}(\rho_{b,k-1}) + \gamma_4 |s_{\phi,k}|, \ \rho_{b,-1} = 0, \tag{61}$$

The values of learning gain and control parameters in (58)-(61) are set to the same as the the corresponding ones in (54)-(57), respectively. The convergence history of $s_{\phi,k}$ in this algorithm is shown in Fig.8, where the definition of J_k is the same as that in Fig. 7. By contrast, the maximum of $|s_{\phi,k}|$ during each iteration of no-constraint ILC does not possess the barrier property.

Comparison B: Traditional D-type learning law [38] is adopted to simulation as follows:

$$u_k = u_k + \gamma_5(\dot{x}_{1,d} - \dot{x}_{1,k}) \tag{62}$$

where $\gamma_5 = 0.9$ is the learning gain. The position tracking and velocity tracking during the 15 iteration are illustrated in Figs. 9 and 10, respectively. From them, we can see the tracking error can not converge to zero or the small neighborhood of zero even if after so many iterations. The maximum of $s_{\phi,k}$ during each iteration is shown in Fig. 11, where the definition of J_k is the same as that in Fig. 7. According to Figs. 9-11, we conclude that the D-type ILC algorithm is not suitable for the considered time-iteration-varying parametric system with nonzero initial errors. The above simulation results verify the effectiveness of theoretical analysis in this work.

VI. CONCLUSION

A filtering-error constrained adaptive ILC scheme is proposed to solve the tracking problem for nonlinear systems with nonzero initial errors and time-iteration-varying parameters generated by HOIM in this paper. To achieve the filtering error constraint during each iteration, a barrier Lyapunov function is introduced for controller design. The problem of nonzero initial state errors is handled by using the technique of time-varying boundary layer. The state tracking errors can asymptotically converge to a tunable residual set as the iteration number increases. In the future, we will study the adaptive ILC for time-iteration-varying parametric systems with nonsymmetric deadzone.

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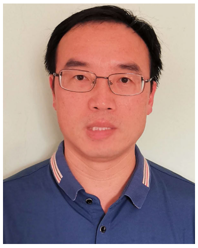

ZHI YANG received the B.S. degree in computer application technology from Zhejiang Sci-Tec University, in 2000, and the M.S. degree in software engineering from Hangzhou Dianzi University, in 2017. He is currently an Associate Professor with the College of Information Engineering, Zhejiang University of Water Resources and Electric Power. His current research interests include computer assisted design (CAD) and iterative learning control.

QIUZHEN YAN received the M.S. degree in computer science and the Ph.D. degree in control science and engineering from the Zhejiang University of Technology, Hangzhou, China, in 2005 and 2015, respectively. Since 2005, he has been with the College of Information Engineering, Zhejiang University of Water Resources and Electric Power, where he is currently a Lecturer. His research interests include iterative learning control and repetitive control. He is a Senior Member of the Chinese Association of Automation.

WEI WANG received the B.S. degree in mathematics from the Hebei Normal University, China, in 1999, and the M.S. degree in mathematics from Xidian University, China, in 2006. She is currently an Assistant Professor with the School of Mathematics and Physics, Hebei GEO University. Her current research interests include applied mathematics and nonlinear systems.

YUNTAO ZHANG received the B.S. and M.S. degrees in computer science from the Zhejiang University of Technology, Hangzhou, in 2002 and 2005, respectively. Since 2005, he has been with the College of Information Engineering, Zhejiang University of Water Resources and Electric Power, where he is currently an Associate Professor. His research interests include adaptive control and computer control algorithms.

JIANPING CAI was born in 1975. He received the Ph.D. degree from Zhejiang University, in 2014. He is currently a Professor with the Zhejiang University of Water Resources and Electric Power. His research interests include nonlinear systems and adaptive control.