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Neural Network-Based Adaptive Controller for Trajectory Tracking of Wheeled Mobile Robots

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ABSTRACT Trajectory tracking control is indispensable for a wheeled mobile robot to achieve successful navigation. The classical tracking control systems that are used in wheeled mobile robots do not compensate for the parameter uncertainties and external disturbances. For control strategy, this paper presents a novel hybrid approach, combining a neural network-based kinematic controller and a model reference adaptive control. The controller parameters are adaptively determined online using neural networks. The adaptively tuned kinematic controller ensures a fast convergence to the desired trajectory. The model reference adaptive controller retains the desired tracking performance when parameter and model uncertainties occur. The Lyapunov stability method is used to obtain the adaptive gains which guarantee the asymptotic stability of the error dynamics, where the error is the difference between the outputs of the reference model and the actual plant. The performance of the proposed controller is compared with that of the PID controller, kinematic controller, and adaptive dynamic controller using different performance analysis indices such as integral absolute error, integral squared error, and mean absolute error. Simulation studies demonstrate that the proposed controller achieves high tracking accuracy and fast convergence as compared to the PID, kinematic, and adaptive dynamic controllers considering parameter uncertainties and slip disturbances. The outcomes of the simulation studies also illustrate that the proposed controller achieves the best transient performance. Experiments using real-world tests based on a two-wheeled differential drive robot architecture have elucidated the feasibility of the developed controller regarding tracking accuracy, total control effort, and robustness against uncertainties.

INDEX TERMS Trajectory tracking, wheeled mobile robot, neural networks, adaptive controller, dynamics.

I. INTRODUCTION

In the past few decades, the usage of unmanned ground vehicles (UGVs) is increasing rapidly because of their high performance in various applications such as military, agricultural imaging, surveillance, transport, and logistics. Based on the type of locomotion, UGVs are classified as wheeled mobile robots (WMRs) and legged robots. When compared to legged robots, WMRs are easier to manufacture, control, and are less expensive. The major areas of research in the field of WMRs consists of navigation, path planning, trajectory generation, trajectory control, and stability. The determination of a control law which enables the WMR to follow a time parameterized reference path is termed as the trajectory tracking problem. The main objective of a tracking

controller is to enable the WMR to track the desired trajectory with minimum tracking error and by following a reference velocity. An effective controller should produce the desired performance even in the presence of external disturbances, uncertainties, and measurement noises.

Among wheeled mobile robots, differential drive WMRs are more popular, as their motion can be controlled and programmed easily. The number of actuators of a WMR is less than the number of degrees of freedom and so the WMRs are categorized as under-actuated systems which makes the trajectory tracking problem a highly challenging task. The constraints imposed on the motion of WMRs are non-integrable and are classified as non-holonomic systems. Various control algorithms are developed in the last few decades for the trajectory tracking of WMRs. The control algorithms can be divided into (i) algorithms considering only robot kinematics and (ii) algorithms considering both kinematics

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and dynamics. Authors of [1] employed two PID controllers for the trajectory tracking of a unicycle mobile robot. The parameters of the PID controllers are determined using PSO algorithm. The authors were able to ensure that the tracked trajectory is close to the reference trajectory. However, the dynamic model is developed by neglecting parameters like slip velocity, wheel diameter, parameters of motors and their servos, and model uncertainties. A more accurate dynamic model is used in [2], which considered both known and uncertain parameters. This work proposed separate kinematic and dynamic controllers for the trajectory tracking of the robot. Saturation and failures of actuators are not considered in this work. A predictive control technique with friction compensation is proposed in [3] to solve the trajectory tracking problem for a three-wheeled omnidirectional mobile robot. An inverse kinematics block is utilized in a cascade control structure to compute the velocity references at each step based on information about the robot position and a desired trajectory. The use of a simplified friction model allowed a portion of the control effort is utilized to linearize the system, allowing the use of an efficient algorithm for linear MPC with constraints. The approach is efficient in tracking predefined trajectories. However, the closed-loop stability analysis for nonlinear system with MPC is a difficult task. Even though the strategy proposed in [3] is successful in dealing with frictional uncertainty, it is silent on the variations of other system parameters and also on the sensor noises. For non-holonomic mobile robot tracking control, the authors of [4] suggest a typical backstepping control technique. The system's kinetic equations are first extracted, then appropriate robot reference pathways are generated, and backstepping method control is used to build a controller that offers control input values to reduce the tracking error [4]. However, the transient performance is characterized by overshoots, and the velocity generated is not smooth. In [5], a simple adaptive control strategy including actuator dynamics is proposed for path tracking of uncertain nonholonomic mobile robots. The adaptive control technique is used to deal with parametric uncertainties and disturbances, and the DSC technique is used in [5] to develop the controller. A tracking control strategy that combines a biologically inspired backstepping controller and a torque controller with an unscented Kalman filter (UKF) and a Kalman filter (KF) is presented in [6]. The bioinspired backstepping controller and torque controller can avoid and reduce the velocity jumps and overshoots that occur in conventional backstepping control, resulting in smooth velocity commands. The linear quadratic control (LQR) discussed in [7] considered the multi-input multi output complexity of the WMR and this controller is implemented in KHEPERA IV. The tracking precision is achieved with an oscillation. The authors of [8] presented an optimized LQR controller using fire fly optimization algorithm. The simulation results are satisfactory. However, the tracking accuracy is reduced when applied to real robots. The above mentioned control algorithms may not give satisfactory results with external disturbances and uncertainties. The sliding mode controller developed in [9] is a dual loop controller

consisting of a kinematic controller and a sliding mode controller. The controller is capable of tracking a mobile robot under parameter uncertainties. However, the method exhibited 'chattering effect'. A novel backstepping and fuzzy sliding mode controller (BFSMC) is proposed in [10] where due considerations are given to external disturbances and model uncertainties. The BFSMC produced less chattering as compared to conventional SMC. In [11], a control structure for nonholonomic mobile robots is created that allows the integration of a kinematic controller with an adaptive fuzzy controller for trajectory tracking. A fuzzy logic system (FLS) estimates the system uncertainty, which includes mobile robot parameter change and unknown nonlinearities. The online tuning of FLS parameters allowed the real-time control of mobile robots. Adaptive PD controller is proposed in [12]. Here the variation in the plant parameters are compensated by adjusting the parameters K_p and K_d . A neural network controller that can track a reference trajectory in a shorter time is proposed in [13]. The performance of this controller is not satisfactory during high speed movements. The dynamics of the wheeled mobile robot is highly non linear. The neural networks have the capability of approximating any non linear systems. It is illustrated in [14]–[16] that neural network based adaptive compensation schemes can cancel the effect of uncertainties in nonlinear systems. Authors of [17] proposed a backstepping controller based on an adaptive Elman neural network which could approximate the uncertainties leading to improved controller performance. However, this method has the problem of over-fitting. A nonlinear controller is presented by the authors of [18] for the eye-in-hand visual trajectory tracking of WMRs. In [18], the control signal is generated using the estimated errors of position by considering the kinematic model. However, kinematic imperfections cause odometric error. Also, for applications requiring high speeds, consideration of robot dynamics is required to achieve better control accuracy. A robust virtual controller is proposed in [19] by considering both robot kinematics and dynamics for the tracking of differential drive robots. The dynamic model is designed in [19] by assuming that the inertia matrix, M is fixed. However, for the cases where payload exist, the elements of M matrix can vary and will affect the control accuracy. An adaptive Fuzzy neural network control scheme is proposed by the authors of [20] for coordinated movements of multiple robots with unknown dynamics and time-varying constraints. The unknown dynamics are approximated using Fuzzy neural networks. However, the control input is oscillatory which may cause damage to motor. An adaptive neural network controller is developed in [21] for the trajectory tracking of n link robot with unknown dynamics. Simulation results prove that the adaptive neural network controller has excellent performance as compared to the PD controller.

The dynamics of a WMR is highly nonlinear and complex. Both linear and nonlinear controllers are proposed in the literature for trajectory tracking. Because of their simple structure, small amount of calculation, and good real-time performance, linear controllers such as PID and LQR

controllers are still widely used in actual robot control. However, linear controllers give satisfactory results only near the operating points [22]–[25]. Also, classical PID controllers do not demonstrate global stability. Nonlinear saturation functions have been put forward to attain global stability [26]. Their performance degrades on account of parameter variations, disturbances, and noisy sensor data. Model uncertainties, variations of plant parameters, and external disturbances are the challenges to be addressed while designing a tracking controller for mobile robots. An adaptive nonlinear controller is capable of providing optimum and desired tracking performance as the control parameters are tuned online and no linearised approximation is used. Due to the learning capabilities and ability to approximate any nonlinear system, neural networks are gaining wide attention in adaptive tuning of controller parameters. Motivated by the above findings, this work focuses on solving the trajectory tracking control problem of wheeled mobile robot which is under the influence of parameter uncertainties, measurement noises, and external disturbances, using a neural network based controller which is adaptive and nonlinear. In this work, a neural network is used to adaptively determine the parameters of the kinematic controller. A model reference adaptive controller (MRAC) is also added to the proposed controller to ensure a stable tracking performance in the presence of parameter variations and uncertainties. The adaptive gains of the MRAC are designed using Lyapunov method satisfying the asymptotic stability of the error between the outputs of actual plant and reference model. Since the developed control technique provides satisfactory performance under external disturbances and uncertainties, they can be applied in wheeled mobile robots that are used in ware houses, factories, surveillance, space exploration, etc. The major contributions of this work are:-

- The proposed controller is capable of tracking a desired trajectory with minimum tracking error even under parameter and model uncertainties, and with noisy measured data
- Comparing with [1], [2], [27], it is observed that the proposed controller achieves best transient performance due to its fast converging and least overshoot.
- The proposed controller can provide oscillation free response.
- The control input (speed) generated by the controller should be within limits imposed by the bounded constraints of the actuators, which is satisfied by the proposed controller.
- Simulation results and real world experiments illustrate that the neural network based adaptive controller brings significant improvement in terms of tracking accuracy, control input, and total control effort as compared to the PID, kinematic, and adaptive dynamic controllers.

The rest of this paper is organized as follows. Section II explains the preliminary concepts of trajectory tracking, mobile robot kinematics, and dynamics. The proposed controller is described in Section III. Simulations and results are

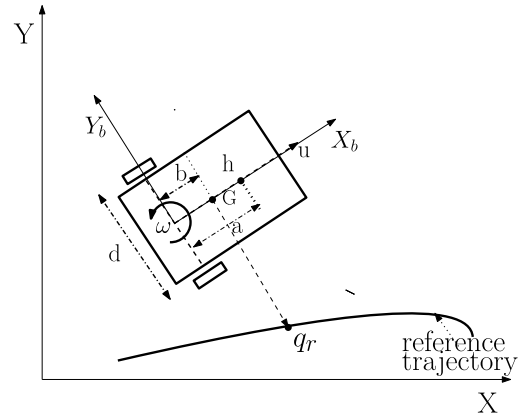


FIGURE 1. 2D representation of WMR.

discussed in Section IV. Experimental tests and results are given in section V and the concluding remarks are presented in section VI.

II. PRELIMINARIES

A. MOBILE ROBOT KINEMATICS

Consider a mobile robot moving in a 2 dimensional plane, shown in Figure 1. Let

$$q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \tag{1}$$

be the WMR position in the XOY inertial reference frame and let

$$q_b = \begin{bmatrix} x_b \\ y_b \\ \theta_b \end{bmatrix} \tag{2}$$

represents the robot position in the body attached frame $X_bO_bY_b$.

The relation between the global frame and robot frame can be expressed as

$$q = R(\theta)q_b \tag{3}$$

where

$$R(\theta) = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Kinematics refers to the study of the motion of a robot without considering the forces that cause its motion. The forward kinematics of a mobile robot can be obtained as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v \cos \theta \\ v \sin \theta \\ \omega \end{bmatrix} \tag{4}$$

The rotational velocity of the left and right wheels, ω_l and ω_r , respectively are expressed in terms of the control input v and ω as

$$\omega_l = \frac{v}{R} + \frac{\omega d}{2R} \tag{5}$$

$$\omega_r = \frac{v}{R} - \frac{\omega d}{2R} \tag{6}$$

where R is the radius of the wheel and d is the distance between two wheels.

The position and orientation of the robot at any instant t can be obtained by integrating Equation (4) as

$$\begin{cases} x(t) = \int v(t) \cos \theta(t) dt \\ y(t) = \int v(t) \sin \theta(t) dt \\ \theta(t) = \int \omega(t) dt \end{cases} \tag{7}$$

Consider a desired pose $q_r = [x_r \ y_r \ \theta_r]^T$ on the reference trajectory of the figure shown in Figure 1. The error vector is defined as the difference between reference pose $q_r(x_r, y_r, \theta_r)$ and current pose $q(x, y, \theta)$ which is found from

$$e(t) = \begin{bmatrix} e_x(t) \\ e_y(t) \\ e_\theta(t) \end{bmatrix} = \begin{bmatrix} \cos \theta(t) & \sin \theta(t) & 0 \\ -\sin \theta(t) & \cos \theta(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r(t) - x(t) \\ y_r(t) - y(t) \\ \theta_r(t) - \theta(t) \end{bmatrix} \tag{8}$$

The trajectory control problem of a mobile robot refers to finding the angular velocity and linear velocity such that the distance error and the angular error are brought to zero which ultimately enables a robot to follow a desired reference trajectory. By taking the derivative of Equation (8), the error dynamics is computed as

$$\dot{e}(t) = \begin{bmatrix} \dot{e}_x(t) \\ \dot{e}_y(t) \\ \dot{e}_\theta(t) \end{bmatrix} = \begin{bmatrix} 0 & \omega(t) & 0 \\ -\omega(t) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_x(t) \\ e_y(t) \\ e_\theta(t) \end{bmatrix} + \begin{bmatrix} v_r(t) \cos e_\theta(t) - v(t) \\ v_r(t) \sin e_\theta(t) \\ \omega_r(t) - \omega(t) \end{bmatrix} \tag{9}$$

The reference trajectory can be tracked with a controller which drives the error in distance and deviation in orientation to zero, which is achieved by

$$\lim_{t \rightarrow \infty} e(t) = 0 \tag{10}$$

and by

$$\lim_{t \rightarrow \infty} \dot{e}(t) = 0 \tag{11}$$

B. MOBILE ROBOT DYNAMICS

Dynamics of a mobile robot refers to the study of motion of the system considering the forces that cause the motion. The WMR dynamic model is more complicated and so the kinematic model is considered for the design of most of the motion control algorithms. However, dynamic model is inevitable if the applications require high speed movements and heavy payload. During navigation, most of the autonomous mobile robots use odometry to localize their positions. Slip at the wheel-ground contact points results in accumulation of odometric error and can affect the performance of sensor based navigation algorithms.

Hence, to achieve an improved navigation performance, the wheel-slip information should be considered for WMR modeling. The assumption of no-slip condition is not satisfied in most of the practical applications. Maneuverability of the mobile robots can be improved by controlling the slip magnitude. Hence, this work considers a WMR model by incorporating the slip dynamics. The mathematical model presented in [27], [28] which integrates both the kinematic and dynamic models that incorporates the slip dynamics is considered in this paper which is stated as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{v} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} u \cos \theta - a\omega \sin \theta \\ u \sin \theta + a\omega \cos \theta \\ \omega \\ \frac{\alpha_3}{\alpha_1} \omega^2 - \frac{\alpha_4}{\alpha_1} u \\ \frac{\alpha_5}{\alpha_2} u\omega - \frac{\alpha_6}{\alpha_2} \omega \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{\alpha_1} & 0 \\ 0 & \frac{1}{\alpha_2} \end{bmatrix} \begin{bmatrix} u_r \\ \omega_r \end{bmatrix} + \begin{bmatrix} \delta_x \\ \delta_y \\ 0 \\ \delta_v \\ \delta_\omega \end{bmatrix} \tag{12}$$

where a is the distance between the point of interest h and the central point of virtual axis linking the two wheels, u_r and ω_r are the desired linear and angular velocities. The parameter $\alpha = [\alpha_1 \ \alpha_2 \ \alpha_3 \ \alpha_4 \ \alpha_5 \ \alpha_6]^T$ is a function of robot mass m , moment of inertia I_z , electrical resistance of motors R_a , coefficient of friction B_e , electromotive constant k_b , radius of the wheel r , torque constant K_a , proportional gains K_{PL} and K_{PR} , derivative gains K_{DL} and K_{DR} of the PD controllers, the moment of inertia of rotor-reduction gear-wheel I_e , and the nominal radius of tires R_t . The vector α is given by

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{bmatrix} = \begin{bmatrix} \frac{R_a}{K_a} (mR_t r + 2I_e) + 2rK_{DL} \\ \frac{R_a}{K_a} (I_e d^2 + 2R_t r (I_z + mb^2)) + 2rdK_{DR} \\ 2rK_{PR} \\ \frac{R_a mb R_t}{2K_a K_{PL}} \\ \frac{R_a}{K_a} \left(\frac{K_a K_b}{R_a} + B_e \right) \\ \frac{R_a mb R_t}{K_a d K_{PR}} \\ \frac{R_a}{K_a} \left(\frac{K_a K_b}{R_a} + B_e \right) \frac{d}{2rK_{PR}} + 1 \end{bmatrix} \tag{13}$$

The parameters δ_x and δ_y are uncertainties on velocities and are functions of lateral slip and robot orientation along x and y directions, respectively. These parameters can be modeled as

$$\delta_x = -v^s \sin \theta \tag{14}$$

$$\delta_y = v^s \cos \theta \tag{15}$$

The parameter δ_v is a function of robot mass and longitudinal forces exerted by the castor which is expressed as

$$\delta_v = \frac{R_l R_a}{2\alpha_1 K_{PL} K_a} [m\omega v^s + F_{ex'} + F_{cx'}] + \frac{\alpha_4}{2\alpha_1} (v_l^s + v_r^s) + \frac{I_e R_a + r K_a K_{DL}}{2\alpha_1 r K_{PL} K_a} (\dot{v}_r^s + \dot{v}_l^s) \quad (16)$$

and δ_ω is a function of robot mass and lateral forces exerted by the castor which can be computed from

$$\delta_\omega = \frac{\alpha_6}{\alpha_2 d} (v_r^s - v_l^s) + \frac{I_e R_a d + 2r K_a K_{DR}}{2\alpha_2 r K_{PR} K_a d} (v_r^s - v_l^s) - \frac{\alpha_5}{\alpha_2} v^s + \frac{R_l R_a}{\alpha_2 d K_{PR} K_a} [e F_{ey'} + c F_{cy'} + \tau_e] \quad (17)$$

where v_l^s and v_r^s are the longitudinal slip speeds of left and right wheels, respectively, τ_e is the moment exerted by the tool, $F_{cx'}$ and $F_{cy'}$ are the longitudinal and lateral force exerted by the castor, $F_{ex'}$ and $F_{ey'}$ are the longitudinal and lateral forces exerted by the tool.

III. TRAJECTORY TRACKING USING NEURAL NETWORK BASED ADAPTIVE CONTROLLER

The mathematical model given by Equation (12) illustrates that the robot trajectory tracking system is highly nonlinear and strongly coupled. A controller designed with a linearised model are expected to give accurate results near the equilibrium points. As the states deviate from the equilibrium point, the achieved output also can deviate from the desired response. For trajectory tracking problems, WMRs are subjected to unstructured modelling such as measurement noise, wind and parameter variations owing to possible changes in mass, frictional coefficient, and moment of inertia. A promising solution is to vary the controller parameters in tune with these uncertainties and system parameter variation such that the desired performance is achieved. Neural networks are powerful tools to approximate any nonlinear functions. This capability of neural networks motivates us to propose a nonlinear adaptive kinematic controller whose parameters are adapted online using a neural network. The nonlinear kinematic controller determines the velocities for tracking a given trajectory. When a neural network is employed to obtain the parameters of the nonlinear controller, a fast convergence to the desired trajectories with more accurate tracking performance is achieved. The neural networks are trained using back propagation algorithm. To retain the desired performance under external disturbances and un-modeled parameter uncertainties, a model reference adaptive controller is also added to the proposed control strategy. The MRAC ensures that the system follows a reference model. The reference model is designed by integrating the WMR model and the LQR controller response. The errors in the velocities determined by the neural network based kinematic controller are compensated by the model reference adaptive controller. Thus, by combining the neural network based kinematic controller and the MRAC, a stable tracking performance is

achieved under the presence of model uncertainties, disturbances, and sensor noises. The control architecture is shown in Figure 2. The reference model, neural network based kinematic controller, and the adaptive controller are the three major blocks of the proposed system.

A. REFERENCE MODEL

For a conventional trajectory tracking controller, initially, the error will be very high and so the controller demand high adaptive gains which may results in a large overshoot followed by an oscillatory response. This can be avoided by using a model reference adaptive controller. This paper uses a closed loop model with an LQR controller as the reference model. The reference model is designed such that its response is similar to that of a WMR model controlled by an LQR. The state space representation of the reference model can be represented as

$$\dot{w}_m = A_m w_m + B_m u_r \quad (18)$$

where

$$A_m = \begin{bmatrix} 0 & 0 & -\cos(\omega_r t) \\ 0 & 0 & -\sin(\omega_r t) \\ 0 & 0 & 0 \end{bmatrix},$$

$$B_m = \begin{bmatrix} -\sin(\omega_r t) & 0 \\ 0 & \cos(\omega_r t) \\ 0 & 1 \end{bmatrix},$$

$$w_m^T = [w_{m1}(t) \quad w_{m2}(t) \quad w_{m3}(t)]^T$$

is the pose of the WMR and $u_r^T = [u_{r1} \quad u_{r2}]^T$ represents the control inputs.

An LQR controller minimizes a performance function

$$J = \frac{1}{2} \int_0^\infty (w^T Q w + u^T R u) dt \quad (19)$$

where $w \in \mathfrak{R}^{3 \times 1}$, $u \in \mathfrak{R}^{2 \times 1}$, $Q \in \mathfrak{R}^{3 \times 3}$ and $R \in \mathfrak{R}^{2 \times 2}$ such that the desired trajectory is tracked with minimum control. The LQR controller gain, K_{lqr} can be computed from

$$K_{lqr} = R^{-1} B^T P \quad (20)$$

where P is a positive definite matrix and is obtained from the algebraic Ricatti equation defined by

$$A^T P + PA - PBR^{-1}B^T + Q = 0 \quad (21)$$

The feedback control law of the reference model is

$$u = -K_{lqr} * e \quad (22)$$

where e is the error between the output of the reference model and the reference trajectory.

The objective of model reference adaptive controller is to make sure that the plant output follows the output of the reference model. This can be achieved by minimizing the error between outputs of the plant and the reference model, which is carried out by an adaptive mechanism.

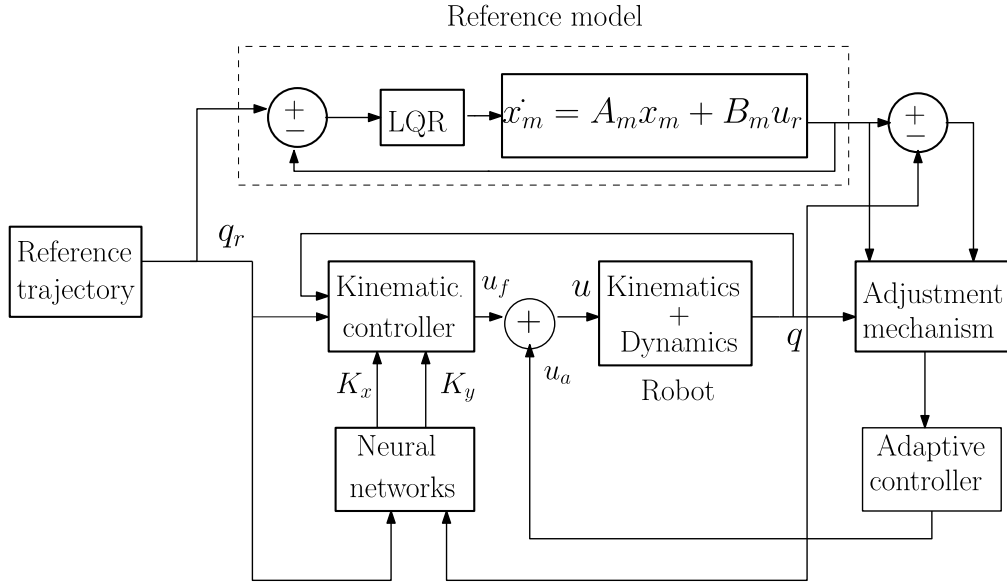


FIGURE 2. Proposed neural network based adaptive controller.

B. ADAPTIVE MECHANISM

The plant model is represented as

$$\dot{w} = Aw + Bu \tag{23}$$

where A and B are obtained from the linearised model of Equation (12) as

$$A = \begin{bmatrix} 0 & 0 & -u\sin\theta - a\omega\cos\theta \\ 0 & 0 & u\cos\theta - a\omega\sin\theta \\ 0 & 0 & 0 \end{bmatrix}$$

and

$$B = \begin{bmatrix} \cos\theta & -a\sin\theta \\ \sin\theta & a\cos\theta \\ 0 & 1 \end{bmatrix}$$

This paper proposes a control input u as

$$u = u_a + u_f \tag{24}$$

where u_a is the output of an adaptive controller and u_f is the output of a feedback controller. The adaptive part u_a, is obtained from

$$u_a = \hat{\beta}_1 w + \hat{\beta}_2 u_r \tag{25}$$

where \hat{\beta}_1 and \hat{\beta}_2 are the estimated adaptive gains. The feedback part of the controller is computed as

$$u_f = -Kw \tag{26}$$

where K is the gain of the kinematic controller.

The closed loop consisting of the robot and its controller can be represented as

$$\dot{w} = A_c w + B_c u_r \tag{27}$$

where A_c = (A + B\hat{\beta}_1 - BK) and B_c = B\hat{\beta}_2

The error between the reference and the actual state is computed as

$$e = w - w_m \tag{28}$$

Differentiating Equation (28) and substituting for \dot{w} and \dot{w}_m from Equation (27) and Equation (18) respectively yields

$$\dot{e} = A_m e + (A_c - A_m)w + (B_c - B_m)u_r \tag{29}$$

Thus the closed loop error dynamics can be obtained as

$$\dot{e} = A_m e + (A - BK + B\hat{\beta}_1 - A_m)w + (B\hat{\beta}_2 - B_m)u_r \tag{30}$$

Assume that ideal gains \beta_1 and \beta_2 exists such that

$$\begin{aligned} A - BK + B\beta_1 &= A_m \\ B\beta_2 &= B_m \end{aligned}$$

and are referred as the matching conditions. Then Equation (30) will be reduced to

$$\dot{e} = A_m e - B\tilde{\beta}_1 w - B\tilde{\beta}_2 u_r \tag{31}$$

where \tilde{\beta}_1 and \tilde{\beta}_2 are the adaptive gain estimation errors and are given by

$$\begin{aligned} \tilde{\beta}_1 &= \beta_1 - \hat{\beta}_1 \\ \tilde{\beta}_2 &= \beta_2 - \hat{\beta}_2 \end{aligned}$$

Adaptation Law: The adaptation law states that the choice of adaptive gain variation

$$\dot{\hat{\beta}}_1 = -B^T P e w^T \tag{32}$$

$$\dot{\hat{\beta}}_2 = -B^T P e u_r^T \tag{33}$$

make sure that

$$\lim_{t \rightarrow \infty} e(t) = 0 \tag{34}$$

where $e(t)$ is error between the actual plant output and reference model output.

Proof: Consider a Lyapunov function

$$V = \frac{1}{2}e^T P e + \frac{1}{2}tr(\tilde{\beta}_1^T \tilde{\beta}_1) + \frac{1}{2}tr(\tilde{\beta}_2^T \tilde{\beta}_2) \quad (35)$$

where P is a positive definite matrix. Then

$$\begin{aligned} \dot{V} = & \frac{1}{2}\dot{e}^T P e + \frac{1}{2}e^T P \dot{e} + \frac{1}{2}tr(\dot{\tilde{\beta}}_1^T \tilde{\beta}_1) \\ & + \frac{1}{2}tr(\tilde{\beta}_1^T \dot{\tilde{\beta}}_1) + \frac{1}{2}tr(\dot{\tilde{\beta}}_2^T \tilde{\beta}_2) + \frac{1}{2}tr(\tilde{\beta}_2^T \dot{\tilde{\beta}}_2) \end{aligned} \quad (36)$$

Substituting Equation (31) in Equation (36) and on further calculation

$$\begin{aligned} \dot{V} = & \frac{1}{2}e^T (A_m^T P + P A_m) e - \left(\beta_1^T (B^T P e w^T + \dot{\hat{\beta}}_1) \right) \\ & - \left(\beta_2^T (B^T P e u_r^T + \dot{\hat{\beta}}_2) \right) \end{aligned} \quad (37)$$

The positive definite matrix P is computed from

$$A_m^T P + P A_m = -Q \quad (38)$$

where Q is a positive definite matrix.

Substituting Equation (38) in Equation (37) and choosing

$$\dot{\hat{\beta}}_1 = -B^T P e w^T \quad (39)$$

$$\dot{\hat{\beta}}_2 = -B^T P e u_r^T \quad (40)$$

Equation (37) reduces to

$$\frac{dV}{dt} = -\frac{1}{2}e^T Q e$$

which is negative definite which implies that the error dynamics is asymptotically stable. \square

C. NEURAL NETWORK BASED KINEMATIC CONTROLLER

In this work, a kinematic controller proposed in [27] is considered as the base controller to generate the desired angular and linear velocity profiles for tracking the reference trajectory. The kinematic control law can be written as

$$\begin{bmatrix} v_f \\ \omega_f \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\frac{1}{a} \sin \theta & \frac{1}{a} \cos \theta \end{bmatrix} \begin{bmatrix} \dot{x}_d + l_x \tanh \left[\frac{k_x}{l_x} x_e \right] \\ \dot{y}_d + l_y \tanh \left[\frac{k_y}{l_y} y_e \right] \end{bmatrix} \quad (41)$$

where a is the distance between the point of interest $h(x,y)$ and the center of the virtual line connecting the two wheels, l_x and l_y are saturation constants, x_e is the difference between the desired x position and actual x position of the robot, y_e is the difference between the desired y position and actual y position of the robot, k_x and k_y are the controller gains.

In [27], the controller gains k_x and k_y are found by trial and error methods. Even though, is simple as it does not involve any complicated procedure, it takes a lot of time and an optimal response cannot be guaranteed under system uncertainties and external disturbances. A solution is to find the controller parameters adaptively which can be done by using neural networks as they have the capability to approximate

TABLE 1. Parameters for neural network training.

Parameters	Values
Number of input neurons	6
Number of output neurons	2
Number of hidden layers	2
Number of neurons in hidden layers	50
Activation function	Sigmoid
Learning rate	0.02

any nonlinear function. In this paper, a neural network is used to get the gains of the kinematic controllers to achieve a fast convergence to the desired trajectory.

NEURAL NETWORK TRAINING

The neural network structure which finds the controller gains (K_x, K_y) is shown in Figure 3. Inputs to the neural network are reference state q_r , current state q . The outputs are controller gains k_x and k_y . The parameters for neural network training are given in Table 1.

The training data is obtained as follows: A set of reference trajectories are applied as the inputs to the closed loop tracking system which employs the kinematic controller given by Equation (41). The gains are adjusted by trial and error until the error between actual and desired trajectory are approximately zero. The controller gains are used as the target data for the training purpose. The neural networks are trained using back propagation algorithm. The plot performance which is the variation of the training record error values against the number of training epochs is shown in Figure 4. The neural networks proposed in this work require training only once, for a given mobile robot. Once trained, the saved networks can be used to compute the gains. The neural network can approximate any nonlinear system, and so the outputs provided by the neural networks are more accurate, if it is trained properly. Since the robot is trained properly and the neural network structure is not too complex, the proposed neural network has the ability to find the optimum values of controller parameters quickly and accurately. Hence, it gives fast and more accurate performance, which is illustrated by the simulation and experimental results. The controller gains are determined using back propagation, which ensures nearly zero tracking error as the weights of the neural network are updated until the error reaches an acceptable value. The controller gains determined by the neural networks are given to the kinematic controller. The sum of the control inputs generated by the kinematic controller and the adaptive controller is applied to the WMR to track the desired trajectory.

The performance of the proposed control system is evaluated by comparing it with a conventional PID controller, kinematic controller, and an adaptive dynamic controller. Various performance indices such as integral of absolute error (IAE), integral squared error (ISE), and mean absolute error (MAE) are taken for evaluating the effectiveness of the proposed controller in tracking different types of trajectories.

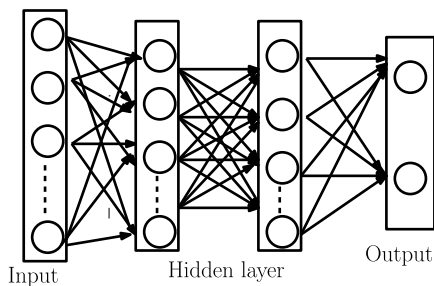


FIGURE 3. Neural network structure.

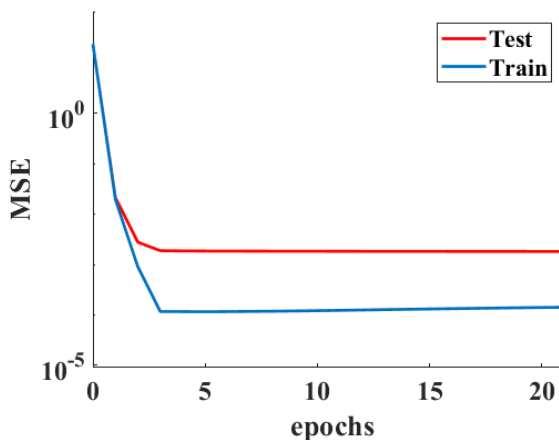


FIGURE 4. Performance plot of neural networks.

The integral squared error is defined as

$$ISE = \int_0^t e(t)^2 dt \tag{42}$$

The accumulated error is denoted by the integral of absolute error and is obtained by

$$IAE = \int_0^t |e(t)| dt \tag{43}$$

where $e(t) = [e_x \ e_y \ e_\theta]^T$.

Efficacy of the proposed adaptive controller is validated and the simulation results are discussed in the following section.

IV. RESULTS AND DISCUSSIONS

Simulations are performed to evaluate the efficacy of the proposed control strategy. The key findings and results are explained in this section. The WMR model described in Section II is implemented using MATLAB/SIMULINK 2020a. The parameters of WMR chosen for simulation are given in Table 2. A comparative assessment of the performance of the proposed neural network based adaptive controller is prepared by comparing it with PID controller, adaptive dynamic controller, and kinematic controller that are cited in the literature [1], [2], [27].

TABLE 2. Parameters for simulation.

Parameters	Values
Radius of the wheel	5cm
Mass of the robot	3kg
Moment of inertia	0.006kg.m ²
Distance between two wheels	0.3m
Maximum linear velocity	1.75m/s
Maximum angular velocity	1.75rad/s
Saturation constant	0.1

A. TRAJECTORY SCENARIOS

Different trajectory tracking scenarios with distinct curvature profiles are used to demonstrate the performance of the proposed control algorithm. The following trajectories are used

- Circle:

$$(x_r(t), y_r(t)) = (\cos t, \sin t) \tag{44}$$

- Lemniscate

$$(x_r(t), y_r(t)) = (\sin t \cos t, 0.5 \cos t - 0.5) \tag{45}$$

- Line

$$(x_r(t), y_r(t)) = (t, mt) \tag{46}$$

where m is the slope. In this paper, m is chosen as 1.

- Step change

$$\begin{cases} x_r(t) = t \\ y_r(t) = 0, & \text{for } 0 < t < 35 \\ = 1 & \text{for } 35 < t < 65 \\ = -1 & \text{for } 65 < t < 90 \end{cases} \tag{47}$$

B. NEED FOR AN ADAPTIVE CONTROLLER

Tracking of a lemniscate trajectory using a PID controller, with and without model uncertainties, is considered. Let $s(0, 0)$ be the initial position of the WMR and $\delta_x = -1.5 \sin t \sin \theta$, $\delta_y = 1.5 \sin t \cos \theta$ be the uncertainties in the kinematic model of the WMR. The PID controller parameters are set as $k_p = 10.76$, $k_d = 8.54$, and $k_i = -0.253$ using the Zeiger Nicholas tuning method. Figure 5a shows the tracked and reference trajectory for both with and without uncertainties. It is obvious from this figure that, the PID controller is capable of tracking the reference trajectory with less tracking error in the absence of kinematic model uncertainties. However, under the influence of kinematic uncertainties, tracking error is very high and the plant becomes unstable at the point $g(-0.5, 0)$. It is evident from Figure 5b that linear and angular velocities (the control inputs) are not following the reference speeds and are diverging at $t = 5.2s$, resulting in an unstable condition. Next section illustrates that with an adaptive controller, a stable tracking performance with least tracking error can be achieved even in the presence of model uncertainties and external disturbances.

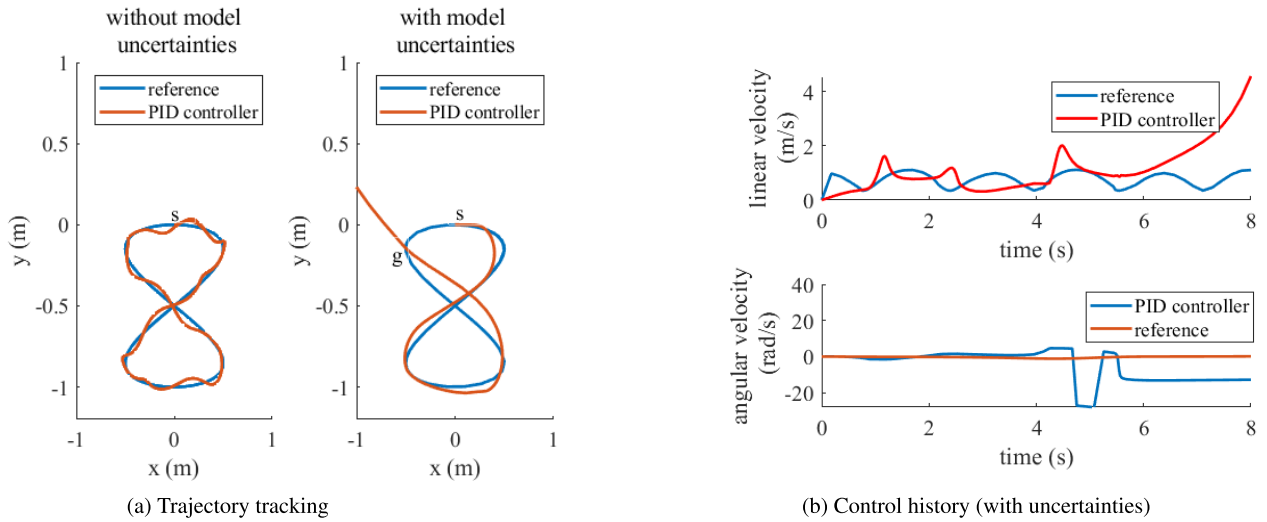


FIGURE 5. Performance of PID controller.

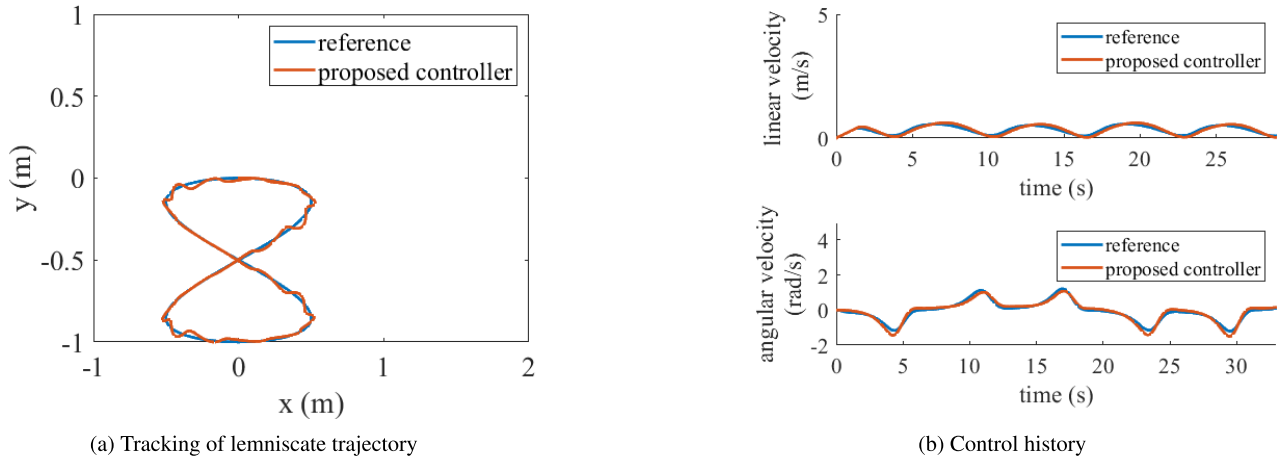


FIGURE 6. Performance of proposed controller considering model uncertainties.

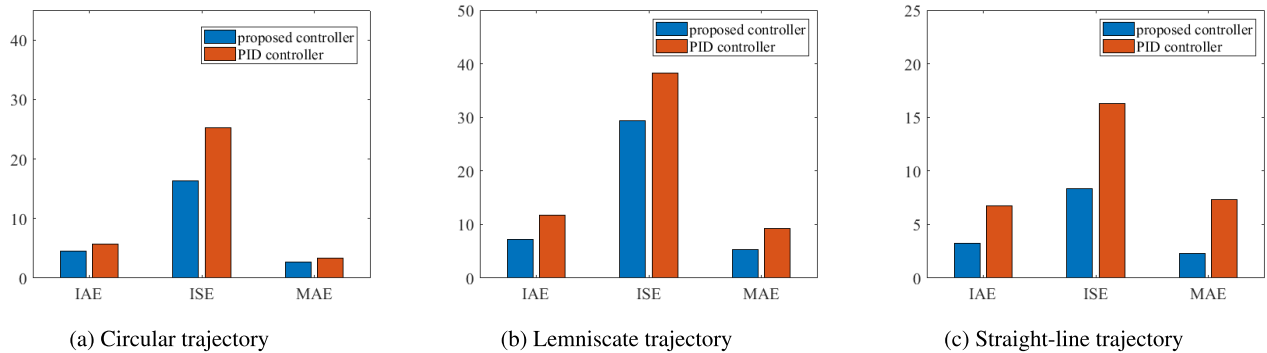


FIGURE 7. Summary of trajectory tracking performances (Effect of noise).

C. ROBUSTNESS OF THE PROPOSED CONTROLLER

The robustness of the proposed controller is validated by tracking the lemniscate trajectory under model uncertainties

and sensor noise. Both kinematic and dynamic uncertainties $\delta_x = -1.5 \sin t \sin \theta$, $\delta_y = 1.5 \sin t \cos \theta$, $\delta_v = 2 \sin t + \cos t$, and $\delta_\omega = 2 \cos t - \sin t$ are considered. From Figure 6a, it can

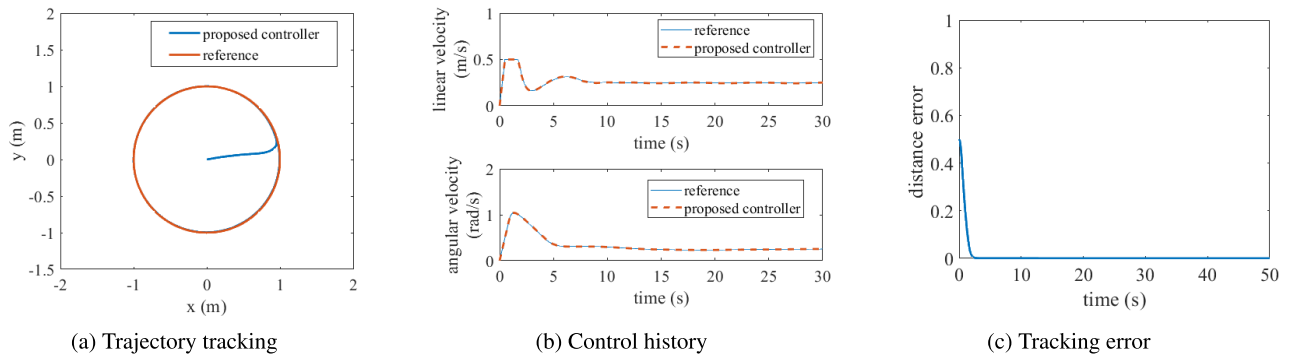


FIGURE 8. Circular trajectory.

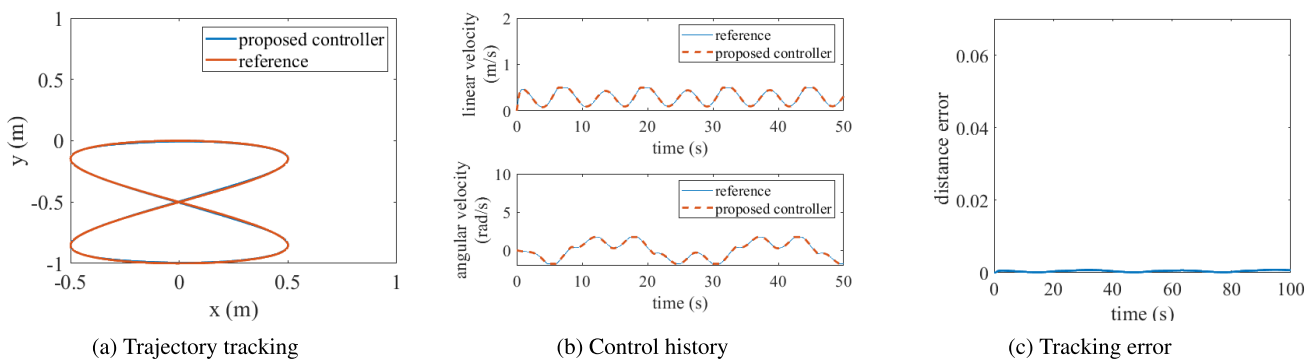


FIGURE 9. Lemniscate trajectory.

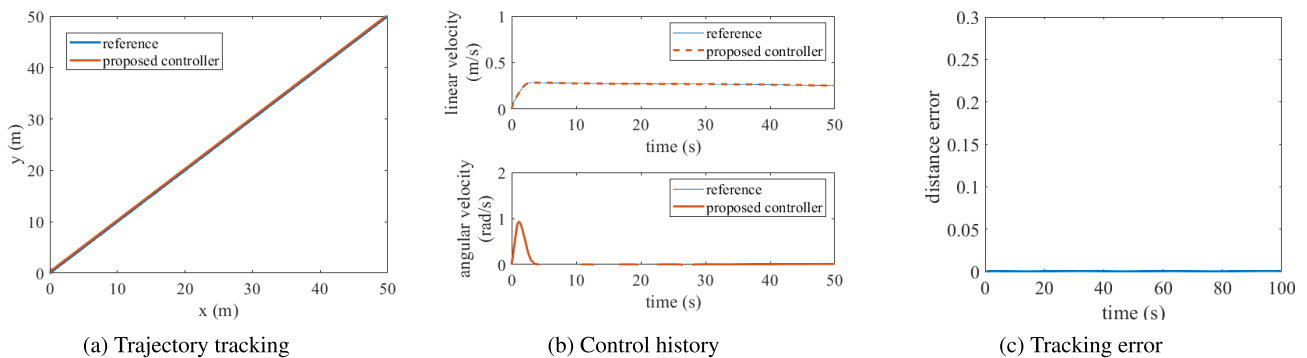


FIGURE 10. Straight line trajectory.

be demonstrated that the proposed controller succeeds in tracking the lemniscate trajectory even under model uncertainties with negligible tracking error. It is elucidated from Figure 6b that the proposed controller is able to follow the reference velocities, both linear and angular, considering the uncertainties. In order to evaluate the efficacy of the developed controller in the presence of sensor noise, a Gaussian noise with standard deviation 1 is added as the sensor noise to the x and y positions of the WMR as follows: $x = x + \delta$ and $y = y + \delta$. Figures 7a- 7c illustrates a comparative analysis of various performance indices IAE, ISE and MAE

with PID and the proposed controller for different trajectories (a) circular, (b) lemniscate, and (c) straight-line, respectively. These figures illustrate that the tracking performance of the proposed controller is superior, under noisy measured data, as compared to PID controller.

D. PERFORMANCE EVALUATION OF THE PROPOSED CONTROLLER

This section analyses the simulation results obtained with the proposed controller for (i) circular (ii) lemniscate (iii) straight-line trajectories. The desired trajectory is

provided to the plant and the reference model. The velocity for tracking the desired trajectory is determined using the kinematic controller. The gains of the kinematic controller for tracking the reference trajectories are determined using the multi layer neural networks. Back propagation algorithm is used here to train the neural networks. The adaptive gains are adjusted so that the plant output follows the reference model and thus the error vanish asymptotically. The distance error is calculated as follows:

$$d_e = \sqrt{(x_r - x)^2 + (y_r - y)^2}$$

The distance error is calculated for both the cases, with and without kinematic model uncertainties. A circle of radius of 1m given by Equation (44) is used as the reference trajectory. The circle is centered at the origin. The initial position of the robot is (0,0). The reference and the actual circular trajectories are shown in Figure 8a. Figure 8b shows that the proposed controller is exactly following the reference speeds and from Figure 8c, it is evident that the distance error converges to zero for the proposed controller.

Figures 9a-9c illustrate the performance of the proposed controller while tracking a lemniscate trajectory defined by Equation (45). The neural network based adaptive controller is capable of tracking the desired trajectory with minimum tracking error and the control inputs exactly follow the reference velocities.

A straight-line trajectory computed by Equation (46) is considered as the reference trajectory. From Figure 10a, it is observed that the desired straight-line trajectory can be perfectly tracked by the proposed controller. The error in tracking a straight-line trajectory is negligible as compared to the error that occurred while tracking circular trajectory and lemniscate trajectory. The linear velocity and angular velocity attained by the proposed controller are shown in Figure 10b. From Figures 10b and 10c it is clear that the proposed controller can track the straight line trajectory with zero error and follows the reference velocities.

E. COMPARISON OF THE PROPOSED CONTROLLER WITH KINEMATIC CONTROLLER AND ADAPTIVE DYNAMIC CONTROLLER

To illustrate the effectiveness of the proposed method as compared to other methods, a comparative study is carried out. A fixed gain kinematic controller and an adaptive dynamic controller are considered for the comparison. The kinematic controller is capable of tracking the reference trajectories when there is no model uncertainties or external noise. Both kinematic uncertainties δ_x , δ_y and dynamic model uncertainties δ_v , δ_ω are considered here. The circular trajectory tracked by the different controllers in the presence of uncertainties is shown in Figure 11a. It is clear from Figure 11b that the tracking error is high for kinematic controller as compared to the other two controllers and is least for proposed controller. It is evident from Figures 11c and 11d that the control history is oscillating for kinematic controller. Also, the instantaneous control input is less for the proposed neural

network based adaptive controller. A step trajectory with amplitude 1 whose step value changes at $t = 35s$ and $t = 65s$ is also considered to analyze and compare the performance of the three controllers. The trajectories tracked by the different controllers are depicted in Figure 11e. It is elucidated from Figure 11e that the robot cannot make sharp turns. The kinematic and adaptive dynamic controllers fail to track the reference step trajectory and produce oscillations. The transient performance of the three controllers considering the two step changes are given in Tables 3 (a) and (b). It is obvious from these two tables and Figure 11e that the proposed controller is able to achieve the final desired steady state value with least time, least overshoot, and without oscillations. The linear and angular velocities are shown in Figures 11f and 11g. It is clear from these figures that the kinematic controller fails to follow the reference control inputs. This simulation study demonstrates that the proposed controller outperforms the kinematic and adaptive dynamic controllers in terms of tracking error, control inputs, and transient performance considering both model uncertainties and external noises.

TABLE 3. Transient response parameters.

(a) First step change			
Parameters	Kinematic controller	Adaptive dynamic controller	Proposed controller
Peak overshoot	11.4%	9.1%	6.3%
Settling time	22.5	21.4	9.5
Oscillations	yes	yes	no
(b) Second step change			
Parameters	Kinematic controller	Adaptive dynamic controller	Proposed controller
Peak overshoot	11.1%	10.2%	5.4%
Settling time	15.53	15.12	7.93
Oscillations	yes	yes	no

A summary of trajectory tracking performance indices of kinematic controller, adaptive controller and, the proposed controller for the three trajectories are shown in Table 4. It is obvious from these tables that the IAE, ISE and, MAE is minimum for the neural network based adaptive controller. From this comparative study, it is clear that the proposed neural network based adaptive controller performs better than the other two controllers.

V. EXPERIMENTAL TESTS

In order to evaluate the performance of the proposed neural network based adaptive controller on a real-world scenario relevant to trajectory tracking, a two wheeled differential drive mobile robot shown in Figure 12 is used. For the real-world experiments the same trajectory scenarios given by Equations (44), (45), and (46) are chosen as the reference trajectories. The working environment is a laptop with specifications: an Intel(R) Core(TM) i7-6500U CPU @2.50GHz, 64 GB RAM, 2 physical cores, 4 logical processors, and

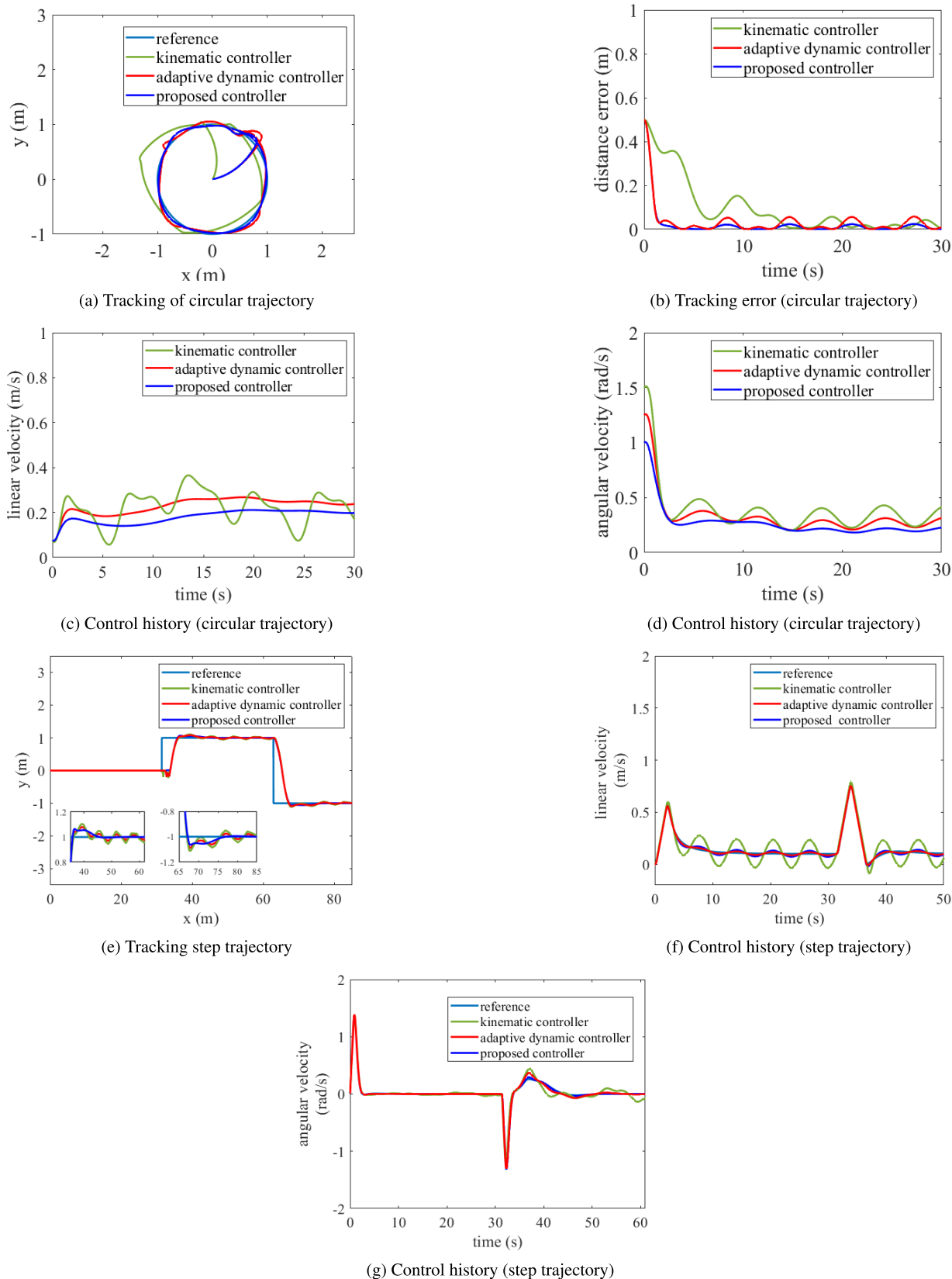


FIGURE 11. Comparison of controllers.

TABLE 4. Comparison of controllers.

(a) Circular trajectory				
Controller	IAE	ISE	MAE	Total control effort
Kinematic controller	4.73	7.51	0.42	15.674
Adaptive dynamic controller	3.64	5.28	0.35	12.324
Proposed controller	2.21	4.72	0.25	10.996
(b) Lemniscate trajectory				
Controller	IAE	ISE	MAE	Total control effort
Kinematic controller	4.9	8.2	0.48	34.981
Adaptive dynamic controller	3.8	7.5	0.39	29.782
Proposed controller	3.1	6.2	0.32	24.672
(c) Straight-line trajectory				
Controller	IAE	ISE	MAE	Total control effort
Kinematic controller	2.4	3.9	0.31	15.721
Adaptive dynamic controller	2.1	3.5	0.21	14.532
Proposed controller	1.5	3.2	0.18	12.342

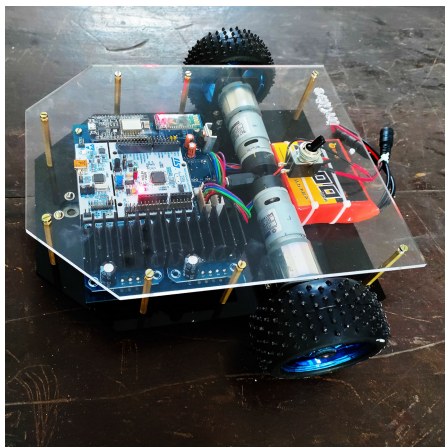


FIGURE 12. Differential drive wheeled mobile robot.

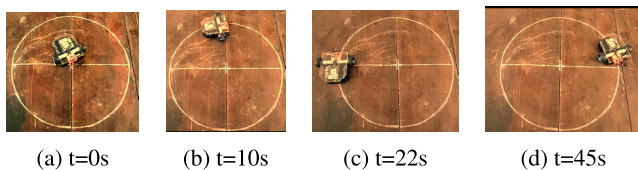


FIGURE 13. Snapshots of circular trajectory tracking.

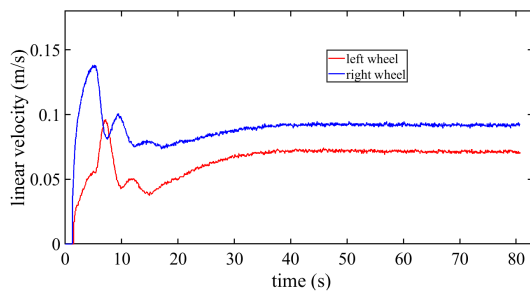


FIGURE 14. Left and right wheel velocities.

Windows 10, 64 bit. The laptop is communicated with the robot through a UART interface. Maximum speed of the wheel is 60 rpm. A two channel Hall effect encoder is used

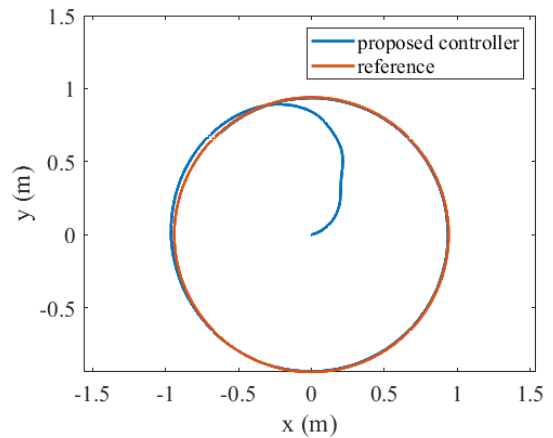


FIGURE 15. Tracking circular trajectory (with load and sand).

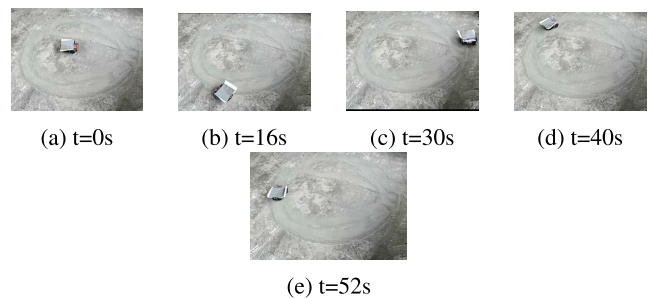


FIGURE 16. Snapshots of circular trajectory tracking (with load and sand).

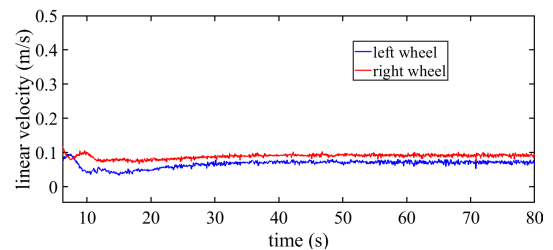
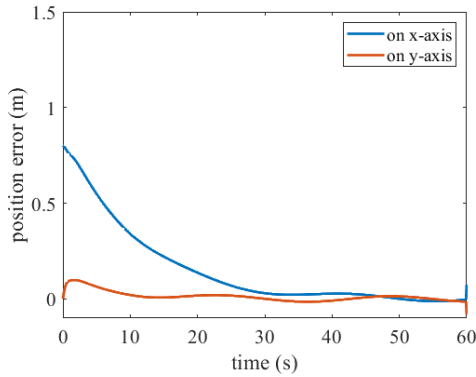


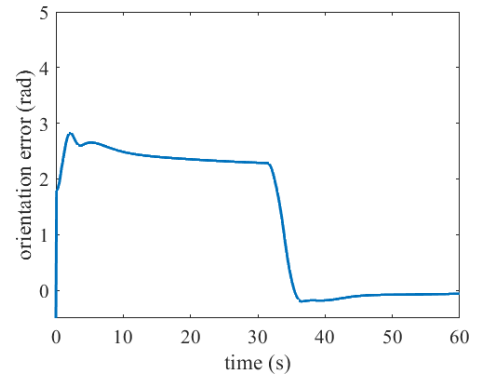
FIGURE 17. Left and right wheel velocities (with load and sand).

here to encode the rotation angle. For distributing the power and interconnecting with the attached modules, a STM32 micro-controller is used. The power supply is provided by a 12V-220 mAH battery. The chassis is equipped with two wheels and each wheel is actuated using a separated DC geared motor. In this test, MATLAB 2020a is employed for implementing the controller. The control inputs, linear and angular velocities are generated by the MATLAB/SIMULINK program which are send to the robot via bluetooth interface.

The sample time is set as 0.1s. The initial pose of the robot is taken as (0,0). The snapshots of trajectory tracking is shown in Figure 13. By adjusting the velocities of the left and right wheels, the robot motion is controlled. The velocity profile of the right and left wheels while tracking the circular trajectory is given in Figure 14. To evaluate the robustness of the developed algorithm against parameter uncertainties, a body of mass of 400g is placed on the robot. By laying sand on the



(a) Position error(with load and sand)



(b) Orientation error(with load and sand)

FIGURE 18. Tracking error(with load and sand).

ground where robot is supposed to track the desired circular trajectory, the slipping disturbance is achieved. The actual trajectory of the robot with added load and sand is depicted in Figure 15. In Figure 16, the snapshots of tracking the circular trajectory considering the external mass and sand is given. The left and right wheel velocities of the robot for tracking circular trajectory in the presence of parameter uncertainties and external disturbance is depicted in Figure 17. It is obvious from Figures 18a and 18b that the position and orientation errors are negligible and converges to zero.

By considering the parameter uncertainties and external disturbances, neural network based adaptive controller is shown to have better performance in real-world experiments and the real trajectories are tracked with minimal errors.

VI. CONCLUSION

In this paper, a neural network based model reference adaptive controller is proposed for the trajectory tracking of wheeled mobile robots. The adaptive gains are derived satisfying the condition that the error between the outputs of actual plant and reference model tends to zero asymptotically, which is guaranteed as the gains are derived using Lyapunov stability method. The parameters of kinematic controller are determined online using neural networks. Since the parameters are tuned online the proposed controller is capable of eliminating the drawbacks of conventional controller under the influence of model uncertainties and disturbances. Simulation studies using kinematic controller and adaptive dynamic controller considering model uncertainties proved that it fails to track the desired trajectory. However, the proposed controller tracks the desired trajectory with nearly zero error even in the presence of model uncertainties. Our in-depth research using both simulations and real-world experiments based on the two wheeled differential drive mobile robot illustrated the outstanding efficacy and the feasibility of neural network based adaptive controller in terms of accuracy, total control effort, and robustness compared to the related controllers such as adaptive dynamic controller, PID controller, and kinematic

controller for trajectory tracking in navigation scenarios with a relevant set of curvature profiles.

**APPENDIX
STABILITY ANALYSIS OF ADAPTATION LAW**

The derivation of adaptation law given in section III and its stability analysis is explained here. The linearized plant model is represented as

$$\dot{w} = Aw + Bu$$

The control input u is the sum of adaptive controller and feedback controller

$$u = u_a + u_f$$

The adaptive control law is given by

$$u_a = \hat{\beta}_1 w + \hat{\beta}_2 u_r$$

The feedback part of the controller is computed as

$$u_f = -Kw$$

The closed loop consisting of the robot and its controller can be represented as

$$\begin{aligned} \dot{w} &= Aw + B(u_a + u_f) \\ \dot{w} &= Aw + B(\hat{\beta}_1 w + \hat{\beta}_2 u_r) + B(-kw) \end{aligned}$$

Let $A_c = (A + B\hat{\beta}_1 - BK)$ and $B_c = B\hat{\beta}_2$ Then

$$\dot{w} = A_c w + B_c u_r$$

Error e

$$\begin{aligned} e &= w - w_m \\ \dot{e} &= \dot{w} - \dot{w}_m \\ \dot{e} &= A_c w + B_c u_r - A_m w_m - B_m u_r \end{aligned}$$

Adding and subtracting $A_m w$ gives

$$\begin{aligned} \dot{e} &= A_c w + B_c u_r - A_m w_m - B_m u_r - A_m w + A_m w \\ \dot{e} &= A_m e + (A_c - A_m)w + (B_c - B_m)u_r \end{aligned}$$

Substituting for A_c and B_c

$$\dot{e} = A_m e + (A - BK + B\hat{\beta}_1 - A_m)w + (B\hat{\beta}_2 - B_m)u_r$$

Assume that ideal gains β_1 and β_2 exists such that

$$\begin{aligned} A - BK + B\beta_1 &= A_m \\ B\beta_2 &= B_m \end{aligned}$$

and are referred as the matching conditions. Then error dynamics is given by

$$\dot{e} = A_m e - B\tilde{\beta}_1 w - B\tilde{\beta}_2 u_r$$

where $\tilde{\beta}_1$ and $\tilde{\beta}_2$ are the adaptive gain estimation errors and is given by

$$\begin{aligned} \tilde{\beta}_1 &= \beta_1 - \hat{\beta}_1 \\ \tilde{\beta}_2 &= \beta_2 - \hat{\beta}_2 \end{aligned}$$

To make the closed loop error asymptotically stable the adaptive gains variations have to be chosen as

$$\begin{aligned} \dot{\hat{\beta}}_1 &= -B^T P e w^T \\ \dot{\hat{\beta}}_2 &= -B^T P e u_r^T \end{aligned}$$

so that

$$\lim_{t \rightarrow \infty} e(t) = 0$$

where $e(t)$ is error between the actual plant output and reference model output.

Consider a Lyapunov function $V(e, \beta_1, \beta_2)$

$$V = \frac{1}{2} e^T P e + \frac{1}{2} \text{tr}(\tilde{\beta}_1^T \tilde{\beta}_1) + \frac{1}{2} \text{tr}(\tilde{\beta}_2^T \tilde{\beta}_2)$$

where P is a positive definite matrix. Then

$$\begin{aligned} \dot{V} &= \frac{1}{2} \dot{e}^T P e + \frac{1}{2} e^T P \dot{e} + \frac{1}{2} \text{tr}(\dot{\tilde{\beta}}_1^T \tilde{\beta}_1) \\ &\quad + \frac{1}{2} \text{tr}(\tilde{\beta}_1^T \dot{\tilde{\beta}}_1) + \frac{1}{2} \text{tr}(\dot{\tilde{\beta}}_2^T \tilde{\beta}_2) + \frac{1}{2} \text{tr}(\tilde{\beta}_2^T \dot{\tilde{\beta}}_2) \\ \dot{V} &= \frac{1}{2} \dot{e}^T P e + \frac{1}{2} e^T P \dot{e} + \text{tr}(\tilde{\beta}_1^T \dot{\tilde{\beta}}_1) + \text{tr}(\tilde{\beta}_2^T \dot{\tilde{\beta}}_2) \end{aligned}$$

Substitute the equation for \dot{e} , we get

$$\begin{aligned} \dot{V} &= \frac{1}{2} \left(e^T A_m - \omega^T \tilde{\beta}_1^T B^T - u_r^T \tilde{\beta}_2^T B^T \right) P e \\ &\quad + \frac{1}{2} e^T P \left(A_m e - B\tilde{\beta}_1 w - B\tilde{\beta}_2 u_r \right) \\ &\quad - \text{tr}(\tilde{\beta}_1^T \dot{\tilde{\beta}}_1) - \text{tr}(\tilde{\beta}_2^T \dot{\tilde{\beta}}_2) \end{aligned}$$

Rearranging the above equation

$$\begin{aligned} \dot{V} &= \frac{1}{2} e^T (A_m^T P + P A_m) e - \omega^T \tilde{\beta}_1^T B^T P e - u_r^T \tilde{\beta}_2^T B^T P e \\ &\quad - \text{tr}(\tilde{\beta}_1^T \dot{\tilde{\beta}}_1) - \text{tr}(\tilde{\beta}_2^T \dot{\tilde{\beta}}_2) \end{aligned}$$

Substitute

$$A_m^T P + P A_m = -Q$$

in \dot{V} and choosing

$$\begin{aligned} \dot{\hat{\beta}}_1 &= -B^T P e w^T \\ \dot{\hat{\beta}}_2 &= -B^T P e u_r^T \\ \dot{V} &= \frac{1}{2} e^T (-Q) e - \omega^T \tilde{\beta}_1^T B^T P e - u_r^T \tilde{\beta}_2^T B^T P e \\ &\quad + \text{tr}(\tilde{\beta}_1^T B^T P e w^T) + \text{tr}(\tilde{\beta}_2^T B^T P e u_r^T) \end{aligned}$$

Using the property of trace

$$\dot{V} = -\frac{1}{2} e^T Q e$$

which is negative definite.

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