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Adoption of Virtual and Augmented Reality for Mathematics Education: A Scoping Review

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ABSTRACT Augmented reality and virtual reality, collectively called extended reality (XR), has made substantial strides in the education sector in both theory and practice. Existing active research focuses on implementation by educators to teach real-world phenomena, and for students to learn through an immersive experience. This article surveys existing research in XR with special focus on the implications of immersive extended realities for teaching and learning engineering mathematics in institutes of higher learning. We also survey various interactive multimedia associated with XR before examining the implications of XR as an educational tool for existing mathematics pedagogy. Finally, the contribution of this scoping review is to provide an adaptable framework on XR implementation for educators, and potential academic advances for researchers.

INDEX TERMS Mathematics education research, extended reality, education technology, pedagogy, pedagogical framework, virtual reality, augmented reality.

I. INTRODUCTION AND OVERVIEW

Augmented reality and virtual reality, collectively called extended reality (XR), as a concept, has existed since the popularisation of creating illusions of alternative realities in science fiction. With the development of immersive technology, XR has been actively used for professionals in well-known areas such as flight simulation, design, and the humanities, and also in other areas such as medical [1]–[5], the languages [6], [7], and various vocational training [8]–[10]. With training as its main usage, the pathway of adoption of such technologies naturally finds its way into educational settings [11], [12]. New possibilities for teaching and learning emerge with the advances of XR and have been widely acknowledged as beneficial by educational researchers [13]–[15]. These educational benefits have made XR one of the key emerging technologies for education.

There are several similarities between XR and interactive multimedia. Both technologies can deploy visual-audio-kinesthetic learning experiences. The ease of use for simple XR applications in a smart device has enabled XR to catch up with interactive multimedia for asynchronous learning. The availability of XR toolkits like ARKit, Vuforia Engine,

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Unity, and Google VR has opened opportunities for educators to learn and deploy their choice of XR content. Nevertheless, an acute demand to fully express the differences between XR and other forms of interactive multimedia is critical and necessary. The lack of a formal framework to distinguish the two have unfortunately led to XR being unjustifiably implemented in many areas, including for pedagogy [16]. The introduction of such a framework can minimize instances where XR is implemented in a poor or sensationalist approach, which severely hampers the proliferation of a useful technology for education. While interactive educational multimedia has been present for decades, readily available XR technologies have only been available for the last decade.

A key difference and advantage of XR is the representation of three-dimensional (3D) objects embedded in a 3D world. 3D thinking can be enhanced, and the mental transformation of information, not available on 2D interactive multimedia, can be facilitated [17]. For example, in a course of multivariable calculus where most of the work is done in 2D (plotting software can plot 3D objects, but they are still represented on a 2D display), can be translated into a 3D representation, reducing the cognitive load on the user and facilitating the visualisation of complicated functions. Learners' interaction with a 3D virtual object is intuitive as

it is naturally how they perceive objects. Furthermore, 3D learning objects within XR educational systems have the unique advantage to provide opportunities to go beyond traditional face-to-face or computer-based simulation activities, providing the learner with a sense of in-situ, active engagement [18]. Although some studies have shown this may not necessarily lead to a comparative grade advantage over a learner who learns from interactive multimedia [19], [20], XR can significantly enhance the learning experience and have shown added intrinsic benefits such as improving student motivation [21], [22].

While mixed reality is the latest immersive technology that allows for digital and real-world objects to interact, it is a developing technology and relatively costly, with few researches being done in the education sector. Furthermore, some aspects of mixed reality have applications in both virtual reality (VR) and augmented reality (AR) [23]. Therefore, in this work, we focus on the VR and AR aspects of XR.

XR as a technology presents the potential of offering learners the opportunity to participate with an augmented or virtual reality, and hence invoke full participation. Immersive participation allows the transference of concepts to new contexts. However, mathematics as a subject, especially in higher education, has traditionally not been taught in an immersive environment. The potential usefulness of XR and the current extent of XR use in mathematics education in higher education presents an opportunity for enhancing the teaching and learning [24], [25]. While capitalising on the benefits of XR is important for the advancement of mathematics education, educators ought to use XR to solve an existing pedagogical problem instead of deciding on implementing XR before going in search for a problem. This is evidenced via a review by Mirkropoulos and Natsis [26], studying the use of VR in designing virtual environments for education in the years 1999-2009, revealing that only limited studies have a clear pedagogical framework. Subsequent studies [27], [28] conducted in the 2010s and early 2020s arrive at the same conclusion, notwithstanding attempts to evaluate the effectiveness of learning technologies [14], [29]. Beyond education, in the context of training, there is limited empirical evidence to show that the use of VR leads to better learning performance; instead, the advantage over interactive multimedia stems from a high sense of presence during VR simulation, which has shown to lead to increased skills learning [30]. The aim to introduce XR as a robust pedagogy tool motivates the content of our work here.

The remaining of this section seeks to introduce the XR learning environments, in particular, VR and AR. Section II focuses on the current use of XR in education and the theoretical potential of XR in education. In essence, the approach here is an attempt to infuse XR into existing pedagogical frameworks. This way, XR as a learning environment, can take a more pedagogical description that leads to achieved learning outcomes. While there are many ways to classify VR and AR, we have focused on one classification based on space and hardware considerations for adoption in the classroom.

Readers can refer to the cited references to be apprised of and consider other means of classification, which include marker-based, marker-less, GPS based amongst others to learn more about the technical implementation of such technologies. Next, Section III contextualises the implementation of XR in mathematics, at the level of higher education. We synthesize existing studies in mathematics educational XR research at this level, and provide a perspective on how improvements can be made on existing research. Section IV concludes our work here. This scoping review therefore aimed to explore the body of literature pertaining to existing framework to integrate XR technologies with pedagogical considerations. Following which, existing higher education mathematics research articles obtained from Google Scholar and Education Resources Information center (ERIC) databases are evaluated against the framework to identify alignment to the framework.

A. WHAT IS VIRTUAL REALITY?

Virtual Reality (VR) is multiply defined, however, one common definition is by Biocca and Levy [31], where they defined VR as “the sum of the hardware and software systems that seek to perfect an all-inclusive, sensory illusion of being present in another environment”. One of the great benefits of VR, touted by proponents of the technology, is the promotion of an *immersive* experience. Users are immersed into a virtual environment, but the extent of that, which constitutes as VR, is broad. For example, there exist some primitive implementation of VR, which consist combinations of sound and visuals, however, when pushed to its full potential, it includes a complete range of sensory experiences such as olfactory, aural, and tactile immersion [17]. The appropriate hardware needs to be acquired in order for a complete immersive experience.

The typical VR experience, comes in three categories shown in Fig. 1. (1) Non-immersive VR. the most common form, are often largely forgotten as VR experiences. They involve a motion sensor that detects a user’s motion, which is then translated on screen, in a virtual world. Video games like Wii Sports is a common example of non-immersive VR. These may not require any head-mounted devices, but tactile controls can be used as inputs for the virtual world. (2) Semi-immersive VR provides a partially virtual environment, and is commonly used for training and educational purposes. As an example, in a flight simulator, the controls in the simulator give real inputs to the simulator, but the instrument read back and window screens are displaying virtual content. As with non-immersive VR, user’s interaction with the virtual environment. (3) Fully-immersive VR. Fully-immersive VR provides users the most realistic immersive experience possible. The use of head mounted display is required to provide sensory content with a wide field of view, and can even be programmable to provide full-body haptic feedback.

VR platforms are also key in translating the immersive experience to the end user. Controls can come in the form of hand-held controls where actions are mapped to buttons, or full control panels, where users are required to physically








Virtual Reality Experiences			Virtual Reality Platforms			
Non-immersive VR	Semi-immersive VR	Fully-immersive VR	Stationary Displays		Head-mounted Displays	
			Fish tank display	Surround VR display	Occlusive head-mounted display	Smart device VR display
						

FIGURE 1. Types of virtual reality experiences and accompanying platforms. See section on ‘Acknowledgement’ for the image attributions.

perform the actions. Displays provide visual feedback to the users. Stationary displays generally catering to less immersive implementations of VR while head mounted displays being able to provide a greater immersive experience [32], [33]. Displays are often complemented with audio, and high-end immersive experience also provides gyroscopic and haptic feedback.

B. WHAT IS AUGMENTED REALITY?

Augmented Reality (AR) is defined as a real-time direct or indirect view of a physical real-world environment that has been augmented by adding virtual computer-generated information to it [34]. AR allows for virtual information to be placed in the immediate surroundings of the user, which enhances perception of and interaction with the real world. Similar to VR implementation, AR is not limited to sight, and can potentially apply to all senses. AR has the added advantage as a substitute for impairment of some senses by augmenting the real world with assisting information [35]–[38]. Furthermore, since interaction with the real-world is its main advantage, AR can be deployed both indoors and outdoors.

The typical AR experience can be loosely categorised into three AR systems shown in Fig. 2. (1) Fixed AR is a system that is fixed in a locale. Users are confined to a location and with little flexibility in changing locations, which requires a relocation of the entire system setup. There are advantages to such systems as it does not require location tracking, and a fixed setup allows for more controls and computing power to bring more realism to the AR experience. Fixed AR can be deployed both indoors and outdoors. (2) Mobile AR, not to be confused with AR deployed on mobile phones, enables users to deploy the AR system at any location. The hardware required to deploy the AR system is portable which gives users the flexibility to use this interface anywhere with a trade-off for lesser computational power. As most people in the developed world have access to a “computer” in their smartphones, mobile AR are often deployed, using smart devices as its main platform. (3) Mixed AR is a combination of both fixed AR and mobile AR. While the ease of

transporting the AR system is possible, it is not as convenient as mobile AR because it also requires advancements to wearable technology, as the computer system needs to be carried with the user. These can also be alleviated if the equipment is mounted on a vehicle. However, advances in portable technology has significantly increase the prospects of deployable mixed AR.

Since AR augments virtual objects onto the real-world, or substitutes real-world objects with virtual objects, a view finder (often a camera) is necessary to capture environmental features. Then a user can view the AR in two forms, mounted spatial view which enables spatial AR, and smart device display, where virtual objects only appear on screen. However, advances in technology has led to development of state-of-the-art AR targeting lightweight handheld displays [39] that seeks to amalgamate the advantages of both. Spatial AR makes use of optical elements, such as holograms or video-projection, to display information directly onto real-world objects without the need for a display. Such a technology is useful as it can be scaled up to a group of users, interacting and collaborating on a single virtual object [40].

II. XR AND THE DEVELOPMENT OF EDUCATIONAL THEORY AND METHODOLOGY

In this section, we synthesize the existing frameworks that are available and envision how they can be put together to produce a robust pedagogical framework that best amalgamates learning theories with educational technology. The challenge with the incorporation of XR in teaching and learning is the alignment of educational technology with the intended learning outcomes. As mentioned previously, poor or sensationalist implementation of learning technologies, not limited to XR, can severely hamper the proliferation of a useful technology for education. Therefore, educators need to form an alignment between the class content and use of XR, as with the introduction of any other learning activities.

How can one prevent the aforementioned and appropriately use XR in pedagogy? In Fowler’s design for learning framework [41], Fowler performed a critical review of

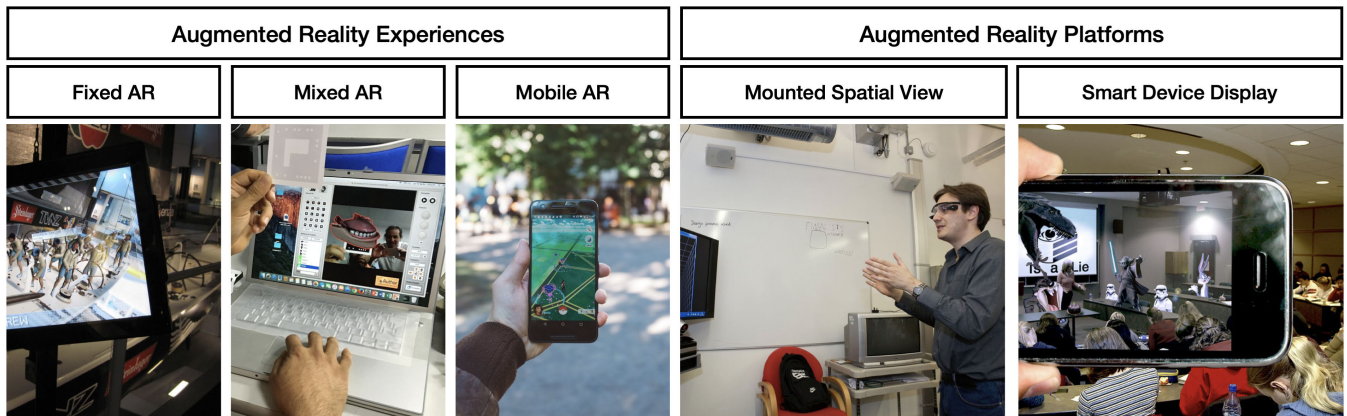


FIGURE 2. Types of augmented reality experiences and accompanying platforms. See section on 'Acknowledgement' for the image attributions.

the work introduced by Dalgarno and Lee [42] on incorporating 3D virtual environments into education. One of Fowler's critique is that, more often than not, educational technology research is based on the technology affordances with limited consideration for the pedagogical aspects such as learning outcomes and objectives. As such, Fowler proposes an alignment between Mayes and Fowler's pedagogical framework [43] and Dalgarno and Lee's considerations for virtual learning environments. This alignment is an attempt to achieve good pedagogical design, described by Biggs [44] to be an interactive system of factors including fixed student-related factors such as ability; teaching-related factors such as curriculum, teaching methods, and assessment; and the approaches to learning through tasks aimed to achieve an outcome. Fowler's framework calls for an alignment between pedagogical requirements at various stages of learning with the technology affordances made available by XR. In essence, the process of alignment is a unifying procedure that is central in *design for learning*. This concept is also previously advocated by Sharpe and Beetham when considering the pedagogy of e-learning [45].

The pedagogical framework introduced by Fowler and further enhanced in our work here can be summarised (Fig. 3) into four key stages: Design, Prototype, Validate, and Iterate. The *design* process begins with the identification of intended learning outcomes. Next is to decide the learning stages, by considering two frameworks, each classified into three stages: Mayes' pedagogical framework and Conole's learning activities [46]. Mayes' pedagogical framework proposes three learning stages. Conceptualisation are the sets of information students should understand, this can be done through construction of knowledge by scaffolding the learning process; finally, creating opportunities for dialogue sustains students' interest and motivation in the learned content. Likewise, learning activities need to be aligned to these three stages. A broad-based approach to ensure that the learning activities optimally engage the student through adequate knowledge representations need to be coupled with experiential, contextual, and if required, collaborative learning.

The use of XR should help students experience the in the appropriate context. Matching Conole's learning activities to Mayes' pedagogical framework (colour-coded in shades of blue in Fig 3), the use of XR to present knowledge representations should be aligned with conceptualisation. Construction and scaffolding of learning are done through experiential and contextual learning. Finally, engagement and collaborative learning create the dialogue for further curiosity.

The process of design for learning has two process outcomes, identifying learning requirements, and task affordances, leading to creation of a *prototype*. While the commonly used framework is the taxonomy introduced by Bloom [47], [48], this can be substituted for any preferred frameworks such as the Structure of the Observed Learning Outcome (SOLO) taxonomy introduced by Biggs *et al.* [49], or Fink's taxonomy [50]. The process of prototyping is as important as the design process. The prototyping stage ensures that the learning requirements are identified through a meticulous study of what stage the student is at, and what stage of the learning taxonomy can activate the student's cognition to achieve the learning required. The work by Yang *et al.* [51] provides a comprehensive description and guide to designing XR systems for education. The learning environment, such as XR, can facilitate these learning requirements, as current XR technology is able to provide information at a suitable level of representational fidelity through virtual text, audio cues, video clips, and immersion experiences. Such experiences have varying level of interaction dependent on the level of immersion and complexity of the technology. Thus, learning with XR needs to balance the complexity of learner interaction with representational fidelity.

The process to *validate* is another key step in ensuring that what has been theoretically designed and prototyped translates into achieved learning outcomes. This can be done in several ways, often observed in educational research. Tools for validation generally include appropriate use of questionnaires, attitude scales, interviews, case-studies, observational studies, etc. The right use of which is largely dependent on

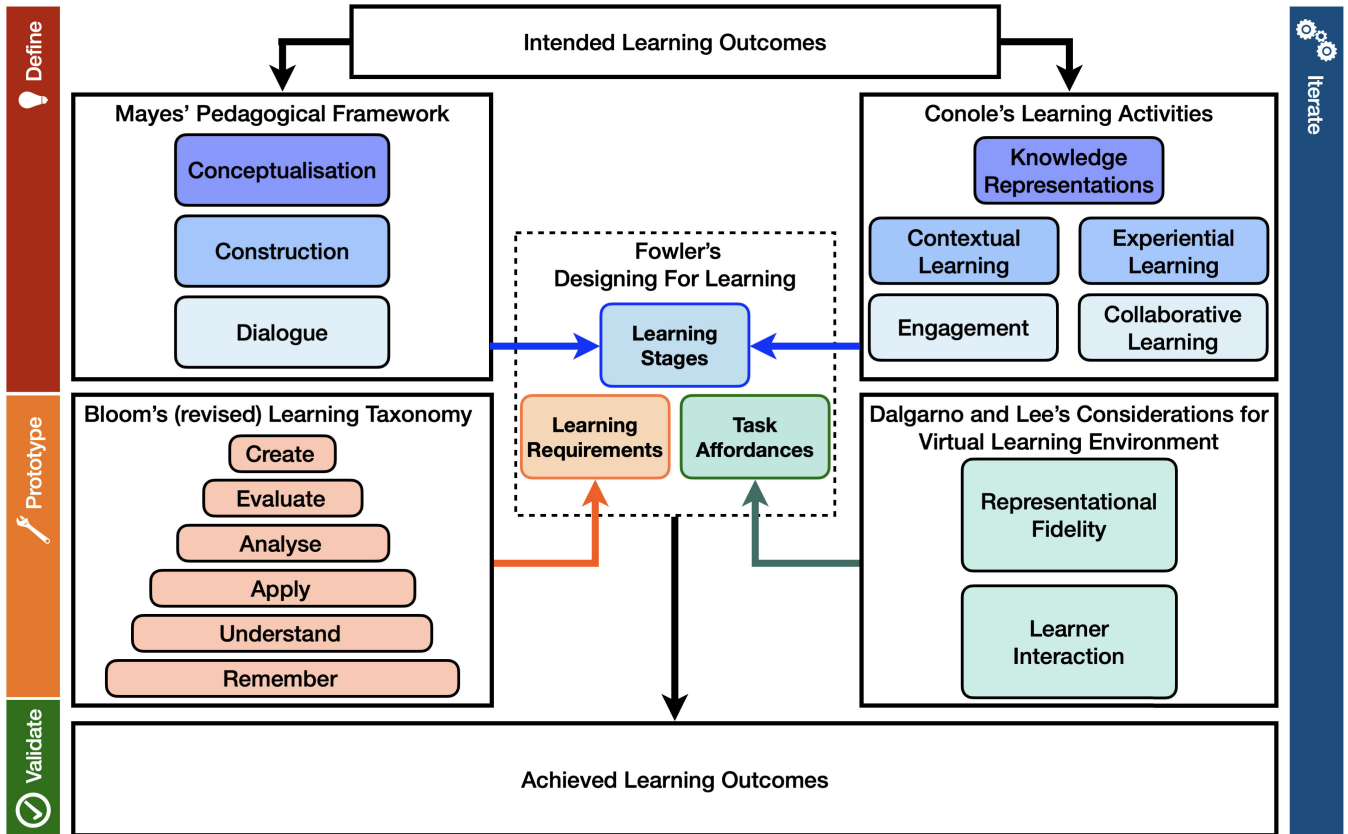


FIGURE 3. An enhanced framework for extended reality learning environments based on Ref [41].

the research question. Regardless, soliciting feedback from students and documenting the teaching process would help further define the use of XR, and its appropriate use for learning activities. We will see some examples of this in Section III. Lastly, *iterate*. As with all teaching and learning tools, the process of iteration seeks to identify additions, subtractions, and substitutions to the methodology implemented. This iteration process is a refining process that relevancy of XR technology and prevents sensationalist or haphazard introduction of XR in learning spaces, and continues to be a key step in educational research. As with all experiments that have to be refined and replicated, the process of iteration ensures that any research done on XR in the context of education continues to be a work-in-progress, instead of an isolated attempt under a very specific set of conditions that are difficult to replicate.

A. POTENTIAL DISADVANTAGES OF XR

Regardless of the advantages of XR in educational context, it is also vital to consider the associated disadvantages that commonly arise from XR implementation. High costs are often associated with creating an appropriate educational station using XR technology based on professional hardware and software. As an emerging technology, the availability of such technology often comes with associated start-up cost

and developmental cost. Thus, despite the advantages associated with XR, the large scale implementation of XR as an educational tool is often hindered by the lack of administrative and financial support. This often leads to compounding issues, which also includes slow adoption and lack of enthusiasm from educators to implement XR technology in the classroom [52], [53].

Irrefutably, XR requires much background work to create a virtual environment with many test scenarios and details. The prototyping and iteration process require both educators and students to be familiar with new technical skills, which often lead to longer implementation duration. In many recent research in the field of training and education, technical issues are often cited as one of the reasons that hinders learning [53]–[55] and this may distract students from the actual learning. However, much like other learning tools, it is through such iterative process that the teaching tool is continuously being improved.

XR has a high probability of acquiring routine in the actions taken, reducing the affective input of users. It is important for the emotive state of student to be activated in the learning process due to the strong effect of emotions in learning environments [56]. The use of VR could potentially suppress socio-emotive expression of students, which is vital in the learning process [57]. Thus, the task affordances are

key in ensuring that any design for learning goes beyond rote and invoke the socio-emotive learning amongst students. XR learning environments also require a period of adaptation. XR controls are not conventional and require some efforts and time to learn, there are multiple reports of motion sickness when using XR. Virtual environment tolerance is not equal for all users which in itself is a challenge to educational research as well [58], [59].

III. IMPLICATIONS OF XR FOR MATHEMATICS EDUCATIONAL PRACTICE

Mathematics has traditionally been learnt using non-digital means, such as paper and pen, and taught on matching hardware, such as the chalkboard or whiteboard. The digital age brought about advances, where the teaching tool has upgraded to incorporate PowerPoint, and recitations have been incorporated into undergraduate teaching to match the technological competence of the modern learner [60]. However, despite taking steps with technological advances, while educational technology have made notable strides, the use of more advanced technology has stagnated in mathematics education. While new technologies may not address students' struggle with mathematics problem-solving skills, any inactivity will continue to stall advances in mathematics pedagogy. Thus, educators are challenged to develop innovative teaching and learning approaches to facilitate conceptual understanding, scaffold learning, and create dialogue opportunities for solving mathematics problems applied to real-life applications.

XR is not necessarily applicable for all subject areas. This is largely due to the benefits of visualisation being significantly more important in some subjects than others. XR has the advantage of being useful for topics where spatial arrangement is important or there are dynamic changes. When it comes to mathematics education especially at the early undergraduate levels, various levels of cognitive function are required. Namely, qualitative review, quantitative thinking with accuracy, and abstract relational thinking. Students apply the axioms, theorems, and definitions to build complex relations, and prove mathematical concepts [61]. These are notably skills that XR may not have a direct applicable usage for. This makes the alignment process in Fowler's framework slightly more challenging for educators. Regardless, we advocate that knowledge gain should not be the only aim of XR learning experiences [62]. Furthermore, if successfully done, XR as a virtual learning experience has the potential to enhance mathematical literacy, especially at the level of abstract relational thinking, which is a transferable skill, highlighted as one of the important competencies in the 21st century [63].

With the complexity of considering pedagogical frameworks and incorporation of learning technologies, it is no surprise that, both Fernandez [64] and Reeves and Lin [65] identified the lack of alignment between pedagogy and technology infrastructure as a major barrier to adoption of XR technologies in education. A separate study identified five

present challenges to AR implementation, which can be extended to XR implementation. They are: (i) lack of teacher training, (ii) lack of educational experience, (iii) lack of conceptual foundation, (iv) lack of educational research, and (v) lack of institutional support [66]. Each party has its own technical knowledge, an educator with pedagogy and developers with programming. The main problem is that the developed material is not adapted to the curriculum but is based on experiences that are presumed interesting. Similarly, what educators might envision XR is capable of doing, often is unscalable from the technological perspective. Thus, a compatible and nonexclusive approach must be taken. There needs to be a bridge to explore opportunities where AR can be injected into pedagogy beyond the basic level, this task is performed by the educational architect, which can be a person from the institution's pedagogy department [64]. Fernandez proposes a six-step methodology to aid adoption of AR technology and are quintessential elements: (i) training teachers; (ii) developing conceptual prototypes; (iii) teamwork involving the teacher, a technical programmer, and an educational architect; (iv) producing the experience; (v) training teachers to apply AR solutions within their teaching methodology; and (vi) implementing the use of the experience with students. The introduction of an educational architect closes the gap between the expertise from the educators with pedagogy and the technology experts with XR infrastructure.

In the context of Fig. 3, the educator supplies information for the left side of the diagram, namely, the learning requirement and part of the learning stages, while the technology expert provides information to complement the learning stages and address the task affordances of XR. The role of the educational architect is to blend the learning stages, learning requirements, and task affordances together to achieve the learning outcomes.

A. CASE STUDIES IN XR IMPLEMENTATION FOR MATHEMATICS EDUCATION

In the remaining of this section, we will highlight existing research performed for which XR is implemented, where attempts have been made to narrow the gap between technology use for undergraduate mathematics education. A summary of existing studies, obtained through keyword search on Google Scholar, and Education Resources Information center (ERIC) databases. Keywords include combinations of Mathematics, undergraduate, virtual reality, augmented reality, and mixed reality. Only recent works post-2015 are considered. To synthesize the information, we identify if existing research methodology include the pedagogical alignment proposed in Section II, and the level of implementation. Specifically, for learning stages, the aspects of learning activities achieved based on the corresponding levels in Mayes' pedagogical framework; and for learning requirement, we specify the level of learning taxonomy achieved. This is carried out by a critical review of research articles and identifying ideas or keywords synonymous with those in Fig. 3. Table 2 contains the summary of our findings.

TABLE 1. XR implementation in undergraduate mathematics education, list of work reviewed.

Reference	XR Type	Hardware	Concept	Identified Pedagogy Alignment										
				Learning Requirement					Task Affordances					
				R	U	AP	AN	E	C	KR	CON	EX	EN	COL
[67], [68]	AR	Smart device	Calculus	✓	✓	✓	✓				✓	✓	✓	
[69]	AR	Smart device	Algebra	✓	✓	✓					✓		✓	
[70]	AR	Computer	Vectors	✓	✓	✓	✓				✓	✓		
[71], [72]	VR	Computer	Modelling	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
[73], [74]	AR	Smart device	Calculus	✓	✓	✓	✓	✓			✓		✓	✓
[75]	VR	HMD	Modelling	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
[76]	AR	Smart device	Calculus	✓	✓	✓					✓	✓		✓
[19]	VR	HMD	Calculus	✓	✓	✓					✓		✓	

Abbreviations. AR: Augmented Reality, VR: Virtual reality, HMD: Head-mounted devices;

R: Remember, U: Understand, AP: Apply, AN: Analyse, E: Evaluate, C: Create;

KR: Knowledge representations, CON: Contextual, EX: Experiential, EN: Engagement, COL: Collaborative.

Since various research work chooses to use different types of XR, and hardware platform, we will not be comparing the task affordances of these works. In all the works reviewed, the lower echelons of Bloom’s learning taxonomy are actively considered in the prototype when incorporating XR into the teaching and learning of various topics. However when comparing the various learning stages and requirements, only the works by Orey and Rosa [71], Herrera *et al.* [73], [74], and Ríos *et al.* [75], [77], implemented XR for the higher echelons of the learning taxonomy. Comparatively, these works considered a greater number of dimensions of learning activities, in particular creating opportunities for dialogue through student-student and teacher-student engagement, as well as collaborative learning. For example, if the intended use of XR is to allow students to ‘Evaluate’ or ‘Create’ in a particular subject matter, then on the side of learning activities, the XR platform would naturally involve ‘Dialogue’ through engagement and collaborative learning. Inversely, if the intended use of XR is to allow students to ‘Remember’, ‘Understand’, or ‘Apply’, then often this translates only to ‘Conceptualisation’ and ‘Construction’ being achieved, leaving out important ‘Dialogue’ to consolidate students’ learning.

To close this section, we discuss several aspects from two recent works [19], [75] to show the difference between a research work that seeks to align the use of XR with pedagogy with one that focuses on the application of XR to substitute existing methods. While the content is on different mathematical concepts, both of these works use VR as their primary platform, with the intention of facilitating engineering students’ learning in the topics of mathematical modelling and calculus, respectively.

In a project for third year undergraduate students on teaching and learning mathematical modelling [75], the authors stated their aim as “to develop statistical thinking in our students by relating an industrial process with mathematical modelling using 3D videos, VR, and AR”. The topics identified for implementation is hypotheses tests, simulation methods, and queuing theory. The use of 3D videos in the

initial stages were intentionally chosen to supplement the initial learning stages, where it was student-centered learning. Students were afforded the tools necessary to build up statistical thinking through instructor empowerment and engagement. A particular problem statement in the context of the topics has been identified and students are organised in teams where they are to propose a solution, which is then visualised using VR. The research methodology involves key words such as “choice, collaboration, communication, critical thinking, and creativity”. Alongside the intended learning outcomes, the choice of using VR was to impart soft skills of statistical thinking to the students were concisely articulated. López Ríos *et al.* had also meticulously aligned the learning stages by first introducing key concepts (conceptualisation) through an alternative media before bringing in VR to allow the students to experiment (construction) and collaborate (dialogue). While the authors did not report the VR system that was used, the learning stages and learning requirements were clearly defined, and improvements to the work were documented with subsequent publications [77]. The introduction of an evaluation framework (Five S-C strategy) [77] further provide the alignment analogous to the framework introduced in our work here.

We now turn to another project targeting first year undergraduate students on teaching and learning multivariable calculus [19]. The authors stated their aim “to test the effectiveness of VR as a medium to visualise the partial derivatives of two-variable functions, and to see if we could replace part of the standard classroom environment with a flipped classroom with the aid of VR”. As specified, the topics identified for implementation is partial derivatives and interpreting contour lines. Based on the design of the experiment, the learning stages are identified to include conceptualisation and construction. In the experimental design, students in the treatment group were given the time to accustom to the VR platform and a self-test was administered before the actual quiz. No instructor-led teaching session was administered for the treatment group, instead students

TABLE 2. The experimental details, reported results and benefits of the works reviewed.

Reference	Experimental Group	Reported Results / Benefits
[67]	30 engineering college students	Students were invited to obtain feedback for the AR application for Mathematics. AR technology in education found to lead to increase in the current motivation to learn by students
[68]	—	The future the uses of AR technology will increase offering positive changes for the development of the spatial visualization skill. Thus, bringing a special access to the graphical symbolic representation in mathematics.
[69]	59 undergraduate students	The AR experience is positive. It can help enhance the teaching-learning process. The authors reported that AR is a valuable complimentary teaching tool for topics that benefit from contextual learning experience and multipoint visualization.
[70]	18 undergraduate students and 2 lecturers	AR system facilitates the explanation and conception of abstract ideas that cannot be easily visualised with traditional pedagogical strategies. Most users had a positive attitude towards using the AR system.
[71], [72]	104 and 76 prospective mathematics teachers, respectively	Participants who traditionally had limited or no access to higher education, were given the opportunity, and the mathematical tools to develop sophisticated models to solve real-world problems through mathematical modelling. Being in a virtual learning environment helped students to interact, collaboratively inquire, and investigate in accordance with their own interests pertaining to the real-world problems that they were resolving.
[73]	1229 engineering students	The mean final grade of students increased from 77 to 84 (maximum 100 points). And the percentage of failure has gone from 20% to 5%. The use of the AR computational tools have improved interaction between teacher and students in a remote environment. The AR tools have simplified the explanations and has allowed students to concentrate on important aspects of the problems to be solved.
[74]	442 mathematics students	Same experimental results as reported by Ref. [73]. In addition, 442 mathematics students in the experimental group obtained 15 points more than the control group, and the percentage of students achieving the minimum spatial skills level required to pass the mathematics course increased 36%. This research shows a positive impact in the use of 3D tools to develop spatial skills.
[75]	60 third-year engineering students (30 in experimental, and 30 in control group)	There was an undisclosed improvement in student performance in the final exam for the experimental group over the control group. The learning improvement in the experimental group shows that when students experiment the VR technology on their courses, their technical and soft skills can be improved.
[76]	Engineering and environmental sciences students were being surveyed	For the students who used AR, the levels of interaction and participation had an average increase of 15% and 22%, respectively, compared to the control group. The authors also reported that the AR platform seemed to have allowed students to learn according to their own personal style.
[19]	312 first-year undergraduate students (125 in experimental, 187 in control group)	Students perform worse on some questions after using the VR application, and for some other questions students have similar performance to the treatment group. The authors reported that while VR learning may be perceived as beneficial, it does not necessarily translate in better understanding. Students in the treatment group felt that their understanding and visualization skills improved, but it was not translated to better results in the quiz.

were to “set their own pace” and “take control of their own learning”. Students in the control group were given a paper version of the self-test and quiz, supplemented by an

instructor-led session before the self-test and quiz. As part of the research methodology, “Understand” from the learning taxonomy was explicitly mentioned, whereas other levels like

“Remember” and “Apply” were implied in the methodology. As compared to the paper and pen scenario, the VR environment did not provide any more information other than the third dimension for the multivariable functions and spatial manipulation. With a limited application of VR in such context, it may be too early to conclude that “VR is not always (our emphasis) suitable as a replacement to lectures and classroom learning...” [19]. A possible extension of their project could be in ensuring that the higher attainment levels of the learning requirements and learning stages are clearly defined and aligned, the inclusion of learner interaction, or even a virtual instructor can also be included, before the true potential of VR can be observed.

IV. CONCLUSION

In this scoping review, we have assimilated a versatile framework for XR implementation in education, summarised in Fig. 3. The unified framework presented in this work, is an attempt to provide educators with an overview of the pedagogical and technical considerations for successful implementation of XR enhanced teaching and learning. We reviewed current literature on XR in undergraduate mathematics pedagogy to show the trend that most of the current works are pushing the boundaries with the potential of XR as a tool for learning activities to achieve the higher echelons of the learning taxonomy. Considering that learning technologies continue to evolve, application of such technologies need to catch up through discerned implementation. This calls for a separate role - the education architect - who is pedagogically trained, and has working knowledge of XR technology to form the alignment between the learning requirement and task affordances. Furthermore, technology enhanced pedagogy is an on-going process that requires multiple experimentation and iteration. Finally, we encourage aspiring teaching teams that wish to implement XR initiatives in pedagogy to use the framework that has been introduced in here.

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