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A Train Frequency Optimization Model for the Joint Operation With Two Intersecting Metro Lines

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ABSTRACT With the increase of travel demands in the networked metro system, more and more metro transfer stations are suffering from oversaturated situations, leading to the accumulation of passengers in the transfer corridor and boarding area with potential accident risks. To further improve the transfer efficiency and passenger accumulation safety at transfer stations, the joint operation strategy is adopted. Metro trains are allowed to run across different metro lines and passengers switch other lines without transfer. This paper proposes 1) a meaningful, yet simple, way for defining the running strategy of trains for joint operation with two intersecting metro lines and 2) an optimal model for the train frequency to minimize indicators associated with the passenger service and train capacity. Regulation constraints such as waiting time of through and transfer passengers, number of available trains, load factor, and departure interval are taken into account. Finally, a case study of the joint operation in the Beijing metro system is implemented to demonstrate the performance and effectiveness of the proposed approaches.

INDEX TERMS Metro system, joint operation, train frequency optimization.

I. INTRODUCTION

With the characteristics of high capacity, reliability, and low energy consumption, metro transit has been developed as the most important public transportation mode to solve road congestion and associated environmental pollutions. It plays a negligible role in metropolitan cities all over the world. Due to the large scale of the network, there is a rapidly increasing transfer demand in the metro network. In the metro of Beijing, for example, the transfer volume in the Xizhimen station during the morning peak hours of a workday in June 2016 was 33,735 passengers per hour. Moreover, there are 25 stations with more than 15000 passengers per hour, which accounts for 50% of the transfer volume of all transfer stations. A large scale of transfer passengers may cause severe congestion in the transfer station and lead to safety problems. Meanwhile, the transfer inconvenience of different train lines affects the service level to a large extent.

To improve the service level of overcrowding metro transfer station, many researchers focus on i) passenger flow control, in which passengers are firstly required to queue at the corridor or wait outside [1], e.g., Tiyu West Road station in Guangzhou, Tiantongyuan station in Beijing; ii) stop-skipping, no stop at the transfer station, e.g., Hujialou station in Beijing. However, these strategies cannot reduce the volume of transfer passengers, and only delay the gathering of passengers at the transfer station and increase the travel time of passengers. To alleviate congestion in the transfer station, the joint operation has been implemented in some large-scale metro systems. A joint operation is an operating approach for trains on one line share another line.

With a joint operation between different metro lines, metro trains are allowed to run across different rail transit lines and passengers switch other lines without transfer, which brings great convenience to transfer passengers. Significantly, in this way, passenger accumulation and congestion on platforms can be prevented, and operational safety can be improved greatly. Nowadays, the joint operation in China has been

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only used in Chongqing and Beijing. More lines will be implemented joint operation in the future.

To our knowledge, the current joint operation methods are often implemented considering signal, civil engineering and other infrastructure, but not taking transfer passengers and through passengers into consideration. In this paper, we mainly focus on optimizing passenger demand-oriented operation- the train frequency for joint operation between two different lines- to reduce the number of transfer passengers but meet the needs of other origin-destination (OD) matrix demands in the metro system.

The rest of this paper is organized as follows. In Section II, the literature review is given to introduce the state of art in this field. Section III describes the problem statement. Section IV the problem is formulated as an integer linear programming model. Section V provides a case study, in which two lines train frequencies based on the Beijing metro in China are considered to evaluate the performance of the proposed mathematical model and algorithm. Finally, concluding remarks and future research directions are given in Section VI.

II. LITERATURE REVIEW

Generally, several train operation control strategies are implemented to relieve the station pressure. Train operation strategies including all-stop, stop-skipping, and short-turning which are characterized by operational flexibility, can ensure sufficient capacity to accommodate all passengers [2]. Where all-stop is a conventional train operation strategy, but stop-skipping, short-turning are nonconventional operation approaches that have often been adopted for unbalanced passenger flow [3], [4]. While stop-skipping consists of allowing trains to skip certain low-demand stations and it is an important train operation strategy to reduce both operating costs and passenger travelling time [5], [6]. This operation strategy was first developed for the Chicago Metro system in 1947, and later implemented in Philadelphia, New York, and Santiago [7]. Meanwhile, short-turning is another type of train operation control strategy, which is applied when there is a low passenger demand along part of the line [8]. It allows a train to turn around in an intermediate station on a rail line without operating along the full length of the line [9]. However, the above two nonconventional operation approaches are often implemented in a single line and cannot effectively alleviate the transfer passenger problem. Thus, a new train operation approach is desired to develop to reduce the transfer volumes in transfer stations, especially the hub stations connecting two different lines. Since the joint operation allows train operation on two different lines, passengers can travel easily to other lines without transfer, resulting in fewer transfer volumes in transfer stations. A TCRP (Transit Cooperative Research Program) Report defines the joint operation as ‘‘commingled, simultaneous train operation on shared track by railroad trains (freight and/or passenger) and rail transit vehicles’’ [10]. For example, Tokyo provides services linking subway and railway suburban lines. Most metro lines now offer through connections, the exceptions being the TRTA Ginza and Marunouchi lines, which were constructed in the early days, and the TMG Oedo Line, which uses the latest linear motor technology [11]. In aviation, there are also cases of joint operation: Vlachou and Lovell [12] introduced an alternative concept into airspace flow, the collaborative trajectory options program, in which aircraft operators are allowed to submit sets of alternative trajectory options for their flights, with accompanying cost.

Since it is relatively complex for transit agencies to make optimization decisions on aligning the joint operation line and to select the number and location of stations along with it in rail infrastructure projects, few cities are adopting joint operation in practice [13]. In addition, it is difficult to coordinate standards of guideways, switches and crossovers, signal management, communication, power supply, and other infrastructure elements. In most cases, if the track has been built for only one line from end to end, it's very expensive to add a second line that requires redesigning and building more track, extensive modifications of the existing infrastructure (signal equipment, power supply, etc.) and operations management. Thus, it's usually to consider the joint lines in the construction project, not in the operation stage.

To meet the travel demands in the metro system, optimizing rail capacity allocation is an important way, and affected by train frequency, train stop strategy, etc. At present, there are many optimization methods and solution algorithms to optimize rail capacity. Carey [14] used mathematical programming methods to solve the train operation path with a choice of lines, platforms, and routes. Nesheli and Ceder [15] proposed a multi-routing planning model with aims at minimizing the number of vehicles and the number of short-turn trips, and a strategy that combines short-turning, holding, and stop-skipping was put forwarded to reduce the passenger travel time and the number of transfer passengers. Ceder [16] analyzed the effect of selecting a turnaround station in the short route on the result of the operation. Minimizing the number of operation trains and the operation complexity were set as the objective function in their route optimization model. Chowdhury and Chien [17] considered the real-time dispatch of vehicles to minimize the total costs of vehicle holding, connection delay, and missed connections. Chen, Li, and Liu [18] described a way of calculating the turning-back capacity according to the type of turning-back station, taking turn-back, and tracking interval times into account. Zhao *et al.* [19] proposed a train running strategy for Y-type urban rail transit aimed at minimizing the passenger travel time and train operating distance. The decision variables of the model were the turn-back locations and the departure frequencies of the train routings in the multi-routing planning.

As for solution algorithms, Cevallos and Zhao [20] developed a genetic algorithm (GA) to solve the problem of minimizing transfer times in public transit. Tsai, Chien, and Wei [21] presented an approach to jointly optimize temporal service headway and differential fare for an intercity transit system, considering heterogeneous demand elasticity, by developing a GA to search for the optimal solution.

Xiong *et al.* [22] analyzed the relationship among route position, relative costs, and headway in the optimal routing design problem, using a depth-first search algorithm (DFS) and a GA to find the solution.

While there exists sufficient literature on rail capacity optimization for single lines, not enough studies have been conducted under joint operation. For the complexity of the joint operation, the existing studies on single lines are not suitable for joint lines. As the rail capacity is usually based on the maximum number of trains that can be operated in a section of a track in a given period [23], the train frequency is an important factor affecting rail capacity. Therefore, in this paper, we mainly focus on how to optimize the train frequency for joint operation in the operation stage, which reduces the number of transfer passengers but meets the needs of other origin-destination (OD) matrix demands.

III. PROBLEM DESCRIPTION

Since the metro multi-line joint operation allows a train to run on different lines, a reasonable joint operation scheme can effectively reduce the transfer volume and travel time, which can improve the service level. To realize the metro joint operation, one should first determine the area for joint operation depending on the passenger demands of the refereed line. Then, put forward an optimal train running strategy for the maximum reduction of the transfer flow and little impact on other passenger flows.

A. TRAIN RUNNING STRATEGY

To realize the joint operation, there are two train running strategies. One is inserting new trains into the original fleet, and the other is replacing part of the original trains with new trains. As shown in Figure 1, S is a transfer station for the Blue and Red lines. Areas 1 and 2 are joint operation sections, while Areas 3 and 4 are non-joint operation sections, which are determined by the passenger demands OD matrix. Blue and red trains represent the original fleet running on the Blue and Red lines, respectively. While green trains are the new vehicles (extra available trains from the Blue line or the Red line). Both of the strategies mentioned above should be set to satisfy the multidirectional and unbalanced characteristics of the passenger demand, as specified by the OD matrix.

1) INSERTING NEW TRAINS INTO THE ORIGINAL FLEET

Green trains are inserted into the original Blue and Red line fleets, as illustrated in Figure 2 for the Blue line. This train running strategy has the following characters.”

- Applicable conditions: Passenger flow is even in Areas 1 and 3 (or even in Areas 2 and 4), and the original departure interval is large enough to insert new trains. In addition, the headway t_s between the inserting green trains and the original blue trains should be smaller than the minimum interval $t_{\min}$ requirement.
- Advantages: The departure interval is shortened and the waiting time of passengers in Areas 1 and 2 is reduced; as a consequence, the volume pressure of the transfer station is also reduced.

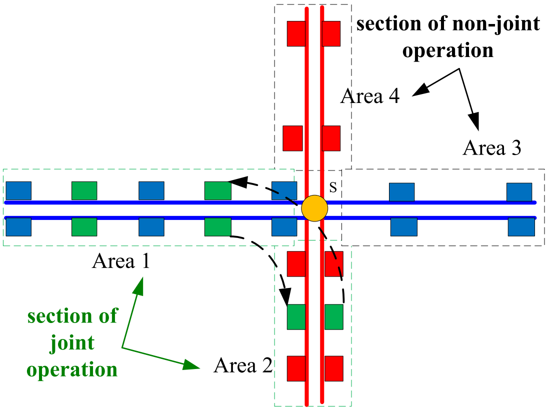

FIGURE 1. Train running strategy.

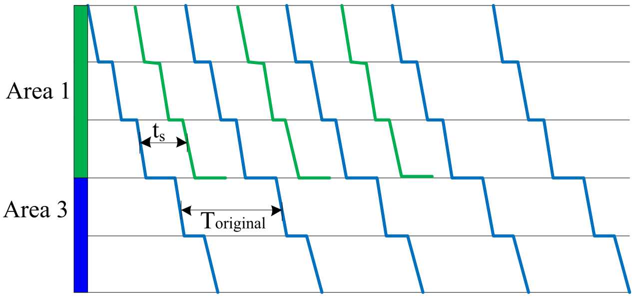

FIGURE 2. Time-space diagrams of joint operation under inserting new trains.

- Disadvantages: The operating costs of the railway corporation are greatly increased; the load factor of trains in Areas 1 and 2 is less than the original, resulting in a waste of capacity.

Therefore, this relatively simple train running strategy will not be discussed in this article.

2) REPLACING PART OF THE ORIGINAL TRAINS

Part of the blue or red trains is replaced by green trains. Figure 3 shows an example on the Blue line. This train running strategy has the following characters.

- Applicable conditions: If the maximum passenger flow U is uneven in Areas 1 and 3, assuming $U_1 \geq n * U_3$ $\left(n = 4/3, 3/2, 2, 3\right)$, then green trains could replace part

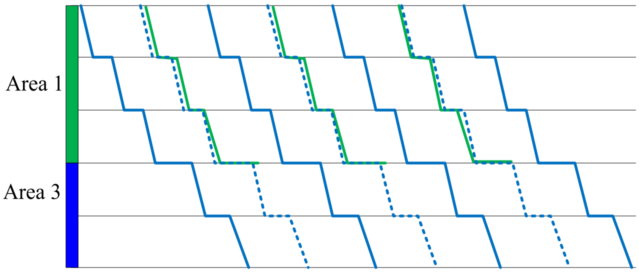

FIGURE 3. Time-space diagrams of rail transit operation under replacing trains.

of the original trains in Area 1. The passenger demand in Area 3 should be simultaneously satisfied.

- Advantages: The vehicle turnaround speeds up, and the waste of capacity in Area 3 is reduced; as a consequence, the volume pressure of the transfer station is also reduced.
- Disadvantages: The waiting time in Area 3 increases; through passengers (whose destination is on the Blue line, travelling from Area 1 to 3) need to get off a green train and wait for a blue train to their destination.

This type of train running strategy is more complex and its adoption may make the train routing planning better satisfy the multidirectional and unbalanced characteristics of passenger demand. Hence, we mainly consider this train running strategy in our optimization formulation for train joint operation.

B. CLASSIFICATION OF AFFECTED PASSENGERS

Let's assume that there are two lines as shown in Figure 4, where the Blue line constitutes of m stations, denoted as $L_{blue} = \{l_1, \cdots l_i, \cdots l_s, \cdots, l_m\}$, and the Red line constitutes of n stations, denoted as $L_{red} = \left\{l_{1'}, \cdots, l_{s'}, \cdots, l_{j'}, \cdots, l_{n'}\right\}$.

Among these stations, l_s and $l_{s'}$ standing for the same station, i.e., transfer station. A new joint operation route is established between two lines. Namely, there are three operation routes, i.e., $(l_1 - l_m)$, $(l_{1'} - l_{n'})$, *and* $\left(l_{1'} - l_{s(s')} - l_{n'}\right)$ (joint operation section, illustrated as the green line in Figure 1).

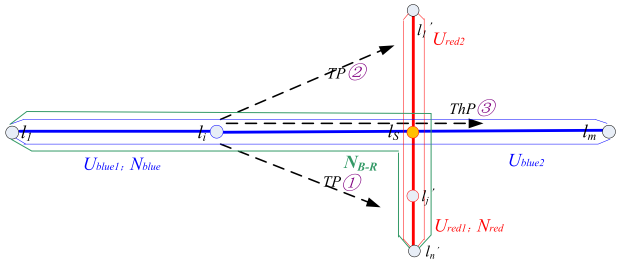

FIGURE 4. **Diagram of joint operation.**

The joint operation trains and regular trains share the same infrastructure. Fewer trains are running in the non-joint operation section than in the joint operation section. However, this operation approach is acceptable because of the uneven passenger flow between Areas 1 and 3 (or Areas 2 and 4). For this train running strategy, the waiting time of passengers whose initial or destination station is located in a non-joint operation section is increased; and the through passengers also experience additional waiting time in practice [24]. In previous studies, only transfer passengers were taken into account in the joint operation but not additional passengers. In this study, further constraints should be introduced into the model to reduce the waiting time of these passengers.

To better estimate the affection of the multi-line joint operation on passengers, the affected passengers are classified into two types, listed as follows.

Transfer passengers (TP), who transfer from one line to the other line in the transfer station located in the joint operation area. There are two types of passenger flows TP① and TP② in Fig. 4. The travel time of TP① will be reduced benefited from the joint operation, whereas the passenger flow TP② is not affected.

Through passengers (ThP), whose destination is on the Blue line, i.e., they travel between the joint and non-joint operation sections, are denoted as the passenger flow ThP③. If the ThP③ passengers on a joint operation (green) train, they should get off the joint operation (green) train in the transfer station to wait there for a train running the blue line, which results in travel time increment. The situation of passengers who start in a Red line station of the joint section.

IV. OPTIMIZATION MODEL FOR TRAIN FREQUENCY OF JOINT OPERATION

The paper mainly focuses on optimizing the train frequency of joint operation to balance the travel time of through and transfer passengers, subject to the section through capacity limitation and departure interval requirement. Since the train capacity is mainly related to train frequency, the decision variables in our optimal model are train frequency on each line under the train running strategy of replacing part of the original trains. Before presenting our optimization model for train frequency of train multiline joint operation, we assume that passengers arrive at a station with a uniform distribution, and the average waiting time of each passenger is half of the departure interval [25].

A. NOTATIONS AND DECISION VARIABLES

A series of notations, symbols, and decision variables are introduced in Table 1.

B. OBJECTIVE FUNCTION

The purpose of the joint operation is to effectively reduce the transfer volume, that is, alleviate the congestion in the transfer station. It means to transport the transfer passenger to their destination as soon as possible by shortening the transfer time in the transfer station. However, as more Green trains (fewer transfer passengers) share through the capacity of the Blue line, there will be fewer Blue trains, and thus through passengers will wait longer. Similarly, the through passengers of the Red line are in the same situation. Therefore, the wait time of all affected passengers, including the transfer passenger, and the through passengers, should be taken into account in our formulation.

1) TRANSFER PASSENGERS

The number of transfer passengers in TP① who enter at a station located in the joint section of the Blue line and exit at a station located in the joint section of the Red line. The flow TP① can be divided into two categories: one is formed by the passengers who take the green train along the line without transfer, whose number is proportional to $N_{B-R} \big/ (N_{blue} + N_{B-R})$, and there is no affection on this

TABLE 1. Parameters and notations.

Symbol	Detailed definition
Necessary Sets	
L	Set of metro stations of the Blue line and the Red line, $L=\{L_{blue},L_{red}\}$.
L_{blue},L_{red}	$L_{blue}=\{l_1,\cdots l_i,\cdots l_s,\cdots,l_m\}$, where m is the total number of stations in the Blue line. $L_{red}=\{l_{1'},\cdots,l_{s'},\cdots,l_{j'},\cdots,l_{n'}\}$, where n is the total number of stations in the Red line. i,j are the index of the station.
Parameters	
$l_s,l_{s'}$	Transfer station of the Blue line and Red line.
Q_{ij}	The number of passengers from station *i* to *j*.
N^a_{blue},N^a_{red}	The number of available trains on the Blue line and Red line, respectively.
$t^{walk}_{l_s}$	Average walking time of the transfer station l_s.
$t^{turn}_{blue},t^{turn}_{red}$	The turnaround time of the Blue and Red line, respectively.
t^{turn}_{B-R}	The turnaround time of the joint section.
$t^{\min intervals}_{blue}$ $t^{\max intervals}_{blue}$	The maximum and minimum headway on the Blue line, respectively.
$t^{\min intervals}_{red}$ $t^{\max intervals}_{red}$	The maximum and minimum headway on the Red line, respectively.
$U^{joint}_{blue},U^{joint}_{red}$	The maximum passenger flow within the joint section on the Blue and Red line, respectively.
$U^{nojoint}_{blue},U^{nojoint}_{red}$	The maximum passenger flow within the non-joint sections on the Blue and Red line, respectively.
C_{blue},C_{red}	The maximum number of passengers available on the trains belonging to the Blue and Red line, respectively.
$\eta^{\max},\eta^{\min}$	The maximum and minimum load ratio of the trains, respectively.
Decision Variables	
N_{blue},N_{red},N_{B-R}	The number of trains to be operated only on the Blue line, Red line, and trains between joint sections within each hour, respectively.

category of transfer passengers since they are transported by the joint trains; and the others should take time to walk from the Blue line to the Red line as well as wait at the platform for a train on the Red line, whose number is proportional to $N_{blue}/(N_{blue}+N_{B-R})$. The number of the affected transfer passenger from the Blue line to the Red line is $TP^{affected}_{blue}$, which can be calculated by (1). For this category of transfer passengers, the additional travel time is the sum of the walking time in the transfer station and waiting time on the platform for a train on the Red line. Usually, the average walking time $t^{walk}_{l_s}$ can be surveyed in the transfer station in advance, and waiting time $t^{wait}_{blue-tr}$ can be calculated through (2), which is highly affected by the train frequency.

$$TP^{affected}_{blue}=\sum_{i=1}^{s-1}\sum_{j=s'+1}^{n'}Q_{ij}\cdot N_{B-R}\big/(N_{blue}+N_{B-R}) \tag{1}$$

$$t^{wait}_{blue-tr}=60\big/2(N_{red}+N_{B-R}) \tag{2}$$

where Q_{ij} is the number of passengers from station *i* to *j*.

2) THROUGH PASSENGERS

The number of through passengers in ThP③ who enter at a station located in the joint section of the Blue line and exit at a station located in the non-joint section of the Blue line. The flow ThP③ can also be divided into two categories: some of them get on a blue train, so they do not need to wait at a station; their number is proportional to $N_{blue}/(N_{blue}+N_{B-R})$. Others may get on a green train, so they need to wait for a blue train, no matter at which station (at a transfer station or a station between the transfer and initial stations); their number is proportional to $N_{B-R}/(N_{blue}+N_{B-R})$, and so the number of the affected through passenger on the Blue line is $ThP^{affected}_{blue}$ which can be calculated by (3). The waiting time for through passengers who take a Green train and wait for a blue train is $t^{wait}_{blue-thp}$, which can be found through (4).

$$ThP^{affected}_{blue}=\sum_{i=1}^{s}\sum_{j=s+1}^{m}Q_{ij}\cdot N_{B-R}\big/(N_{blue}+N_{B-R}) \tag{3}$$

$$t^{wait}_{blue-thp}=60\big/2N_{blue} \tag{4}$$

The situation is similar for passengers who enter a station on the Red line. While the affected passengers on both lines are considered in the proposed optimization model. For simplicity, we only discuss the Blue Line as an example in this section.

Therefore, the objective function can be expressed as (5).

$$\min(Z_{TP}+Z_{ThP}) \tag{5}$$

where,

$$Z_{TP}=TP^{affected}_{blue}\cdot\left(t^{walk}_{l_s}+t^{wait}_{blue-tr}\right)+TP^{affected}_{red}\cdot\left(t^{walk}_{l_s}+t^{wait}_{blue-tr}\right) \tag{6}$$

$$Z_{ThP}=ThP^{affected}_{blue}\cdot t^{wait}_{blue-thp}+ThP^{affected}_{red}\cdot t^{wait}_{blue-thp} \tag{7}$$

C. CONSTRAINTS

1) LIMITATION OF AVAILABLE TRAINS

Since the trains cannot keep working all the time, if a train reaches the standard mileage for prescribed maintenance, it has to enter into the depot to undergo the corresponding maintenance operations. Taking trains maintenance (not-in-service trains) into account, the number of running trains should not exceed the number of available trains in the Blue line N^a_{blue} and Red line N^a_{red}. If there is insufficient capacity in a particular line, the missing capacity can be supplemented by another line in joint operation as the vehicle organization is also flexible in joint operation in our optimization model. The model assumes that Green trains are provided by the Red line. The number of running trains is restricted by the turnaround time and the number of available trains, and the resulting limit is given by (8)–(9).

$$N_{B-R}\cdot t^{turn}_{B-R}\big/60+N_{red}\cdot t^{turn}_{red}\big/60\le N^a_{red} \tag{8}$$

$$N_{blue}\cdot t^{turn}_{blue}\big/60\le N^a_{blue} \tag{9}$$

Parameters of N_{blue}, N_{red}, N_{B-R} are the number of trains to be operated only on the Blue and Red line and joint section within each hour, respectively. Parameters of t_{blue}^{turn}, t_{red}^{turn} and t_{B-R}^{turn} are the turnaround time of in the Blue line, Red line and joint section, respectively.

2) DEPARTURE INTERVAL REQUIREMENT

Since the waiting time of passengers in the non-joint section is mainly influenced by the train departure interval, a smaller train departure interval is desired to decrease passenger waiting time. But that may need more vehicles that result in higher operation costs. From the perspective of the operation manager, the greater departure interval means saving operation costs. From the perspective of safety operation, two adjacent trains should satisfy a certain departure interval. Taking technical, safety factors, and passenger demand into account, the departure interval in each line is limited by the maximum departure interval and minimum departure interval within the line. Therefore, for the joint operation of two intersecting lines, there will be four different headways that should be required in the joint and non-joint sections, equated by (10)–(13).

$$t_{blue}^{\min intervals} \le 60\big/(N_{blue} + N_{B-R}) \le t_{blue}^{\max intervals} \tag{10}$$

$$t_{blue}^{\min intervals} \le 60\big/N_{blue} \le t_{blue}^{\max intervals} \tag{11}$$

$$t_{red}^{\min intervals} \le 60\big/(N_{red} + N_{B-R}) \le t_{red}^{\max intervals} \tag{12}$$

$$t_{red}^{\min intervals} \le 60\big/N_{red} \le t_{red}^{\max intervals} \tag{13}$$

3) LOAD FACTOR CONSTRAINTS

Since the load factor η of a train, defined as the ratio of the average load over a designated period of time to the peak load occurring in that period. It not only reflects the service quality of metro network but also the operating cost, so it should be also taken into account in this paper. In order to improve the quality of passenger service, trains should not be overcrowded, and the load factor cannot be too small either to save operating costs. Therefore, the ratio of passenger volume to capacity within each area should have a certain limit. Similar to the departure interval requirement, there will be four different load factors that should be satisfied when considering the joint operation of two intersecting lines, equated by (14)–(17).

$$\eta^{\min} \le U_{blue}^{joint}\Big/(N_{blue} \cdot C_{blue} + N_{B-R} \cdot C_{red}) \le \eta^{\max} \tag{14}$$

$$\eta^{\min} \le U_{red}^{joint}\Big/(N_{red} + N_{B-R}) \cdot C_{red} \le \eta^{\max} \tag{15}$$

$$\eta^{\min} \le U_{blue}^{nojoint}\Big/N_{blue} \cdot C_{blue} \le \eta^{\max} \tag{16}$$

$$\eta^{\min} \le U_{red}^{nojoint}\Big/N_{red} \cdot C_{red} \le \eta^{\max} \tag{17}$$

To sum it up, the optimization model for the train frequency in a metro network with joint operation can be formulated as the following model, which is subjected to limitation of available trains (8)-(9), departure interval requirement (10)-(13), and load factor constraints (14)-(17).

$$\begin{cases} Min\ Z = Z_{TP} + Z_{ThP} \\ s.t.\ constraints\ (8)\text{-}(17) \\ N_{blue}, N_{red}, N_{B-R} \in Z^{+} \end{cases} \tag{18}$$

V. CASE STUDY

A. CASE SETTINGS

In reality, the joint operation in the metro system is applicable in many countries, especially in Y-type metro networks, where the transfer stations experience more volume as all the passengers of the feeder line need to wait there for the trunk line. Therefore, we consider a Y-type metro network in Beijing metro system to verify the effectiveness of our proposed optimization model for joint operation in the metro system.

Since the passenger flow mainly consists of commuters in Beijing metro system during the morning peak hour (7:00–9:00), who are from suburbs to the city center and transfer in the intersection between suburban lines and urban lines, the transfer station usually suffers from severe congestion. Thus, we consider a typical Y-type metro network in our case study, as shown in Figure 5, which consists of the Blue line in the urban and the Red line in the suburban. On the Red and Blue lines, there are 12 and 16 stations, respectively, and XEQ is a transfer station connecting the Red line and Blue line.

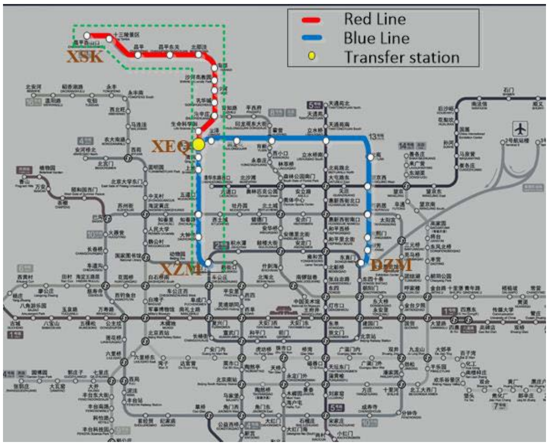

FIGURE 5. Red and Blue lines of Beijing metro system.

The passenger demand data was collected on a workday of June 2021 to represent the dynamics of passenger flow at each section. The demand profiles for each section on each line are illustrated in Figure 6.

In the Y-type metro networks, the number of transfer passengers during the morning rush hour of a workday is 19531 per hour. The passenger flow from XSK to XZM is 14539 persons per hour, which accounts for 74% of the total transfer flow. Therefore, XSK–XEQ–XZM can be considered as the joint operation section, circled by the green dash line in Fig. 5. During the morning peak hour, the departure time

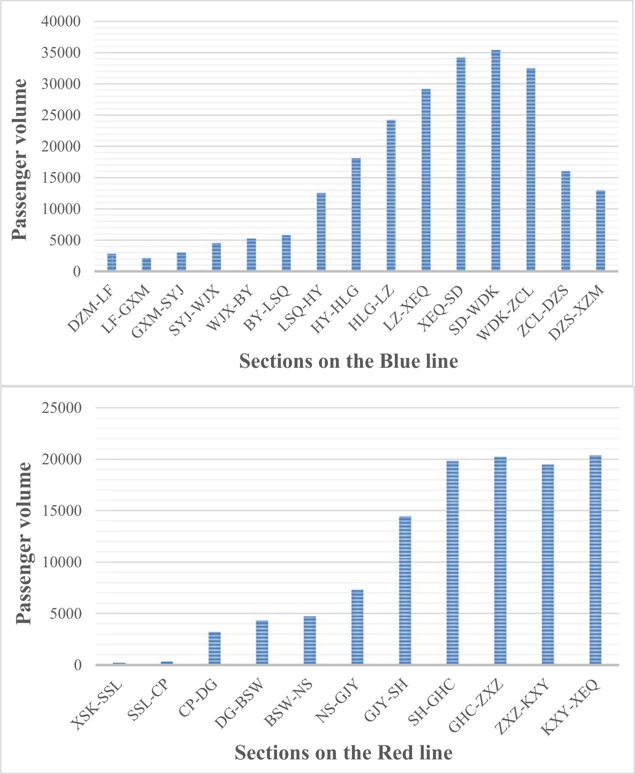

FIGURE 6. Passenger data.

TABLE 2. Values of some parameters in case.

Parameters	Blue Line	Red Line
Turnaround Time	120'15"	58'30"
Maximum headway (min)	4'	5'
Minimum headway (min)	2'30"	3'
Train capacity (person)	1428	1460
Number of Available Trains (train)	40	16

interval of trains is shorter and the full-load rate is quite higher than off-peak. The corresponding operation parameters are listed in Table 2, including turnaround time, max/min headway, the number of available trains, etc.

Additionally, the turnaround time of XSK–XEQ–XZM is 100 min, while the minimum and maximum load factors of all trains are 70% and 120%, respectively. The walking time to the transfer station is 4 min.

Since the proposed objective function and constraints are all linear, the presented problem is an integer linear programming problem that can be solved by existing commercial solvers. We used solver CPLEX 12.6 to solve the presented mathematical model with the OPL language. The numerical tests are performed on a PC with Windows 7 platform, Intel(R) Core(TM) i7-2130 with 2.7 GHz CPU and 8.00 GB memory.

B. RESULTS ANALYSIS

1) OPTIMIZATION RESULTS

The turnaround time for a Blue train is two times longer than for a Red train, so the required number of operating trains is larger in the Blue line. Taking into account the fact that the number of available trains is limited, the Blue line is not suitable for providing Green trains, and all Green trains are provided by the Red line.

The results of train frequency for the Blue and Red lines in joint operation are shown in Figure 7.

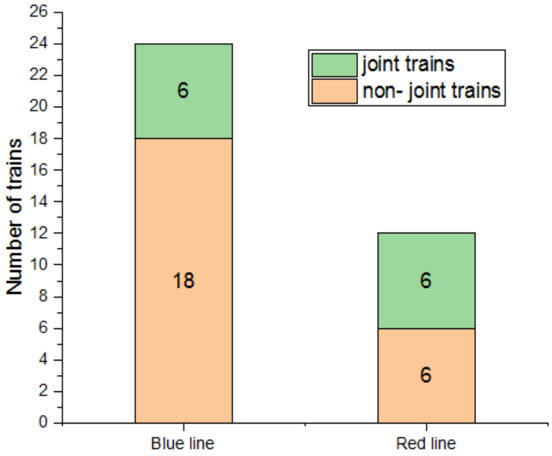

FIGURE 7. Model results for the train frequency.

In Figure 7, the train frequency of the Blue line is 24, 18 for Blue and 6 for Green; whereas the train frequency of the Red line is 12, 6 for Red and 6 for Green.

The ratio of passenger volume to capacity (load factor) in each section is shown in Figure 8.

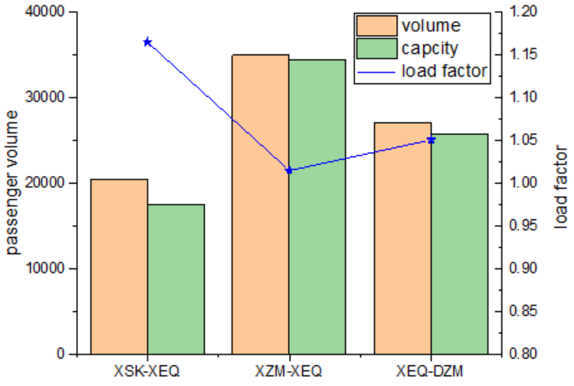

FIGURE 8. Diagram for the passenger volume and train capacity in each section.

Figure 8 shows that the passenger volume of the Red line (XSK-XEQ) and the Blue line (XEQ-DZM) exceeds their capacity, with their load factors greater than 1. While it's near to 1 in the Green Line (XZM-XEQ). Actually, the Red line has the highest load factor during the morning rush hour in Beijing and is as high as 1.4 for the Red line, and 1.12 for the Blue line. Since we set the maximum load factor is set as 120% in this study, the optimization results show that the maximum load factor appears in the Red line, i.e., 115%.

Under joint operation, the total transfer time during the morning rush hour at XEQ Station is 50192 min. However, if the Red and Blue lines operate independently, the total transfer time increases to 92365 min. Hence, the total transfer time can be reduced by 45.7% through applying joint operation, and the number of transfer passengers is reduced by half.

2) EFFECT OF THE NUMBER OF AVAILABLE TRAINS

The number of running trains is restricted by that of available trains, given by N^a_{blue} and N^a_{red}. The number of trains available in the Red line has been set to 16 so far, even though we should note that choosing a different value may yield different solutions. In this section, we will study the influence of the N^a_{red} on the solutions. The number of available trains is changed from 12 to 18, the corresponding result is shown Figure 9.

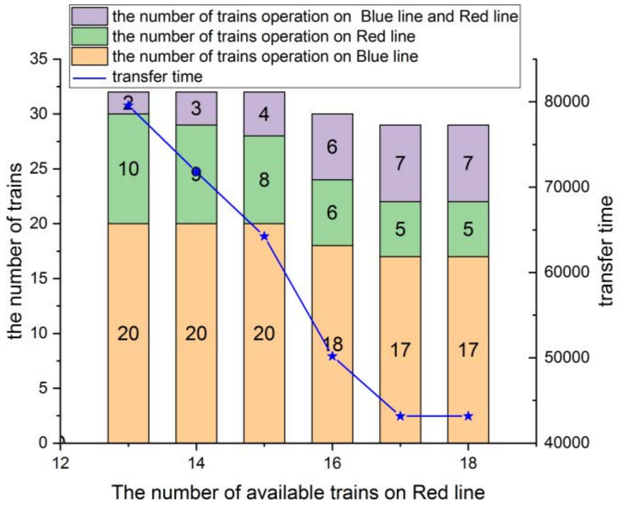

FIGURE 9. **Train frequency and total transfer time with different available trains in the Red line.**

Figure 9 shows that (i) if the number of available trains in the Red line is 12, there is no solution. It is because the capacity is insufficient and the load rate is over 1.2; (ii) if the number of trains available in the Red line is greater than 17, the capacity and the objective function do not change. the reason is that the trains from the Red line cannot enter the Blue line subjected to the train headway and load factor constraints. Meanwhile, the number of joint trains is also limited by these constraints; (iii) the objective function is approximately 79563 *min* when $N^a_{red} = 13$. If the number of available trains increased from 13 to 17, the value of the objective function decreases from 79563 *min* to 43163 *min.* In this case, the travel time of transfer passengers can be reduced by 45.8%. Thus it can be seen that the number of available trains on the Red line directly affects the efficiency of the joint operation.

From the above analysis, we can see that a reasonable N^a_{red} can reduce both transfer time and operation cost. In other words, the number of available trains cannot be too small to avoid insufficient capacity while it cannot be too large either to save operating costs. As shown in the above case, when the number of available trains of the Red line N^a_{red} increases from 16 to 17, the minimum transfer time, the minimum number of trains, and the maximum number of available trains could be achieved, and when N^a_{red} is greater than 17, the cost will increase but the capacity will no longer improve. That means the optimal N^a_{red} for the joint operation in this case is 17 and the company does not need to buy more trains than this threshold.

VI. CONCLUSION

In this paper, two types of train running strategies are firstly proposed and studied. Based on the analysis of the two train running strategies, we consider that the train running strategies of replacing part of the original trains with new trains are suitable for joint operation. After that, we analyze the affected passengers under this train running strategy. Taking the limitation of available trains, departure intervals, service quantity, and operation cost into account, we propose an optimization model for the train frequency in the joint operation to balance the travel time of through and transfer passengers as well as relieve the congestion in transfer stations.

Through the case study analysis, we can see that i) the joint operation strategy can reduce the total transfer time by 45.7%. Besides, under certain conditions, almost half of the transfer passengers no longer need to transfer; ii) by increasing the number of available trains in the Red line, both the affected passengers and the total number of trains decreased. However, when this number was greater than a certain threshold, the service capacity of the metro system did not change because of other line available trains limitations. Therefore, for joint operation, the train frequencies of each line should be coordinated with each other; iii) the threshold for the number of available trains can be obtained by our model. With a reasonable threshold, the metro operation company does not need to provide more trains than that threshold, which allows them to reduce the expenses.

Compared to the conventional metro operation approach, the joint operation can provide substantial benefits from more flexible track utilization. What's more important is that it can help to reduce the volume pressure in transfer stations that may avoid terrible stampedes. Our optimization model for train joint operation can help metro operation companies execute better train running strategies and train frequency schemes, thereby leading to a more efficient and sustainable railway system.

REFERENCES

[1] J. Shi, L. Yang, J. Yang, F. Zhou, and Z. Gao, "Cooperative passenger flow control in an oversaturated metro network with operational risk thresholds," *Transp. Res. C, Emerg. Technol.*, vol. 107, pp. 301–336, Oct. 2019, doi: 10.1016/j.trc.2019.08.008.

[2] E. Hassannayebi, A. Sajedinejad, and S. Mardani, "Disruption management in urban rail transit system: A simulation based optimization approach," in *Handbook of Research on Emerging Innovations in Rail Transportation Engineering*. Hershey, PA, USA: IGI Global, 2016, doi: 10.4018/978-1-5225-0084-1.ch018.

[3] C. E. Cortés, S. Jara-Díaz, and A. Tirachini, "Integrating short turning and deadheading in the optimization of transit services," *Transp. Res. A, Policy Pract.*, vol. 45, no. 5, pp. 419–434, Jun. 2011, doi: 10.1016/j.tra.2011.02.002.

[4] J. Lee, "Mathematical modeling for optimizing skip-stop rail transit operation strategy using genetic algorithm," Dept. Transp. Urban Infrastruct. Stud., Morgan State Univ., Baltimore, MD, USA, Tech. Rep., 2012. [Online]. Available: https://www.researchgate.net/publication/339004326_Mathematical_Modeling_for_Optimizing_Skip-Stop_Rail_Transit_Operation_Strategy_Using_Genetic_Algorithm

[5] Y. Wang, B. De Schutter, T. van den Boom, B. Ning, and T. Tang, "Real-time scheduling for single lines in urban rail transit systems," in *Proc. IEEE Int. Conf. Intell. Rail Transp. Process.*, Beijing, China, Aug. 2013, pp. 1–6, doi: 10.1109/ICIRT.2013.6696258.

[6] H. Zhang and S. Z. Zhao, "Design of integrated limited-stop and short-turn services for a bus route," *Math. Problems Eng.*, vol. 2016, May 2016, Art. no. 7901634, doi: 10.1155/2016/7901634.

[7] M. Freyss, R. Giesen, and J. C. Muñoz, "Continuous approximation for skip-stop operation in rail transit," *Transp. Res. C, Emerg. Technol.*, vol. 36, pp. 419–433, Jul. 2013, doi: 10.1016/j.sbspro.2013.05.012.

[8] A. Deckoff, "Master dissertations: The short-turn as a real time transit operating strategy," Massachusetts Inst. Technol., Cambridge, MA, USA, Tech. Rep., 1990. [Online]. Available: http://dspace.mit.edu/handle/1721.1/41774

[9] T. Parkinson and I. Fisher, *TCRP Report 13: Rail transit capacity*, vol. 118. Washington, DC, USA: Transportation Research Board, 1996. [Online]. Available: https://www.apta.com/research-technical-resources/tcrp/

[10] S. Phraner, R. Roberts, P. Stangas, K. Karach, J. Shortreed, and G. Thompson. (1999). *TCRP Report 52: Joint Operation of Light Rail Transit or Diesel Multiple Unit Vehicles With Railroads*. [Online]. Available: https://www.apta.com/research-technical-resources/tcrp/

[11] M. Aoki, "Railway operators in Japan 4 central Tokyo," *Jpn. Railway Transp. Rev.*, vol. 30, pp. 42–53, May 2002. [Online]. Available: https://trid.trb.org/view/605528.

[12] K. Vlachou and D. J. Lovell, "Mechanisms for equitable resource allocation when airspace capacity is reduced," *Transp. Res. Record*, vol. 2325, no. 10, pp. 97–102, 2013, doi: 10.3141/2325-10.

[13] M. W. Kang, M. K. Jha, and R. Buddharaju, "Rail transit route optimization model for rail infrastructure planning and design: Case study of Saint Andrews, Scotland," *J. Transp. Eng.*, vol. 140, no. 1, pp. 1–11, Jan. 2014, doi: 10.1061/(ASCE)TE.1943-5436.0000445.

[14] M. Carey, "A model and strategy for train path with choice of lines, platforms, and routes," *Transp. Res. B*, vol. 28, no. 5, pp. 333–353, 1994, doi: 10.1016/0191-2615(94)90033-7.

[15] M. M. Nesheli and A. Ceder, "A robust, tactic-based, real time framework for public-transport transfer synchronization," *Transp. Res. C*, vol. 60, pp. 105–123, Oct. 2015, doi: 10.1016/j.trc.2015.08.008.

[16] A. Ceder, "Optimal design of transit short-turn trips," *Transp. Res. Rec.*, vol. 1221, pp. 8–22, Oct. 1989.

[17] M. S. Chowdhury and S. I. J. Chien, "Dynamic vehicle dispatching at intermodal transfer station," *Transp. Res. Rec.*, vol. 1753, pp. 61–68, Dec. 2001, doi: 10.3141/1753-08.

[18] L. Chen, X. Li, and G. Liu, "Analysis of the capacity of the turning-back station in the railway system," in *Proc. ICTE*, Chengdu, China, Oct. 2013, pp. 2761–2765, doi: 10.1061/9780784413159.401.

[19] X. M. Zhao, "Multi-routing planning design of Y-type urban rail transit," *Adv. Mech. Eng.*, vol. 8, no. 8, pp. 1–12, 2016, doi: 10.1177/1687814016667385.

[20] F. Cevallos and F. Zhao, "Minimizing transfer times in a public transit network with a genetic algorithm," *Transp. Res. Rec.*, vol. 1971, pp. 74–79 May 2013, doi: 10.3141/1971-11.

[21] F. M. Tsai, S. Chien, and C. H. Wei, "Joint optimization of temporal headway and differential fare for transit systems considering heterogeneous demand elasticity," *J. Transp. Eng.*, vol. 139, no. 1, pp. 30–39, Jan. 2013, doi: 10.1061/(ASCE)TE.1943-5436.0000468.

[22] J. Xiong, W. Guan, and L. Y. Song, "Optimal routing design of a community shuttle for metro stations," *J. Transp. Eng.*, vol. 139, no. 12, pp. 1211–1223, Dec. 2013, doi: 10.1061/(ASCE)TE.1943-5436.0000608.

[23] Y. C. Lai and Y. J. Lin, "Using profit-maximizing capacity framework and models for railway capacity management," *J. Transp. Eng.*, vol. 142, no. 10, 2016, Art. no. 04016045, doi: 10.1061/(ASCE)TE.1943-5436.0000879.

[24] R. Goverde, "Synchronization control of scheduled train services to minimize passenger waiting times," in *Proc. Annu. Congr.*, vol. 2, Delft, The Netherlands, 1988, pp. 1–35, doi: 10.1.1.18.109.

[25] D. Clarke, *Doctoral Dissertations: An examination of railroad capacity and its implications for rail-highway intermodal transportation*. Knoxville, TN, USA: University of Tennessee, 1995. [Online]. Available: https://trace.tennessee.edu/utk_graddiss/2668/

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