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Wireless Localization Method Using the Distributed Iterative Stochastic-Resonance-Based Signal Spectral Combination

DI HE[1], (Senior Member, IEEE), LIEEN GUO[2], JIAQING QU[3], AND CHIH-CHUN HO[4]

[1]Shanghai Key Laboratory of Navigation and Location-Based Services, Shanghai Jiao Tong University, Shanghai 200240, China
[2]School of Mechanical and Electrical Engineering, Nanchang University, Nanchang 330029, China
[3]Shanghai Radio Equipment Research Institute, Shanghai 200235, China
[4]Longfor Group, Beijing Longfor Blue Engine Industrial Park, Chaoyang, Beijing 100029, China

Corresponding authors: Di He (dihe@sjtu.edu.cn) and Lieen Guo (guolieen@163.com)

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ABSTRACT In this study, a kind of nonlinear stochastic-resonance (SR) signal enhancement technique combined with the distributed receiving array signal power spectrum combination approach is proposed. By utilizing the signal spectral power improvement technique, the fitness of the distributed SR system with the array structure, and the information combination of the receiving power spectra, the corresponding objective of wireless localization can be realized successfully. An iterative nonlinear SR-based processing structure and corresponding algorithm are also proposed, which guarantee the applicability of the proposed approach. Computer simulations give the better performances of the proposed method compared with the conventional direction of arrival (DoA) estimation approaches and corresponding wireless localization estimation results.

INDEX TERMS Stochastic resonance (SR), distributed power spectral combination, wireless localization.

I. INTRODUCTION

Recently, the high-precision localization has become a serious demand in many applications. Accordingly a lot of approaches have been proposed, for example the Extended-Kalman-Filter-based (EKF-based) approach [1], global navigation satellite system (GNSS) with real-time-kinematic-based (RTK-based) approach [2], visible light localization approach [3], data fusion of vision and RF sensors technique [4], deep-neural-networks-based device-free WiFi localization [5], simultaneous-localization-and-mapping-based (SLAM-based) approach [6], cooperative localization with deep reinforcement learning [7], direct localization for massive multiple-input-multiple-output (MIMO) system [8], and so on. Meanwhile, to realize the high-precision localization objective, some extra facilities are necessary, such as the RTK receivers, high precision cameras, visible light communication or localization systems. But in the wireless localization, it shows specific advantages that the outdoor base stations or indoor WiFi access points (AP) generally exist to fulfill the demand of wireless communications coverage. Simultaneously, the higher localization precision of traditional wireless localization methods also needs to be enhanced especially under some complex environments and low signal-to-noise (SNR) conditions.

Many research methods have been proposed to achieve the high-precision localization under complex wireless transmission environments and low SNR, for example the high-precision off-grid direction of arrival (DoA) estimation method [9], [10] and the co-prime linear array DoA estimation method [11], passive location estimation using time of arrival (ToA) measurement [12], time differential of arrival (TDoA) localization method involving the known positions of base stations and the power of the received signal [13], and the fusion of DoA/ToA or DoA/TDoA measurements [14]–[16] are mostly effective and applicable technologies. Recently, some gridless one-bit DoA estimation approach [17] is also proposed using atomic norm denoising.

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While in the above wireless localization methods, it still has some serious restrictions. As for the ToA-based approaches, the time synchronization between the base stations and the receivers or the synchronization within different base stations is required. As for the DoA-based approaches, the multiuser interference and none-line-of-sight (NLOS) transmission are the most unpredicted factors which can easily make the conventional DoA-estimation-based localization invalid. Due to the above restrictions, some more reliable and effective wireless localization methods need to be considered.

In our study, a novel wireless localization approach using the modern wireless communication facilities is proposed and discussed, which is based on the distributed nonlinear stochastic resonance (SR) enhanced signal power spectral combination technique and corresponding iterative processing algorithm. This work is also the extension of our previous research work using the SR-enhanced power spectrum fusion to realize wireless positioning [18]. In this proposed approach, by introducing the nonlinear SR signal pre-processing method and the distributed SR processing structure, the signal power spectrum could be improved to compensate the path fading and the unpredictable interference in the wireless communication channel; and by introducing the receiving signal power spectral combination, the NLOS transmission problem can also be overcome effectively, which is familiar in the wireless communication channel. Simultaneously, the introduced iterative SR-based processing algorithm can help to reinforce the real application of this proposed approach. Computer simulation results certify that high-precision localization performances can be achieved even under low SNR and NLOS environment, which shows good advantages in the real applications.

The main contributions of this study are listed as follows: (1) A novel wireless localization approach with high-precision performance is proposed; (2) The corresponding SR-enhancing structure and the combination algorithm are presented clearly, which are very helpful for the complex wireless-localization-related applications; (3) A special iterative nonlinear SR-based processing structure and corresponding algorithm are also given clearly, which can further guarantee the applicability of the proposed approach; (4) The wireless signal DoA estimation results and the corresponding final wireless localization performances are evaluated comparing with traditional wireless localization methods, which also ensure the validity and applicability of the proposed approach.

The remaining of the paper is arranged as follows: Section II describes the wireless signal transmission model together with the traditional wireless localization approaches, and the main problems in the wireless localization are also given. In Section III, the proposed wireless localization approach is introduced in details, which is based on the distributed SR-based signal power spectral combination technique, and the iterative SR-based processing structure with corresponding algorithm. The computational simulation results are presented in Section IV, while the performance results are compared between the proposed approach and conventional wireless localization methods. Finally, the concluding remarks are given in Section V to summarize the whole paper.

II. TRADITIONAL WIRELESS LOCALIZATION APPROACHES

Wireless localization system is one of the most popular-used localization systems in the applications which just relies on the wireless transmitted signal. Especially in recent years, with the fast development of the public wireless communication systems including the 4G or upcoming ubiquitous 5G wireless communication systems and Wi-Fi communication systems for indoor and outdoor applications, the wireless localization has become a special localization application means in different fields, such as IoT localization, smart and intelligent manufacturing, self-driving cars, unmanned aerial vehicles (UAVs), and so on.

Generally speaking, in the certain wireless localization system, suppose that there is a wireless baseband source signal $s(t)$ at the transmitter, then the corresponding receiving signal $r(t)$ at the receiver can be written as

$$r(t) = \alpha_0 \cdot s(t) \cdot e^{j\omega t} + \sum_{k=1}^{K} \alpha_k \cdot s(t-\tau_k) \cdot e^{j\omega(t-\tau_k)} + n(t) \tag{1}$$

While on the right side of Eq.(1), the first item represents the receiving LOS component, the second item represents the multipath component, and the last item is the additive channel noise. $s(t)$ is with mean *0* and variance σ_s^2; α_0 is the amplitude fading factor corresponding to the LOS component; ω is the carrier angular frequency of the source signal; K is the multipath number at the receiver; α_k and τ_k ($k = 1, 2, \cdots, K$) are the corresponding amplitude fading factor and the time delay related to the k^{th} multipath signal, respectively; $n(t)$ is the additive channel noise (including the uncorrelated interferences) with mean *0* and variance σ^2.

Among various wireless localization methods, the DoA-based methods have attracted a lot of interests in the researchers due to the fact that it is easy to be implemented [19]–[21]. And for the traditional DoA-based wireless localization method, to achieve a super-resolution DoA estimation performance, some special receiving array structure may be used at the receiver, for example the uniform linear array (ULA) structure, the uniform circular array (UCA) structure, the uniform plane array (UPA) structure, and the receiving signal at the array can then be expressed as

$$r_m(t) = \alpha_{0,m} \cdot s(t) \cdot e^{j\omega t} + \sum_{k=1}^{K} \alpha_{k,m} \cdot s(t-\tau_{k,m}) \cdot e^{j\omega(t-\tau_{k,m})} + n_m(t) \quad (m = 1, 2, \cdots, M) \tag{2}$$

where M is the total antenna number in the receiving array structure; $r_m(t)$ is the receiving signal at the m^{th} receiving antenna ($m = 1, 2, \cdots, M$); $\alpha_{0,m}$ is the amplitude fading factor corresponding to the LOS component at the m^{th} receiving antenna; $\alpha_{k,m}$ and $\tau_{k,m}$ ($k = 1, 2, \cdots, K$) are the corresponding amplitude fading factor and the time delay related to the k^{th} multipath signal at the m^{th} receiving antenna corresponding to the LOS signal, respectively; and $n_m(t)$ is the combination of additive channel noise and uncorrelated interferences at the m^{th} receiving antenna with mean *0* and variance σ_n^2.

In the traditional super-resolution DoA estimation approaches, the eigenstructure signal-subspace and noise-subspace expression and eigenvalue decomposition approach is the most important one, in which the multiple signal classification (MUSIC) method [22], the estimating signal parameters via rotational invariance techniques (ESPRIT) [23], and their improved algorithms [24], [25] are the most representative and applicable approaches, which may have good performance and can realize high-precision DoA estimation results. At the same time, the final DoA estimation performances are also closely related to the receiving array structure, and the basic ESPRIT approach is only fit for the ULA receiving structure.

For example, in the basic MUSIC algorithm, the MUSIC space spectrum can be expressed by

$$P_{MUSIC}(\theta) = \frac{\boldsymbol{a}^H(\theta)\boldsymbol{a}(\theta)}{\boldsymbol{a}^H(\theta)\hat{\Pi}^{\perp}\boldsymbol{a}(\theta)} \tag{3}$$

where $\boldsymbol{a}(\theta)$ is the direction vector or steering vector of DoA which may be changed with the array structure, and θ is the source signal incidence angle which may include the azimuth angle and the elevation angle in the three dimensional space; $\hat{\Pi}^{\perp} = \hat{\boldsymbol{U}}_n\hat{\boldsymbol{U}}_n^H$ is the projection of sample estimate of received signal covariance matrix on the noise-subspace; $\boldsymbol{U}_n$ is the eigenvector of the noise-subspace, and $\hat{\boldsymbol{U}}_n$ is the estimate of $\boldsymbol{U}_n$. Generally, the θ value which reaches the peak value of MUSIC space spectrum $P_{MUSIC}(\theta)$ can be regarded as the final estimate of DoA. In Eq.(3), $(\cdot)^H$ is the complex conjugation operator, and $(\cdot)^{\perp}$ is the orthogonal complement operator.

At the same time, in the basic ESPRIT algorithm, by calculating the space cross-covariance matrix between the receiving signal vector and its corresponding translation vector, it can also get the estimated DoA information of the source signal based on the rotation matrix resulting from the eigenvalue decomposition of the space cross-covariance matrix. While here to reduce the complexity of the paper, we don't give the detailed description for the basic ESPRIT algorithm and its many improvement algorithms, which is easy to find in the related literatures.

Although the traditional DoA estimation methods including MUSIC, ESPRIT and most of their improvement algorithms show very good super-resolution DoA estimation performances especially compared with some classical beamforming methods, it still has a serious problem in the real application that they cannot guarantee the estimated DoA result is exactly the direction of LOS from the source signal. That is to say, if the LOS signal is blocked, or there are only NLOS signal at the receiver, or when the power of some multipath signal is higher than that of the LOS signal, the DoA estimation result and the corresponding wireless localization result may possess large errors, which restrict the real application of these traditional DoA estimation methods seriously.

III. PROPOSED APPROACH BASED ON DISTRIBUTED SR-ENHANCED POWER SPECTRAL COMBINATION

To solve the above-mentioned problem in real applications, here we try to propose a new wireless localization approach based on the idea of distributed nonlinear SR-enhanced signal power spectrum combination technique, together with its iterative nonlinear processing structure and corresponding algorithm, and discuss the applicability of this proposed approach.

A. DISTRIBUTED SR-ENHANCED POWER SPECTRAL COMBINATION

As discussed in Section II, when the LOS signal at the receiver has serious path fading or even blocked by some obstacles, it will be very difficult to find the correct location of the source signal which also corresponds to the direction of the LOS component at the receiver under low SNR or NLOS circumstances. In other words, both conditions above can be regarded as one situation that the power of the LOS signal is seriously submerged into the NLOS components and the additive channel noise, and in the worst case the power of the LOS signal may become 0 when it is totally blocked by the obstacles.

From the aspect of wireless localization, suppose some narrowband pilot signal is transmitted in the wireless communication system for the localization purpose as the integration of communication and navigation, or the bandwidth of the pilot signal is much smaller than that of the communication carrier frequency, the baseband signal at each receiving antenna will have the following property as

$$s(t - \tau_{k,m}) \approx s(t) \tag{4}$$

so Eq.(2) can be rewritten as

$$r_m(t) = \alpha_{0,m} \cdot s(t) \cdot e^{j\omega t} + \sum_{k=1}^{K} \alpha_{k,m} \cdot s(t) \cdot e^{j\omega(t-\tau_{k,m})} + n_m(t) \tag{5}$$

It can be found that $r_m(t)$ is a complex signal, and both its real part and imaginary part contain the phase information related to DoA estimation. Without loss of generality, here we consider the signal processing of the real part of $r_m(t)$, while the same processing can also be applied to the imaginary part. By multiplying the real part of $r_m(t)$ with a product term $\cos(\omega - \Delta\omega)t$ where $\Delta\omega$ is a relative small enough angular frequency compared with ω, and we have

$$\begin{aligned} r_m^R(t) &= \alpha_{0,m} \cdot s(t) \cdot \frac{1}{2}\left[\cos(\Delta\omega \cdot t) + \cos(2\omega t - \Delta\omega \cdot t)\right] \\ &\quad + \sum_{k=1}^{K} \alpha_{k,m} \cdot s(t) \cdot \frac{1}{2}\Big[\cos(\Delta\omega \cdot t - \omega\tau_{k,m}) \\ &\quad + \cos(2\omega t - \Delta\omega \cdot t - \omega\tau_{k,m})\Big] \\ &\quad + n_m^R(t) \cdot \cos(\omega - \Delta\omega)t \end{aligned} \tag{6}$$

where $r_m^R(t)$ is the output signal after the above-mentioned processing, $n_m^R(t)$ is the real part of $n_m(t)$. In the imaginary part processing, we could just instead all the $\cos(\cdot)$ function by $\sin(\cdot)$ function in Eq.(6). In the meantime, it needs to mention that the corresponding following signal processing may also be doubled if the imaginary part processing is introduced. So to reduce the computational complexity and simplify the description of the following proposed approach, we here only consider the real part processing of the receiving signal, which will not influence the performance evaluation of the following proposed approach.

In the modern wireless communications especially 4G, 5G systems and Wi-Fi system, the carrier angular frequencies ω are relatively high over 1.5×10^{10} *rad/s*. In Eq.(6), it can be found that the baseband information contains in both high-frequency band and low-frequency band. To reduce the computational complexity, we can try to let the signal $r_m^R(t)$ pass through a low-pass filter first. While in some real conditions, the multipath item $\alpha_{k,m}\cdot s(t)\cdot\frac{1}{2}\cdot\cos(\Delta\omega\cdot t-\omega\tau_{k,m})$ in Eq.(6) may not be filtered clearly and ideally if the time delay $\tau_{k,m}$ is also relatively small. Therefore, the multipath item can be regarded as another low frequency component in the signal besides $\Delta\omega$, or it can be regarded as another independent component of additive noise. Based on that, we can let $r_m^R(t)$ pass through an ideal low-pass filter which can filter the frequency components higher than $\omega/2$, then it becomes

$$\begin{aligned} r_m^{R-LP}(t) &= \frac{1}{2}\alpha_{0,m} \cdot s(t) \cdot \cos(\Delta\omega \cdot t) \\ &\quad + \frac{1}{2}\sum_{k=1}^{K}\left[\alpha_{k,m} \cdot s(t) \cdot \cos(\Delta\omega \cdot t - \omega\tau_{k,m})\right] \\ &\quad + LPF\left[n_m^R(t) \cdot \cos(\omega - \Delta\omega)t\right] \\ &\triangleq \frac{1}{2}\alpha_{0,m} \cdot s(t) \cdot \cos(\Delta\omega \cdot t) + n_m^{R-LP}(t) \end{aligned} \tag{7}$$

where $r_m^{R-LP}(t)$ is the signal $r_m^R(t)$ passing through the low-pass filter, $LPF\,[\cdot]$ is a low-pass filter, and $n_m^{R-LP}(t)$ is defined as the additive noise signal composed of $LPF\left[n_m^R(t) \cdot \cos(\omega - \Delta\omega)t\right]$ and all other multipath components after passing through the low-pass filter with mean 0 and variance $\sigma_{n_m}^2$, say $\frac{1}{2}\sum_{k=1}^{K}\left[\alpha_{k,m} \cdot s(t) \cdot \cos(\Delta\omega \cdot t - \omega\tau_{k,m})\right]$.

For the signal $r_m^{R-LP}(t)$ appeared in Eq.(7), although it may still have the problems of low SNR and NLOS as described before, it can be found that the signal can be regarded as the sum of a low-frequency signal containing the desired baseband information and an additive noise, and it is obvious that this signal form just fits the input signal or driving signal requirement of the traditional SR system very well [26]. For the SR system, it stands for phenomena for which the ordered response of a system with respect to weak input signals can be significantly increased by appropriately tuning the noise intensity to an optimal but nonvanishing value [26]. The SR system has now been applied in different applications, such as weak signal estimation [27], wireless receiver performance enhancement [28], and so on. So if the SR system can be introduced, the signal spectrum power amplification (SPA) enhancement performance in the SR system can be utilized to enhance the SNR of the baseband signal [26], which can try to solve one of the problems discussed before.

Without loss of generality, the state equation of the introduced SR system driven by the external signal $r_m^{R-LP}(t)$ can be expressed by

$$\dot{x}_m(t) = f_m[x_m(t), r_m^{R-LP}(t), \gamma_m(t)] \tag{8}$$

where $f_m[\cdot]$ is the nonlinear function of the m^{th} SR system which corresponds to the m^{th} receiving antenna; $x_m(t)$ is the status variable of the SR system at the m^{th} receiving antenna; $\gamma_m(t)$ is the inner SR noise of the SR system with mean *0* and variance $\sigma_{\gamma_m}^2$, which can be used to optimize the whole SR system to an ideal condition such as to reach the maximal output SNR [27].

In this study, we use the traditional discrete bistable SR system in [26] as the distributed SR system $f_m[\cdot]$ in Eq.(8), and its state equation can be written as

$$\begin{aligned} \frac{x_m(t+\Delta t) - x_m(t)}{\Delta t} &= a \cdot x_m(t) + b \cdot x_m^3(t) + g_1 \cdot r_m^{R-LP}(t) \\ &\quad + g_2 \cdot \gamma_m(t) \end{aligned} \tag{9}$$

where Δt is the time sampling interval; a, b, g_1 and g_2 are the system parameters of the discrete bistable SR system. In a lot of applications, $a = 2$ and $b = -1$ are mostly selected parameters which can give some better system performance than other parameter values [26].

In Eq.(9), as mentioned above, $r_m^{R-LP}(t)$ can be regarded as the external driving signal of the discrete bistable SR system, and $\gamma_m(t)$ can be regarded as the inner SR noise. According to the Linear Response Theory of SR system [26], when the dynamic evolution of the whole SR system comes into the dynamic steady state, the status variable $x_m(t)$ can be separated into the following two components as

$$x_m(t) = r_{m(SR)}^{R-LP}(t) + \gamma_{m(SR)}(t) \tag{10}$$

where $r_{m(SR)}^{R-LP}(t)$ and $\gamma_{m(SR)}(t)$ can be regarded as the nonlinear SR system responses corresponding to the external driving signal $r_m^{R-LP}(t)$ and the inner SR noise $\gamma_m(t)$, respectively.

Therefore according to the SPA Theory of SR system [26], the input SNR (defined by $SNR_{i(m)}$, representing the SNR of $r_m^{R-LP}(t)$ in Eq.(7)) and output SNR (defined by $SNR_{o(m)}$, representing the SNR of $x_m(t)$ in Eq.(10) when the SR system comes into the dynamic steady state) of the traditional discrete bistable SR system under the assumption that $n_m^{R-LP}(t)$ and $\gamma_{m(SR)}(t)$ are white Gaussian noise signals can be calculated by

$$SNR_{i(m)} = \frac{\alpha_{0,m}^2 \sigma_s^2}{8\sigma_{n_m}^2} \tag{11}$$

$$SNR_{o(m)} = \frac{\sqrt{2} \cdot g_1^2 \alpha_{0,m}^2 \sigma_s^2}{\left(g_1^2 \sigma_{n_m}^2 + g_2^2 \sigma_{\gamma_m}^2\right)^2} e^{-\frac{2}{g_1^2 \sigma_{n_m}^2 + g_2^2 \sigma_{\gamma_m}^2}} \tag{12}$$

While in some application case for example a complex noisy environment instead of additive white Gaussian noise (AWGN), the nonlinear response of the SR system can be calculated by

$$\langle x_m(t) \rangle_{asy} = \sum_{l=-\infty}^{+\infty} G_l \cdot \exp\left[i\left(l\omega t + \varphi_0\right)\right] \tag{13}$$

where $\langle x_m(t) \rangle_{asy}$ is the asymptotic response of the SR system when $t \rightarrow +\infty$, G_l is the complex-valued amplitudes connected with the multiple frequencies of the external force signal, and φ_0 is the corresponding phase shift of the driving signal after the SR system [26].

The output power at the original driving signal frequency is then defined by $|G_1|^2$ at $\pm\omega$. And the SPA η of the driving signal can then be calculated by [26]

$$\eta = \left(\frac{2\,|G_1|}{\alpha_{0,m}\sigma_s/2}\right)^2 = \left(\frac{4\,|G_1|}{\alpha_{0,m}\sigma_s}\right)^2 \tag{14}$$

In the above expression, it is obvious that the final SPA effect is determined by $|G_1|$ seriously, which is also both related to the driving signal and complex noise power at $\pm\omega$, so the explicit expression of (14) can only be written or calculated when the frequency property of background noise can be determined, otherwise it is impossible to give a uniform evaluation to the proposed approach. Based on the above reasons and without loss of generality, here we still assume that $n_m^{R-LP}(t)$ and $\gamma_{m(SR)}(t)$ are white Gaussian noise.

So in our proposed SR-enhanced approach, by selecting the above $SNR_{o(m)}$ in Eq.(12) as an objective function of the signal power spectrum combination optimization problem especially under the constraint $SNR_{o(m)} > SNR_{i(m)}$, the signal power spectrum enhancement corresponding to the external driving signal $r_m^{R-LP}(t)$ can be realized. And for the detailed nonlinear SR-based processing structure, another corresponding novel SR-based iterative processing algorithm is also proposed in our study, which will be described clearly in the following subsection *B*.

Based on the analyses above, an SR-enhanced signal power spectrum combination approach can be proposed to improve the DoA estimation and wireless localization performances. And Fig.1 plots the block diagram structure of the proposed approach.

In the real applications, it always happens that there exist multiple wireless base stations or Wi-Fi access points (APs)

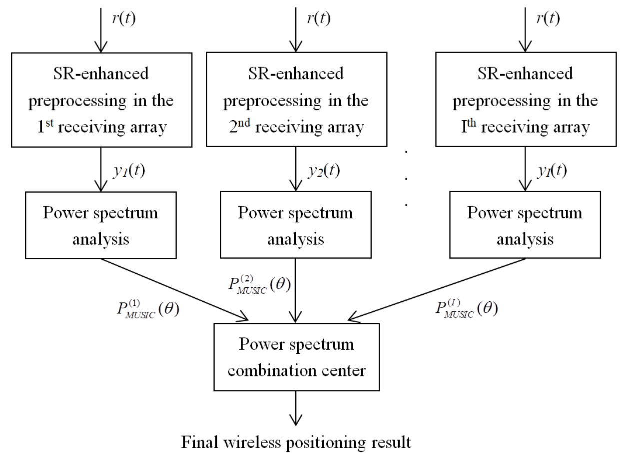

FIGURE 1. Block diagram structure of the proposed approach.

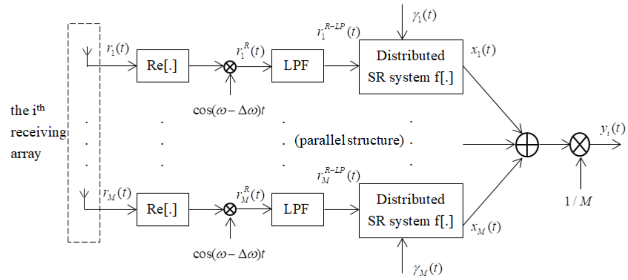

FIGURE 2. Distributed SR signal processing structure in each receiving array.

which can receive the uplink communication signal from the transmitter or the mobile terminal. To realize the distributed wireless localization and to solve the problem that the LOS signal is seriously faded or even blocked, at each base station or AP a receiving antenna array can be set up so that the array-based signal processing and power spectrum analysis such as basic MUSIC algorithm could be performed. As described above, to improve the SNR of the receiving signal, the SR-based signal power enhancement or data preprocessing method is firstly implemented based on Eqs.(6)~(8). Simultaneously, to further utilize the advantage of distributed parallel SR network over the single SR processing at each of the receiving array, a kind of distributed SR signal processing structure is introduced in Fig.2. As can be seen, in each antenna of the receiving array, there exists a distributed SR processing unit which is used to perform the corresponding nonlinear SR processing as in Eqs.(6)~(8). So the distributed parallel SR network structure is established. Previous studies have seen that the signal detection performance of the distributed parallel SR network shows better signal enhancement performance with the increasing of the distributed parallel SR system number [28]. So we introduce this distributed structure in our study, while the multiple receiving antennas in the array structure can certainly guarantee the performance of the distributed parallel SR system.

After the parallel SR processing, we can take the average of the output signal at the i^{th} receiving array as

$$y_i(t) = \frac{1}{M}\sum_{m=1}^{M} x_m(t) \tag{15}$$

So by analyzing or calculating the power spectrum of signal $y_i(t)$ at each of the receiving array, we can then get the corresponding spectrum information $P_{MUSIC}^{(i)}(\theta)$ under a unified coordinate system. Here to simplify the following expressions and analyses, we still use the conventional MUSIC method for the spectrum analysis as in Eq.(3). Some other spectrum analysis methods can also be used here.

Finally, when all the spectrum information corresponding to the same mobile user or mobile terminal at each base station or AP can be collected, a kind of spectrum combination method is introduced to solve the NLOS problem mentioned before. Still for the simplicity and to reduce the computational complexity, we just consider the combination technique as follows

$$P_{total}\left|_{(x,y,z)}\right. = \sum_{i=1}^{I} \eta_i \cdot P_{MUSIC}^{(i)}\left(\theta \left|_{(x,y,z)}\right.\right) \tag{16}$$

where $P_{total}\left|_{(x,y,z)}\right.$ is the final combined power spectrum information at some certain position (x, y, z), I is the total number of base stations or APs which can be used to realize the combination and wireless localization purpose, η_i is the power spectrum combination coefficient corresponding to the i^{th} receiving array. For the power spectrum combination, it requires that all the APs coordinates are set under the same consistent unified coordinate system, so that the final spectrum combination could be easily realized by using the linear weighted summarization in Eq.(16). According to the fact that the final combined spectrum information $P_{total}\left|_{(x,y,z)}\right.$ can also be regarded as a kind of weighted spectral sum to all the distributed array at different locations, the corresponding spatial signal power spectral diversity could guarantee to solve the NLOS problem at several receiving array, and the global spectrum property of the source signal could be presented more clearly.

In other words, the spatial correlation between the multipath signals received by different space-distributed base stations is much weaker than that of the LOS signals, because in real applications there should exists large differences within the reflected power in different directions when the wireless signals are reflected by the irregular surrounding reflective surfaces. According to this, the multipath and NLOS wave suppression effect can be further realized by the power spectrum combination processing introduced here, and then the high-precision localization performance can be achieved.

B. ITERATIVE NONLINEAR SR-BASED PROCESSING STRUCTURE AND CORRESPONDING ALGORITHM

Considering the problems mentioned in the last subsection, we continue to propose a kind of novel signal subspace decomposition approach by introducing the nonlinear SR processing together with the iterative processing structure.

In the nonlinear dynamic processing methods, SR is a very special technique which possesses the following advantages [29]–[31]: (1) It can try to improve the SNR of the system input signal especially under low SNR and certain SR optimization conditions. (2) The traditional SR process is a kind of dynamical procedure, which indicates that the signal SNR improvement will be realized gradually with the increasing of receiving signal snapshots.

To utilize the SR system properties and solve the problems in the conventional subspace-decomposition-based methods, based on the above advantages and the conventional SR system structure, we construct the block diagram of the proposed approach based on the iterative SR system shown in Fig.3 as follows.

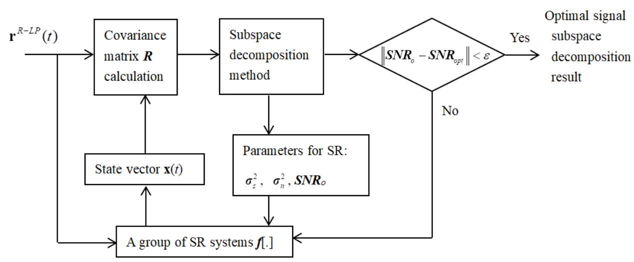

FIGURE 3. Block diagram of the proposed approach based on the iterative SR system.

In Fig.3, a group of dynamic state equations of the SR system can be expressed by

$$\dot{\mathbf{x}}(t) = \boldsymbol{f}[\mathbf{x}(t), \mathbf{r}^{R-LP}(t), \boldsymbol{\gamma}(t)] \tag{17}$$

where

$$\mathbf{x}(t) = [x_1(t), x_2(t), \cdots, x_M(t)]^T \tag{18}$$

$$\boldsymbol{f}[\cdot] = [f_1[\cdot], f_2[\cdot], \cdots, f_M[\cdot]]^T \tag{19}$$

$$\mathbf{r}^{R-LP}(t) = \left[r_1^{R-LP}(t), r_2^{R-LP}(t), \cdots, r_M^{R-LP}(t)\right]^T \tag{20}$$

$$\boldsymbol{\gamma}(t) = [\gamma_1(t), \gamma_2(t), \cdots, \gamma_M(t)]^T \tag{21}$$

In the above equations, $\boldsymbol{f}[.]$ is the dynamic state functions of a group of SR systems with dimension M × 1, $\mathbf{x}(t)$ is the state vector of this group of SR system with dimension M × 1, $\dot{\mathbf{x}}(t)$ is the first derivative vector of $\mathbf{x}(t)$, $\mathbf{r}^{R-LP}(t)$ is the input signal vector of this group of SR system with dimension M × 1, $\gamma(t)$ is the optimization noise vector of this group of SR system with mean $\mathbf{0}$ and dimension M × 1, and $[\cdot]^T$ is the transpose operation.

Considering there are K multiple source signals, and $\boldsymbol{\sigma}_s^2$ is the signal variance vector defined as $\boldsymbol{\sigma}_s^2 = [\sigma_{s_1}^2, \sigma_{s_2}^2, \cdots, \sigma_{s_K}^2]^T$, $\boldsymbol{\sigma}_n^2$ is the noise variance vector defined as $\boldsymbol{\sigma}_n^2 = [\sigma_n^2, \sigma_n^2, \cdots, \sigma_n^2]^T$ with dimension (M − K) × 1 where we suppose M > K, ε is a fixed relative small enough positive value, $\|\cdot\|$ is the modulus operator, $\boldsymbol{SNR}_{opt}$ is the optimal SR system output SNR vector with dimension M × 1 which will be described later in this subsection, and $\boldsymbol{SNR}_o$ is the output signal SNR vector with dimension M × 1 defined by

$$\boldsymbol{SNR}_o = \left[\frac{\sigma_{s_1}^2}{\sigma_n^2}, \frac{\sigma_{s_2}^2}{\sigma_n^2}, \cdots, \frac{\sigma_{s_K}^2}{\sigma_n^2}\right]^T \tag{22}$$

In the proposed algorithm, to ensure the user fairness and to reduce the analyses complexity, we suppose that the corresponding parameters for different users (different source signal) remain the same except the source signal variance, that is

$$\alpha_0 = \alpha_{0,1} = \alpha_{0,2} = \cdots = \alpha_{0,M} \tag{23}$$

$$\sigma_n^2 = \sigma_{n_1}^2 = \sigma_{n_2}^2 = \cdots = \sigma_{n_M}^2 \tag{24}$$

$$\sigma_\gamma^2 = \sigma_{\gamma_1}^2 = \sigma_{\gamma_2}^2 = \cdots = \sigma_{\gamma_M}^2 \tag{25}$$

So to reach the optimal performance enhancement, we define the optimal SR system output SNR vector $\boldsymbol{SNR}_{opt}$ as

$$\boldsymbol{SNR}_{opt} = \begin{bmatrix} \dfrac{\sqrt{2}\cdot g_1^{*2}\alpha_0^2\sigma_{s_1}^2}{\left(g_1^{*2}\sigma_n^2 + g_2^{*2}\sigma_\gamma^{*2}\right)^2} e^{-\frac{2}{g_1^{*2}\sigma_n^2+g_2^{*2}\sigma_\gamma^{*2}}} \\ \dfrac{\sqrt{2}\cdot g_1^{*2}\alpha_0^2\sigma_{s_2}^2}{\left(g_1^{*2}\sigma_n^2 + g_2^{*2}\sigma_\gamma^{*2}\right)^2} e^{-\frac{2}{g_1^{*2}\sigma_n^2+g_2^{*2}\sigma_\gamma^{*2}}} \\ \vdots \\ \dfrac{\sqrt{2}\cdot g_1^{*2}\alpha_0^2\sigma_{s_K}^2}{\left(g_1^{*2}\sigma_n^2 + g_2^{*2}\sigma_\gamma^{*2}\right)^2} e^{-\frac{2}{g_1^{*2}\sigma_n^2+g_2^{*2}\sigma_\gamma^{*2}}} \end{bmatrix} \tag{26}$$

where the parameters g_1^*, g_2^*, and σ_γ^* are the optimal parameters of g_1, g_2, and σ_γ to reach the maximal SNR of the SR system output signal, say

$$\left(g_1^*, g_2^*, \sigma_\gamma^*\right) = \arg \max_{(g_1, g_2, \sigma_\gamma)} \frac{\sqrt{2}\cdot g_1^2\alpha_0^2\sigma_{s_k}^2}{\left(g_1^2\sigma_n^2 + g_2^2\sigma_\gamma^2\right)^2} e^{-\frac{2}{g_1^2\sigma_n^2+g_2^2\sigma_\gamma^2}} \quad (k = 1, 2, \cdots, K) \tag{27}$$

while the optimization process of the above problem could be realized easily by conventional optimization methods.

To give a more detailed description and explanation to the proposed approach, we now give the corresponding algorithm operation steps as shown in the following Table 1.

In the above proposed algorithm, some details in the calculation or processing could be explained more clearly as follows:

(1) In Step 3, we need to use the traditional subspace decomposition method to get the estimated values of $\boldsymbol{\sigma}_s^2$, $\boldsymbol{\sigma}_n^2$, and $\boldsymbol{SNR}_o$, respectively. For example in the traditional MUSIC method, by utilizing the eigenvalue decomposition, a group of eigenvalues in the descending order can be obtained as $\left(\sigma_1^2, \sigma_2^2, \cdots, \sigma_M^2\right)$ where $\sigma_1^2 \geq \sigma_2^2 \geq \cdots \geq \sigma_M^2$. Theoretically speaking, when M > K, the last M − K eigenvalues may have the same result which equals to σ_n^2, such that the signal subspace and the noise subspace could be distinguished and the corresponding vectors $\boldsymbol{\sigma}_s^2$ and $\boldsymbol{\sigma}_n^2$ could be estimated. In the real applications, a threshold ε_1 can be set such that when the errors within the last few eigenvalues are less than ε_1 they can be regarded as the eigenvalues corresponding to the noise subspace, and the mean value of these eigenvalues $\bar{\sigma}_n^2$ can be taken as the estimate of σ_n^2, while the other eigenvalues just correspond to those of the signal subspace so that $\boldsymbol{\sigma}_s^2$ can be estimated. And finally the parameter $\boldsymbol{SNR}_o$

TABLE 1. Iterative nonlinear SR-based signal-subspace decomposition and array signal processing algorithm.

Step 1: Collect the receiving signal vector $\mathbf{r}^{R-LP}(t)$ from the receiving array;

Step 2: Calculate the covariance matrix $\boldsymbol{R}$ of $\mathbf{r}^{R-LP}(t)$;

Step 3: Use the traditional subspace decomposition method (such as MUSIC and ESPRIT) to get the estimated values of $\boldsymbol{\sigma}_s^2$, $\boldsymbol{\sigma}_n^2$, and $\boldsymbol{SNR}_o$;

Step 4: When a certain SR system has been selected, the parameter $\boldsymbol{SNR}_{opt}$ is a fixed optimal output SNR vector corresponding to the selected SR system, and if a fixed relative small enough positive value ε is also chosen, we can decide whether the condition $\left\|\boldsymbol{SNR}_o - \boldsymbol{SNR}_{opt}\right\| < \varepsilon$ is true or false: if it is true, the previous signal subspace decomposition result is the optimal result and the whole algorithm finishes; otherwise, go to **Step 5** to start the SR processing while introducing the estimated values of $\boldsymbol{\sigma}_s^2$, $\boldsymbol{\sigma}_n^2$, and $\boldsymbol{SNR}_o$ in **Step 3** as the inputting parameters of the SR system;

Step 5: Use Eq.(8) in the SR system processing to realize the signal subspace enhancement. Because the whole algorithm is a kind of iterative calculation, so if it is the first round SR processing, just use the receiving signal vector $\mathbf{r}^{R-LP}(t)$ as the input signal vector, while $\boldsymbol{\gamma}(t)$ is the optimization noise vector; otherwise, make a piecewise combination of both $\mathbf{r}^{R-LP}(t)$ and normalized state vector $\mathbf{x}(t)$ in the last iteration round, say $\tilde{\mathbf{x}}(t)$ which will be explained later, as the new input signal vector $\mathbf{r}^{R-LP}(t)$ in this iteration round. Finally, set the normalized state vector $\mathbf{x}(t)$ of this group of SR system as the output parameter of the SR system processing;

Step 6: Go back to **Step 2**, and use the normalized state vector $\mathbf{x}(t)$ of the output parameter in **Step 5** to calculate the covariance matrix $\boldsymbol{R}$ of $\tilde{\mathbf{x}}(t)$, then start the whole iterative processing till the condition $\left\|\boldsymbol{SNR}_o - \boldsymbol{SNR}_{opt}\right\| < \varepsilon$ in **Step 4** can be fulfilled and the iterative processing can be finished.

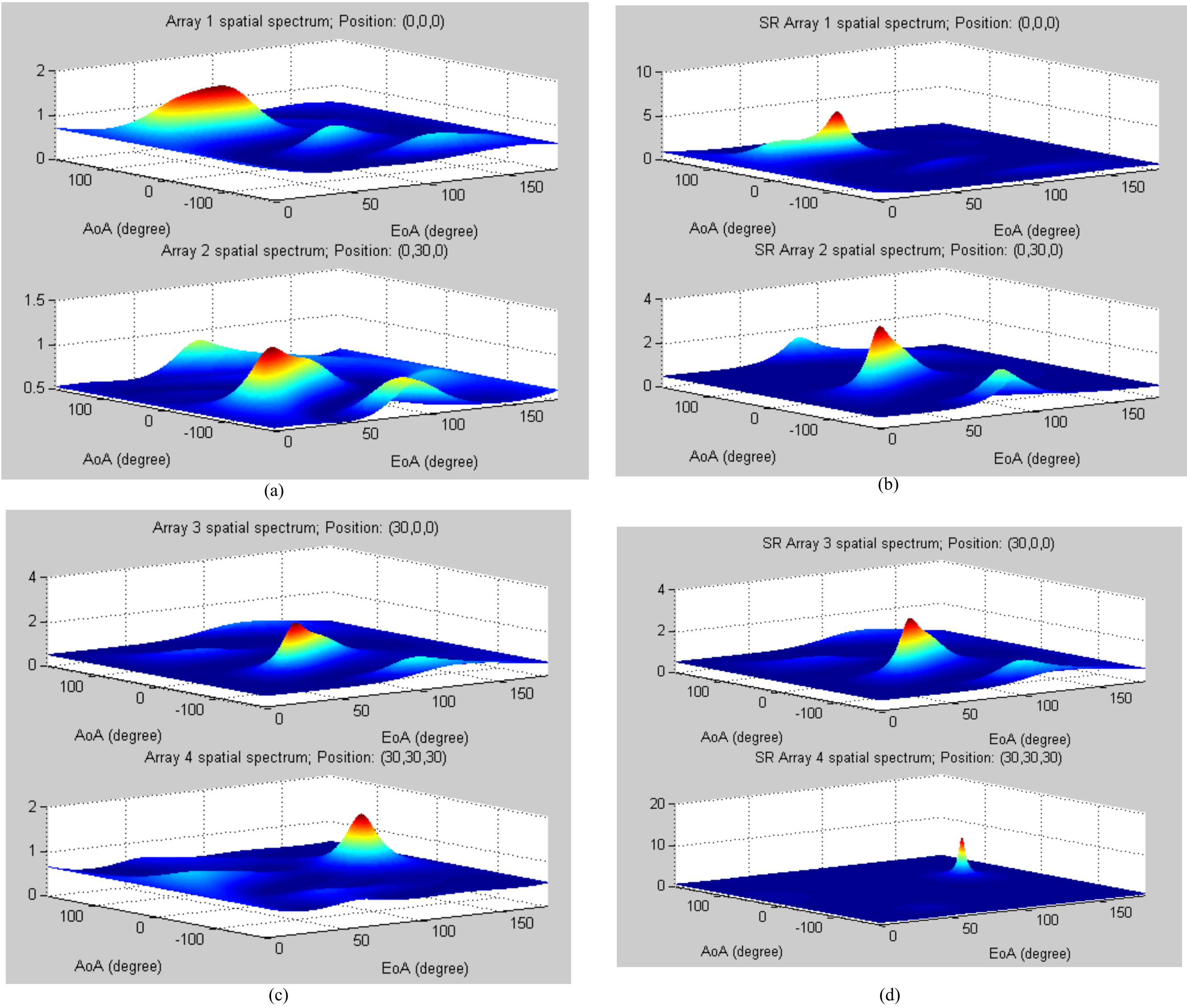

FIGURE 4. **Spatial spectrum at each array before and after the SR processing.**

can be calculated by

$$\boldsymbol{SNR}_o = \left[\frac{\sigma_1^2 - \bar{\sigma}_n^2}{\bar{\sigma}_n^2}, \frac{\sigma_2^2 - \bar{\sigma}_n^2}{\bar{\sigma}_n^2}, \cdots, \frac{\sigma_K^2 - \bar{\sigma}_n^2}{\bar{\sigma}_n^2} \right]^T \tag{28}$$

(2) In Step 5, $\boldsymbol{\gamma}(t)$ is the optimization noise vector in the SR system, while it can be optimized based on the selected group of SR systems. In this research, due to the fact that in the receiving signal vector, each receiving signal at certain receiving antenna $r_m^{R-LP}(t)$ (m $=$ $1, 2, \ldots,$ M) may have the same property which can be expressed as the summary of the channel noise and K independent source signal $\{s_1(t), s_2(t), \cdots, s_K(t)\}$ which are multiplied with the components in the direction vector, we suppose the same SR system can be used in each branch of the selected group of SR systems. For example if the traditional discrete bistable SR system in Eq.(9) is used as the distributed SR system, and when the source signal is the single-frequency sinusoidal signal and the channel noise is AWGN, the corresponding output SNR can be calculated by Eq.(12).

(3) In Step 5, the signal $\tilde{\mathbf{x}}(t)$ is a linear combination of both $\mathbf{r}^{R-LP}(t)$ and normalized state vector $\mathbf{x}(t)$ in the last iteration round. For example, in the real applications the calculation of the covariance matrix can be written as

$$\begin{aligned} \boldsymbol{R} &= E\left[\mathbf{r}^{R-LP}(t)\left(\mathbf{r}^{R-LP}(t)\right)^H\right] \\ &= \frac{1}{T}\sum_{i=0}^{T-1}\frac{\mathbf{r}^{R-LP}(t+i\cdot\Delta t)\left(\mathbf{r}^{R-LP}(t+i\cdot\Delta t)\right)^H}{\left\|\mathbf{r}^{R-LP}(t)\right\|^2} \end{aligned} \tag{29}$$

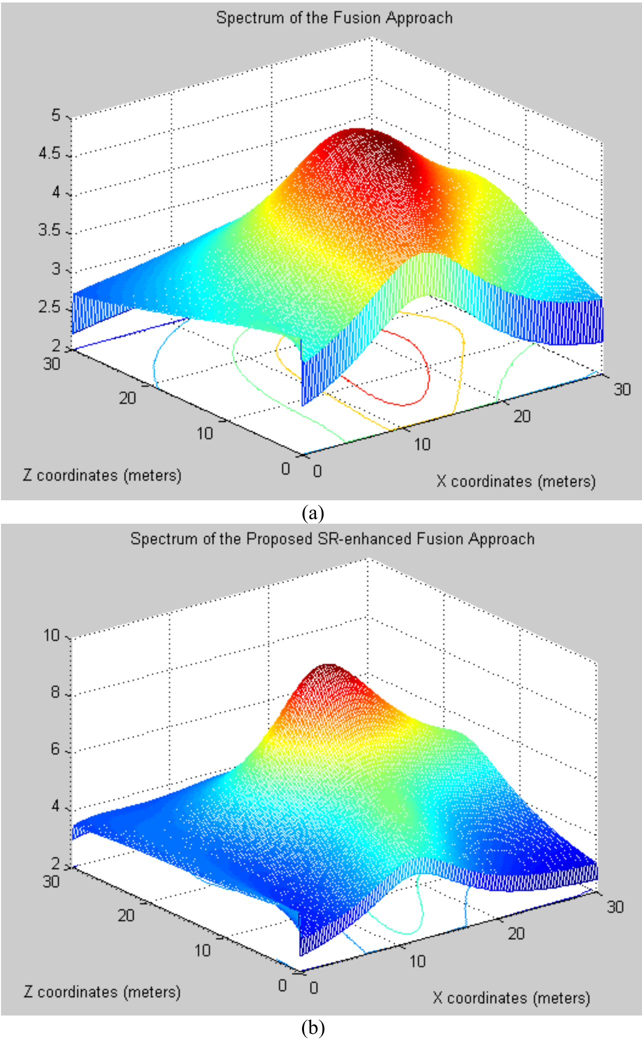

FIGURE 5. Spatial spectrum of the combination approach and the proposed SR-enhanced combination approach under receiving signal SNR = −10dB.

where T is the total sampling times for averaging. And then the signal $\tilde{\mathbf{x}}(t)$ can be expressed by

$$\tilde{\mathbf{x}}(t) = \begin{cases} \mathbf{r}^{R-LP}(t), & t_0 \le t < t_0 + F \cdot \Delta t \\ \mathbf{x}(t), & t_0 + F \cdot \Delta t \le t \le t_0 + (T-1) \cdot \Delta t \end{cases} \tag{30}$$

where t_0 is the starting time for the averaging of covariance matrix calculation in Eq.(29), and F is a fixed separating factor for the piecewise operation. So that the updated $\boldsymbol{R}$ based on the signal $\tilde{\mathbf{x}}(t)$ except the first round iteration calculation can be written as

$$\begin{aligned} \boldsymbol{R} &= E\left[\tilde{\mathbf{x}}(t)\tilde{\mathbf{x}}^H(t)\right] \\ &= \frac{1}{T}\sum_{i=0}^{T-1} \frac{\tilde{\mathbf{x}}(t+i\cdot\Delta t)\tilde{\mathbf{x}}^H(t+i\cdot\Delta t)}{\left\|\tilde{\mathbf{x}}(t)\right\|^2} \end{aligned} \tag{31}$$

Based on the algorithm and the technical details above, the proposed signal subspace decomposition approach based on the iterative SR technique can be realized, and it can be applied in many different areas.

IV. COMPUTER SIMULATIONS

To evaluate the performances of the proposed approach and compare with the traditional wireless localization methods, we carry out some computer simulations and give the comparison results in this section.

In the computer simulations, set $s(t) = 1$ as the constant pilot signal, $\omega = 1.9 \times 10^{10}$rad/s, $\Delta\omega = 1$rad/s. The multipath number is set as $K = 2$. Define that there are 4 base stations or APs ($I = 4$) which can receive the user signal while 2 of the 4 LOS signal are blocked, and 8-antenna UCA structure ($M = 8$) is applied in each array. The traditional discrete bistable SR system shown in Eq.(9) is used as the distributed SR system $f[\cdot]$ where the parameters are chosen as the sampling interval $\Delta t = 2.5 \times 10^{-6}$s, $a = 2$ and $b = -1$, and $\gamma_m(t)$ is set as the AWGN signal with mean 0 and variance 1. In the SR system $f[\cdot]$, the other two parameters g_1 and g_2 can be achieved through the optimization process in [26]. The combination coefficients are pre-determined as $\eta_i = \frac{1}{M} = 0.125$ (i $= 1, 2, \ldots, 8$), which correspond to the even combination coefficients to all M distributed parallel branches in Fig.2. In the covariance matrix calculation and the piecewise separating operation, we set the total sampling times $T = 100$, the separating factor $F = 50$, and the parameter $\varepsilon = 0.1$. The locations of 4 base stations or APs are (0, 0, 0), (0, 30, 0), (30, 0, 0) and (30, 30, 30), and the units are meters. The user (transmitter) position is (15, 20, 10), and 2 multipath signal are generated randomly in the area.

Based on the above simulation parameter settings, Fig.4 gives the spatial spectrum at each array before the SR processing and after the SR processing, respectively. The SNR at the receiving array is −10dB. It can be found that the spectrum peaks after the SR processing are much sharper and easier to be detected than those before the SR processing. Fig.5 shows the spatial spectrum of the combination approach without SR processing or SR power spectrum enhancement and the proposed SR-enhanced combination approach. It can also be discovered that the peak appears more clearly after the SR processing.

To compare the performances with the traditional wireless DoA estimation methods and wireless localization methods, Fig.6 shows the RMSE performances of DoA estimation error under different SNR by using the traditional MUSIC method, the combination method without SR-enhancement and the proposed SR-enhanced combination approach, while the MUSIC method estimates the DoA directly based on the spectral maximal value at the base stations or APs, and the other two combination methods estimate the DoA result based on the final spectral combination localization result and not based on the spectral maximal value at each base station or AP, which is also the most important difference between these two kinds of methods. It reveals that the proposed approach can get lower RMSE results

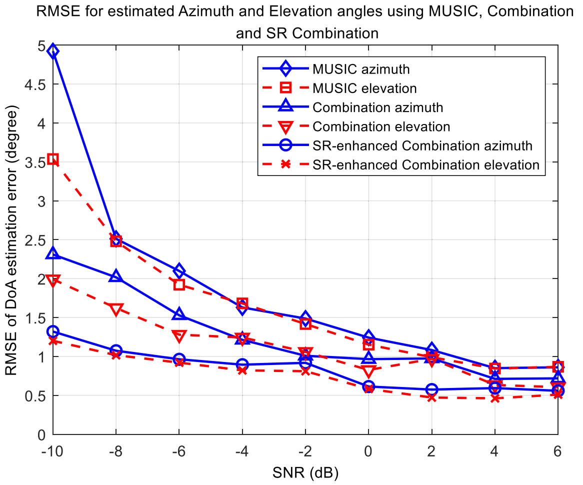

FIGURE 6. RMSE of DoA estimation error under different SNR while 2 of 4 LOS signal are blocked.

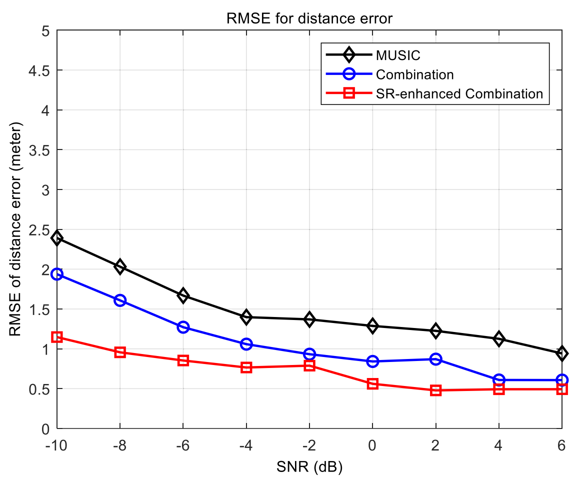

FIGURE 7. RMSE of wireless localization estimation error under different SNR while 2 of 4 LOS signal are blocked.

than the other two methods especially under low SNR, and the combination method without SR-enhancement performs better than the traditional MUSIC method. Fig.7 shows the RMSE performances of wireless localization estimation error under different SNR, and the final localization estimation performances also accord with those in Fig.6. Both figures show the validity of the proposed approach especially under low SNR and NLOS conditions.

To give a more persuasive comparison with the traditional methods, the following simulations are based on the condition that there are 4 base stations or APs which can receive the user signal while 3 of the 4 LOS signal are blocked, and the other parameters and conditions are as the same as in the above simulations. Figs. 8 and 9 show the RMSE performances of DoA estimation and wireless localization estimation error under different SNR, respectively. It can be found that the RMSEs of the traditional MUSIC method are much bigger than those of the combination methods, which also reflects that the traditional MUSIC method cannot work efficiently,

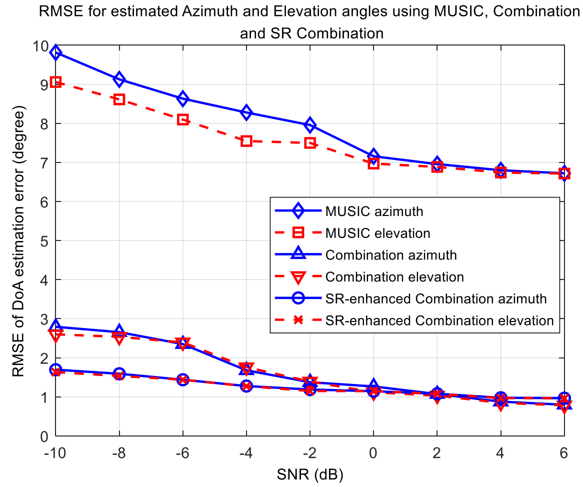

FIGURE 8. RMSE of DoA estimation error under different SNR while 3 of 4 LOS signal are blocked.

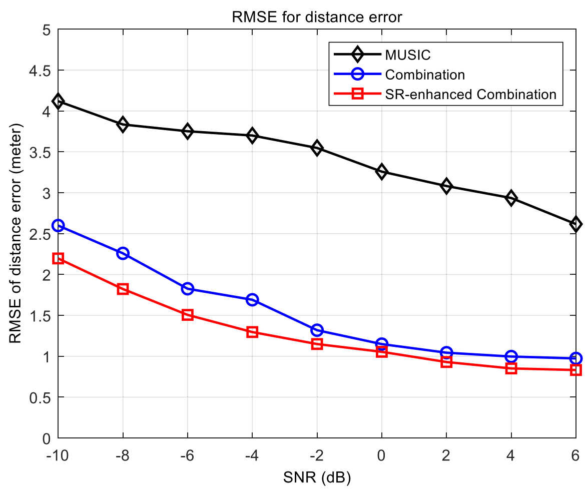

FIGURE 9. RMSE of localization estimation error under different SNR while 3 of 4 LOS signal are blocked.

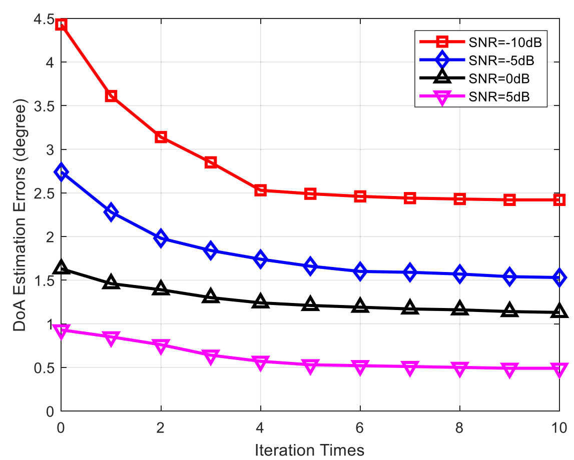

FIGURE 10. DoA estimation error versus the iteration times of the proposed approach under different SNR from −10dB to 5dB.

while the proposed approach still performs well under this serious condition.

Finally, to evaluate the performance of the proposed approach with the increasing of the iteration times, Fig.10 plots the curves which show the DoA estimation error versus the iteration times of the proposed approach under different SNR values from -10dB to 5dB, where the iteration times last till 10. In this figure, it can be discovered that the signal DoA estimation error will decrease gradually with the increasing of the iteration times, while the DoA estimation performances almost keep stable after 5 iteration times, which shows good convergence property of the proposed approach. And it is also clear that the higher the SNR is, the better the final DoA estimation performance will be when the iteration times increase.

V. CONCLUSION

In this study, a novel wireless localization method based on the combination of distributed SR-enhanced signal power spectrum information and corresponding iterative processing algorithm is proposed. By introducing a kind of nonlinear SR system to improve the receiving signal power spectrum and combining the distribute power spectrum at the receiving array structure, the NLOS problem and the low SNR problem can be solved successfully. And the iterative processing algorithm also promotes the applicability of the proposed approach. Computer simulations also show that the DoA estimation error and the wireless localization estimation error have been reduced compared with the traditional methods, which also verifies the validity of this novel approach. Due to the good properties and good performances, it can have potential application advantages in real wireless environments.

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DI HE (Senior Member, IEEE) received the Ph.D. degree in circuits and systems from Shanghai Jiao Tong University, Shanghai, China, in 2002. From 2002 to 2004, he was with the Department of Electrical and Computer Engineering, University of Calgary, Calgary, AB, Canada, as a Postdoctoral Fellow. He is currently an Associate Professor with the School of Electronic, Information and Electrical Engineering, Shanghai Jiao Tong University. His research interests include wireless positioning, wireless communications, and non-linear dynamics and its applications in wireless positioning and communications.

LIEEN GUO is currently a Professor with the School of Mechanical and Electrical Engineering, Nanchang University. His research interest includes digital signal processing.

JIAQING QU received the Ph.D. degree from Harbin Engineering University, Harbin, China. He is currently a Senior Engineer with the Shanghai Radio Equipment Research Institute. His research interests include satellite navigation and anti-interference communication.

CHIH-CHUN HO received the Ph.D. degree in communication engineering from Peking University, in 2016. Since 2019, he has been with the Longfor Group as the Head of the AIoT Laboratory. His main research interests include wireless communication and optical communication technology.