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Performance Prediction for Coherent Noise Radars Using the Correlation Coefficient

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ABSTRACT Noise radars, as well as certain types of quantum radar, can be understood in terms of a correlation coefficient which characterizes their detection performance. Although most results in the noise radar literature are stated in terms of the signal-to-noise ratio (SNR), we show that it is possible to carry out performance prediction in terms of the correlation coefficient. To this end, we derive the range dependence of the correlation coefficient under the assumption that all external noise is additive white Gaussian noise. We then combine our result with a previously-derived expression for the receiver operating characteristic (ROC) curve of a coherent noise radar, showing that we can obtain ROC curves for varying ranges. A comparison with corresponding results for a conventional radar employing coherent integration shows that our results are sensible. The aim of our work is to show that the correlation coefficient is a viable adjunct to SNR in understanding noise radar performance.

INDEX TERMS Noise radar, covariance matrix, correlation coefficient, quantum radar, radar performance prediction, range.

I. INTRODUCTION

Noise radar, as the name suggests, uses a noise waveform as its transmit signal [1]–[5]. As in other radars, noise radars retain a copy of the transmitted signal as a reference for matched filtering. Due to the presence of extraneous noise such as thermal noise and system noise, the reference signal is not necessarily a perfect copy of the signal that was transmitted through free space. Relatively little attention has been paid to the degradation of the reference signal used for matched filtering. Often, the degradation is assumed to be arbitrarily small, as was done in [6] for example.

This motivates the use of the Pearson correlation coefficient between the free-space signal and the reference signal, or the *correlation coefficient* for short. Although this correlation has appeared in previous publications on noise radar [5]–[7], most results in noise radar have been stated in terms of the signal-to-noise ratio (SNR) of the free-space signal. This is because SNR is far more familiar to engineers.

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However, the correlation coefficient takes into account both the degradation of the free-space signal and the degradation of the reference signal, the latter of which is not captured by the SNR [6], [8]. We feel, therefore, that the correlation coefficient is a sensible metric for evaluating the performance of noise radars—and perhaps of other radars, too. Although the correlation coefficient will never replace SNR as a performance metric, we will show that it has merits of its own and can stand alongside SNR as an additional lens through which to view noise radars.

A. WHY THE CORRELATION COEFFICIENT?

One of the most surprising attractions of the correlation coefficient is that it forms a bridge between noise radar and the new field of quantum radar [9]–[11]. In fact, the class of quantum radars known as *quantum two-mode squeezing radar* is essentially a noise radar that can achieve high correlation coefficients, at least as far as target detection performance is concerned. Moreover, quantum radars particularly excel when low signal powers are required [12]. In this regime, the correlation coefficient is of particular value because the degradation of the reference signal cannot be neglected.

In this paper, we show how performance prediction for a coherent noise radar can be carried out in terms of the correlation coefficient. We use the radar range equation to derive the dependence of the correlation coefficient on the range of a given target; this equation holds irrespective of whether the radar is coherent or not. We then exploit a previous result, which relates the receiver operating characteristic (ROC) curve of a coherent noise radar to the correlation coefficient, to show how the ROC curve for a noise radar varies with range.

We emphasize that our work is not intended to supplant the standard SNR-based approach to the analysis of noise radars. Our contribution is merely to show that a full analysis of detection performance based on the correlation coefficient is sensible, easy to carry out, and has the potential to be a valuable adjunct to an SNR-based analysis. In particular, we show how the correlation coefficient depends on range, an important piece of the puzzle when analyzing the performance of noise radars and quantum two-mode squeezing radars.

II. THE NOISE RADAR PROTOCOL

In our analysis, we will consider radars which work as follows:

1) Produce two correlated zero-mean Gaussian random noise signals.
2) Retain one of the signals and send it directly to the receiver for use as a reference for matched filtering. Transmit the other signal toward a target.
3) Measure the in-phase and quadrature voltages at the receiver.
4) Correlate the received and recorded signals. Declare a detection if the correlation exceeds a given threshold.

Normally, the reference signal in step 2 would be digitized immediately upon generation if it were not generated digitally in the first place. In this case, the received signal in step 3 would also be digitized and the correlation would be performed via digital signal processing. An example of the practical implementation of such a scheme can be seen in [13]. It is interesting to note, however, that the reference signal could be sent to the receiver in analog form and the correlation in step 4 performed via analog signal processing, as done in [6]. Our theoretical results would not change in either case, but for simplicity we will assume that we are working with digital signal processing.

III. THE NOISE RADAR CORRELATION COEFFICIENT

Let us denote the time series of in-phase and quadrature voltages of the received signal by $I_1(t)$ and $Q_1(t)$, respectively. Similarly, let us denote the voltages of the reference signal by $I_2(t)$ and $Q_2(t)$. We model these voltage time series as stationary, zero-mean, real-valued Gaussian white noise processes that are mutually uncorrelated when the time difference between the signals is nonzero. Therefore, we will drop the time variable for simplicity and assume that the time difference is always zero. We will assume further that the target is stationary, the phase shift between transmit and receive is a constant (which implies that the radar is coherent), and that any system or external noise is additive white Gaussian noise.

Under these assumptions, the four voltages are completely characterized by the covariance matrix $\mathrm{E}[\mathbf{x}\mathbf{x}^T]$ where $\mathbf{x} = [I_1, Q_1, I_2, Q_2]^T$. It was shown in [6] (though in different notation) that, if the reference signal is a direct copy of the transmitted signal, the covariance matrix can be written in block matrix form as

$$\boldsymbol{\Sigma}(P_1, P_2, \rho, \phi) = \begin{bmatrix} P_1\mathbf{1}_2 & \rho\sqrt{P_1P_2}\mathbf{R}(\phi) \\ \rho\sqrt{P_1P_2}\mathbf{R}(\phi)^T & P_2\mathbf{1}_2 \end{bmatrix} \quad (1)$$

where P_1 and P_2 are the powers (or, equivalently, the variances) of the received and reference signals respectively, $\mathbf{1}_2$ is the 2×2 identity matrix, ρ is a parameter such that $0 \leq \rho \leq 1$, ϕ is the phase, and $\mathbf{R}(\phi)$ is the rotation matrix

$$\mathbf{R}(\phi) = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}. \quad (2)$$

Alternatively, the transmitted and reference signals can be generated by mixing a single source of bandlimited Gaussian noise with a carrier signal. This results in two sidebands of correlated noise, one of which can be transmitted and the other retained. This case was considered in [11], where it was shown that the form of the resulting correlation matrix is the same as (1) except that instead of a rotation matrix, a reflection matrix appears instead:

$$\mathbf{R}'(\phi) = \begin{bmatrix} \cos\phi & \sin\phi \\ \sin\phi & -\cos\phi \end{bmatrix}. \quad (3)$$

The results in this paper do not depend on whether $\mathbf{R}(\phi)$ or $\mathbf{R}'(\phi)$ is used.

Although it is usual to form the complex voltages $z_1 = I_1 + jQ_1$ and $z_2 = I_2 + jQ_2$, we prefer to use real-valued voltages here because the real-valued covariance matrix (1) connects more easily with the covariance matrices used in quantum optics. See, for example, [11], [14], [15].

A. TARGET DETECTION AND THE CORRELATION COEFFICIENT

The parameter ρ in (1) is the focus of this paper. We call it the *correlation coefficient* because it characterizes the strength of the correlation between the received and transmitted signals. This can be seen by noting that, when the phase shift ϕ is zero, $\mathrm{E}[I_1I_2] = \rho\sqrt{P_1P_2}$. In this case, ρ is simply the Pearson correlation coefficient between I_1 and I_2. The effect of ϕ is to "distribute" the correlation among the cross-covariances $\mathrm{E}[I_1I_2]$, $\mathrm{E}[I_1Q_2]$, $\mathrm{E}[Q_1I_2]$, and $\mathrm{E}[Q_1Q_2]$. Note that we can always choose $\rho \geq 0$ because its sign can be absorbed into $\mathbf{R}(\phi)$ or $\mathbf{R}'(\phi)$.

The correlation coefficient is strongly related to the problem of target detection. At one extreme, if $\rho = 1$, then the received and reference signals are perfectly correlated. This would occur in the ideal case where there is absolutely no noise introduced into the signals. On the other hand, if $\rho = 0$, then the two signals are completely uncorrelated. This would be the case if there were no target at all, so the received signal is not an echo of the transmitted signal.

The above discussion suggests that detecting a target with a noise radar reduces to distinguishing between the following two hypotheses:

$$\begin{aligned} H_0 &: \rho = 0 \quad \text{Target absent} \\ H_1 &: \rho > 0 \quad \text{Target present} \end{aligned} \tag{4}$$

There are various test statistics, or detectors, that can be used to test these hypotheses. For example, the detector described in [6] effectively calculates the function

$$\frac{\sqrt{(I_1 I_2 + Q_1 Q_2)^2 + (I_1 Q_2 - Q_1 I_2)^2}}{4}. \tag{5}$$

If we replace $I_1 I_2$ with $\mathrm{E}[I_1 I_2]$ and do the same with the other terms, this quantity evaluates to $\rho\sqrt{P_1 P_2}/2$. This intuitively shows that the detector function (5) is a reasonable test statistic for the hypothesis test on ρ.

In this paper, however, we will focus on another detector, namely ρ itself, though our methods will apply to any test statistic for the above hypothesis test. When ρ is used as a detector, step 4 of the protocol described in Sec. II can be more concretely stated as follows: calculate an estimate $\hat{\rho}$ of the correlation coefficient of the radar's received and recorded signals, set a threshold, and declare a detection if $\hat{\rho}$ lies above the threshold.

One way to estimate ρ is described in [16]; the paper also gives approximations for ROC curves as a function of the four parameters in (1) when ρ is used for target detection. The estimate is obtained by performing the minimization

$$\min_{P_1, P_2, \rho, \phi} \left\| \mathbf{\Sigma}(P_1, P_2, \rho, \phi) - \hat{\mathbf{S}} \right\|_F \tag{6}$$

subject to the constraints $0 \leq P_1$, $0 \leq P_2$, $0 \leq \rho \leq 1$, and $0 \leq \phi \leq 2\pi$. Here $\hat{\mathbf{S}}$ is the sample covariance matrix calculated directly from the radar's voltage measurements, and F denotes the Frobenius norm. The value of ρ that minimizes this expression is taken as the estimate $\hat{\rho}$. Note that this minimization is where the coherence of the radar comes into play: if the radar were incoherent, minimizing (6) over ϕ would not be meaningful and the resulting $\hat{\rho}$ may not be accurate. In future work we will show that this minimization can be performed with much less computational power than the form (6) would suggest, but for the purposes of this paper it is enough to know that this is the method being used to obtain an estimate of ρ from radar data.

IV. CORRELATION COEFFICIENT AS A FUNCTION OF RANGE

Given the importance of the correlation coefficient, its range dependence is of considerable interest. We therefore present a theoretical derivation here. In order to simplify the analysis, it is convenient to assume that the received and reference signals can be decomposed into perfectly correlated and perfectly uncorrelated parts. That is, we assume that each signal is the sum of a component which is common to both signals and a component which is independent of the other signal. More explicitly, if we assume $\phi = 0$ for clarity's sake, we may write

$$I_1 = \sqrt{\eta} I_{\text{corr}} + I_{n1} \tag{7a}$$

$$I_2 = I_{\text{corr}} + I_{n2} \tag{7b}$$

where I_{corr} is the component of the in-phase voltages I_1 and I_2 that the two signals have in common, η is a factor that accounts for gains and losses in the received signal relative to the reference signal, while I_{n1} and I_{n2} are components which are totally uncorrelated with I_{corr} and with each other. We can think of I_{n1} and I_{n2} as added white Gaussian noise. The corresponding expressions for Q_1 and Q_2 have the same forms as (7a) and (7b), respectively. (The expressions for arbitrary ϕ are more complicated, but would lead to the same final result for the range dependence.) By squaring the above expressions and taking expectation values, we obtain

$$P_1 = \eta P + P_{n1} \tag{8a}$$

$$P_2 = P + P_{n2} \tag{8b}$$

where $P \equiv \mathrm{E}[I_{\text{corr}}^2]$ is the power of the perfectly correlated part, while P_{n1} and P_{n2} are the powers of the uncorrelated parts I_{n1} and I_{n2}, respectively.

Note that the decomposition above is a mathematical abstraction, as the signals may be tainted with noise at the very source and there may exist no perfectly correlated *physical* signal. In other words, I_{corr} may not correspond to any signal physically generated by the radar.

From (7a) and (7b), it is easy to calculate that $\mathrm{E}[I_1 I_2] = \sqrt{\eta} P$. Equating this with the corresponding entry in the covariance matrix (1) results in $\rho\sqrt{P_1 P_2} = \sqrt{\eta} P$, which immediately implies

$$\rho = \sqrt{\left(\frac{\eta P}{P_1}\right)\left(\frac{P}{P_2}\right)}. \tag{9}$$

The next step is to use (8a) to eliminate the first factor of P and (8b) to eliminate the second, giving

$$\begin{aligned} \rho &= \sqrt{\left(\frac{P_1 - P_{n1}}{P_1}\right)\left(\frac{P_2 - P_{n2}}{P_2}\right)} \\ &= \sqrt{\left(1 - \frac{P_{n1}}{P_1}\right)\left(1 - \frac{P_{n2}}{P_2}\right)}. \end{aligned} \tag{10}$$

It is interesting that the SNR-like quantities P_{n1}/P_1 and P_{n2}/P_2 appear here. This ties in with the idea that noise radars have two SNRs associated with them, as described in [8].

Next, we assert on physical grounds that $P_{n1} \geq P_{n2}$. The meaning of this assertion is that the signal received by the radar contains at least as much noise as the internal reference signal. This is reasonable because the received signal has passed through free space and is contaminated with both system noise and external noise (e.g. atmospheric noise), whereas the reference signal is contaminated only with system noise. We also assume that, if there were no external noise, the two signals would be contaminated with the *same* amount of noise, except that the noise in the received signal would be attenuated by a factor of η. Therefore,

$$P_{n1} = \eta P_{n2} + P_n, \tag{11}$$

where P_n is the power of the external noise added to the received signal. From (11), we can rewrite (8a) as $P_1 = \eta(P + P_{n2}) + P_n$, which implies

$$P_1 = \eta P_2 + P_n \tag{12}$$

by (8b).

It follows from (8a), (8b), and (10) that, if there were no external noise ($P_n = 0$), the correlation coefficient would be

$$\begin{aligned}\rho_0 &= \sqrt{\left(1 - \frac{\eta P_{n2}}{\eta P_2}\right)\left(1 - \frac{P_{n2}}{P_2}\right)} \\ &= 1 - \frac{P_{n2}}{P_2}.\end{aligned} \tag{13}$$

This represents the maximum correlation that can be observed by the radar. The fact that it is less than unity is a reflection of the fact that the radar contains system noise.

Substituting (11), (12), and (13) into (10), we can rewrite in terms of ρ_0, η, P_n, and P_2 as

$$\rho = \frac{\rho_0}{\sqrt{1 + P_n/(\eta P_2)}}. \tag{14}$$

From this form of ρ, it is very easy to derive the range dependence. We need only recognize that η, which we introduced to account for gains or losses between the transmit and reference signals, is really nothing more than the radar range equation (up to a gain factor if the transmit signal was amplified relative to the reference signal). According to one form of the radar range equation [17, eq. (1.6)], the received power P_1 can be written as

$$P_1 = \frac{G A_e \sigma}{(4\pi)^2 R^4} P_2 + P_n, \tag{15}$$

where G is the gain of the transmit antenna, A_e is the effective area of the receive antenna, σ is the target's radar cross section (RCS), and R is the range. Comparing this with (12), we see that

$$\eta(R) = \frac{G A_e \sigma}{(4\pi)^2 R^4}, \tag{16}$$

which can be substituted into (14) to obtain the range dependence of ρ. (We are obviously not bound to this exact form of the radar range equation; if another form of the radar range equation is desired, it may be introduced in exactly the same way.) It is possible to simplify matters, however, by introducing a characteristic range R_c defined as

$$R_c = \left(\frac{G A_e \sigma P_2}{(4\pi)^2 P_n}\right)^{1/4}. \tag{17}$$

In terms of R_c, we finally obtain

$$\rho(R) = \frac{\rho_0}{\sqrt{1 + (R/R_c)^4}}. \tag{18}$$

This is the desired expression for the correlation coefficient as a function of range.

The characteristic range R_c has the following interpretation: it is the range at which the received signal power P_2 is equal to the external noise P_n—in other words, the range at which SNR is unity (0 dB). For pulsed radars that do not rely on matched filtering, R_c would be the theoretical maximum range (though of course a coherent noise radar would still be able to detect at longer ranges). It is also the range at which the correlation coefficient is reduced to $1/\sqrt{2}$ of its maximum value. For the example parameters given in Table 1, which were inspired by the values given in [13], we find that $R_c = 1.0$ km.

TABLE 1. Example noise radar parameters.

Parameter	Variable	Value
Tx antenna gain	G	28 dB
Rx antenna effective area	A_e	0.1 m^2
Target RCS	σ	10 m^2
Tx signal power	P_2	20 dBm
Rx noise power	P_n	−94 dBm

The equation (18) shows that the correlation coefficient factorizes in an aesthetically pleasing manner, with one factor being the initial correlation ρ_0 and the other factor a simple function of the normalized range R/R_c. The initial correlation is a measure of the best possible performance that the radar can deliver, and does not depend on anything outside the radar. It can be thought of as the quality of the matched filtering performed by the radar. The characteristic range is essentially the radar range equation, and can be thought of as a measure of how quickly the performance of the radar decays with range. Equation (18) nicely separates these two aspects of radar operation.

Fig. 1 plots the correlation coefficient as a function of range for varying values of R_c. Since ρ_0 appears in (18) as a multiplicative constant, our plots show only the case where $\rho_0 = 1$.

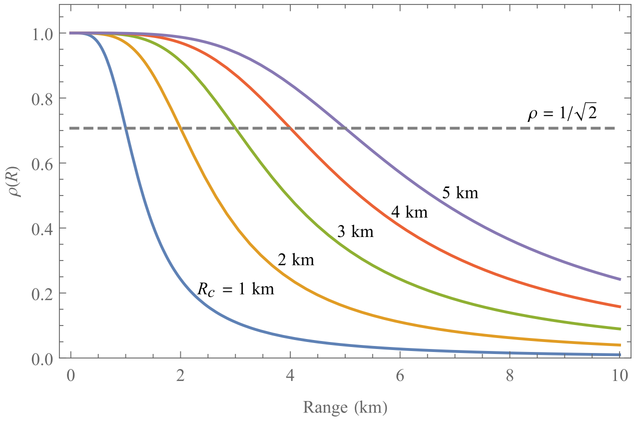

FIGURE 1. Correlation coefficient as a function of range for varying values of the characteristic length R_c, assuming $\rho_0 = 1$. Dashed line indicates $\rho = 1/\sqrt{2}$.

It is worth noting that, because the derivations in this section do not depend on the phase between the transmitter and receiver, (18) holds for all noise radars, whether coherent or not.

V. PERFORMANCE PREDICTION USING THE CORRELATION COEFFICIENT

As mentioned in Sec. III-A, ρ can be considered a detector function for use in the problem of distinguishing between the presence or absence of a target. In this section, we study the ROC curves that are obtained when ρ is used as the detector function.

The procedure we use to estimate ρ from the voltage time series I_1, Q_1, I_2, and Q_2 was briefly described in Sec. III-A. For that procedure, there exists an explicit expression for the ROC curve [16]:

$$p_{\mathrm{D}}(p_{\mathrm{FA}}|\rho, N) = Q_1\left(\frac{\rho\sqrt{2N}}{1-\rho^2}, \frac{\sqrt{-2\ln p_{\mathrm{FA}}}}{1-\rho^2}\right). \quad (19)$$

Here p_{D} is the probability of detection, p_{FA} is the probability of false alarm, N is the number of voltage samples over which to integrate, and Q_1 denotes the Marcum Q-function (not to be confused with the quadrature voltage Q_1 of the signal received by the radar). This is an approximate expression that holds when N is greater than approximately 100.

Fig. 2 shows a plot of p_{D} as a function of ρ for various values of p_{FA} when $N = 150$. This plot is analogous to the more common plots of p_{D} as a function of single-pulse SNR, an example of which is given in Fig. 4.4 of [18]. The only difference is that Fig. 2 is not for the single-pulse case, since it assumes 150 samples were integrated. Nevertheless, Fig. 2 gives some idea of the values of ρ needed to achieve a preset detection performance. For example, we can read from Fig. 2 that, if a noise radar is to achieve $p_{\mathrm{D}} = 0.9$ and $p_{\mathrm{FA}} = 10^{-6}$, it is necessary to have $\rho = 0.364$ when $N = 150$. Of course, the required ρ decreases as N increases, and vice versa.

The range dependence of the ROC curve is obtained simply by substituting (18) into (19). Representative plots are shown in Fig. 3. We take $R_c = 1$ km because that is the result obtained from the parameters in Table 1. As might be expected, the probability of detection falls precipitously as R becomes significantly larger than R_c.

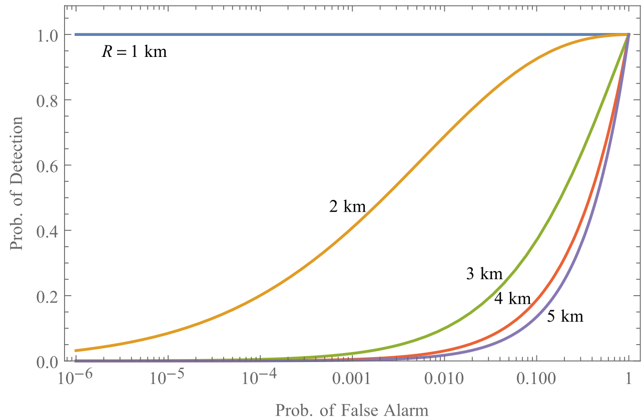

FIGURE 3. ROC curves for a noise radar detecting a target at various ranges, assuming $\rho_0 = 0.8$, $R_c = 1.0$ km, and $N = 150$.

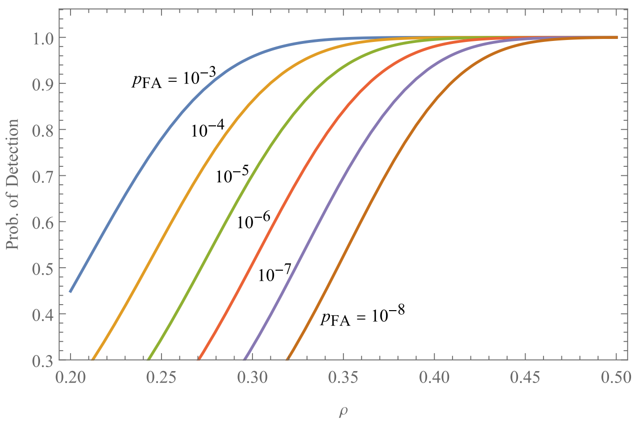

FIGURE 2. Probability of detection as a function of the correlation coefficient ρ for various values of p_{FA}, assuming $N = 150$.

We note that there are various other detector functions which could be used, such as the envelope detector studied in [6] as well as the one described in [19]. The ROC curves for both of these also depend on the correlation coefficient, so it is possible to determine detector performance as a function of range for both detectors by substituting (18). In fact, the ROC curve expressions in [6] have the merit of being valid for all N, not just for large N as described above. We do not analyze it here because the expressions in that paper are difficult to work with, even numerically.

As a check on the plausibility of our expressions, we compare the ROC curves obtained here with those for a conventional coherent radar using a sinusoidal waveform, as described in, e.g., [18]. We would expect to see a rough correspondence because the performance of a noise radar should not differ too widely from that of a conventional radar. Of course, we cannot expect more than a *rough* correspondence because, after all, the waveforms are different. Recalling that R_c is the range at which $SNR = 1$ and that received signal power varies inversely with the fourth power of the range, it follows that the single-pulse SNR is

$$SNR = \left(\frac{R_c}{R}\right)^4. \quad (20)$$

It is known that the ROC curve for such a conventional radar, assuming perfect coherent integration, is given by

$$p_{\mathrm{D}}(p_{\mathrm{FA}}|SNR, N) = Q_1\left(\sqrt{2N \cdot SNR}, \sqrt{-2\ln p_{\mathrm{FA}}}\right). \quad (21)$$

A derivation of (21) can be found in [18]. Curiously, the Marcum Q-function appears both in (21) and in (19), though we repeat that the latter is an approximate result which holds only for large N, whereas (21) is exact. Plots of ROC curves for various SNRs, for both noise radar and conventional radar, are given in Fig. 4. Note that, in the derivation of (21), noise is assumed to be added only to the received signal [18]. We therefore take $\rho_0 = 1$ in our comparison of conventional radar with noise radar, so the noise radar performs perfect matched filtering and there is no noise in the reference signal.

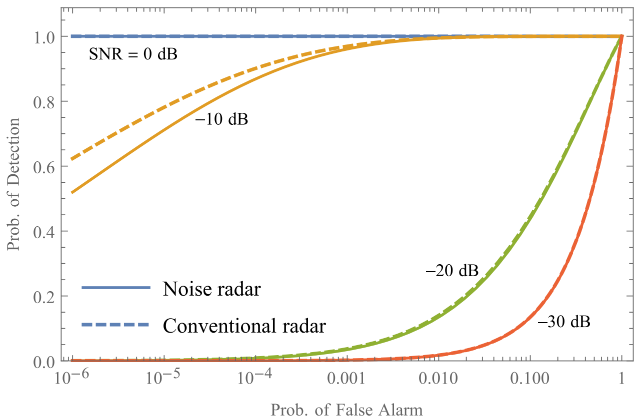

FIGURE 4. Comparison of ROC curves between a noise radar and a conventional radar at various SNRs, assuming $\rho_0 = 1$ and $N = 150$.

It can be seen that, for the most part, the ROC curves are comparable between the two radars. The differences between the two arise from the different waveforms employed by the two radars and the use of ρ as a detector function for the noise radar, which is not analogous to the envelope detector of a conventional radar. As noted above, we expect only a rough correspondence between the ROC curves of the two types of radars. Therefore, the deviation between the ROC curves for the two radars at -10 dB and 0 dB (though the deviation in the latter is not visible in Fig. 4) is expected. Nevertheless, the two cases are similar enough to show that the results we have obtained are reasonable.

VI. CONCLUSION

In this paper, we saw that noise radars can be described in terms of a certain correlation coefficient ρ which is intimately related to detection performance. We then derived the range dependence of this coefficient. This result holds whether or not the radar is coherent. Finally, we showed that when the radar is coherent, we can obtain ROC curves for varying target ranges by combining the range dependence with a previously derived expression for the ROC curve.

We again emphasize that our work is not meant to supplant the standard SNR-based analysis methods, but only to present an alternative figure of merit which allows noise radars to be viewed in a slightly different light than hitherto. Through the results in this paper, we believe we have made a strong case that the correlation coefficient is a viable lens through which the performance of noise radars can be understood.

Much exploratory work remains to be done to show how changes in ρ would affect radar performance. For example, it would be of interest to determine the Cramér–Rao bound for bearing estimation in terms of ρ. Another important question is the exact relationship between ρ and SNR. We also aim to calculate the values of ρ and N required to achieve desired values of p_{D} and p_{FA}, in a similar fashion to what was done in [6]. Other directions for future work include generalizing our results to cases where the assumptions listed in Sec. III do not hold, such as fluctuating targets, moving targets, time-varying phase shifts between transmit and receive, non-Gaussian additive noise, and multiplicative noise scenarios.

We also plan to explore the applicability of noise radar to various sensing applications. For example, biomedical sensors may benefit from the fact that noise waveforms are less likely to interfere with other medical equipment compared to the sinusoidal waveforms used in many radars. In particular, we could explore the applicability of noise radar to fall detection [20]. The performance prediction framework presented above could help us to understand the detection performance we could expect from a fall detector based on noise waveforms. Our work could also help us decide whether the enhanced detection performance of quantum radars would be helpful for fall detection or other sensing applications.

Finally, it would be interesting to determine whether the correlation coefficient could profitably be applied to the analysis of other types of radars, e.g. pulse radars or frequency-modulated continuous wave radars.

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