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Barrier Adaptive Iterative Learning Control for Tank Gun Control Systems Under Nonzero Initial Error Condition

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ABSTRACT In this paper, a barrier adaptive iterative learning control scheme is proposed to solve the trajectory-tracking problem for tank gun control systems under nonzero initial error condition. A novel construction method of rectified reference trajectory is presented for dealing with the initial position problem of iterative learning control for tank gun control systems. With a quadratic form barrier Lyapunov function adopted to controller design, the quadratic form of system error is constrained within the preset range during each iteration. Adaptive iterative learning control technique and robust control technique are jointly used to compensate for the parametric/nonparametric uncertainties and nonsymmetric deadzone nonlinearity. As the iteration number increases, the system state of tank gun control systems may accurately track the rectified reference trajectory, which leads to a excellent tracking performance during the part operation interval of tank gun control systems. Simulation results are presented to verify the effectiveness of the proposed barrier adaptive iterative control scheme.

INDEX TERMS Tank gun control systems, adaptive iterative learning control, barrier Lyapunov function, initial position problem.

I. INTRODUCTION

Iterative learning control (ILC) works well in coping with repetitive tracking tasks or periodic disturbance rejection [1]–[5]. By updating the control input gradually according to the system error in previous iterative cycle, perfect tracking may be achieved over the whole time interval as the iteration number increases, even where it is very hard to carry out system modelling. Due to these advantages, ILC has been applied in the precision control deign for various devices and processes, such as robotics systems, hard disk drives, servo control systems and high-speed trains [6]–[11]. Contraction-mapping ILC acts as the mainstream in the early research and application stage of ILC. So far, it is still a promising technology. Adaptive ILC may be seen as the combination of ILC and adaptive control. For the convenience in dealing with parametric uncertainties, adaptive ILC has received increasing attention in this century [1], [12].

Tanks can help troops enhance the efficiency of artillery firepower and improve the surviving ability of soldiers. Therefore, tanks are consider as a class of useful weapons in modern battle for its effective function of attack and defense meanwhile. Whether in an attacking combat or in a defensive combat, tanks should accomplish missions fleetly and accurately, despite of the existence of friction, uncertainties and external disturbances. Therefore, high stability and high control accuracy are indispensable for the tank gun control system. The accurate control for tank gun control systems have been studied for long, with some control strategies considered, such as variable structure control [13], PID control [14], optimal control [15], direct adaptive control [16], adaptive fuzzy control [17], adaptive robust control [18], active disturbance rejection control [19], disturbance observer based sliding mode control [20], adaptive neural network control [21]

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and event-triggered adaptive control [22]. In recent years, some scholars have explored the ILC algorithms of tank gun control systems, but the research in this field is not mature yet.

We will consider two important aspects of ILC algorithm designs for tank gun control systems in this work. The first aspect is about the initial position problem of tank gun control systems. In traditional ILC algorithms, the initial errors of controlled systems are assumed to be zero at each iteration [23]–[25]. Otherwise, a slight nonzero initial error may lead to the divergence of tracking error, which is the so-called initial position problem of ILC. It is impossible to reset the initial system states in industrial applications exactly to the initial values of reference trajectories. This bring about the need to develop ILC algorithms for systems subjected to arbitrary nonzero initial errors. As response, a few solutions have been proposed during the past twenty years, such as time-varying boundary layer technique [26], error-tracking method [27], initial rectifying action [29], [30], etc. As far as ILC for tank gun control systems is concerned, the studies on its initial position problem is still at a preliminary stage. In [31], a robust learning control scheme is proposed to solve the trajectory-tracking problem for tank gun control systems, with alignment condition adopted as one remedy to the initial position problem of ILC. In [32], a time-varying boundary layer is constructed to overcome the nonzero initial error during the iterative learning controller design for tank gun control systems. In [33], a novel neural network-based error-track iterative learning control scheme is proposed to tackle trajectory tracking problem for tank gun control systems. However, none of these works consider the issue of initial rectification ILC for tank gun control systems. How to develop an effective initial rectification ILC scheme to meet the requirement of arbitrary nonzero initial errors for tank gun control systems is still unclear.

The second issue we will address in this work is about system constraints of tank gun control systems during operation. For a real ILC application systems, in order to improve the robustness or safety, it is necessary to constrain the system output, the system state, or the output tracking error during the operation in some situations. Inspired by the constraint design strategy in barrier adaptive control [34], [35], barrier adaptive ILC has been studied in the past decade. In [36], an output-constrained adaptive ILC scheme is proposed to solve the tracking problem for a class of nonlinear systems under alignment condition. In [37], the ILC with error constraint for MIMO systems under alignment condition is investigated. In [38], state-constrained ILC is studied for a class of nonparametric uncertain systems under nonzero initial error condition, with an error-tracking strategy used to deal with the initial position problem of ILC. Later on, an neural network based adaptive ILC algorithm is proposed for nonlinear uncertain systems with state constraints in [39]. The joint position constrain problem for robotic ILC systems is discussed in [40]. The research results on constrained spatial adaptive ILC are reported in [41] and [42], respectively. To the best of authors' knowledge, so far, no literature has discussed the system-constraint ILC for tank gun control systems. How to develop an effective initial rectification ILC scheme to meet the requirement of error constraints during each iteration and the requirement of arbitrary nonzero initial errors for tank gun control systems, has not been addressed yet.

Motivated by the above discussion, this paper focuses on the barrier adaptive ILC algorithm design for tank gun control systems under arbitrary nonzero initial error condition. The main results and contributions are given as follows.

(1) A novel construction method of rectified reference trajectory is put forward to overcome the nonzero initial error problem in the ILC design for tank gun control systems. A new adaptive ILC law is developed to make the system state follow the rectified reference trajectory over the whole time interval, such that the system state can accurately track the reference trajectory during the predetermined part time interval.

(2) With a quadratic form barrier Lyapunov function adopted to controller design, the quadratic form of system error is constrained within the preset range during each iteration.

(3) A compensating strategy to nonsymmetric deadzone input is given, under a relaxed assumption on deadzone parameters.

The rest of this paper is organized as follows. Section II describes the system definition and problem formulation. The construction of rectified reference trajectory and the design process of adaptive iterative learning controller for tank gun control systems are addressed in Section III. The convergence analysis of closed loop tank gun control systems is given in Section IV. Section V presents simulation results. The conclusion of this work is given in Section VI.

II. PROBLEM FORMULATION

High-performance tank gun control systems is indispensible for helping tanks work at full capacity. With the rapid development of machinery manufacturing and modern control technology, the power system and control method of tank gun control systems tend to be fully electrified and digitalized. Compared to traditional electro-hydraulic/full-hydraulic tank gun control systems, full-electric ones own such advantages as simple structure, high efficiency and easy maintenance. These advantages help full-electric tank gun control become the mainstream of tank gun control systems. A full-electric tank gun control system consists of two subsystems, vertical subsystem and horizontal subsystem. This vertical subsystem is actually an AC servo driving system, which is composed of an AC motor, a deceleration device, a barrel, etc. The structure diagram of vertical subsystem is shown in Fig. 1. The block diagram of tank gun control systems, a careful reduction of a complex nonlinear simulation model, is shown in Fig. 2. The definitions of symbols in Fig. 2 are given in Table 1.

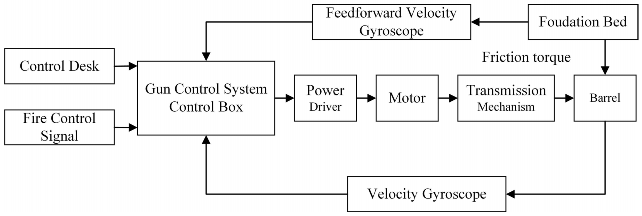

FIGURE 1. Structure diagram of vertical servo system of full-electrical tank gun.

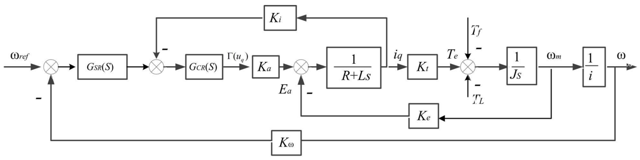

FIGURE 2. Block diagram of tank gun control systems.

TABLE 1. The definitions of symbols.

Symbol	Definition
ω_{ref}	desired angular velocity of cannon
ω_m	angular velocity of PMSM
ω	angular velocity of cannon
$G_{CR}(s)$	velocity regulator
$G_{CR}(s)$	current regulator
R	resistance of the motor armature circuit
L	inductance of the motor armature circuit
K_a	amplifier gain
E_a	armature back electromotive force of motor
K_i	current feedback coefficient of q axis
E_a	armature back electromotive force of motor
K_e	electric torque coefficient
T_e	motor torque
T_L	load torque disturbance
T_f	friction torque disturbance
T_e	motor torque
T_L	load torque disturbance
T_f	friction torque disturbance
K_ω	angular velocity feedback coefficient of cannon
J	total moment of inertia to the rotor
B	viscous friction coefficient
i	moderating ratio
s	Laplace operator

From Fig. 2, we can get the following dynamics of tank gun control control systems.

$$\begin{cases} \dot{i}_q = -\frac{R}{L} i_q - \frac{K_e i}{L}\omega_m + \frac{K_a}{L}\Gamma(u_p), \\ \dot{\omega}_m = \frac{K_t}{Ji} i_q - \frac{1}{Ji} T_{Ls}, \end{cases} \tag{1}$$

where $T_{Ls} = T_L + T_f$. $\Gamma(u_q)$ is the output of nonsymmetric deadzone, which can be described as (2).

$$\Gamma(u_p) = \begin{cases} m_r(u_q - b_r) & \text{for } u_q \geq b_r \\ 0 & \text{for } b_l \leq u_q < b_r \\ m_l(u_q - b_l) & \text{for } u_q < b_l, \end{cases} \tag{2}$$

in which m_r, m_l and b_r are strictly positive and unknown, and b_l is strictly negative and unknown. We assume that $m_r \neq m_l$, and $m_{\rm m} = \min(m_l, m_r)$ is unknown, which can be regarded as a relaxation to the traditional assumption $m_r = m_l$ required in some control algorithms [32].

Then, (2) can be rewritten as follows:

$$\Gamma(u_p) = m_{\rm m} u_q + m_\delta u_q + \delta, \tag{3}$$

where

$$m_\delta = \begin{cases} m_l - m_{\rm m}, \text{ if } m_l \geq m_r = m_{\rm m} \\ m_r - m_{\rm m}, \text{ if } m_r > m_l = m_{\rm m} \end{cases} \tag{4}$$

and

$$\delta = \begin{cases} -m_r b_r, \text{ for } u_q \geq 0 \\ -m_l b_l, \text{ for } u_q < 0. \end{cases} \tag{5}$$

It is obvious that both m_δ and δ in (3)-(5) are bounded. From (1) and (3), we obtain

$$\ddot{\omega}_m = -\frac{R}{L}\dot{\omega}_m - \frac{K_t K_e}{LJ}\omega_m + \frac{K_a K_t}{LJi}(m_{\rm m} u_q + m_\delta u_q + \delta) - (\frac{R}{LJi}T_{Ls} + \frac{1}{Ji}\dot{T}_{Ls}). \tag{6}$$

Define $x_1 = \omega_m$ and $x_2 = \dot{\omega}_m$. From (6), the dynamics of tank gun control systems at the kth iteration can be described as

$$\begin{cases} \dot{x}_{1,k} = x_{2,k}, \\ \dot{x}_{2,k} = -\frac{R}{L}x_{2,k} - \frac{K_t K_e}{LJ}x_{1,k} + \frac{K_a K_t}{LJi}(m_{\rm m} u_{q,k} \\ \quad + m_{\delta,k} u_{q,k} + \delta_k) + f(\boldsymbol{x}_k, t), \end{cases} \tag{7}$$

where $\boldsymbol{x}_k = [x_{1,k}, x_{2,k}]^T$ and $f(\boldsymbol{x}_k, t) = -(\frac{R}{LJi}T_{Ls} + \frac{1}{Ji}\dot{T}_{Ls})$. Without loss of generality, we assume $f(\boldsymbol{x}_k, t) = f_1(\boldsymbol{x}_k) + f_2(\boldsymbol{x}_k, t)$, where $f_1(\boldsymbol{x}_k)$ satisfies the Liphitz-like continuous condition as

$$|f_1(\boldsymbol{x}_k) - f_1(\boldsymbol{x}_d)| \leq \alpha(\boldsymbol{x}_k, \boldsymbol{x}_d)\|\boldsymbol{x}_k - \boldsymbol{x}_d\|, \tag{8}$$

with $\alpha(\boldsymbol{x}_k, \boldsymbol{x}_d, t)$ being a known continuous function, and $f_2(\boldsymbol{x}_k, t)$ satisfies

$$|f_2(\boldsymbol{x}_k, t)| \leq \epsilon_f(t), \tag{9}$$

with $\epsilon_f(t)$ being an unknown time-varying parameter.

The control task in this work is to find a sequence of control input $u_{q,k}$ such that the system error $\boldsymbol{e}_k = [e_{1,k}, e_{2,k}]^T = \boldsymbol{x}_k - \boldsymbol{x}_d$ converges to zero or its small bounded neighborhood, as the iteration number increases.

III. CONTROL SYSTEM DESIGN

A. CONSTRUCTION OF RECTIFIED REFERENCE TRAJECTORY

As mentioned above, $\boldsymbol{e}_k(0) \neq 0$ is an obstacle of designing ILC systems. In this work, we will construct rectified reference trajectory to overcome this obstacle.

In order to achieve the tracking objective, our control strategy is to make $\boldsymbol{x}_k(t)$ accurately track the initial-rectification reference trajectory $\boldsymbol{x}^*_k(t) = [x^*_{1,k}(t), x^*_{2,k}(t)]$ for $t \in [0, T]$, which is formed as follows:

$$x^*_{1,k}(t) = x_{1,d}(t) + h(t)e_{1,k}(0) + h(t)\sin(t)e_{2,k}(0), \tag{10}$$

$$x^*_{2,k}(t) = \dot{x}_{1,d}(t) + \dot{h}(t)e_{1,k}(0) + [h(t)\cos(t) + \dot{h}(t) \times \sin(t)]e_{2,k}(0), \tag{11}$$

where

$$h(t) = \begin{cases} \frac{10(t_\delta - t)^3}{t_\delta^3} - \frac{15(t_\delta - t)^4}{t_\delta^4} + \frac{6(t_\delta - t)^5}{t_\delta^5}, & \text{if } 0 \le t \le t_\delta, \\ 0, & \text{if } t_\delta < t \le T. \end{cases}$$

Remark 1: Note that $h(0) = 1, \dot{h}(0) = 0, h(t_\delta) = 0, \dot{h}(t_\delta) = 0, \ddot{h}(t_\delta) = 0$. According to (10)-(11), we can see $\boldsymbol{x}_k^*(0) = \boldsymbol{x}_d(0)$ and $\boldsymbol{x}_k^*(t) = \boldsymbol{x}_d(t)$ for $t \in [t_\delta, T]$. $x_{1,k}^*(t)$ is twice differential with respect to t.

B. CONTROLLER DESIGN

Let $\boldsymbol{e}_k^*(t) = [e_{1,k}^*(t), e_{2,k}^*(t)]^T = \boldsymbol{x}_k(t) - \boldsymbol{x}_k^*(t)$. From (10) and (11), we can see $\boldsymbol{e}_k^*(t) = \boldsymbol{e}_k(t)$ for $t \in [t_\delta, T]$. Then, it follows from (7) that

$$\begin{cases} \dot{e}_{1,k}^* = e_{2,k}^*, \\ \dot{e}_{2,k}^* = -\frac{R}{L}x_{2,k} - \frac{K_t K_p}{LJ}x_{1,k} + \frac{K_a K_t}{LJi}(m_{\mathrm{m}} u_{q,k} \\ \qquad + m_{\delta,k} u_{q,k} + \delta_k) + f_1(\boldsymbol{x}_k) + f_2(\boldsymbol{x}_k, t) - \ddot{x}_{1,k}^*, \end{cases} \tag{12}$$

whose vector form is

$$\begin{aligned} \dot{\boldsymbol{e}}_k^* &= A\boldsymbol{e}_k^* + \boldsymbol{b}[e_{1,k}^* + 2e_{2,k}^* - \frac{R}{L}x_{2,k} - \frac{K_t K_p}{LJ}x_{1,k} \times \\ &\quad + \frac{K_a K_t}{LJi}(m_{\mathrm{m}} u_{q,k} + m_{\delta,k} u_{q,k} + \delta_k) + f_1(\boldsymbol{x}_k) \\ &\quad + f_2(\boldsymbol{x}_k, t) - \ddot{x}_{1,k}^*], \end{aligned} \tag{13}$$

with $\boldsymbol{b} = [0, 1]^T$ and

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}. \tag{14}$$

For such a matrix A, there must exist symmetric positive definite matrices P and Q, which satisfy $PA + A^T P = -Q$. Define a candidate barrier Lyappunov function

$$V_k(t) = \frac{\boldsymbol{e}_k^{*T} P \boldsymbol{e}_k^*}{2\varpi(b_e - \boldsymbol{e}_k^{*T} P \boldsymbol{e}_k^*)}, \tag{15}$$

where $\varpi = \frac{K_a K_t m_{\mathrm{m}}}{LJi}$, $b_e > 0$ is the constraint bound of $\boldsymbol{e}_k^{*T} P \boldsymbol{e}_k^*$. Taking the time derivative of V_k, we obtain the following expression:

$$\begin{aligned} \dot{V}_k &= \frac{\sigma_k}{2\varpi}(\dot{\boldsymbol{e}}_k^{*T} P \boldsymbol{e}_k^* + \boldsymbol{e}_k^{*T} P \dot{\boldsymbol{e}}_k^*) \\ &= -\frac{\sigma_k}{2\varpi}\boldsymbol{e}_k^{*T} Q \boldsymbol{e}_k^* + \frac{\sigma_k}{\varpi}\boldsymbol{e}_k^{*T} P\boldsymbol{b}[e_{1,k}^* + 2e_{2,k}^* - \frac{R}{L}x_{2,k} \\ &\quad - \ddot{x}_{1,k}^* - \frac{K_t K_p}{LJ}x_{1,k} + f_1(\boldsymbol{x}_k) + f_2(\boldsymbol{x}_k, t)] \\ &\quad + \sigma_k \boldsymbol{e}_k^{*T} P\boldsymbol{b}[u_{q,k} + \frac{m_{\delta,k}}{m_{\mathrm{m}}}u_{q,k} + \frac{\delta_k}{m_{\mathrm{m}}}], \end{aligned} \tag{16}$$

where $\sigma_k = \frac{b_e}{(b_e - \boldsymbol{e}_k^T P \boldsymbol{e}_k)^2}$. On the basis of (8), we have

$$\begin{aligned} &\boldsymbol{e}_k^{*T} P\boldsymbol{b}\frac{\sigma_k}{\varpi}(f_1(\boldsymbol{x}_k) - f_1(\boldsymbol{x}_d)) \\ &\le \frac{\sigma_k}{\varpi}|\boldsymbol{e}_k^{*T} P\boldsymbol{b}|\alpha(\boldsymbol{x}_k, \boldsymbol{x}_d)\|\boldsymbol{e}_k\| \\ &\le \frac{\sigma_k}{2\varpi}\|\boldsymbol{e}_k\|^2 + \frac{\sigma_k}{2\varpi}\alpha^2(\boldsymbol{x}, \boldsymbol{x}_d)(\boldsymbol{e}_k^{*T} P\boldsymbol{b})^2. \end{aligned} \tag{17}$$

By utilizing (17), we can rewrite (16) as follows:

$$\begin{aligned} \dot{V}_k &\le -\frac{\sigma_k}{2\varpi}\boldsymbol{e}_k^{*T} Q \boldsymbol{e}_k^* + \frac{\sigma_k}{\varpi}\boldsymbol{e}_k^{*T} P\boldsymbol{b}[e_{1,k}^* + 2e_{2,k}^* - \frac{R}{L}x_{2,k} \\ &\quad - \ddot{x}_{1,k}^* - \frac{K_t K_p}{LJ}x_{1,k} + f_1(\boldsymbol{x}_d) + f_2(\boldsymbol{x}_k, t)] \\ &\quad + \sigma_k \boldsymbol{e}_k^{*T} P\boldsymbol{b}(u_{q,k} + \frac{m_{\delta,k}}{m_{\mathrm{m}}}u_{q,k} + \frac{\delta_k}{m_{\mathrm{m}}}) \\ &\quad + \frac{\sigma_k}{2\varpi}\alpha^2(\boldsymbol{x}, \boldsymbol{x}_d)(\boldsymbol{e}_k^{*T} P\boldsymbol{b})^2 + \frac{\sigma_k}{2\varpi}\|\boldsymbol{e}_k\|^2 \\ &\le -\frac{\sigma_k}{2\varpi}\boldsymbol{e}_k^{*T} Q \boldsymbol{e}_k^* + \sigma_k \boldsymbol{e}_k^{*T} P\boldsymbol{b}\boldsymbol{\theta}^T\boldsymbol{\varphi}_k + \frac{\sigma_k}{\varpi}|\boldsymbol{e}_k^{*T} P\boldsymbol{b}|\epsilon_f \\ &\quad + \frac{\sigma_k}{2\varpi}\|\boldsymbol{e}_k\|^2 + \sigma_k \boldsymbol{e}_k^{*T} P\boldsymbol{b}(u_{q,k} + \frac{m_{\delta,k}}{m_{\mathrm{m}}}u_{q,k} + \frac{\delta_k}{m_{\mathrm{m}}}), \end{aligned} \tag{18}$$

where

$$\begin{aligned} \boldsymbol{\theta} &= [\frac{1}{\varpi}, \frac{1}{2\varpi}, -\frac{R}{\varpi L}, -\frac{K_t K_p}{\varpi LJ}, \frac{1}{\varpi}f_1(\boldsymbol{x}_d)]^T, \\ \boldsymbol{\varphi}_k &= [e_{1,k}^* + 2e_{2,k}^* - \ddot{x}_{1,k}^*, \frac{1}{2}\alpha^2(\boldsymbol{x}, \boldsymbol{x}_d)\boldsymbol{e}_k^{*T} P\boldsymbol{b}, x_{2,k}, x_{1,k}, 1]^T. \end{aligned}$$

Based on (18), the control law and learning laws are designed as

$$u_{q,k} = -\gamma_1 \boldsymbol{e}_k^{*T} P\boldsymbol{b} + u_{p1,k} + u_{\rho 1,k} + u_{\rho 2,k}, \tag{19}$$

$$u_{p1,k} = -\sigma_k \boldsymbol{\theta}_k^T \boldsymbol{\varphi}_k, \tag{20}$$

$$u_{\rho 1,k} = -\rho_{1,k} u_{p1,k} \tanh(\sigma_k \rho_{1,k} u_{p1,k}(k+1)^2 \boldsymbol{e}_k^{*T} P\boldsymbol{b}), \tag{21}$$

$$u_{\rho 2,k} = -\rho_{2,k} \tanh(\sigma_k \rho_{2,k}(k+1)^2 \boldsymbol{e}_k^{*T} P\boldsymbol{b}), \tag{22}$$

$$\boldsymbol{\theta}_k = \mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1}) + \gamma_2 \sigma_k \boldsymbol{e}_k^{*T} P\boldsymbol{b}\boldsymbol{\varphi}_k, \boldsymbol{\theta}_{-1} = 0, \tag{23}$$

$$\rho_{1,k} = \mathrm{sat}_{0,\bar{\rho}_1}(\rho_{1,k-1}) + \gamma_3 \sigma_k |\boldsymbol{e}_k^{*T} P\boldsymbol{b}||u_{p1,k}|, \rho_{1,-1} = 0, \tag{24}$$

$$\rho_{2,k} = \mathrm{sat}_{0,\bar{\rho}_2}(\rho_{2,k-1}) + \gamma_4 \sigma_k |\boldsymbol{e}_k^{*T} P\boldsymbol{b}|, \rho_{2,-1} = 0, \tag{25}$$

where $\gamma_1 > 0, \gamma_2 > 0, \gamma_3 > 0, \gamma_4 > 0$, and $\boldsymbol{\theta}_k$, $\rho_{1,k}$ and $\rho_{2,k}$ are used to estimate $\boldsymbol{\theta}$, ρ_1 and ρ_2, respectively. The saturation function $\mathrm{sat}_{\cdot,\cdot}(\cdot)$ is defined as follows:

For a scalar $\hat{a}$,

$$\mathrm{sat}_{\underline{a},\bar{a}}(\hat{a}) = \begin{cases} \bar{a} & \hat{a} > \bar{a} \\ \hat{a} & \underline{a} \le \hat{a} \le \bar{a} \\ \underline{a} & \hat{a} < \underline{a} \end{cases};$$

for a vector $\hat{\boldsymbol{a}} = [\hat{a}_1, \hat{a}_2, \cdots, \hat{a}_m] \in \boldsymbol{R}^m$, $\mathrm{sat}_{\underline{a},\bar{a}}(\hat{\boldsymbol{a}}) = \big[\mathrm{sat}_{\underline{a},\bar{a}}(\hat{a}_1), \mathrm{sat}_{\underline{a},\bar{a}}(\hat{a}_2), \cdots, \mathrm{sat}_{\underline{a},\bar{a}}(\hat{a}_m)\big]^T$.

Substituting (19) into (18), we have

$$\begin{aligned} \dot{V}_k &\le -\frac{\sigma_k}{2\varpi}\boldsymbol{e}_k^{*T} Q \boldsymbol{e}_k^* + \sigma_k \boldsymbol{e}_k^{*T} P\boldsymbol{b}\tilde{\boldsymbol{\theta}}_k^T \boldsymbol{\varphi}_k + \sigma_k |\boldsymbol{e}_k^{*T} P\boldsymbol{b}|\rho_2 \\ &\quad + \frac{\sigma_k}{2\varpi}\|\boldsymbol{e}_k\|^2 + \sigma_k \boldsymbol{e}_k^{*T} P\boldsymbol{b}\frac{m_{\mathrm{m}} + \sigma_k \delta_m}{m_{\mathrm{m}}}(u_{\rho 1,k} + u_{\rho 2,k}) \\ &\quad + \sigma_k |\boldsymbol{e}_k^{*T} P\boldsymbol{b}|\rho_1 |u_{p1,k}|, \end{aligned} \tag{26}$$

where ρ_1 is the upper bound of $\frac{m_{\delta,k}}{m_{\mathrm{m}}}$, ρ_2 is the upper bound of $\frac{1}{\varpi}\epsilon_f + \frac{|\delta_k|}{m_{\mathrm{m}}}$, and $\tilde{\boldsymbol{\theta}}_k = \boldsymbol{\theta} - \boldsymbol{\theta}_k$.

IV. CONVERGENCE ANALYSIS

Theorem 1: Given the closed loop tank gun control system (7) and the control law and learning laws (19)-(25), the following facts will hold: (1)

$$\lim_{k\to\infty} \boldsymbol{e}_k(t) = 0, \quad \forall t \in [t_\delta, T]. \tag{27}$$

(2) $\boldsymbol{e}_k^{*T}(t)P\boldsymbol{e}_k^*(t) < b_e$ and $\|\boldsymbol{e}_k^*(t)\| < \frac{\sqrt{b_e}}{\sqrt{\lambda_P}}$ hold during each iteration, where λ_P is the minimum eigenvalue of matrix P.

(3) Moreover, all system variables to be bounded at each iteration.

Proof: i). In this part, we will check the constraint characteristic by taking the time derivative of Lyapunov functional.

Firstly, let us define a Lyapunov functional as follows:

$$L_k = V_k + \frac{1}{2\gamma_2}\int_0^t \tilde{\boldsymbol{\theta}}_k^T\tilde{\boldsymbol{\theta}}_k d\tau + \frac{1}{2\gamma_3}\int_0^t \tilde{\rho}_{1,k}^2 d\tau + \frac{1}{2\gamma_4}\int_0^t \tilde{\rho}_{2,k}^2 d\tau, \tag{28}$$

where $\tilde{\rho}_{1,k} = \rho_1 - \rho_{1,k}$, $\tilde{\rho}_{2,k} = \rho_2 - \rho_{2,k}$. The time derivative of L_k is

$$\dot{L}_k = \dot{V}_k + \frac{1}{2\gamma_2}\tilde{\boldsymbol{\theta}}_k^T\tilde{\boldsymbol{\theta}}_k + \frac{1}{2\gamma_3}\tilde{\rho}_{1,k}^2 + \frac{1}{2\gamma_4}\tilde{\rho}_{2,k}^2. \tag{29}$$

By the property $0 \le |\upsilon| - \upsilon\tanh(\frac{\upsilon}{\epsilon}) \le 0.2785\epsilon$, we obtain

$$\begin{aligned}
&\sigma_k|u_{p1,k}|\rho_1|\boldsymbol{e}_k^{*T}P\boldsymbol{b}| - \sigma_k u_{p1,k}\,\boldsymbol{e}_k^{*T}P\boldsymbol{b}\frac{m_{\mathrm{m}}+\delta_m}{m_{\mathrm{m}}}u_{\rho_1,k}\\
&= \sigma_k|u_{p1,k}|\rho_1|\boldsymbol{e}_k^{*T}P\boldsymbol{b}| - \sigma_k|u_{p1,k}|\rho_{1,k}|\boldsymbol{e}_k^{*T}P\boldsymbol{b}| + \sigma_k|u_{p1,k}|\\
&\quad\times \rho_{1,k}|\boldsymbol{e}_k^{*T}P\boldsymbol{b}| - \sigma_k u_{p1,k}\boldsymbol{e}_k^{*T}P\boldsymbol{b}\frac{m_{\mathrm{m}}+m_{\delta,k}}{m_{\mathrm{m}}}\rho_{1,k}\tanh(\sigma_k\\
&\quad\times \rho_{1,k}u_{p1,k}(k+1)^2\boldsymbol{e}_k^{*T}P\boldsymbol{b})\\
&\le \sigma_k|u_{p1,k}|\tilde{\rho}_{1,k}|\boldsymbol{e}_k^{*T}P\boldsymbol{b}| + \frac{0.2785}{(k+1)^2}.
\end{aligned} \tag{30}$$

and

$$\begin{aligned}
&\sigma_k\rho_2|\boldsymbol{e}_k^{*T}P\boldsymbol{b}| - \sigma_k\boldsymbol{e}_k^{*T}P\boldsymbol{b}\frac{m_{\mathrm{m}}+\delta_m}{m_{\mathrm{m}}}u_{\rho_2,k}\\
&= \sigma_k\rho_2|\boldsymbol{e}_k^{*T}P\boldsymbol{b}| - \sigma_k\rho_{2,k}|\boldsymbol{e}_k^{*T}P\boldsymbol{b}| + \sigma_k\rho_{2,k}|\boldsymbol{e}_k^{*T}P\boldsymbol{b}| - \sigma_k\\
&\quad\times \boldsymbol{e}_k^{*T}P\boldsymbol{b}\frac{m_{\mathrm{m}}+m_{\delta,k}}{m_{\mathrm{m}}}\rho_{2,k}\tanh(\sigma_k\rho_{2,k}(k+1)^2\boldsymbol{e}_k^{*T}P\boldsymbol{b})\\
&\le \sigma_k\tilde{\rho}_{2,k}|\boldsymbol{e}_k^{*T}P\boldsymbol{b}| + \frac{0.2785}{(k+1)^2}.
\end{aligned} \tag{31}$$

Substituting (30) and (31) into (26), we have

$$\dot{V}_k \le -\frac{\lambda_q - 1}{2\varpi}\sigma_k\boldsymbol{e}_k^{*T}\boldsymbol{e}_k^* + \sigma_k\boldsymbol{e}_k^{*T}P\boldsymbol{b}\tilde{\boldsymbol{\theta}}_k^T\boldsymbol{\varphi}_k + \sigma_k|\boldsymbol{e}_k^{*T}P\boldsymbol{b}|\tilde{\rho}_2 + \sigma_k|\boldsymbol{e}_k^{*T}P\boldsymbol{b}|\tilde{\rho}_{1,k}|u_{p1,k}| + \frac{0.557}{(k+1)^2}, \tag{32}$$

where λ_q is the minimum eigenvalue of positive definite symmetric matrix Q. It is easy to make $\lambda_q - 1 > 0$ hold by choosing a proper P.

Substituting (32) into (29) leads to

$$\begin{aligned}
\dot{L}_k \le& -\frac{\lambda_q - 1}{2\varpi}\sigma_k\boldsymbol{e}_k^{*T}\boldsymbol{e}_k^* + \sigma_k\boldsymbol{e}_k^{*T}P\boldsymbol{b}\tilde{\boldsymbol{\theta}}_k^T\boldsymbol{\varphi}_k + \sigma_k|\boldsymbol{e}_k^{*T}P\boldsymbol{b}|\tilde{\rho}_2\\
&+ \sigma_k|\boldsymbol{e}_k^{*T}P\boldsymbol{b}|\tilde{\rho}_{1,k}|u_{p1,k}| + \frac{0.557}{(k+1)^2} + \frac{1}{2\gamma_2}\tilde{\boldsymbol{\theta}}_k^T\tilde{\boldsymbol{\theta}}_k\\
&+ \frac{1}{2\gamma_3}\tilde{\rho}_{1,k}^2 + \frac{1}{2\gamma_4}\tilde{\rho}_{2,k}^2.
\end{aligned} \tag{33}$$

By using (23), we have

$$\begin{aligned}
&\sigma_k\boldsymbol{e}_k^{*T}P\boldsymbol{b}\tilde{\boldsymbol{\theta}}_k^T\boldsymbol{\varphi}_k + \frac{1}{2\gamma_2}\tilde{\boldsymbol{\theta}}_k^T\tilde{\boldsymbol{\theta}}_k\\
&= \frac{1}{2\gamma_2}(\boldsymbol{\theta} - \boldsymbol{\theta}_k)^T(2\boldsymbol{\theta}_k - 2\mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1}) + \boldsymbol{\theta} - \boldsymbol{\theta}_k)\\
&= \frac{1}{2\gamma_2}[-\boldsymbol{\theta}_k^T\boldsymbol{\theta}_k + \boldsymbol{\theta}^T\boldsymbol{\theta} - 2\boldsymbol{\theta}^T\mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1})\\
&\quad + 2\boldsymbol{\theta}_k^T\mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1})]\\
&= -\frac{1}{2\gamma_2}[\boldsymbol{\theta}_k - \mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1})]^T[\boldsymbol{\theta}_k - \mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1})]\\
&\quad + \frac{1}{2\gamma_2}[\mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1}^T)\mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1}) + \boldsymbol{\theta}^T\boldsymbol{\theta}\\
&\quad - 2\boldsymbol{\theta}^T\mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1})]\\
&\le \frac{1}{2\gamma_2}[\mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1}^T)\mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1}) + \boldsymbol{\theta}^T\boldsymbol{\theta}\\
&\quad - 2\boldsymbol{\theta}^T\mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1})].
\end{aligned} \tag{34}$$

Since both $\mathrm{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1})$ and $\boldsymbol{\theta}$ are bounded, there exists a there exits a positive number m_1, which satisfies

$$\sigma_k\boldsymbol{e}_k^{*T}P\boldsymbol{b}\tilde{\boldsymbol{\theta}}_k^T\boldsymbol{\varphi}_k + \frac{1}{2\gamma_2}\tilde{\boldsymbol{\theta}}_k^T\tilde{\boldsymbol{\theta}}_k \le m_1 \tag{35}$$

Similarly, by using (24), we can see that there exists positive numbers m_2 and m_3, which satisfy

$$\begin{aligned}
&\sigma_k|\boldsymbol{e}_k^{*T}P\boldsymbol{b}|\tilde{\rho}_{1,k}|u_{p1,k}| + \frac{1}{2\gamma_3}\tilde{\rho}_{1,k}^2\\
&= \frac{1}{2\gamma_3}[-\rho_{1,k}^2 + \rho_1^2 - 2\rho_1\mathrm{sat}_{\underline{\rho_1},\bar{\rho}_1}(\rho_{1,k-1})\\
&\quad + 2\rho_{1,k}\mathrm{sat}_{\underline{\rho_1},\bar{\rho}_1}(\rho_{1,k-1})]\\
&= \frac{1}{2\gamma_3}[\mathrm{sat}_{\underline{\rho_1},\bar{\rho}_1}(\rho_{1,k-1})\mathrm{sat}_{\underline{\rho_1},\bar{\rho}_1}(\rho_{1,k-1}) + \rho_1^2\\
&\quad - 2\rho_1\mathrm{sat}_{\underline{\rho_1},\bar{\rho}_1}(\rho_{1,k-1})] - \frac{1}{2\gamma_3}[\rho_{1,k} - \mathrm{sat}_{\underline{\rho_1},\bar{\rho}_1}(\rho_{1,k-1})]^2\\
&\le \frac{1}{2\gamma_3}[\mathrm{sat}_{\underline{\rho_1},\bar{\rho}_1}(\rho_{1,k-1})\mathrm{sat}_{\underline{\rho_1},\bar{\rho}_1}(\rho_{1,k-1}) + \rho_1^2\\
&\quad - 2\rho_1\mathrm{sat}_{\underline{\rho_1},\bar{\rho}_1}(\rho_{1,k-1})]\\
&\le m_2,
\end{aligned} \tag{36}$$

and

$$\begin{aligned}
&\sigma_k|\boldsymbol{e}_k^{*T}P\boldsymbol{b}|\tilde{\rho}_2 + \frac{1}{2\gamma_4}\tilde{\rho}_{2,k}^2\\
&= \frac{1}{2\gamma_4}[-\rho_{2,k}^2 + \rho_2^2 - 2\rho_2\mathrm{sat}_{\underline{\rho_2},\bar{\rho}_2}(\rho_{2,k-1})\\
&\quad + 2\rho_{2,k}\mathrm{sat}_{\underline{\rho_2},\bar{\rho}_2}(\rho_{2,k-1})]\\
&= \frac{1}{2\gamma_4}[\mathrm{sat}_{\underline{\rho_2},\bar{\rho}_2}(\rho_{2,k-1})\mathrm{sat}_{\underline{\rho_2},\bar{\rho}_2}(\rho_{2,k-1}) + \rho_2^2\\
&\quad - 2\rho_2\mathrm{sat}_{\underline{\rho_2},\bar{\rho}_2}(\rho_{2,k-1})] - \frac{1}{2\gamma_4}[\rho_{2,k} - \mathrm{sat}_{\underline{\rho_2},\bar{\rho}_2}(\rho_{2,k-1})]^2
\end{aligned}$$

$$\leq \frac{1}{2\gamma_4}[\text{sat}_{\underline{\rho_2},\bar{\rho}_2}(\rho_{2,k-1})\text{sat}_{\underline{\rho_2},\bar{\rho}_2}(\rho_{2,k-1}) + \rho_2^2 - 2\rho_2\text{sat}_{\underline{\rho_2},\bar{\rho}_2}(\rho_{2,k-1})] \leq m_3, \quad (37)$$

respectively. Combining (35)-(37) with (33), we have

$$\dot{L}_k \leq -\frac{\lambda_q - 1}{2\varpi}\sigma_k \boldsymbol{e}_k^{*T}\boldsymbol{e}_k^* + m_1 + m_2 + m_3 + \frac{0.557}{(k+1)^2} \leq m_1 + m_2 + m_3 + \frac{0.557}{(k+1)^2}. \quad (38)$$

In light of $L_k(0) = 0$ and (38), the boundedness of $L_k(t)$ may be derived during $t \in [0, T]$. Further, we have

$$\frac{\boldsymbol{e}_k^{*T}(t)\boldsymbol{P}\boldsymbol{e}_k^*(t)}{2\varpi(b_e - \boldsymbol{e}_k^{*T}(t)\boldsymbol{P}\boldsymbol{e}_k^*(t))} \leq t(m_1 + m_2 + m_3 + \frac{0.557}{(k+1)^2}). \quad (39)$$

Note that $\boldsymbol{e}_k^{*T}(0)\boldsymbol{P}\boldsymbol{e}_k^*(0) = 0$ for any k. Once $\boldsymbol{e}_k^{*T}(t)\boldsymbol{P}\boldsymbol{e}_k^*(t)$ increases nearly to b_e for any $t \in (0, T]$, $\frac{\boldsymbol{e}_k^{*T}(t)\boldsymbol{P}\boldsymbol{e}_k^*(t)}{2\varpi(b_e - \boldsymbol{e}_k^{*T}(t)\boldsymbol{P}\boldsymbol{e}_k^*(t))} \to +\infty$ will happen, which is contrary to the inequality (39). Therefore,

$$\boldsymbol{e}_k^{*T}(t)\boldsymbol{P}\boldsymbol{e}_k^*(t) < b_e \quad (40)$$

holds for $t \in [0, T]$, which implies that

$$\|\boldsymbol{e}_k^*(t)\| < \frac{\sqrt{b_e}}{\sqrt{\lambda_P}} \quad (41)$$

for $t \in [0, T]$ and for any k.

ii) In this part, we will analyze the convergence of tracking error by calculating the difference of $L_k(t)$ between two adjacent iteration index.

From (32), we obtain

$$V_k \leq \int_0^t \boldsymbol{e}_k^{*T}\boldsymbol{P}\boldsymbol{b}\tilde{\boldsymbol{\theta}}_k^T\boldsymbol{\varphi}_k d\tau + \int_0^t |\boldsymbol{e}_k^{*T}\boldsymbol{P}\boldsymbol{b}|\tilde{\rho}_2 d\tau + \int_0^t |\boldsymbol{e}_k^{*T}\boldsymbol{P}\boldsymbol{b}|\tilde{\rho}_{1,k}|u_{p1,k}|d\tau + \frac{0.557t}{(k+1)^2}. \quad (42)$$

While $k > 0$, by utilizing (28) and (42), we can derive

$$\begin{aligned} L_k - L_{k-1} &= \int_0^t \sigma_k \boldsymbol{e}_k^{*T}\boldsymbol{P}\boldsymbol{b}\tilde{\boldsymbol{\theta}}_k^T\boldsymbol{\varphi}_k d\tau + \int_0^t \sigma_k|\boldsymbol{e}_k^{*T}\boldsymbol{P}\boldsymbol{b}|\tilde{\rho}_{2,k}d\tau \\ &+ \int_0^t \sigma_k|\boldsymbol{e}_k^{*T}\boldsymbol{P}\boldsymbol{b}|\tilde{\rho}_{1,k}|u_{p1,k}|d\tau + \frac{0.557t}{(k+1)^2} - V_{k-1} \\ &+ \frac{1}{2\gamma_2}\int_0^t (\tilde{\boldsymbol{\theta}}_k^T\tilde{\boldsymbol{\theta}}_k - \tilde{\boldsymbol{\theta}}_{k-1}^T\tilde{\boldsymbol{\theta}}_{k-1})d\tau + \frac{1}{2\gamma_3}\int_0^t (\tilde{\rho}_{1,k}^2 \\ &- \tilde{\rho}_{1,k-1}^2)d\tau + \frac{1}{2\gamma_4}\int_0^t (\tilde{\rho}_{2,k}^2 - \tilde{\rho}_{2,k-1}^2)d\tau. \end{aligned} \quad (43)$$

By the relationship $(a-b)^2 - (a-\hat{b})^2 \leq (a-b)^2 - (a - \text{sat}_{\underline{b},\bar{b}}(\hat{b}))^2$, it follows from (23) that

$$\begin{aligned} &\frac{1}{2\gamma_2}(\tilde{\boldsymbol{\theta}}_k^T\tilde{\boldsymbol{\theta}}_k - \tilde{\boldsymbol{\theta}}_{k-1}^T\tilde{\boldsymbol{\theta}}_{k-1}) + \sigma_k \boldsymbol{e}_k^{*T}\boldsymbol{P}\boldsymbol{b}\tilde{\boldsymbol{\theta}}_k^T\boldsymbol{\varphi}_k \\ &\leq \frac{1}{2\gamma_2}(2\boldsymbol{\theta}^* - \boldsymbol{\theta}_k - \text{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1}))^T(\text{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1}) - \boldsymbol{\theta}_k) + \sigma_k \boldsymbol{e}_k^{*T}\boldsymbol{P}\boldsymbol{b}\tilde{\boldsymbol{\theta}}_k^T\boldsymbol{\varphi}_k \\ &\leq \frac{1}{\gamma_2}(\boldsymbol{\theta}^* - \boldsymbol{\theta}_k)^T(\text{sat}_{\underline{\theta},\bar{\theta}}(\boldsymbol{\theta}_{k-1}) - \boldsymbol{\theta}_k) + \gamma_2\sigma_k \boldsymbol{e}_k^{*T}\boldsymbol{P}\boldsymbol{b}\boldsymbol{\varphi}_k) \\ &= 0. \end{aligned} \quad (44)$$

In a similar way, from (24) and (25), we can obtain

$$\begin{aligned} &\frac{1}{2\gamma_3}(\tilde{\rho}_{1,k}^2 - \tilde{\rho}_{1,k-1}^2) + \sigma_k|\boldsymbol{e}_k^{*T}\boldsymbol{P}\boldsymbol{b}|\tilde{\rho}_{1,k} \\ &\leq \frac{1}{\gamma_3}(\rho_1 - \rho_{1,k}) \\ &\times (\text{sat}_{0,\bar{\rho}_1}(\rho_{1,k-1}) - \rho_{1,k} + \gamma_3\sigma_k|\boldsymbol{e}_k^{*T}\boldsymbol{P}\boldsymbol{b}||\tilde{u}_{q,k}|) \\ &= 0 \end{aligned} \quad (45)$$

and

$$\begin{aligned} &\frac{1}{2\gamma_4}(\tilde{\rho}_{2,k}^2 - \tilde{\rho}_{2,k-1}^2) + \sigma_k|\boldsymbol{e}_k^{*T}\boldsymbol{P}\boldsymbol{b}|\tilde{\rho}_{2,k} \\ &\leq \frac{1}{\gamma_4}(\rho_2 - \rho_{2,k})(\text{sat}_{0,\bar{\rho}_2}(\rho_{2,k-1}) - \rho_{2,k} + \gamma_4\sigma_k|\boldsymbol{e}_k^{*T}\boldsymbol{P}\boldsymbol{b}|) \\ &= 0, \end{aligned} \quad (46)$$

respectively. Substituting (44)-(46) into (43) yields

$$L_k - L_{k-1} \leq -V_{k-1} + \frac{0.557t}{(k+1)^2}, \quad (47)$$

which further implies

$$L_k(t) \leq L_0(t) - \frac{1}{2\varpi}\sum_{j=0}^{k-1}\frac{\boldsymbol{e}_j^{*T}\boldsymbol{P}\boldsymbol{e}_j^*}{b_e - \boldsymbol{e}_j^{*T}\boldsymbol{P}\boldsymbol{e}_j^*} + \sum_{j=1}^{j=k+1}\frac{0.557t}{j^2}. \quad (48)$$

Due to

$$\lim_{k\to\infty}\sum_{j=1}^{j=k+1}\frac{0.557t}{j^2} = \frac{0.557\pi^2 t}{3}, \quad (49)$$

from (48), we get

$$\begin{aligned} L_k(t) &\leq L_0(t) - \frac{1}{2\varpi b_e}\sum_{j=0}^{k-1}\boldsymbol{e}_k^{*T}\boldsymbol{P}\boldsymbol{e}_k^* + \frac{0.557\pi^2 t}{3} \\ &\leq L_0(t) + \frac{0.557\pi^2 t}{3} - \frac{\lambda_P}{2\varpi b_e}\sum_{j=0}^{k-1}\|\boldsymbol{e}_k^*\|^2 \end{aligned} \quad (50)$$

Let $M_L = t(m_1+m_2+m_3+0.557)$. According to (38), we can see

$$0 \leq L_0(t) \leq M_L, \quad \forall t \in [0, T]. \quad (51)$$

It follows from (50) and (51) that

$$L_k(t) \leq M_L + \frac{0.557\pi^2 t}{3} - \frac{\lambda_P}{2\varpi b_e}\sum_{j=0}^{k-1}\|\boldsymbol{e}_k^*(t)\|^2, \quad (52)$$

which implies

$$\lim_{k\to\infty}\boldsymbol{e}_k^*(t) = 0, \quad \forall t \in [0, T]. \quad (53)$$

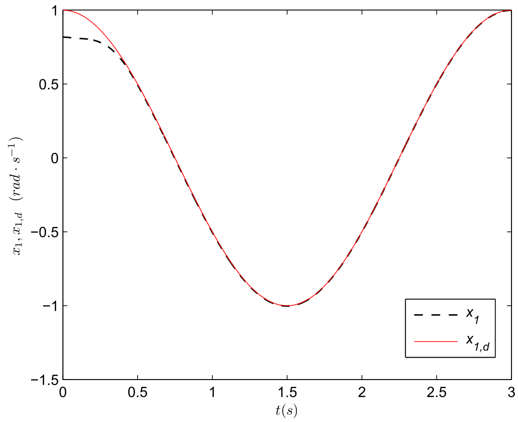

FIGURE 3. x_1 and its reference signal $x_{1,d}$ (constraint ILC).

According to the construction strategy described in (10)-(11), from (53), we have

$$\lim_{k \to +\infty} \boldsymbol{e}_k(t) = 0, \quad \forall t \in [t_\delta, T]. \tag{54}$$

Meanwhile, $\boldsymbol{\varphi}_k$ and $\boldsymbol{\theta}_k$ can be verified to be bounded by using the boundedness of $\|\boldsymbol{e}_k^*\|$ and the property of saturation functions. Further, u_k and other signals can be guarantee to be bounded.

■

In this work, a quadratic form Lyapunov function is used to design iterative learning controller to constrain the quadratic form of system error within the preset range during each iteration. In addition, hyperbolic tangent function is used to replace signum function for relieving the chattering phenomenon.

V. NUMERICAL SIMULATION

In this simulation example, the adopted simulation parameters in the tank gun control systems are given as $R = 0.4\Omega$, $J = 0.0067$ kg $\cdot$ m^2, $i = 1039$, $L = 2.907 \times 10^{-3}$H, $K_t = 0.195N \cdot$ m/A, $K_e = 0.197\ V/($ rad $\cdot\ s^{-1})$, $B = 1.43 \times 10^{-4}$ N $\cdot$ m, $K_a = 2$, $T = 3$. $\boldsymbol{x}_k(0) = [0.8 + 0.1\text{rand1}(k), -0.02 + 0.02\text{rand2}(k)]^T$. $\boldsymbol{x}_d = [\cos(\frac{2}{3}\pi t), -\frac{2}{3}\pi \sin(\frac{2}{3}\pi t)]^T$. The control objective is to make $\boldsymbol{x}_k$ accurately track $\boldsymbol{x}_d$. $\Delta f(\boldsymbol{x}_k, t) = 13.2 + 0.1\ x_{1,k} + 0.2\ x_{2,k} + 0.2\text{sign}(x_{2,k}) + 0.2\text{rand3}(k)\sin(0.5t)$. Here, rand1$(\cdot)$ $-$ rand3$(\cdot)$ are random numbers between 0 and 1. The values of deadzone parameters are set as $b_r = 0.2$, $b_l = -0.3$, $m_r = 1.5$ and $m_l = 2$.

The proposed adaptive ILC law (19) and adaptive learning laws (23)-(25) are adopted in this simulation, with $\gamma_1 = 5$, $\gamma_2 = 1.5$, $\gamma_3 = 0.1$, $\gamma_4 = 0.1$, $\underline{\theta} = -100$, $\bar{\theta} = 100$, $\bar{\rho}_1 = 20$, $\bar{\rho}_2 = 20$,

$$P = \begin{bmatrix} 7.5 & 2.5 \\ 2.5 & 2.5 \end{bmatrix}. \tag{55}$$

The trajectory-tracking profiles of angular velocity and angular acceleration of the tank gun servo system at the 10th cycle are shown in Figs. 3-4, respectively, with the tracking error profiles illustrated in Fig. 5-6. From Figs. 3-6, we can see that $\boldsymbol{x}_k(t)$ follows $\boldsymbol{x}_d(t)$ for $t \in [t_\delta, T]$, as the iteration number increases. The convergence history of $\boldsymbol{e}_k^{*T} P \boldsymbol{e}_k^*$ is given in Fig. 7, where $J_k := \max_{t \in [0,T]} \boldsymbol{e}_k^{*T}(t) P \boldsymbol{e}_k^*(t)$. From Fig. 7, we can see that $\boldsymbol{e}_k^{*T}(t) P \boldsymbol{e}_k^*(t) < b_e$ holds during each iteration. The control input signal the 10th iteration cycle is shown in Fig. 8. As shown in Figs. 3-7, the closed-loop tank gun servo system can obtain good tracking performance. The above simulation results show that the adaptive iterative learning controller can obtain an excellent control performance for the tank gun control system.

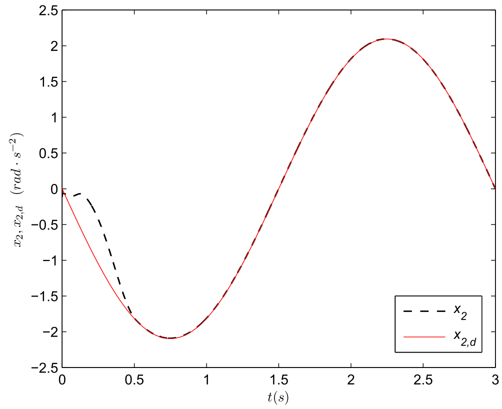

FIGURE 4. x_2 and its reference signal $x_{2,d}$ (constraint ILC).

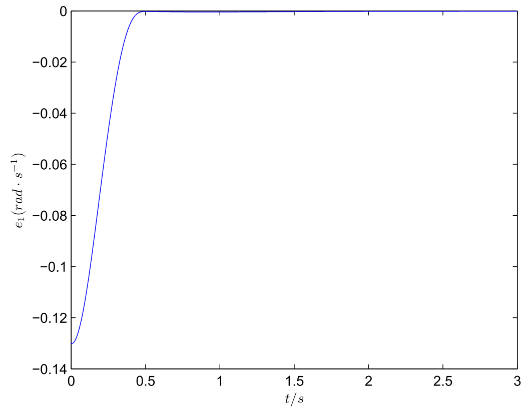

FIGURE 5. The error e_1 (constraint ILC).

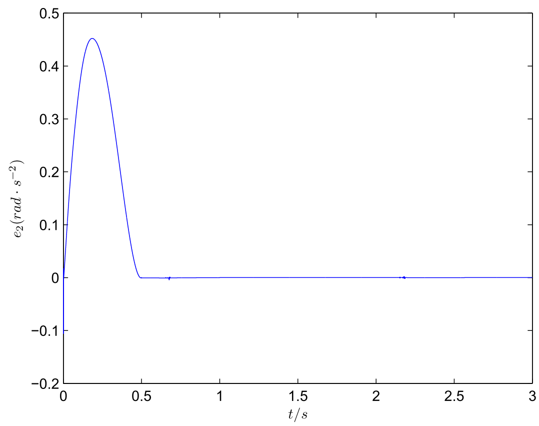

FIGURE 6. The error e_2 (constraint ILC).

For comparison, the no-constraint adaptive ILC algorithm is adopted to simulation as follows:

$$u_{q,k} = -\gamma_1 \boldsymbol{e}_k^{*T} P \boldsymbol{b} + u_{p1,k} + u_{\rho1,k} + u_{\rho2,k}, \tag{56}$$

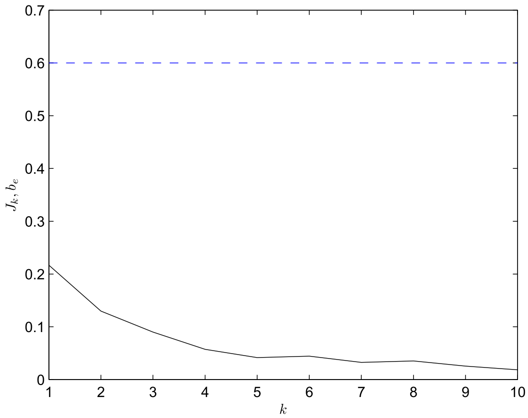

FIGURE 7. Convergence history of $e_k^{*T}Pe_k^*$ (constraint ILC).

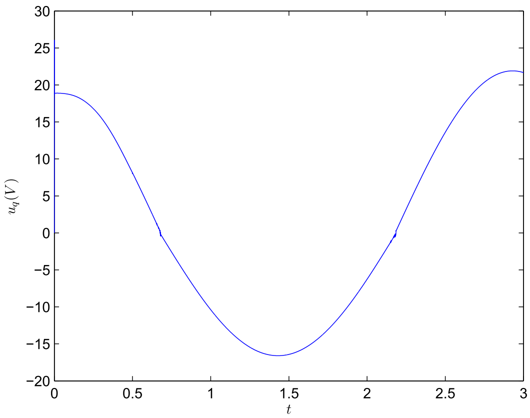

FIGURE 8. Control input(constraint ILC).

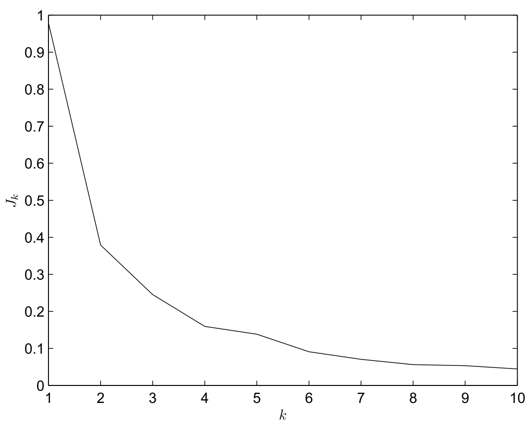

FIGURE 9. Convergence history of $e_k^{*T}Pe_k^*$ (no-constraint ILC).

$$u_{p1,k} = -\theta_k^T \varphi_k, \quad (57)$$
$$u_{\rho 1,k} = -\rho_{1,k} u_{p1,k} \tanh(\rho_{1,k} u_{p1,k} (k+1)^2 e_k^{*T} Pb), \quad (58)$$
$$u_{\rho 2,k} = -\rho_{2,k} \tanh(\rho_{2,k} (k+1)^2 e_k^{*T} Pb), \quad (59)$$
$$\theta_k = \mathrm{sat}_{\underline{\theta},\bar{\theta}}(\theta_{k-1}) + \gamma_2 e_k^{*T} Pb\varphi_k, \theta_{-1} = 0, \quad (60)$$
$$\rho_{1,k} = \mathrm{sat}_{0,\bar{\rho}_1}(\rho_{1,k-1}) + \gamma_3 |e_k^{*T} Pb||u_{p1,k}|, \rho_{1,-1} = 0, \quad (61)$$
$$\rho_{2,k} = \mathrm{sat}_{0,\bar{\rho}_2}(\rho_{2,k-1}) + \gamma_4 |e_k^{*T} Pb|, \rho_{2,-1} = 0, \quad (62)$$

The values of learning gain and control parameters in (56)-(62) are set to be the same as the corresponding ones in (19)-(25), respectively. The convergence history of $e_k^{*T} Pe_k^*$ in this algorithm is shown in Fig. 9, where the definition of J_k is the same as in Fig. 7. Comparing Fig. 9 with Fig. 7, we can see the maximum of $e_k^{*T} Pe_k^*$ in no-constraint ILC does not observe the barrier property. The above simulation results verify the effectiveness of theoretical analysis in this work.

VI. CONCLUSION

In order to solve the trajectory-tracking problem for tank gun control systems with quadratic error constrained and arbitrary initial errors, an initial-rectification adaptive ILC scheme has been proposed in this paper. For removing the zero initial error condition in ILC design, the control strategy of this work is to let the system state track the rectified reference trajectory. To improve the robustness and system safety, the quadratic form of system error is constrained within a preset range by using barrier Lyapunov approach. Adaptive ILC and robust control are jointly used to compensate for the parametric/nonparametric uncertainties and nonsymmetric deadzone nonlinearity in tank gun control systems.

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