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Particle-Swarm-Optimization-Based 2D Output Feedback Robust Constraint Model Predictive Control for Batch Processes

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ABSTRACT For the input and output constraints and uncertainties in batch processes, a 2D output feedback robust constrained model predictive control (MPC) method is designed by combining iterative learning control (ILC), MPC and output feedback. Firstly, an equivalent 2D-FM closed-loop prediction model is established by combining with the proposed output feedback controller. Then an optimization performance index function with terminal constraint is constructed to study its control optimization. According to the designed optimization performance index and Lyapunov stability theory, the feasible MPC problem is obtained by solving the linear matrix inequalities (LMIs). At the same time, the gain of the new output feedback control law is given to ensure that the performance index reaches the minimum upper bound under the constraints of input and output. In order to solve the manual adjustment problem of some parameters in the performance index function, the particle swarm optimization (PSO) algorithm is introduced, and a better solution is found near the controller by using the search optimization method. Finally, taking the injection molding process as an example and comparing with the existing method without using PSO algorithms, it is proved that the above method is more feasible.

INDEX TERMS Batch processes, input–output constraint, iterative learning output feedback predictive control, particle swarm optimization algorithm.

I. INTRODUCTION

In the past 50 years, in order to adapt to the development of industrialization, the control technology of the chemical industry's production process has made great progress [1]–[7]. The process targeted by the research results is roughly based on continuous processes and batch processes. The early design of control technology was obviously based on a continuous process. Such as PID control, adaptive control, etc., [1]–[6]. During the research process, it is discovered that the products produced by some types of chemical production processes are typical batch production processes, whose characteristics are significantly different from the continuous process, such as multiple time-varying characteristics in injection molding processes. Following this, new control technologies are constantly being proposed. It is roughly divided into two stages of development: iterative learning control(ILC) stage [7]–[9] and two-dimensional(2D) control algorithm stage [10]–[11]. ILC is a good choice for repetitive process controller design [12]. But due to the changes of environmental factors and operating conditions, especially the chemical batch process can not be well repeated, that is, it has a certain degree of non repeatability (or uncertainty), single ILC is no longer applicable. In order to deal with the

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above problems, a 2D control algorithm combining feedback control and ILC was proposed [13]. Reference [13] first proposed the batch process control algorithm to ensure two-dimensional stability. At the same time, it also proposed the general method and idea of feedback control combined with ILC under the framework of the two-dimensional system theory. Many results are currently available [11], [14]–[16], including system control under normal conditions or in case of failure.

With the deepening of scientific research, people have found that model predictive control (MPC) is well-applied in industrial processes because of its ''receding optimization'' ability, especially in processes that cannot be accurately modeled. Based on the advantages of MPC, the combination design of ILC and MPC in a 2D framework was very meaningful for batch processes [17]–[26]. In these designs, the design of combining ILC and MPC was included [17]–[22], as well as the design of feedback control combined with ILC and MPC [23]–[28]. The control strategy combining feedback, ILC and MPC was proposed [23], multi-step prediction in the time direction and batch direction was given, and it was verified that multi-step prediction in the batch direction could further improve the convergence speed. A feedback control based on iterative learning predictive updating law was constructed and the 'worst' case linear quadratic function was also designed. Optimization of the controller was obtained using the worst-case objective function along the infinite moving horizon [24]. A two-stage optimization method was designed to reduce the sensitivity of the method to nonrepetitive disturbances, and a 2D Lyapunov function was used to ensure the robust stability of the system [25]. Reference [26] extended the method to nonlinear systems with unknown nonlinear inputs. Moreover, the predictive control results of multi-stage batch process models have also appeared [27], [28]. Most of the above researches are based on state feedback. However, in the actual production process, the state of the system is often not easy to measure directly, or due to the economic and usability limitations of the measuring equipment, the state feedback can not be realized physically. Therefore, the relevant control methods for output feedback have attracted attention. At present, there have been many achievements in the research of robust predictive control with output feedback [29]–[31]. For linear systems with norm-bounded uncertainties and disturbances, a new state estimator was designed by using the output feedback robust MPC method, and all the parameter matrices were optimized online, which improved the control performance of the system [29]. The Lyapunov matrix without structural constraints was chosen, and an iterative cone complementary approach was adopted to optimize the control law parameters [30]. In actual industrial processes, due to the limitation of physical conditions or safety reasons, the input and output variables often have constraint conditions and cannot be infinitely valued. The existence of constraints seriously affects the control performance. How to address the constraints in complex industrial systems is very important to improve the safety and efficiency of production. When the controlled process is constrained, if the unconstrained control technology is used, the calculated control effect will not only reduce the performance of the closed-loop system, but also cause the instability of the system. Especially in the case of multi-variable control, how to deal with various constraints in the control system quickly and effectively is very important work in industrial process control. For the problem of output and input constraints, it is usually converted to the linear matrix inequalities (LMIs) [32], [33].

Because the essence of predictive control optimization is to achieve optimization by manually adjusting the parameter variables of the performance index, which is not in line with today's high efficiency and high precision control. It is urgent to find an intelligent optimization algorithm to optimize control systems. As an intelligent search algorithm, the genetic algorithm has strong advantages in batch process optimization. In the description of the optimization problem, only a simple expression of the target is needed, the operation object is the coded population individual, and the target has no continuous and differentiable restrictive constraints, which reduces the processing difficulty in the problem [34], [35]. Compared with the genetic algorithm, the particle swarm optimization (PSO) algorithm [36] has the characteristics of simplicity, fewer parameters, easy implementation, no coding, fast convergence, etc., and has been applied to many engineering fields [37]–[39]. However, few scholars have used the PSO algorithm to optimize control parameters in the field of batch process control.

In this paper, a 2D output feedback robust constrained MPC method for batch processes based on PSO is proposed under a 2D theoretical framework. The advantages of this method are as follows: (1) In the framework of 2D system theory, combined with ILC and MPC, the controller with extended information is designed, which has better tracking performance and robustness, stronger resistance to system uncertainty and external interference. (2) Considering the output and input constraints and the unmeasurable state in the actual production, the output feedback controller achieves its control goal and satisfies the actual production requirements. (3) A PSO algorithm is used to search for a better solution near the traditional controller, which makes up for the influence that some parameters of the performance index function may not be optimal due to manual adjustment. Finally, the feasibility and superiority of the proposed method are verified by modeling and simulation. Compared with the existing results, the control effect obtained based on the method proposed in this paper is indeed better.

II. PROBLEM DESCRIPTION AND THE ESTABLISHMENT OF A NEW STATE SPACE MODEL

A. PROBLEM DESCRIPTION

For batch processes, a normal-bounded uncertain 2D system model is established based on the state space model for the controlled object. The following discrete time repeated processes with uncertain parameter perturbations is considered:

$$\begin{cases} x(T_t+1,T_k) = [A+\Delta A(T_t,T_k)]x(T_t,T_k) \\ \qquad\qquad + [B+\Delta B(T_t,T_k)] \\ \qquad\qquad u(T_t,T_k) + \upsilon(T_t,T_k) \\ y(T_t,T_k) = Cx(T_t,T_k), \quad 0 \le T_t \le T_p \\ x(0,T_k) = x(0), \qquad T_k = 0,1,\ldots; \end{cases} \tag{1}$$

where T_t is time; T_k is the batch index; $x(T_t,T_k) \in \Re^{n_x}$, $y(T_t,T_k) \in \Re^{l_y}$, $u(T_t,T_k) \in \Re^{m_u}$, and $\upsilon(T_t,T_k) \in \Re^{n_x}$ are the state, output, input and disturbance signals of batch T_k at time T_t, respectively; and $\{A,B,C\}$ are constant matrices with appropriate dimensions. $\Delta A(T_t,T_k)$ and $\Delta B(T_t,T_k)$ represent the uncertainty of the system parameters, and it is assumed that they have the following structure: $[\Delta A(T_t,T_k), \Delta B(T_t,T_k)] = E\Delta(T_t,T_k)[F_1,F_2]$, where E, F_1, F_2 are constant matrices with appropriate dimensions and $\Delta(T_t,T_k)$ is the unknown parameter perturbation that satisfies $\Delta^T(T_t,T_k)\Delta(T_t,T_k) \le I$.

B. EQUIVALENT 2D SYSTEM REPRESENTATION

Let $x(T_t+l|T_t,T_k+m|T_k)$, $y(T_t+l|T_t,T_k+m|T_k)$, and $u(T_t+l|T_t,T_k+m|T_k)$ denote the predicted values of the state variable, output variable and input variable at time T_t of batch T_k, respectively.

For the uncertain system model presented in (1), the ILC strategy is adopted to design the following controller:

$$\begin{cases} u(T_t+l|T_t,T_k+m|T_k) \\ \quad = u(T_t+l|T_t,T_k+m-1|T_k) \\ \qquad + U(T_t+l|T_t,T_k+m|T_k) \\ u(T_t|T_t,T_k|T_k) = u(T_t,T_k) \end{cases}$$
$$T_t = 0,1,2,\ldots,T_p;\ l,m = 0,1,2,\ldots, \tag{2}$$

where $U(T_t+l|T_t,T_k+m|T_k) \in \Re^{m_u}$ represents the iterative updating law to be designed at time T_t of batch T_k, and $U(T_t|T_t,T_k|T_k) = U(T_t,T_k)$. $u(T_t,0)$ represents the initial value of the iteration.

Introduce a variable Υ, where Υ can be an input, an output, a state variable or other correlated variables. Let $\Upsilon_{\hbar,\lambda\!\!\!-} = \Upsilon(T_t+\hbar|T_t,T_k+\lambda\!\!\!-|T_k)$. The output tracking error of the system is designed as follows:

$$e_{l,m} = y_{l,m} - y_r(T_t) \tag{3}$$

where $y_r(T_t)$ is the set-point trajectory for each cycle. The error function in the batch direction is defined as follows:

$$\sigma^e_{l,m} = \sigma_{l,m} - \sigma_{l,m-1} \tag{4}$$

where σ can be an input, an output, or a state variable, and $U_{l,m} = u^e_{l,m}$. From (1), (2), (3) and (4), we can obtain:

$$x^e_{l+1,m} = [A+\Delta A_{l,m}]x^e_{l,m} + [B+\Delta B_{l,m}]U_{l,m} + \tilde{\upsilon}_{l,m} \tag{5}$$

$$e_{l+1,m} = e_{l+1,m-1} + C\{[A+\Delta A_{l,m}]x^e_{l,m} + [B+\Delta B_{l,m}]\mathrm{U}_{l,m} + \tilde{\upsilon}_{l,m}\} \tag{6}$$

$$\tilde{\upsilon}_{l+1,m} = [\Delta A_{l,m} - \Delta A_{l,m-1}]x_{l,m-1} + [\Delta B_{l,m} - \Delta B_{l,m-1}]u_{l,m-1} + \upsilon^e_{l,m} \tag{7}$$

When $\tilde{\upsilon}_{l,m} = 0$, it is a repetitive disturbance; otherwise, it is a nonrepetitive disturbance.

A new state variable is introduced as follows:

$$\bar{\chi}_{l,m} = \sum_{n=0}^{l-1} e_{n,m} \tag{8}$$

An extended equivalent 2D system is as follows:

$$\begin{cases} \hat{\chi}_{l+1,m} = [A_1+\Delta A_1]\hat{\chi}_{l,m} + A_2\hat{\chi}_{l,m-1} \\ \qquad\qquad + [B_1+\Delta B_1]U_{l,m} + D_1\tilde{\upsilon}_{l,m} \\ y^e_{l,m} = \begin{bmatrix} e_{l,m-1} \\ \bar{\chi}_{l,m} + e_{l,m} \\ e_{l,m} \end{bmatrix} = G\hat{\chi}_{l,m}, \\ \quad 1 \le T_t \le T_p \\ \bar{\chi}(0,T_k) = 0, \quad T_k = 0,1,\ldots; \\ \quad l = 1,2,3\ldots; \quad m = 1,2,3\ldots \end{cases} \tag{9}$$

where

$$\hat{\chi}_{l,m} = \begin{bmatrix} x^e_{l,m} \\ \bar{\chi}_{l,m} \\ e_{l,m} \end{bmatrix}, \ A_1 = \begin{bmatrix} A & 0 & 0 \\ 0 & I & I \\ CA & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{bmatrix},$$

$$B_1 = \begin{bmatrix} B \\ 0 \\ CB \end{bmatrix}, \quad D_1 = \begin{bmatrix} I \\ 0 \\ C \end{bmatrix}, \ G = \begin{bmatrix} -C & I & 0 \\ 0 & I & I \\ 0 & 0 & I \end{bmatrix},$$

$$\Delta A_1 = \hat{E}\Delta(T_t,T_k)\hat{F}_1, \quad \Delta B_1 = \hat{E}\Delta(T_t,T_k)\hat{F}_2,$$
$$\hat{E} = [(E)^T, 0, (CE)^T]^T, \quad \hat{F}_1 = [F_1,0,0], \ \hat{F}_2 = F_2$$

When only the output can be measured in the process, based on the above 2D output feedback ILC framework, the

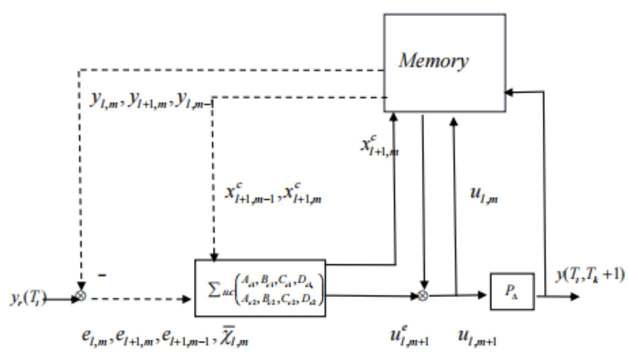

FIGURE 1. Output feedback structure diagram.

following controller can be designed:

$$\begin{cases} x^c_{l+1,m} = A_{c1}x^c_{l,m} + A_{c2}x^c_{l+1,m-1} + B_{c1}y^e_{l,m} \\ \qquad + B_{c2}y^e_{l+1,m-1} \\ U_{l,m} = C_{c1}x^c_{l,m} + C_{c2}x^c_{l+1,m-1} \\ \qquad + D_{c1}y^e_{l,m} + D_{c2}y^e_{l+1,m-1} \end{cases} \tag{10}$$

where $x^c_{l,m} \in \Re^{n_x+2l_y}$ is the internal state of the controller and $\{A_{ci}, B_{ci}, C_{ci}, D_{ci}\}_{i=1,2}$ is the appropriate dimension controller parameter. Let $Z_{l+1,m} = \begin{bmatrix} \hat{\chi}_{l+1,m} \\ x^c_{l+1,m} \end{bmatrix}$. When (10) is substituted into (9), the 2D control system is obtained:

$$\begin{cases} Z_{l+1,m} = (\bar{A}_1 + \Delta\bar{A}_1)Z_{l,m} + \bar{A}_2 Z_{l+1,m-1} \\ \qquad + (\bar{B}_1 + \Delta\bar{B}_1)U_{l,m} + \bar{D}\tilde{\upsilon}_{l,m} \\ y^e_{l,m} = \bar{G}Z_{l,m} \\ l = 1, 2, 3 \ldots; m = 1, 2, 3 \ldots \end{cases} \tag{11}$$

where

$$\bar{A}_1 = \begin{bmatrix} A_1 & 0 \\ B_{c1}G & A_{c1} \end{bmatrix}, \Delta\bar{A}_1 = \begin{bmatrix} \Delta A_1 & 0 \\ 0 & 0 \end{bmatrix} = \bar{E}\Delta(T_t, T_k)\bar{F}_1,$$

$$\bar{A}_2 = \begin{bmatrix} A_2 & 0 \\ B_{c2}G & A_{c2} \end{bmatrix}, \quad \bar{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \ \Delta\bar{B}_1 = \begin{bmatrix} \Delta B_1 \\ 0 \end{bmatrix}$$
$$= \bar{E}\Delta(T_t, T_k)\bar{F}_2,$$

$$\bar{E} = \begin{bmatrix} \hat{E} \\ 0 \end{bmatrix}, \quad \bar{F}_1 = \begin{bmatrix} \hat{F}_1 & 0 \end{bmatrix},$$

$$\bar{F}_2 = \hat{F}_2, \quad \bar{D} = \begin{bmatrix} D_1 \\ 0 \end{bmatrix}, \ \bar{G} = \begin{bmatrix} G & 0 \end{bmatrix}$$

III. PSO-BASED 2D OUTPUT FEEDBACK ROBUST PREDICTIVE TRACKING CONTROLLER DESIGN

A. PERFORMANCE INDEX FUNCTION

According to the characteristics of the batch process, it can be divided into repetitive interference and nonrepetitive interference. Therefore, the definitions of the performance indicators are also different. When the interference is nonrepetitive interference, in infinite time domains and, the "worst" case indicator at time of batch of the uncertain system is defined as follows:

$$\min_{U_{l,m}, l,m \geq 0} \ \max_{[A,B,C]\in\Omega} J_\infty(T_t, T_k)$$

$$J_\infty(T_t, T_k) = \sum_{l=0}^{\infty}\sum_{m=0}^{\infty} \ell_{l,m} = \sum_{l=0}^{N-1}\sum_{m=0}^{N-1} \ell_{l,m} + V_m(Z_{N,N}) \tag{12}$$

where $V_m(Z_{l,m})$ is called a terminal constraint and

$$\ell_{l,m} = Z^T_{l,m}Q_1 Z_{l,m} + Z^T_{l+1,m-1}Q_2 Z_{l+1,m-1} + U^T_{l,m}RU_{l,m} - (\gamma)^2\tilde{\upsilon}^T_{l,m}\tilde{\upsilon}_{l,m}$$

The constraint condition is as follows:

$$\begin{cases} Z_{l+1,m} = (\bar{A}_1 + \Delta\bar{A}_1)Z_{l,m} + \bar{A}_2 Z_{l+1,m-1} \\ \qquad + (\bar{B}_1 + \Delta\bar{B}_1)U_{l,m} + \bar{D}\tilde{\upsilon}_{l,m} \\ \qquad l = 1, 2, 3 \ldots; \quad m = 1, 2, 3 \ldots \end{cases}$$

$$\begin{cases} \left\| U_{l,m} \right\| \leq r_m \\ \left\| y^e_{l,m} \right\| \leq y_m \end{cases} \tag{13}$$

where Q_1, Q_2, R is the corresponding weight matrix; $\gamma > 0$ and r_m, y_m are the upper bound values of variables $U_{l,m}$ and $y^e_{l,m}$, respectively; $\begin{bmatrix} A & B & C \end{bmatrix} \in \Omega$; and Ω denotes uncertain sets.

B. STABILITY ANALYSIS AND CONTROLLER DESIGN OF A 2D SYSTEM

To solve the optimization problem in (12), a new predictive law (10) is designed with predictive control theory, and the robust stability of the system is studied. Under controller (10), the closed-loop prediction model can be expressed as follows:

$$\begin{cases} Z_{l+1,m} = (\bar{A}_1 + \Delta\bar{A}_1 + \bar{B}_1 Y_1 + \Delta\bar{B}_1 Y_1)Z_{l,m} \\ \qquad + (A_2 + \bar{B}_1 Y_2 + \Delta\bar{B}_1 Y_2)Z_{l+1,m-1} + \bar{D}\tilde{\upsilon}_{l,m} \\ y^e_{l,m} = \bar{G}Z_{l,m} \\ l = 1, 2, 3 \ldots; \quad m = 1, 2, 3 \ldots \end{cases} \tag{14}$$

where $Y_1 = \begin{bmatrix} D_{c1}G & C_{c1} \end{bmatrix}$, $Y_2 = \begin{bmatrix} D_{c2}G & C_{c2} \end{bmatrix}$.

The Lyapunov function is defined as follows:

$$\begin{aligned} V(Z_{l,m}) &= V_h(Z_{l,m}) + V_v(Z_{l,m}), \\ V_h(Z_{l,m}) &= Z^T_{l,m}P_1 Z_{l,m} = Z^T_{l,m}\theta_1 L_1^{-1} Z_{l,m}, \\ V_v(Z_{l,m}) &= Z^T_{l,m}P_2 Z_{l,m} = Z^T_{l,m}\theta_1 L_2^{-1} Z_{l,m} \end{aligned} \tag{15}$$

where P_1, P_2 are positive definite matrices to be determined and satisfy:

$$\alpha P_1 + \beta P_2 < P, \quad \alpha > 1, \ \beta > 1, \ P > 0, \ P = \theta_1 L^{-1},$$
$$\alpha L_1^{-1} + \beta L_2^{-1} < L^{-1} \tag{16}$$

To ensure the robust stability of the system and the solvability of optimization problem, the following Lyapunov inequality constraints are required to hold:

$$\begin{aligned} &V_h(Z_{l+1,m}) - V_h(Z_{l,m}) + V_v(Z_{l+1,m}) - V_v(Z_{l+1,m-1}) \\ &\quad \leq -\Big[Z^T_{l,m}Q_1 Z_{l,m} + Z^T_{l+1,m-1}Q_2 Z_{l+1,m-1} \\ &\qquad + U^T_{l,m}RU_{l,m} - (\gamma)^2\tilde{\upsilon}^T_{l,m}\tilde{\upsilon}_{l,m}\Big] \end{aligned} \tag{17}$$

For the closed-loop prediction model in (14), it is assumed that there are a series of initial conditions, there are two positive integers T_l, K_m, and

$$Z(T_t + l, T_k) = 0, \quad l \geq s_1; \ Z(T_t, T_k + m) = 0, \ m \geq s_2$$

where $s_1 < \infty$ and $s_2 < \infty$ are positive integers, and the corresponding $Z(T_t + l, T_k)$ and $Z(T_t, T_k + m)$ are the boundaries of the time direction and the batch direction, respectively. $s = \max\{s_1, s_2\}$.

By superimposing

$$\begin{aligned} &V_h(Z_{l+1,m}) - V_h(Z_{l,m}) + V_v(Z_{l+1,m}) - V_v(Z_{l+1,m-1}) \\ &\quad \leq -\Big[Z^T_{l,m}Q_1 Z_{l,m} + Z^T_{l+1,m-1}Q_2 Z_{l+1,m-1} \\ &\qquad + U^T_{l,m}RU_{l,m} - (\gamma)^2\tilde{\upsilon}^T_{l,m}\tilde{\upsilon}_{l,m}\Big] \end{aligned} \tag{18}$$

from $l, m = 0$ to $l, m = \infty$, the following inequality is obtained:

$$\max J_\infty(T_t, T_k) \le sV(Z(T_t, T_k)) \le \theta$$

where θ is the upper bound of $J_\infty(T_t, T_k)$.

Lemma 1 ([40]): For the given matrices W, L and V with appropriate dimensions, where W and V are positive definite matrices, then

$$L^T VL - W < 0 \tag{19}$$

if and only if

$$\begin{bmatrix} -W & L^T \\ L & -V^{-1} \end{bmatrix} < 0 \text{ or } \begin{bmatrix} -V^{-1} & L \\ L^T & -W \end{bmatrix} < 0 \tag{20}$$

Lemma 2 ([41]): Let E, F, D and M denote real matrices with appropriate dimensions and M satisfy $M = M^T$; then, for all $F^T F \le I$,

$$M + DFE + E^T F^T D^T < 0 \tag{21}$$

if and only if there exists $\xi > 0$ such that

$$M + \xi^{-1}DD^T + \xi E^T E < 0 \tag{22}$$

Theorem 1: Assume $\tilde{\upsilon}(T_t, T_k) = 0$ holds. For given positive definite matrices $Q_1, Q_2 \in \Re^{(n_x+l_y)\times(n_x+l_y)}$, and $R \in \Re^{m_u \times m_u}$, and positive numbers $\alpha > 1, \beta > 1$, the 2D-FM system in (11) is solvable if there exist positive definite symmetric matrices $X > 0, Y > 0, \bar{P} > 0, S_1 > 0, S_2 > 0$, appropriate dimension matrices $\widehat{A}_{ci}, \widehat{B}_{ci}, \widehat{C}_{ci}, \widehat{D}_{ci} (i = 1, 2)$, and scalars $\varepsilon > 0$ and $\theta_1 > 0$ such that the following LMIs hold:

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} \\ * & \Theta_{14} & 0 \\ * & * & \Theta_{15} \end{bmatrix} < 0 \tag{23}$$

$$\alpha S_1 + \beta S_2 < \Pi_{11} \tag{24}$$

$$\begin{bmatrix} -1 & Z(T_t|T_t, T_k|T_k) \\ * & -\bar{P}^{-1} \end{bmatrix} \le 0 \tag{25}$$

$$\begin{bmatrix} -r_m^2 \Pi_{21} & 0 & \prod_{23} \\ * & -r_m^2 \Pi_{31} & \prod_{33} \\ * & * & -I \end{bmatrix} < 0 \tag{26}$$

$$\begin{bmatrix} -y_m^2 \bar{P}^{-1} & \bar{G} \\ * & -I \end{bmatrix} \le 0 \tag{27}$$

where

$$\Theta_{11} = \begin{bmatrix} -\Pi_{11} & \Pi_{12} & \Pi_{13} \\ * & -S_1 & 0 \\ * & * & -S_2 \end{bmatrix}, \Theta_{12} = \begin{bmatrix} 0 & 0 & 0 \\ \Pi_{22} & 0 & \Pi_{23} \\ 0 & \Pi_{32} & \Pi_{33} \end{bmatrix}$$

$$\Theta_{13} = \begin{bmatrix} 0 & \Pi_{14} \\ \Pi_{24} & 0 \\ \Pi_{34} & 0 \end{bmatrix},$$

$$\Theta_{14} = \begin{bmatrix} -\theta_1 Q_1^{-1} & 0 & 0 \\ * & -\theta_1 Q_2^{-1} & 0 \\ * & * & -\theta_1 R^{-1} \end{bmatrix},$$

$$\Theta_{15} = \begin{bmatrix} -\varepsilon I & 0 \\ * & -\varepsilon^{-1} I \end{bmatrix}$$

$$\prod_{11} = \begin{bmatrix} X & I \\ I & Y \end{bmatrix}, \prod_{12} = \begin{bmatrix} A_1 X + B_1 \widehat{C}_{c1} & A_1 + B_1 \widehat{D}_{c1} G \\ \widehat{A}_{c1} & YA_1 + \widehat{B}_{c1} G \end{bmatrix},$$

$$\prod_{13} = \begin{bmatrix} A_2 X + B_1 \widehat{C}_{c2} & A_2 + B_1 \widehat{D}_{c2} G \\ \widehat{A}_{c2} & YA_2 + \widehat{B}_{c2} G \end{bmatrix}, \prod_{14} = \begin{bmatrix} X\hat{E} \\ \hat{E} \end{bmatrix},$$

$$\prod_{21} = S_1, \quad \prod_{22} = \prod_{32} = \begin{bmatrix} X & M \\ I & 0 \end{bmatrix},$$

$$\prod_{23} = \begin{bmatrix} \widehat{C}_{c1} & \widehat{D}_{c1} G \end{bmatrix}^T,$$

$$\prod_{24} = \begin{bmatrix} \hat{F}_1 X + \hat{F}_2 \widehat{C}_{c1} & \hat{F}_1 X + \hat{F}_2 \widehat{D}_{c1} G \end{bmatrix}^T$$

$$\prod_{31} = S_2, \quad \prod_{33} = \begin{bmatrix} \widehat{C}_{c2} & \widehat{D}_{c2} G \end{bmatrix}^T,$$

$$\prod_{34} = \begin{bmatrix} \hat{F}_2 \widehat{C}_{c2} & \hat{F}_2 \widehat{D}_{c2} G \end{bmatrix}^T$$

If $X, Y, \widehat{A}_{ci}, \widehat{B}_{ci}, \widehat{C}_{ci}, \widehat{D}_{ci} (i = 1, 2)$ are the feasible solutions of matrix inequalities (23)-(27), then the parameters of controller (10) with output feedback can be designed as follows:

$$\begin{cases} D_{ci} = \widehat{D}_{ci} \\ C_{ci} = (\widehat{C}_{ci} - D_{ci} GX)(M^T)^{-1} \\ B_{ci} = N^{-1}(\widehat{B}_{ci} - YB_1 D_{ci}) \\ A_{ci} = N^{-1}(\widehat{A}_{ci} - YA_i X + YB_1 D_{ci} GX + \\ NB_{ci} GX + YB_1 C_{ci} M^T)(M^T)^{-1} \end{cases} \quad (\text{i} = 1, 2) \tag{28}$$

If M, N are full rank matrices that satisfy the condition $XY + MN^T = I$, then they can be obtained by a singular value decomposition of the matrix $I - XY$.

Proof: When $\tilde{\upsilon}(T_t, T_k) = 0$, in infinite time domains $[T_t, \infty)$ and $[T_k, \infty)$, the "worst" case indicator at time T_t of batch T_k of the uncertain system is defined as follows:

$$\min_{U_{l,m}, l,m \ge 0} \max_{[A,B,C] \in \Omega} J_\infty(T_t, T_k)$$

$$J_\infty(T_t, T_k) = \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \ell_{l,m} = \sum_{l=0}^{N-1} \sum_{m=0}^{N-1} \ell_{l,m} + V_m(Z_{N,N}) \tag{29}$$

where

$$\ell_{l,m} = Z_{l,m}^T Q_1 Z_{l,m} + Z_{l+1,m-1}^T Q_2 Z_{l+1,m-1} + U_{l,m}^T R U_{l,m}$$

The increment function is designed as follows:

$$\begin{aligned} \Delta V(Z_{l+1,m}) &= \Delta V_h(Z_{l+1,m}) + \Delta V_v(Z_{l+1,m}) \\ &= V_h(Z_{l+1,m}) - V_h(Z_{l,m}) \\ &\quad + V_v(Z_{l+1,m}) - V_v(Z_{l+1,m-1}) \end{aligned}$$

$$\begin{aligned} V(Z_{l,m}) &= V_h(Z_{l,m}) + V_v(Z_{l,m}) \\ V_h(Z_{l,m}) &= Z_{l,m}^T P_1 Z_{l,m} = Z_{l,m}^T \theta_1 L_1^{-1} Z_{l,m} \\ V_v(Z_{l,m}) &= Z_{l,m}^T P_2 Z_{l,m} = Z_{l,m}^T \theta_1 L_2^{-1} Z_{l,m} \\ \Delta V_h(Z_{l+1,m}) &= Z_{l+1,m}^T P_1 Z_{l+1,m} - Z_{l,m}^T P_1 Z_{l,m} \\ &= Z_{l+1,m}^T \theta_1 (L_1)^{-1} Z_{l+1,m} - Z_{l,m}^T \theta_1 (L_1)^{-1} Z_{l,m} \\ \Delta V_v(Z_{l+1,m}) &= Z_{l+1,m}^T P_2 Z_{l+1,m} - Z_{l+1,m-1}^T P_2 Z_{l+1,m-1} \\ &= Z_{l+1,m}^T \theta_1 (L_2)^{-1} Z_{l+1,m} \\ &\quad - Z_{l+1,m-1}^T \theta_1 (L_2)^{-1} Z_{l+1,m-1} \end{aligned} \tag{30}$$

The following variables are defined as follows:

$$\phi_{l,m} = \phi(T_t + l|T_t, T_k + m|T_k) = \begin{bmatrix} Z_{l,m} \\ Z_{l+1,m-1} \end{bmatrix}$$

The sufficient condition for the robust stability of the closed-loop system in (11) is the existence of some positive definite symmetric matrices P, P_1, P_2 that make (31)true,

$$\begin{aligned} &V_h(Z_{l+1,m}) - V_h(Z_{l,m}) + V_v(Z_{l+1,m}) - V_v(Z_{l+1,m-1}) \\ &= Z_{l+1,m}^T (P_1 + P_2) Z_{l+1,m} - Z_{l,m}^T P_1 Z_{l,m} - Z_{l,m}^T P_2 Z_{l,m} \\ &\le Z_{l+1,m}^T P Z_{l+1,m} - Z_{l,m}^T P_1 Z_{l,m} - Z_{l,m}^T P_2 Z_{l,m} \\ &\le -\left[Z_{l,m}^T Q_1 Z_{l,m} + Z_{l+1,m-1}^T Q_2 Z_{l+1,m-1} + U_{l,m}^T R U_{l,m} \right] \end{aligned} \tag{31}$$

Eq. (31) can be translated into

$$\phi^T \psi \phi < 0 \Leftrightarrow \psi < 0 \tag{32}$$

where $U_{l,m} = Y_1 Z_{l,m} + Y_2 Z_{l+1,m-1}$.

$$\begin{aligned} \psi &= \zeta_1^T \theta_1 L^{-1} \zeta_1 - \zeta_2^T \theta_1 L_1^{-1} \zeta_2^T - \zeta_3^T \theta_1 L_2^{-1} \zeta_3^T + \zeta_2^T Q_1 \zeta_2^T \\ &\quad + \zeta_3^T Q_2 \zeta_3^T - \zeta_4^T R \zeta_4^T \\ \zeta_1 &= \begin{bmatrix} \bar{A}_1 + \Delta\bar{A}_1 + \bar{B}_1 Y_1 + \Delta\bar{B}_1 Y_1 & \bar{A}_2 + \bar{B}_1 Y_2 + \Delta\bar{B}_1 Y_2 & \bar{D} \end{bmatrix} \\ \zeta_2 &= \begin{bmatrix} I & 0 & 0 \end{bmatrix}, \quad \zeta_3 = \begin{bmatrix} 0 & I & 0 \end{bmatrix}, \\ \zeta_4 &= \begin{bmatrix} Y_1 & Y_2 & 0 \end{bmatrix} \end{aligned}$$

Applying Lemma 1 and Lemma 2 to (32), we obtain

$$\begin{bmatrix} \Theta_{21} & \Theta_{22} & \Theta_{23} \\ * & \Theta_{14} & 0 \\ * & * & \Theta_{15} \end{bmatrix} < 0 \tag{33}$$

where

$$\Theta_{21} = \begin{bmatrix} -L^{-1} & L^{-1}(\bar{A}_1 + \bar{B}_1 Y_1) & L^{-1}(\bar{A}_2 + \bar{B}_1 Y_2) \\ * & -L_1^{-1} & 0 \\ * & * & -L_2^{-1} \end{bmatrix},$$

$$\Theta_{22} = \begin{bmatrix} 0 & 0 & 0 \\ I & 0 & Y_1^T \\ 0 & I & Y_1^T \end{bmatrix},$$

$$\Theta_{23} = \begin{bmatrix} 0 & L^{-1}\bar{E} \\ \bar{F}_1^T + Y_1^T \bar{F}_2^T & 0 \\ Y_2^T \bar{F}_2^T & 0 \end{bmatrix}$$

Let $L^{-1} = \bar{P}, \bar{P} = \begin{bmatrix} Y & N \\ N^T & W \end{bmatrix}, \bar{P}^{-1} = \begin{bmatrix} X & M \\ M^T & K \end{bmatrix}, \Lambda_1 = \begin{bmatrix} X & I \\ M^T & 0 \end{bmatrix}$. Multiplying (33) on the left by $\bar{\Lambda}_1^T = diag(\Lambda_1^T, \Lambda_1^T, \Lambda_1^T, I, I, I, I, I)$ and on the right by $\bar{\Lambda}_1$, we can obtain:

$$\begin{bmatrix} \Theta_{31} & \Theta_{32} & \Theta_{33} \\ * & \Theta_{14} & 0 \\ * & * & \Theta_{15} \end{bmatrix} < 0 \tag{34}$$

where Θ_{31}, as shown at the bottom of the next page.

Notably, (34) is equivalent to (23). According to (24), $\alpha L_1^{-1} + \beta L_2^{-1} < L^{-1}$, so $\alpha P_1 + \beta P_2 < P$. Therefore, (23) and (24) are sufficient conditions for Eq. (31).

According to (31),

$$\begin{aligned} \Delta V(Z_{l,m}) < -\Big[& Z_{l+1,m-1}^T Q_2 Z_{l+1,m-1} \\ & + Z_{l,m}^T Q_1 Z_{l,m} + U_{l,m}^T R U_{l,m} \Big] < 0 \end{aligned}$$

Then, the closed-loop system in (11) is solvable.

According to (18), when the system is superimposed from $l, m = 1$ to $l, m = \infty$, we obtain (35), as shown at the bottom of the next page. Then,

$$-sV(Z(T_t, T_k)) \le -\sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \ell_{l,m} \tag{36}$$

From (25), we can obtain $Z_{0,0}^T L^{-1} Z_{0,0} \le 1$.

If $P = \theta_1 L^{-1}$ and $\psi_l = P_1 + P_2$, then $Z_{0,0}^T P Z_{0,0} \le \theta_1$ is true; hence,

$$V(Z(T_t, T_k)) = Z^T(T_t, T_k) \psi_l Z(T_t, T_k) \le \theta_1 \tag{37}$$

where θ_1 is the upper bound of $V(Z(T_t, T_k))$. Therefore, $J_\infty(T_t, T_k) = \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \ell_{l,m} \le sV(Z(T_t, T_k)) \le s\theta_1 \le \theta$ is true. Thus, when perturbations are involved, the traditional asymptotic stability cannot converge to the origin. In contrast, there exists the following robust positive definite invariant set:

$$\Psi := \{Z | V(Z) \le \theta_1\} \tag{38}$$

It is satisfied that the system states converge to this set.

For $\forall Z \in \Psi$, there is:

$$\alpha_1 V(|Z|) \le V(Z) \le \alpha_2 V(|Z|) \tag{39}$$

where $\alpha_1, \alpha_2 \in K_\infty$, and

$$\begin{aligned} &V(Z)^+ - V(Z) \\ &\le -\begin{bmatrix} Z^T(T_t, T_k) Q_1 Z(T_t, T_k) + Z^T(T_t + 1, T_k - 1) Q_2 \\ Z(T_t + 1, T_k - 1) + U^T(T_t, T_k) R U(T_t, T_k) \end{bmatrix} \end{aligned} \tag{40}$$

$$(\lambda_{\min})^* |Z|^2 \le V^*(Z) \le (\lambda_{\max})^* |Z|^2 \tag{41}$$

where $(\lambda_{\min})^* := \min\{p_{\min}(\psi_l)^*\}$, $(\lambda_{\max})^* := \max\{p_{\max}(\psi_l)^*\}$, $p_{\min}(\cdot)$ and $p_{\max}(\cdot)$ are the minimum and maximum eigenvalues respectively, $V^*(Z)$ is the optimal value

$$\Theta_{31} = \begin{bmatrix} -\Lambda_1^T \bar{P} \Lambda_1 & \Lambda_1^T \bar{P}(\bar{A}_1 + \bar{B}_1 Y_1)\Lambda_1 & \Lambda_1^T \bar{P}(\bar{A}_2 + \bar{B}_1 Y_2)\Lambda_1 \\ * & -\Lambda_1^T L_1^{-1} \Lambda_1 & 0 \\ * & * & -\Lambda_1^T L_2^{-1} \Lambda_1 \end{bmatrix},$$

$$\Theta_{32} = \begin{bmatrix} 0 & 0 & 0 \\ \Lambda_1^T & 0 & \Lambda_1^T Y_1^T \\ 0 & \Lambda_1^T & \Lambda_1^T Y_2^T \end{bmatrix},$$

$$\Theta_{33} = \begin{bmatrix} 0 & \Lambda_1^T \bar{E} \\ \Lambda_1^T(\bar{F}_1^T + Y_1^T \bar{F}_2^T) & 0 \\ \Lambda_1^T Y_2^T \bar{F}_2^T & 0 \end{bmatrix}$$

$$\Lambda_1^T \bar{P} \Lambda_1 = \begin{bmatrix} X & I \\ I & Y \end{bmatrix},$$

$$\Lambda_1^T \bar{P} = \begin{bmatrix} I & 0 \\ Y & N \end{bmatrix}$$

$$\Lambda_1^T \bar{P}(\bar{A}_1 + \bar{B}_1 Y_1)\Lambda_1 = \begin{bmatrix} A_1 X + B_1 \widehat{C}_{c1} & A_1 + B_1 \widehat{D}_{c1} G \\ \widehat{A}_{c1} & Y A_1 + \widehat{B}_{c1} G \end{bmatrix},$$

$$\Lambda_1^T \bar{P}(\bar{A}_2 + \bar{B}_1 Y_2)\Lambda_1 = \begin{bmatrix} A_2 X + B_1 \widehat{C}_{c2} & A_2 + B_1 \widehat{D}_{c2} G \\ \widehat{A}_{c2} & Y A_2 + \widehat{B}_{c2} G \end{bmatrix},$$

$$\Lambda_1^T Y_1^T = \begin{bmatrix} \widehat{C}_{c1} & \widehat{D}_{c1} G \end{bmatrix}^T, \quad \Lambda_1^T Y_2^T = \begin{bmatrix} \widehat{C}_{c2} & \widehat{D}_{c2} G \end{bmatrix}^T,$$

$$\Lambda_1^T(\bar{F}_1^T + Y_1^T \bar{F}_2^T) = \begin{bmatrix} \hat{F}_1 X + \hat{F}_2 \widehat{C}_{c1} & \hat{F}_1 X + \hat{F}_2 \widehat{D}_{c1} G \end{bmatrix}^T$$

$$\Lambda_1^T(Y_2^T \bar{F}_2^T) = \begin{bmatrix} \hat{F}_2 \widehat{C}_{c2} & \hat{F}_2 \widehat{D}_{c2} G \end{bmatrix}^T,$$

$$\Lambda_1^T \bar{E} = \begin{bmatrix} X\hat{E} \\ \hat{E} \end{bmatrix}$$

$$\begin{cases} \widehat{D}_{ci} = D_{ci} \\ \widehat{C}_{ci} = D_{ci} G X + C_{ci} M^T \\ \widehat{B}_{ci} = Y B_1 D_{ci} + N B_{ci} & \qquad (i = 1, 2) \\ \widehat{A}_{ci} = Y A_i X + Y B_1 D_{ci} G X \\ \qquad + N B_{ci} G X + Y B_1 C_{ci} M^T + N A_{ci} M^T \end{cases}$$

$$S_1 = \Lambda_1^T L_1^{-1} \Lambda_1, \quad S_2 = \Lambda_1^T L_2^{-1} \Lambda_1, \ \Pi_{11} = -\Lambda_1^T \bar{P} \Lambda_1$$

$$\begin{aligned} &\sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \{V_h(Z_{l+1,m}) - V_h(Z_{l,m}) + V_v(Z_{l+1,m}) - V_v(Z_{l+1,m-1})\} \\ &= \sum_{m=0}^{\infty} \begin{pmatrix} V_h(Z_{1,m}) - V_h(Z_{0,m}) + V_v(Z_{1,m}) - V_v(Z_{1,m-1}) + V_h(Z_{2,m}) \\ -V_h(Z_{1,m}) + V_v(Z_{2,m}) - V_v(Z_{2,m-1}) + \cdots + V_h(Z_{\infty+1,m}) \\ - V_h(Z_{\infty,m}) + V_v(Z_{\infty+1,m}) - V_v(Z_{\infty+1,m-1}) \end{pmatrix} \\ &= \sum_{m=0}^{\infty} (V_h(Z_{\infty+1,\infty}) - V_h(Z_{l+1,-1})) + \sum_{l=0}^{\infty} (V_v(Z_{l+1,\infty}) - V_v(Z_{0,m})) \\ &= -\sum_{m=0}^{s_2} V_h(Z_{0,m}) - \sum_{l=0}^{s_1} V_v(Z_{l+1,-1}) \\ &\geq -s_2 V_h(Z_{0,0}) - s_1 V_v(Z_{1,-1}) \\ &\geq -sV(Z_{0,0}) = -sV(Z(T_t, T_k)) \end{aligned} \tag{35}$$

of $V(Z)$ at the time T_t of batch T_k, ψ_l^* is the optimal value of ψ_l at the time T_t of batch T_k.

Define $Z_{0,0} = Z(T_t, T_k)$, $U_{0,0} = U(T_t, T_k)$, equation (29) is equivalent to

$$\begin{aligned} J_\infty(T_t, T_k) &= \sum_{l=0}^{\infty}\sum_{m=0}^{\infty} \ell_{l,m} = \ell_{0,0} + \sum_{l=1}^{\infty}\sum_{m=1}^{\infty} \ell_{l,m} \\ &\quad + \sum_{l=1}^{\infty} \ell_{l,0} + \sum_{m=1}^{\infty} \ell_{0,m} \\ &= Z^T(T_t, T_k)Q_1 Z(T_t, T_k) + Z^T(T_t+1, T_k-1) \\ &\quad \times Q_2 Z(T_t+1, T_k-1) \\ &\quad + U^T(T_t, T_k)RU(T_t, T_k) + \sum_{l=1}^{\infty}\sum_{m=1}^{\infty} \ell_{l,m} \\ &\quad + \sum_{l=1}^{\infty} \ell_{l,0} + \sum_{m=1}^{\infty} \ell_{0,m} \end{aligned} \tag{42}$$

Acorrding to (30), superpose:

$$\begin{aligned} &V_h(Z_{l+1,m}) - V_h(Z_{l,m}) + V_v(Z_{l+1,m}) - V_v(Z_{l+1,m-1}) \\ &\quad \le -\left[Z_{l,m}^T Q_1 Z_{l,m} + Z_{l+1,m-1}^T Q_2 Z_{l+1,m-1} + U_{T_l,K_m}^T R U_{l,m}\right] \end{aligned} \tag{43}$$

From $l, m = 1$ to $l, m = \infty$, we can get

$$\sum_{l=1}^{\infty}\sum_{m=1}^{\infty} \ell_{l,m} < \sum_{m=1}^{\infty} V_h(Z_{1,m}) + \sum_{l=1}^{\infty} V_v(Z_{l+1,0}) \tag{44a}$$

From $l = 1$ to $l = \infty$, $m = 0$, we can get

$$\sum_{l=1}^{\infty} \ell_{l,0} < \sum_{l=1}^{\infty} V_v(Z_{l+1,-1}) - \sum_{l=1}^{\infty} V_v(Z_{l+1,0}) + V_h\left(Z_{1,0}\right) \tag{44b}$$

From $m = 1$ to $m = \infty$, $l = 0$, we can get

$$\sum_{m=1}^{\infty} \ell_{0,m} < \sum_{m=1}^{\infty} V_h(Z_{0,m}) - \sum_{m=1}^{\infty} V_h(Z_{1,m}) + V_v\left(z_{1,-1}\right) \tag{44c}$$

According to (44a), (44b), (44c),

$$\begin{aligned} &\sum_{l=1}^{\infty}\sum_{m=1}^{\infty} \ell_{l,m} + \sum_{l=1}^{\infty} \ell_{l,0} + \sum_{m=1}^{\infty} \ell_{0,m} \\ &< \sum_{l=1}^{\infty} V_v(Z_{l+1,m}) + V_h\left(Z_{1,0}\right) + \sum_{m=1}^{\infty} V_h(Z_{0,m}) + V_v\left(Z_{1,-1}\right) \\ &< V(Z_{1,0}) + \sum_{l=1}^{S_1} V_v(Z_{l+1,-1}) \\ &\quad + \sum_{m=1}^{S_2} V_h(Z_{0,m}) \le sV(Z_{1,0}) \le s\theta_1 \end{aligned} \tag{45}$$

Then

$$\begin{aligned} &J_\infty(T_t, T_k) \\ &\quad \le \begin{bmatrix} Z^T(T_t, T_k)Q_1 Z(T_t, T_k) + Z^T(T_t+1, T_k-1)Q_2 \\ Z(T_t+1, T_k-1) + U^T(T_t, T_k)RU(T_t, T_k) + s\theta_1 \end{bmatrix} \end{aligned} \tag{46}$$

The optimization problem in the time T_t of batch T_k can be solved by the following formula:

$$\min_{U_{l,m}, l,m\ge 0} \varphi \tag{47}$$

where $J_\infty(T_t, T_k) \le \varphi$.

The optimization problem can be solved by transforming it into a linear matrix inequality

$$\begin{aligned} &[Z^T(T_t, T_k)Q_1 Z(T_t, T_k) + Z^T(T_t+1, T_k-1) \\ &\quad \times Q_2 Z(T_t+1, T_k-1) \\ &\quad + U^T(T_t, T_k)RU(T_t, T_k) + s\theta_1] \le \varphi \end{aligned} \tag{48}$$

(48) is equivalent to

$$\begin{bmatrix} -\varphi + \Xi & Z^T(T_t, T_k) & Z^T(T_t+1, T_k-1) & U^T(T_t, T_k) \\ * & -Q_1^{-1} & 0 & 0 \\ * & * & -Q_2^{-1} & 0 \\ * & * & * & -R^{-1} \end{bmatrix} \le 0 \tag{49}$$

where $\Xi = s\theta_1$,

$$\min_{U_{l,m}, l,m\ge 0} \varphi \tag{50}$$

The constraints are (25) and (49).

For the input constraint of equation (26), combined with Lemma 1 and (24), the following exists:

$$\begin{aligned} &\left\| U_{l,m} \right\|^2 \\ &\quad = \begin{bmatrix} Z_{l,m} \\ Z_{l+1,m-1} \end{bmatrix}^T \begin{bmatrix} Y_1 & Y_2 \end{bmatrix}^T \begin{bmatrix} Y_1 & Y_2 \end{bmatrix} \begin{bmatrix} Z_{l,m} \\ Z_{l+1,m-1} \end{bmatrix} \\ &\quad \le \begin{bmatrix} Z_{l,m} \\ Z_{l+1,m-1} \end{bmatrix}^T \begin{bmatrix} r_m^2 L_1^{-1} & 0 \\ 0 & r_m^2 L_2^{-1} \end{bmatrix} \begin{bmatrix} Z_{l,m} \\ Z_{l+1,m-1} \end{bmatrix} \\ &\quad \le Z_{l,m}^T r_m^2 L^{-1} Z_{l,m} \le r_m^2 \end{aligned} \tag{51}$$

For the output constraint of equation (27), combined with Lemma 1, the following exists:

$$\begin{aligned} \left\| y_{l,m}^e \right\|^2 &= \left\| \bar{G} Z_{l,m} \right\|^2 = Z_{l,m}^T \bar{G}^T \bar{G} Z_{l,m} \\ &\le Z_{l,m}^T y_m^2 L^{-1} Z_{l,m} \le y_m^2 \end{aligned} \tag{52}$$

Theorem 2: Assume $\tilde{\upsilon}(T_t, T_k) \ne 0$ holds. For given positive definite matrices $Q_1, Q_2 \in \Re^{(n_x+l_y)\times(n_x+l_y)}$, and $R \in \Re^{m_u \times m_u}$, and positive numbers $\alpha > 1, \beta > 1, \gamma > 0$, if the 2D-FM system in (11) is solvable if there exist positive definite symmetric matrices $X > 0, Y > 0, \bar{P} > 0, S_1 > 0, S_2 > 0$, appropriate dimension matrices $\widehat{A}_{ci}, \widehat{B}_{ci}, \widehat{C}_{ci}, \widehat{D}_{ci} (i = 1, 2)$

and scalars $\varepsilon > 0$, $\theta_1 > 0$, and $\lambda > 0$ such that the following LMIs hold:

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{41} \\ * & \Theta_{14} & 0 \\ * & * & \Theta_{42} \end{bmatrix} < 0 \tag{53}$$

$$\alpha S_1 + \beta S_2 < \Pi_{11} \tag{54}$$

$$\begin{bmatrix} -\Pi_{1,1} & \Pi_{12} & t_1\Pi_{13} & 0 & \Pi_{14} & \Pi_{15} \\ * & -(1-\lambda)S_1 & 0 & \Pi_{24} & 0 & 0 \\ * & * & -(1-\lambda)S_2 & \Pi_{34} & 0 & 0 \\ * & * & * & -\varepsilon I & 0 & 0 \\ * & * & * & * & -\varepsilon^{-1} & 0 \\ * & * & * & * & * & -\frac{\lambda}{\gamma^2}I \end{bmatrix} < 0 \tag{55}$$

$$\begin{bmatrix} -1 & Z_{0,0} \\ * & -\bar{P}^{-1} \end{bmatrix} \leq 0 \tag{56}$$

$$\begin{bmatrix} -r_m^2\Pi_{21} & 0 & \prod_{23} \\ * & -r_m^2\Pi_{31} & \prod_{33} \\ * & * & -I \end{bmatrix} < 0 \tag{57}$$

$$\begin{bmatrix} -y_m^2\bar{P}^{-1} & \bar{G} \\ * & -I \end{bmatrix} \leq 0 \tag{58}$$

where

$$\Theta_{41} = \begin{bmatrix} 0 & \Pi_{14} & \Pi_{15} \\ \Pi_{24} & 0 & 0 \\ \Pi_{34} & 0 & 0 \end{bmatrix},$$
$$\Theta_{42} = \begin{bmatrix} -\varepsilon I & 0 & 0 \\ * & -\varepsilon^{-1}I & 0 \\ * & * & -\theta_1^{-1}\gamma^2 I \end{bmatrix},$$
$$\prod_{15} = \begin{bmatrix} XD_1 \\ D_1 \end{bmatrix}.$$

If $X, Y, \widehat{A}_{ci}, \widehat{B}_{ci}, \widehat{C}_{ci}, \widehat{D}_{ci} (i = 1, 2)$ are the feasible solutions of matrix inequalities (53)-(58), then the parameters of controller (10) with output feedback can be designed as (28).

If M, N are full rank matrices that satisfy the condition $XY + MN^T = I$, then they can be obtained by a singular value decomposition of the matrix $I - XY$.

Proof: When $\tilde{\upsilon}(T_t, T_k) \neq 0$, Similar to the Theorem 1:

$$\min_{U_{l,m}, l,m\geq 0} \max_{[A,B,C]\in\Omega} \bar{J}_\infty(T_t, T_k)$$
$$\bar{J}_\infty(T_t, T_k) = \sum_{l=0}^{\infty}\sum_{m=0}^{\infty} \bar{\ell}_{l,m} = \sum_{l=0}^{N-1}\sum_{m=0}^{N-1} \bar{\ell}_{l,m} + V_m(Z_{N,N}) \tag{59}$$

where

$$\bar{\ell}_{l,m} = Z_{l,m}^T Q_1 Z_{l,m} + Z_{l+1,m-1}^T Q_2 Z_{l+1,m-1} + U_{l,m}^T R U_{l,m} - (\gamma)^2 \tilde{\upsilon}_{l,m}^T \tilde{\upsilon}_{l,m}$$

The following variables are defined as follows:

$$\bar{\phi}_{l,m} = \begin{bmatrix} Z_{l,m} \\ Z_{l+1,m-1} \\ \tilde{\upsilon}_{l,m} \end{bmatrix} \tag{60}$$

The sufficient condition for the robust stability of the closed-loop system in (11) is the existence of positive definite symmetric matrices P, P_1, P_2 that make (61) true.

$$\begin{aligned} & V_h(Z_{l+1,m}) - V_h(Z_{l,m}) + V_v(Z_{l+1,m}) - V_v(Z_{l+1,m-1}) \\ &= Z_{l+1,m}^T (P_1 + P_2) Z_{l+1,m} \\ &\quad - Z_{l,m}^T P_1 Z_{l,m} - Z_{T_l+1,K_m-1}^T P_2 Z_{l+1,m-1} \\ &\leq Z_{l+1,m}^T P Z_{l+1,m} - Z_{l,m}^T P_1 Z_{l,m} - Z_{l+1,m-1}^T P_2 Z_{l+1,m-1} \\ &\leq -\Big[Z_{l,m}^T Q_1 Z_{l,m} + Z_{l+1,m-1}^T Q_2 Z_{l+1,m-1} \\ &\quad + U_{l,m}^T R U_{l,m} - (\gamma)^2 \tilde{\upsilon}_{l,m}^T \tilde{\upsilon}_{l,m}\Big] \end{aligned} \tag{61}$$

(61) can be translated into

$$\bar{\phi}_{l,m}^T \bar{\psi} \bar{\phi}_{l,m} < 0 \Leftrightarrow \bar{\psi} < 0 \tag{62}$$

where $U_{l,m} = Y_1 Z_{l,m} + Y_2 Z_{l+1,m-1}$,

$$\begin{aligned} \bar{\psi} &= \zeta_1^T \theta_1 L^{-1} \zeta_1 - \zeta_2^T \theta_1 L_1^{-1} \zeta_2^T - \zeta_3^T \theta_1 L_2^{-1} \zeta_3^T + \zeta_2^T Q_1 \zeta_2^T \\ &\quad + \zeta_3^T Q_2 \zeta_3^T - \zeta_4^T R \zeta_4^T + \zeta_5^T \gamma^2 \zeta_5^T, \\ \zeta_1 &= \begin{bmatrix} \bar{A}_1 + \Delta\bar{A}_1 + \bar{B}_1 Y_1 + \Delta\bar{B}_1 Y_1 & \bar{A}_2 + \bar{B}_1 Y_2 + \Delta\bar{B}_1 Y_2 & \bar{D} \end{bmatrix}, \end{aligned}$$

and

$$\zeta_2 = \begin{bmatrix} I & 0 & 0 \end{bmatrix}, \quad \zeta_3 = \begin{bmatrix} 0 & I & 0 \end{bmatrix}, \; \zeta_4 = \begin{bmatrix} Y_1 & Y_2 & 0 \end{bmatrix},$$
$$\zeta_5 = \begin{bmatrix} 0 & 0 & I \end{bmatrix}.$$

Applying **Lemma 1** and **Lemma 2** to Equation (62), we can obtain

$$\begin{bmatrix} \Theta_{21} & \Theta_{22} & \Theta_{51} \\ * & \Theta_{14} & 0 \\ * & * & \Theta_{42} \end{bmatrix} < 0 \tag{63}$$

where

$$\Theta_{51} = \begin{bmatrix} 0 & L^{-1}\bar{E} & L^{-1}\bar{D} \\ \bar{F}_1^T + Y_1^T \bar{F}_2^T & 0 & 0 \\ Y_2^T \bar{F}_2^T & 0 & 0 \end{bmatrix}$$

Let $L^{-1} = \bar{P}, \bar{P} = \begin{bmatrix} Y & N \\ N^T & W \end{bmatrix}, \bar{P}^{-1} = \begin{bmatrix} X & M \\ M^T & K \end{bmatrix}, \Lambda_1 = \begin{bmatrix} X & I \\ M^T & 0 \end{bmatrix}$ and multiply (63) on the left by $\tilde{\Lambda}_1^T = diag\,(\Lambda_1^T, \Lambda_1^T, \Lambda_1^T, I, I, I, I, I, I)$ and on the right by $\tilde{\Lambda}_1$; then, we can obtain

$$\begin{bmatrix} \Theta_{31} & \Theta_{32} & \Theta_{61} \\ * & \Theta_{14} & 0 \\ * & * & \Theta_{42} \end{bmatrix} < 0 \tag{64}$$

where

$$\Theta_{61} = \begin{bmatrix} 0 & \Lambda_1^T\bar{E} & \Lambda_1^T\bar{D} \\ \Lambda_1^T(\bar{F}_1^T + Y_1^T\bar{F}_2^T) & 0 & 0 \\ \Lambda_1^T Y_2^T \bar{F}_2^T & 0 & 0 \end{bmatrix},$$

$$\Lambda_1^T\bar{D} = \begin{bmatrix} XD_1 \\ D_1 \end{bmatrix}$$

Notably, (64) is equivalent to (54). According to (24), $\alpha L_1^{-1} + \beta L_2^{-1} < L^{-1}$, so $\alpha P_1 + \beta P_2 < P$. Therefore, (54) and (55) are sufficient conditions for Equation (61). Similar to Theorem 1, the closed-loop system in (11) is robustly stable.

From the superposition of inequality (61), we can obtain (65), as shown at the bottom of the page. Then,

$$-sV(Z(T_t, T_k)) \leq -\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\bar{\ell}_{l,m} \tag{66}$$

Referring to Theorem 1, the following can be obtained from (56):

$$V(Z(T_t, T_k)) = Z^T(T_t, T_k)\psi_l Z(T_t, T_k) \leq \theta_1 \tag{67}$$

where θ_1 is the upper bound of $V(Z(T_t, T_k))$. Then,

$$\bar{J}_\infty(T_t, T_k) = \sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\bar{\ell}_{l,m} \leq sV(Z(T_t, T_k)) \leq s\theta_1 \leq \theta$$

Applying **Lemma 1** and **Lemma 2** to Equation (55), we can obtain

$$\zeta_1^T L^{-1}\zeta_1 - \zeta_2^T(1-\lambda)L_1^{-1}\zeta_2 - \zeta_3^T(1-\lambda)L_2^{-1}\zeta_3 - \zeta_5^T\frac{\lambda}{\gamma^2}\zeta_5 < 0$$

When (63) is multiplied on the left by $\bar{\phi}_{l,m}^T$ and on the right by $\bar{\phi}_{l,m}$, we can obtain:

$$\frac{1}{\theta_1}\left\{Z_{l+1,m}^T P Z_{l+1,m} - (1-\lambda)\left[V_h(Z_{l,m}) + V_v(Z_{l+1,m-1})\right]\right\} - \lambda\frac{1}{\gamma^2}\tilde{\upsilon}^T\tilde{\upsilon} < 0 \tag{68}$$

The following can be obtained from the above equation:

$$\begin{aligned}\frac{1}{\theta_1}V(Z_{l+1,m}) &\leq (1-\lambda)\frac{1}{\theta_1}\begin{bmatrix} V_h(Z_{l,m}) \\ +V_v(Z_{l+1,m-1}) \end{bmatrix} + \lambda\frac{1}{\gamma^2}\tilde{\upsilon}^T\tilde{\upsilon} \\ &\leq (1-\lambda)\frac{1}{\theta_1}V(Z_{l,m}) + \lambda\frac{1}{\gamma^2}\tilde{\upsilon}^T\tilde{\upsilon} \\ &\leq (1-\lambda) + \lambda = 1\end{aligned} \tag{69}$$

Thus, when perturbations are involved, the traditional asymptotic stability cannot converge to the origin. In contrast, there exists a robust positive definite invariant set:

$$\Psi := \{Z | V(Z) \leq \theta_1\} \tag{70}$$

It is satisfied that the system states converge to this set.

For $\forall Z \in \Psi$, there exists

$$\alpha_3 V(|Z|) \leq V(Z) \leq \alpha_3 V(|Z|) \tag{71}$$

where $\alpha_3, \alpha_4 \in K_\infty$ and

$$\begin{aligned}&V(Z)^+ - V(Z) \\ &\quad \leq -\begin{bmatrix} Z_{l,m}^T Q_1 Z_{l,m} + Z_{l+1,m-1}^T Q_2 Z_{l+1,m-1} \\ + U_{l,m}^T R U_{l,m} - (\gamma)^2\tilde{\upsilon}_{l,m}^T\tilde{\upsilon}_{l,m} \end{bmatrix}\end{aligned} \tag{72}$$

$$(\lambda_{\min})^* |Z|^2 \leq V^*(Z) \leq (\lambda_{\max})^* |Z|^2 \tag{73}$$

When $Z_{0,0} = Z(T_t, T_k)$ and $U_{0,0} = U(T_t, T_k)$, equation (59) is equivalent to

$$\begin{aligned}&\bar{J}_\infty(T_t, T_k) \\ &= \sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\bar{\ell}_{l,m} = \bar{\ell}_{0,0} + \sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\bar{\ell}_{l,m} + \sum_{l=1}^{\infty}\bar{\ell}_{l,0} + \sum_{m=1}^{\infty}\bar{\ell}_{0,m} \\ &= Z^T(T_t, T_k)Q_1 Z(T_t, T_k) + Z^T(T_t+1, T_k-1) \\ &\quad \times Q_2 Z(T_t+1, T_k-1) \\ &\quad + U^T(T_t, T_k)RU(T_t, T_k) - (\gamma)^2\tilde{\upsilon}^T(T_t, T_k)\tilde{\upsilon}(T_t, T_k) \\ &\quad + \sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\bar{\ell}_{T_l,K_m} + \sum_{l=1}^{\infty}\bar{\ell}_{l,0} + \sum_{m=1}^{\infty}\bar{\ell}_{0,m}\end{aligned} \tag{74}$$

$$\begin{aligned}&\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\{V_h(Z_{l+1,m}) - V_h(Z_{l,m}) + V_v(Z_{l+1,m}) - V_v(Z_{l+1,m-1})\} \\ &= \sum_{m=0}^{\infty}\begin{pmatrix} V_h(Z_{1,m}) - V_h(Z_{0,m}) + V_v(Z_{1,m}) - V_v(Z_{1,m-1}) + V_h(Z_{2,m}) \\ - V_h(Z_{1,m}) + V_v(Z_{2,m}) - V_v(Z_{2,m-1}) + \ldots + V_h(Z_{\infty+1,m}) \\ - V_h(Z_{\infty,m}) + V_v(Z_{\infty+1,m}) - V_v(Z_{\infty+1,m-1}) \end{pmatrix} \\ &= \sum_{m=0}^{\infty}(V_h(Z_{\infty+1,\infty}) - Z_{l+1,-1}) + \sum_{l=0}^{\infty}(V_v(Z_{l+1,\infty}) - V_v(Z_{0,m}) \\ &= -\sum_{m=0}^{s_2}V_h(Z_{0,m}) - \sum_{l=0}^{s_1}V_v(Z_{l+1,-1}) \\ &\geq -s_2 V_h(Z_{0,0}) - s_1 V_v(Z_{1,-1}) \\ &\geq -sV(Z_{0,0}) = -sV(Z(T_t, T_k))\end{aligned} \tag{65}$$

Similar to **theorem 1**, we can obtain:

$$\bar{J}_\infty(T_t,T_k) < \begin{bmatrix} Z^T(T_t,T_k)Q_1Z(T_t,T_k)+Z^T(T_t+1,T_k-1)Q_2 \\ Z(T_t+1,T_k-1)+U^T(T_t,T_k)RU(T_t,T_k) \\ -(\gamma)^2\tilde{\upsilon}^T(T_t,T_k)\tilde{\upsilon}(T_t,T_k)+s\theta_1 \end{bmatrix} \tag{75}$$

The optimization problem at time T_t of batch T_k can be solved by the following formula:

$$\min_{U_{l,m},l,m\geq 0} \varphi \tag{76}$$

where $\bar{J}_\infty(T_t,T_k) \leq \varphi$.

The optimization problem can be solved by transforming it into a linear matrix inequality:

$$\begin{aligned}[&Z^T(T_t,T_k)Q_1Z(T_t,T_k)+Z^T(T_t+1,T_k-1)\\ &\quad\times Q_2Z(T_t+1,T_k-1)\\ &\quad+U^T(T_t,T_k)RU(T_t,T_k)+s\theta_1]\leq\varphi\end{aligned} \tag{77}$$

(77) is equivalent to

$$\begin{bmatrix} -\varphi+\Xi & Z^T(T_t,T_k) & Z^T(T_t+1,T_k-1) & U^T(T_t,T_k) \\ * & -Q_1^{-1} & 0 & 0 \\ * & * & -Q_2^{-1} & 0 \\ * & * & * & -R^{-1} \end{bmatrix} \leq 0 \tag{78}$$

where $\Xi = s\theta_1$ and

$$\min_{U_{l,m},l,m\geq 0} \varphi \tag{79}$$

The constraints are (56) and (78).

C. CONTROLLER PARAMETER OPTIMIZATIONBASED ON THE PSO ALGORITHM

The basic principle of the PSO algorithm can be described as: a group is composed of multiple particles in high-dimensional space flight at a certain speed, and each particle in the search considers the best particle and the best group in the search history of the other particles on the basis of the speed and position updates.

$$\begin{aligned} v^{i,k+1} &= w\,v^{i,k}+c_1\xi(p^{i,k}-x^{i,k})+c_2\eta(p^{g,k}-x^{i,k}) \\ x^{i,k+1} &= x^{i,k}+v^{i,k+1} \end{aligned} \tag{80}$$

where $v^{i,k}$ is the velocity vector of the ith particle in the kth iteration, $v^{i,0}$ is initial iteration speed of the ith particle, $x^{i,k}$ is the position of the ith particle in the kth iteration, w is the inertia weight, c_1 and c_2 are the learning factors or acceleration coefficients, ξ and η are uniformly distributed random numbers between [0, 1], $p^{i,k}$ is the optimal location of the ith particle in the kth iteration, and $p^{i,k}$ is the global optimal position of all particles in the kth iteration.

Let x be the vector composed of all the elements in controller $A_{c1}, B_{c1}, C_{c1}, D_{c1}, A_{c2}, B_{c2}, C_{c2}, D_{c2}$, andit is the decision variable in the optimization problem, where $x \in R^{40}$.

Our goal is to obtain $\min_{x_j^*-|x_j^*|\leq x_j\leq x_j^*+|x_j^*|} J(x), j=1,2,\ldots,40$, where x_j is the jth component of x, x_j^* is the jth component of x^*, and x^* is the vector composed of all elements in the controller parameters $A_{c1}^*, B_{c1}^*, C_{c1}^*, D_{c1}^*, A_{c2}^*, B_{c2}^*, C_{c2}^*, D_{c2}^*$ that are obtained by the traditional method. x^* is the initial iteration position in (80). The optimal position mentioned above refers to the position when the performance index is minimized, and the same constraint $x_j^*-|x_j^*|\leq x_j^{i,k}\leq x_j^*+|x_j^*|$ is applied to each particle position in (80), where $x_j^{i,k}$ is the jth component of the ith particle in the kth iteration.

In simple terms, the PSO algorithm is used to find a better solution near the controller obtained by the traditional method to make the function value of the performance index J as small as possible.

In the PSO algorithm with the above constraints, the initial total number of particles is set as 100, the inertia weight is set as 0.5, the learning factors c_1 and c_2 are set as 1.5, and $k \leq 300$ (i.e., the number of iterations is 300).

IV. SIMULATION

The injection molding process is a typical multistage production process in the chemical industry. Each product production mainly includes five steps, that is, the Clamping period → Injection period → Packing period → Cooling period → Mold opening period. The injection speed and other parameters of the injection period need to be controlled with high precision to achieve an increase in the final product yield. Here, we take the injection period as an example to consider its control effect.

The control speed parameters are taken as the research object. First, the response to the injection speed (output) of the proportional valve (input) is determined as an autoregressive model, and the mathematical model in the frequency domain of the injection section of the injection molding process is established as follows:

$$\frac{IV}{VO}=\frac{1.239z^{-1}-0.9282z^{-2}}{1-(1.607+0.1\delta)z^{-1}+(0.6089+0.1\delta)z^{-2}} \tag{81}$$

where IV is the injection speed and VO is the valve opening.

The state space model of model (81) is expressed as

$$\begin{cases} x(T_t+1,T_k)=\left(\begin{bmatrix} 1.607 & 1 \\ -0.6089 & 0 \end{bmatrix} + \begin{bmatrix} 0.1\delta(T_t,T_k) & 0 \\ 0.1\delta(T_t,T_k) & 0 \end{bmatrix}\right)x(T_t,T_k) \\ \qquad + \left(\begin{bmatrix} 1.239 \\ -0.9282 \end{bmatrix} + \begin{bmatrix} 0.1\delta(T_t,T_k) \\ 0.1\delta(T_t,T_k) \end{bmatrix}\right) \\ \qquad u(T_t,T_k)+\upsilon(T_t,T_k) \\ y(T_t,T_k)=\begin{bmatrix} 1 & 0 \end{bmatrix}x(T_t,T_k) \end{cases} \tag{82}$$

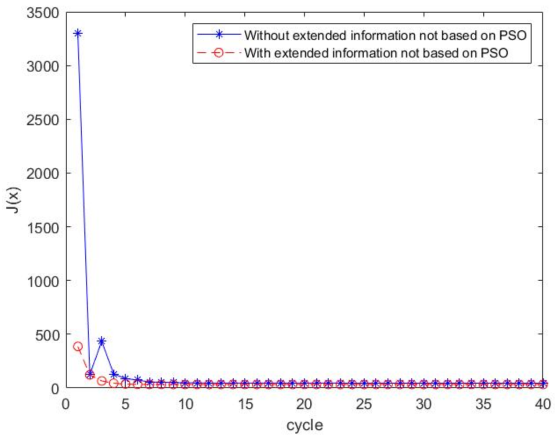

FIGURE 2. Tracking performance comparison with/without extended information.

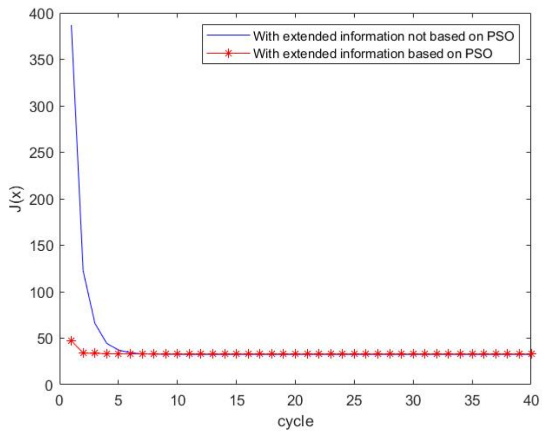

FIGURE 3. Tracking performance comparison with/without PSO.

where the variable $\delta(T_t, T_k)$ varies randomly in the range [0, 1].

The set-point under every batch is set as:

$$\begin{cases} y_r(T_t) = 15 & 0 \le T_t \le 141 \\ y_r(T_t) = 30 & 141 < T_t \le 282 \end{cases} \tag{83}$$

In order to verify the effectiveness of the control method proposed in this paper, under repetitive disturbance and non-repetitive disturbance, the control effects without extended information, with extended information, and with extended information combined with PSO algorithm are compared.

A. CASE 1: ROBUSTNESS TO REPETITIVE DISTURBANCE

As seen from the above simulation experiments, the state space model of the injection period is (82), where $\upsilon(T_t, T_k)$ is the disturbance of the injection period and satisfies $\upsilon = \cos(T_t) \times \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}^T$. In this case, the external disturbance is only determined by time.

As shown in Fig.2, the tracking performance of the system with or without extended information under repetitive disturbance is compared. It can be seen from Fig.2 that the tracking performance of the system with extended information is significantly better than that of the system without extended information, and it can converge to the stable state faster.

In Fig.3, the tracking performance of the system optimized by PSO is compared with that of the system not optimized by PSO when the system has extended information under repetitive disturbance. It can be seen from Fig.3 that the tracking performance of the system optimized by PSO is also better than that of the system not optimized by PSO, and the same is true when converging to a steady state.

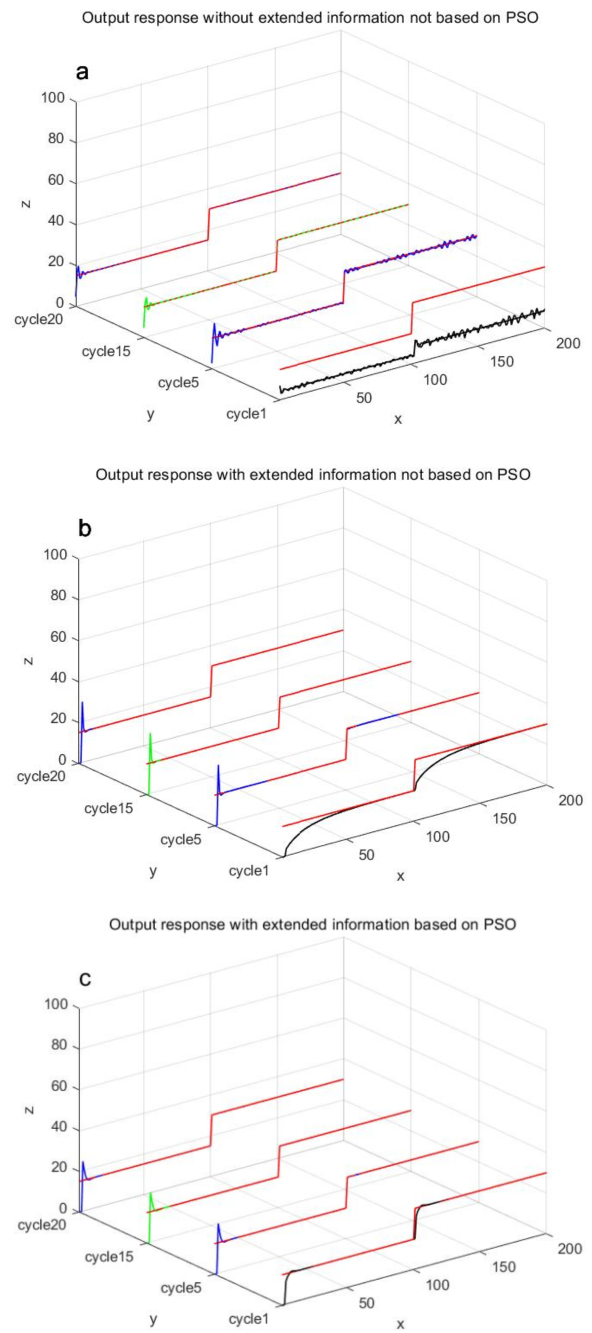

FIGURE 4. Output response of systems under repetitive disturbances.

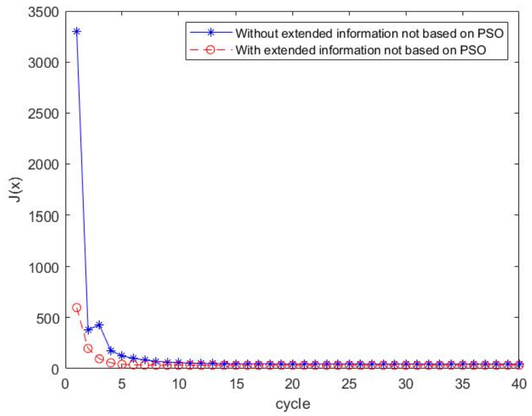

FIGURE 5. Tracking performance comparison with and without extended information.

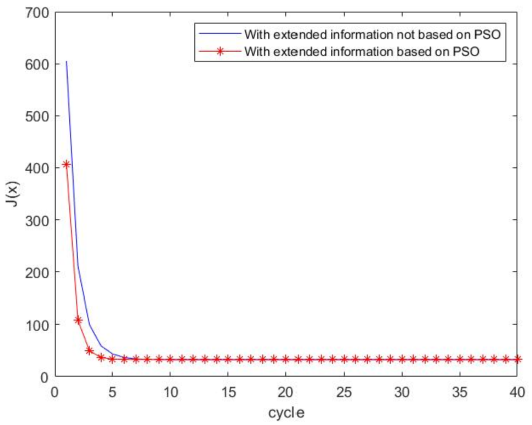

FIGURE 6. Tracking performance comparison with/without PSO.

Fig.4 shows output response of systems under repetitive disturbances. Figs.4(a, b and c) show the output response without extended information, with extended information, and with extended information combined with PSO algorithm in the 1st, 5th, 10th and 20th batches, respectively. Compared Fig.4(a) with Fig.4(b), Fig.4(b) can almost track the given output trajectory in the first batch, and the fluctuation time in subsequent batches is shorter. Compared Fig.4(b) with Fig.4(c), it is obvious that the output tracking using the optimized algorithm is faster, as shown in Fig.4(c), the first batch of output trajectories. Not only that, the fluctuation in the initial time of subsequent batches is also small. Under repetitive disturbance, the tracking performance with extended information is better than that without extended information, and the tracking performance of PSO with extended information is better than that only has extended information.

B. CASE 2: ROBUSTNESS TO NON-REPETITIVE DISTURBANCE

In this case, robustness against non-repetitive disturbances is shown. The real-time dynamics of the system are given in (82), where the nonrepeated disturbance $\upsilon(T_t, T_k)$ satisfies

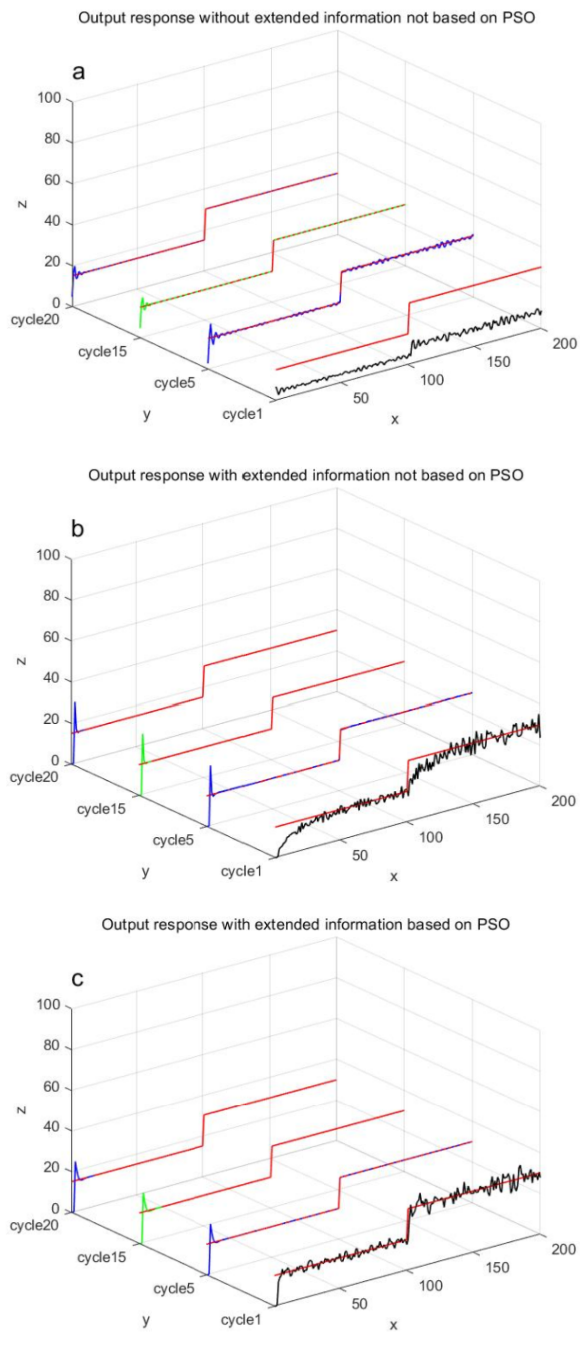

FIGURE 7. Output response of systems under nonrepetitive disturbances.

$\upsilon = 0.3 \times \left[\Delta_1 \ \Delta_2 \right]^T$ and $\Delta_i \, (i = 1, 2, 3)$ varies randomly in the range [0, 1] and $\upsilon(T_t, T_k)$ depends on both T_t and T_k.

As shown in Fig.5, the tracking performance of the system with or without extended information under non-repetitive disturbances is compared. It can be seen from Fig.5 that the tracking performance of the system with extended information is significantly better than that of the system without extended information, and it can converge to the stable state faster in about four batches, and is close to zero error tracking.

Similar to the results given in Fig.5, under non-repetitive disturbances, the control performance is obviously better with the optimization algorithm, as shown in Fig.5.

Similar to Fig.4. Fig.7 still shows output response of the system, but in the case of non-repetitive disturbances. As shown in Fig.4, the control effect is the best by applying PSO with extended information. Of course, affected by non-repetitive disturbances, the output curve obviously fluctuates near the given trajectory.

V. CONCLUSION

For a single-phase batch process with uncertainties and unknown disturbances, a PSO-based two-dimensional output feedback robust constrained MPC method is proposed by introducing a new model formed by extended information, combining ILC and MPC. We optimize and adjust performance index parameters through PSO algorithm such that the designed output feedback predictive controller achieves better control effects. Finally, take the injection molding process as an example, compared with the existing results, this fact is also proved.

REFERENCES

[1] S. Erdogan, G. Özkan, and M. Alpbaz, "Self-tuning control of batch polymerization reactor," *J. Chem. Eng. Jpn.*, vol. 31, no. 4, pp. 499–505, 1998.

[2] D. Bonvin, "Control and optimization of batch processes," *IEEE Control Syst. Mag.*, vol. 26, no. 6, pp. 34–45, Dec. 2006.

[3] E. Korovessi and A. Linninger, *Batch Processes*. Boca Raton, FL, USA: CRC Press, 2006.

[4] Y. Yang and F. Gao, "Adaptive control of the filling velocity of thermoplastics injection molding," *Control Eng. Pract.*, vol. 8, no. 11, pp. 1285–1296, Nov. 2000.

[5] R. Zhang, S. Wu, Z. Cao, J. Lu, and F. Gao, "A systematic min–max optimization design of constrained model predictive tracking control for industrial processes against uncertainty," *IEEE Trans. Control Syst. Technol.*, vol. 26, no. 6, pp. 2157–2164, Nov. 2018.

[6] R. Zhang, R. Lu, A. Xue, and F. Gao, "New minmax linear quadratic fault-tolerant tracking control for batch processes," *IEEE Trans. Autom. Control*, vol. 61, no. 10, pp. 3045–3051, Oct. 2016.

[7] J. E. Kurek and M. B. Zaremba, "Iterative learning control synthesis based on 2-D system theory," *IEEE Trans. Autom. Control*, vol. 38, no. 1, pp. 121–125, Jan. 1993.

[8] J. H. Lee and K. S. Lee, "Iterative learning control applied to batch processes: An overview," *Control Eng. Pract.*, vol. 15, no. 10, pp. 1306–1318, 2007.

[9] Y. Wang, F. Gao, and F. J. Doyle, "Survey on iterative learning control, repetitive control, and run-to-run control," *J. Process Control*, vol. 19, no. 10, pp. 1589–1600, 2009.

[10] J. Lu, Z. Cao, and F. Gao, "Batch process control-overview and outlook," *Acta Autom. Sinica*, vol. 43, no. 7, pp. 933–943, 2017.

[11] L. Wang, R. Zhang, and F. Gao, *Iterative Learning Stabilization and Fault-Tolerant Control for Batch Processes*. Singapore: Springer, 2020.

[12] J. Lu, Z. Cao, R. Zhang, and F. Gao, "Nonlinear monotonically convergent iterative learning control for batch processes," *IEEE Trans. Ind. Electron.*, vol. 65, no. 7, pp. 5826–5836, Jul. 2018.

[13] J. Shi, F. Gao, and T.-J. Wu, "Robust design of integrated feedback and iterative learning control of a batch process based on a 2D Roesser system," *J. Process Control*, vol. 15, no. 8, pp. 907–924, Dec. 2005.

[14] T. Liu and F. Gao, "Robust two-dimensional iterative learning control for batch processes with state delay and time-varying uncertainties," *Chem. Eng. Sci.*, vol. 65, no. 23, pp. 6134–6144, Dec. 2010.

[15] T. Liu and Y. Wang, "A synthetic approach for robust constrained iterative learning control of piecewise affine batch processes," *Automatica*, vol. 48, no. 11, pp. 2762–2775, Nov. 2012.

[16] Y. Wang, J. Shi, D. Zhou, and F. Gao, "Iterative learning fault-tolerant control for batch processes," *Ind. Eng. Chem. Res.*, vol. 45, no. 26, pp. 9050–9060, 2006.

[17] D. Li, S. He, Y. Xi, T. Liu, F. Gao, Y. Wang, and J. Lu, "Synthesis of ILC–MPC controller with data-driven approach for constrained batch processes," *IEEE Trans. Ind. Electron.*, vol. 67, no. 4, pp. 3116–3125, Apr. 2020.

[18] L. Zhou, L. Jia, Y.-L. Wang, D. Peng, and W. Tan, "An integrated robust iterative learning control strategy for batch processes based on 2D system," *J. Process Control*, vol. 85, pp. 136–148, Jan. 2020.

[19] S.-K. Oh and J. M. Lee, "Iterative learning model predictive control for constrained multivariable control of batch processes," *Comput. Chem. Eng.*, vol. 93, no. 4, pp. 284–292, Jul. 2016.

[20] S. Lucia, T. Finkler, and S. Engell, "Multi-stage nonlinear model predictive control applied to a semi-batch polymerization reactor under uncertainty," *J. Process Control*, vol. 23, pp. 1306–1319, Oct. 2013.

[21] S.-K. Oh and J. M. Lee, "Iterative learning control integrated with model predictive control for real-time disturbance rejection of batch processes," *J. Chem. Eng. Jpn.*, vol. 50, no. 6, pp. 415–421, Jun. 2017.

[22] J. Lu, Z. Cao, and F. Gao, "Multi-point iterative learning model predictive control," *IEEE Trans. Ind. Electron.*, vol. 66, no. 8, pp. 6230–6240, Aug. 2019.

[23] J. Shi, F. Gao, and T.-J. Wu, "Single-cycle and multi-cycle generalized 2D model predictive iterative learning control (2D-GPILC) schemes for batch processes," *J. Process Control*, vol. 17, no. 9, pp. 715–727, Oct. 2007.

[24] L. Wang, J. Yu, P. Li, H. Li, and R. Zhang, "A 2D-FM model-based robust iterative learning model predictive control for batch processes," *ISA Trans.*, vol. 110, pp. 271–282, Apr. 2021.

[25] J. Lu, Z. Cao, Z. Wang, and F. Gao, "A two-stage design of two-dimensional model predictive iterative learning control for nonrepetitive disturbance attenuation," *Ind. Eng. Chem. Res.*, vol. 54, pp. 5683–5689, May 2015.

[26] D. Li, Y. Xi, J. Lu, and F. Gao, "Synthesis of real-time-feedback-based 2D iterative learning control–model predictive control for constrained batch processes with unknown input nonlinearity," *Ind. Eng. Chem. Res.*, vol. 55, no. 51, pp. 13074–13084, Dec. 2016.

[27] Y. Wang, D. Zhou, and F. Gao, "Iterative learning model predictive control for multi-phase batch processes," *J. Process Control*, vol. 18, pp. 543–557, Jul. 2008.

[28] L. M. Wang, J. X. Yu, R. D. Zhang, P. Li, and F. R. Gao, "Iterative learning control for multiphase batch processes with asynchronous switching," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 51, no. 4, pp. 2536–2549, Apr. 2021.

[29] J. Hu and B. Ding, "Output feedback robust MPC of uncertain norm-bounded linear systems with disturbance," *Int. J. Control*, vol. 94, no. 9, pp. 2388–2395, Sep. 2021.

[30] B. Ding, J. Hu, X. Tang, and J. Wang, "A synthesis approach to output feedback MPC for LPV model with bounded disturbance," *IEEE Access*, vol. 8, pp. 228337–228348, 2020.

[31] J. Hu and B. Ding, "A periodic approach to dynamic output feedback MPC for quasi-LPV model," *IEEE Trans. Autom. Control*, vol. 66, no. 5, pp. 2257–2264, May 2021.

[32] B. Ding, "Constrained robust model predictive control via parameter-dependent dynamic output feedback," *Automatica*, vol. 46, no. 9, pp. 1517–1523, 2010.

[33] B. Ding, X. Ping, and H. Pan, "On dynamic output feedback robust MPC for constrained quasi-LPV systems," *Int. J. Control.*, vol. 86, no. 12, pp. 2215–2227, Jun. 2013.

[34] Y. Zhang and L. Jia, "Particle swarm optimization based optimization for batch processes," in *Proc. Chin. Autom. Congr. (CAC)*, Nov. 2019, pp. 4797–4802.

[35] R. Zhang and J. Tao, "A nonlinear fuzzy neural network modeling approach using improved genetic algorithm," *IEEE Trans. Ind. Electron.*, vol. 65, no. 7, pp. 5882–5892, Jul. 2018.

[36] Q. Wu, C. Cole, and T. McSweeney, "Applications of particle swarm optimization in the railway domain," *Int. J. Rail Transp.*, vol. 4, no. 3, pp. 167–190, Apr. 2016.

[37] L. Tang and X. Wang, "An improved particle swarm optimization algorithm for the hybrid flowshop scheduling to minimize total weighted completion time in process industry," *IEEE Trans. Control Syst. Technol.*, vol. 18, no. 6, pp. 1303–1314, Nov. 2010.

[38] M. R. AlRashidi and M. E. El-Hawary, "A survey of particle swarm optimization applications in electric power systems," *IEEE Trans. Evol. Comput.*, vol. 13, no. 4, pp. 913–918, Aug. 2009.

[39] N. K. Jain, U. Nangia, and A. Jain, "PSO for multiobjective economic load dispatch (MELD) for minimizing generation cost and transmission losses," *J. Inst. Eng. (India), Ser. B*, vol. 97, no. 2, pp. 185–191, Jun. 2016.
[40] S. Boyd, L. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequality in System and Control Theory*. Philadelphia, PA, USA: SIAM, 1994.
[41] K.-W. Yu and C.-H. Lien, "Stability criteria for uncertain neutral systems with interval time-varying delays," *Chaos, Solitons Fractals*, vol. 38, no. 3, pp. 650–657, Nov. 2008.

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