Dynamic Multi-Attribute Decision-Making Based on Interval-Valued Picture Fuzzy Geometric Heronian Mean Operators

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ABSTRACT For the dynamic multi-attribute decision-making problem, the decision information is usually given in the form of the interval-valued picture fuzzy number (IVPFN), and the attributes are also usually related to each other, a decision method based on the interval-valued picture fuzzy geometric weighted Heronian average mean (IVPFGWHM) operator is proposed. First, the algorithms of IVPFN are defined by combining the picture fuzzy number (PFN) with the algorithms of the interval-valued intuitionistic fuzzy number (IVIFN). Then, using the algorithms of IVPFN and geometric Heronian average mean operators, four Heronian mean operators for IVPFN are proposed: the interval-valued picture fuzzy geometric Heronian average mean (IVPFGHM) operator, the interval-valued picture fuzzy geometric weighted Heronian average mean (IVPFGWHM) operator, and the dynamic interval-valued picture fuzzy geometric weighted Heronian average mean (DIVPFGWHM) operator. Then some properties of these operators are studied. Furthermore, a multi-attribute decision-making process based on DIVPFGWHM is proposed. At the same time, with the aid of the best-worst method (BWM), we obtained the attribute weights. Finally, by analyzing the current situation of logistics industry and using the proposed method to select logistics companies, and by comparing with the other methods to illustrate the effectiveness and advantages of the developed method.

INDEX TERMS Interval-valued picture fuzzy number, interval-valued picture fuzzy geometric weight Heronian average mean operator, best-worst method, dynamic multi-attribute decision-making.

I. INTRODUCTION

Since Zadeh proposed fuzzy sets [1], it has been widely used to describe fuzzy and uncertain decision information. However, because the fuzzy set have only degree of membership, it cannot describe some complex decision-making problems. Then Atanassov [2] extended it to the Intuitionistic Fuzzy Set (IFS). IFS assigns to each element a membership degree and a non-membership degree, and it can describe and characterize the essence of fuzzy decision information in more detail. However, in the decision-making process, decision makers usually have more than just ''support'' and ''opposition'' attitudes. In addition to expressing ''consistency'' and ''inconsistency'', the traditional IFS does

not consider other possibilities such as refusal. In order to overcome this shortcoming, Cuong and Kreinovich [3] proposed the concept of picture fuzzy set (PFS). PFS is a generalization of IFS. Its advantage is to use three membership functions to describe the behavior of decision makers, including positive membership, neutral membership and negative membership. For example, in voting activities, voters can be divided into four groups: ''support'', ''abstain'', ''against'', ''refusal of the voting''. It conforms to the situation in real life, so it has attracted the attention of many researchers in the field. Wu et al [4] proposed the concept of SVN2TL set (SVN2TLS) and single valued neutrosophic 2-tuple linguistic element (SVN2TLE), Later, Wu *et al.* [5] combined hesitant fuzzy sets and Pythagorean fuzzy sets, proposed the concept of hesitant Pythagorean fuzzy sets, and based on these concepts, some multi-attribute

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The associate editor coordinating the review of this manuscript and approving it for publication was Francisco J. Garcia-Penalvo.

decision methods are given. Wei *et al.* [6]–[8] defined several procedures for calculating the similarity between PFSs. Wang [9] applied the VIKOR method to the PFS, Liu [10] proposed the concept of complex picture fuzzy sets (CPFS) which is a generalization of picture fuzzy sets. Wang *et al.* [11] put forward a multi-criteria group decision-making method to solve building energy efficiency retrofitting (BEER) project selection problem. Tian and Peng [12] defined the picture fuzzy score and accuracy function, and developed a corresponding comparative method between two picture fuzzy numbers. Ashraf *et al.* [13] introduced a series of picture fuzzy weighted geometric aggregation operators by using t-norm and t-conorm, and proposed the TOPSIS method to aggregate the picture fuzzy information. Lin *et al.* [14] proposed a novel picture fuzzy multi-criteria decision making (MCDM) model to solve the problem of car sharing stations. Jiang *et al.* combined the picture fuzzy numbers, extended TODIM, and cumulative prospect theory (PF-CPT-TODIM), and use it to evaluate food companies. Moreover, various decision-making methods based on PFS have been studied [15], [16].

However, in real life, it's difficult to describe the degree of membership and non-membership with crisp numbers, so Atanassov and Gargov [17] proposed the concept of interval valued intuitionistic fuzzy sets (IVIFS) to deal with the problem. IVIFS is an extension of intuitionistic fuzzy set. It can effectively describe and process the uncertainty of decision information. Therefore, it has attracted wide attention. Wan and Dong [18] defined the ordered weighted average operator and hybrid weighted average operator for IVIFNs based on the Karnik-Mendel algorithms and employed to solve multi-attribute group decision making problems with IVIFNs. Xu and Yager [19] developed a new similarity measure between intuitionistic fuzzy sets, and applied it for consensus analysis in group decision making based on intuitionistic fuzzy preference relations, and finally further extend it to the interval-valued intuitionistic fuzzy set theory. Many multi-attribute decision-making methods based on IVIF have appeared one after another. Xu [20], [21] studied the algorithm of IVIFS and put forward some IVIFWA aggregation operators, IVIFGA aggregation operators, IVIFOWA aggregation operators based on IVIFS, and gave the ranking method of score function, accuracy function and IVIFNs, Li [22] combined TOPSIS method and nonlinear programming theory. Garg and Kumar [23] gave a multi-attribute decision-making method for the interval-valued intuitionistic fuzzy set using the set pair analysis (SPA) theory. Lu and Wei [24] extended TODIM method to the MADM with the interval-valued intuitionistic fuzzy numbers. Zhao *et al.* [25] proposed the TODIM method based on the cumulative prospect theory to solve the MAGDM problem under IVIFS. Wu *et al.* [26], [27] use interval type 2 fuzzy sets (IT2FSs) to solve the portfolio allocation and the selection of green suppliers problem. However, there are currently few studies that combine the interval number with the PFN to deal with the situation

where the decision information is given in the form of the IVPFN.

In dealing with multi-attribute decision-making problems, the aggregation operator is a very effective method, Wang and Garg [28] define some Pythagorean fuzzy interaction aggregation operators with the aid of Archimedean t-conorm and t-norm (ATT), but the above aggregation operator only considers the independence of the attributes, however in actual situations, different attributes will have different degrees of connection. Such as complementarity, redundancy, preference relations, etc. The Heronian mean (HM) operator is an aggregation operator that deals with the interrelationships between attributes. In the past, some scholars' research on HM operator mainly focused on the theory and application of inequalities. Yu [29], [30] proposed geometric Heronian mean (GHM) operator, and then combined IFS and GHM operators in an intuitionistic fuzzy environment, and proposed the intuitionistic fuzzy geometric Heronian mean operator (IFGHM) and the intuitionistic fuzzy geometric weighted Heronian mean operator (IFGWHM), at the same time, the properties are studied, and finally the effectiveness and practicability of the IFGWHM operator are verified by MADM examples. Then, Yu extended the HM operator to the interval-valued intuitionistic fuzzy environment, and proposed a generalized interval-valued intuitionistic fuzzy HM (GIIFHM) and an approach to multi-criteria decision making based on weighted GIIFHM (GIIFWHM). Liu and Chen [31] proposed the intuitionistic fuzzy Archimedean Heronian aggregation (IFAHA) operator and the intuitionistic fuzzy weight Archimedean Heronian aggregation (IFWAHA) operator. Luo and Xing [32] combined the partitioned HM operator with the PFN to deal with the problem of hotel selection. Liu [33] proposed some new intuitionistic uncertain language HM operators to deal with the situation where both the attribute weights and the expert weights take the form of crisp numbers, and attribute values take the form of intuitionistic uncertain linguistic variables. Zhou *et al.* [34] proposed a decision-making method based on interval-valued intuitionistic fuzzy geometric weighted Heronian average operator for the multi-attribute group decision-making problem where the decision information is IVIFNs and the attributes are related to each other. Liu *et al.* [35] proposed the partitioned Heronian mean (PHM) operator which assumes that all attributes are partitioned into several parts and members in the same part are interrelated while in different parts there are no interrelationships among members. Yang *et al.* [36] proposed an online shopping support model by using q-rung orthopair fuzzy interaction weighted Heronian mean operators. Lin *et al.* [37] proposed the partitioned geometric Heronian mean (PGHM) operator based on the linguistic q-rung orthopair fuzzy sets to solve the interrelationship problem in decision-making. Wu *et al.* [38] proposed some Dombi Heronian mean operators with interval-valued intuitionistic fuzzy numbers, and proposed two MADM methods based on these operators. However, there is no research to combine the HM operator with IVPFN.

In real life, decision-making information often be collected at different periods, which is a dynamic multi-attribute decision-making problem. In the dynamic IVIF multi-attribute decision-making problem, Xu and Yager [39] studied the dynamic multi-attribute decision making problems with intuitionistic fuzzy information, and proposed two new aggregation operators dynamic intuitionistic fuzzy weighted averaging (DIFWA) operator and uncertain dynamic intuitionistic fuzzy weighted averaging (UDIFWA) operator, Guo *et al.* [40] gave a dynamic comprehensive evaluation method to reflect the dynamics of decision-making from the perspective of index and time dimensions; Chen *et al.* [41] proposed a multi-attribute decision-making process that dynamically handles the selection of service methods for large servers Problem. Yang *et al.* [42] gave a multi-attribute decision-making model of dynamic intuitionistic fuzzy normal aggregate settlement based on time preference. Liu *et al.* [43] proposed a new DIF-MADM method and some new dynamic intuitionistic fuzzy weighted geometric operators to address some limits of the existing methods. However, these dynamic methods are not combined with IVPF theory, nor do they consider the relationship between aggregated data.

The above studies on IVIF have been very well developed, and a complete set of systems and methods have been developed for the multi-attribute decision-making problem in the IVIFS environment, however, most of the research directions tend to combine the interval number with IFNs when the decision information is not accurate, and few of them combine the interval number with PFNs to deal with the multi-attribute decision problem. Based on this, This paper presents a weighted GHM operator based on IVPF to deal with dynamic multi-attribute decision making problems, extends the real number GWHM operator to the field of IVPF, and defines the IVPFGWHM operator that can handle dynamic multi-attribute decision-making problems. Use the non-linear characteristics of the IVPFGWHM operator to link the relationship between the aggregated data, so that the decision result is closer to the actual situation.

The main contributions are as follows.

(1) Combining the interval number with the PFN to propose the IVPFN, and the algorithms, score function, accuracy function of IVPFN, and the ranking method are studied, finally they are applied to actual dynamic decision-making problems.

(2) Extend the weighted GHM operator to the multi-attribute decision-making in the IVPF environment, and prove that under this condition, the result of the operator aggregation is also an IVPFN; at the same time, the relevant properties of the operator are studied. Then under this background, the IVPFGWHM operator under dynamic conditions is defined.

(3) Again, we use the best worst method (BWM) introduced by Rezaei [44] to determine the weight information of the attributes.

(4) The detailed process of the IVPF multi-attribute decision-making is designed. The dynamic IVPFGWHM operator is used to aggregate a single aggregate value at each time point to obtain a comprehensive aggregate value, and the decision alternatives are ranked according to the score function and the accuracy function.

The remainder of this paper is organized as follows. In section 2, the preliminary knowledge of IVPFN and GHM operators are introduced in brief, the algorithms of IVPFNs are defined, several interval-valued picture fuzzy GHM operators are proposed on the basis of these algorithms, and their properties are discussed, then the BWM method is briefly introduced. Then, a new decision-making method based on these aggregation operators is proposed in section 3. In section 4, a case of logistics company selection is studied to show the advantages of IVPFNs and the practicality of our proposed method, and demonstrates the superiority of the proposed method by comparing with other existing methods. Finally, some necessary conclusions are given.

II. PRELIMINARIES

In this section, we briefly introduced the preliminary knowledge of IVPFN and GHM operators, and defined the algorithms of IVPFNs. Several interval-valued picture fuzzy GHM operators are proposed based on these algorithms, and their properties are discussed. Then the BWM method is briefly reviewed.

A. INTERVAL-VALUED PICTURE FUZZY NUMBERS
Definition 1 ([3]):
Let X be a non-empty finite set. Then a picture fuzzy set (PFS) in X is defined by

$$A = \{\langle x, \mu A(x), \eta A(x), \nu A(x) | x \in X \rangle\}, \quad (1)$$

where, $\mu A(x), \eta A(x), \nu A(x)$ represent the positive membership, neutral membership and negative membership function of the element x in X belonging to set A respectively. In addition, $\pi A(x) = 1 - \mu A(x) - \eta A(x)$ is called the refusal degree of element x belonging to A, which satisfy the condition.

$$0 \leq \mu A(x) \leq 1, 0 \leq \eta A(x) \leq 1, 0 \leq \nu A(x) \leq 1,$$
$$0 \leq \pi A(x) \leq 1, 0 \leq \mu A(x) + \eta A(x) + \nu A(x) \leq 1,$$

then $(\mu A(x), \eta A(x), \nu A(x))$ is called a picture fuzzy number (PFN).

Definition 2: Let X be a non-empty finite set. Then an interval-valued picture fuzzy set (IVPFS) in X is defined by

$$A = \{\langle x, \mu A(x), \eta A(x), \nu A(x) | x \in X \rangle\}, \quad (2)$$

where, $\mu A(x) = [\mu^L A(x), \mu^U A(x)], \eta A(x) = [\eta^L A(x), \eta^U A(x)], \nu A(x) = [\nu^L A(x), \nu^U A(x)]$, and $\mu A(x), \eta A(x), \nu A(x)$ are all interval numbers in [0, 1], represent positive membership, neutral membership and negative membership respectively. which satisfy

the condition.

$$0 \le \mu^U A(x) + \eta^U A(x) + \nu^U A(x) \le 1,$$
$$\mu^L A(x) \ge 0, \eta^L A(x) \ge 0, \nu^L A(x) \ge 0.$$

Then, $\left(\left[\mu^L A(x), \mu^U A(x)\right]\left[\eta^L A(x), \eta^U A(x)\right], \left[\nu^L A(x), \nu^U A(x)\right]\right)$ is called an interval-valued picture fuzzy number (IVPFN). Marked as $\left(\left[a, b\right], \left[c, d\right], \left[e, f\right]\right)$.

B. THE OPERATIONS OF IVPFNS

In this part, we defined the arithmetic operations of two picture fuzzy numbers, Let $\alpha_1 = ([a_1, b_1], [c_1, d_1], [e_1, f_1])$ and $\alpha_2 = ([a_2, b_2], [c_2, d_2], [e_2, f_2])$ be the IVPFNs, $\forall \lambda > 0$, then the operations of IVPFNs are defined as follows.

(1) $\bar{\tilde{\alpha}}_1 = ([e_1, f_1], [c_1, d_1], [a_1, b_1]);$

(2) $\tilde{\alpha}_1 \cap \tilde{\alpha}_2 = ([\min(a_1, a_2), \min(b_1, b_2)], [\max(c_1, c_2), \max(d_1, d_2)], [\max(e_1, e_2), \max(f_1, f_2)]);$

$\tilde{\alpha}_1 \cup \tilde{\alpha}_2 = ([\max(a_1, a_2), \max(b_1, b_2)], [\min(c_1, c_2), \min(d_1, d_2)], [\min(e_1, e_2), \min(f_1, f_2)]);$

(3) $\tilde{\alpha}_1 \oplus \tilde{\alpha}_2 = ([a_1 + a_2 - a_1 a_2, b_1 + b_2 - b_1 b_2], [c_1 c_2, d_1 d_2], [e_1 e_2, f_1 f_2]);$

(4) $\tilde{\alpha}_1 \otimes \tilde{\alpha}_2 = ([a_1 a_2, b_1 b_2], [c_1 + c_2 - c_1 c_2, d_1 + d_2 - d_1 d_2], [e_1 + e_2 - e_1 e_2, f_1 + f_2 - f_1 f_2]);$

(5) $\lambda \tilde{\alpha}_1 = ([1 - (1 - a_1)^\lambda, 1 - (1 - b_1)^\lambda], [c_1^\lambda, d_1^\lambda], [e_1^\lambda, f_1^\lambda]);$

(6) $\tilde{\alpha}_1^\lambda = ([a_1^\lambda, b_1^\lambda], [1 - (1 - c_1)^\lambda, 1 - (1 - d_1)^\lambda], [1 - (1 - e_1)^\lambda, 1 - (1 - f_1)^\lambda]).$

It is easy to know that all of the above operation results are still IVPFNs. According to these operation results, the following operations can be obtained.

(7) $\alpha_1 \oplus \alpha_2 = \alpha_2 \oplus \alpha_1;$

(8) $\alpha_1 \otimes \alpha_2 = \alpha_2 \otimes \alpha_1;$

(9) $\lambda(\alpha_1 \oplus \alpha_2) = \lambda \alpha_1 \oplus \lambda \alpha_2;$

(10) $(\alpha_1 \otimes \alpha_2)^\lambda = \alpha_1^\lambda \otimes \alpha_2^\lambda;$

(11) $\lambda_1 \alpha_1 \oplus \lambda_2 \alpha_1 = (\lambda_1 + \lambda_2) \alpha_1;$

(12) $\alpha_1^{\lambda_1} \otimes \alpha_1^{\lambda_2} = (\alpha_1)^{\lambda_1 + \lambda_2}.$

Definition 3: Let $\alpha = ([a, b], [c, d], [e, f])$ be an IVPFN, a score function $S(\alpha)$ and an accuracy function $H(\alpha)$ of the IVPFN can be defined by

$$S(\alpha) = \frac{a - c - e + b - d - f}{2}, \quad S(\alpha) \in [-1, 1]; \quad (3)$$

$$H(\alpha) = \frac{a + b + c + d + e + f}{2}, \quad H(\alpha) \in [0, 1]. \quad (4)$$

Based on the score function $S(\alpha)$ and the accuracy function $H(\alpha)$, two IVPFNs can be compared the order relation, which is defined as follows.

(1) If $S(\alpha_1) < S(\alpha_2)$, then, $\alpha_1 \prec \alpha_2;$

(2) If $S(\alpha_1) = S(\alpha_2)$, then,

① If $H(\alpha_1) < H(\alpha_2)$, then $\alpha_1 \prec \alpha_2;$

② If $H(\alpha_1) = H(\alpha_2)$, then $\alpha_1 = \alpha_2.$

If the value of $S(\alpha)$ is larger, the corresponding IVPFN is larger.

C. GEOMETRIC HERONIAN MEAN OPERATOR

Let $I = [0, 1], p, q > 0, HM^{p,q} : I^n \rightarrow I$ and a non-negative real number set $\{a_1, a_2, \ldots, a_n\}$, The GHM operator

is defined by

$$GHM^{p,q}(a_1, a_2, \ldots, a_n) = \frac{1}{p + q}$$
$$\times \left(\prod_{i=1, j=i}^{n} (pa_i + qa_j)^{\frac{2}{n(n+1)}} \right) \quad (5)$$

We combine the GHM operator and propose the IVPFGHM operator to deal with the situation where the information of the multi-attribute decision-making problem is given in the form of PFN.

D. INTERVAL-VALUED PICTURE FUZZY GEOMETRIC HERONIAN MEAN OPERATOR Definition 5:

Let $\alpha_i = ([a_i, b_i], [c_i, d_i], [e_i, f_i]) (i = 1, 2, \ldots, n)$ be a collection of IVPFNs, and $p, q > 0$, and $IVPFGHM^{p,q} : \Omega^n \rightarrow \Omega$, then the IVPFGHM operator is defined by

$$IVPFGHM^{p,q}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \frac{1}{p + q}$$
$$\times \left(\overset{n}{\underset{i=1, j=i}{\otimes}} (p\alpha_i \oplus q\alpha_j)^{\frac{2}{n(n+1)}} \right). \quad (6)$$

Lemma 1: Let $\gamma_{ij} = (l\gamma_{ij}, m\gamma_{ij}, n\gamma_{ij})$ be an IVPFN, and $i, j = 1, 2, \ldots, n$, then the formula (7) is true.

$$\overset{n}{\underset{i=1, j=i}{\otimes}} \gamma^{\frac{2}{n(n+1)}} = \left(\prod_{i=1, j=i}^{n} l_{\gamma_{ij}}^{\frac{2}{n(n+1)}}, 1 - \prod_{i=1, j=i}^{n} (1 - m\gamma_{ij})^{\frac{2}{n(n+1)}}, \right.$$
$$\left. \times 1 - \prod_{i=1, j=i}^{n} (1 - n\gamma_{ij})^{\frac{2}{n(n+1)}} \right) \quad (7)$$

where, $l\gamma_{ij}$, $m\gamma_{ij}$ and $n\gamma_{ij}$ represent the positive membership, neutral membership and negative membership Respectively.

Theorem 1: Let $\alpha_i = ([a_i, b_i], [c_i, d_i], [e_i, f_i]) (i = 1, 2, \ldots, n)$ be a collection of IVPFNs, and $p, q > 0$, then, the result aggregated from (6) is still an IVPFN, and even

$$IVPFGHM^{p,q}(\alpha_1, \alpha_2, \ldots, \alpha_n)$$
$$= \left([1 - (1 - \prod_{i=1, j=i}^{n} (1 - (1 - a_i)^p (1 - a_j)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}}, \right.$$
$$\times 1 - (1 - \prod_{i=1, j=i}^{n} (1 - (1 - b_i)^p (1 - b_j)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}}],$$
$$\times [(1 - \prod_{i=1, j=i}^{n} (1 - (c_i)^p (c_j)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}},$$
$$\times (1 - \prod_{i=1, j=i}^{n} (1 - (d_i)^p (d_j)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}}],$$
$$\times [(1 - \prod_{i=1, j=i}^{n} (1 - (e_i)^p (e_j)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}},$$
$$\left. \times (1 - \prod_{i=1, j=i}^{n} (1 - (f_i)^p (f_j)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}}] \right). \quad (8)$$

Proof: Since $\alpha_i = ([a_i, b_i], [c_i, d_i], [e_i, f_i])$, and $\alpha j = ([a_j, b_j], [c_j, d_j], [e_j, f_j])$, from Eq (5), we have

$$p\alpha_i = ([1 - (1 - a_i)^p, 1 - (1 - b_i)^p], [c_i^p, d_i^p], [e_i^p, f_i^p]),$$
$$q\alpha_j = ([1 - (1 - a_j)^q, 1 - (1 - b_j)^q], [c_j^q, d_j^q], [e_j^q, f_j^q]).$$

From Eq (3) we have

$$p\alpha_i \oplus q\alpha_j$$
$$= ([1 - (1 - a_i)^p(1 - a_j)^q, 1 - (1 - b_i)^p(1 - b_j)^q],$$
$$\times [(c_i)^p(c_j)^q, (d_i)^p(d_j)^q], [(e_i)^p(e_j)^q, (f_i)^p(f_j)^q]).$$

Then from Eq (6) we have

$$(p\alpha_i \oplus q\alpha_j)^{\frac{2}{n(n+1)}}$$
$$= ([[1 - (1 - a_i)^p(1 - a_j)^q]^{\frac{2}{n(n+1)}},$$
$$\times (1 - (1 - b_i)^p(1 - b_j)^q)^{\frac{2}{n(n+1)}}],$$
$$\times [1 - (1 - (c_i)^p(c_j)^q)^{\frac{2}{n(n+1)}}, 1 - (1 - (d_i)^p(d_j)^q)^{\frac{2}{n(n+1)}}],$$
$$\times [1 - (1 - (e_i)^p(e_j)^q)^{\frac{2}{n(n+1)}}, 1 - (1 - (f_i)^p(f_j)^q)^{\frac{2}{n(n+1)}}]). \quad (9)$$

Replace γ_{ij} in formula (7) with $p\alpha_i \oplus q\alpha_j$ in formula (9), replace $l\gamma_{ij}$ with $[(1 - (1 - a_i)^p(1 - a_j)^q)^{\frac{2}{n(n+1)}}, (1 - (1 - b_i)^p(1 - b_j)^q)^{\frac{2}{n(n+1)}}]$, replace $m\gamma_{ij}$ with $[1 - (1 - (c_i)^p(c_j)^q)^{\frac{2}{n(n+1)}}, 1 - (1 - (d_i)^p(d_j)^q)^{\frac{2}{n(n+1)}}]$, replace $n\gamma_{ij}$ with $[1 - (1 - (e_i)^p(e_j)^q)^{\frac{2}{n(n+1)}}, 1 - (1 - (f_i)^p(f_j)^q)^{\frac{2}{n(n+1)}}]$.

Combination Eq (3), (4), (5), (6) can prove that the following formula is true.

$$\overset{n}{\underset{i=1,j=i}{\otimes}} (p\alpha_i \oplus q\alpha_j)^{\frac{2}{n(n+1)}}$$
$$= \left([\prod_{i=1,j=i}^{n} (1 - (1 - a_i)^p(1 - a_j)^q)^{\frac{2}{n(n+1)}}, \right.$$
$$\times \prod_{i=1,j=i}^{n} (1 - (1 - b_i)^p(1 - b_j)^q)^{\frac{2}{n(n+1)}}],$$
$$\times 1 - \prod_{i=1,j=i}^{n} (1 - (c_i)^p(c_j)^q)^{\frac{2}{n(n+1)}},$$
$$\times 1 - \prod_{i=1,j=i}^{n} (1 - (d_i)^p(d_j)^q)^{\frac{2}{n(n+1)}}],$$
$$\times 1 - \prod_{i=1,j=i}^{n} (1 - (e_i)^p(e_j)^q)^{\frac{2}{n(n+1)}},$$
$$\left. \times 1 - \prod_{i=1,j=i}^{n} (1 - (d_i)^p(d_j)^q)^{\frac{2}{n(n+1)}}] \right).$$

So,

$$IVPFGHM^{p,q}(\alpha_1, \alpha_2, \ldots, \alpha_n)$$
$$= \frac{1}{p+q} \left(\overset{n}{\underset{i=1,j=i}{\otimes}} (p\alpha_i \oplus q\alpha_j)^{\frac{2}{n(n+1)}} \right)$$

$$= \left([1 - (1 - \prod_{i=1,j=i}^{n} (1 - (1 - a_i)^p(1 - a_j)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}}, \right.$$
$$\times 1 - (1 - \prod_{i=1,j=i}^{n} (1 - (1 - b_i)^p(1 - b_j)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}}],$$
$$\times [(1 - \prod_{i=1,j=i}^{n} (1 - (c_i)^p(c_j)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}},$$
$$\times (1 - \prod_{i=1,j=i}^{n} (1 - (d_i)^p(d_j)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}}],$$
$$\times [(1 - \prod_{i=1,j=i}^{n} (1 - (e_i)^p(e_j)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}},$$
$$\left. \times (1 - \prod_{i=1,j=i}^{n} (1 - (f_i)^p(f_j)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}}] \right).$$

Therefore formula (8) is proved.

Following, we will study some properties of IVPFGHM operator.

Theorem 2 (Idempotency): Let $\alpha_i = ([a_i, b_i], [c_i, d_i], [e_i, f_i])$ be a collection of IVPFNs, if

$$\alpha_i = ([a_i, b_i], [c_i, d_i], [e_i, f_i])$$
$$= \alpha = ([a, b], [c, d], [e, f]) \ (i = 1, 2, \ldots, n),$$

then

$$IVPFGHM^{p,q}(\alpha_1, \alpha_2, \ldots, \alpha_n)$$
$$= IVPFGHM^{p,q}(\alpha, \alpha, \ldots, \alpha) = \alpha. \quad (10)$$

Proof: Since

$$\alpha_i = ([a_i, b_i], [c_i, d_i], [e_i, f_i]) = \alpha = ([a, b], [c, d], [e, f])$$

$(i = 1, 2, \ldots, n)$, then,

$$IVPFGHM^{p,q}(\alpha_1, \alpha_2, \ldots, \alpha_n)$$
$$= IVPFGHM^{p,q}(\alpha, \alpha, \ldots, \alpha)$$
$$= \left([1 - (1 - \prod_{i=1,j=i}^{n} (1 - (1 - a)^p(1 - a)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}}, \right.$$
$$\times 1 - (1 - \prod_{i=1,j=i}^{n} (1 - (1 - b)^p(1 - b)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}}],$$
$$\times [(1 - \prod_{i=1,j=i}^{n} (1 - (c)^p(c)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}},$$
$$\times (1 - \prod_{i=1,j=i}^{n} (1 - (d)^p(d)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}}],$$
$$\times [(1 - \prod_{i=1,j=i}^{n} (1 - (e)^p(e)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}},$$
$$\left. \times (1 - \prod_{i=1,j=i}^{n} (1 - (f)^p(f)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}}] \right)$$

$$= \left([1 - (1 - \prod_{i=1,j=i}^{n} (1 - (1-a)^{p+q})^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}} \right. ,$$

$$\times 1 - (1 - \prod_{i=1,j=i}^{n} (1 - (1-b)^{p+q})^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}}] ,$$

$$\times [(1 - \prod_{i=1,j=i}^{n} (1-c^{p+q})^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}},$$

$$\times (1 - \prod_{i=1,j=i}^{n} (1 - d^{p+q})^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}}],$$

$$\times [(1 - \prod_{i=1,j=i}^{n} (1-e^{p+q})^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}},$$

$$\left. \times (1 - \prod_{i=1,j=i}^{n} (1 - f^{p+q})^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}}] \right)$$

$$= \left([1 - (1 - (1 - (1 - a)^{p+q}))^{\frac{1}{p+q}} \right.$$

$$\times , 1 - (1 - (1 - (1 - b)^{p+q}))^{\frac{1}{p+q}}],$$

$$\times [(1 - (1 - c^{p+q}))^{\frac{1}{p+q}},$$

$$\times (1 - (1 - d^{p+q}))^{\frac{1}{p+q}}], [(1 - (1 - e^{p+q}))^{\frac{1}{p+q}},$$

$$\left. \times (1 - (1 - f^{p+q}))^{\frac{1}{p+q}}] \right)$$

$$= ([a, b], [c, d], [e, f]).$$

Now, the proof is completed.

Theorem 3 (Permutation): Let $(\alpha_1, \alpha_2, \ldots, \alpha_n)$ and $(\alpha'_1, \alpha'_2, \ldots, \alpha'_n)$ be two collections of IVPFNs, If $(\alpha'_1, \alpha'_2, \ldots, \alpha'_n)$ is an arbitrary permutation of $(\alpha 1, \alpha_2, \ldots, \alpha_n)$, then,

$$IVPFGHM^{p,q}(\alpha_1, \alpha_2, \ldots, \alpha_n)$$
$$= IVPFGHM^{p,q}(\alpha 1', \alpha'_2, \ldots, \alpha'_n). \quad (11)$$

Proof: By the operations of IVPFN, we have

$$IVPFGHM^{p,q}(\alpha_1, \alpha_2, \ldots, \alpha_n)$$
$$= \frac{1}{p+q} \left(\overset{n}{\underset{i=1,j=i}{\otimes}} (p\alpha_i \oplus q\alpha_j)^{\frac{2}{n(n+1)}} \right)$$
$$= \frac{1}{p+q} \left(\overset{n}{\underset{i=1,j=i}{\otimes}} (p\alpha'_i \oplus q\alpha'_j)^{\frac{2}{n(n+1)}} \right)$$
$$= IVPFGHM^{p,q}(a'_1, a'_2, \ldots, a'_n).$$

which completes the proof.

Theorem 4 (Monotonicity): Let $\alpha_i = ([a_i, b_i], [c_i, d_i], [e_i, f_i])$ and $\alpha''_i = ([a''_i, b''_i], [c''_i, d''_i], [e''_i, f''_i])(i = 1, 2, \ldots, n)$ be two collections of IVPFNs, If $\alpha_i \leq \alpha''_i \alpha_i \leq \alpha i''$, for all i, then,

$$IVPFGHM^{p,q}(\alpha_1, \alpha_2, \ldots, \alpha_n)$$
$$\leq IVPFGHM^{p,q}(\alpha''_1, \alpha''_2, \ldots, \alpha''_n). \quad (12)$$

Proof: Let $IVPFGHM^{p,q}(\alpha_1, \alpha_2, \ldots, \alpha_n) = ([a, b], [c, d], [e, f])$, and $IVPFGHM^{p,q}(\alpha''_1, \alpha''_2, \ldots, \alpha n'') = [a'', b''], [c'', d''], [e'', f'']$.

Since $a_i \leq a''_i, a_j \leq a''_j$, then we have $1 - (1 - a_i)^p(1 - a_j)^q \leq 1 - (1 - a''_i)^p(1 - a''_j)^q$, thereafter,

$$1 - \prod_{i=1,j=i}^{n} (1 - (1-a_i)^p(1 - a_j)^q)^{\frac{2}{n(n+1)}}$$

$$\geq 1 - \prod_{i=1,j=i}^{n} (1 - (1-a''_i)^p(1 - a''_j)^q)^{\frac{2}{n(n+1)}},$$

thus,

$$a = 1 - (1 - \prod_{i=1,j=i}^{n} (1 - (1-a_i)^p(1 - a_j)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}} \leq a''$$

$$= 1 - (1 - \prod_{i=1,j=i}^{n} (1 - (1-a''_i)^p(1 - a''_j)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}}.$$

Similarly,

$$1 - (1 - b_i)^p(1 - b_j)^q$$
$$\leq 1 - (1 - b''_i)^p(1 - b''_j)^q,$$

$$1 - \prod_{i=1,j=i}^{n} (1 - (1-b_i)^p(1 - b_j)^q)^{\frac{2}{n(n+1)}}$$

$$\geq 1 - \prod_{i=1,j=i}^{n} (1 - (1-b''_i)^p(1 - b''_j)^q)^{\frac{2}{n(n+1)}},$$

$$b = 1 - (1 - \prod_{i=1,j=i}^{n} (1 - (1-b_i)^p(1 - b_j)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}} \leq b''$$

$$= 1 - (1 - \prod_{i=1,j=i}^{n} (1 - (1-b''_i)^p(1 - b''_j)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}}.$$

Since $e_i \geq e''_i, e_j \geq e''_j$, then we have

$$1 - (1 - e_i)^p(1 - e_j)^q \geq 1 - (1 - e''_i)^p(1 - e''_j)^q,$$

thereafter,

$$1 - \prod_{i=1,j=i}^{n} (1 - (1-e_i)^p(1 - e_j)^q)^{\frac{2}{n(n+1)}}$$

$$\leq 1 - \prod_{i=1,j=i}^{n} (1 - (1-e''_i)^p(1 - e''_j)^q)^{\frac{2}{n(n+1)}},$$

thus,

$$e = 1 - (1 - \prod_{i=1,j=i}^{n} (1 - (1-e_i)^p(1 - e_j)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}} \geq e''$$

$$= 1 - (1 - \prod_{i=1,j=i}^{n} (1 - (1-e''_i)^p(1 - e''_j)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}}.$$

Similarly,

$$1 - (1 - f_i)^p(1 - f_j)^q$$
$$\geq 1 - (1 - f''_i)^p(1 - f''_j)^q,$$

$$1 - \prod_{i=1,j=i}^{n} (1 - (1-f_i)^p (1 - f_j)^q)^{\frac{2}{n(n+1)}}$$

$$\leq 1 - \prod_{i=1,j=i}^{n} (1 - (1-f_i'')^p (1 - f_j'')^q)^{\frac{2}{n(n+1)}},$$

$$f = 1 - (1 - \prod_{i=1,j=i}^{n} (1 - (1-f_i)^p (1 - f_j)^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}} \geq f''$$

$$= 1 - (1 - \prod_{i=1,j=i}^{n} (1 - (1-f_i'')^p (1 - f_j'')^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}}.$$

So,

$$IVPFGHM^{p,q}(\alpha_1, \alpha_2, \ldots, \alpha_n)$$
$$\leq IVPFGHM^{p,q}(a_1'', a_2'', \ldots, a_n''),$$

Therefore, the proof of Theorem 4 is completed.

Theorem 5 (boundedness): Let $\alpha_i = ([a_i, b_i], [c_i, d_i], [e_i, f_i])$ $(i = 1, 2, \ldots, n)$ be a collection of IVPFNs, and $\alpha_i^- = \min(a_1, a_2, \ldots, a_n)$, $\alpha_i^+ = \max(a_1, a_2, \ldots, a_n)$, then we have

$$\alpha_i^- \leq IVPFGHM^{p,q}(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq \alpha i^+. \quad (13)$$

Proof: According to Theorem 2, we have

$$IVPFGHM^{p,q}(\alpha_i^-, \alpha_i^-, \ldots, \alpha_i^-)$$
$$= \alpha_i^-, IVPFGHM^{p,q}(\alpha_i^+, \alpha_i^+, \ldots, \alpha_i^+) = \alpha_i^+.$$

Further, According to Theorem 4, we have

$$IVPFGHM^{p,q}(\alpha_i^-, \alpha_i^-, \ldots, \alpha_i^-)$$
$$\leq IVPFGHM^{p,q}(\alpha_1, \alpha_2, \ldots, \alpha_i)$$
$$\leq IVPFGHM^{p,q}(\alpha_i^+, \alpha_i^+, \ldots, \alpha_i^+).$$

So, $\alpha_i^- \leq IVPFGHM^{p,q}(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq \alpha_i^+$.
Theorem 5 is completed.

E. INTERVAL-VALUED PICTURE FUZZY GEOMETRIC WEIGHTED HERONIAN MEAN OPERATOR

Based on the above research, the IVPFGHM operator considers the interrelationships between attributes, but in practical applications, different attributes have different degrees of importance. For this reason, we propose the IVPFGWHM operator.

Definition 6: Let $\alpha_i = ([a_i, b_i], [c_i, d_i], [e_i, f_i])$ $(i = 1, 2, \ldots, n)$ be a collections of IVPFNs, and $p, q > 0$, $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ is the weight vector of $\alpha_i (i = 1, 2, \ldots, n)$, satisfying $\omega_\iota \geq 0$, $\iota = 1, 2, \ldots, n$, $\sum_{i=1}^{n} \omega_i = 1$, then the IVPFGWHM operator is defined by

$$IVPFGWHM^{p,q}(\alpha_1, \alpha_2, \ldots, \alpha_n)$$
$$= \frac{1}{p+q} \left(\bigotimes_{i=1,j=i}^{n} \left((p\alpha_i)^{\omega_i} \oplus (q\alpha_j)^{\omega_j} \right)^{\frac{2}{n(n+1)}} \right). \quad (14)$$

Theorem 6: Let $\alpha_i = ([a_i, b_i], [c_i, d_i], [e_i, f_i])$ $(i = 1, 2, \ldots, n)$ be a collections of IVPFNs, and $p, q > 0$, $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ is the weight vector of $\alpha_i (i =$

$1, 2, \ldots, n)$, satisfying $\omega_\iota \geq 0$, $\iota = 1, 2, \ldots, n$, $\sum_{i=1}^{n} \omega_i = 1$, then, the result aggregated from (14) is still an IVPFN, and even

$$IVPFGWHM^{p,q}(\alpha_1, \alpha_2, \ldots, \alpha_n)$$
$$= \left(\left[1 - (1 - \prod_{i=1,j=i}^{n} (1 - (1 - (a_i)^{\omega_i})^p \right. \right.$$
$$\times (1 - (a_j)^{\omega_j})^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}},$$
$$\times 1 - (1 - \prod_{i=1,j=i}^{n} (1 - (1 - (b_i)^{\omega_i})^p$$
$$\left. \times (1 - (b_j)^{\omega_j})^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}} \right],$$
$$\times \left[(1 - \prod_{i=1,j=i}^{n} (1 - (1 - (1 - c_i)^{\omega_i})^p \right.$$
$$\times (1 - (1 - c_j)^{\omega_j})^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}},$$
$$\times (1 - \prod_{i=1,j=i}^{n} (1 - (1 - (1 - d_i)^{\omega_i})^p$$
$$\left. \times (1 - (1 - d_j)^{\omega_j})^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}} \right],$$
$$\times \left[(1 - \prod_{i=1,j=i}^{n} (1 - (1 - (1 - e_i)^{\omega_i})^p \right.$$
$$\times (1 - (1 - e_j)^{\omega_j})^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}},$$
$$\times (1 - \prod_{i=1,j=i}^{n} (1 - (1 - (1 - f_i)^{\omega_i})^p$$
$$\left. \left. \times (1 - (1 - f_j)^{\omega_j})^q)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}} \right] \right). \quad (15)$$

The proof of Theorem 6 is similar to Theorem 1, which is omitted here.

F. DYNAMIC INTERVAL-VALUED PICTURE FUZZY GEOMETRIC WEIGHTED HERONIAN MEAN OPERATOR

In order to solve the problem of dynamic multi-attribute decision-making, Definition 7 defines the DIVPFGWHM operator.

Definition 7: Let $\alpha_j(t_1), \alpha_j(t_2), \ldots, \alpha_j(t_p)(j = 1, 2, \ldots, m)$ be the attribute value of the a certain attribute in the period $t_1, t_2, \ldots, t_p$, Where $\alpha_j(t_k)$ is represented by IVPFN, at the same time, assume that $\theta(t) = (\theta(t_1), \theta(t_2), \ldots, \theta(t_m))^T$, $\theta(t_k) \geq 0$, $\sum_{k=1}^{m} \theta(t_k) = 1$, $k = 1, 2, \ldots, m$ are the time weight vectors of each period, then the DIVPFGWHM operator is defined by

$$DIVPFGWHM_{\theta(t)}^{p,q} \left(\alpha_j(t_1), \alpha_j(t_2), \ldots, \alpha_j(t_p) \right)$$

$$= \frac{1}{p+q} \left(\overset{m}{\underset{i=1,k=i}{\otimes}} ((p\alpha_j(t_i))^{\theta(t_i)} \oplus (q\alpha_j(t_k))^{\theta(t_k)})^{\frac{2}{n(n+1)}} \right), \tag{16}$$

The DIVPFGWHM operator also satisfies Theorems 2, 3, 4, and 5. Although this operator is similar to the IVPFGWHMM operator, the weight here is the time weight, so it is a new operator.

G. STEPS OF THE BWM METHOD

The weight of attributes is very important in the aggregation and ranking of alternatives. In order to ensure that our proposed method has better results, we need to objectively determine the weight information of attributes. Based on the new best-worst method (BWM) proposed by Rezae for multi-attribute decision-making technology, we calculate the weight of the attribute set. The advantages of the BWM method are as follows.

(1) Compared to matrix-based MCDM methods, it requires fewer comparisons.

(2) The BWM method provides more consistent comparisons and more reliable results.

(3) It can derive the weights independently or in combination with other MCDM methods.

(4) In this method, only integers are used, making it easier to use.

Step 1. Determine a set of decision-making criteria. In this step, the decision maker develops a set of criteria to evaluate various alternatives.

Step 2. Choose the best criterion C_b and the worst criterion C_w from the criteria determined in the first step.

Step 3. Use a number between 1 and 9 to determine the preference of the best standard relative to other standards. The result is called the best-other vector, denoted as $A_B = (a_{B1}, a_{B2}, \ldots, a_{Bn})^T$, where a_{Bj} represents the preference degree of the best criterion C_b over the criterion C_j, and $a_{BB} = 1$.

Step 4. Use a number between 1 and 9 to determine the preference of the worst standard relative to other standards. The result is called the other - worst vector, denoted as $A_W = (a_{1W}, a_{2W}, \ldots, a_{nW})^T$, where a_{Bj} represents the preference degree of the best criterion C_b over the criterion C_j, and $a_{BB} = 1$.

Step 5. Find the optimal weights $(w_1^*, w_2^*, \ldots, w_n^*)$, Construct the following non-linear programming model.

$$\min \max \left\{ \left| w_B - a_{Bj} w_j \right|, \left| w_j - a_{jW} w_w \right| \right\}$$

$$s.t. \begin{cases} \sum_{j=1}^{n} w_j = 1 \\ w_j \geq 0, \ \forall j. \end{cases} \tag{17}$$

Problem (17) can be transferred to the following problem:

$$\min \xi^L$$

$$s.t. \begin{cases} \left| w_B - a_{Bj} w_j \right| \leq \xi^L, & \forall j \\ \left| w_j - a_{jW} w_W \right| \leq \xi^L, & \forall j \\ \sum_{j=1}^{n} w_j = 1 \\ w_j \geq 0, & \forall j. \end{cases} \tag{18}$$

ξ^L represents the consistency index, the closer ξ^L is to 0, the higher the consistency.

III. DYNAMIC MULTI-ATTRIBUTE DECISION-MAKING BASED ON IVPFGWHM OPERATOR

For the dynamic multi-attribute decision-making problem, here we use the IVPFGWHM operator as a tool to solve it.

A. PROBLEM DESCRIPTION

Assume that there are m alternatives, denoted as $A = \{A_1, A_2, \ldots, A_m\}$, and n criteria, denoted as $C = \{C_1, C_2, \ldots, C_n\}$, The weight vector of these criteria is $w = (w_1, w_2, \ldots, w_n)^T$, which satisfies the condition $w_j \in [0, 1]$, $\sum_{j=1}^{n} w_j = 1$, The weight vector of periods is $\lambda(t) = (\lambda(t_1), \lambda(t_2), \ldots, \lambda(t_q))^T$, which also satisfies $\sum_{k=1}^{q} \lambda(t_k) = 1$, $\lambda(t_k) \in [0, 1](k = 1, 2, \ldots, q)$, where $\lambda(t_k)$ represents the weight of the period k.

B. DECISION MAKING PROCEDURES

In this section, the decision-making method based on the proposed aggregation operator is to propose to solve the decision-making problem described in section 4. The specific decision-making frame diagram and procedures are described as Figure 1.

Step1. Normalize the initial decision-making matrix.

Convert all cost criteria into benefit criteria, the standardized decision matrix constructed is as follows.

$$D(t_k) = (d_{ij}(t_k))m \times n = \begin{cases} a_{ij}, & for \ benefit \ criteria \\ \overline{a_{ij}}, & for \ \cos t \ criteria. \end{cases} \tag{19}$$

where, $\overline{a_{ij}}$ is the complement of a_{ij}, which satisfies

$$\overline{a_{ij}} = \left(\left[e_{ij}, f_{ij} \right], \left[a_{ij}, b_{ij} \right], \left[c_{ij}, d_{ij} \right] \right)$$
$$(i = 1, 2, \ldots, m, j = 1, 2, \ldots, n).$$

Step2. Calculate the comprehensive IVPF for a single period separately.

For the Decision matrix $D(t_k)$ of different periods, use IVPFGWHM operator to aggregate the evaluation value $d_{ij}(t_k)$ of alternative i under different attributes in period k. The attribute aggregation values of each alternatives in different periods are as follows.

$$IVPFGWHM(d_{i1}(t_k), d_{i2}(t_k), \ldots, d_{im}(t_k)).$$

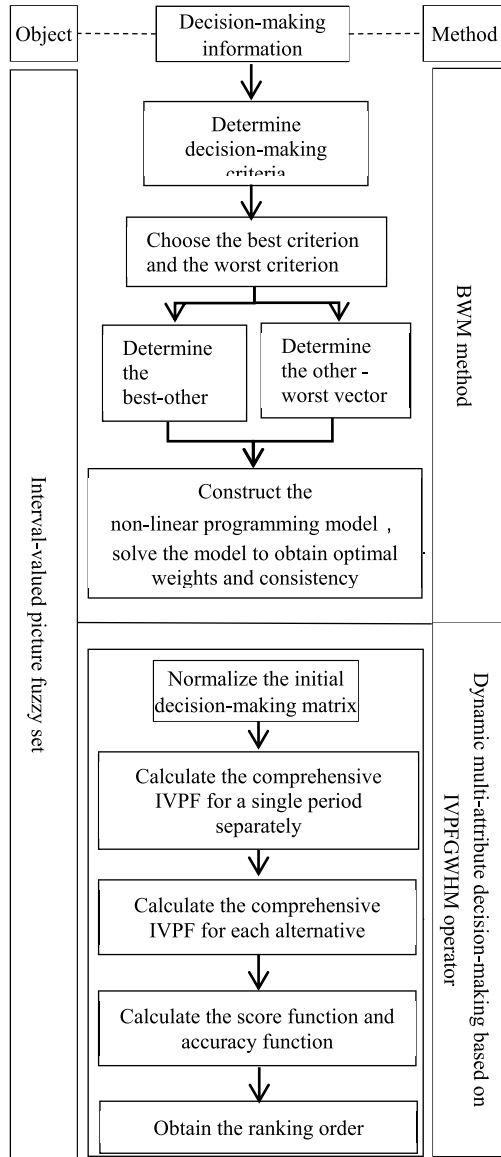***Step3***. Calculate the comprehensive IVPF for each alternative.

FIGURE 1. Frame diagram for BWM method and DIVPFGWHM operator.

TABLE 1. Indicators and meanings.

Indicators		meaning
C_1	Infrastructure Development	Railroad operating mileage, number of civilian cars, number of warehouses, etc.
C_2	Logistics economic situation	Company earnings, investment in fixed assets, and research and experimental funding for the transportation industry
C_3	Operations Management	The number of employees and the total amount of freight transported by the company, etc.
C_4	"Smart" innovation results	The scale of smart delivery cabinets, drones and smart stereo warehouses

Use the DIVPFGWHM operator to aggregate the IVPF of each alternative in different periods, the comprehensive IVPFN of each alternative is obtained as follows.

$$DIVPFGWHM(d_i(t_1), d_i(t_2), \ldots, d_i(t_q)).$$

Step4. Calculate the score function and accuracy function of the comprehensive evaluation value of each alternative.

Based on Eq (13), (14), we calculate the score function $S(\alpha)$ and accuracy function $H(\alpha)$.

Step5. Obtain the ranking order.

According to the comparison method proposed in Definition 3, we can get the final ranking order of all alternatives.

IV. CASE STUDY

with the development of market economy and the continuous innovation of science and technology, many enterprises are eager to realize the automatic operation and efficient management of the material transportation process through advanced logistics network technology, and the emergence of intelligent logistics through intelligent hardware and software, Internet of Things, big data and other technical means, the logistics industry and the Internet combined, changing the original market environment and business processes of the logistics industry. The intelligence of logistics industry has a positive role in promoting China's logistics industry to improve profits and reduce logistics costs. For enterprise decision makers, it is crucial to choose a suitable logistics company to improve resource utilization and management. Through the review of the literature related to ''intelligent logistics performance evaluation in China'', this paper decided to use the following indicators as the attributes of logistics company selection, and the specific definition of each attribute is shown in Table 1.

This section will analyze how companies select logistics companies based on these indicators.

An enterprise wants to realize the automatic operation and efficient management of material transportation process through the advanced logistics network technology. After research and investigation, four candidate companies $A = \{A_1, A_2, A_3, A_4\}$ were identified, after discussion by the board of directors, it was decided to investigate and evaluate these 4 companies based on the above mentioned indicators. In order to compare the four companies more reasonably, the board of directors decided to evaluate the performance of these four companies in 2017-2019 under these four standards. The evaluation result is given in the form of IVPFN.

A. DECISION MAKING ANALYSIS

In this section, we use the IVPFGWHM operator to solve the partner selection problem mentioned above. First, we use the best-worst method (BWM) to objectively calculate the weights of attributes. Then, we use the DIVPFGWHM operator to aggregate the decision information and sort the alternatives. The detailed decision analysis is shown as follows.

1) DETERMINATION OF ATTRIBUTES WEIGHT

Step1. Determine four decision criteria $\{C_1, C_2, C_3, C_4\}$.

Step2. After research and discussion by the company's board of directors, it was decided that C_1 was the best criterion and C_4 was the worst criterion.

Step3. Construct a pairwise comparison vector for the best criterion, which are shown in Table 2.

Step4. Construct a pairwise comparison vector for the worst criterion. which are shown in Table 3.

Step5. According to Table 2 and Table 3, we derive the weights of the four attributes based on the above-mentioned nonlinear programming model, shown as follows.

$$\min \xi^L$$

$$s.t. \begin{cases} |w_1 - w_1| \le \xi^L, \\ |w_1 - 4\,w_2| \le \xi^L, \\ |w_1 - 3\,w_3| \le \xi^L, \\ |w_1 - 8\,w_1| \le \xi^L, \\ |w_1 - 8\,w_4| \le \xi^L, \\ |w_2 - 2\,w_4| \le \xi^L, \\ |w_3 - 3\,w_4| \le \xi^L, \\ |w_4 - w_4| \le \xi^L, \\ w_1 + w_2 + w_3 + w_4 = 1, w_1, w_2, w_3, w_4 \ge 0 \end{cases} \quad (20)$$

By solving this nonlinear programming model, we obtain the optimal attribute weight vector as $w^* = \{0.58, 0.15, 0.20, 0.07\}$, and $\xi^L = 0.014$.

2) MULTI-ATTRIBUTE DECISION MAKING

In order to get the best alternative, the following steps are performed.

TABLE 2. Pairwise comparison vector for the best criterion.

Criteria	Best criterion : C_1
C_1	1
C_2	4
C_3	3
C_4	8

TABLE 3. Pairwise comparison vector for the best criterion.

Criteria	Worst criterion : C_4
C_1	8
C_2	2
C_3	3
C_4	1

Interval-valued picture fuzzy decision matrixes $D(t_k)(k = 1, 2, 3)$ are shown as Table4-Table6, where the weight vector of period t_k is $\lambda(t_k) = (1/6, 1/3, 1/2)^T (k = 1, 2, 3)$.

Step1. Normalized the decision matrix. Because all attributes are benefit, this step is skipped.

Step2. Calculate the comprehensive IVPF for a single period separately. According to the decision matrix $D(t_k)$, the IVPFGWHM operator is used to aggregate the elements of different alternatives under each attribute, and the aggregate value of IVPFs of alternative A_i is obtained (21)–(23), as shown at the bottom of the next page.

Step3. Calculate the comprehensive IVPF for each alternative. use the DIVPFGWHM operator to aggregate the IVPFN $d_i(t_k)$ of each alternative A_i at different periods t, obtain the comprehensive IVPFN of each alternative as follows (24), as shown at the bottom of the next page.

Step4. Calculate the score function $S(r_i)$ $(i = 1, 2, 3, 4)$ of the comprehensive evaluation value of each alternative, the results are shown as follows.

$$S(r_1) = 0.8573, \quad S(r_2) = 0.8753,$$
$$S(r_3) = 0.8580, \quad S(r_4) = 0.8821. \quad (25)$$

Step5. Obtain the ranking order.

According to the score function $S(r_i)(i = 1, 2, 3, 4)$, the ranking order of the alternatives $\{A_1, A_2, A_3, A_4\}$ is: $A_4 \succ A_2 \succ A_3 \succ A_1$, where "$\succ$" means superior. Therefore, the best solution is A_4.

B. COMPARATIVE ANALYSIS

In order to further illustrate the advantages of the proposed method, IVPFWA operator, IVPFGA operator and IVPF-EDAS method are used to handle the decision information of this case, and the score function value, accuracy function value and the Appraisal Score (AS) of IVPF-EDAS method is shown in the table 7.

It can be seen from Table 8 that the ordering result of the DIVPFGWHM operator is slightly different from

TABLE 4. IVPF decision matrix $D(t_1)$.

	C_1	C_2	C_3	C_4
A_1	([0.2,0.3],[0.1,0.2], [0.2,0.4])	([0.3,0.4],[0.2,0.3] ,[0.1,0.2])	([0.1,0.2],[0.1,0.3] ,[0.2,0.5])	([0.3,0.4],[0.2,0.3], [0.2,0.3])
A_2	([0.3,0.4],[0.2,0.3], [0.2,0.3])	([0.1,0.2],[0.3,0.5] ,[0.1,0.3])	([0.3,0.4],[0.1,0.3] ,[0.2,0.3])	([0.2,0.5],[0.1,0.2], [0.1,0.2])
A_3	([0.1,0.3],[0.2,0.5], [0.1,0.2])	([0.1,0.2],[0.3,0.5] ,[0.1,0.3])	([0.3,0.4],[0.4,0.5] ,[0,0.1])	([0.2,0.3],[0.2,0.3], [0.1,0.3])
A_4	([0.2,0.4],[0.4,0.5], [0,0.1])	([0.2,0.3],[0.1,0.2] ,[0.2,0.4])	([0.1,0.4],[0,0.3], [0.1,0.2])	([0.2,0.3],[0.4,0.5], [0.1,0.2])

TABLE 5. IVPF decision matrix $D(t_2)$.

	C_1	C_2	C_3	C_4
A_1	([0.2,0.3],[0.4,0.5], [0.1,0.2])	([0.3,0.4],[0.4,0.5], [0,0.1])	([0.1,0.2],[0.3,0.5] [0.1,0.3])	([0.2,0.3],[0.1,0.2] ,[0.2,0.4])
A_2	([0.1,0.4],[0,0.3], [0.1,0.2])	([0.1,0.2],[0.3,0.5] [0.1,0.3])	([0.3,0.4],[0.2,0.3], [0.2,0.3])	([0.3,0.4],[0.2,0.3] ,[0.1,0.2])
A_3	([0.2,0.3],[0.1,0.2], [0.2,0.4])	([0.1,0.3],[0.2,0.5] [0.1,0.2])	([0.2,0.5],[0.1,0.2], [0.1,0.2])	([0.1,0.2],[0.1,0.3] ,[0.2,0.5])
A_4	([0.2,0.4],[0.4,0.5], [0,0.1])	([0.2,0.3],[0.2,0.3], [0.1,0.3])	([0.1,0.4],[0.2,0.3], [0,0.3])	([0.2,0.4],[0.4,0.5] ,[0,0.1])

TABLE 6. IVPF decision matrix $D(t_3)$.

	C_1	C_2	C_3	C_4
A_1	([0.1,0.2],[0.3,0.5], [0.1,0.3])	([0.1,0.2],[0.3,0.5], [0.1,0.3])	([0.2,0.5],[0.1,0.2], [0.1,0.2])	([0.1,0.3],[0.2,0.5] ,[0.1,0.2])
A_2	([0.3,0.4],[0.2,0.3], [0.2,0.3])	([0.3,0.4],[0.4,0.5], [0,0.1])	([0.1,0.3],[0.2,0.5], [0.1,0.2])	([0.2,0.4],[0.4,0.5] ,[0,0.1])
A_3	([0.2,0.4],[0,0.1], [0.3,0.4])	([0.1,0.3],[0.3,0.4], [0.1,0.2])	([0.1,0.2],[0.1,0.3], [0.2,0.4])	([0.3,0.4],[0.1,0.4] ,[0,0.2])
A_4	([0.3,0.5],[0,0.2], [0.1,0.2])	([0.3,0.4],[0.2,0.3], [0.2,0.3])	([0.1,0.4],[0.2,0.3], [0.1,0.3])	([0.1,0.3],[0.2,0.5] ,[0.1,0.2])

$$(d_i(t_1))_{4\times1} = \begin{pmatrix} [0.6696, 0.7402] , [0.0330, 0.0686] , [0.0528, 0.1193] \\ [0.7089, 0.7786] , [0.0542, 0.0968] , [0.0512, 0.0861] \\ [0.6215, 0.7428] , [0.0667, 0.1490] , [0.0273, 0.0627] \\ [0.6547, 0.7539] , [0.0946, 0.1340] , [0.0313, 0.0660] \end{pmatrix} \tag{21}$$

$$(d_i(t_2))_{4\times1} = \begin{pmatrix} [0.6648, 0.7366] , [0.1061, 0.1492] , [0.0290, 0.0666] \\ [0.6259, 0.7757] , [0.0608, 0.0987] , [0.0329, 0.0667] \\ [0.6518, 0.7569] , [0.0313, 0.0743] , [0.0479, 0.1047] \\ [0.6547, 0.7539] , [0.1023, 0.1377] , [0.0230, 0.0667] \end{pmatrix} \tag{22}$$

$$(d_i(t_3))_{4\times1} = \begin{pmatrix} [0.6015, 0.7133] , [0.0759, 0.1423] , [0.0273, 0.0805] \\ [0.6892, 0.7726] , [0.0702, 0.1180] , [0.0464, 0.0765] \\ [0.6424, 0.7611] , [0.0362, 0.0761] , [0.0716, 0.1100] \\ [0.6913, 0.8169] , [0.0385, 0.0737] , [0.0313, 0.0667] \end{pmatrix} \tag{23}$$

$$(d_i)_{4\times1} = \begin{pmatrix} [0.8570, 0.8971] , [0.0280, 0.0478] , [0.0109, 0.0286] \\ [0.8740, 0.9173] , [0.0226, 0.0383] , [0.0148, 0.0260] \\ [0.8616, 0.9103] , [0.0135, 0.0304] , [0.0202, 0.0358] \\ [0.8753, 0.9218] , [0.0241, 0.0368] , [0.0099, 0.0231] \end{pmatrix} \tag{24}$$

TABLE 7. IVPFWA operator and IVPFGA operator aggregation results.

Alternative	IVPFWA		IVPFGA		IVPF-EDAS method
	Score function	Accuracy function	Score function	Accuracy function	Appraisal Score
A_1	0.0351	0.7446	0.0072	0.7554	0.6030
A_2	0.0982	0.8191	0.0705	0.7742	0.2867
A_3	0.0147	0.7051	-0.0225	0.7307	0.9285
A_4	0.1895	0.7704	0.1166	0.7540	0

TABLE 8. The comparison of the sorting results of several methods.

Method	Ranking order
DIVPFGWHM	$A_4 \succ A_2 \succ A_3 \succ A_1$
IVPFWA	$A_4 \succ A_2 \succ A_1 \succ A_3$
IVPFGA	$A_4 \succ A_2 \succ A_1 \succ A_3$
IVPF-EDAS method	$A_3 \succ A_1 \succ A_2 \succ A_4$

TABLE 9. The effect of parameters p, q on sorting results in DIVPFGWHM operator.

DIVPFGWHM	Score function	Ranking order
p=0, q=0.5	(0.8560, 0.8711, 0.8596, 0.8839)	$A_4 \succ A_2 \succ A_3 \succ A_1$
p=0.5, q=0	(0.8482, 0.8791, 0.8654 ,0.8503)	$A_2 \succ A_3 \succ A_4 \succ A_1$
p=q=0.5	(0.8515, 0.8831, 0.8555,0.8702)	$A_2 \succ A_4 \succ A_3 \succ A_1$
p=0, q=1	(0.8668, 0.8834, 0.8701, 0.8924)	$A_4 \succ A_2 \succ A_3 \succ A_1$
p=1, q=0	(0.8432, 0.8704, 0.8628, 0.8550)	$A_2 \succ A_3 \succ A_4 \succ A_1$
p=0.5, q=1	(0.8513, 0.8730, 0.8648, 0.8801)	$A_4 \succ A_2 \succ A_3 \succ A_1$
p=1, q=0.5	(0.8376, 0.8689, 0.8398, 0.8490)	$A_2 \succ A_4 \succ A_3 \succ A_1$
p=q=1	(0.8573, 0.8753, 0.8580, 0.8821)	$A_4 \succ A_2 \succ A_3 \succ A_1$
p=2, q=3	(0.8671, 0.8824, 0.8725, 0.9124)	$A_4 \succ A_2 \succ A_3 \succ A_1$
p=3, q=2	(0.8698, 0.8901, 0.8796, 0.9235)	$A_4 \succ A_2 \succ A_3 \succ A_1$
p=q=4	(0.8877, 0.9214, 0.8997, 0.9342)	$A_4 \succ A_2 \succ A_3 \succ A_1$

that of other methods, which is shown in the ordering of alternatives A_3 and A_1. Use the DIVPFGWHM operator, A_3 is better than A_1, while using IVPFWA operator and IVPFGA operator the result obtained is that A_1 is better than A_3, using the EDAS method, the sorting result is completely different, but the optimal alternative of the three operators is A_4, and the other three methods fail to consider the relationship between the attributes. When there is a relationship of complementarity, redundancy, preference, etc. between attributes, the DIVPFGWHM operator has more advantages. Even if the values of multiple parameters are changed, the obtained score function value and the accuracy

function value are quite different, which can effectively overcome the subjectivity brought by the decision maker. At the same time, it is simple and easy to operate in terms of operator aggregation operations. Based on the above analysis, the conclusion drawn in this article is reasonable.

C. SENSITIVITY ANALYSIS
In this section, we discuss the effect of the parameters p, q in the IVPFGWHM operator on the aggregation results. We assign different values to the parameters p, q, respectively, and the results are shown in the following table 9.

As can be seen from Table 9, changes in parameters p, q will affect the ranking results, but the optimal solution is still A_4 or A_2. When p =0 or q =0, the aggregation results do not reflect the interactions between attributes, and thus changes in p, q cause fluctuations in the ranking results; when p, q is small, the interactions between attributes are weak, so changes in p, q will also cause fluctuations in the ranking results; when p, q is large, the interactions between attributes is strong, even if p, q changes, the aggregation results will not make large differences, thus making the ranking results stable. In the actual decision problem, the decision maker can choose appropriate parameters according to the actual needs, but it is not recommended to choose too large or too small parameters.

V. CONCLUSION
In multi-attribute decision-making problems in real life, decision-making information often appears in the form of inaccurate interval numbers, and traditional IVIF fails to consider other information except "consistent" and "inconsistent". In this paper, the interval number and the picture fuzzy number are combined to study the properties and calculation rules of IVPFS. In addition, the decision attributes are often related to each other to varying degrees. The IVPF information is combined with the GHM operator to study IVPFGHM operator and IVPFGWHM operator and IVPFGWHM operator in dynamic environment, and study some properties of IVPFGHM operator, design the detailed process of decision algorithm and simulation example, in dynamic multi-attribute decision problem, use BWM method to solve the weight of each attribute is calculated, and the DIVPFGWHM operator is used to solve it. The result of the calculation example shows the validity and correctness of the operator. Compared with the traditional method, this method considers the interrelationships between

decision attributes, makes the decision analysis closer to the actual situation of the decision problem, the decision result is more reasonable, and provides a new idea for solving the dynamic multi-attribute decision-making problem. The method proposed in this paper has the following advantages.

(1) This method combines the interval number with the PFN to propose the IVPFN, so it describes cognitive information more widely and accurately.

(2) The IVPFGWHM operator proposed by this method has two parameters, p and q, the decision maker can adjust these two parameters according to the input data and subjective preferences to obtain the ranking order. Therefore, this method is more flexible.

(3) The method combines IVPFS with GHM operator, hence it considers the correlation between attributes, and reveals the interaction of factors and the influence of the interaction in the decision-making process and results.

At the same time, the method has the following disadvantages.

(1) The proposed algorithms of IVPFN are not perfect and need further improvement.

(2) The proposed operator only contains the advantage that the HM operator can handle the problem of inter-correlation between attributes and does not consider the effect of anomalous data on the aggregation results.

On the basis of this study, subsequent studies can focus on the following points. In the basic theory, the operation rules and related theories of IVPFN can be further expanded, for example, combining IVPFs with power operators or other operators, or studying the similarity and distance measures between IVPFs to check other types of cognitive information. In terms of application, the method can be used in business decision, medical diagnosis, pattern recognition and other fields.

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