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A Novel Normalized Subband Adaptive Filter Algorithm Based on the Joint-Optimization Scheme

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ABSTRACT Herein, we propose a normalized subband adaptive filter (NSAF) algorithm that adjusts both the step size and regularization parameter. Based on the random-walk model, the proposed algorithm is derived by minimizing the mean-square deviation of the NSAF at each iteration to calculate the optimal parameters. We also propose a method for estimating the uncertainty in an unknown system. Consequently, the proposed algorithm improves performance in terms of tracking speed and misalignment. Simulation results show that the proposed NSAF outperforms existing algorithms in system identification scenarios.

INDEX TERMS Adaptive filter, normalized subband adaptive filter, variable step size, variable regularization parameter, mean-square deviation.

I. INTRODUCTION

Adaptive filter algorithms have been used in a wide range of signal processing applications, such as acoustic echo cancellation, system identification, and channel equalization [1]–[5]. The normalized least-mean-square (NLMS) algorithm is one of the most widely used adaptive filter algorithms owing to its low computational complexity and ease of implementation [6], [7]. However, it exhibits substantial performance degradation in terms of the convergence rates for highly correlated input signals. To address this problem, affine projection (AP) algorithm and normalized subband adaptive filter (NSAF) algorithm were introduced [3], [8], [9]. The AP algorithm achieves fast convergence speeds using multiple input vectors; thus, its computational complexity increases for higher-order systems, such as acoustic echo cancellation. However, despite the lower computational complexity than the AP algorithm, the NSAF algorithms have better performance in terms of tracking speed owing to their self-whitening property. Moreover, some variants of the NSAF, such as sign subband adaptive filter (SSAF) and its

variants [10]–[12], M-estimate NSAF [13], [14], sparsity-aware SSAF and NSAF [15], [16], and bias-compensated NSAF [17]–[19], have been proposed to improve performance.

The NSAF algorithms have two important parameters that affect the performance in terms of convergence rates and steady-state errors, i.e., step size and regularization parameter. The fixed step size and the regularization parameter lead to a trade-off between the convergence rate and misalignment in the NSAF. To control these parameters, variable step-size (VSS) algorithms and variable regularization-parameter (VR) algorithms have been proposed for the NSAF [20]–[28]. Unlike other variable parameter NSAF algorithms, the joint-optimization step size and regularization parameter NSAF (JOSR-NSAF) algorithm controls both the step size and regularization parameter [28]. The JOSR-NSAF algorithm is derived from joint-optimization (JO) scheme [29] based on minimizing the mean-square deviation (MSD). The result of the MSD analysis from the JOSR-NSAF algorithm is similar to that of the VR-NSAF algorithm developed by Jeong *et al.* [27] under stationary environments. However, this analysis method does not accurately reflect the change in the regularization parameters for colored input signals well [26].

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In this study, we propose an NSAF algorithm that controls both the step size and regularization parameter owing to improvements in terms of tracking speed and misalignment. The contribution of this study is threefold. First, an MSD analysis of the NSAF algorithm is provided for both the step size and regularization parameter based on the random-walk model. The proposed method is derived from a method similar to that of the adaptive regularization NSAF algorithm [26]; however, we propose a method that considers the step size and non-stationary systems to obtain both the optimal step size and regularization parameter at each iteration. Second, an optimized NSAF algorithm is derived from the MSD analysis, which control both the step size and regularization parameter to minimize the MSD of the NSAF algorithm at each iteration. Third, a method for estimating the uncertainty of an unknown system is proposed. Simulations tested in the system identification scenario and the performance of the proposed algorithm were compared with those of existing algorithms.

The remainder of this paper is organized as follows. A brief review of the NSAF algorithm is presented in Section II. The MSD performance analysis of the NSAF algorithm is developed in Section III. Section IV introduces an optimal NSAF algorithm. Practical issues are presented for the proposed algorithm in Section V. Finally, in Section VI, the performance of the proposed NSAF algorithm is verified through various computer simulations.

II. REVIEW OF CONVENTIONAL NSAF ALGORITHM

We consider a system identification problem such as acoustic echo cancellation. The weight coefficient vector of the unknown system and input signal at the discrete-time index, n , are denoted by:

$$\mathbf{w}_o = [w_0, w_1, \dots, w_{M-1}]^T, \quad (1)$$

$$\mathbf{u}(n) = [u(n), u(n-1), \dots, u(n-M+1)]^T, \quad (2)$$

where M is filter lengths. The desired signal derived from an unknown system is

$$d(n) = \mathbf{u}^T(n)\mathbf{w}_o + v(n), \quad (3)$$

where $v(n)$ represents measurement noise with zero mean and variance σ_v^2 . Figure 1 shows the structure of the NSAF. In this figure, the desired signal, $d_i(n)$, and input signal, u_i , are derived by the analysis filter, $H_i(z)$, which is defined as

$$H_i(z) = \sum_{j=0}^{L-1} h_i(j)z^{-j}, \quad i = 0, 1, \dots, N, \quad (4)$$

where N represents the number of subbands, L is the length of the analysis filter, and $h_i(j)$ is the j th coefficient of the analysis filter for the i th subband. $d_{i,D}$ and $y_{i,D}$ are obtained by critically sub-sampling d_i and y_i , respectively. Therefore, the downsampled output signal of the i th subband is defined as $y_{i,D}(k) = \mathbf{u}_i^T(k)\widehat{\mathbf{w}}(k)$, where $\mathbf{u}_i(k) = [u_i(kN), u_i(kN-1), \dots, u_i(kN-M+1)]^T$, $\widehat{\mathbf{w}}(k) = [\widehat{w}_0(k), \widehat{w}_1(k), \dots, \widehat{w}_{M-1}(k)]^T$ indicates the estimation of \mathbf{w}_o

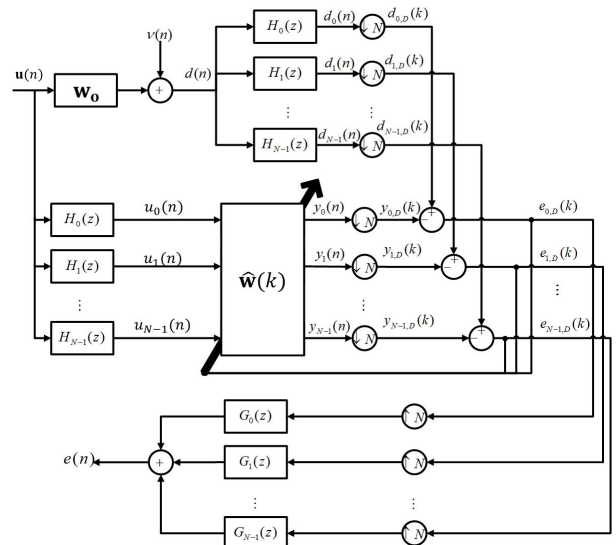


FIGURE 1. Structure of the NSAF.

at index k , and k is used to index the decimated signal. We define the decimated output error signal of the i th subband as follows:

$$\begin{aligned} e_{i,D}(k) &= d_{i,D}(k) - y_{i,D}(k) \\ &= \mathbf{u}_i^T(k)\widehat{\mathbf{w}}(k) + v_{i,D}(k), \end{aligned} \quad (5)$$

where $\widehat{\mathbf{w}}(k) \triangleq \mathbf{w}_o - \mathbf{w}(k)$, and $v_{i,D}(k)$ is the i th subband noise with zero-mean white Gaussian noise with variance $\sigma_{v_i}^2$. Therefore, the update equation of the conventional NSAF algorithm is:

$$\widehat{\mathbf{w}}(k) = \widehat{\mathbf{w}}(k-1) + \sum_{i=0}^{N-1} \frac{\mu_i \mathbf{u}_i(k) e_{i,D}(k)}{\mathbf{u}_i^T(k)\mathbf{u}_i(k) + \beta_i}, \quad (6)$$

where μ_i and β_i are the step size and regularization parameter of the i th subband, respectively.

III. MSD ANALYSIS OF NSAF WITH THE RANDOM-WALK MODEL

A. RANDOM-WALK MODEL

We assume that an unknown weight vector \mathbf{w}_o has the following random-walk model [29], [30]

$$\mathbf{w}_o(n) = \mathbf{w}_o(n-1) + \mathbf{q}(n), \quad (7)$$

where $\mathbf{q}(k)$ is a white Gaussian noise vector with zero mean and variance $m\sigma_q^2 = E\{\mathbf{q}^T(k)\mathbf{q}(k)\}$ and covariance matrix $E\{\mathbf{q}(k)\mathbf{q}^T(k)\} = \sigma_q^2 \mathbf{I}_M$ with \mathbf{I}_M being an $M \times M$ identity matrix. We have an unknown weight vector at the k th iteration as follows:

$$\mathbf{w}_o(k) = \mathbf{w}_o(k-1) + \sum_{j=1}^N \mathbf{q}(kN-j+1). \quad (8)$$

B. PRELIMINARIES

Let us consider the following four assumptions as proposed in [26], [31]–[34] for analysis of the MSD performance of the NSAF with step-size and regularization parameters.

Assumption 1: The input signal vector $\mathbf{u}(n)$ has zero mean and an independent and identically distributed (i.i.d.) and covariance matrix:

$$\mathbf{R} = E \left\{ \mathbf{u}(n)\mathbf{u}^T(n) \right\} = \mathbf{V}\Lambda\mathbf{V}^T, \quad (9)$$

where $\Lambda = \text{diag}(\lambda_1 \lambda_2 \cdots \lambda_M)$ are the eigenvalues of covariance matrix \mathbf{R} , and $\mathbf{V} = [\mathbf{v}_1 \mathbf{v}_2 \cdots \mathbf{v}_M]$ are the corresponding orthonormal eigenvectors ($\mathbf{V}^T\mathbf{V} = \mathbf{I}$).

Assumption 2: The signals $v_{i,D}(k)$, $\mathbf{u}_i(k)$, $\mathbf{q}(k)$, and $\tilde{\mathbf{w}}(k)$ are statistically independent.

Assumption 3: The i th subband input vector $\mathbf{u}_i(k)$ is the product of three independent variables s_i , r_i , \mathbf{v}_i that are i.i.d., such that

$$\mathbf{u}_i(k) = s_i r_i \mathbf{v}_i, \quad \text{where} \begin{cases} P(s_i = \pm 1) = \frac{1}{2} \\ r_i \sim \|\mathbf{u}_i(k)\| \\ P(\mathbf{v}_i = \mathbf{v}_j) = p_j, \quad j = 1, 2, \dots, m \end{cases} \quad (10)$$

Here, $r_i \sim \|\mathbf{u}_i(k)\|$ indicates that r_i has the same distribution as the norm of $\mathbf{u}_i(k)$. Note that $\sum_{j=1}^m p_j = 1$.

C. MSD ANALYSIS OF NSAF

The NSAF update equation (6) can be rewritten in terms of $\tilde{\mathbf{w}}$ using (8) as follows:

$$\tilde{\mathbf{w}}(k) = \tilde{\mathbf{w}}(k-1) - \sum_{i=0}^{N-1} \frac{\mu_i \mathbf{u}_i(k) e_{i,D}(k)}{\mathbf{u}_i^T(k) \mathbf{u}_i(k) + \beta_i} + \sum_{j=1}^N \mathbf{q}(kN+j-1), \quad (11)$$

where $\tilde{\mathbf{w}}(k) \triangleq \mathbf{w}_o(k) - \mathbf{w}(k)$. By subtracting (5) from (8), the decimated output error signals become

$$\begin{aligned} e_{i,D}(k) &= \mathbf{u}_i^T(k) \mathbf{w}_o(k) - \mathbf{u}_i^T(k) \widehat{\mathbf{w}}(k-1) + v_{i,D}(k) \\ &= \mathbf{u}_i^T(k) \tilde{\mathbf{w}}(k-1) + \mathbf{u}_i^T(k) \sum_{j=1}^N \mathbf{q}(kN+j-1) \\ &\quad + v_{i,D}(k). \end{aligned} \quad (12)$$

Considering (12), we rearrange (11) as

$$\begin{aligned} \tilde{\mathbf{w}}(k) &= \left(\mathbf{I}_M - \sum_{i=0}^{N-1} \frac{\mu_i \mathbf{u}_i(k) \mathbf{u}_i^T(k)}{\mathbf{u}_i^T(k) \mathbf{u}_i(k) + \beta_i} \right) \tilde{\mathbf{w}}(k-1) \\ &\quad - \sum_{i=0}^{N-1} \frac{\mu_i \mathbf{u}_i(k) v_{i,D}(k)}{\mathbf{u}_i^T(k) \mathbf{u}_i(k) + \beta_i} \\ &\quad - \sum_{i=0}^{N-1} \frac{\mu_i \mathbf{u}_i(k) \mathbf{u}_i^T(k) \sum_{j=1}^N \mathbf{q}(kN+j-1)}{\mathbf{u}_i^T(k) \mathbf{u}_i(k) + \beta_i} \\ &\quad + \sum_{j=1}^N \mathbf{q}(kN+j-1), \end{aligned} \quad (13)$$

Applying Assumption 2 and 3, the covariance matrix of $\tilde{\mathbf{w}}(k)$ can be obtained as follows:

$$\begin{aligned} \mathbf{P}(k) &= E \left\{ \Phi(k) \mathbf{P}(k-1) \Phi^T(k) \right\} \\ &\quad + E \left\{ \sum_{i=0}^{N-1} \frac{\mu_i^2 \sigma_{v_i}^2 r_i^2}{(r_i^2 + \beta_i)^2} \mathbf{v}_i \mathbf{v}_i^T \right\} \\ &\quad + E \left\{ \sum_{i=0}^{N-1} \frac{\mu_i^2 N \sigma_q^2 r_i^4}{(r_i^2 + \beta_i)^2} \mathbf{v}_i \mathbf{v}_i^T \mathbf{v}_i \mathbf{v}_i^T \right\} \\ &\quad - E \left\{ \sum_{i=0}^{N-1} \frac{2\mu_i N \sigma_q^2 r_i^2}{r_i^2 + \beta_i} \mathbf{v}_i \mathbf{v}_i^T \right\} + N \sigma_q^2 \mathbf{I}_M, \end{aligned} \quad (14)$$

where $\Phi(k) = \mathbf{I}_M - \sum_{i=0}^{N-1} \frac{\mu_i r_i^2 \mathbf{v}_i \mathbf{v}_i^T}{r_i^2 + \beta_i}$.

If we define $\pi_j(k) \triangleq \mathbf{v}_j^T \mathbf{P}(k) \mathbf{v}_j$, by multiplying \mathbf{v}_j^T and \mathbf{v}_j on both side of (14), we obtain:

$$\begin{aligned} \pi_j(k) &= E \left\{ \mathbf{v}_j^T \Phi(k) \mathbf{P}(k-1) \Phi^T(k) \mathbf{v}_j \right\} \\ &\quad + E \left\{ \sum_{i=0}^{N-1} \frac{\mu_i^2 \sigma_{v_i}^2 r_i^2}{(r_i^2 + \beta_i)^2} \mathbf{v}_j^T \mathbf{v}_i \mathbf{v}_i^T \mathbf{v}_j \right\} \\ &\quad + E \left\{ \sum_{i=0}^{N-1} \frac{\mu_i^2 N \sigma_q^2 r_i^4}{(r_i^2 + \beta_i)^2} \mathbf{v}_j^T \mathbf{v}_i \mathbf{v}_i^T \mathbf{v}_i \mathbf{v}_i^T \mathbf{v}_j \right\} \\ &\quad - E \left\{ \sum_{i=0}^{N-1} \frac{2\mu_i N \sigma_q^2 r_i^2}{r_i^2 + \beta_i} \mathbf{v}_j^T \mathbf{v}_i \mathbf{v}_i^T \mathbf{v}_j \right\} + N \sigma_q^2 \mathbf{v}_j^T \mathbf{v}_j \\ &= \sum_{q=1}^M p_q E \left\{ \mathbf{v}_j^T \left(\mathbf{I} - \sum_{i=0}^{N-1} \alpha_i r_i^2 \mathbf{v}_q \mathbf{v}_q^T \right) \mathbf{P}(k-1) \right. \\ &\quad \times \left. \left(\mathbf{I} - \sum_{i=0}^{N-1} \alpha_i r_i^2 \mathbf{v}_q \mathbf{v}_q^T \right)^T \mathbf{v}_j \right\} \\ &\quad + \sum_{q=1}^M p_q E \left\{ \sum_{i=0}^{N-1} \alpha_i^2 \sigma_{v_i}^2 r_i^2 \mathbf{v}_j^T \mathbf{v}_q \mathbf{v}_q^T \mathbf{v}_j \right\} \\ &\quad + \sum_{q=1}^M p_q E \left\{ \sum_{i=0}^{N-1} \alpha_i^2 N \sigma_q^2 r_i^4 \mathbf{v}_j^T \mathbf{v}_q \mathbf{v}_q^T \mathbf{v}_q \mathbf{v}_q^T \mathbf{v}_j \right\} \\ &\quad - \sum_{q=1}^M p_q E \left\{ \sum_{i=0}^{N-1} 2\alpha_i N \sigma_q^2 r_i^2 \mathbf{v}_j^T \mathbf{v}_q \mathbf{v}_q^T \mathbf{v}_j \right\} + N \sigma_q^2 \end{aligned} \quad (15)$$

where $\alpha_i \triangleq \mu_i / (r_i^2 + \beta_i)$. From [2, eq.(18) and (20)], the recursion of $\pi_j(k)$ for the i th subband can be expressed as follows:

$$\begin{aligned} \pi_{i,j}(k) &= \left[1 + \frac{N}{M} \left\{ (\alpha_i r_i^2)^2 - 2\alpha_i r_i^2 \right\} \right] (\pi_{i,j}(k-1) + N \sigma_q^2) \\ &\quad + \frac{N \alpha_i^2 r_i^2 \sigma_{v_i}^2}{M}, \end{aligned} \quad (16)$$

when $M \gg 1$.

To conduct the MSD analysis of the NSAF, the MSD is defined as $\text{MSD}(k) \triangleq E(\tilde{\mathbf{w}}^T(k) \tilde{\mathbf{w}}(k)) = \text{Tr}(\mathbf{P}(k))$. Finally,

we obtain

$$\begin{aligned} \text{MSD}(k) &= \text{Tr}(\mathbf{P}(k)) = \frac{1}{N} \text{Tr}(\mathbf{V}^T \mathbf{P}(k) \mathbf{V}) \\ &= \frac{1}{N} \sum_{i=0}^{N-1} \sum_{j=1}^M \pi_{i,j}(k) \\ &= \frac{1}{N} \sum_{i=0}^{N-1} f_i(k), \end{aligned} \quad (17)$$

where

$$\begin{aligned} f_i(k) &\triangleq \sum_{j=1}^M \pi_{i,j}(k) \\ &= \left[1 + \frac{N}{M} \left\{ (\alpha_i r_i^2)^2 - 2\alpha_i r_i^2 \right\} \right] (f_i(k-1) + MN\sigma_q^2) \\ &\quad + N\alpha_i^2 r_i^2 \sigma_{v_i}^2. \end{aligned} \quad (18)$$

IV. OPTIMIZED NSAF ALGORITHM BASED ON THE JOINT-OPTIMIZATION SCHEME

To rapidly minimize the MSD of the NSAF at each iteration, a joint-optimization strategy can be derived by $\partial \text{MSD}(k) / \partial \mu_i(k) = \partial \text{MSD}(k) / \partial \beta_i(k) = 0$. Therefore, we can obtain a novel update equation of the NSAF with optimal step size $\mu_i(k)$ and regularization parameter $\beta_i(k)$ as follows:

$$\hat{\mathbf{w}}(k) = \hat{\mathbf{w}}(k-1) + \sum_{i=0}^{N-1} \alpha_i^*(k) \mathbf{u}_i(k) e_{i,D}(k), \quad (19)$$

where

$$g_i(k) \triangleq f_i(k-1) + MN\sigma_q^2, \quad (20)$$

$$\alpha_i^*(k) \triangleq \frac{\mu_i(k)}{r_i^2(k) + \beta_i(k)} = \frac{g_i(k)}{r_i^2 g_i(k) + M\sigma_{v_i}^2}, \quad (21)$$

$$f_i(k) = \left\{ 1 - \frac{N}{M} (r_i^2 \alpha_i^*(k)) \right\} g_i(k). \quad (22)$$

Substituting (20) and (21) into (22), we obtain the quadratic equation for the steady-state value of $f_i(k)$ as follows:

$$\begin{aligned} r_i^2 (f_{i,ss} + MN\sigma_q^2)^2 - M^2 r_i^2 \sigma_q^2 (f_{i,ss} + MN\sigma_q^2) \\ - M^3 \sigma_q^2 \sigma_{v_i}^2 = 0, \end{aligned} \quad (23)$$

where $f_{i,ss} \triangleq \lim_{k \rightarrow \infty} f_i(k)$. Therefore, the steady-state value of MSD can be obtained as

$$\lim_{k \rightarrow \infty} \text{MSD}(k) = \lim_{k \rightarrow \infty} \text{Tr}(\mathbf{P}(k)) = \frac{1}{N} \sum_{i=0}^{N-1} f_{i,ss}, \quad (24)$$

where

$$f_{i,ss} = \frac{M^2 \sigma_q^2}{2} \left(1 - \frac{2N}{M} + \sqrt{1 + \frac{4\sigma_{v_i}^2}{M r_i^2 \sigma_q^2}} \right). \quad (25)$$

The performance limit of the proposed algorithm is related to the system parameters (N , M), measurement noise, (σ_v^2), and non-stationary environments, (σ_q^2).

V. PRACTICAL CONSIDERATIONS

The proposed NSAF algorithm depends on the parameters r_i , $\sigma_{v_i}^2$, and σ_q^2 . Therefore, practical considerations are necessary.

The first parameter, r_i , is related to the subband input variance $\sigma_{u_i}^2$. Therefore, we can use the subband input variance instead of r_i^2 as follows:

$$r_i^2 \simeq M\sigma_{u_i}^2(k) = E \left\{ \mathbf{u}_i^T(k) \mathbf{u}_i(k) \right\}. \quad (26)$$

Moreover, the input variance is easily estimated as follows [28], [29]:

$$\hat{\sigma}_{u_i}^2(k) = \frac{1}{M} \mathbf{u}_i^T(k) \mathbf{u}_i(k). \quad (27)$$

The second parameter is the subband noise variance, $\sigma_{v_i}^2$, which can be easily estimated [31]–[34]. We assume that the subband noise variance, $\sigma_{v_i}^2$, is already known, because it is out of the scope of this study.

The third parameter, σ_q^2 , denotes the uncertainties in unknown system $\mathbf{w}_o(n)$. The performance of the proposed NSAF algorithm is affected by the value of σ_q^2 . Therefore, it is important to estimate the value of σ_q^2 for implementing the proposed NSAF algorithm. A method for estimating σ_q^2 using the random-walk model (8) and replacing $\mathbf{w}_o(k)$ with $\hat{\mathbf{w}}(k)$ is as follows:

$$\sum_{j=1}^N \|\mathbf{q}(kN-j+1)\|^2 = \|\hat{\mathbf{w}}(k) - \hat{\mathbf{w}}(k-1)\|^2. \quad (28)$$

We define the estimated parameter, $\sigma_{q,1}^2$, which is estimated using the above method as

$$\hat{\sigma}_{q,1}^2(k) \triangleq \frac{\|\hat{\mathbf{w}}(k) - \hat{\mathbf{w}}(k-1)\|^2}{NM}. \quad (29)$$

When a system undergoes an abrupt change, the parameter $\hat{\sigma}_{q,1}^2(k)$ has large values, which provides a fast convergence speed. However, an error in the estimation of the parameter, $\hat{\sigma}_{q,1}^2(k)$ causes an error in deriving the optimal parameter, $\alpha_i^*(k)$, which leads to significant misalignment. Thus, we propose a method to estimate σ_q^2 using multiband mean-squared error (MSE), which is defined as follows:

$$\zeta(k) \triangleq \frac{1}{N} \sum_{i=0}^{N-1} E \left(\|e_{i,D}(k)\|^2 \right) \quad (30)$$

Applying Assumption 2, the multiband MSE can be rewritten as

$$\begin{aligned} \zeta(k) &= \frac{1}{N} \sum_{i=0}^{N-1} E \left(\|\mathbf{u}_i^T(k) \tilde{\mathbf{w}}(k-1)\|^2 \right) \\ &\quad + \frac{1}{N} \sum_{i=0}^{N-1} E \left(\left\| \mathbf{u}_i^T(k) \sum_{j=1}^N \mathbf{q}(kN+j-1) \right\|^2 \right) \\ &\quad + \frac{1}{N} \sum_{i=0}^{N-1} E \left(\|v_{i,D}(k)\|^2 \right) \end{aligned} \quad (31)$$

TABLE 1. Summary of the proposed algorithm.

Initialization.
$\hat{\mathbf{w}}(0) = \mathbf{0}, f_i(0) = 1$
Parameters.
γ , predefined
$\sigma_{v_i}^2$, known or estimated
Adaptation.
For time index $k = 1, 2, \dots$
$M\hat{\sigma}_{u_i}^2(k) = \mathbf{u}_i^T(k)\mathbf{u}_i(k)$
$\sigma_{e_i}^2(k) = \gamma\sigma_{e_i}^2(k-1) + (1-\gamma)e_{i,D}(k)^2$
$g_i(k) = f_i(k-1) + MN\hat{\sigma}_q^2(k-1)$
$\alpha_i^*(k) = \frac{\mu_i(k)}{\hat{\sigma}_{u_i}^2(k) + \beta_i(k)} = \frac{g_i(k)}{\hat{\sigma}_{u_i}^2(k)g_i(k) + M\sigma_{v_i}^2}$
$f_i(k) = \left\{1 - \frac{N}{M}(\hat{\sigma}_{u_i}^2(k)\alpha_i^*(k))\right\}g_i(k)$
$\hat{\mathbf{w}}(k) = \hat{\mathbf{w}}(k-1) + \sum_{i=0}^{N-1}\alpha_i^*(k)\mathbf{u}_i(k)e_{i,D}(k)$
$\hat{\sigma}_{q,1}^2(k) = \frac{\ \hat{\mathbf{w}}(k) - \hat{\mathbf{w}}(k-1)\ ^2}{NM}$
$\hat{\sigma}_{q,2}^2(k) = \max\left\{\frac{\sum_{i=0}^{N-1}(\sigma_{e_i}^2(k) - \sigma_{v_i}^2 - \hat{\sigma}_{u_i}^2(k)f_i(k-1))}{NM\sum_{i=0}^{N-1}\hat{\sigma}_{u_i}^2(k)}, 0\right\}$
$\hat{\sigma}_q^2(k) = \min\{\hat{\sigma}_{q,1}^2(k), \hat{\sigma}_{q,2}^2(k)\}.$

From [[26], eq(33)]

$$\frac{1}{N} \sum_{i=0}^{N-1} \sigma_{e_i}^2(k) = \frac{1}{N} \sum_{i=0}^{N-1} \left(\sigma_{u_i}^2(k)f_i(k-1) + NM\sigma_{u_i}^2(k)\sigma_q^2(k) + \sigma_{v_i}^2 \right), \quad (32)$$

where $\sigma_{e_i}^2(k) = \gamma\sigma_{e_i}^2(k-1) + (1-\gamma)e_{i,D}(k)^2$, and γ is the smoothing factor ($0 \ll \gamma < 1$). We define the estimated parameter $\hat{\sigma}_{q,2}^2$, which is estimated using the above equation (32) as

$$\hat{\sigma}_{q,2}^2(k) \triangleq \max\left\{\frac{\sum_{i=0}^{N-1}(\sigma_{e_i}^2(k) - \sigma_{v_i}^2 - \hat{\sigma}_{u_i}^2(k)f_i(k-1))}{NM\sum_{i=0}^{N-1}\hat{\sigma}_{u_i}^2(k)}, 0\right\} \quad (33)$$

When a system is stable in a stationary environment, the subband error variance $\sigma_{e_i}^2(k)$ and $\sigma_{u_i}^2(k)f_i(k-1) + \sigma_{v_i}^2$ become equal; thus, $\hat{\sigma}_{q,2}^2(k)$ takes small values leading to small misalignment under a stationary environment. To reduce performance degradation, the parameter $\hat{\sigma}_q^2(k)$ is estimated as

$$\hat{\sigma}_q^2(k) \triangleq \min\left\{\hat{\sigma}_{q,1}^2(k), \hat{\sigma}_{q,2}^2(k)\right\}. \quad (34)$$

The proposed algorithm is summarized in Table 1.

VI. SIMULATION RESULTS

Computer simulations were performed using the system identification model to verify the MSD performance of the

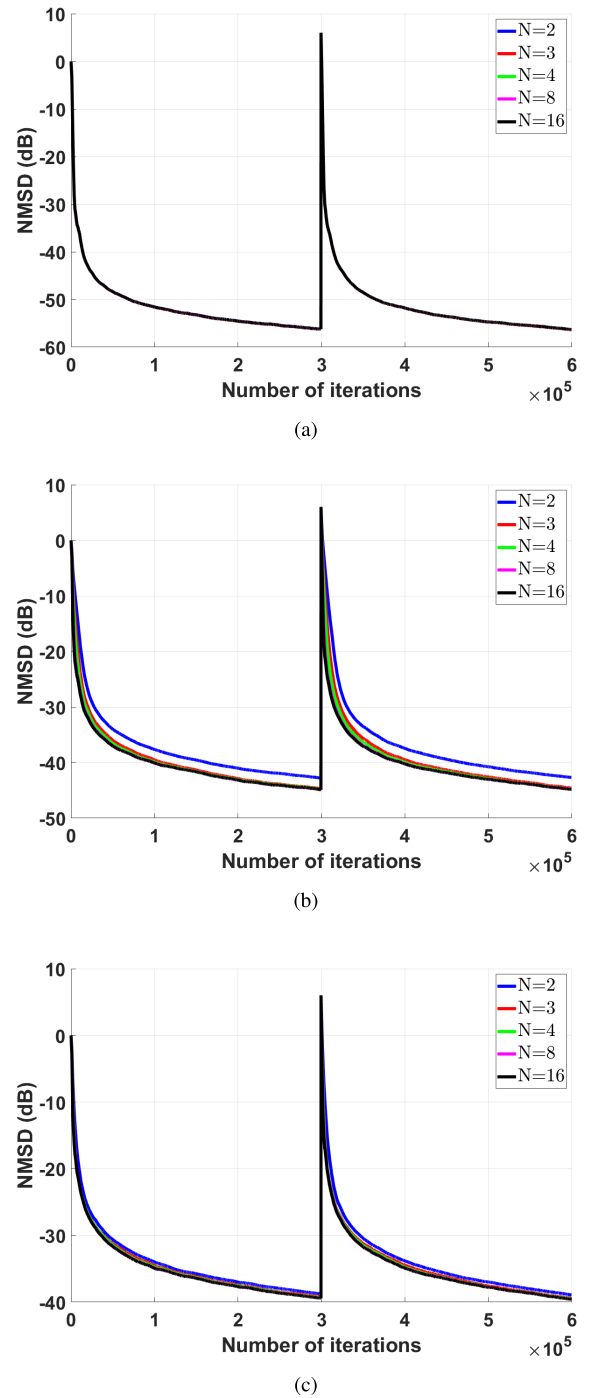
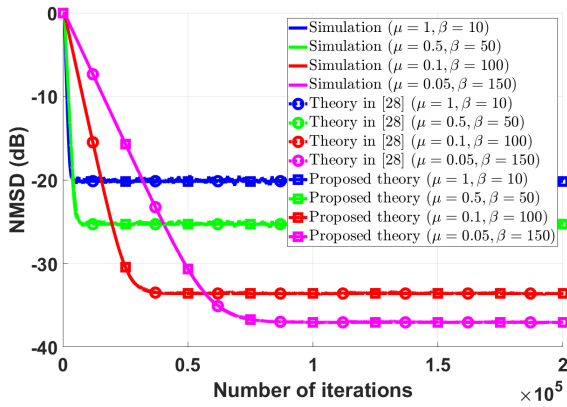
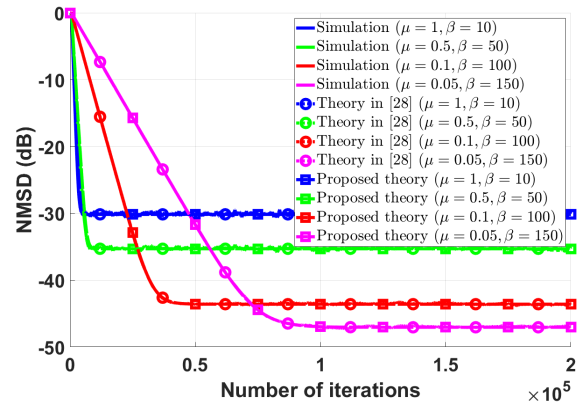


FIGURE 2. NMSD learning curves of proposed NSAFs according to the number of subbands N with sudden system change environment (SNR = 30dB, $L = 8 \times N$). (a) white Gaussian input (b) AR₁ input (c) AR₂ input.

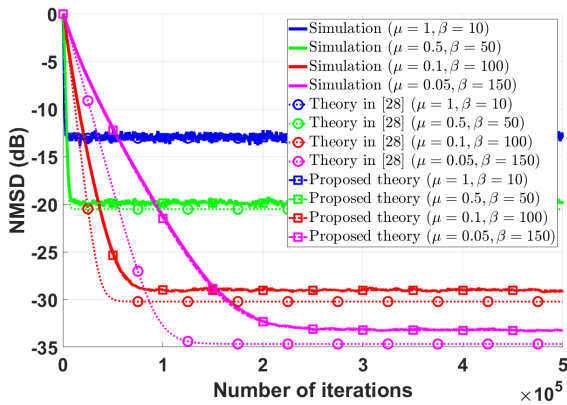
proposed algorithm. The unknown system coefficient, \mathbf{w}_o was randomly generated with unit variance for the MSD analysis comparison and the acoustic impulse response of a room for performance comparison. It was assumed that the adaptive filter and the unknown system have the same number of filter length, which was set $M = 512$ for computer simulations.



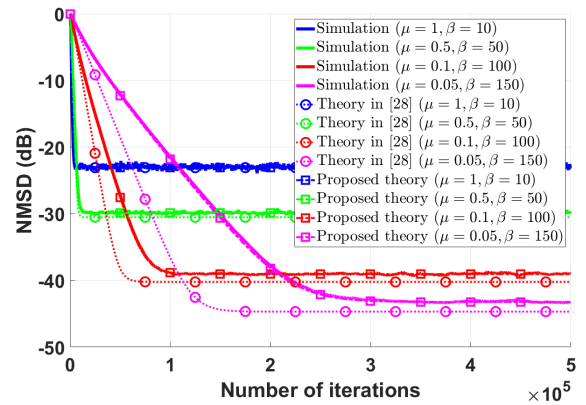
(a)



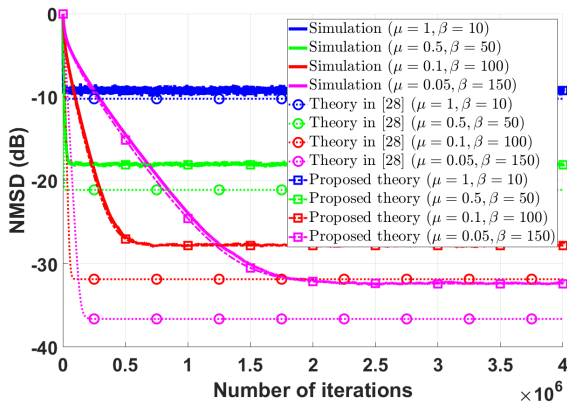
(a)



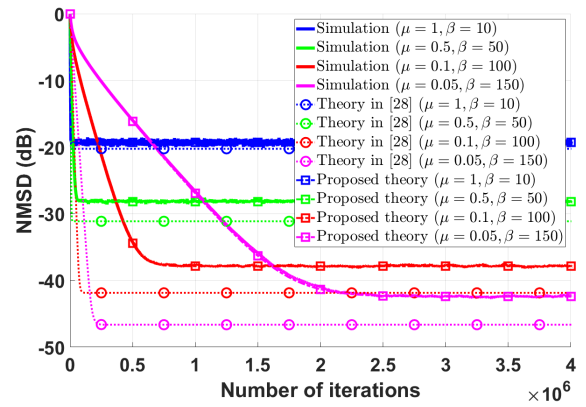
(b)



(b)



(c)



(c)

FIGURE 3. NMSD learning curves of the experimental results (solid lines), theoretical analysis results, and the proposed method (SNR = 20dB). (a) whit Gaussian input (b) AR₁ input (c) AR₂.

The colored input signals were generated by filtering the white Gaussian noise through

$$G_1(z) = \frac{1}{1 - 0.95z^{-1}}, \quad (35)$$

$$G_2(z) = \frac{1}{1 - 1.6z^{-1} + 0.81z^{-2}}, \quad (36)$$

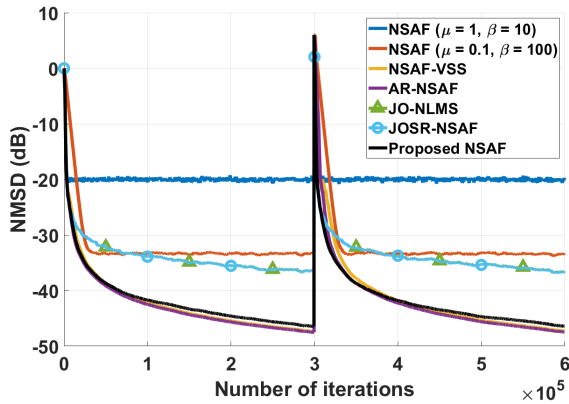
which are referred to as AR₁, and AR₂. The signal-to-noise ratio (SNR) was set to 20dB or 30dB to add the

FIGURE 4. NMSD learning curves of the experimental results (solid lines), theoretical analysis results, and the proposed method (SNR = 30dB). (a) whit Gaussian input (b) AR₁ input (c) AR₂.

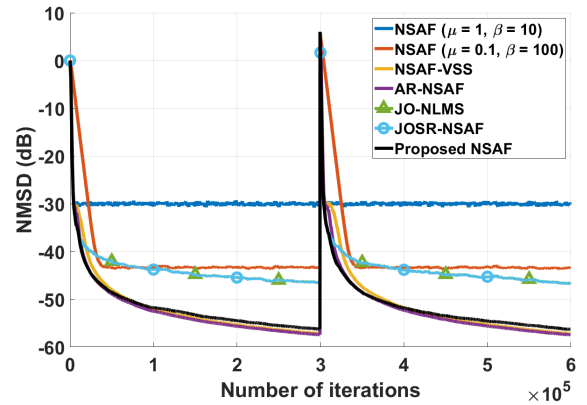
measurement noise at the output signal y_i where the SNR is defined as

$$\text{SNR} \triangleq 10 \log_{10} \frac{E \left\{ (\mathbf{u}(n)^T \mathbf{w}_o)^2 \right\}}{E \left\{ v(n)^2 \right\}}. \quad (37)$$

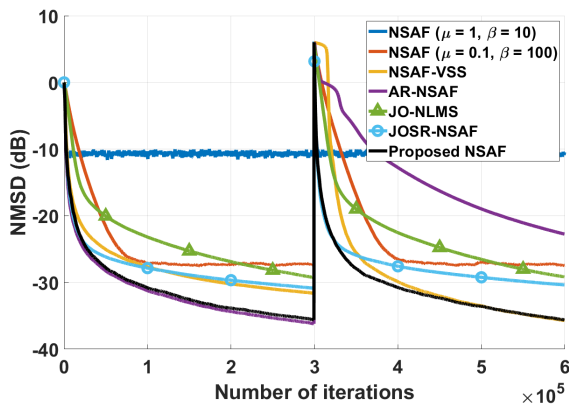
In addition, we assumed that the noise variance was known. The measure of performance is the normalized mean squared



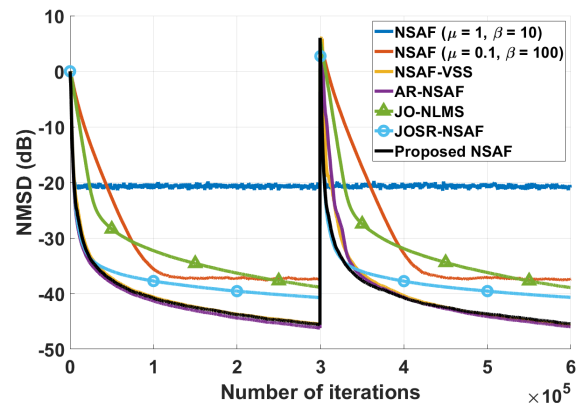
(a)



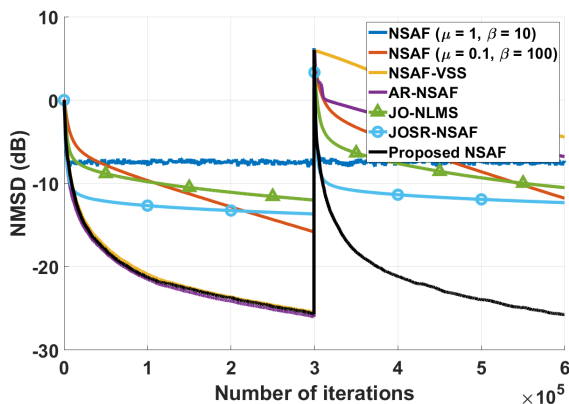
(a)



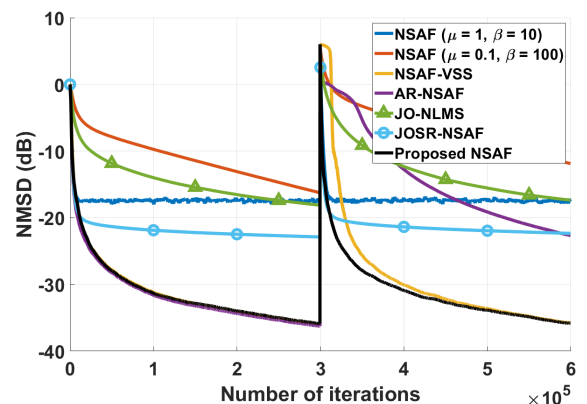
(b)



(b)



(c)



(c)

FIGURE 5. NMSD learning curves of proposed NSAFs, JO-NLMS, JOSR-NSAF, and standard NSAFs with various step sizes (SNR = 20dB). (a) whit Gaussian input (b) AR₁ input (c) AR₂.

FIGURE 6. NMSD learning curves of proposed NSAFs, JO-NLMS, JOSR-NSAF, and standard NSAFs with various step sizes (SNR = 30dB). (a) whit Gaussian input (b) AR₁ input (c) AR₂.

deviation (NMSD), which is defined as:

$$\text{NMSD} \triangleq 10 \log_{10} E \left\{ \frac{\tilde{\mathbf{w}}^T(k) \tilde{\mathbf{w}}(k)}{(\mathbf{w}_o^T \mathbf{w}_o)} \right\}. \quad (38)$$

The simulation results were obtained by ensemble averaging over 10 trials. For the proposed algorithm, we set $\gamma = 1 - \kappa N/M$, where $\kappa = 2$.

A. SELF WHITENING EFFECT DEPENDING ON THE NUMBER OF SUBBANDS

The convergence performance of the subband adaptive filter is almost same for white Gaussian input signal; the proposed NSAF algorithm is shown in Figure 2(a). However, the convergence rate of the subband adaptive filter is further improved with an increased number of subbands owing to the self-input whitening effect for colored input signals.

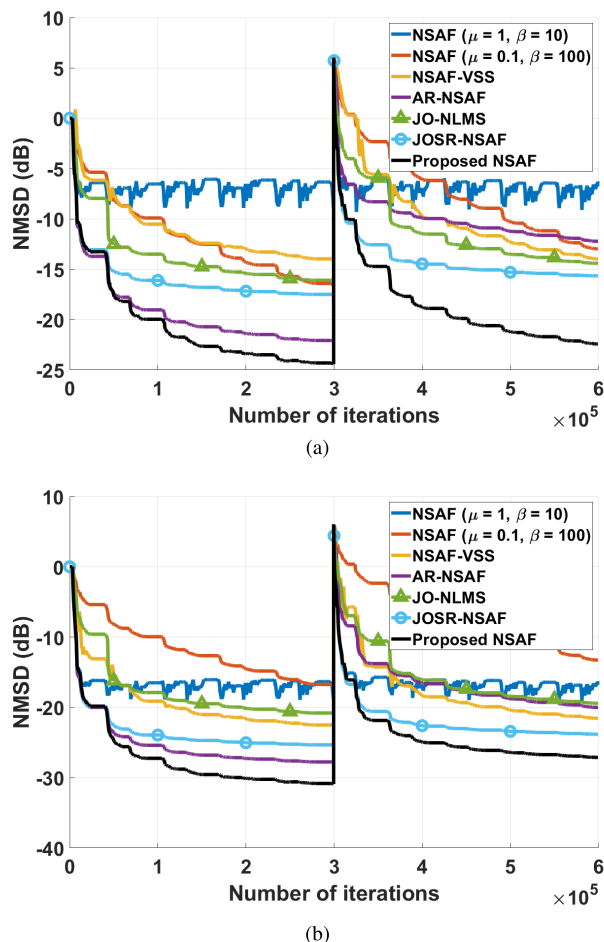


FIGURE 7. NMSD learning curves of proposed NSAFs, JO-NLMS, JOSR-NSAF, and standard NSAFs with various step sizes for speech input signal. (a) SNR = 20dB; (b) SNR = 30dB.

Figure 2(b) and 2(c) show that the NMSD learning curves of the proposed NSAFs according to the number of subbands in AR₁ and AR₂ input environments, respectively. As can be seen, the convergence performance of the proposed NSAF is saturated at $N = 8$, implying that the colored input signals are sufficiently whited at $N = 8$.

B. MSD ANALYSIS COMPARISON

Figure 3 and 4 show the NMSD learning curves of the experimental results and theoretical analysis results for white Gaussian and colored input signals. As shown in Figure 3(a) and 4(a), the analysis methods of [28] and proposed are match well with the experimental results for white Gaussian input signals. However, Figure 3(b), 4(b), 3(c), and 4(c) show a discrepancy between the theoretical analysis results that are obtained by [28] and the experimental results for both convergence and steady-state behaviors. The proposed analysis can properly estimate the NMSD learning curves of the NSAFs that have various step sizes and regularization parameters in colored input environments as shown in Figure 3 and 4.

C. PERFORMANCE COMPARISON

Figure 5 and 6 show the NMSD learning curves for the conventional NSAFs, NSAF-VSS [21], AR-NSAF [26], JO-NLMS [29], and JOSR-NSAF [28] with white Gaussian, AR₁, and AR₂ input signals. All algorithms needed to tune several parameters that were set according to the recommendations provided in [21], [26], [28], [29]. The NSAF-VSS algorithm controls only step-size values with zero regularization parameters and the AR-NSAF algorithm controls only regularization parameters with a unit step size. However, the proposed algorithm and JO algorithms control both parameters. As can be seen in figure 5 and 6, the JO algorithms have low convergence rates and high misalignments than the existing VSS and VR algorithms, especially when the input signal is white Gaussian. The proposed algorithm has steady-state errors performance similar to VSS and VR algorithms, and provides faster convergence rates than existing algorithms when the unknown system changes abruptly. In Figure 7, the proposed algorithm outperforms the existing VSS, VR, and JO algorithms in terms of the steady-state error and the convergence speed when the input signal is a speech sequence.

VII. CONCLUSION

In this paper, we presented an NSAF algorithm that controls both the step size and regularization parameter by minimizing the MSD performance of the NSAF at each iteration. To deal with optimal parameters, the MSD analysis method for step size and regularization parameter was proposed based on the random-walk model. We then proposed an estimation algorithm for calculating the uncertainty of the unknown system to improve the performance of the proposed NSAF algorithm in terms of misalignments and convergence rates. Simulation results verified that the proposed NSAF with adjusting parameters had a fast convergence rate and low misalignment as compared with the existing algorithms in the system identification scenario.

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