IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received December 18, 2021, accepted January 7, 2022, date of publication January 12, 2022, date of current version January 18, 2022.

Digital Object Identifier 10.1109/ACCESS.2022.3142539

Consensus of Heterogeneous Multi-Agent Systems Under Directed Topology

HUA GENG[1], HONGYANG WU[1], JINGLI MIAO[1], SHUAI HOU[1], AND ZENGQIANG CHEN[2]
[1]School of Information and Electrical Engineering, Hebei University of Engineering, Handan 056038, China
[2]College of Artificial Intelligence, Nankai University, Tianjin 300350, China

Corresponding author: Hua Geng (huahua27102710@163.com)

This work was supported by the Natural Science Foundation of Hebei Province under Grant F2019402419.

ABSTRACT In this paper, the consensus problem of heterogeneous multi-agent systems under directed topology is investigated. Specifically, this system is composed of three classes of agents respectively described by first-order, second-order and third-order integrator dynamics. By the aid of linear filter, graph theory and matrix theory, the consensus problem is realized based on the two proposed consensus protocols. Moreover, group consensus can also be solved by adjusting parameters. Finally, some examples were presented to illustrate the theoretical results. Our study is expected to establish a more realistic model and provide a method to solve the consensus problem of heterogeneous multi-agent systems in more complex situations.

INDEX TERMS Consensus, directed topology, multi-agent systems, heterogeneous, linear filter.

I. INTRODUCTION

In recent years, much attention has been received for multi-agent systems because of its wide application, such as military [1], energy management [2], sensor network [3], transport [4], etc. In practice, agents often need to have a common goal, reach the same place or work in the same state, etc. Therefore, it is meaningful to study the consensus problem of multi-agent systems. As a basic problem of multi-agent systems, consensus was first investigated in [5], [6]. The theoretical frameworks of the consensus problem and the basic method of solving consensus were presented. Subsequently, second-order integrator agent systems [7], high-order integrator agent systems [8], nonlinear agent systems [9] and multi-agent systems in different application scenarios [10], [11] have been studied and some meaningful conclusions have been obtained. Inspired by the prominent work, the consensus problem in some special application scenarios was studied. He etc. [12] investigated the consensus problem in a more realistic scenario, which associated with noisy environments, leader-following networks, high-order nonlinear dynamical, switching topology and communication delay. The proposed consensus protocol was proved that it was robust against the bounded communication delay in noisy environments. In [13], the consensus of partial differential equation (PDE) agents was discussed. Based on adaptive distributed unit-vector control law and Lyapunov direct method, the condition of asymptotical consensus and synchronization for uncertain parabolic partial differential equation (PDE) agents with Neumann boundary condition was obtained. It could solve the leader-following tracking problem of uncertain parabolic partial differential equation (PDE) agents. Recently, experts have obtained some insightful results in this field. For example, novel consensus protocol using only relative state was proposed in [14]. It was effective for the discrete-time second-order integrator system with time-varying delay and switching topology, in which the velocity of each agent was not required to zero value. In [15], the containment control problem was solved for general linear multi-agent systems (MASs) under the asynchronous setting. An asynchronous distributed algorithm was proposed for the linear multi-agent systems (MASs). By the aid of non-negative matrix theory and the composition of binary relations, the asynchronous containment control problem was well solved. In [16], based on the properties of sub-stochastic matrix and super-stochastic matrix, the tracking issues of multi-agent systems (MASs) over signed networks was investigated in different scenarios, such as first-order MASs, second-order MASs, general linear MASs, time delays, switching topologies, random

The associate editor coordinating the review of this manuscript and approving it for publication was Hosam El-Ocla.

networks and so on. In [17], the bipartite tracking consensus was studied over directed cooperation-competition networks. These conclusions were valid for both fixed and switching topology networks.

Most of the above results are mainly focused on solving the consensus problem for the homogeneous multi-agent systems, which composed of a class of agents with homogeneous dynamics. These agents work together to complete a complex task through coordination, cooperation, competition and scheduling. When agents are described as the same dynamics, multi-agent systems can be modeled as homogeneous systems with first-order, second-order or higher-order agents respectively. However, in reality, many complex systems are composed of various individuals with different characteristics due to various constraints, such as Robot World Cup. Different kinds of robots have different tasks, and the models are different. When the control input of the agents is velocity, acceleration or jerk, the agents can be described as first-order, second-order or third-order dynamics respectively. Obviously, in this case, modeling multi-agent system as a heterogeneous system is a better choice. It is more practical and challenging. Also, there is a growing literature on this meaningful issue. In [18], the consensus problem of heterogeneous multi-agent systems composed of first-order and second-order integrator agents was first investigated. By the aid of Lyapunov direct method, some consensus protocols were presented when the graph was undirected connected. Considering the special situation that the speed state of second-order agents was not measurable, some consensus protocols without velocity information were proposed in the scenarios of input constraints and leader-following network [19]. In [20], based on the proposed consensus protocols with input saturation, consensus problem, aggregation problem and tracking problem were solved when the communication topology was undirected connected. Instead of the linear dynamics case, the nonlinear dynamics systems were studied in [21]. The consensus protocols with adaptive control law were feasible in both nonlinear first-order system case and nonlinear second-order system case. It was independent of any global information and avoided the continuous monitoring issue. In [22], by designing distributed observers and tracking controllers, the fixed-time consensus problem of heterogeneous multi-agent systems with first-order nonlinear agents and second-order nonlinear agents was solved. It was a more challenging topic than the other consensus problems, such as mean consensus, finite-time consensus and min consensus, etc. In [23], the heterogeneous multi-agent systems with first-order and second-order agents were dealt with as quasi-consensus problem of homogenous multi-agent networks. Under undirected networks, some important results were obtained. In addition to the research on undirected networks scenario, the past work has also made a great contribution to the research on consensus problem of heterogeneous systems under directed network. For example, by the aid of the finite-time stability theory and LaSalle's invariance principle, the proposed nonlinear protocols could solve the consensus problem for mixed-order systems in finite time under directed topology [24]. In [25], the group consensus was discussed for discrete-time mixed-order multi-agent systems under fixed and directed interactive topology. Furthermore, due to the needs of some specific application scenarios, the consensus of high-order multi-agent systems with different dynamics has also been studied. In [26], the finite-time consensus problem for a general class of high-order nonlinear heterogeneous multi-agent systems was investigated. The integral sliding-mode control technique and a new variable-gain finite-time observer were used to help solve the output finite-time consensus of high-order multi-agent systems with heterogeneous dynamics. In [27], a fully-distributed Proportional-Integral-Derivative (PID) control strategy was proposed to solve the leader-tracking and the containment control problems for heterogeneous high-order multi-agent systems. Each follower would track the leader and converge to the convex hull spanned by the multiple leaders in the case of containment control.

Up to now, the researches on heterogeneous multi-agent systems are mainly about mixed-order system which composed of first-order and second-order integrator agents or homogeneous-order multi-agent systems with different dynamics. More general and complex situations have not been addressed. However, in reality, heterogeneous systems may be composed of various individuals with different characteristics, structure and dynamics. A special case is the multi intelligent vehicle cooperative control system. When some intelligent vehicles can carry people, comfort is an important parameter, which is determined by jerk. Obviously, it is appropriate to describe these agents as third-order integrator dynamics. So the intelligent vehicle cooperative control system is modeled as a heterogeneous multi-agent system composed of first-order (input is velocity), second-order (input is acceleration) and third-order agents (input is jerk). In addition, since the communication ability of different agents is different, the directed case is more realistic and more reasonable. In this paper, motivated by these issues, we investigate the consensus of a heterogeneous multi-agent system composed of first-order, second-order and third-order integrator agents under directed topology. The proposed protocols can solve the consensus problem for this more complex system and they can be extended to the more complex heterogeneous multi-agent system composed of arbitrary order integrator agents by adjusting the parameters of control law. Moreover, by selecting different control parameters, the group consensus can also be solved. Our study is expected to establish a more realistic model and provide a method to solve the consensus problem of heterogeneous multi-agent systems in more complex situations.

The rest of the paper is organized as follows. In Section 2, some basic knowledge, definition and lemma are briefly outlined. In Section 3, two consensus protocols are proposed. Theoretical analysis shows that the system can achieve exponential consensus with the protocols. Moreover, group consensus can also be solved by adjusting parameters.

In Section 4, some examples are given. Finally, we get the conclusion in Section 5.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. GRAPH THEORY

The communication among agents is represented by graph. A directed graph G is composed of a pair (V, E), where $V = \{1, 2, \ldots, n\}$ is a finite, nonempty set of nodes and $E \subseteq V \times V$ is a set of ordered pairs of nodes. An edge (i, j) denotes node i can receive information from node j. Directed spanning tree is at least one node can transmit information along the edge $\{i, j\}, \{j, k\} \ldots$ sequence path to any other node. The weighted adjacency matrix of G is denoted by $A = [a_{ij}] \in R^{n\times n}$, where $a_{ii} = 0$ and $a_{ij} = 1$ (if there is an edge from node j to node i). Its degree matrix $D = diag\{d_1, d_2, \ldots, d_n\} \in R^{n\times n}$ is a diagonal matrix, where diagonal elements $d_i = \sum_{j=1}^{n} a_{ij}$ for $i = 1, 2, \ldots, n$. Then the Laplacian matrix of the weighted graph is denoted by $L = D - A$, which is asymmetric and row sum is zero.

The next lemma is given to establish the relationship between graph and Laplacian matrix.

Lemma 1 [28]: Let G be a graph on n vertices with Laplacian L, then L has at least one zero eigenvalue and all of the nonzero eigenvalues are in the open right half plane. Furthermore, L has exactly one zero eigenvalue if and only if the directed graph associated with L has a spanning tree. Moreover, $\mathbf{1} = [1, \ldots, 1]^{\mathrm{T}}$ is the eigenvector corresponding to the zero eigenvalue.

B. HETEROGENEOUS MULTI-AGENT SYSTEMS

In this subsection, the composition of heterogeneous multi-agent systems, the definition of consensus and an important lemma are given.

In this paper, the heterogeneous multi-agent system is composed of three classes of agents respectively described by first-order, second-order and third-order integrator dynamics. The number of first-order agents, second-order agents and third-order agents are $l, m - l, n - m$ respectively, where $0 < l < m < n$.

The first-order agent dynamics can be expressed as follows.

$$\dot{x}_i(t) = u_i(t) \tag{1}$$

where $x_i(t) \in R$ is the position information and $u_i(t) \in R$ is the input.

The second-order agent dynamics can be expressed as follows.

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t) \end{cases} \tag{2}$$

where $x_i(t) \in R$ is the position information, $v_i(t) \in R$ is the velocity information and $u_i(t) \in R$ is the input.

The third-order agent dynamics can be expressed as follows.

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = z_i(t) \\ \dot{z}_i(t) = u_i(t) \end{cases} \tag{3}$$

where $x_i(t) \in R$ is the position information, $v_i(t) \in R$ is the velocity information, $z_i(t) \in R$ is the acceleration information and $u_i(t) \in R$ is the input.

Definition 1: The heterogeneous multi-agent system composed of (1)-(3) is said to reach consensus if for any initial condition, we have

$$\begin{cases} \lim_{t\to\propto} \left|x_j - x_i\right| = 0 \\ \lim_{t\to\propto} \left|v_j - v_i\right| = 0 \\ \lim_{t\to\propto} \left|z_j - z_i\right| = 0. \end{cases}$$

The next lemma is given to help us solve the consensus problem.

Lemma 2 [29]: The linear filter is given as follows:

$$r(t) = \dot{e}(t) + \alpha e(t) \tag{4}$$

where $\alpha \in R^+$ is a positive real number.

$\dot{e}(t)$ and $e(t)$ are exponential convergent if $r(t)$ is exponential convergent.

III. MAIN RESULTS

In this section, the distributed control law of heterogeneous multi-agent systems composed of first-order, second-order and third-order integrator agent is presented. By the linear filter, two consensus protocols are proposed when the communication topology is fixed and directed. We suppose that there are n agents in the heterogeneous multi-agent systems. The first group agents, labeled from 1 to l, are first-order integrator agents. The second group agents, labeled from $l + 1$ to m, are second-order integrator agents. The third group agents, labeled from $m + 1$ to n, are third-order integrator agents.

For every second-order agent, an appropriate linear filter is designed as follows.

$$r_{1i}(t) = \dot{x}_i(t) + \alpha_1 x_i(t), \quad i \in \{l + 1, \ldots, m\} \tag{5}$$

where $\alpha_1 > 0$.

For every third-order agent, two appropriate linear filters are designed as follows.

$$\begin{cases} r_{2i}(t) = \dot{x}_i(t) + \alpha_2 x_i(t) \\ r_{3i}(t) = \dot{r}_{2i}(t) + \alpha_3 r_{2i}(t), \end{cases} \quad i \in \{m + 1, \ldots n\} \tag{6}$$

where $\alpha_2 > 0, \alpha_3 > 0$.

Motivated by [28], a consensus protocol is proposed as follows.

$$\begin{cases} u_i = \sum_{j=1}^{n} a_{ij}(\varepsilon_j - \varepsilon_i) \\ \qquad i \in \{1, \ldots, l\} \\ u_i = \sum_{j=1}^{n} a_{ij}(\varepsilon_j - \varepsilon_i) - \alpha_1 v_i \\ \qquad i \in \{l+1, \ldots, m\} \\ u_i = \sum_{j=1}^{n} a_{ij}(\varepsilon_j - \varepsilon_i) - (\alpha_2 + \alpha_3) z_i - \alpha_2\alpha_3 v_i \\ \qquad i \in \{m+1, \ldots, n\} \end{cases} \tag{7}$$

where $A = [a_{ij}]_{n \times n}$ is weighted adjacency matrix, $\alpha_1 > 0, \alpha_2 > 0, \alpha_3 > 0$ are the parameter of linear filter and $\varepsilon = [x_1, \ldots, x_l, r_{1(l+1)}, \ldots, r_{1(m)}, r_{3(m+1)}, \ldots r_{3(n)}]^T$ is state vector.

Based on the proposed consensus protocol (7), the consensus problem is solved as Theorem 1.

Theorem 1: Suppose communication topology with a directed spanning tree is fixed and directed, based on the linear filter (5)-(6) and the protocol (7), the consensus of a heterogeneous multi-agent system composed of (1)-(3) is solved if $\alpha_1 = 1, \alpha_2 - \alpha_3 \neq 0, \alpha_2\alpha_3 = 1$ hold.

Proof: The heterogeneous multi-agent system is described as follows.

$$\begin{cases} \dot{x}_i = u_i & i \in \{1, \ldots, l\} \\ \dot{x}_i = v_i & \\ \dot{v}_i = u_i & i \in \{l+1, \ldots, m\} \\ \dot{x}_i = v_i & \\ \dot{v}_i = z_i & \\ \dot{z}_i = u_i & i \in \{m+1, \ldots, n\}. \end{cases} \tag{8}$$

Substituting the linear filter (5)-(6) and consensus protocol (7) into (8), the heterogeneous multi-agent system is written as follows.

$$\begin{cases} \dot{x}_i = \sum_{j=1}^{n} a_{ij}(\varepsilon_j - \varepsilon_i) \\ \qquad i \in \{1, \ldots, l\} \\ \dot{r}_{1i} = \sum_{j=1}^{n} a_{ij}(\varepsilon_j - \varepsilon_i) \\ \qquad i \in \{l+1, \ldots, m\} \\ \dot{r}_{3i} = \sum_{j=1}^{n} a_{ij}(\varepsilon_j - \varepsilon_i) \\ \qquad i \in \{m+1, \ldots, n\} \end{cases} \tag{9}$$

Let (9) be a vector form.

$$\dot{\varepsilon} = -L\varepsilon \tag{10}$$

where $\varepsilon = [x_1, \ldots, x_l, r_{1(l+1)}, \ldots, r_{1(m)}, r_{3(m+1)}, \ldots, r_{3(n)}]^T$.

It follows from Lemma 1 that system (10) could solve consensus and the equilibrium state is $[k, \ldots, k]^T, k \in R$, that is

$$\begin{cases} x_1 = x_2 = \ldots = x_l = k \\ r_{1(l+1)} = r_{1(l+2)} = \ldots = r_{1(m)} = k \\ r_{3(l+1)} = r_{3(l+2)} = \ldots = r_{3(m)} = k \end{cases} \tag{11}$$

The linear filter state in (5) and (6) satisfies the following equation.

$$\begin{cases} r_{1i}(t) = \dot{x}_i(t) + \alpha_1 x_i(t) = k & i \in \{l+1, \ldots, m\} \\ r_{3i}(t) = \dot{r}_{2i}(t) + \alpha_3 r_{2i}(t) = k & i \in \{m+1, \ldots n\} \end{cases} \tag{12}$$

Solving (12), we have

$$x_i(t) = \frac{k}{\alpha_1} - \frac{k}{\alpha_1} e^{-\alpha_1 t} + x_i(0) e^{-\alpha_1 t}, \quad i \in \{l+1, \ldots, m\} \tag{13}$$

$$\begin{aligned} x_i(t) = &\frac{k}{\alpha_2\alpha_3} + \frac{k}{\alpha_2(\alpha_2 - \alpha_3)} e^{-\alpha_2 t} - \frac{k}{\alpha_3(\alpha_2 - \alpha_3)} e^{-\alpha_3 t} \\ &+ \frac{\alpha_2 x_i(0)}{\alpha_2 - \alpha_3} e^{-\alpha_2 t} - \frac{\alpha_3 x_i(0)}{\alpha_2 - \alpha_3} e^{-\alpha_3 t} - \frac{\dot{x}_i(0)}{\alpha_2 - \alpha_3} e^{-\alpha_2 t} \\ &+ \frac{\dot{x}_i(0)}{\alpha_2 - \alpha_3} e^{-\alpha_3 t} - \frac{(\alpha_2 + \alpha_3) x_i(0)}{\alpha_2 - \alpha_3} e^{-\alpha_2 t} \\ &+ \frac{(\alpha_2 + \alpha_3) x_i(0)}{\alpha_2 - \alpha_3} e^{-\alpha_3 t}, \quad i \in \{m+1, \ldots n\} \end{aligned} \tag{14}$$

$\alpha_1 = 1, \alpha_2 - \alpha_3 \neq 0$ and $\alpha_2\alpha_3 = 1$ hold from Theorem 1, so we have

$$\begin{cases} \lim_{t \to \infty} x_i(t) = k & i \in \{1, \ldots, n\} \\ \lim_{t \to \infty} v_i(t) = 0 & i \in \{l+1, \ldots, n\} \\ \lim_{t \to \infty} z_i(t) = 0 & i \in \{m+1, \ldots, n\} \end{cases} \tag{15}$$

Based on Definition 1, the consensus problem is solved.

Remark 1: Based on the protocol (7), the velocities of all second-order integrator agents and third-order integrator agents tend to zero. The acceleration of third-order integrator agents tend to zero. It means that all agents in the heterogeneous multi-agent systems will converge to one point and stay there. In directed case, the consensus state depends on the right and left eigenvectors of L corresponding to zero eigenvalue and the parameters of the linear filter. Further, the value of the consensus state is the product of the left eigenvector and the initial value of the ε vector.

Remark 2: The protocol (7) and linear filter (5)-(6) can be extended to solve consensus problem of heterogeneous multi-agent systems composed of arbitrary n-order agent. It means that designing n-1 linear filter for every n-order agent and these filter parameter need to match. These parameters can be determined according to the constant term and exponential term by Laplace transform and inverse Laplace transform. The order of agent is higher, the calculation is more difficult.

Remark 3: The proposed protocol (7) is different from the work in [23], [30]. The method in [23] is for the undirected network cases. It is only for heterogeneous system with first-order and second-order integrator agents. The proposed protocol (7) based on linear filter can solve the consensus of heterogeneous multi-agent systems composed of first-order, second-order and third-order agents. Furthermore, it is easier to get a solution to the consensus problem of heterogeneous multi-agent systems composed of arbitrary n-order agents. It could not be applied to the study of other heterogeneous systems, such as nonlinear second-order systems with different dynamics in [30]. However, with this issue, the higher-order agent can be reduced to a first-order agent, so as to simplify the design of the controller. Furthermore, By the aid of non-smooth control law and ADRC control law, the consensus problem of heterogeneous system with non-integrator dynamics can be solved.

It follows from the proof of Theorem 1 that the consensus is different by adjusting $\alpha_1, \alpha_2, \alpha_3$. So the conclusion is given as Corollary 1.

Corollary 1: Suppose communication topology with a directed spanning tree is fixed and directed, based on the linear filter (5)-(6) and the protocol (7), the group consensus of a heterogeneous multi-agent system composed of (1)-(3) is solved if $\alpha_1 \neq 1, \alpha_2 - \alpha_3 \neq 0, \alpha_2\alpha_3 \neq 1$ hold.

Proof: It follows from proof of theorem 1 that the position of first-order agents, second-order agents and third-order agents will tend to $k, k/\alpha_1, k/\alpha_2\alpha_3$ respectively. The velocity of second-order agents and third-order agents, the acceleration of third-order agents will tend to zero. So, let $\alpha_1 \neq 1, \alpha_2 - \alpha_3 \neq 0, \alpha_2\alpha_3 \neq 1$, the group consensus is solved.

By the aid of linear filter and matrix theory, another protocol is proposed as follows.

$$\begin{cases} u_i = \sum_{j=1}^{n} a_{ij}(\varepsilon_j - \varepsilon_i) - x_i \\ \qquad i \in \{1, \ldots, l\} \\ u_i = \sum_{j=1}^{n} a_{ij}(\varepsilon_j - \varepsilon_i) - \alpha_1 v_i - r_{1i} \\ \qquad i \in \{l+1, \ldots, m\} \\ u_i = \sum_{j=1}^{n} a_{ij}(\varepsilon_j - \varepsilon_i) - (\alpha_2 + \alpha_3) z_i - \alpha_2\alpha_3 v_i - r_{3i} \\ \qquad i \in \{m+1, \ldots, n\} \end{cases} \tag{16}$$

where $A = [a_{ij}]_{n\times n}$ is weighted adjacency matrix, $\alpha_1 > 0, \alpha_2 > 0, \alpha_3 > 0$ are the parameter of linear filter and $\varepsilon = [x_1, \ldots, x_l, r_{1(l+1)}, \ldots, r_{1(m)}, r_{3(m+1)}, \ldots r_{3(n)}]^T$ is state vector.

Based on the proposed consensus protocol (16), the consensus problem is solved as Theorem 2.

Theorem 2: Suppose communication topology with a directed spanning tree is fixed and directed, based on the linear filter (5)-(6) and the protocol (16), the consensus of a heterogeneous multi-agent system composed of (1)-(3) is solved.

Proof: With the protocol (16) and linear filter (5)-(6), the heterogeneous multi-agent systems can be written as follows.

$$\dot{\varepsilon} = -L\varepsilon - \varepsilon \tag{17}$$

where $\varepsilon = [x_1, \ldots, x_l, r_{1(l+1)}, \ldots, r_{1(m)}, r_{3(m+1)}, \ldots, r_{3(n)}]^T$.

Convert matrix $(-L - I)$ into diagonal matrix or Jordan matrix.

$$-TLT^{-1} - TT^{-1} = -\Lambda - I. \tag{18}$$

$$-TLT^{-1} - TT^{-1} = -J - I \tag{19}$$

It follows from Lemma 1 that the real parts of all the eigenvalues of $(-\Lambda - I)$ or $(-J - I)$ are negative. It means that (17) is exponentially convergent, i.e., all states of ε are exponentially convergent. From Lemma 2, the linear states, second-order agent states and third-order agent states are also exponentially convergent. So we have

$$\begin{cases} \lim_{t\to\infty} x_i(t) = 0 & i \in \{1, \ldots, n\} \\ \lim_{t\to\infty} v_i(t) = 0 & i \in \{l+1, \ldots, n\} \\ \lim_{t\to\infty} z_i(t) = 0 & i \in \{m+1, \ldots, n\}. \end{cases} \tag{20}$$

Based on Definition 1, the consensus problem is solved.

Remark 4: The consensus protocol (16) has a stronger effect than the former protocol (7), i.e., all first-order, second-order and third-order integrator agents will eventually tend to zero and stay there.

IV. SIMULATION

In this section, three examples are given to illustrate the effectiveness of the proposed consensus protocols.

Suppose the heterogeneous multi-agent system is composed of first-order, second-order and third-order integrator agents. Number 1 and 2 are first-order integrator agents. Number 3 and 4 are second-order integrator agents. Number 5 and 6 are third-order integrator agents. If the consensus protocols are still feasible for the heterogeneous multi-agent systems with minimum communication cost, the communication topology can be described as figure 1. Obviously, the adjacent matrix can be given as follows.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Example 1: Let $\alpha_1 = 1, \alpha_2 = 2, \alpha_3 = 0.5$, i.e., $\alpha_1 = 1, \alpha_2 - \alpha_3 \neq 0, \alpha_2\alpha_3 = 1$. It means that the initial conditions of Theorem 1 hold. Suppose the position initial value is $[x_1, \ldots, x_6] = [1, 2, 3, 4, 5, 6]$, velocity initial value is $[v_3, \ldots, v_6] = [2, 3, 4, 5]$, and acceleration initial value is $[z_5, z_6] = [1, 2]$. The trajectory of all agents is shown in figure 2, figure 3 and figure 4.

From figure 2, figure 3 and figure 4, we know that the acceleration of number 5 and 6 agents will tend to zero, the

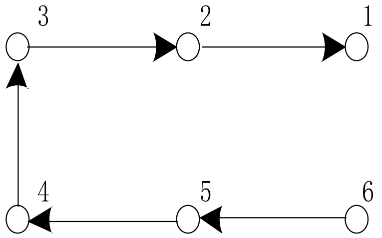

FIGURE 1. Communication topology.

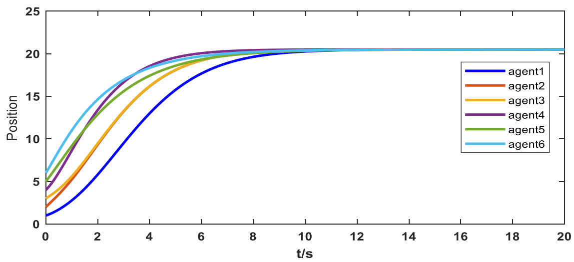

FIGURE 2. Position trajectory with theorem 1.

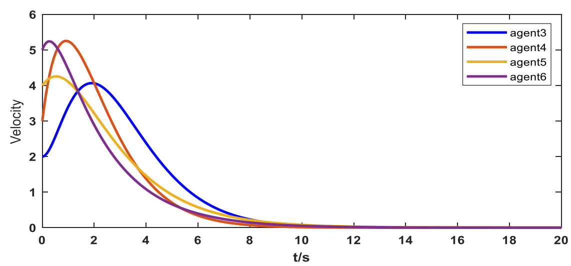

FIGURE 3. Velocity trajectory with theorem 1.

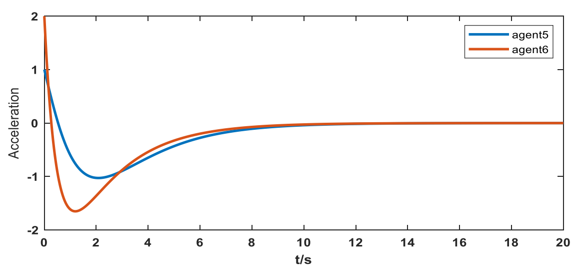

FIGURE 4. Acceleration trajectory with theorem 1.

velocity of number 3-6 agents will tend to zero and the position of number 1-6 agents will tend to same value. It means that the consensus has been obtained and Theorem 1 is correct. Moreover, from figure 1, we can get the right and left eigenvectors of $-L$ corresponding to zero eigenvalue are $[1, 1, 1, 1, 1, 1]^T$ and $[0, 0, 0, 0, 0, 1]$ respectively. Based on the initial conditions in Example 1, the initial value of this vector ε is $[1, 2, 5, 7, 16, 20.5]^T$. So we know that the value of consensus state is 20.5. As can be seen from figure 2, it illustrates the theoretical results in Remark 1.

Example 2: Let $\alpha_1 = 2, \alpha_2 = 4, \alpha_3 = 1$, i.e., $\alpha_1 = 2, \alpha_2 - \alpha_3 \neq 0, \alpha_2\alpha_3 \neq 1$. It means that the initial conditions of Corollary 1 hold. Suppose the position initial value is $[x_1, \ldots, x_6] = [1, 2, 3, 4, 5, 6]$, velocity initial value is $[v_3, \ldots, v_6] = [2, 3, 4, 5]$, and acceleration initial value is $[z_5, z_6] = [1, 2]$. The trajectory of all agents is shown in figure 5, figure 6 and figure 7.

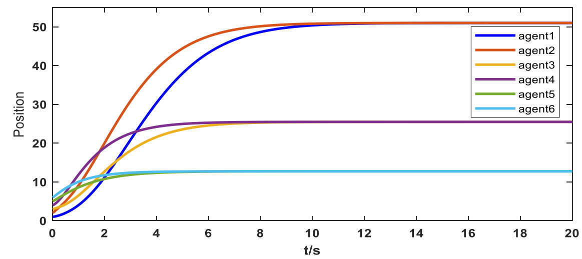

FIGURE 5. Position trajectory with corollary 1.

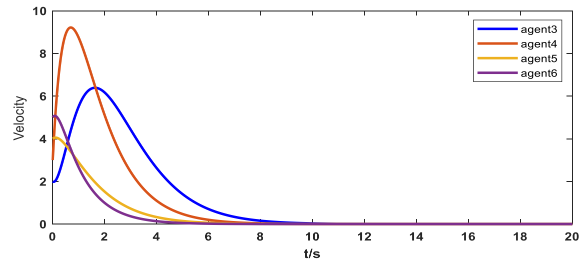

FIGURE 6. Velocity trajectory with corollary 1.

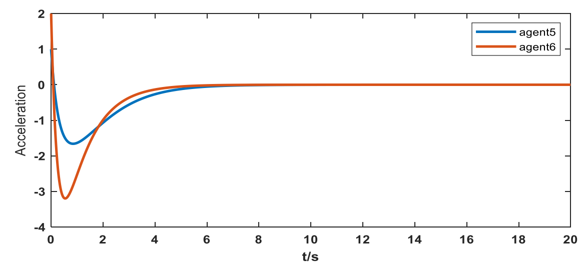

FIGURE 7. Acceleration trajectory with corollary 1.

From figure 5, figure 6 and figure 7, it is shown that the acceleration of number 5 and 6 agents will tend to zero, the velocity of number 3-6 agents will tend to zero and the position of number 1-6 agents will tend to three different values respectively. The group consensus is reached. It implies that different classes of agents can be grouped to complete their own tasks. The stable position value of second-order and third-order agents is one half and a quarter of first-order agents, and it is determined by $1/\alpha_1$ and $1/\alpha_2\alpha_3$ respectively. Obviously, the result in figure 5 matches the initial conditions $\alpha_1 = 2, \alpha_2\alpha_3 = 4$.

Example 3: Let $\alpha_1 = 1, \alpha_2 = 1, \alpha_3 = 1$, i.e., the initial conditions of Theorem 2 hold. Suppose the position initial value is $[x_1, \ldots, x_6] = [1, 2, 3, 4, 5, 6]$, velocity initial value is $[v_3, \ldots, v_6] = [2, 3, 4, 5]$, and acceleration initial value is $[z_5, z_6] = [1, 2]$. The trajectory of all agents is shown in figure 8, figure 9 and figure 10.

It can be seen from figure 8, figure 9 and figure 10, the acceleration of number 5 and 6 agents, the velocity of number 3-6 agents and the position of number 1-6 agents will tend to zero. With the protocol (16), all agents are driven to zero and stay there. The consensus is solved.

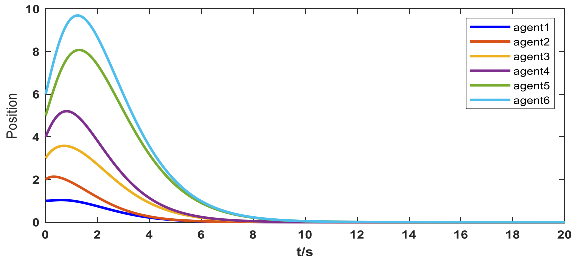

FIGURE 8. Position trajectory with theorem 2.

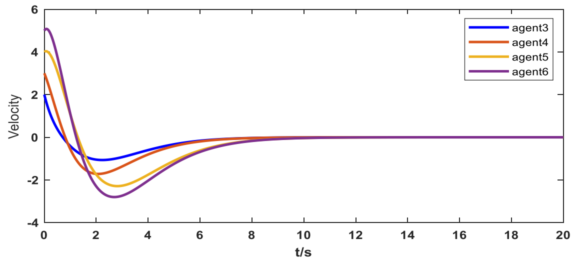

FIGURE 9. Velocity trajectory with theorem 2.

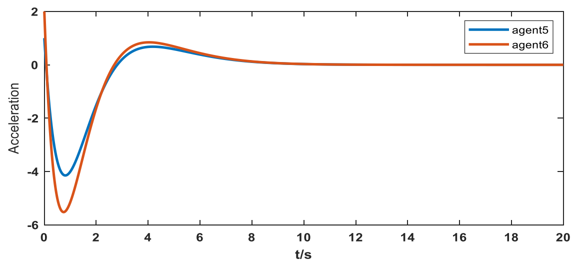

FIGURE 10. Acceleration trajectory with theorem 2.

From the simulation results of three examples, the consensus can be solved with protocol (7), the value of consensus state is not zero which depends on the right and left eigenvectors of L corresponding to zero eigenvalue and the parameters of the linear filter. Further, group consensus can also be solved by adjusting parameters. Compared with protocol (7), protocol (16) has a stronger effect. It not only solves the consensus problem, but also makes the state tend to zero. The value of the consensus state must be zero. It is different from protocol (7).

V. CONCLUSION

In this paper, by the method of designing linear filter, the consensus problem of heterogeneous multi-agent system is investigated which composed of first-order, second-order and third-order agents. Based on the proposed protocols, some filter states will match the position state and will not change i.e. both the velocity and acceleration of agents will tend to zero. It means that consensus is solved. Moreover, group consensus can also be solved by adjusting filter parameters. Furthermore, this method can be used to deal with high-order integrator agents. It is easy to solve the consensus problem of heterogeneous multi-agent systems composed of arbitrary classed of order agents. Our study is expected to establish a more realistic model and provide a method to solve the consensus problem of heterogeneous multi-agent systems in more complex situations. With the help of this issue, it is meaningful to study the consensus of heterogeneous systems in different scenarios that are more reasonable and practical, such as without state measurements, time-delay, leader-following, fixed-time consensus and so on.

REFERENCES

[1] T. Ichimura, T. Uemoto, and A. Hara, "Emergence of altruism behavior for multi feeding areas in army ant social evolutionary system," in *Proc. IEEE Int. Conf. Syst., Man, Cybern. (SMC)*, San Diego, CA, USA, Oct. 2014, pp. 170–175.

[2] M. A. Mohamed, T. Jin, and W. Su, "Multi-agent energy management of smart islands using primal-dual method of multipliers," *Energy*, vol. 208, Oct. 2020, Art. no. 118306.

[3] X.-C. Jia, S. Gao, X. You, and B. Li, "Output consensus of heterogeneous multi-agent systems with a multi-sensor multi-rate sampling mechanism," *J. Franklin Inst.*, vol. 357, no. 17, pp. 12640–12669, Nov. 2020.

[4] R. Liao, L. Han, X. Dong, Q. Li, and Z. Ren, "Finite-time formation-containment tracking for second-order multi-agent systems with a virtual leader of fully unknown input," *Neurocomputing*, vol. 415, pp. 234–246, Nov. 2020.

[5] A. Jadbabaie, J. Lin, and A. S. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," in *Proc. 41st IEEE Conf. Decis. Control*, Las Vgas, NV, USA, Dec. 2002, pp. 2953–2958.

[6] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1520–1533, Sep. 2004.

[7] X. Dong, Y. Zhou, Z. Ren, and Y. Zhong, "Time-varying formation tracking for second-order multi-agent systems subjected to switching topologies with application to quadrotor formation flying," *IEEE Trans. Ind. Electron.*, vol. 64, no. 6, pp. 5014–5024, Jun. 2017.

[8] C. P. Bechlioulis and G. A. Rovithakis, "Decentralized robust synchronization of unknown high order nonlinear multi-agent systems with prescribed transient and steady state performance," *IEEE Trans. Autom. Control*, vol. 62, no. 1, pp. 123–134, Jan. 2017.

[9] Y. Tian and C. D. Li, "Finite-time consensus of second-order nonlinear multi-agent systems with impulsive effects," *Mod. Phys. Lett. B*, vol. 34, no. 35, Dec. 2020, Art. no. 2050406.

[10] X. Li, X. Yi, and L. Xie, "Distributed online optimization for multi-agent networks with coupled inequality constraints," *IEEE Trans. Autom. Control*, vol. 66, no. 8, pp. 3575–3591, Aug. 2021.

[11] X. Li, Y. Tang, and H. R. Karimi, "Consensus of multi-agent systems via fully distributed event-triggered control," *Automatica*, vol. 116, Jun. 2020, Art. no. 108898.

[12] P. He, Y. Li, and J. H. Park, "Noise tolerance leader-following of high-order nonlinear dynamical multi-agent systems with switching topology and communication delay," *J. Franklin Inst.*, vol. 353, no. 1, pp. 108–143, Jan. 2016.

[13] P. He, "Consensus of uncertain parabolic PDE agents via adaptive unit-vector control scheme," *IET Control Theory Appl.*, vol. 12, no. 18, pp. 2488–2494, Dec. 2018.

[14] C. J. Li, G. P. Liu, P. He, F. Q. Deng, and H. Li, "Dynamic consensus of second-order networked multiagent systems with switching topology and time-varying delays," *IEEE Trans. Cybern.*, early access, Jun. 7, 2021, doi: 10.1109/TCYB.2021.3079717.

[15] L. Shi, Y. Xiao, J. Shao, and W. X. Zheng, "Containment control of asynchronous discrete-time general linear multiagent systems with arbitrary network topology," *IEEE Trans. Cybern.*, vol. 50, no. 6, pp. 2546–2556, Jun. 2020.

[16] L. Shi, W. X. Zheng, J. Shao, and Y. Cheng, "Sub-super-stochastic matrix with applications to bipartite tracking control over signed networks," *SIAM J. Control Optim.*, vol. 59, no. 6, pp. 4563–4589, Jan. 2021.

[17] J. Shao, W. X. Zheng, L. Shi, and Y. Cheng, "Bipartite tracking consensus of generic linear agents with discrete-time dynamics over cooperation–competition networks," *IEEE Trans. Cybern.*, vol. 51, no. 11, pp. 5225–5235, Nov. 2021.

[18] Y. Zheng, Y. Zhu, and L. Wang, "Consensus of heterogeneous multi-agent systems," *IET Control Theory Appl.*, vol. 5, no. 16, pp. 1881–1888, Apr. 2011.

[19] Y. Zheng and L. Wang, "Consensus of heterogeneous multi-agent systems without velocity measurements," *Int. J. Control*, vol. 85, no. 7, pp. 906–914, 2012.
[20] H. Geng, Z.-Q. Chen, Z.-X. Liu, and Q. Zhang, "Consensus of a heterogeneous multi-agent system with input saturation," *Neurocomputing*, vol. 166, pp. 382–388, Oct. 2015.
[21] H. Geng, Z. Q. Chen, C. Y. Zhang, Z. X. Liu, and Q. Zhang, "Consensus of heterogeneous multi-agent systems with linear and nonlinear dynamics," *J. Appl. Anal. Comput.*, vol. 6, no. 1, pp. 1–11, Feb. 2016.
[22] H. B. Du, G. H. Wen, D. Wu, Y. Y. Cheng, and J. H. Lu, "Distributed fixed-time consensus for nonlinear heterogeneous multi-agent systems," *Automatica*, vol. 113, Mar. 2020, Art. no. 108797.
[23] Y. Z. Song and M. Q. Gu, "Quasi-average consensus in undirected networks of multi-agents with mixed order integrators," *Control Eng. China*, vol. 16, no. 2, pp. 220–222, 2009.
[24] Y. Feng and X. Tu, "Consensus analysis for a class of mixed-order multi-agent systems with nonlinear consensus protocols," *Trans. Inst. Meas. Control*, vol. 37, no. 2, pp. 147–153, Feb. 2015.
[25] M. Liu and Y. Feng, "Group consensus of mixed-order multi-agent systems with fixed and directed interactive topology," *IEEE Access*, vol. 7, pp. 179712–179719, 2019.
[26] S. Shi, H. Feng, W. Liu, and G. Zhuang, "Finite-time consensus of high-order heterogeneous multi-agent systems with mismatched disturbances and nonlinear dynamics," *Nonlinear Dyn.*, vol. 96, no. 2, pp. 1317–1333, Apr. 2019.
[27] D. G. Lui, A. Petrillo, and S. Santini, "An optimal distributed PID-like control for the output containment and leader-following of heterogeneous high-order multi-agent systems," *Inf. Sci.*, vol. 541, pp. 166–184, Dec. 2020.
[28] W. Ren and R. W. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Trans. Autom. Control*, vol. 50, no. 5, pp. 655–661, May 2005.
[29] Y. C. Fang and G. Z. Lu, *Nonlinear System Theory*. Beijing, China: Tsinghua, 2009, pp. 60–61.
[30] Z. Wang and H. He, "Asynchronous quasi-consensus of heterogeneous multiagent systems with nonuniform input delays," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 50, no. 8, pp. 2815–2827, Aug. 2020.

HUA GENG received the M.S. degree in control theory and control engineering from the Tianjin University of Technology and Education, in 2013, and the Ph.D. degree in control theory and control engineering from Nankai University, in 2016. He has been at the Hebei University of Engineering, where he is currently a Lecturer with the School of Information and Electrical Engineering. His main research interests complex networks and multi-agent systems.

HONGYANG WU received the B.S. degree in automation from the Liren College, Yanshan University, in 2014. His main research interest includes multi-agent systems.

JINGLI MIAO received the M.S. degree in control theory and control engineering from the Hebei University of Science and Technology, and the Ph.D. degree in control theory and control engineering from the University of Science and Technology Beijing. She has been at the Hebei University of Engineering, where she is currently a Professor with the School of Information and Electrical Engineering. Her main research interests include nonlinear systems and intelligent control.

SHUAI HOU received the M.S. and Ph.D. degrees in control theory and control engineering from Northeastern University. He has been at the Hebei University of Engineering, where he is currently an Associate Professor with the School of Information and Electrical Engineering. His main research interests include artificial intelligence and intelligent control.

ZENGQIANG CHEN received the B.S. degree in mathematics, and the M.S. and Ph.D. degrees in control theory and control engineering from Nankai University, Tianjin, China, in 1987, 1990, and 1997, respectively. He has been at Nankai University, where he is currently a Professor with the Department of Automation. His main research interests include neural network control, complex networks, and multi-agents systems.