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Exponential Stability of Impulsive Cascaded Systems and Its Application in Robot Control

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ABSTRACT In this paper, we study the problem of exponential stability of impulsive cascaded systems. In particular, we provide some sufficient conditions that guarantee the exponential stability of the cascaded systems, provided that the two subsystems are also exponentially stable. The proof of the stability of the cascaded systems is based on the second Lyapunov method and the existence of converse theorems for the stability of impulsive systems. Finally, the usefulness of our results is illustrated by its application to the problem of trajectory tracking for a wheeled robot.

INDEX TERMS Cascade impulsive systems, second Lyapunov method, impulsive control.

I. INTRODUCTION

The present study of cascaded impulsive systems finds its motivation for two main reasons: the importance of impulsive systems and the importance of cascaded systems.

First, it is known that many biological systems, optimal control models in economics, theoretical physics, ecology, and industrial robotics have a sudden change in the form of disturbances in their states [1]–[4]. These short-term perturbations have a duration that is negligible compared to the duration of the processes. Therefore, it is natural to assume that these perturbations act instantaneously or as impulses. Consequently, impulsive differential equations appear to be a natural description of these processes. On the other hand, impulsive systems appear naturally in certain control strategies of dynamic systems, such as the impulsive control strategy or the intermittent control strategy. Impulsive control is a control theory based on impulsive differential equations. The key idea of impulsive control is to change the state instantaneously at certain instants, and the information is transmitted only at certain discrete times. Thus, compared with the continuous control strategy in which the information is transmitted continuously, the control cost and quantity of transmitted information can be effectively minimized. In addition, in some cases, impulsive control can be an efficient method for treating systems that cannot withstand continuous disturbances [5]–[10].

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Initially, the impulsive control approach was used in ordinary differential equations to stabilize networked systems and multi-agent systems [11]–[16] and the references therein. Recently, impulsive control has also been used to control partial differential equations [17]–[19]. Most recently, an impulsive control was designed via the event-triggered method to establish the rapid exponential stabilization of a class of damped wave equations derived from brain activity in [20], as well as to achieve the same stability result for the dynamic population Lotka-McKendrick equation in [21].

However, unlike in impulsive control where the control signal is instantaneous, in intermittent control, the control signal is on for some non-zero periods of time but is off for other periods. This type of control is thus a transition between continuous and impulsive control and has often been the focus of the study of synchronization in complex network systems [22]–[25].

Second, a large class of nonlinear systems can be decomposed into cascaded subsystems that are less complex than the original system. Moreover, cascaded systems may appear in many control applications. Most remarkably, in some cases, a system can be split into two subsystems for which control inputs can be designed with the aim that the closed-loop will have a cascaded structure [26]–[29]. In the literature, cascaded continuous systems were originally introduced for autonomous systems; their roots can be traced back to [30], where the authors offered some sufficient conditions for the global stabilizability of two cascaded connected nonlinear autonomous systems. In this study, using the LaSalle-Krasovskii invariance principle, the authors have shown that a cascaded autonomous system is globally asymptotically stable (GAS) if all orbits are bounded and if both subsystems are GAS. The boundedness condition on the solutions was later eased in [31] for some particular cases. Sufficient conditions for the interconnection term to prove the global stabilization of partially linear cascade systems were proposed in [32]. Sufficient growth conditions were also given in [33] for the global stabilization of nonlinear cascade systems. In [34], sufficient conditions for global smooth stabilization of cascaded nonlinear systems were achieved when making the first system strictly passive for an output that spans the unstable part of the vector field of the second system. In [35], an added integrating technique was used to prove the global asymptotic stabilization for the cascaded system, while in [29], an adaptive controller was developed using feedback passivation together with an explicit Lyapunov function built for cascaded systems. In [36], some theoretical challenges for the stability analysis of cascaded nonlinear systems were presented.

For the case of a class of cascaded nonautonomous systems, it was shown in [37]; that a cascaded nonautonomous system is globally uniformly exponentially stable if and only if each isolated subsystem is globally uniformly exponentially stable. This statement fails, for example, for asymptotic stability properties, an additional property on the boundedness of solutions can be added to recover the asymptotic stability for the entire system. It has been shown in [27], [28], that the compound cascaded system of two globally uniformly asymptotically stable subsystems is globally uniformly asymptotically stable provided that its solutions are globally uniformly bounded. This boundedness assumption of solutions was also used to show a similar result for the semi-global stability in [38]; and practical stability properties in [39]. A survey of some analyses and designs of cascaded nonlinear nonautonomous systems is given in [40].

The stability of discrete-time cascaded systems was also discussed in [41]–[46]. In [41], a partial state feedback controller design scheme was considered for the study of the global stabilization problem for a class of cascaded nonlinear systems with a time-varying delay. In [42], the output tracking problem for cascaded switched nonlinear systems was studied based on the mean residence time method. In the case where the zero dynamics were not stabilizable under an arbitrary switching signal, sufficient conditions for that problem were established. In [43], the dwell time method was used to study the stabilization problem of a class of cascaded switched nonlinear systems in the presence of actuator saturation. Sufficient conditions have been proposed for cascaded switched systems to be exponentially stable by designing state feedback controllers. Later, in [44], using the forwarding technique and some recently developed tools for the input-to-state system (ISS), a global state feedback controller was constructed to solve the global stabilization problem for a class of cascaded nonlinear systems with upper triangular structures. Reference [45] used the idea of the cross-term constructed Lyapunov function, first introduced in [29], to present some sufficient conditions for semi-global and practical asymptotic stability as well as the construction of a controller stabilizing such systems.

However, the possibility of impulses has been excluded from the above-mentioned works. In general, the stability of impulsive differential equations has been extensively developed in the last decades; and many results of this theory can be found in the literature [1]–[4] and the references therein. Recently, using a converse theorem for the practical exponential stability of impulsive systems, [47] established the practical exponential stability of cascaded impulsive systems. More recently, and in a similar spirit, the practical asymptotic stability of cascaded impulsive systems was elaborated in [48].

In this study, as a continuation of previous works, we established the uniform exponential stability of cascaded impulsive systems. The present work is an attempt to lay a foundation for the study of the exponential stability of cascaded impulsive systems and to explore the eventual benefits of their application in the impulsive control of continuous systems. Thus, the main purpose of this work; is to use the results obtained on the exponential stability of cascaded impulsive systems to control continuous cascaded systems via an impulsive controller.

The remainder of this paper is organized as follows. Section 2 contains some definitions and preliminary results. In Section 3, sufficient conditions are provided to guarantee the uniform exponential stability of cascaded systems. In Section 4, an example of a pursuit problem for a mobile wheel robot that moves in a plane with numerical simulations is presented as an illustration. Section 5 presents the conclusions of this study.

II. PRELIMINARIES

Let us consider the cascaded impulsive system in the following form:

$$\dot{x}_1 = f_1(t, x_1) + g(t, x)x_2, \quad t \neq t_k, \tag{1}$$
$$\dot{x}_2 = f_2(t, x_2), \tag{2}$$
$$\Delta x_1 = I_k(x_1) + L_k(x)x_2, \quad t = t_k,\ k = 1, 2, .. \tag{3}$$
$$\Delta x_2 = J_k(x_2). \tag{4}$$

The two isolated subsystems are given by

$$\dot{x}_1 = f_1(t, x_1), \quad t \neq t_k \tag{5}$$
$$\Delta x_1 = I_k(x_1), \quad t = t_k,\ k = 1, 2, .. \tag{6}$$

and

$$\dot{x}_2 = f_2(t, x_2), \quad t \neq t_k \tag{7}$$
$$\Delta x_2 = J_k(x_2), \quad t = t_k,\ k = 1, 2, .. \tag{8}$$

where $t \in \mathbb{R}_+$, $x_1 \in \mathbb{R}^n$, $x_2 \in \mathbb{R}^m$, and $x = [x_1, x_2]^T$. For $i = 1, 2$, the jumps $\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-)$, where $x_i(t_k^+) = \lim_{h\to 0^+} x_i(t_k + h)$, $x_i(t_k^-) = \lim_{h\to 0^+} x_i(t_k - h)$.

Throughout this paper, we suppose that the following assumptions hold:

($\mathcal{H}_1$) The functions $f_1 : [t_0, \infty) \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$ $f_2 : [t_0, \infty) \times \mathbb{R}^m \longrightarrow \mathbb{R}^m$ and $g : [t_0, \infty) \times \mathbb{R}^{n+m} \longrightarrow \mathbb{R}^{n\times m}$ are continuous in their arguments, locally Lipschitz in x, uniformly in t, $f_1(.,.), f_2(.,.)$ are continuously differentiable in both arguments and $f_1(t,0) = f_2(t,0) \equiv 0$. The functions $I_k : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ $J_k : \mathbb{R}^m \longrightarrow \mathbb{R}^m$ and $L_k : \mathbb{R}^{n+m} \longrightarrow \mathbb{R}^{n\times m}$ are locally Lipschitz and $I_k(0) = J_k(0) = 0$.

($\mathcal{H}_2$) The fixed sequence of times $(t_k)_{k\in\mathbb{N}}$ satisfies the conditions $0 \leq t_0 < t_1 < \ldots < t_k < \ldots$, and $t_k \to +\infty$ as $k \to +\infty$.

Note that the standard assumptions $(\mathcal{H}_1)-(\mathcal{H}_2)$ guarantee, for any initial condition $x_0 = [x_{10}, x_{20}]^T$, the existence and uniqueness of the solution $x(t) = [x_1(t), x_2(t)]^T$ defined in the interval $[t_0, +\infty)$ for system (1)-(4). According to the classical assumptions in the theory of impulsive differential equations, we assume that $x({t_k}^-) = x(t_k)$. The solution $x(t)$ is then continuously differentiable for all $t \neq t_k, k = 1, 2, \ldots$ and left-continuous at the discontinuity points $t_k, k = 1, 2, \ldots$ [1], [2].

The following notations and definitions will be needed for later use.

Notation 1: $\mathcal{K}$ denotes the class of continuous functions $\alpha : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ such that α is continuous, strictly increasing, and $\alpha(0) = 0$.

Notation 2: $PC[\mathbb{R}_+, \mathbb{R}^n]$ denotes the set of functions $h : \mathbb{R}_+ \longrightarrow \mathbb{R}^n$, which are continuous for $t \in \mathbb{R}_+, t \neq t_k$, have discontinuities of the first kind at points t_k and are left continuous.

Notation 3: $\mathcal{V}_0$ (resp., $\mathcal{V}_1$) denotes the class of functions $V : \mathbb{R}_+ \times \mathbb{R}^n \longrightarrow \mathbb{R}_+$ such that $V(t, 0) = 0$ for all $t \in \mathbb{R}_+$ and $V(t, x)$ is locally Lipschitz in x, continuous everywhere except possibly at a sequence of points $\{t_k\}$, where $V(t, x)$ is left continuous and the right limit $V(t_k^+, x)$ exists for all $x \in \mathbb{R}^n$ (resp. the class of functions $V \in \mathcal{V}_0$ and it is, moreover, continuously differentiable).

Definition 1: The equilibrium point $x = 0$ of system (1)-(4) is said to be uniformly exponentially stable (UES) if there exist positive constants $r, k,$ and λ such that

$$\|x(t)\| \leq k\|x(t_0)\|e^{-\lambda(t-t_0)}, \quad \forall t \geq t_0, \ \forall \ \|x_0\| \leq r. \tag{9}$$

Definition 2: The equilibrium point $x = 0$ of system (1)-(4) is said to be uniformly bounded (UB) if there exist a class $\mathcal{K}$ function α and a constant $c > 0$ such that

$$\|x(t)\| \leq \alpha(\|x(t_0)\|) + c, \ \forall \ t \geq t_0. \tag{10}$$

Definition 3: We define the Dini derivative or the upper right-hand generalized derivative of a function $V(t, x_1)$ along the solutions of (1) as follows:

$$D^+V_{(1)}(t, x_1) = \limsup_{h\to 0^+} \frac{1}{h}[V(t+h, x_1 + hf_1(t, x_1)) - V(t, x_1)]$$

III. MAIN RESULT

In this section, we state and prove our main result. First, we recall an auxiliary result which is taken from [2], and a converse theorem concerning the exponential stability of impulsive systems which can be found in [1], which will be used later.

Lemma 1: Assume that

1) $v \in \mathcal{V}_0$,
2) for $k = 1, 2.., t \geq t_0$
$$D^+v(t) \leq a(t)v(t) + b(t), \ t \neq t_k,$$
$$v(t_k^+) \leq c_k v(t_k) + d_k,$$
where $a, b \in PC[\mathbb{R}_+, \mathbb{R}]$, $c_k \geq 0$ and d_k are constants.

Then

$$\begin{aligned} v(t) \leq{} & v(t_0)\Big(\prod_{t_0<t_k<t} c_k\Big)e^{\int_{t_0}^t a(s)ds} \\ &+ \int_{t_0}^t \Big(\prod_{s<t_k<t} c_k\Big)e^{\int_s^t a(u)du}b(s)ds \\ &+ \sum_{t_0<t_k<t,} \Big(\prod_{t_k<t_j<t} c_j\Big)e^{\int_{t_k}^t a(s)ds}d_k. \end{aligned}$$

The following converse theorem for the exponential stability of impulsive systems is taken from [1].

Proposition 1: Let conditions $(\mathcal{H}_1) - (\mathcal{H}_3)$ hold and let solution x_1 of system (5)- (6) be exponentially stable.

Then, there exist positive constants $\rho, c_1, c_2, c_3, c_4, K$ and a function $V \in \mathcal{V}_1$ such that the following conditions are satisfied for all $x_1, y_1 \in \mathcal{B}_\rho = \{y \in \mathbb{R}^n;\ \|y\| \leq \rho\}$ and $t \geq 0$,

(i) $c_1\|x_1\|^2 \leq V(t, x_1) \leq c_2\|x_1\|^2$,
(ii) $\dot{V}_{(5)}(t, x_1) \leq -c_3V(t, x_1), \ t \neq t_k$,
(iii) $\|\frac{\partial V}{\partial x}(t, x_1)\| \leq c_4\|x_1\|, \ t \neq t_k$,
(iv) $|V(t, x_1) - V(t, y_1)| \leq K\|x_1 - y_1\|$,
(v) $V\left(t_k^+, x_1(t_k) + I_k(x_1(t_k))\right) \leq V(t_k, x_1(t_k))$.

Now, we are in a position to introduce our main results.

Theorem 1: Assume that systems (5)-(6) and (7)-(8) are UES and the interconnection terms $g(.,.)$ and $L_k(.)$ satisfy the following conditions, for all $t \geq 0, x \in \mathbb{R}^{n+m}$,

(a) $\|g(t, x)\| \leq M + \varepsilon\|x\|$,
(b) $\|L_k(x)\| \leq m_k + \epsilon_k\|x\|$,

where M, ε are positive constants and, m_k and ϵ_k are supposed to be bounded with respect to k. Then, the impulsive cascaded system (1)-(4) is UES.

Before proving Theorem 1, let us first quote the following remarks.

Remark 1: First, note that because (7)-(8) is exponentially stable, there are positive real constants r_2, k_2, and μ such that:

$$\|x_2(t)\| \leq k_2\|x_{20}\|e^{-\mu(t-t_0)}, \quad \forall \ t \geq t_0, \tag{11}$$

for all x_{20} such that $\|x_{20}\| \leq r_2$, where $x_2(t)$ is the solution of the impulsive subsystem (7)-(8) with initial condition x_{20}. Because $\|x_{20}\| \leq r_2$ and $e^{-\mu(t-t_0)} \leq 1, \ \forall \ t \geq t_0$, it follows from (11) that

$$\|x_2(t)\| \leq \sigma_2 := k_2r_2. \tag{12}$$

Additionally, we can suppose that c_3 in (*ii*) in Proposition 1 and μ in (11) are equal. Indeed, if this is not the case, by replacing either c_3 in (*ii*) or μ in (11) by $\min\{c_3, \mu\}$ we get the wished result.

$$-c_3 V(t, x_1) \leq -\min\{c_3, \mu\} V(t, x_1)$$
$$e^{-\mu(t-t_0)} \leq e^{-\min\{c_3,\mu\}(t-t_0)}$$

Remark 2: Second, to simplify the proof, we will prove Theorem 1 under the following hypotheses (a') and (b') instead of hypotheses (a) and (b) respectively.

(a') $\|g(t, x)\| \leq M + \varepsilon \|x_1\|$,
(b') $\|L_k(x)\| \leq m_k + \epsilon_k \|x_1\|$.

In fact,

$$\begin{aligned}\|g(t, x)\| &\leq M + \varepsilon\|x\|,\\ &\leq M + \varepsilon\|x_1\| + \varepsilon\|x_2\|,\\ &\leq M + \varepsilon\|x_1\| + \sigma_2,\\ &\leq M' + \varepsilon\|x_1\|,\end{aligned}$$

where $M' = M + \sigma_2$.

Proof: We break up the proof into two steps. In the first step, we prove that solutions $x_1(.)$ of the impulsive cascaded system (1)-(4) are bounded, while in the second step, we establish the UES of the impulsive cascaded system (1)-(4).

A. FIRST STEP: BOUNDEDNESS OF SOLUTIONS

Subsystem (5)-(6) is UES, then by Proposition 1, there exists a Lyapunov function $V \in \mathcal{V}_1$ satisfying all conditions (i)-(vi). For $t \neq t_k$, the derivative of V along the solutions of system (1)-(4) is given by

$$\begin{aligned}\dot{V}_{(1)}(t, x_1) &= \dot{V}_{(5)}(t, x_1) + \frac{\partial V}{\partial x} g(t, x) x_2,\\ &\leq -c_3 V(t, x_1) + c_4 \|x_1\| \, \|g(t, x)\| \, \|x_2\|,\\ &\leq -c_3 V(t, x_1) + c_4 \|x_1\| \,(M + \varepsilon \|x_1\|)\, \|x_2\|,\\ &\leq -c_3 V(t, x_1) + c_4\, \varepsilon \|x_2\| \, \|x_1\|^2\\ &\quad + c_4\, M \|x_2\| \, \|x_1\|.\end{aligned}$$

Using (12) and the fact that $\|x_1\|^2 \leq \frac{1}{c_1} V(t, x_1)$, it follows that

$$\begin{aligned}\dot{V}_{(1)}(t, x_1) &\leq -c_3 V(t, x_1) + c_4\, \varepsilon \sigma_2 \|x_1\|^2 + c_4\, M \sigma_2 \|x_1\|,\\ &\leq -\Big(c_3 - \frac{c_4 \varepsilon}{c_1}\|x_2\|\Big) V(t, x_1)\\ &\quad + \frac{c_4 M \sigma_2}{\sqrt{c_1}} \sqrt{V(t, x_1)},\\ &\leq -\Big(c_3 - \lambda_1 e^{-c_3(t-t_0)}\Big) V(t, x_1)\\ &\quad + \beta_1 \sqrt{V(t, x_1)},\end{aligned} \tag{13}$$

where $\lambda_1 = \frac{c_4 \varepsilon \sigma_2}{c_1}$ and $\beta_1 = \frac{c_4 M \sigma_2}{\sqrt{c_1}}$.

On the other hand, for $t = t_k$, we have

$$\begin{aligned}&V(t_k^+, x_1(t_k^+))\\ &= V(t_k^+, x_1(t_k) + I_k(x_1(t_k)) + L_k(x(t_k)) x_2(t_k)),\\ &= V(t_k^+, x_1(t_k) + I_k(x_1(t_k))\\ &\quad + L_k(x(t_k)) x_2(t_k))\\ &\quad - V(t_k^+, x_1(t_k) + I_k(x_1(t_k))\\ &\quad + V(t_k^+, x_1(t_k) + I_k(x_1(t_k)),\end{aligned}$$

From (iv) and (v) in Proposition1, we obtain successively

$$\begin{aligned}&V(t_k^+, x_1(t_k) + I_k(x_1(t_k)) + L_k(x(t_k)) x_2(t_k))\\ &\quad V(t_k^+, x_1(t_k) + I_k(x_1(t_k))\\ &\leq K \; \|L_k(x(t_k))\| \, \|x_2(t_k)\|,\end{aligned}$$

and

$$V(t_k^+, x_1(t_k) + I_k(x_1(t_k))) \leq V(t_k, x_1(t_k)).$$

This gives

$$V(t_k^+, x_1(t_k^+)) \leq V(t_k, x_1(t_k)) + K \; \|L_k(x(t_k))\| \, \|x_2(t_k)\|,$$

and by assumption (b'), we obtain

$$\begin{aligned}V(t_k^+, x_1(t_k^+)) &\leq V(t_k, x(t_k)) + K\,(m_k\\ &\qquad + \epsilon_k \|x_1(t_k)\|)\, \|x_2(t_k)\|,\\ &\leq V(t_k, x(t_k)) + K \epsilon_k \|x_2(t_k)\| \, \|x_1(t_k)\|\\ &\qquad + K m_k \, \|x_2(t_k)\|,\\ &\leq V(t_k, x(t_k)) + \frac{K \epsilon_k}{2} \|x_2(t_k)\| \|x_1(t_k)\|^2\\ &\qquad + \frac{K \epsilon_k}{2} \|x_2(t_k)\| + K m_k \, \|x_2(t_k)\|,\\ &\leq \Big(1 + \frac{K \epsilon_k}{2 c_1} \|x_2(t_k)\|\Big) V(t_k, x(t_k))\\ &\qquad + K \left(\frac{\epsilon_k}{2} + m_k\right) \; \|x_2(t_k)\|,\\ &\leq \Big(1 + \alpha e^{-c_3(t_k - t_0)}\Big) V(t_k, x(t_k))\\ &\qquad + d e^{-c_3(t_k - t_0)},\end{aligned} \tag{14}$$

where $\alpha = \frac{K \sigma_2 \bar{\epsilon}}{2 c_1}$ and $d = K \sigma_2 \left(\frac{\bar{\epsilon}}{2} + \overline{m}\right)$.

Let $w(t) = \sqrt{V(t, x_1)}$. Then, from (13) and (14), $w(t)$ satisfies

$$\dot{w}(t) \leq -\Big(\frac{c_3}{2} - \frac{\lambda_1}{2} e^{-c_3(t-t_0)}\Big) w(t) + \frac{\beta_1}{2}, \; t \neq t_k, \tag{15}$$

$$\begin{aligned}w(t_k^+) &\leq \Big(1 + \sqrt{\alpha} e^{-\frac{c_3}{2}(t_k - t_0)}\Big) w(t_k)\\ &\quad + \sqrt{d} e^{-\frac{c_3}{2}(t_k - t_0)}.\end{aligned} \tag{16}$$

It follows from the Lemma 1 that for $\forall t \in (t_k, t_{k+1}]$ we have

$$\begin{aligned}w(t) \leq\;& w(t_0) \prod_{t_0 < t_k < t} \big(1 + \sqrt{\alpha} e^{-\frac{c_3}{2}(t_k - t_0)}\big)\\ &\times e^{\int_{t_0}^{t} -\left(\frac{c_3}{2} - \frac{\lambda_1}{2} e^{-c_3(s-t_0)}\right) ds}\\ &+ \int_{t_0}^{t} \Big(\prod_{t_0 < t_s < t} \big(1 + \sqrt{\alpha} e^{-\frac{c_3}{2}(t_k - t_0)}\big)\Big)\\ &\quad \times e^{\int_{s}^{t} -\left(\frac{c_3}{2} - \frac{\lambda_1}{2} e^{-c_3(u-t_0)}\right) du} \, \frac{\beta_1}{2} ds\end{aligned}$$

$$+ \sum_{t_0<t_k<t,} \Big(\prod_{t_k<t_j<t} (1+\sqrt{\alpha} e^{-\frac{c_3}{2}(t_j-t_0)}) \Big) \times e^{\int_{t_k}^{t} -\left(\frac{c_3}{2} - \frac{\lambda_1}{2} e^{-c_3(s-t_0)}\right) ds} \times \sqrt{d} e^{-\frac{c_3}{2}(t_k-t_0)}. \tag{17}$$

Now let us give estimates of the three integrals on the right-hand side of the inequality (17). Firstly,

$$\begin{aligned} &e^{\int_{\tau}^{t} -\left(\frac{c_3}{2} - \frac{\lambda_1}{2} e^{-c_3(s-t_0)}\right) ds} \\ &= e^{-\frac{c_3}{2}(t-\tau)} \times e^{\frac{\lambda_1}{2c_3}\left[e^{-c_3(\tau-t_0)} - e^{-c_3(t-t_0)}\right]} \\ &\leq e^{-\frac{c_3}{2}(t-\tau)} e^{\frac{\lambda_1}{2c_3}}. \end{aligned} \tag{18}$$

Secondly, the series $\sum_{k\geq 1} e^{-\frac{c_3}{2}(t_k-t_0)}$ is convergent according to the comparison criterion for numerical series (let us note S its limit), and as it has the same nature as the infinite product $\prod_{k\geq 1} \big(1+\sqrt{\alpha} e^{-\frac{c_3}{2}(t_k-t_0)}\big)$, it follows that $\prod_{k\geq 1} \big(1+\sqrt{\alpha} e^{-\frac{c_3}{2}(t_k-t_0)}\big)$ is also convergent to a limit which we will note P. Then the inequality (17) becomes

$$\begin{aligned} w(t) &\leq w(t_0) P e^{-\frac{c_3}{2}(t-t_0)} e^{\frac{\lambda_1}{2c_3}} \\ &\quad + \frac{\beta_1}{2} P e^{\frac{\lambda_1}{2c_3}} \int_{t_0}^{t} e^{-\frac{c_3}{2}(t-s)} ds \\ &\quad + P\sqrt{d} e^{\frac{\lambda_1}{2c_3}} \sum_{t_0<t_k<t} e^{-\frac{c_3}{2}(t-t_k)} e^{-\frac{c_3}{2}(t_k-t_0)}, \\ &\leq w(t_0) P e^{-\frac{c_3}{2}(t-t_0)} e^{\frac{\lambda_1}{2c_3}} \\ &\quad + \beta_1 P e^{\frac{\lambda_1}{2c_3}} \frac{1}{c_3} \big[1 - e^{-\frac{c_3}{2}(t-t_0)}\big] \\ &\quad + P\sqrt{d} e^{\frac{\lambda_1}{2c_3}} \sum_{t_0<t_k<t} e^{-\frac{c_3}{2}(t_k-t_0)}, \\ &\leq w(t_0) P e^{\frac{\lambda_1}{2c_3}} + \frac{1}{c_3}\beta_1 P e^{\frac{\lambda_1}{2c_3}} + P\sqrt{d} e^{\frac{\lambda_1}{2c_3}} S, \end{aligned}$$

In the light of (i) in Proposition 1, this leads to

$$\|x_1(t)\| \leq \gamma_1 \|x_{10}\| + \gamma_2,$$

where $\gamma_1 = \sqrt{\frac{c_2}{c_1}} P e^{\frac{\lambda_1}{2c_3}}$ and $\gamma_2 = \frac{1}{\sqrt{c_1}} \big(\frac{1}{c_3}\beta_1 P e^{\frac{\lambda_1}{2c_3}} + P\sqrt{d} e^{\frac{\lambda_1}{2c_3}} S\big)$. The uniform boundedness of solutions x_1 of system (1)-(4) is then proved, and consequently, there exists a constant $\sigma_1 > 0$ such that: $\|x_1(t)\| \leq \sigma_1, \ \forall, t \geq t_0$.

B. SECOND STEP: EXPONENTIAL STABILITY OF SOLUTIONS

Using the calculations we did for the proof of the boundedness of solutions, we obtain that the derivative of V along the trajectories of equation (1) for $t \neq t_k$ is given by

$$\begin{aligned} \dot{V}_{(1)}(t, x_1) &\leq -c_3 V(t, x_1) + c_4 \|x_1\| \|g(t,x)\| \|x_2\|, \\ &\leq -c_3 V(t, x_1) + c_4 \|x_1\| (M + \varepsilon \|x_1\|) \|x_2\|, \\ &\leq -c_3 V(t, x_1) + \beta_2 \|x_{20}\| e^{-c_3(t-t_0)}, \end{aligned}$$

where $\beta_2 = c_4\sigma_1(M + \varepsilon\sigma_1)k_2$. And for $t = t_k$ we have

$$V(t_k^+, x_1(t_k^+)) \leq \big(1 + \alpha e^{-c_3(t_k-t_0)}\big) V(t_k, x(t_k)) + \tilde{d} \|x_{20}\| e^{-c_3(t_k-t_0)},$$

where $\tilde{d} = \frac{K k_2 \bar{\epsilon}}{2} + K k_2 \overline{m}$. According to the Lemma 1, we get for all $t \geq t_0$

$$\begin{aligned} V(t, x_1) &\leq V(t_0, x_{10}) \Pi e^{-c_3(t-t_0)} \\ &\quad + P\beta_2 \|x_{20}\| \int_{t_0}^{t} e^{-c_3(t-s)} e^{-c_3(s-t_0)} ds \\ &\quad + \tilde{d} P \|x_{20}\| \sum_{t_0<t_k<t,} e^{-c_3(t-t_k)} e^{-c_3(t_k-t_0)}, \\ &\leq c_2 P \|x_{10}\|^2 e^{-c_3(t-t_0)} + \beta_2 \|x_{20}\| (t-t_0) e^{-c_3(t-t_0)} \\ &\quad + \tilde{d} P \|x_{20}\| \sum_{t_0<t_k<t} e^{-\frac{c_3}{2}(t-t_0)} e^{-\frac{c_3}{2}(t_k-t_0)}, \\ &\leq c_2 P \|x_{10}\|^2 e^{-c_3(t-t_0)} + \frac{2\beta_2 \|x_{20}\|}{c_3 e} e^{-\frac{c_3}{2}(t-t_0)} \\ &\quad + \tilde{d}\, S P \|x_{20}\| e^{-\frac{c_3}{2}(t-t_0)} \\ &\leq C\big(\|x_{10}\|^2 + \|x_{20}\|\big) e^{-\frac{c_3}{2}(t-t_0)}, \end{aligned} \tag{19}$$

where C is a positive constant that is independent of the initial conditions. Consequently, we obtain

$$\begin{aligned} \|x_1\| &\leq \frac{\sqrt{C}}{\sqrt{c_1}} \big(\|x_{10}\| + \sqrt{\|x_{20}\|}\big) e^{-\frac{c_3}{4}(t-t_0)}, \\ &\leq \tilde{\alpha}(\|x_0\|) e^{-\frac{c_3}{4}(t-t_0)}, \end{aligned}$$

where $\tilde{\alpha}$ is a class $\mathcal{K}$ function; thus the proof of Theorem 1 is achieved.

Remark 3: In the available results concerning continuous systems, the boundedness of the solutions is a necessary condition to guarantee the asymptotic stability of the cascaded system of forms (1)-(2). Therefore, the question was how to ensure this uniform boundedness. Additional assumptions were imposed on the growth rate of the interconnection term g as

$$\|g(t,x)\| \leq \theta_1(\|x_2\|) + \theta_2(\|x_2\|)\|x_1\| \tag{20}$$

where $\theta_1, \theta_1 : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ are two continuous functions [27], [28], [33], [38]–[40], [47], [48]. Contrary to what we mentioned above, in Theorem 5 the boundedness of the solutions is not a necessary condition to guarantee the convergence of the cascaded system; moreover, the condition imposed on the interconnection term $g(t,x)$ between the two subsystems is more general than condition (20), and than the one mentioned in [37].

IV. APPLICATION: TRACKING CONTROLLER FOR ROBOT MANIPULATORS

To illustrate our theoretical results, we consider the problem of tracking a wheeled robot that moves in a plane. The motion of this tracking robot is given by the following system of differential equations [26], [49];

$$\dot{x}(t) = v(t)\cos(\theta(t)), \tag{21}$$

$$\dot{y}(t) = v(t)\sin(\theta(t)), \tag{22}$$
$$\dot{\theta}(t) = \omega(t), \tag{23}$$

where v is the forward velocity, ω is the angular velocity, (x, y) are the coordinates of the center of the vehicle, and θ is the angle between the heading direction and the x-axis (see Figure 1). The problem of tracking a reference robot as done in [26] and [51]

$$\begin{aligned} \dot{x}_r(t) &= v_r(t)\cos(\theta_r(t)), \\ \dot{y}_r(t) &= v_r(t)\sin(\theta_r(t)), \\ \dot{\theta}_r(t) &= \omega_r(t). \end{aligned}$$

If we define the error coordinates as in [26], [49]–[51]

$$\begin{bmatrix} x_e(t) \\ y_e(t) \\ \theta_e(t) \end{bmatrix} = \begin{bmatrix} \cos(\theta(t)) & \sin(\theta(t)) & 0 \\ -\sin(\theta(t)) & \cos(\theta(t)) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r(t) - x(t) \\ y_r(t) - y(t) \\ \theta_r(t) - \theta(t) \end{bmatrix},$$

then the coordinates of the error equations in the moving frame can be expressed in the following form

$$\dot{x}_e(t) = \omega(t)y_e(t) - v(t) + v_r(t)\cos(\theta_e(t)), \tag{24}$$
$$\dot{y}_e(t) = -\omega(t)x_e(t) + v_r(t)\sin(\theta_e(t)), \tag{25}$$
$$\dot{\theta}_e(t) = \omega_r(t) - \omega(t). \tag{26}$$

The trajectory tracking problem of system (24)-(26) has been studied in depth using cascaded systems in several papers, for example, in [50] and [26] (where the control represents a function that depends on two variables of the triplet (x_e, y_e, θ_e) and an additional variable defined by a certain differential equation. This control was nonlinear in [26], and was reduced to linear in [50]). The same problem was treated later for delay systems in [49]. In the following, we study this problem using impulsive control such that the following closed error equation

$$\dot{z}_e(t) = f(t, z_e(t)) + g(t, z_e(t), \theta_e(t)) \tag{27}$$
$$\dot{\theta}_e(t) = \omega_r(t) - \omega(t) \tag{28}$$
$$z_e(t_k^+) = (1+\lambda)z_e(t_k) + L_k(z_e(t), \theta_e(t)) \tag{29}$$
$$\theta_e(t_k^+) = (1+\lambda)\theta_e(t_k) \tag{30}$$
$$z_e(t_0^+) = z_0, \quad \theta_e(t_0^+) = \theta_0 \tag{31}$$

is exponentially stable, where $\lambda \in (-1, 0)$ and

$$z_e = \begin{bmatrix} x_e \\ y_e \end{bmatrix}, \quad f(t, z_e(t)) = \begin{bmatrix} \omega(t)y_e(t) - v(t) \\ -\omega(t)x_e(t) \end{bmatrix},$$
$$g(t, z_e(t)) = \begin{bmatrix} v_r(t)\cos(\theta_e(t)) \\ v_r(t)\sin(\theta_e(t)) \end{bmatrix},$$

where λ is a positive constant that is strictly smaller than 1. Clearly, system (27)-(31) can be considered as a cascaded system of the form (1)-(4), whose subsystems (5)-(6) and (7)-(8) are respectively:

$$\dot{z}_e(t) = f(t, z_e(t)), \tag{32}$$
$$z_e(t_k^+) = (1+\lambda)z_e(t_k), \tag{33}$$
$$z_e(t_0^+) = z_0, \tag{34}$$

and

$$\dot{\theta}_e(t) = \omega_r(t) - \omega(t), \tag{35}$$
$$\theta_e(t_k^+) = (1+\lambda)\theta_e(t_k), \tag{36}$$
$$\theta_e(t_0^+) = \theta_0. \tag{37}$$

The cross terms $g(t, z_e, \theta_e) = (v_r(t)\cos(\theta_e(t)), v_r(t)\sin(\theta_e(t)))$ and $L(t, z_e(t), \theta_e(t))$ are considered to satisfy conditions (a) and (b) in Theorem 1.

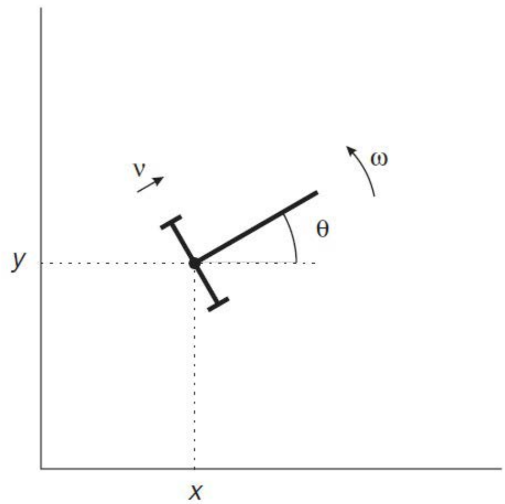

FIGURE 1. Wheeled robot coordinates.

A. EXPONENTIAL STABILITY OF SUBSYTEM (23)-(24)

The system (32)-(33) can be written as

$$\dot{z}_e(t) = A(t)z_e(t) + \varphi(t), \tag{38}$$

where $\varphi(t) = [-v(t) + v_r(t), 0]^T$ and $A(t) = \begin{pmatrix} 0 & \omega(t) \\ -\omega(t) & 0 \end{pmatrix}$.

This system can be interpreted as a perturbed linear time-varying system with an exponentially stable nominal system. The derivative of the Lyapunov function $V(t) = z_e(t)^T z_e(t) = x_e^2(t) + y_e^2(t)$ along the trajectories of the nominal system of (32)-(33) gives $\dot{V}(t) = 0, \ \forall\, t \in (t_k, t_{k+1}]$ and then

$$V(t) = V(t_k^+), \quad \forall\, t \in (t_k, t_{k+1}]. \tag{39}$$

Furthermore, for all $t = t_k$

$$\begin{aligned} V(t_k^+) &= \|z(t_k^+)\|^2, \\ &= (1+\lambda)^2\|z(t_k)\|^2, \\ &= (1+\lambda)^2\|z(t_{k-1}^+)\|^2, \\ &= (1+\lambda)^2 V(t_{k-1}^+). \end{aligned}$$

It yields that

$$V(t_k^+) = (1+\lambda)^{2k} V(t_0^+),$$

which gives

$$V(t_k^+) \le V(t_0^+)e^{2k\ln(1+\lambda)}. \tag{40}$$

As for all $t \in (t_k, t_{k+1}]$ we have $t - t_0 \leq (k+1)\Delta$. It follows from (39) and (40) that $\forall\, t \in (t_k, t_{k+1}]$

$$V(t) \leq V(t_0^+)e^{-2\ln(1+\lambda)}e^{\frac{2}{\Delta}\ln(1+\lambda)(t-t_0)},$$

and consequently

$$\|z(t)\| \leq C_0\|z_0\|e^{-\gamma_0(t-t_0)}, \tag{41}$$

where $C_0 = e^{-\ln(1+\lambda)}$ and $\gamma_0 = -\frac{2}{\Delta}\ln(1+\lambda) > 0$, respectively. In view of the foregoing, the exponential stability of the nominal system of (32)-(33) results.

Concerning the perturbed system (32)-(33), let us suppose that the velocities v and v_r satisfy the following assumption

$$|v(t) - v_r(t)| \leq C_1 e^{-\gamma_1 t}, \tag{42}$$

for some positive constants C_1 and γ_1. Let W be the Lyapunov function given by Proposition 1 for the nominal system of (32)-(33). The derivative of W along the trajectories of system (32)-(33) gives, for $t \neq t_k$

$$\begin{aligned}\dot{W}(t) &\leq -c_3 W(t) + c_4 C_1 e^{-\gamma t}\,\|z\|,\\ &\leq -c_3 W(t) + \frac{c_4 c}{\sqrt{c_1}}\sqrt{W(t)}e^{-\gamma t},\end{aligned}$$

and for $t = t_k,\ W(t_k^+) \leq W(t_k)$. If we pose $w(t) = \sqrt{W(t)}$, we obtain

$$\begin{aligned}w(t) &\leq -\frac{c_3}{2}w(t) + c_5 e^{-\gamma_1 t}, \quad t \neq t_k\\ w(t_k^+) &\leq w(t_k).\end{aligned}$$

Due to the Lemma 1, we obtain

$$\begin{aligned}w(t) &\leq w(t_0^+)e^{-\frac{c_3}{2}(t-t_0)} + c_5\int_{t_0}^{t} e^{-\frac{c_3}{2}(t-s)}e^{-\gamma_2 s}ds,\\ &\leq w(t_0^+)e^{-\frac{c_3}{2}(t-t_0)} + c_5\int_{t_0}^{t} e^{-\frac{c_3}{2}(t-s)}e^{-\gamma_2(s-t_0)}ds,\\ &\leq w(t_0^+)e^{-\tilde{\gamma}(t-t_0)} + c_5\int_{t_0}^{t} e^{-\tilde{\gamma}(t-s)}e^{-\tilde{\gamma}(s-t_0)}ds,\\ &\leq w(t_0^+)e^{-\tilde{\gamma}(t-t_0)} + c_5(t-t_0)e^{-\tilde{\gamma}(t-t_0)},\\ &\leq w(t_0^+)e^{-\tilde{\gamma}(t-t_0)} + \frac{2c_5}{\tilde{\gamma}}e^{-\frac{\tilde{\gamma}}{2}(t-t_0)},\\ &\leq (\sqrt{c_2}\|z_0\| + \frac{2c_5}{\tilde{\gamma}})e^{-\frac{\tilde{\gamma}}{2}(t-t_0)},\end{aligned}$$

this gives

$$\|z_e(t)\| \leq (\sqrt{\frac{c_2}{c_1}}\|z_0\| + \frac{2c_5}{\sqrt{c_1}\tilde{\gamma}})e^{-\frac{\tilde{\gamma}}{2}(t-t_0)}$$

and the exponential stability of the system (32)-(33) results. Concerning the subsystem (35)-(37), we assume that, for $t \in (t_k, t_{k+1}]$, the following hypothesis is verified

$$\lim_{k\to+\infty}\Big(k\ln|1+\lambda| + \int_{t_0}^{t}\omega_r(s) - \omega(s)ds\Big) = -\infty. \tag{43}$$

This assumption guarantees the exponential convergence of subsystem (35)-(37), therefore, condition (a) of Theorem 1 is fulfilled. One can easily ensure that if

$$v_r(t) \leq \bar{v} < +\infty. \tag{44}$$

The function $g(t, z_e, \theta_e)$ satisfies condition (b) with $M = \bar{v}$ and $\varepsilon = 0$. Hence, in view of Theorem 1, the solution of system (1)-(4) is UES.

Remark 4: We would like to draw attention to the fact that assumption (42) is not restrictive. Indeed, the nominal system of (32)-(33) is exponentially stable, and by adding the element $\varphi(t)$, the system becomes non-homogeneous, which makes its exponential stabilization more difficult. To maintain the exponential convergence of the system (32)-(33) and to limit the effect of the $\varphi(t)$ perturbation on it, we must either add a term to the instants of the impulses, or make these perturbations themselves exponentially close to zero. To see this more clearly, let us take the following scalar example where the $\varphi(t)$ term is close to zero (but not exponentially close to zero) and yet does not preserve the exponential stability of the system (32)-(33):

$$\begin{aligned}x'(t) &= \frac{\sin(t)}{t^{\frac{1}{4}}}, \quad t \neq k, &(45)\\ x(k^+) &= \frac{1}{2}x(k^+), &(46)\\ x(0) &= 1. &(47)\end{aligned}$$

The solution of the nominal system of (45)-(47) is given by $x(t) = \frac{1}{2^k}$, for any $t \in (k, k+1], k \geq 0$. This solution was clearly exponentially stable (red in Figure 2). The solution of (45-47) is not exponentially stable (blue in Figure 2).

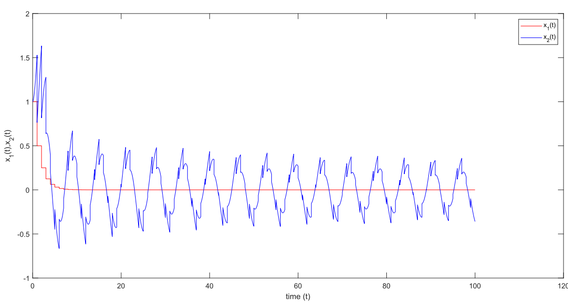

FIGURE 2. **Solutions of nominal system and system (45)-(47).**

For the simulation, we take $v(t) = 1.25\ e^{-0.5t}$, $v_r(t) = e^{-0.5t}$, $\omega(t) = \sin(t)$, $\omega_r(t) = \frac{1}{2}\sin(t/5) + \sin(t)$, $\lambda = -0.6$, while the perturbed term at the jump instants is taken equal to $L_k(z_e(t), \theta_e(t)) = (\frac{x_e\sin(\theta_e)}{1+x_e^2+y_e^2}, -\frac{y_e\sin(\theta_e)}{1+x_e^2+y_e^2})$. Finally, the pulse step taken is equal to 0.01 and the initial condition $(-\frac{\pi}{2}, \frac{\pi}{4}, \pi)^T$. It can be verified that all conditions (42), (43) and (44) are satisfied and the system is UES. The state trajectories of system (27)-(31) without impulses are depicted in Figure 3. The convergence of the trajectories of the impulsive subsystems Equations (32)-(34) and (35)-(37) is shown in Figure 4, which illustrates the exponential convergence of the isolated subsystems, while Figure 5 reveals the exponential convergence of the solution of the cascaded impulsive system (27)-(31) to zero.

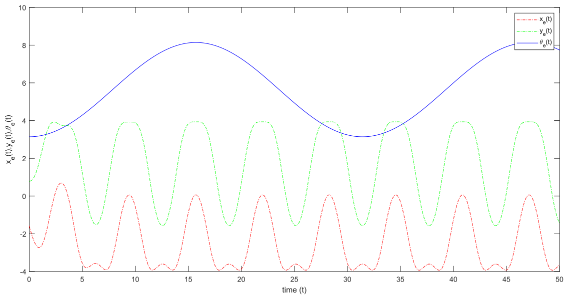

FIGURE 3. Solution of (27)-(31) without impulses. .

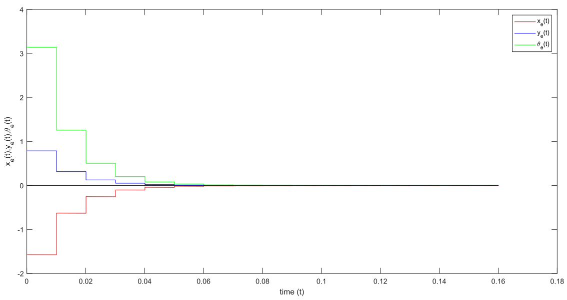

FIGURE 4. Solutions of subsytems (32)-(34) and (35-37) without interconnection.

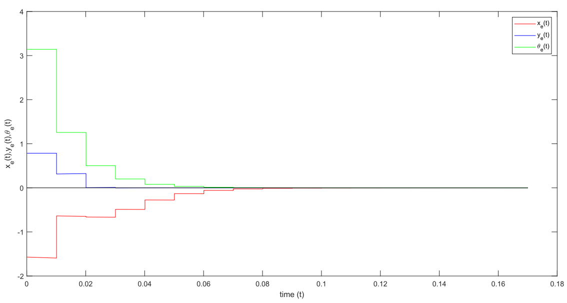

FIGURE 5. Solution of (27)-(31) under impulsive control.

V. CONCLUSION

In this paper, we addressed the problem of the exponential stability of impulsive cascade systems. A stability criterion, which covers a large class of such systems, was established using the second Lyapunov method. The proven result was applied to the trajectory-tracking problem for a simple dynamic model of a mobile robot. By combining the theory of cascaded systems and the theory of impulsive systems, we have proved that it is possible to design an impulsive controller that allows trajectory tracking.

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