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A Contribution in the Observer Design for a Class of Bilinear Singular Delayed Systems

FATMA HAMZAOUI, MALEK KHADHRAOUI, AND HASSANI MESSAOUD
Research Laboratory LARATSI, National Engineering School of Monastir (ENIM), University of Monastir Tunisia, Monastir 5000, Tunisia

Corresponding author: Fatma Hamzaoui (hamzaoui.ff@gmail.com)

ABSTRACT This paper proposes a functional observer for a bilinear singular systems with constant time delay present in both state and known input vectors. The considered system has transformed to an updated form in order to estimate both a state vector and an unknown inputs functional vector independently from the considered time delay. The necessary conditions for the existence of the functional observer are formulated. The proposed observer design procedure is based on the Lyapunov–Krasovskii theory. Numerical example is given to prove the new approach.

INDEX TERMS Singular, bilinear, time delay, unknown inputs, functional observer.

I. INTRODUCTION

Bilinear systems have been striking in the recent decade [7], [10], [14]. In fact, numerous areas of engineering, socio-economic, biology and ecology can be labelled by bilinear models while classical ones are insufficient such as [13] and [11].

Moreover, bilinear system can be viewed as a special non-linear one. A great part of published researches have been interesting in the controllability, observability ([8], [9], [10]) and identification. However less attention has been given to the development of observer for unknown inputs bilinear models ([12], [14]).

On the one hand, for a class of linear systems, observers are mainly excited by the system input and output and its existence depends on the system observability characteristic ([15]-[18]). On the other hand, bilinear class systems observability is usually affected by the input. Its dynamic performance depends on the chosen known input ([14] and [19]).

Furthermore, highly important has been given to delayed system ([3], [6], [15], [19], [22] and [23]). Time delay, presented in state and known input vectors, gives rise to an instability system. Thus, stability criteria was considered and tested using different tools (LMI tests) ([5] and [18]) and according to the Lyapunov Krasovskii theory [21].

Motivated by these facts, we have suggested an observer in order to estimate both state functional vector and unknown input functional vector for a class of a bilinear system with the presence of an unknown input and constant time delay. Based on Lyapunov-Krasovskii [21], Linear Matrix Inequalities (LMI) approach is developed to determine the observer optimum gain.

The present paper is organized as follows. Section II presents the difference between current work and recent published work. Section III introduces the problem that we seek propose to solve. Section IV gives a time domain solution for the unknown input functional observer design problem. An LMI approach is then applied to optimize the gain implemented in the observer. Section V drafts a numerical example to illustrate the approach and then section VI concludes the paper.

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II. RELATED WORK

In this framework, we develop a functional observer for bilinear system. In recent references ([7], [8] and [9]) the authors introduce an approach which solves the observer problem even when the system presents unknown inputs ([14] and [20]). However, in this present paper, we consider a singular system with an additional constant time delay in both state and inputs vectors. And we suggest to estimate both a state functional and an unknown input functional vectors. The time delay is a source of instability [22], so control of delayed systems is practically important.

III. PROBLEM FORMULATION

Considering the following bilinear descriptor system presented as:

$$E\dot{x}(t) = Ax(t) + A_d x(t-\tau) + \sum_{i=1}^{m} D_i u_i x(t) + Bu(t) + B_d u(t-\tau) + Fv(t) \tag{1}$$

$$y(t) = Cx(t) + Gv(t) \tag{2}$$
$$z(t) = Lx(t) \tag{3}$$

$x \in \mathbb{R}^n$, $u^T = [u_1, u_2, \ldots, u_m] \in \mathbb{R}^m$, $y \in \mathbb{R}^p$ and $z \in \mathbb{R}^r$ represent the state, the input, the output and the state functional vectors. Thus, $v \in \mathbb{R}^q$ is the unknown input vector. $E, A, A_d, B, B_d, F, G, C$ and $D_i (i = 1, 2 \ldots .m)$ are the known matrices of appropriate dimensions.
$\tau(t)$ is the variable delay in the state and the input vectors.

Starting from:

Assumption [14]:

1) $rank\ (E) < n$
2) $rank \begin{bmatrix} F \\ G \end{bmatrix} = q$
3) $rank\ [G] = \bar{q} < q$

By the way, there will be an orthogonal matrix $R \in \mathbb{R}^{(p \times p)}$ and a non singular matrix $W \in \mathbb{R}^{(q \times q)}$, such as [14]:

$$R^T G W = \begin{pmatrix} I_{\bar{q}} & 0 \\ 0 & 0 \end{pmatrix} \tag{4}$$

Multiplying equation (2) by R^T, we will obtain:

$$y_1(t) = C_1 x(t) + v_1(t) \tag{5}$$
$$y_2(t) = C_2 x(t) \tag{6}$$

with

$$R^T y(t) = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, \quad R^T C = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \tag{7}$$

After using equation (4), equation (1) can be reported as:

$$\begin{aligned} E\dot{x}(t) = A_1 x(t) + A_d x(t-\tau) + \sum_{i=1}^{m} D_i u_i x(t) + Bu(t) \\ + B_d u(t-\tau) + F_1 y_1(t) + F_2 v_2(t) \end{aligned} \tag{8}$$

with

$$FW = \begin{bmatrix} F_1 & F_2 \end{bmatrix}, \tag{9}$$
$$W^{-1} v(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}, \tag{10}$$
$$A_1 = A - F_1 C_1 \tag{11}$$

Then the system (1-3) switches to:

$$\begin{aligned} E\dot{x}(t) &= A_1 x(t) + A_d x(t-\tau) + \sum_{i=1}^{m} D_i u_i x(t) + Bu(t) \\ &\quad + B_d u(t-\tau) + F_1 y_1(t) + F_2 v_2(t) \end{aligned} \tag{12}$$
$$y(t) = C_1 x(t) + v_1(t) \tag{13}$$
$$y_2(t) = C_2 x(t) \tag{14}$$
$$z(t) = Lx(t) \tag{15}$$

Yet, the measurement $y(t)$ has been divided into two parts. $y_1(t)$ is affected by the unknown input $v_1(t)$. $y_2(t)$ is the disturbance-free part of the measurement $y(t)$. So that we can use this part in observer design like output vector.

Based on (4)-(6), assumptions and hypothesis become:

Hypothesis [17]:

1) $rank\ (E) = r < n$
2) $rank \begin{bmatrix} E \\ C_2 \end{bmatrix} = n$

Objective: We aim at developing a functional observer for bilinear singular delayed system. The considered system is affected by an unknown input acting on the dynamic and output equations. The functional observer estimates a unknown input functional $v_1(t)$ and a linear combination of the state vector components x_i, $i = 1 \ldots n$, using only the system input and output measurements.

We suppose that:

$$\bar{z}(t) = \begin{bmatrix} Lx(t) \\ v_1(t) \end{bmatrix} \in \mathbb{R}^{(r+\bar{q})}, \quad L \in \mathbb{R}^{(r \times n)} \tag{16}$$

By replacing $v_1(t)$ by its expression derived from (13), $z(t)$ can be written as:

$$\bar{z}(t) = \bar{L}x(t) + \bar{I}y_1(t) \tag{17}$$

with

$$\bar{L} = \begin{bmatrix} L \\ C_1 \end{bmatrix}, \quad \bar{I} = \begin{bmatrix} 0_{r \times \bar{q}} \\ I_{\bar{q}} \end{bmatrix}, \tag{18}$$

$rank(\bar{L}) = r + \bar{q} < n$

According to [1] there exists a non singular matrix T such as:

$$T = \begin{pmatrix} a_0 & b_0 \\ c_0 & d_0 \end{pmatrix} \tag{19}$$

with

$$a_0 E + b_0 C_2 = \bar{L} \tag{20}$$
$$c_0 E + d_0 C_2 = 0 \tag{21}$$

The proposed observer is presented as follows:

$$\begin{aligned} \dot{\eta}(t) &= M\eta(t) + M_d \eta(t-\tau) + Hu(t) \\ &\quad + H_d u(t-\tau) + L_1 y_1(t) + L_2 y_1(t) \\ &\quad + L_d(t - \tau(t)) + \sum_{i=1}^{m} N_i u_i y_2(t) \end{aligned} \tag{22}$$
$$\hat{\bar{z}}(t) = \eta(t) + P_1 y_1(t) + P_2 y_2(t) \tag{23}$$

$\eta(t)$ is the state vector of the observer and $\hat{\bar{z}}(t)$ is the estimation of state functional $\bar{z}(t)$.
The matrices M, M_d, H, H_d, L_1, L_2, L_d, N_i, P_1 and P_2 will be determined using the LMI approach.

IV. TIME DOMAIN DESIGN OF THE FUNCTION OBSERVER

A. CONDITIONS OF UNKNOWN INPUT OBSERVER

If we consider $e(t)$ as the time estimation error:

$$e(t) = \hat{\bar{z}}(t) - \bar{z}(t) \tag{24}$$

Using (17) and (23) we will have:

$$e(t) = \eta(t) + (P_1 - \bar{I})y_1(t) + (P_2 C_2 - \bar{L})x(t) \tag{25}$$

In order to get rid on the effect of the unknown input on the observer dynamic, we impose:

$$P_1 = \bar{I} \tag{26}$$

Using (20) and (21) we can write:

$$e(t) = \eta(t) - (a_0 + \beta c_0)Ex(t) \tag{27}$$
$$+(P_2 - b_0 - \beta d_0)C_2x(t) \tag{28}$$

To obtain an unbiased estimation error dynamic we suppose that:

$$P_2 = b_0 + \beta d_0 \tag{29}$$

The equation (27) will be:

$$e(t) = \eta(t) - SEx(t) \tag{30}$$

with

$$S = a_0 + \beta c_0 \tag{31}$$

Theorem 1: System (22-23) is a functional observer for system (1-3) if even the following equations are attached:

i. $\dot{e}(t) = Me(t) + M_de(t-\tau)$ is asymptotically stable.
ii. $MGE + L_2C_2 - SA_1 = 0$
iii. $M_dGE + L_dC_2 - SA_d = 0$
iv. $SB - H = 0$
v. $SB_d - H_d = 0$
vi. $L_1 - SF_1 = 0$
vii. $N_iC_2 - SD_i = 0, i = 1, \dots, m$
viii. $SF_2 = 0$

▲

Proof 1: The time derivative of $e(t)$ given in (30) is:

$$\dot{e}(t) = \dot{\eta}(t) - SE\dot{x}(t) \tag{32}$$

Now, replacing $\dot{\eta}(t)$ and $E\dot{x}(t)$ with its expression (22) and (12) relation (32) will be:

$$\begin{aligned}\dot{e}(t) = {} & Me((t) + M_de(t-\tau) + [H_d - SB_d]u(t-\tau)\\ & +[MGE + L_2C_2 - SA_1]x(t) + [H - SB]u(t)\\ & +[M_dGE + L_dC_2 - SA_d]x(t-\tau) + SF_2v_2(t)\\ & +[L_1 - SF_1]y_1(t) + \sum_{i=1}^{m}(N_iC_2 - SD_i)u_ix(t)\end{aligned} \tag{33}$$

The functional observer (22-23) will estimate asymptotically the real functional of the state $z(t)$ and the unknown input function $v_1(t)$ if conditions i) – viii) are required. ■

In the next part, and according to [16], an equivalent form of constraints vii) and viii) is proposed and conditions for the existence of the desired functional observer are deduced. This equivalent form is based on the matrix ϕ defined as:

$$\phi = \begin{bmatrix} F_2 & D_a\kappa(C_a)\end{bmatrix} \tag{34}$$

with

$$D_a = \begin{bmatrix} D_1 & D_2 & \dots & D_m\end{bmatrix} \tag{35}$$

$$C_a = \begin{bmatrix} D_2 & 0 & .. & 0\\ 0 & C_2 & .. & 0\\ .. & .. & .. & ..\\ 0 & 0 & .. & C_2\end{bmatrix} \tag{36}$$

$$N_a = \begin{bmatrix} N_1 & N_2 & \dots & N_m\end{bmatrix} \tag{37}$$

and $\kappa(.)$ spans the basis of the right-kernel of the corresponding matrix. The constraints given in theorem 1 can be expressed equivalently in terms of matrix ϕ.

Lemma: System (22-23) is a functional observer for system (1-3) if the following equations are set:

i. $\dot{e}(t) = Me(t) + M_de(t-\tau)$ is asymptotically stable.
ii. $MGE + L_2C_2 - SA_1 = 0$
iii. $M_dGE + L_dC_2 - SA_d = 0$
iv. $SB = H$
v. $SB_d = H_d$
vi. $L_1 = SF_1$
vii. $N_a = SD_aC_a^+, i = 1, \dots, m$
viii. $S\phi = 0$

with C_a^+ is a generalized inverse of C_a

Proof 2: Replacing S with its expression (31) in conditions vii) and viii) of theorem 1, we will obtain:

$$\begin{bmatrix} -\beta & N_a\end{bmatrix}\begin{bmatrix} c_0F_2 & c_0D_a\\ 0 & C_a\end{bmatrix} = \begin{bmatrix} a_0F_2 & a_0D_a\end{bmatrix} \tag{38}$$

Since C_2 is of full row rank, the matrix $\begin{bmatrix} C_a^+ & \kappa(C_a)\end{bmatrix}$ is non-singular, then equation (38) is equivalent to:

$$\begin{aligned}&\begin{bmatrix} -\beta & N_a\end{bmatrix}\begin{bmatrix} c_0F_2 & c_0D_a\\ 0 & C_a\end{bmatrix}\begin{bmatrix} I & 0 & 0\\ 0 & C_a^+ & \kappa(C_a)\end{bmatrix}\\ &\quad = \begin{bmatrix} a_0F_2 & a_0D_a\end{bmatrix}\begin{bmatrix} I & 0 & 0\\ 0 & C_a^+ & \kappa(C_a)\end{bmatrix}\end{aligned} \tag{39}$$

Then

$$\begin{aligned}&\begin{bmatrix} -\beta & N_a\end{bmatrix}\begin{bmatrix} c_0F_2 & c_0D_aC_a^+ & c_0D_a\kappa(C_a)\\ 0 & I & 0\end{bmatrix}\\ &\quad = \begin{bmatrix} a_0F_2 & a_0D_aC_a^+ & a_0D_a\kappa(C_a)\end{bmatrix}\end{aligned} \tag{40}$$

After using equation (34), equation (40) will be equivalent to:

$$N_a = (a_0 + \beta c_0)D_aC_a^+ = SD_aD_a^+ \tag{41}$$
$$(a_0 + \beta c_0)\begin{bmatrix} F_2 & D_a\kappa(C_a)\end{bmatrix} = S\phi = 0 \tag{42}$$

So, lemma 1 and theorem 1 are equivalent. ■

B. DETERMINATION OF THE OBSERVER MATRICES

Now, after replacing S with its equation (31) in conditions ii), iii), vi) et vii) of lemma 1 we will have,

$$a_0A_1 = Ma_0E + JC_2 - \beta c_0A_1 \tag{43}$$
$$a_0A_d = M_da_0E + J_dC_2 - \beta c_0A_d \tag{44}$$
$$a_0D_aC_{2a}^+ = N_aN_a - \beta c_0D_aC_{2a}^+ \tag{45}$$

and

$$a_0F_2 = -\beta c_0F_2 \tag{46}$$

with

$$J = L_2 - M\beta d_0 \tag{47}$$

$$J_d = L_d - M_d \beta d_0 \tag{48}$$

We can rewrite (43)-(46) in the following matrix form:

$$\Gamma = \Omega\Theta \tag{49}$$

with

$$\Omega = \begin{bmatrix} M & M_d & J & J_d & \beta & N_a \end{bmatrix} \tag{50}$$

$$\Theta = \begin{pmatrix} a_0E & 0 & 0 & 0 \\ 0 & a_0E & 0 & 0 \\ C_2 & 0 & 0 & 0 \\ 0 & C_2 & 0 & 0 \\ c_0A_1 & c_0A_d & -c_0F_2 & -c0D_a \\ 0 & 0 & 0 & C_a \end{pmatrix} \tag{51}$$

$$\Gamma + \begin{bmatrix} a_0A_1 & a_0A_d & a_0F_2 & a_0D_a \end{bmatrix} \tag{52}$$

We note that the solution (49) exists if:

$$rang \begin{bmatrix} \Theta \\ \Gamma \end{bmatrix} = rang(\Theta) \tag{53}$$

By respecting the latter condition (53), we will have:

$$X = \Gamma\Theta^+ - Z(I - \Theta\Theta^+) \tag{54}$$

Thus Θ^+ is the generalized inverse of matrix Θ and Z is an arbitrary matrix. That will be determined later using the LMI approach.

The unknown matrix M is defined as:

$$M = X \begin{pmatrix} I \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = M_1 + ZM2 \tag{55}$$

Replacing X with (54) in (55), we will have:

$$M_1 = \Gamma\Theta^+ \begin{pmatrix} I \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad M_2 = (I - \Theta\Theta^+) \begin{pmatrix} I \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{56}$$

The unknown matrix M_d is defined as:

$$M_d = X \begin{pmatrix} 0 \\ I \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = M_{d1} + ZMd2 \tag{57}$$

Replacing X with (54) in (57), we will have:

$$M_{d1} = \Gamma\Theta^+ \begin{pmatrix} 0 \\ I \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad M_{d2} = (I - \Theta\Theta^+) \begin{pmatrix} 0 \\ I \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{58}$$

Similarly, we can determine J, J_d, β and N_a.

At this stage, and according to theorem 1 and Lyapunov-Krasovskii stability theory, one can get the gain matrix Z which parametrizes the observer matrices, as proposed in theorem 2.

Theorem 2: System (22-23) is a functional bilinear observer for system (1-3) if there are matrices $P = P^T > 0$, $Q = Q^T > 0$ and Y solution of the following LMI:

$$\begin{pmatrix} \varphi & X \\ * & \Pi \end{pmatrix} < 0 \tag{59}$$

where φ, X and Π are given in the appendix.
The gain Z is given as follows:

$$Z = P^{-1}Y \tag{60}$$

▲

Proof 3: Having considered the following Lyapunov-Krasovskii functional:

$$V(e,t) = e^T(t)Pe(t) + \int_{t-\tau}^{t} e^T(s)\, Q\, e(s)\, ds + \int_0^{\tau}\int_{t-\theta}^{t} \dot{e}^T(s)\, P\, \dot{e}(s)\, ds\, d\theta \tag{61}$$

where P, Q and R are Positive Definite Matrices.
Using condition i) of theorem 1, the time derivative of the functional $V(e,t)$ has to be:

Where

$$\begin{aligned} \dot{V}(e,t) = {} & e^T(t)[PM + M^TP + Q + \tau M^TPM]e(t) \\ & + e^T(t-\tau)[M_d^TP + \tau M_d^TPM]e(t) \\ & + e^T(t)[PM_d + \tau M^TPM_d]e(t-\tau) \\ & + e^T(t-\tau)[\tau M_d^TPM_d - Q]e(t-\tau) \\ & - \int_0^{\tau} \dot{e}^T(t-\theta)\, P\, \dot{e}(t-\theta)\, d\theta \end{aligned} \tag{62}$$

As θ is limited, the quantity $\dot{e}(t-\theta)$ satisfy:

$$\lim_{t\to\infty} \dot{e}(t-\theta) = \lim_{t\to\infty} \dot{e}(t) \tag{63}$$

Therefore:

$$\lim_{t\to\infty} \int_0^{t} \dot{e}^T(t-\theta)P\dot{e}(t-\theta)d\theta = \tau \lim_{t\to\infty} \dot{e}^T(t)P\dot{e}(t) \tag{64}$$

Then we suppose that:

$$\lambda = \lim_{t\to\infty} \dot{e}(t) \tag{65}$$

According to (63)-(65) the equation (62) is equivalent to:

$$\begin{aligned} \dot{V}(e,t) = {} & e^T(t)[PM + M^TP + Q + \tau M^TPM]e(t) \\ & + e^T(t-\tau)[M_d^TP + \tau M_d^TPM]e(t) \\ & + e^T(t)[PM_d + \tau M^TPM_d]e(t-\tau) \\ & + e^T(t-\tau)[\tau M_d^TPM_d - Q]e(t-\tau) - \tau\lambda^TP\lambda \end{aligned} \tag{66}$$

This can be written as follows:

$$\begin{bmatrix} e^T(t) & e^T(t-\tau) & \lambda^T \end{bmatrix} \psi \begin{bmatrix} e(t) \\ e(t-\tau) \\ \lambda \end{bmatrix} < 0 \tag{67}$$

with

$$\psi = \begin{pmatrix} \alpha & PM_d + \tau M^T PM_d & 0 \\ * & \tau M_d^T PM_d - Q & 0 \\ * & * & -\tau P \end{pmatrix} \tag{68}$$

and

$$\alpha = M^T P + PM + Q + \tau M^T PM \tag{69}$$

In order to get rid on the quadratic form which is, present in equation (68), we propose rewriting this equation as follows:

$$\Psi = U - \Phi^T \Xi^{-1} \Phi \tag{70}$$

with

$$\Phi = \begin{pmatrix} \sqrt{\tau} M & \sqrt{\tau} M_d & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{pmatrix} \tag{71}$$

$$\Xi = - \begin{pmatrix} P^{-1} & 0 & 0 \\ * & P^{-1} & 0 \\ * & * & P^{-1} \end{pmatrix} \tag{72}$$

And

$$U = \begin{pmatrix} M^T P + PM + Q & PM_d & 0 \\ * & -Q & 0 \\ * & * & 0 \end{pmatrix} \tag{73}$$

According to the Schur lemma, $\Psi < 0$ and $\Xi < 0$ only if:

$$\Lambda = \begin{pmatrix} \Xi & \Phi \\ \Phi^T & U \end{pmatrix} < 0 \tag{74}$$

Now we propose to apply a congruence transformation [6] to Λ such as:

$$\Sigma^T \Lambda \Sigma < 0 \tag{75}$$

where Σ is a non singular matrix given as:

$$\Sigma = \begin{pmatrix} P & 0 & 0 & 0 & 0 & 0 \\ * & P & 0 & 0 & 0 & 0 \\ * & * & P & 0 & 0 & 0 \\ * & * & * & I & 0 & 0 \\ * & * & * & * & I & 0 \\ * & * & * & * & * & I \end{pmatrix} \tag{76}$$

Replacing M and M_d by their expressions given in (55) and (57) successively in (75), theorem 2 becomes obvious. ■

V. NUMERICAL EXAMPLE

Let's consider system (1-3) such as:

$$E = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \quad A = \begin{pmatrix} -7 & 0 \\ 0 & -1 \end{pmatrix}, A_d = \begin{pmatrix} -0.23 & 0 \\ 0 & -0.5 \end{pmatrix},$$

$$B = \begin{pmatrix} 2 & 0 \\ -1 & 3.2 \end{pmatrix}, \quad B_d = \begin{pmatrix} 5 & -1 \\ 2 & 0 \end{pmatrix}, F = \begin{pmatrix} 4 & 1 \\ 0 & -2 \end{pmatrix},$$

$$D_1 = \begin{pmatrix} -1 & 0 \\ -2 & 0 \end{pmatrix}, \quad D_2 = \begin{pmatrix} 0 & -1 \\ 0 & -2 \end{pmatrix},$$

$$C = \begin{pmatrix} 1 & -1 \\ 0.5 & 3 \end{pmatrix}, \quad G = \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix}, L = \begin{pmatrix} 2 & -1 \end{pmatrix},$$

Delay time is $d = 1s$.

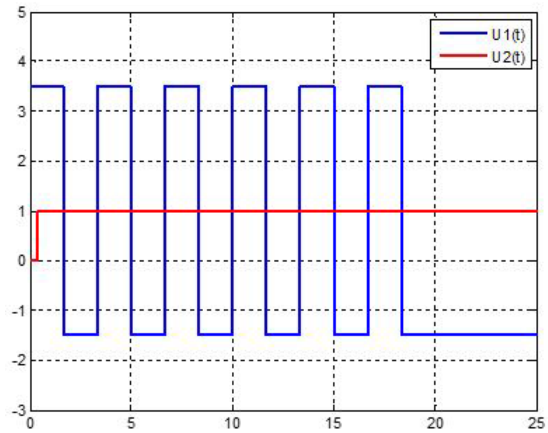

FIGURE 1. Input Signal $u(t)$.

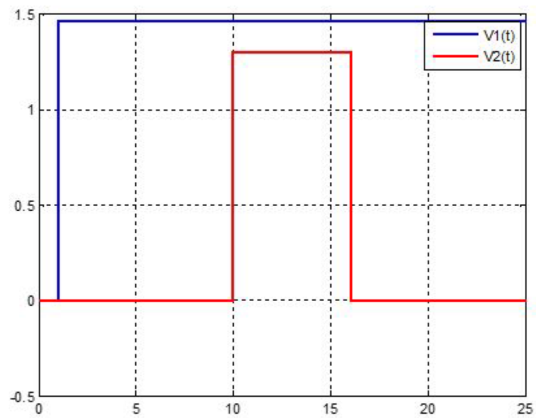

FIGURE 2. Unknown input $v(t)$.

Following the procedure proposed below, we obtain:

$$a_0 = \begin{pmatrix} 2 & 0.1667 \\ -1.1667 & 0 \end{pmatrix}, \quad b_0 = \begin{pmatrix} -0.333 \\ 0.333 \end{pmatrix},$$

$$c_0 = \begin{pmatrix} -1 & 1 \end{pmatrix}, d_0 = \begin{pmatrix} 0 \end{pmatrix}$$

$$R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad W = \begin{pmatrix} 0.4 & -0.2 \\ 0.2 & 0.4 \end{pmatrix},$$

Supposing that $u(t)$, $v(t)$ are given by Figure1 and Figure 2. The resolution of LMI (59) gives:

$$M = \begin{pmatrix} -14.82 & 7.44 \\ 7.44 & -5 \end{pmatrix}, \quad M_d = \begin{pmatrix} -0.43 & 0.21 \\ 0.23 & -0.11 \end{pmatrix},$$

$$L_1 = \begin{pmatrix} 8.66 \\ -4.66 \end{pmatrix}, \quad L_2 = \begin{pmatrix} 3.61 \\ -1.94 \end{pmatrix} L_d = \begin{pmatrix} 0.36 \\ -0.19 \end{pmatrix},$$

$$H = \begin{pmatrix} 10.83 & -6.93 \\ -5.83 & 3.73 \end{pmatrix}, H_d = \begin{pmatrix} 17.33 & -4.33 \\ -9.33 & 2.33 \end{pmatrix}$$

$$N_1 = 10^{-14} \begin{pmatrix} -0.5 \\ 0.26 \end{pmatrix}, N_2 = 10^{-13} \begin{pmatrix} -0.15 \\ 0.08 \end{pmatrix}$$

$$P_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad P_2 = \begin{pmatrix} -0.33 \\ 0.33 \end{pmatrix}$$

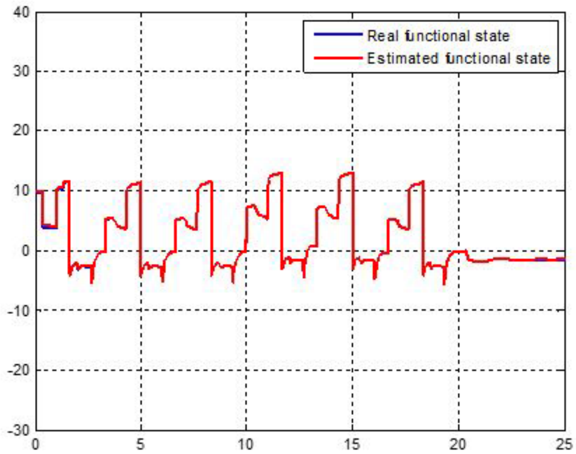

FIGURE 3. State functional vector $z(t)$.

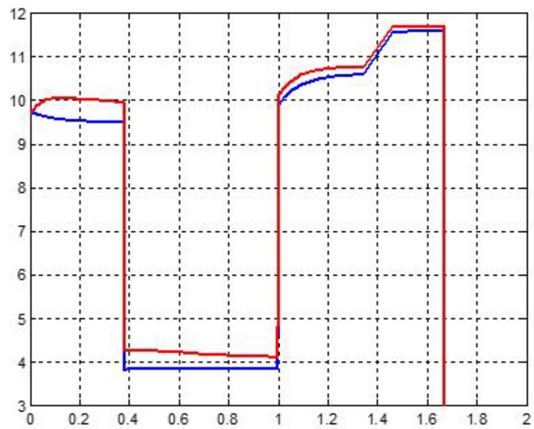

FIGURE 4. Zoom In of state functional vector $z(t)$.

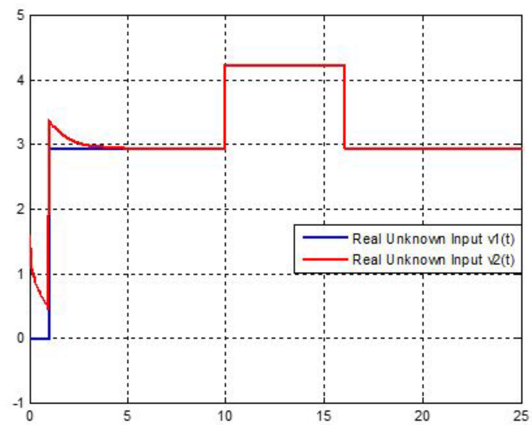

FIGURE 5. Unknown input functional.

Figure 3 demonstrates a suitable estimation dynamic. As shown in a zoomed transitory phase, we note a correct observer dynamic trying to track the real state functional.

In Figure 5 the efficiency of our proposed algorithm. Is noticeable the real unknown input functional and the estimated one are barely the same, except small variations during

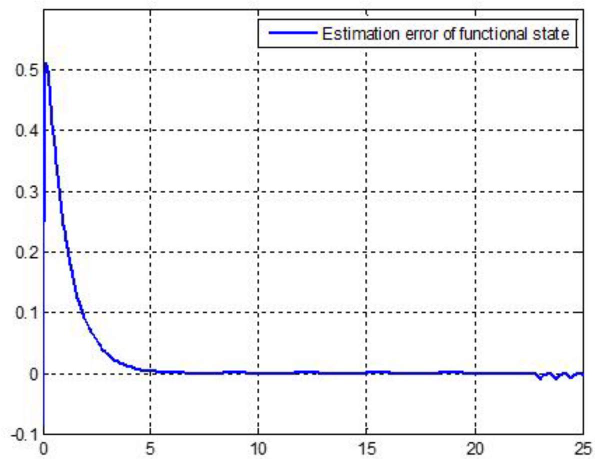

FIGURE 6. Estimation error of functional state.

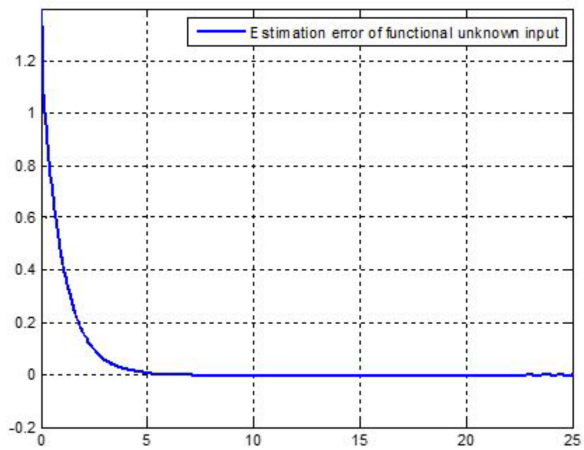

FIGURE 7. Estimation error of unknown input functional.

the transitory phase between instance $0s$ and $4s$ due to the difference between the initial conditions of the real and the estimated unknown input.

Figure 6 and Figure 7 explain an estimation error of, the state functional and the unknown input functional. The proposed observer reaches the permanent phase after a transitory period of $5s$.

Compared to [14] we are added delay in both state and known input vectors and studied systems are singular. The proposed scheme based on [14], [20] and [24] prove its effectiveness.

VI. CONCLUSION

In this paper, we have introduced an efficient observer scheme as a contribution in the estimation of state functional and unknown input functional for a class of bilinear singular delayed systems. The given procedure is independent from the state delay and based on Lyapunov-Krasovskii stability theory. The estimator is characterized by an optimal gain which is established thanks to a set of LMI conditions derived from an unbiased estimation dynamic constraint. A numerical example has been presented and the proposed algorithm proves its efficiency.

APPENDIX

The matrices φ, X and Π used in theorem 2 are defined as:

$$\varphi = \begin{pmatrix} -P & 0 & 0 \\ * & -P & 0 \\ * & * & -P \end{pmatrix},$$

$$X = \begin{pmatrix} \sqrt{\tau}(PM_1 - YM_2) & \sqrt{\tau}(PM_{d1} - YM_{d2}) & 0 \\ * & 0 & 0 \\ * & * & P \end{pmatrix},$$

$$\Pi = \begin{pmatrix} \delta & PM_{d1} - YM_{d2} & 0 \\ * & -Q & 0 \\ * & * & -\tau P \end{pmatrix},$$

$$\delta = PM_1 - YM_2 + (PM_1 - YM_2)^T + Q$$

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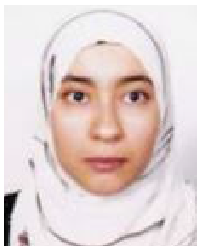
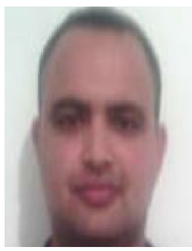


FATMA HAMZAOUI received the degree in electrical engineering from the National School of Engineers of Monastir (ENIM), Tunisia, in 2012, where she is currently pursuing the Ph.D. degree with the Research Laboratory LARATSI. Her research interests include observer design for singular delayed system and affected by an unknown input in time and frequency domains.



MALEK KHADHRAOUI received the Electrical Engineering degree in 2007, and the master's Diploma and Ph.D. degrees from the National School of Engineers of Monastir-Tunisia (ENIM), in 2009 and 2016, respectively. His research interests include robust control and observation for linear system in the time and the frequency domain.

HASSANI MESSAOUD received the Master of Science degree in control engineering from the High Normal School of Technical Education (ENSET), Tunis, Tunisia, in 1985, the Ph.D. degree in control engineering from the University of Nice, Sophia Antipolis, France, in 1993, and the Habilitation Diploma degree from the National School of Engineers of Tunis (ENIT), Tunisia, in 2001. He is currently a Professor with the National School of Engineers of Monastir (ENIM), Tunisia, and the Director of the Research Laboratory LARATSI. His main research interests include robustness in identification and control of nonlinear systems with application to diagnosis and equalization of numerical communication channels.