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Forward Kinematics and Singularity Analyses of an Uncoupled Parallel Manipulator by Algebraic Screw Theory

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ABSTRACT In this contribution, the Jacobian analysis of a four-legged six-degrees-of-freedom decoupled parallel manipulator is carried out through the screw theory. As an intermediate step, for the sake of completeness the inverse/forward displacement analysis as well as the computation of the workspace of the robot are achieved by taking advantage of the decoupled orientation and position of the moving platform. Afterward, the input/output equation of velocity of the parallel robot is obtained by harnessing of the properties of reciprocal screw systems. Once the Jacobian matrices are identified and investigated, the analysis of singularities for the robot manipulator emerges as a natural application of the Jacobian analysis. Numerical examples are included with the purpose to show the practicality and versatility of the method of kinematic analysis. Furthermore, the numerical results obtained by means of the theory of screws are successfully verified with the aid of commercially available software like ADAMS.

INDEX TERMS Kinematics, parallel robot, screw theory, singular posture, uncoupled kinematics.

I. INTRODUCTION

The Jacobian matrices of parallel manipulators has been extensively investigated covering mainly subjects like performance and singularity analysis [1]–[10]. In that regard, screw theory has been successfully employed in the Jacobian analysis of parallel manipulators. Consider for instance that Tsai [11] reviewed the role of reciprocal screws in the Jacobian analysis of non-redundant parallel manipulators concerned with the singularity analysis. By resorting to reciprocal-screw theory, Joshi and Tsai [12] derived full rank Jacobian matrices with the purpose to approach the singularity analysis of limited-dof parallel manipulators. Huang *et al.* [13] introduced a unified mathematical framework based on the theory of screws and the principle of virtual work available for the first order kinematic and static modeling of limited-dof parallel manipulators. In Choi and Ryu [14] the analysis of a expanded full rank Jacobian matrix reveals some singular configurations in a Schonflies parallel manipulator that are not detected when usual defective Jacobian matrices are employed. Dong *et al.* [15] presented a novel docking mechanism for the aircraft industry where its kinematic performance is evaluated through the analysis of a dimensional homogeneous Jacobian matrix which is obtained applying the theory of screws. Hoevenaars *et al.* [16], [17] realized the Jacobian analysis of a parallel manipulator equipped with a reconfigurable platform comprising two end-effectors based on screw theory. Ye *et al.* [18] applied reciprocal-screw theory in the design process of a reconfigurable parallel manipulator performing the Jacobian analysis introducing a unified Jacobian model. Zhang *et al.* [19] introduced a novel parallel manipulator for needle insertion where, based on the theory of screws, the robot brings a workspace free of singularities.

In this work, the Jacobian analysis of a decoupled spatial parallel manipulator with topology 3-R$\underline{P}$RRC+$\underline{RRP}$RU [20] is approached by means of the theory of screws. The rest of the contribution is organized as follows. In the next section, the architecture and geometry of the decoupled robot

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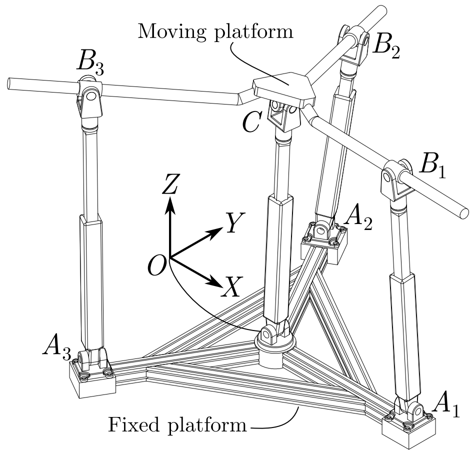

FIGURE 1. Sketch of the parallel manipulator.

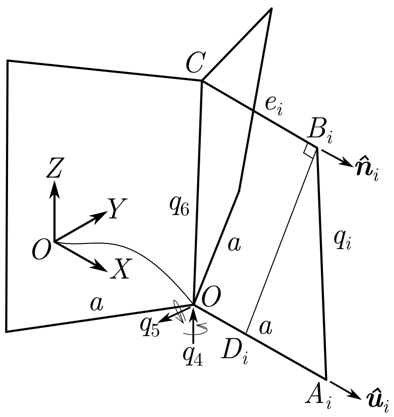

FIGURE 2. Kinematic diagram of the 3-RPRRC+RRPRU manipulator.

manipulator, including the notation used in this contribution, is described. Then, the inverse-forward displacement analysis of the parallel manipulator is performed. The inclusion of two unit vectors allows to formulate six quadratic equations in the unknown components of such vectors which are solved using homotopy continuation method. This strategy does not require the computation of the rotation matrix. In section "Jacobian matrices of the parallel manipulator" the Jacobians are systematically obtained through the formulation of the velocity analysis of the parallel manipulator by means of screw theory. In this regard, the Klein form plays a central role. After, in the later section, the singular postures of the robot are determined by investigating the determinant of the Jacobians. In a further section, numerical examples covering most of the points treated in the work are depicted. Finally, some conclusions are given at the end of the contribution.

II. TOPOLOGY OF THE UNCOUPLED PARALLEL MANIPULATOR

As is shown in Fig. 1, the robot under study consists of two platforms, one fixed and one mobile, linked each other by one internal and three external kinematic chains.

The internal limb is a RRPRU-type kinematic chain, where R, U and P denote, respectively, revolute, universal and prismatic joints. The first revolute is assembled with the base in such a way that its axis is normal to the plane of this platform. In addition, the axis of the second revolute pair is coplanar with the base. A prismatic joint controls the length of the limb, which is connected at its end to the moving platform through a revolute pair, such that the axes of both joints are collinear; finally, an universal joint connects the limb to the moving platform. Note that the assembly of the third revolute and the universal joints is equivalent to a spherical joint with the advantage of alleviating frictional forces. On the other hand, the external limbs are RPRRC-type kinematic chains, where the axis of the revolute joint connecting the limb to the base is in the plane of this platform. Then, the prismatic joint and the second revolute joint of each external kinematic chain are connected in that way that they shares a common axis. Moreover, the external limb is linked to the moving platform by means of a compound joint formed with the third revolute pair and a cylindrical joint (C) whose axes are perpendicular to each other. One more time, the topology of the manipulator conveniently avoids the use of spherical joints. Finally, the nominal positions of the external revolute pairs form an equilateral triangle, whose center is shared with the position of the first revolute of the internal limb. In this way, the nominal positions of these four revolutes lie in the plane of the base. Moreover, the axes of the three outer revolutes intersect at the center of such triangle, as is shown if Fig. 1.

The geometry of the parallel manipulator is detailed in the following. From Fig. 2, let O_XYZ be a reference frame whose origin, denoted by O, is the center of the fixed platform, and its base unit vectors are $\boldsymbol{ijk}$. The nominal position of the first revolute joints for the i-th external limb are denoted by points A_i, and located by vectors $\boldsymbol{a}_i$. Thereafter, $i = 1, 2, 3$ unless otherwise stated. As described in the previous paragraph, these points are the vertices of a circle an equilateral triangle inscribed in a circle with center at O and radius a. In addition, the axes of these revolute pairs are characterized by concurrent unit vectors $\boldsymbol{u}_i$, that are directed from point O to the corresponding point A_i. On the other hand, point B_i, which is located by vector $\boldsymbol{b}_i$, denotes the nominal position of the ith (R+C) compound kinematic pair, and the direction of its axis is defined by a unit vector $\boldsymbol{n}_i$ that points from the nominal position of the universal joint of the central limb, point C, to B_i. Please note that C also denotes the center of the moving platform and its position vector is $\boldsymbol{c}$. Furthermore, vectors $\boldsymbol{n}_i$ are located on the plane of the moving platform. With the purpose to generate screws reciprocal to the screw systems of the outer limbs, let us consider that D_i is a point, for which the position vector is $\boldsymbol{d}_i$, that results of the intersection of the line generated by points O and A_i and a line passing through point B_i that is normal to the line defined by points C and B_i. On the other hand, the actuators of the parallel manipulator are notated as generalized coordinates $q_i (i = 1, 2, 3, \ldots, 6)$, where $q_i (i = 1, 2, 3)$ stand for linear actuators associated to

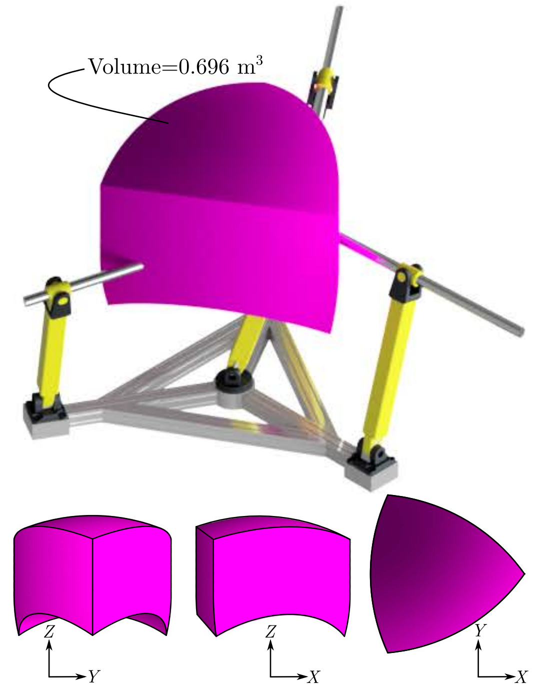

FIGURE 3. The workspace of the manipulator for the orientation defined by the conventional roll-pitch-yaw angles (6°, 3°, 10°).

the prismatic joints of the outer limbs. Meanwhile, $q_i (i = 4, 5, 6)$ are generalized coordinates employed to control the position of the moving platform through the central kinematic chain, e.g., q_4 is used to control the orientation of the lower central revolute pair, which is measured from the $X-$axis. After, the vertical orientation of the central limb is controlled by the generalized coordinate q_5 which is associated to the second revolute joint. Finally, the position of the moving platform is fully specified taking into account q_6 is associated to the extendible length of the central limb. In other words, q_6 is the signed distance between points O and C.

The workspace for the manipulator under study can be obtained by following the procedure described in [21], [22]. An example is depicted in Fig 3.

III. DISPLACEMENT ANALYSIS

For the sake of completeness of the contribution, this section presents the forward and inverse position analyses of the parallel manipulator.

The forward displacement analysis consists of finding the pose of the moving platform with respect to the base when a set of generalized coordinates are given. In other words, it is required to compute the coordinates of C and B_i. With reference to Fig. 2, the position vector $\boldsymbol{c}$, owing the decoupled motion of the moving platform, can be written in a straightforward way in terms of the variables q_4, q_5 and q_6, associated to the central limb, through the next closure equation

$$\begin{aligned} \boldsymbol{c} &= q_6 \boldsymbol{\lambda}, \\ \boldsymbol{\lambda} &= \cos q_4 \cos q_5 \boldsymbol{i} + \sin q_4 \cos q_5 \boldsymbol{j} + \sin q_5 \boldsymbol{k} \end{aligned} \quad (1)$$

where $\boldsymbol{\lambda} = \lambda_X \boldsymbol{i} + \lambda_Y \boldsymbol{j} + \lambda_Z \boldsymbol{k}$ is a unit vector pointed from O to C. Moreover, owing the assemble of the external limb to the base platform through concurrent revolute joints, it follows that for each outer kinematic chain we have necessarily that

$$(\boldsymbol{b}_i - \boldsymbol{a}_i) \cdot \boldsymbol{u}_i = 0 \quad (2)$$

where the position vector $\boldsymbol{b}_i$ may be expressed as follows

$$\boldsymbol{b}_i = \boldsymbol{c} + e_i \boldsymbol{n}_i \quad (3)$$

Hence, with the substitution of (3) into (2) it follows that

$$b_i = (a - \boldsymbol{c} \cdot \boldsymbol{u}_i)/\boldsymbol{n}_i \cdot \boldsymbol{u}_i \quad (4)$$

Furthermore, owing the symmetry of the moving platform it is evident that

$$\boldsymbol{n}_1 + \boldsymbol{n}_2 + \boldsymbol{n}_3 = \boldsymbol{0} \quad (5)$$

where $\boldsymbol{n}_i \cdot \boldsymbol{n}_i = 1$. Within this framework, the forward displacement analysis is focused on the following closure equations

$$\boldsymbol{b}_i \cdot \boldsymbol{b}_i = a^2 + q_i^2 \quad (6)$$

where q_i is the signed distance between A_i and B_i. After a few computations, a system of six quadratic equations in six unknowns, e.g., the components of $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$. In the contribution, this system is solved by using a homotopy continuation method [23]. Once the coordinates of B_i are calculated, see (3), the coordinates of points D_i may be computed taking into account that

$$(\boldsymbol{d}_i - \boldsymbol{b}_i) \cdot \boldsymbol{n}_i = 0 \quad (7)$$

where the component of $\boldsymbol{d}_i$ along the Z-axis vanishes. Additionally, with the purpose of determining the orientation angles of the moving platform consider that the rotation matrix $\mathbf{R}$ may be computed as follows

$$\mathbf{R} = \begin{bmatrix} \boldsymbol{n}_1 & (\boldsymbol{n}_1 \times \boldsymbol{n}_2) \times \boldsymbol{n}_1 & \boldsymbol{n}_1 \times \boldsymbol{n}_2 \end{bmatrix} \quad (8)$$

Also, $\mathbf{R}$ may be expressed considering conventional roll (γ), pitch (β) and yaw (α) angles as follows [24]

$$\mathbf{R} = \begin{bmatrix} c_\alpha c_\beta & c_\alpha s_\beta s_\gamma - s_\alpha c_\gamma & c_\alpha s_\beta c_\gamma + s_\alpha s_\gamma \\ s_\alpha c_\beta & s_\alpha s_\beta s_\gamma + c_\alpha c_\gamma & s_\alpha s_\beta c_\gamma - c_\alpha s_\gamma \\ -s_\beta & c_\beta s_\gamma & c_\beta c_\gamma \end{bmatrix} \quad (9)$$

where $\mathbf{R}$ is calculated following the order of rotations roll, pitch and then yaw.

Finally, the inverse displacement analysis of the robot manipulator is straightforward. In fact, given the pose of the moving platform, e.g. given the vectors $\boldsymbol{c}$ and $\boldsymbol{n}_i$, first the position vectors $\boldsymbol{b}_i$ are obtained directly from (3). After, the generalized coordinates q_i are calculated, based on (6), as follows

$$q_i = \sqrt{\boldsymbol{b}_i \cdot \boldsymbol{b}_i - a^2} \quad (10)$$

Meanwhile, the generalized coordinates q_4, q_5 and q_6 are computed directly upon (1). Indeed, given vector $\boldsymbol{c}$ it is evident that

$$q_6 = \sqrt{\boldsymbol{c} \cdot \boldsymbol{c}},$$

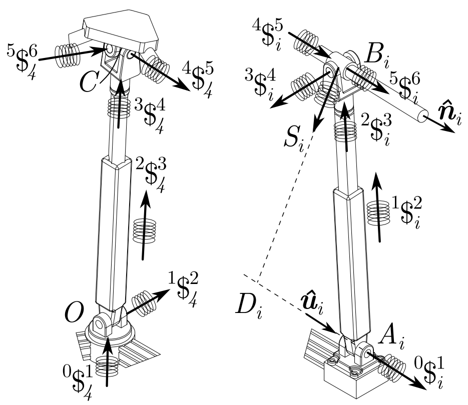

FIGURE 4. Screws of the parallel manipulator.

$$q_5 = \operatorname{atan2}\left(\boldsymbol{c}\cdot\boldsymbol{k},\ \sqrt{(\boldsymbol{c}\cdot\boldsymbol{i})^2+(\boldsymbol{c}\cdot\boldsymbol{j})^2}\right),$$
$$q_4 = \operatorname{atan2}\left(\boldsymbol{c}\cdot\boldsymbol{j},\ \boldsymbol{c}\cdot\boldsymbol{i}\right) \quad (11)$$

Finally, it is worth to note how easy is to formulate the displacement analysis of the parallel manipulator at hand when compared with the classical Gough-Stewart platform [25], [26].

IV. JACOBIAN MATRICES OF THE PARALLEL MANIPULATOR

With the purpose to obtain the Jacobian matrices of the parallel manipulator, in this section the velocity analysis of it is addressed by resorting to the screws theory [27]. Velocity modeling requires to formulate a specific linear map between two vector spaces at a given configuration, i.e., velocity modeling involves the linear map between the velocity state, or twist about a screw, and the actuator rates [28]. In this contribution, the Jacobians of the parallel manipulator emerge combining the theory of screws and the formalities of linear algebra, without doubt an elegant union.

Figure 4 shows the infinitesimal screws of the parallel manipulator where point C is selected as the pole of reference of the screw systems. In what follows a brief explanation of the screws is provided. The screws of the central kinematic chain are determined as follows:

${}^0\$_4^1 = [\boldsymbol{k}, \boldsymbol{k}\times\boldsymbol{c}]$ denotes the screw of constant orientation that models the lower revolute pair;
${}^1\$_4^2 = [\sin q_4\boldsymbol{i} - \cos q_4\boldsymbol{j}, (\sin q_4\boldsymbol{i} - \cos q_4\boldsymbol{j})\times\boldsymbol{c}\,]$ represents the screw of the revolute joint whose primal part remains permanently in the $XY-$plane;
${}^2\$_4^3 = [\boldsymbol{0}, \boldsymbol{\lambda}]$ is the screw dealing with the actuated prismatic joint;
${}^3\$_4^4 = [\boldsymbol{\lambda}, \boldsymbol{0}]$ is the screw attending the revolute joint along the central leg;
${}^4\$_4^5 = [\boldsymbol{n}_1, \boldsymbol{0}]$ and ${}^5\$_4^6 = [(\boldsymbol{n}_1\times\boldsymbol{n}_2)\times\boldsymbol{n}_1, \boldsymbol{0}]$ are the screws simulating the universal joint.

Meanwhile, the screws for the ith external limb are obtained as

${}^0\$_i^1 = [\boldsymbol{u}_i, \boldsymbol{u}_i\times(\boldsymbol{c}-\boldsymbol{a}_i)]$ is the screw associated to the first revolute joint;
${}^1\$_i^2 = [\boldsymbol{0}, \boldsymbol{w}_i]$ is the screw dealing with the actuated prismatic joint;
${}^2\$_i^3 = [\boldsymbol{w}_i, \boldsymbol{w}_i\times(\boldsymbol{c}-\boldsymbol{b}_i)]$ is the screw dealing with the revolute joint along the ith leg;
${}^3\$_i^4 = [\boldsymbol{n}_i\times\boldsymbol{w}_i, (\boldsymbol{n}_i\times\boldsymbol{w}_i)\times(\boldsymbol{c}-\boldsymbol{b}_i)]$ is the screw concerned with the upper revolute joint;
${}^4\$_i^5 = [\boldsymbol{n}_i, \boldsymbol{0}]$ and ${}^5\$_i^6 = [\boldsymbol{0}, \boldsymbol{n}_i]$ are the screws simulating the cylindrical joint, rotation and translation respectively;

where $\boldsymbol{w}_i = (\boldsymbol{b}_i - \boldsymbol{a}_i)/\mid \boldsymbol{b}_i - \boldsymbol{a}_i \mid$ is a unit vector along the ith leg.

The velocity state, or twist about a screw, of the rigid body and its representation through linear combinations of infinitesimal screws is one of the major contributions of the theory of screws [27]. In that regard, for more than four decades the theory of screws has been used successfully in the study of the instantaneous kinematics of parallel manipulators.

Let us consider that $\boldsymbol{\omega}$ is the angular velocity vector of the moving platform as observed from the fixed platform. Furthermore, let be $\boldsymbol{v}_C$ the linear velocity vector of the center C of the moving platform. Then, the velocity state of the moving platform as observed from the fixed platform taking point C as the reference pole, the six-dimensional vector $\boldsymbol{V} = \left[\boldsymbol{\omega}, \boldsymbol{v}_C\right] \in \Re^{6\times 1}$, can be written in screw form through the limbs of the parallel manipulator as follows:

$$\boldsymbol{V} = {}_0\omega_1^i\,{}^0\$_i^1 + {}_1\omega_2^i\,{}^1\$_i^2 + {}_2\omega_3^i\,{}^2\$_i^3 + {}_3\omega_4^i\,{}^3\$_i^4 + {}_4\omega_5^i\,{}^4\$_i^5 + {}_5\omega_6^i\,{}^5\$_i^6 \quad i = 1, 2, 3, 4 \quad (12)$$

where the actuated kinematic joints of the parallel manipulator are ${}_1\omega_2^i = \dot{q}_i (i = 1, 2, 3)$, ${}_0\omega_1^4 = \dot{q}_4$, ${}_1\omega_2^4 = \dot{q}_5$ and ${}_2\omega_3^4 = \dot{q}_6$.

In what follows the input-output equation of velocity of the parallel manipulator is obtained by resorting to reciprocal-screw theory, an elegant strategy that allows to cancel the passive joint rates of the parallel manipulator through the application of the Klein form. Let $\$_1 = [\boldsymbol{s}_1, \boldsymbol{s}_{O1}]$ and $\$_2 = [\boldsymbol{s}_2, \boldsymbol{s}_{O2}]$ be to elements of the Lie algebra $se(3)$ of the Euclidean group $SE(3)$. The Klein form, $\{*; *\}$, is a symmetric bilinear form of $se(3)$ defined as [27]

$$\{\$_1; \$_2\} \equiv \{[\boldsymbol{s}_1, \boldsymbol{s}_{O1}]; [\boldsymbol{s}_2, \boldsymbol{s}_{O2}]\} = \boldsymbol{s}_1\cdot\boldsymbol{s}_{O2} + \boldsymbol{s}_2\cdot\boldsymbol{s}_{O1} \quad (13)$$

Additionally, it is well known that two screws $\$_1$ and $\$_2$ are reciprocal screws if $\{\$_1; \$_2\} = 0$, e.g. when the primal parts of $\$_1$ and $\$_2$ intersect at a common point we have $\{\$_1; \$_2\} = 0$. Similarly, assume that $\$_2 = [\boldsymbol{0}; \boldsymbol{s}_2]$. Said otherwise, $\$_2$ denotes a prismatic joint. If $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ are perpendicular then it follows that $\{\$_1; \$_2\} = 0$. By the way, if $\boldsymbol{s}_1 = \boldsymbol{s}_2$ then $\{\$_1; \$_2\} = 1$.

With the aim of performing the velocity analysis, the screw systems of the outer limbs are first considered. The introduction of the screw reciprocal to the screws representing passive joint rates in the same leg plays a central role. Let

$$\boldsymbol{S}_i = \begin{bmatrix} (\boldsymbol{d}_i - \boldsymbol{b}_i)/\mid \boldsymbol{d}_i - \boldsymbol{b}_i \mid \\ (\boldsymbol{d}_i - \boldsymbol{b}_i) \times (\boldsymbol{c} - \boldsymbol{b}_i)/\mid \boldsymbol{d}_i - \boldsymbol{b}_i \mid \end{bmatrix} \tag{14}$$

be a line in Plücker coordinates directed from point B_i to point D_i. Note that S_i is reciprocal to all the screws of the i-th limb excluding the screw associated to the active prismatic pair. Thus, by applying the Klein form between S_i and both sides of (12) with the reduction of terms, i.e. $\{\boldsymbol{S}_i;\,{}^0\$_i^1\} = \{\boldsymbol{S}_i;\,{}^2\$_i^3\} = \{\boldsymbol{S}_i;\,{}^3\$_i^4\} = \{\boldsymbol{S}_i;\,{}^4\$_i^5\} = \{\boldsymbol{S}_i;\,{}^5\$_i^6\} = 0$, it is possible to write

$$\{\boldsymbol{S}_i;\,\boldsymbol{V}\} = \dot{q}_i\{\boldsymbol{S}_i;\,{}^1\$_i^2\} \tag{15}$$

On the other hand, for the central limb, labelled as the fourth kinematic chain, it is evident that ${}^3\$_4^4$, ${}^4\$_4^5$ and ${}^5\$_4^6$ are reciprocal to the screws associated to the passive joints of this chain. Hence, after applying the Klein form between these screws and both sides of (12) with the reduction of terms yields

$$\{\$_i;\,\boldsymbol{V}\} = \dot{q}_4\{\$_i;\,{}^0\$_4^1\} + \dot{q}_5\{\$_i;\,{}^1\$_4^2\} + \dot{q}_6\{\$_i;\,{}^2\$_4^3\}, \quad \$_i \in \{{}^3\$_4^4,\,{}^4\$_4^5,\,{}^5\$_4^6\} \tag{16}$$

Rearranging (15) and (16) into a matrix-vector form, the input–output equation of velocity of the parallel manipulator is

$$\mathbf{J}_v^{\mathrm{T}}\,\Delta\,\boldsymbol{V} = \mathbf{J}_q\,\dot{\mathbf{Q}} \tag{17}$$

where

$$\mathbf{J}_v = \begin{bmatrix} \boldsymbol{S}_1 & \boldsymbol{S}_2 & \boldsymbol{S}_3 & {}^3\$_4^4 & {}^4\$_4^5 & {}^5\$_4^6 \end{bmatrix} \tag{18}$$

is the *forward Jacobian matrix* of the parallel manipulator. Meanwhile

$$\mathbf{J}_q = \begin{bmatrix} \{\boldsymbol{S}_1;\,{}^1\$_1^2\} & 0 & 0 & 0 & 0 & 0 \\ 0 & \{\boldsymbol{S}_2;\,{}^1\$_2^2\} & 0 & 0 & 0 & 0 \\ 0 & 0 & \{\boldsymbol{S}_3;\,{}^1\$_3^2\} & 0 & 0 & 0 \\ 0 & 0 & 0 & \{{}^3\$_4^4;\,{}^0\$_4^1\} & \{{}^3\$_4^4;\,{}^1\$_4^2\} & \{{}^3\$_4^4;\,{}^2\$_4^3\} \\ 0 & 0 & 0 & \{{}^4\$_4^5;\,{}^0\$_4^1\} & \{{}^4\$_4^5;\,{}^1\$_4^2\} & \{{}^4\$_4^5;\,{}^2\$_4^3\} \\ 0 & 0 & 0 & \{{}^5\$_4^6;\,{}^0\$_4^1\} & \{{}^5\$_4^6;\,{}^1\$_4^2\} & \{{}^5\$_4^6;\,{}^2\$_4^3\} \end{bmatrix} \tag{19}$$

is the *inverse Jacobian matrix* of the parallel manipulator [29]. Moreover,

$$\Delta = \begin{bmatrix} \mathbf{O} & \mathbf{I} \\ \mathbf{I} & \mathbf{O} \end{bmatrix} \tag{20}$$

is an operator of polarity, where $\mathbf{I}$ is a 3×3 identity matrix and $\mathbf{O}$ is the zero matrix. Furthermore,

$$\dot{Q} = \begin{bmatrix} \dot{q}_1 \; \dot{q}_2 \; \dot{q}_3 \; \dot{q}_4 \; \dot{q}_5 \; \dot{q}_6 \end{bmatrix}^T \tag{21}$$

is the first-order driver vector of the parallel manipulator.

V. SINGULARITY ANALYSIS

In this section the inverse and forward Jacobian matrices are studied to determine the singular poses of the robot. In that regard as noted by Coppola *et al.* [30], an advantage of the theory of screws over conventional methods based on the time derivative of vector loop equations is that screw theory avoids tedious parametrization errors and allows precise and complete description of singularity types.

Commonly, in parallel manipulators it is possible to detect three types of singular configurations: i) inverse singularity, ii) forward singularity and iii) combined singularity. An inverse singular posture for the parallel manipulator under study emerges when $\det(\mathbf{J}_q) = 0$, in other words matrix $\mathbf{J}_q$ is singular, and $\mathbf{J}_v$ is non-singular. That is, the inverse velocity analysis is not available. It is interesting to note that in order to elucidate the inverse singularity analysis, as a consequence of the uncoupled motion of the moving platform matrix $\mathbf{J}_q$ may be conveniently rewritten as follows

$$\mathbf{J}_q = \begin{bmatrix} \tilde{\mathbf{J}}_q & O \\ O & \bar{\mathbf{J}}_q \end{bmatrix} \tag{22}$$

where

$$\tilde{\mathbf{J}}_q = diag\begin{bmatrix} \{\boldsymbol{S}_1;\,{}^1\$_1^2\} & \{\boldsymbol{S}_3;\,{}^1\$_3^2\} & \{\boldsymbol{S}_3;\,{}^1\$_3^2\} \end{bmatrix} \tag{23}$$

is a submatrix of $\mathbf{J}_q$ dealing with the orientation of the moving platform. Meanwhile

$$\bar{\mathbf{J}}_q = \begin{bmatrix} \{{}^3\$_4^4;\,{}^0\$_4^1\} & \{{}^3\$_4^4;\,{}^1\$_4^2\} & \{{}^3\$_4^4;\,{}^2\$_4^3\} \\ \{{}^4\$_4^5;\,{}^0\$_4^1\} & \{{}^4\$_4^5;\,{}^1\$_4^2\} & \{{}^4\$_4^5;\,{}^2\$_4^3\} \\ \{{}^5\$_4^6;\,{}^0\$_4^1\} & \{{}^5\$_4^6;\,{}^1\$_4^2\} & \{{}^5\$_4^6;\,{}^2\$_4^3\} \end{bmatrix} \tag{24}$$

is a submatrix of $\mathbf{J}_q$ concerned with the position of the center of the moving platform. Hence, by analysing $\bar{\mathbf{J}}_q$ and $\tilde{\mathbf{J}}_q$ it is possible to approach the inverse singularity analysis. If any of the elements of the diagonal of matrix $\tilde{\mathbf{J}}_q$ vanishes then we obtain $\det(\tilde{\mathbf{J}}_q) = 0$. This condition occurs only when the primal part of S_i is orthogonal to the dual part of ${}^1\$_i^2$ in the same leg. A condition that must be disregarded from the analysis owing the architecture of the outer limbs and therefore an inverse singularity must be credited only to matrix $\bar{\mathbf{J}}_q$. In that concern, taking into account that $\{{}^3\$_4^4;\,{}^0\$_4^1\} = \{{}^3\$_4^4;\,{}^1\$_4^2\} = 0$ and $\{{}^3\$_4^4;\,{}^2\$_4^3\} = 1$, by setting $\det(\bar{\mathbf{J}}_q) = 0$, the condition of inverse singularity leads to

$$\{{}^4\$_4^5;\,{}^0\$_4^1\}\{{}^5\$_4^6;\,{}^1\$_4^2\} = \{{}^5\$_4^6;\,{}^0\$_4^1\}\{{}^4\$_4^5;\,{}^1\$_4^2\} \tag{25}$$

For example, when the central limb is vertical we have $\{{}^4\$_4^5;\,{}^0\$_4^1\} = \{{}^5\$_4^6;\,{}^0\$_4^1\} = 0$ yielding $\det(\bar{\mathbf{J}}_q) = 0$. Finally, according to (25), the general condition of inverse singularity of the parallel manipulator results in

$$[\boldsymbol{n}_1 \cdot (\boldsymbol{k} \times \boldsymbol{c})]\,[(\boldsymbol{n}_1 \times \boldsymbol{n}_2) \times \boldsymbol{n}_1 \cdot (\lambda_X \boldsymbol{i} + \lambda_Y \boldsymbol{j}) \times \boldsymbol{c}] - [(\boldsymbol{n}_1 \times \boldsymbol{n}_2) \times \boldsymbol{n}_1 \cdot (\boldsymbol{k} \times \boldsymbol{c})]\,[\boldsymbol{n}_1 \cdot (\lambda_X \boldsymbol{i} + \lambda_Y \boldsymbol{j}) \times \boldsymbol{c}] = 0 \tag{26}$$

A direct singularity occurs when the forward velocity analysis of the robot is not available, i.e., when $\mathbf{J}_v$ is singular and matrix $\mathbf{J}_q$ is non-singular. In this case the moving platform

TABLE 1. Forward displacement analysis: coordinates of points B_i (in meters).

Sol.	B_1	B_2	B_3
1	(1,.092707,.995825)	(-.110200,1.091076,1.103129)	(-.564114,-.829009,.866558)
2	(1,.278828,.960477)	(-.311829,.974665,1.171441)	(-.295581,-.984046,.837071)
3	(1,-.921997,-.387535)	(-1.092186,.524126,.975656)	(-1.033884,-.557787,-.613482)
4	(1,-.541257,.841013)	(-1.494324,.291952,-.318190)	(.051153,-1.184233,-.592771)

is able to move infinitesimally without changing the value of the inputs. Said otherwise, some degrees of freedom of the parallel manipulator become uncontrollable. Following the trend of the inverse singularity analysis, with the purpose to simplify the forward singularity analysis, it is advisable to consider the uncoupled motion of the moving platform. In fact, the Jacobian matrix $\mathbf{J}_v$ may be conveniently rewritten as follows

$$\mathbf{J}_v = \left[\tilde{\mathbf{J}}_v \; \bar{\mathbf{J}}_v\right] \tag{27}$$

where $\tilde{\mathbf{J}}_v = \left[\boldsymbol{S}_1 \; \boldsymbol{S}_2 \; \boldsymbol{S}_3\right]$ while $\bar{\mathbf{J}}_v = \left[{}^3\$_4^4 \; {}^4\$_4^5 \; {}^5\$_4^6\right]$. Thus, a direct singularity emerges either when $\det(\bar{\mathbf{J}}_v^T\bar{\mathbf{J}}_v) = 0$ or $\det(\tilde{\mathbf{J}}_v^T\tilde{\mathbf{J}}_v) = 0$. The first possibility is easy to elucidate. In fact, note that the screws ${}^3\$_4^4$, ${}^4\$_4^5$ and ${}^5\$_4^6$ models a spherical pair. Thus, taking into account that point C is the reference pole, the screws would be established as ${}^3\$_4^4 = (\boldsymbol{i}, \mathbf{0})$, ${}^4\$_4^5 = (\boldsymbol{j}, \mathbf{0})$ and ${}^5\$_4^6 = (\boldsymbol{k}, \mathbf{0})$. Then it follows that $\det(\bar{\mathbf{J}}_v^T\bar{\mathbf{J}}_v) = 1$, which indicates that the central limb is not responsible for causing a direct singularity. In contrast to the simplicity of the computation of $\det(\bar{\mathbf{J}}_v^T\bar{\mathbf{J}}_v)$, the complexity of the computation of $\det(\tilde{\mathbf{J}}_v^T\tilde{\mathbf{J}}_v)$ is evident due the lack of a closed-form solution for the forward displacement analysis. The derivation of an algebraic expression to accomplish this end is out of scope in this paper, and therefore only evident direct singularities are investigated based on the linear dependency of the elements of matrix $\tilde{\mathbf{J}}_v$. If the lines $\boldsymbol{S}_i$ are coplanar then their dual parts are collinear causing a linear dependency between them and the parallel manipulator is at a singular configuration. Furthermore if the lines $\boldsymbol{S}_i$ are parallel then their primal parts are the same causing again a linear dependency between them and the parallel manipulator is also at a singular configuration. On the other hand, if the lines $\boldsymbol{S}_i$ are concurrent to a point, e.g. point O, then if such point is selected as the reference point the dual parts of the lines $\boldsymbol{S}_i$ vanish producing a direct singularity. The geometry conditions imposed to compute points D_i make this last possibility impossible due to the architecture of the outer limbs and therefore this case must be disregarded from the analysis.

Finally, a combined singularity emerges when both matrices $\mathbf{J}_v$ and $\mathbf{J}_q$ are singular, e.g., when the four limbs of the parallel manipulator are parallel.

VI. NUMERICAL APPLICATION

In order to show the application of the method, in this section numerical examples comprising most of the topics treated in the contribution are provided. Furthermore, with the aim to verify the numerical results of the examples, computer simulations are obtained with the aid of commercially available software like ADAMS˙. To this end consider that the parameters of the parallel robot are $a = 1$m, $\boldsymbol{u}_1 = \boldsymbol{i}$, $\boldsymbol{u}_2 = -0.5\boldsymbol{i} + 0.866\boldsymbol{j}$ and $\boldsymbol{u}_3 = -0.5\boldsymbol{i} - 0.866\boldsymbol{j}$. Thus, the coordinates of points A_i are given by $A_1 = (1, 0, 0)$m, $A_2 = (-0.5, 0.866, 0.0)$m and $A_3 = (-0.5, -0.866, 0.0)$m.

The first part of the exercise is devoted to solve the inverse displacement analysis. Assume that the actual pose of the moving platform is characterized by the position point $C = (0.25, 0.2, 1.0)$m and the orientation angles $\alpha = 6^o$, $\beta = 3^o$ and $\gamma = 10^o$. Thus, we obtain immediately, according to Eq. (1), that the generalized coordinates associated to the central kinematic chain are given by $q_4 = 38.657^o$, $q_5 = 72.247^o$ and $q_6 = 1.05$m. Later on, the variable distances e_i are computed as $e_1 = 0.755$m, $e_2 = 0.972$m and $e_3 = 1.313$m. Finally, length of the external limbs result in $q_1 = 1$m, $q_2 = 1.191$m and $q_3 = 0.869$m.

The next part of the exercise deals with the solution of the forward displacement analysis. To this aim, consider the generalized coordinates obtained in the first part. From (1), the coordinates of the center of the moving platform result as $C = (0.25, 0.2, 1.0)$m. Meanwhile, excluding reflected solutions, the application of the method introduced in the third section of this contribution conduct to 4 real solutions. The corresponding coordinates of points B_i are described in Table 1.

The final part of the numerical example is devoted to solve the velocity analysis for the parallel robot. To this aim, consider solution 2 of Table 1 as the reference configuration of the manipulator. Two examples are considered here.

Example 1: In this case of study, the generalized coordinates are defined by periodical functions as follows:

$$\begin{aligned}
q_1 &= 1 + 0.25\sin(t)\cos(t)\\
q_2 &= 1.191 + 0.3\sin(t)\cos(t)\\
q_3 &= 0.869 + 0.2\sin(t)\cos(t)\\
q_4 &= 0.6747 + \frac{\pi}{18}\sin(t)\\
q_5 &= 1.2609 - \frac{\pi}{9}\sin^2(t)\\
q_6 &= 1.05 + 0.25\sin(t)\cos(t)
\end{aligned}$$

where the time t is confined in the interval $0 \leq t \leq 2\pi$ (s). The resulting temporal behavior of the center of the moving platform, by using the screw theory and the results obtained from the model in ADAMS, are summarized in the plots of Fig. 5 and Fig. 6.

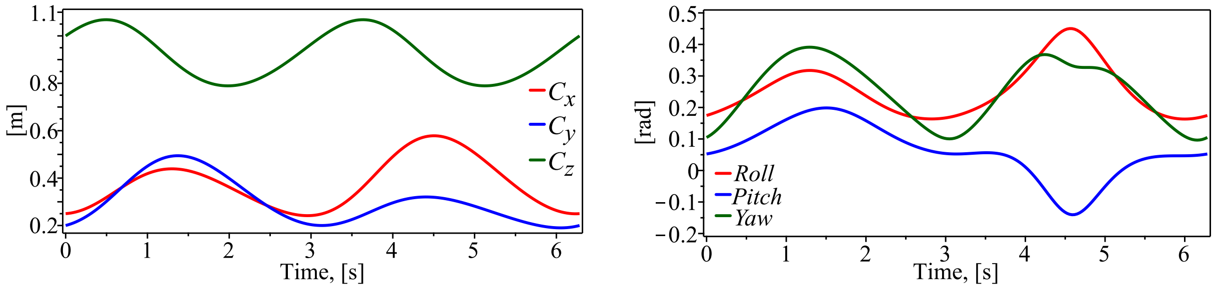

FIGURE 5. Example 1. Position (left) and orientation (right) of the moving platform.

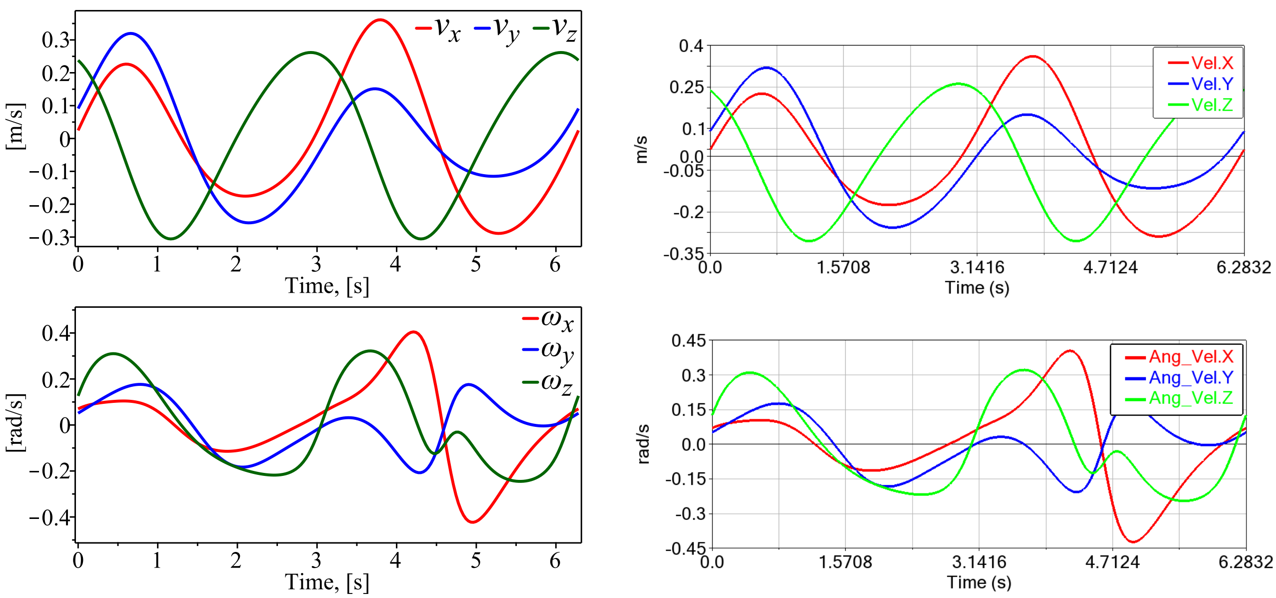

FIGURE 6. Example 1. Time history of the forward kinematics of the center of the moving platform, using the screw theory (left graphics) and using ADAMS® (right graphics).

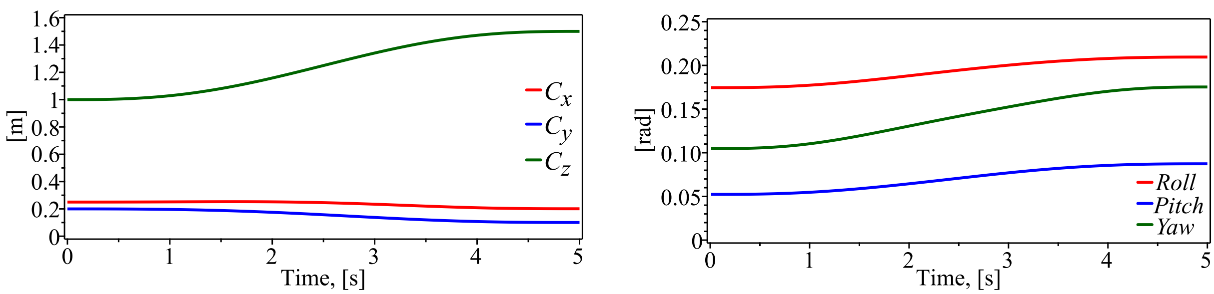

FIGURE 7. Example 2. Position and orientation of the moving platform.

Example 2: Upon the reference configuration of the parallel manipulator, i.e. $C = (0.25, 0.2, 1.0)$, $\alpha = 6^o$, $\beta = 3^o$ and $\gamma = 10^o$, the moving platform must reach the pose characterized by the position $C = (0.2, 0.1, 1.5)$ and the orientation angles $\alpha = 10^o$, $\beta = 5^o$ and $\gamma = 12^o$. Furthermore, consider that at the beginning of the analysis the robot is motionless and is required to move it in a smooth manner in a way that allows to the manipulator reaching the indicated posture after 5 seconds. Quintic polynomial expression of the form $q_i(t) = \mu_{i,0} + \mu_{i,3}t^3 + \mu_{i,4}t^4 + \mu_{i,5}t^5$ are appropriated to achieve this task. The coefficients of these polynomial functions are given in Table 2.

Finally, the temporal behavior of the kinematics of the center of the moving platform by applying the theory of screws of the second example is reported in Fig. 7 and Fig. 8. The left side of Fig. 8 shows the results obtained from simulations in ADAMS in order to compare them with those obtained with the presented method.

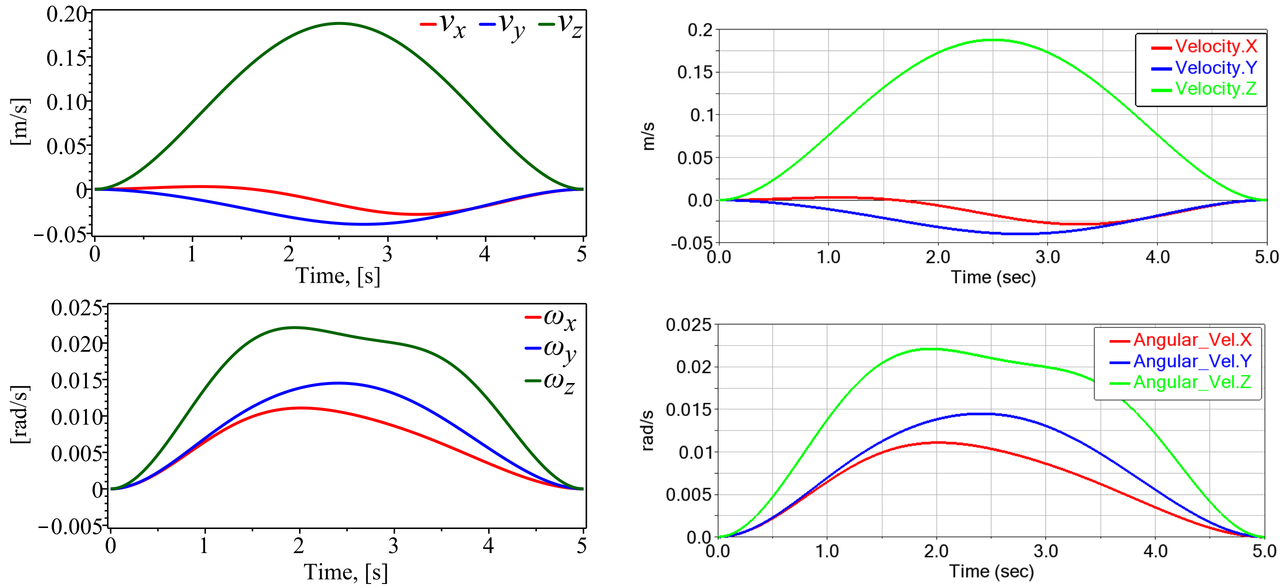

FIGURE 8. Example 2. Time history of the forward kinematics of the center of the moving platform, using the screw theory (left graphics) and using ADAMS® (right graphics).

TABLE 2. Coefficients of the functions for the generalized coordinates $q_i(t)$ for Example 2.

i	$\mu_{i,0}$	$\mu_{i,3}$	$\mu_{i,4}$	$\mu_{i,5}$
1	1.00010	0.035912	-0.010774	0.000862
2	1.19140	0.043552	-0.013066	0.001045
3	0.86971	0.040023	-0.012007	0.000961
4	0.67470	-0.016888	0.005066	-0.000405
5	1.26090	0.012888	-0.003866	0.000309
6	1.05000	0.037320	-0.011196	0.000896

VII. CONCLUSION

Simpler kinematics and improved maneuverability are some advantages of parallel manipulators with uncoupled kinematics over traditional parallel manipulators with coupled kinematics like the Gough-Stewart platform. In this work, the Jacobian analysis of the 3-RPRRC+RRPRU decoupled parallel manipulator is solved by using of the screw theory. The parallel robot studied is composed of a central RRPRU-type kinematic chain whose role is to conduct the center of the moving platform and three external RPRRC-type kinematic chains whose function is to control the orientation of the moving platform.

The displacement analysis is conveniently subdivided into two problems where the pose of the moving platform, as measured from the base, are the main factors to be considered. The inverse-forward position analysis is obtained in closed-form solution owing the decoupled performance of two rotary actuators defining the orientation of the central kinematic chain and one linear actuator representing the extendible length of it. On the other hand, once the position of the moving platform is fixed according to the coordinates of its geometric center, the inverse-forward displacement analysis is completed formulating closure equations based on three unit vectors that are related with the orientation of the moving platform which lead us six quadratic equations instead of the typical seven quadratic equations generated for the forward displacement analysis of the Gough-Stewart manipulator. Afterwards, the input-output equation of velocity of the robot is obtained systematically by taking advantage of the properties of reciprocal screws. This strategy allows to avoid the computation of the passive joint rates of the parallel manipulator owing the properties of the Klein form. Thereafter, the forward and inverse Jacobian matrices associated to the input-output equation of velocity of the robot are employed to determine the singular postures of the parallel manipulator. The singularities are explained using concise vector expressions. However, the singularity analysis in loci form beyond the scope of the contribution due to the lack of closed-form solutions for the forward displacement analysis credited to the external limbs. Numerical examples are included with the aim of showing the versatility and usefulness of the presented methodology.

Finally, some relevant characteristics of the parallel manipulator here considered are listed next: architecture free of spherical and compound joints, uncoupled motion of the moving platform, available decomposed Jacobian matrices, semi-closed form solutions available for the forward displacement analysis and less quadratic equations when compared with the Gough-Stewart platform concerned with the forward displacement analysis.

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