Traffic Modeling and Validation for Intersecting Metro Lines by Considering the Effect of Transfer Stations

FATEMEH KHOSROSERESHKI[ID]1 **AND BIJAN MOAVENI**[ID]2

[1]Control and Signaling Department, School of Railway Engineering, Iran University of Science and Technology, Tehran 1684613114, Iran
[2]Center of Excellence for Modelling and Control of Complex Systems, Faculty of Electrical Engineering, K. N. Toosi University of Technology, Tehran 1631714191, Iran

Corresponding author: Bijan Moaveni (b.moaveni@kntu.ac.ir)

ABSTRACT This paper proposes a nonlinear discrete event state-space model for intersecting metro lines considering the effect of transfer stations. Disturbances such as technical problems in the rolling stock and signaling systems can cause deviations in the predefined train departure times. Any delay in the metro traffic system will increase over time and propagate to other trains, leading to instability which will reduce the efficiency of the system. Transfer stations in metro networks are designed to transfer passengers between trains on different intersecting metro lines. Therefore, traffic modelling of the metro transportation system requires consideration of the effect of such transfer stations. After introducing a discrete event nonlinear model for intersecting metro lines with one or two transfer stations, the accuracy and effectiveness of the introduced model to describe the dynamic behavior of metro traffic system has been evaluated and verified using the results of simulations based on real data from two intersecting lines on the Tehran metro network.

INDEX TERMS Traffic modeling, intersecting metro lines, transfer station, time deviation, delay transmission.

NOMENCLATURE

Symbol	Description
D_k^ℓ	Nominal dwell time at platform k on line ℓ.
$d_{k+1}^{\ell,i}$	Dwell time of train i at platform $k+1$ on line ℓ.
$f_k^{\ell,i}$	Decision function for transferring delay of train i at platform k on line ℓ.
$H_{k+1}^{\ell,i}$	Nominal headway time of train i at platform $k+1$ on line ℓ.
$h_{k+1}^{\ell,i}$	Headway time of train i at platform $k+1$ on line ℓ.
$\bar{h}_{k+1}^{\ell}$	Average headway time over a period of time at platform $k+1$ on line ℓ.
M^L	Number of trains on lines L, $L \in \{\ell, \ell'\}$.
N^L	Number of platforms on lines L, $L \in \{\ell, \ell'\}$.
P_k^L	Platform k on lines L, $L \in \{\ell, \ell'\}$.
$P_{Ave\_c,k+1}^{\ell}$	Average number of passengers traveling between two lines from transfer station $k+1$ for average headway time.

Symbol	Description
$P_{Ave\_p,k+1}^{\ell}$	Average number of passengers arriving from the entry gate at platform of station $k+1$ for average headway on line ℓ.
R_k^ℓ	Nominal running time at platform k on line ℓ.
$r_k^{\ell,i}$	Running time of train i at platform k on line ℓ.
$T_k^{\ell,i}$	Nominal departure time of train i from platform k on line ℓ.
$t_k^{\ell,i}$	Departure time of train i from platform k on line ℓ.
$t_k^{\ell',j}$	Departure time of train j from platform k on line ℓ'.
t_B^L	Buffer time of set of lines L, $L \in \{\ell, \ell'\}$.
$\Delta t_k^{\ell,i}$	Time deviation from nominal departure time of train i at station's platform k on line ℓ.
$u_{d,k}^{\ell,i}$	Dwell time adjustment of train i at platform k on line ℓ (control action).
$u_{r,k}^{\ell,i}$	Running time adjustment of train i at platform k on line ℓ (control action).

The associate editor coordinating the review of this manuscript and approving it for publication was Wen-Sheng Zhao[ID].

$u_k^{\ell,i}$	Control signal to delay recovery of time deviation of train i at platform k on line ℓ.
$w_k^{\ell,i}$	Disturbance of time deviation of train i at platform k on line ℓ.
λ_{k+1}^{ℓ}	Delay rate at platform $k+1$ on line ℓ.
$\lambda_{c,k+1}^{\ell}$	Delay rate at platform $k+1$ of transfer station on line ℓ.
φ_{k+1}^{ℓ}	Time required for a passenger to board on the train in nominal dwell time at platform $k+1$ on line ℓ.
$\Psi_{k+1}^{\ell,i}$	Nominal effect of passengers function of train i from platform $k+1$ on line ℓ.
$\psi_{k+1}^{\ell,i}$	Actual effect of passengers function of train i from platform $k+1$ on line ℓ.

I. INTRODUCTION

The metro is the backbone of public transportation in large cities because it is fast, efficient, and safe. Along with the other modes of transportation, the metro provides a public transportation network to move passengers from any point of origin to any destination in the city. Metro systems consist of intersecting lines and transfer stations that link the key points of the metropolis [1].

In the metro networks, all trains depart according to a predefined train schedule, called the nominal schedule. Unwanted disturbances in the metro system will cause deviation of the train departure times from the nominal schedule [2]. Because the metro traffic system is high-frequency and inherently unstable, the delays will increase in time and propagate to other trains, which decreases the efficiency of a metro traffic system [2]. Consequently, delay recovery and traffic regulation in the metro transportation system are important aspects to improve the quality of transformation services and increase the passenger satisfaction.

A. LITERATURE REVIEW

Campion *et al*. [2] and Van Breusegem *et al*. [3] introduced a valuable model for a metro traffic system. The dynamic model was introduced based on deviations of the departure time from the nominal schedule for open lines and loop lines in a metro traffic system. They proposed an optimal state feedback approach to delay recovery which guaranteed the stability of high-frequency metro traffic systems. Murata [4] and Goodman [5] considered a metro traffic regulation problem and introduced a mathematical evaluation function that considers the effect of passenger expectations. An on-line optimization procedure was used to find the optimum arrival and departure times by minimizing the proposed penalty function. Fernandz *et al*. [6] proposed a predictive traffic regulation model for metro loop lines. They used a convex quadratic programming model to minimize the corresponding cost function in the presence of operational constraints. The main advantages of their proposed approach were its ability

to manage constraints and the solvability of the real-time optimization problem.

Berbey *et al*. [7] presented a Lyapunov-based stability analysis method for a metro traffic system modeled using the definition of the departure time deviation from the nominal schedule. They defined a new stability index to evaluate the effect of saturation on metro lines and predict the need for rescheduling. Also, rescheduling is another method to delay recovery. Gao *et al*. [8] proposed a real-time method to reschedule an over-crowded metro line using a skip-stop pattern during the recovery period to minimize the deviation from the timetable after disruption and reduce the passengers' total waiting time for increasing the passenger satisfaction.

Some researchers have considered the dynamic effect of passengers in the metro traffic model. Lin and Sheu [9], [10] introduced two adaptive optimal control (AOC) and dual heuristic programming (DHP) for delay recovery in a metro traffic system. The results show advantages for the AOC over the DHP when dealing with modelling error. Li and Shutter *et al*. [11] developed a state-space model that considered the safety constraints for the train traffic system. They designed a robust model predictive controller for the traffic regulation to guarantee disturbance attenuation.

Moaveni and Karimi [12] presented a model for a metro traffic system by considering the effect of the number of passengers on the platform and in the trains when calculating the dwell time. In this model, deviations in the departure time of trains and in the number of passengers on the trains from an initial value are defined as state-space variables. They applied model-based predictive control (MPC) to minimize the cost function that included passenger demands in the presence of constraints. Moaveni and Najafi [13], [14] proposed a new nonlinear state-space model that used a knock-on delay concept to modify the transmission delay between sequential trains in an open-loop railway traffic system. They designed a robust model predictive controller (RMPC) to delay recovery and to increase passenger satisfaction.

All of the mentioned studies considered the metro lines to be independent of each other. However, metro lines are clearly connected through transfer stations and, if a delay occurs in one of these lines, passengers can transfer the delay from one line to another line through the transfer stations [15].

Goverde [16] studied complex railway networks when passengers change trains at transfer stations by considering the intermodal connection of a bus service to railway service. He presented optimal buffer times in timetables for scheduled connections by minimizing the total expected transfer waiting times of passengers at a transfer station. Schutter *et al*. [17], [18] considered the effect of the transfer station to model a railway network. They used the switching max-plus method to describe a discrete event model. For delay recovery, they designed an optimal controller for the system by defining a cost function that kept the trains running on schedule and breaking connections. Although transfer stations are considered in this model, it is not suitable for describing the traffic dynamics in metro networks due to

the difference in the distance between the platforms and the running time of trains.

In some researches, the effect of a transfer station was considered as a constraint on the control system design. Li *et al.* [19] proposed a distributed optimal control by considering transfer coordination constraints to synchronize trains' departure at the transfer stations. Since passengers are the main cause of delay transmission between lines through the transfer station, providing appropriate methods to control passenger flow can prevent the spread of delays and alleviate the traffic congestion in the metro network. Yuan *et al.* [20] also use the passenger flow control method to reduce or avoid traffic congestion inside stations. They formulated a model of coordinated passenger flow control as a mixed-integer linear programming model by discretizing the time horizon and minimizing the total waiting time of passengers, including outside stations and on platforms. Wang *et al.* [21] introduced a mixed-integer programming model based on the equivalent time interval to minimize the total number of stranded passengers on a whole metro line by considering the effect of transfer stations. The main shortcoming of the above mentioned studies is that in these studies, the effect of the transfer stations has not been considered in the open-loop dynamic.

B. MAIN CONTRIBUTION

In this paper, the effect of transfer stations in metro traffic modelling to increase the accuracy of a model for high-frequency metro systems is considered. Providing a traffic model for each line of the metro network based on the time deviation method by considering the type of stations on the line can be a solution for decentralized control of the system so that the effect of transfer stations on the lines is also considered. The current study introduces a nonlinear discrete event model for a metro traffic system with intersecting metro lines by considering the effect of transfer stations. The model considers the concept of knock-on delay and buffer times. Based on the proposed approach, the time deviations of trains on lines 2 and 4 in Tehran metro are modeled. The proposed model has been verified using actual data on train departure times on lines 2 and 4.

The rest of this paper is categorized as follows: Section 2 presents the proposed nonlinear discrete event model and includes a state-space model and the effect of the number of passengers at a platform on the model. Section 3 compares the simulation results with actual data. In the last section, the conclusion is presented.

II. PROPOSED METRO TRAFFIC MODEL

In this section, a discrete event nonlinear model for two intersecting metro lines by considering the effect of transfer stations on the traffic dynamic is presented. Modelling has been done according to the assumption that trains have sufficient capacity for transportation. The two intersecting lines have at least one transfer station at which passengers can change lines to travel to their desired destinations. The intersecting
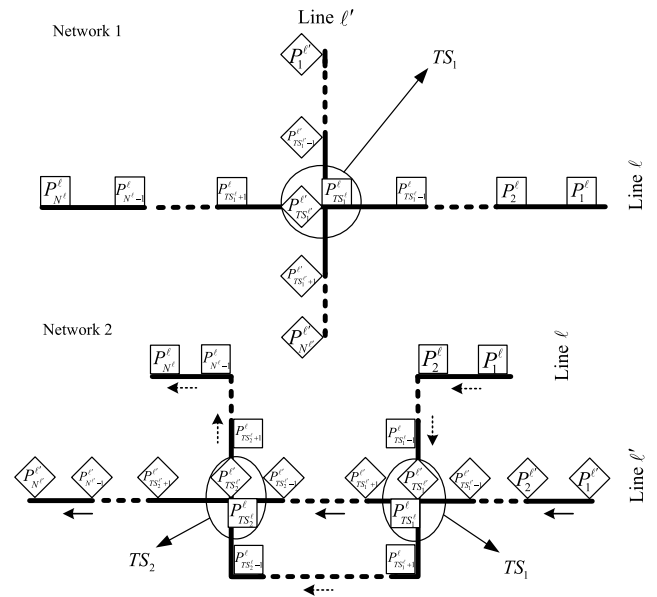
FIGURE 1. Intersecting lines in the metro network. Network 1 has one transfer station, TS_1, and Network 2 has two transfer stations, TS_1 and TS_2. (P_k^L: Platform k on lines L, $L \in \{\ell, \ell'\}$).

lines could include one or more transfer stations, as shown in Fig. 1. Note that the maximum number of transfer stations in Tehran metro network is two stations on intersecting lines. The mathematical model is driven by defining the departure time of train i, $i \in [1, M^\ell]$ at platform $k + 1$, $k \in [1, N^\ell - 1]$ on line ℓ as:

$$t_{k+1}^{\ell,i} = t_k^{\ell,i} + r_k^{\ell,i} + d_{k+1}^{\ell,i} + t_B^\ell \tag{1}$$

The running and dwell times are defined as (2) and (3), respectively.

$$r_k^{\ell,i} = R_k^\ell + u_{r,k}^{\ell,i} \tag{2}$$

$$d_{k+1}^{\ell,i} = D_k^\ell + \psi_{k+1}^{\ell,i} + u_{d,k+1}^{\ell,i} + f_k^{\ell,i} + w_k^{\ell,i} \tag{3}$$

By defining $u_k^{\ell,i}$ as:

$$u_k^{\ell,i} = u_{r,k}^{\ell,i} + u_{d,k+1}^{\ell,i} \tag{4}$$

and substituting (2), (3) and (4) into (1), the dynamic departure time of train i on line ℓ at platform $k + 1$ can be expressed as:

$$t_{k+1}^{\ell,i} = t_k^{\ell,i} + R_k^\ell + u_k^{\ell,i} + f_k^{\ell,i} + w_k^{\ell,i} + D_k^\ell + \psi_{k+1}^{\ell,i} + t_B^\ell \tag{5}$$

As well, the nominal departure times of train i on line ℓ at platforms $k + 1$ satisfy (6).

$$T_{k+1}^{\ell,i} = T_k^{\ell,i} + R_k^\ell + D_k^\ell + \Psi_{k+1}^{\ell,i} + t_B^\ell \tag{6}$$

Therefore, by defining the time deviation as $\Delta t_k^{\ell,i} = t_k^{\ell,i} - T_k^{\ell,i}$ and using (5) and (6), the time deviation of train i on line ℓ at platform $k + 1$ from the nominal departure time can be determined as:

$$\Delta t_{k+1}^{\ell,i} = \Delta t_k^{\ell,i} + (\psi_{k+1}^{\ell,i} - \Psi_{k+1}^{\ell,i}) + u_k^{\ell,i} + f_k^{\ell,i} + w_k^{\ell,i} \tag{7}$$

It is known that, in a metro traffic system, any delay in a train departure time will propagate along the line when the delay time is longer than the buffer time of that line (t_B^L, $L \in \{\ell, \ell'\}$). The decision function for transferring delay, $f_k^{\ell,i}$, is introduced to allow the delay to be correctly transmitted along a line. Also, $\psi_{k+1}^{\ell,i}$ and $\Psi_{k+1}^{\ell,i}$ are the actual and nominal functions to show the effect of passengers on determining the dwell time of the train i at the platform $k+1$ on line ℓ, respectively. Moreover, these two functions are determined based on the type of platform: located at the transfer station or not. Therefore, (7) should be considered in the two following cases:

CASE 1: THE PLATFORM $k + 1$ IS NOT A PLATFORM OF A TRANSFER STATION

In this case, $\psi_{k+1}^{\ell,i}$ and $\Psi_{k+1}^{\ell,i}$, are defined as:

$$\begin{aligned} \psi_{k+1}^{\ell,i} &= \lambda_{k+1}^\ell h_{k+1}^{\ell,i}, \\ \Psi_{k+1}^{\ell,i} &= \lambda_{k+1}^\ell H_{k+1}^{\ell,i} \end{aligned} \quad (8)$$

where,

$$\begin{aligned} h_{k+1}^{\ell,i} &= t_{k+1}^{\ell,i} - t_{k+1}^{\ell,i-1}, \\ H_{k+1}^{\ell,i} &= T_{k+1}^{\ell,i} - T_{k+1}^{\ell,i-1} \end{aligned} \quad (9)$$

and λ_{k+1}^ℓ is the delay rate. The delay rate for a platform which is not in a transfer station is defined as [14]:

$$\lambda_{k+1}^\ell = \frac{\varphi_{k+1}^\ell \times P_{Ave\_p,k+1}^\ell}{\bar{h}_{k+1}^\ell}, \quad \varphi_{k+1}^\ell = \frac{d_{k+1}^\ell}{P_{Act}^\ell} \quad (10)$$

where, P_{Act}^ℓ and d_{k+1}^ℓ are the actual number of passengers at the platform and the dwell time, respectively. Using (8) and (9), (7) can be rewritten as:

$$\Delta t_{k+1}^{\ell,i} = \alpha_{k+1}^\ell \Delta t_k^{\ell,i} + \beta_{k+1}^\ell \Delta t_{k+1}^{\ell,i-1} + \alpha_{k+1}^\ell (u_k^{\ell,i} + f_k^{\ell,i} + w_k^{\ell,i}) \quad (11)$$

where,

$$\alpha_{k+1}^\ell = \frac{1}{1 - \lambda_{k+1}^\ell}, \quad \beta_{k+1}^\ell = -\frac{\lambda_{k+1}^\ell}{1 - \lambda_{k+1}^\ell} \quad (12)$$

In this case, for determining the $f_k^{\ell,i}$, the two following conditions are presented:

(i) $\Delta t_{k+1}^{\ell,i-1} - \Delta t_k^{\ell,i} < t_B^\ell$

In this condition, the delay will not be transferred to the next train on the line. Therefore, $\beta_{k+1}^\ell \Delta t_{k+1}^{\ell,i-1} + \alpha_{k+1}^\ell f_k^{\ell,i}$ in (11) which generates the delay of the next train must be zero as:

$$\beta_{k+1}^\ell \Delta t_{k+1}^{\ell,i-1} + \alpha_{k+1}^\ell f_k^{\ell,i} = 0 \quad (13)$$

So, the decision function for transferring delay can be obtained as (14), using (12) and (13).

$$f_k^{\ell,i} = \lambda_{k+1}^\ell \Delta t_{k+1}^{\ell,i-1} \quad (14)$$

By employing (14), (11) can be rewritten as:

$$\Delta t_{k+1}^{\ell,i} = \alpha_{k+1}^\ell \Delta t_k^{\ell,i} + \alpha_{k+1}^\ell (u_k^{\ell,i} + w_k^{\ell,i}) \quad (15)$$

(ii) $\Delta t_{k+1}^{\ell,i-1} - \Delta t_k^{\ell,i} > t_B^\ell$

In this condition, the delay equal to $\Delta t_{k+1}^{\ell,i-1} - \Delta t_k^{\ell,i} - t_B^\ell$ is transmitted to the next train on the line. Therefore:

$$\beta_{k+1}^\ell \Delta t_{k+1}^{\ell,i-1} + \alpha_{k+1}^\ell f_k^{\ell,i} = \Delta t_{k+1}^{\ell,i-1} - \Delta t_k^{\ell,i} - t_B^\ell \quad (16)$$

and the decision function for transferring delay is presented as:

$$f_k^{\ell,i} = \Delta t_{k+1}^{\ell,i-1} - (1 - \lambda_{k+1}^\ell)(\Delta t_k^{\ell,i} + t_B^\ell) \quad (17)$$

Using (17), (11) can be rewritten as:

$$\Delta t_{k+1}^{\ell,i} = \Delta t_{k+1}^{\ell,i-1} - \beta_{k+1}^\ell \Delta t_k^{\ell,i} + \alpha_{k+1}^\ell (u_k^{\ell,i} + w_k^{\ell,i}) - t_B^\ell \quad (18)$$

Using (14) and (17), when a platform $k+1$ is not a platform of a transfer station, the decision function for transferring delay, $f_k^{\ell,i}$, can be defined as:

$$\begin{aligned} f_k^{\ell,i}(\Delta t_{k+1}^{\ell,i-1}, \Delta t_k^{\ell,i}, t_B^\ell) = g(\delta) \times (1 - \lambda_{k+1}^\ell)[\Delta t_{k+1}^{\ell,i-1} \\ - \Delta t_k^{\ell,i} - t_B^\ell] + \lambda_{k+1}^\ell \Delta t_{k+1}^{\ell,i-1} \end{aligned} \quad (19)$$

where,

$$\delta = \Delta t_{k+1}^{\ell,i-1} - \Delta t_k^{\ell,i} - t_B^\ell, \text{ and } g(\delta) = \begin{cases} 1 & \delta > 0 \\ 0 & \delta \leq 0 \end{cases}$$

CASE 2: THE PLATFORM $k + 1$ IS A PLATFORM OF A TRANSFER STATION

In platforms of transfer stations, there are two sets of passengers: one set is moving toward the platform from the station entrance on line ℓ, $P_{Ave\_p,k+1}^\ell$, and a second set is transfer passengers, $P_{Ave\_c,k+1}^\ell$, which are coming from the another platform of the transfer station. Therefore, $\psi_{k+1}^{\ell,i}$ and $\Psi_{k+1}^{\ell,i}$, as the actual and nominal function of the effect of passengers are defined as:

$$\begin{aligned} \psi_{k+1}^{\ell,i} &= \lambda_{k+1}^\ell h_{k+1}^{\ell,i} + \lambda_{c,k+1}^\ell (t_{k+1}^{\ell,i-1} - t_{k+1}^{\ell',j-1}), \\ \Psi_{k+1}^{\ell,i} &= \lambda_{k+1}^\ell H_{k+1}^{\ell,i} + \lambda_{c,k+1}^\ell (T_{k+1}^{\ell,i-1} - T_{k+1}^{\ell',j-1}) \end{aligned} \quad (20)$$

where, λ_{k+1}^ℓ and $\lambda_{c,k+1}^\ell$ are defined as (10) and (21), respectively.

$$\lambda_{c,k+1}^\ell = \frac{\varphi_{k+1}^\ell \times (P_{Ave\_p,k+1}^\ell + P_{Ave\_c,k+1}^\ell)}{\bar{h}_{k+1}^\ell} \quad (21)$$

Equation (20) shows that any delay in one line results in changing the departure times on the another line at the transfer station, because the number of passengers at the platforms are changed. In other words, any delays in intersecting lines can be transferred to the other lines at the transfer stations.

The dynamic equation of departure time deviation is as (22) using (7), (9) and (20).

$$\begin{aligned} \Delta t_{k+1}^{\ell,i} = \gamma_{k+1}^\ell \Delta t_k^{\ell,i} + \eta_{k+1}^\ell \Delta t_{k+1}^{\ell,i-1} + \mu_{k+1}^\ell \Delta t_{k+1}^{\ell',j-1} \\ + \gamma_{k+1}^\ell (u_k^{\ell,i} + f_k^{\ell,i} + w_k^{\ell,i}) \end{aligned} \quad (22)$$

where,

$$\gamma_{k+1}^\ell = \frac{1}{1 - \lambda_{k+1}^\ell}, \quad \mu_{k+1}^\ell = -\frac{\lambda_{c,k+1}^\ell}{1 - \lambda_{k+1}^\ell},$$

$$\eta_{k+1}^\ell = -\frac{\lambda_{k+1}^\ell - \lambda_{c,k+1}^\ell}{1 - \lambda_{k+1}^\ell} \quad (23)$$

and $f_k^{\ell,i}$, are defined in four following conditions:

(i) $\Delta t_{k+1}^{\ell,i-1} - \Delta t_k^{\ell,i} < t_B^\ell$ and $\Delta t_{k+1}^{\ell',j-1} - \Delta t_k^{\ell',j} < t_B^{\ell'}$

In this condition, the train delays will not be transmitted to the next trains on both lines. Therefore, $\eta_{k+1}^\ell \Delta t_{k+1}^{\ell,i-1} + \mu_{k+1}^\ell \Delta t_{k+1}^{\ell',j-1} + \gamma_{k+1}^\ell f_k^{\ell,i}$ in (22) which generates the delay of the next train must be zero as:

$$\eta_{k+1}^\ell \Delta t_{k+1}^{\ell,i-1} + \mu_{k+1}^\ell \Delta t_{k+1}^{\ell',j-1} + \gamma_{k+1}^\ell f_k^{\ell,i} = 0 \quad (24)$$

and the decision function for transferring delay, $f_k^{\ell,i}$, is obtained as:

$$f_k^{\ell,i} = (\lambda_{k+1}^\ell - \lambda_{c,k+1}^\ell)\Delta t_{k+1}^{\ell,i-1} + \lambda_{c,k+1}^\ell \Delta t_{k+1}^{\ell',j-1} \quad (25)$$

Equation (22) can be rewritten as (26), using (23) and (25).

$$\Delta t_{k+1}^{\ell,i} = \gamma_{k+1}^\ell \Delta t_k^{\ell,i} + \gamma_{k+1}^\ell (u_k^{\ell,i} + w_k^{\ell,i}) \quad (26)$$

(ii) $\Delta t_{k+1}^{\ell,i-1} - \Delta t_k^{\ell,i} < t_B^\ell$ and $\Delta t_{k+1}^{\ell',j-1} - \Delta t_k^{\ell',j} > t_B^{\ell'}$

In this condition, just the delay on line ℓ' will be transferred, thus:

$$\eta_{k+1}^\ell \Delta t_{k+1}^{\ell,i-1} + \mu_{k+1}^\ell \Delta t_{k+1}^{\ell',j-1} + \gamma_{k+1}^\ell f_k^{\ell,i} = \Delta t_{k+1}^{\ell',j-1} - \Delta t_k^{\ell',j} - t_B^{\ell'} \quad (27)$$

and, the decision function for transferring delay, $f_k^{\ell,i}$, is determined as:

$$f_k^{\ell,i} = (\lambda_{k+1}^\ell - \lambda_{c,k+1}^\ell)\Delta t_{k+1}^{\ell,i-1} + \frac{1 - \mu_{k+1}^\ell}{\gamma_{k+1}^\ell}\Delta t_{k+1}^{\ell',j-1}$$
$$- \frac{1}{\gamma_{k+1}^\ell}(\Delta t_k^{\ell',j} + t_B^{\ell'}) \quad (28)$$

Using (28) and (23), (22) can be rewritten as (29).

$$\Delta t_{k+1}^{\ell,i} = \gamma_{k+1}^\ell \Delta t_k^{\ell,i} + \Delta t_{k+1}^{\ell',j-1} - \Delta t_k^{\ell',j} - t_B^{\ell'}$$
$$+ \gamma_{k+1}^\ell (u_k^{\ell,i} + w_k^{\ell,i}) \quad (29)$$

(iii) $\Delta t_{k+1}^{\ell,i-1} - \Delta t_k^{\ell,i} > t_B^\ell$ and $\Delta t_{k+1}^{\ell',j-1} - \Delta t_k^{\ell',j} < t_B^{\ell'}$

In this condition, the delay just will be transferred between the trains on line ℓ, so:

$$\eta_{k+1}^\ell \Delta t_{k+1}^{\ell,i-1} + \mu_{k+1}^\ell \Delta t_{k+1}^{\ell',j-1} + \gamma_{k+1}^\ell f_k^{\ell,i} = \Delta t_{k+1}^{\ell,i-1} - \Delta t_k^{\ell,i} - t_B^\ell \quad (30)$$

and the decision function for transferring delay, $f_k^{\ell,i}$, is determined as:

$$f_k^{\ell,i} = (1 - \lambda_{c,k+1}^\ell)\Delta t_{k+1}^{\ell,i-1} - \frac{1}{\gamma_{k+1}^\ell}(\Delta t_k^{\ell,i} + t_B^\ell) + \lambda_{c,k+1}^\ell \Delta t_{k+1}^{\ell',j-1} \quad (31)$$

Therefore, the dynamic equation of departure time deviation is as (32), using (31) and (23).

$$\Delta t_{k+1}^{\ell,i} = (\gamma_{k+1}^\ell - 1)\Delta t_k^{\ell,i} + \Delta t_{k+1}^{\ell,i-1} - t_B^\ell + \gamma_{k+1}^\ell(u_k^{\ell,i} + w_k^{\ell,i}) \quad (32)$$

(iv) $\Delta t_{k+1}^{\ell,i-1} - \Delta t_k^{\ell,i} > t_B^\ell$, and $\Delta t_{k+1}^{\ell',j-1} - \Delta t_k^{\ell',j} > t_B^{\ell'}$

In this condition, delays between trains on both lines must be considered. Therefore:

$$\eta_{k+1}^\ell \Delta t_{k+1}^{\ell,i-1} + \mu_{k+1}^\ell \Delta t_{k+1}^{\ell',j-1} + \gamma_{k+1}^\ell f_k^{\ell,i}$$
$$= \Delta t_{k+1}^{\ell,i-1} - \Delta t_k^{\ell,i} - t_B^\ell + \Delta t_{k+1}^{\ell',j-1} - \Delta t_k^{\ell',j} - t_B^{\ell'} \quad (33)$$

and the decision function for transferring delay, $f_k^{\ell,i}$, is:

$$f_k^{\ell,i} = (1 - \lambda_{c,k+1}^\ell)\Delta t_{k+1}^{\ell,i-1} - \frac{1}{\gamma_{k+1}^\ell}(\Delta t_k^{\ell,i} + t_B^\ell)$$
$$+ \frac{1 - \mu_{k+1}^\ell}{\gamma_{k+1}^\ell}\Delta t_{k+1}^{\ell',j-1} - \frac{1}{\gamma_{k+1}^\ell}(\Delta t_k^{\ell',j} + t_B^{\ell'}) \quad (34)$$

By considering (34) and (23), (22) can be rewritten as:

$$\Delta t_{k+1}^{\ell,i} = (\gamma_{k+1}^\ell - 1)\Delta t_k^{\ell,i} + \Delta t_{k+1}^{\ell,i-1} - t_B^\ell + \Delta t_{k+1}^{\ell',j-1}$$
$$- \Delta t_k^{\ell',j} - t_B^{\ell'} + \gamma_{k+1}^\ell(u_k^{\ell,i} + w_k^{\ell,i}) \quad (35)$$

Consequently, when a platform $k + 1$ is a platform of the transfer station using (25), (28), (31), and (34), the decision function for transferring delay can be determined for both lines as follows:

$$f_k^{\ell,i}(\Delta t_{k+1}^{\ell,i-1}, \Delta t_k^{\ell,i}, t_B^\ell, \Delta t_{k+1}^{\ell',j-1}, \Delta t_k^{\ell',j}, t_B^{\ell'})$$
$$= g(\delta) \times \frac{1}{\gamma_{k+1}^\ell} \times [\Delta t_{k+1}^{\ell,i-1} - \Delta t_k^{\ell,i} - t_B^\ell] + (\lambda_{k+1}^\ell - \lambda_{c,k+1}^\ell)\Delta t_{k+1}^{\ell,i-1}$$
$$+ \lambda_{c,k+1}^\ell \Delta t_{k+1}^{\ell',j-1} + g(\delta') \times \frac{1}{\gamma_{k+1}^\ell} \times [\Delta t_{k+1}^{\ell',j-1} - \Delta t_k^{\ell',j} - t_B^{\ell'}] \quad (36)$$

where,

$$\delta = \Delta t_{k+1}^{\ell,i-1} - \Delta t_k^{\ell,i} - t_B^\ell, \quad \delta' = \Delta t_{k+1}^{\ell',j-1} - \Delta t_k^{\ell',j} - t_B^{\ell'}$$

and,

$$g(\delta) = \begin{cases} 1 & \delta > 0 \\ 0 & \delta \le 0 \end{cases}, \quad g(\delta') = \begin{cases} 1 & \delta' > 0 \\ 0 & \delta' \le 0 \end{cases}$$

In order to prevent hard nonlinearity in the traffic dynamics in step function $g(\delta)$ and $g(\delta')$, this function is approximated as $\hat{g}(\delta)$ and $\hat{g}(\delta')$ in (37).

$$\hat{g}(\delta) = \frac{1}{1 + e^{-G\delta}}, \quad \hat{g}(\delta') = \frac{1}{1 + e^{-G\delta'}} \quad (37)$$

It is clear that if a large value is selected for G, then the behavior of $\hat{g}(\delta)$ and $\hat{g}(\delta')$ is closer to the behavior of $g(\delta)$ and $g(\delta')$.

Note 1: The departure time deviations for trains of line ℓ' can be determined after substituting ℓ by ℓ' and i by j in all equations. As a result, the dynamic equations of the departure time deviations for the trains on line ℓ' are:

The platform $k + 1$ is not a platform of a transfer station:

$$\Delta t_{k+1}^{\ell',j} = \alpha_{k+1}^{\ell'} \Delta t_k^{\ell',j} + \beta_{k+1}^{\ell'} \Delta t_{k+1}^{\ell',j-1} + \alpha_{k+1}^{\ell'}(u_k^{\ell',j} + f_k^{\ell',j} + w_k^{\ell',j})$$
(38)

where,

$$\alpha_{k+1}^{\ell'} = \frac{1}{1 - \lambda_{k+1}^{\ell'}}, \qquad \beta_{k+1}^{\ell'} = -\frac{\lambda_{k+1}^{\ell'}}{1 - \lambda_{k+1}^{\ell'}}$$

$$f_k^{\ell',j}(\Delta t_{k+1}^{\ell',j-1}, \Delta t_k^{\ell',j}, t_B^{\ell'})$$
$$= \lambda_{k+1}^{\ell'} \Delta t_{k+1}^{\ell',j-1} + g(\delta') \times (1 - \lambda_{k+1}^{\ell'})[\Delta t_{k+1}^{\ell',j-1} - \Delta t_k^{\ell',j} - t_B^{\ell'}]$$
(39)

The platform $k + 1$ is a platform of a transfer station:

$$\Delta t_{k+1}^{\ell',j} = \gamma_{k+1}^{\ell'} \Delta t_k^{\ell',j} + \eta_{k+1}^{\ell'} \Delta t_{k+1}^{\ell',j-1} + \mu_{k+1}^{\ell'} \Delta t_{k+1}^{\ell,i-1}$$
$$+ \gamma_{k+1}^{\ell'}(u_k^{\ell',j} + f_k^{\ell',j} + w_k^{\ell',j})$$
(40)

where,

$$\gamma_{k+1}^{\ell'} = \frac{1}{1 - \lambda_{k+1}^{\ell'}}, \qquad \mu_{k+1}^{\ell'} = -\frac{\lambda_{c,k+1}^{\ell'}}{1 - \lambda_{k+1}^{\ell'}},$$

$$\eta_{k+1}^{\ell'} = -\frac{\lambda_{k+1}^{\ell'} - \lambda_{c,k+1}^{\ell'}}{1 - \lambda_{k+1}^{\ell'}},$$

$$f_k^{\ell',j}(\Delta t_{k+1}^{\ell',j-1}, \Delta t_k^{\ell',j}, t_B^{\ell'}, \Delta t_{k+1}^{\ell,i-1}, \Delta t_k^{\ell,i}, t_B^{\ell})$$
$$= g(\delta') \times \frac{1}{\gamma_{k+1}^{\ell'}} \times [\Delta t_{k+1}^{\ell',j-1} - \Delta t_k^{\ell',j} - t_B^{\ell'}]$$
$$+ (\lambda_{k+1}^{\ell'} - \lambda_{c,k+1}^{\ell'})\Delta t_{k+1}^{\ell',j-1} + \lambda_{c,k+1}^{\ell'}\Delta t_{k+1}^{\ell,i-1}$$
$$+ g(\delta) \times \frac{1}{\gamma_{k+1}^{\ell'}} \times [\Delta t_{k+1}^{\ell,i-1} - \Delta t_k^{\ell,i} - t_B^{\ell}]$$

$$\delta' = \Delta t_{k+1}^{\ell',j-1} - \Delta t_k^{\ell',j} - t_B^{\ell'}, \qquad \delta = \Delta t_{k+1}^{\ell,i-1} - \Delta t_k^{\ell,i} - t_B^{\ell}$$
(41)

A nonlinear state-space model for M^ℓ trains and N^ℓ platforms on line ℓ and $M^{\ell'}$ trains and $N^{\ell'}$ platforms on line ℓ' can be developed by considering time deviation (11) and (22) for line ℓ, and (38) and (40) for line ℓ' as follows:

$$\mathbf{x}(k + 1) = \Xi(k, \mathbf{x}(k), \mathbf{u}(k), \mathbf{f}(k, \mathbf{x}(k)), \mathbf{w}(k))$$
(42)

where, $\Xi(k, \mathbf{x}(k), \mathbf{u}(k), \mathbf{f}(k, \mathbf{x}(k)), \mathbf{w}(k))$ represents the non-linear dynamics of the system and,

$$\mathbf{x}(k) \triangleq [\Delta t_k^{\ell,1}, \Delta t_{k-1}^{\ell,2}, \ldots, \Delta t_{k-(M^\ell-1)}^{\ell,M^\ell}, \Delta t_k^{\ell',1}, \Delta t_{k-1}^{\ell',2},$$
$$\ldots, \Delta t_{k-(M^{\ell'}-1)}^{\ell',M^{\ell'}}]_{1\times(M^\ell+M^{\ell'})}^T$$

$$\mathbf{u}(k) \triangleq [u_k^{\ell,1}, u_{k-1}^{\ell,2}, \ldots, u_{k-(M^\ell-1)}^{\ell,M^\ell}, u_k^{\ell',1}, u_{k-1}^{\ell',2}, \ldots,$$
$$u_{k-(M^{\ell'}-1)}^{\ell',M^{\ell'}}]_{1\times(M^\ell+M^{\ell'})}^T$$

$$\mathbf{w}(k) \triangleq [w_k^{\ell,1}, w_{k-1}^{\ell,2}, \ldots, w_{k-(M^\ell-1)}^{\ell,M^\ell}, w_k^{\ell',1}, w_{k-1}^{\ell',2}, \ldots,$$
$$w_{k-(M^{\ell'}-1)}^{\ell',M^{\ell'}}]_{1\times(M^\ell+M^{\ell'})}^T$$

$$\mathbf{f}(k, \mathbf{x}(k)) \triangleq [f_k^{\ell,1}, f_{k-1}^{\ell,2}, \ldots, f_{k-(M^\ell-1)}^{\ell,M^\ell}, f_k^{\ell',1}, f_{k-1}^{\ell',2}, \ldots,$$
$$f_{k-(M^{\ell'}-1)}^{\ell',M^{\ell'}}]_{1\times(M^\ell+M^{\ell'})}^T$$

$\mathbf{x}(k)$, $\mathbf{u}(k)$, $\mathbf{f}(k, \mathbf{x}(k))$ and $\mathbf{w}(k)$ are state, control, decision function for transferring delay, and disturbance vector, respectively for M^L trains at N^L platforms where $L \in \{\ell, \ell'\}$ and $M^L < N^L$.

III. SIMULATION RESULTS AND DISCUSSION

This section provides the simulation results of the proposed nonlinear metro traffic model by considering the properties of two intersecting lines of Tehran metro network. Fig.2 shows that lines 2 and 4 of the Tehran metro network contained two transfer stations. The actual departure time of the trains of this network was used to validate the model. Data received for this network from the control center is for the movement of the trains from Farhangsara platform, P_1^2, to Sadeghiyeh platform, P_{22}^2, on line 2, and from Kolahdoz platform, P_1^4, to Eram-Sabz platform, P_{19}^4, on line 4. Table 1 presents the parameters of lines 2 and 4 in Tehran metro network.

Fig.3 shows the minimum and maximum number of passengers in all platforms on lines 2 and 4 when the nominal headway is 4 min from $16:00$ to $18:00$ at working days. Also, Fig.4 shows the minimum and maximum delay rates calculated for all platforms on lines 2 and 4 by using the data of Fig.3.

The platforms 11 and 19 on line 2, and 7 and 13 on line 4 are platforms of two transfer stations on the intersecting lines 2 and 4 of Tehran metro network. Evidently, the number of passengers on the platforms of transfer stations are higher in comparison with the other platforms because, in addition to passengers arriving from the entry gates, some passengers also are coming from the platform of another line to this platform (Fig.3). Consequently, the delay rates of the platforms of the transfer stations are higher than the other platforms as it is shown in Fig.4.

Simulations of this section have been performed using the data of Fig.3 and Fig.4. The simulation results are presented for two scenarios. In the first scenario, the effect of transfer stations on the traffic modelling of two intersecting lines of the metro network is studied. Moreover, second scenario is expressed with the aim of validating the introduced model with comparing the actual values of departure times and its simulation results.

A. SCENARIO 1- METRO TRAFFIC MODELLING IN THE PRESENCE OF TRANSFER STATIONS
In this scenario, the simulation results in two following conditions are shown and compared: by considering the effect

TABLE 1. Parameters of lines 2 and 4 in Tehran metro.

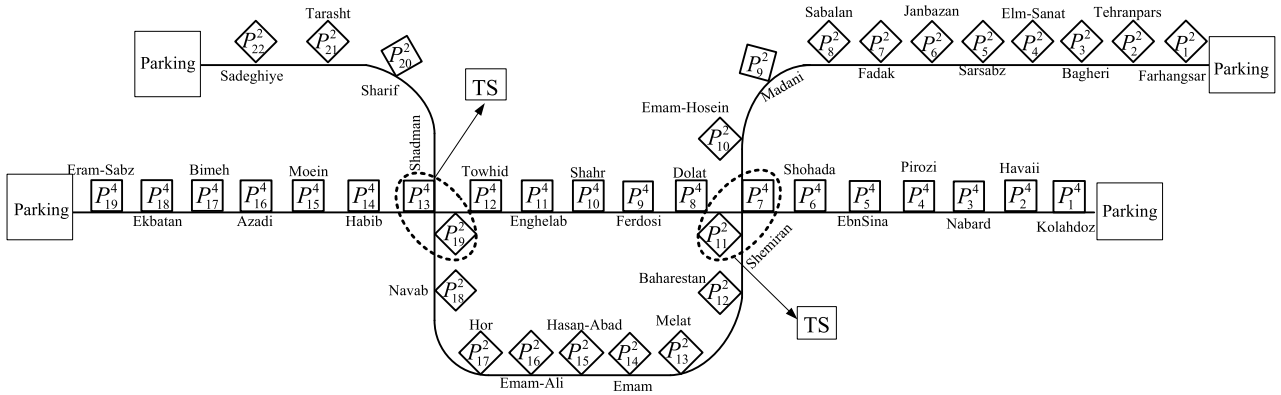| On line $\ell = 2$	Value	On line $\ell' = 4$	Value
N^2	22	N^4	19
M^2	21	M^4	18
t_B^2	$90sec$	t_B^4	$70sec$

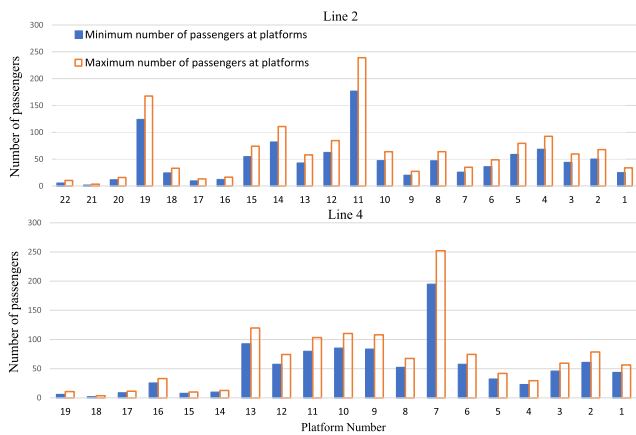**FIGURE 2.** Lines 2 and 4 of Tehran metro network (P_k^L denote platform k on line L).

FIGURE 3. Minimum and maximum number of passengers on platforms of lines 2 and 4 in Tehran metro network for nominal headway (4 min).
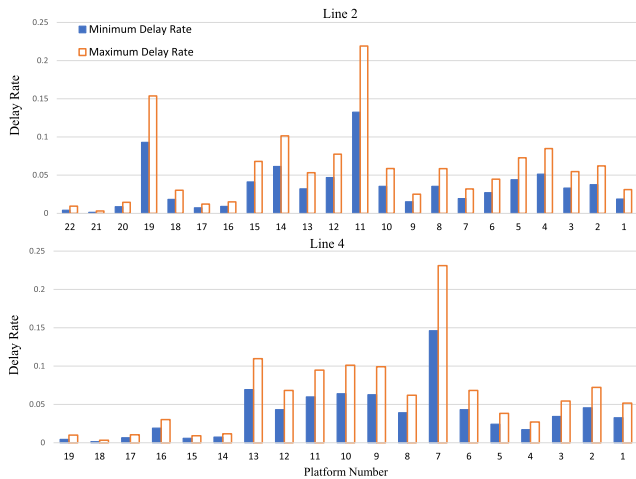

FIGURE 4. Minimum and maximum delay rates for all platforms of lines 2 and 4 in the Tehran metro network for nominal headway (4 min).
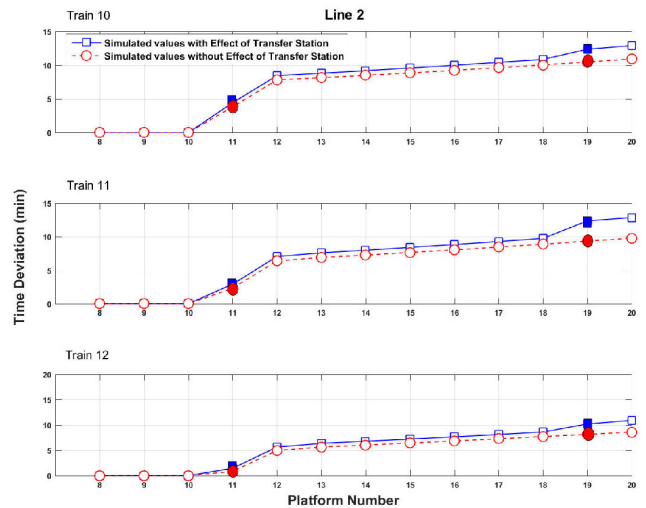

FIGURE 5. Effect of the transfer station on time deviation of trains on line 2. Dashed lines denote the departure time deviation of trains without considering the effect of transfer stations on the model, and the solid lines represent the departure time deviations of trains by considering the effect of transfer stations. Platforms 11 and 19 are transfer stations and their responses are marked in the form of filled geometric shapes.

of transfer stations in the model and without considering the effect of transfer stations.

The delays have occurred for train 10 at platforms 10 and 11 on line 2, $w_{10}^{2,10} = 240 sec$, and $w_{11}^{2,10} = 240 sec$. According to the maximum and minimum delay rates in Fig. 4, their

average value is used in the simulations. Also, $\lambda_{k+1}^2 = 0.041$ and $\lambda_{k+1}^4 = 0.038$ are considered for the simulations without considering the effect of the transfer station. The time deviations for the departure times on metro lines 2 and 4 are shown in Figs. 5 and 6. Evidently, when the effect of the transfer station is not considered in the model, the delay on line 2 has no effect on the time deviations of the train departure times on line 4. As well as, on line 2, the transferred delay coefficient between platforms when the effect of transfer stations has not been considered is smaller in comparison with the condition that the effect of transfer stations has been considered in the model.

B. SCENARIO 2- VALIDATING THE INTRODUCED MODEL USING THE ACTUAL DATA

To validate the introduced model for metro traffic system by considering the effect of transfer stations, a set of data has

TABLE 2. Nominal and actual departure time values for several trains in Tehran metro line 2 in January 2020, The delay has occurred in 17 : 03 : 29 at Shemiran platform.

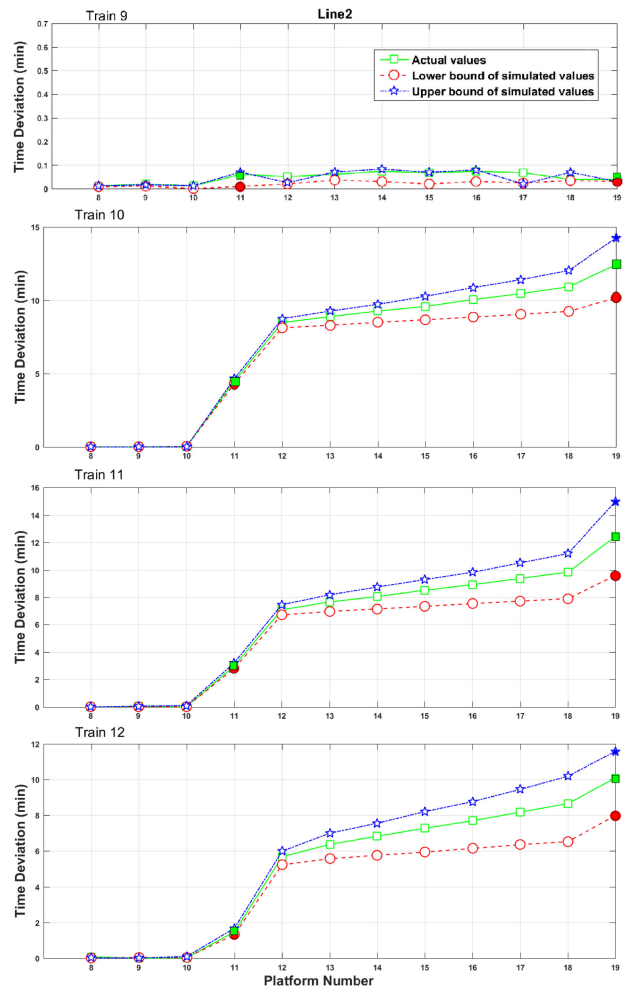| Line 2											
Train number	Platform name	Madani	Emam-Hosein	Shemiran	Baharestan	Melat	Emam	Hasan-Abad	Emam-Ali	Hor	Navab
	Platform number	9	10	11	12	13	14	15	16	17	18
10	Nominal Departure Time	16 : 54 : 00	16 : 56 : 00	16 : 59 : 00	17 : 01 : 00	17 : 03 : 00	17 : 05 : 00	17 : 08 : 00	17 : 10 : 00	17 : 12 : 00	17 : 14 : 00
	Actual Departure Time	16 : 54 : 01	16 : 56 : 03	17 : 03 : 29	17 : 09 : 28	17 : 11 : 54	17 : 14 : 16	17 : 17 : 35	17 : 20 : 03	17 : 22 : 27	17 : 24 : 55
11	Nominal Departure Time	16 : 58 : 00	17 : 00 : 00	17 : 03 : 00	17 : 05 : 00	17 : 07 : 00	17 : 09 : 00	17 : 12 : 00	17 : 14 : 00	17 : 16 : 00	17 : 18 : 00
	Actual Departure Time	16 : 58 : 03	17 : 00 : 02	17 : 05 : 59	17 : 12 : 05	17 : 14 : 39	17 : 17 : 03	17 : 20 : 30	17 : 22 : 55	17 : 25 : 22	17 : 27 : 49
12	Nominal Departure Time	17 : 02 : 00	17 : 04 : 00	17 : 07 : 00	17 : 09 : 00	17 : 11 : 00	17 : 13 : 00	17 : 16 : 00	17 : 18 : 00	17 : 20 : 00	17 : 22 : 00
	Actual Departure Time	17 : 02 : 03	17 : 04 : 01	17 : 08 : 29	17 : 14 : 40	17 : 17 : 22	17 : 19 : 50	17 : 23 : 17	17 : 25 : 42	17 : 28 : 11	17 : 30 : 39
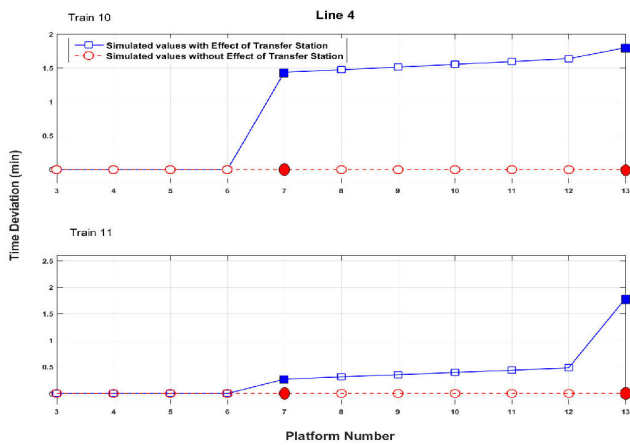
FIGURE 6. Effect of the transfer station on time deviation of trains on line 4. Dashed lines denote the departure time deviation of trains without considering the effect of transfer stations on the model, and the solid lines represent the departure time deviations of trains by considering the effect of transfer stations. Platforms 7 and 13 are transfer stations and their responses are marked in the form of filled geometric shapes.
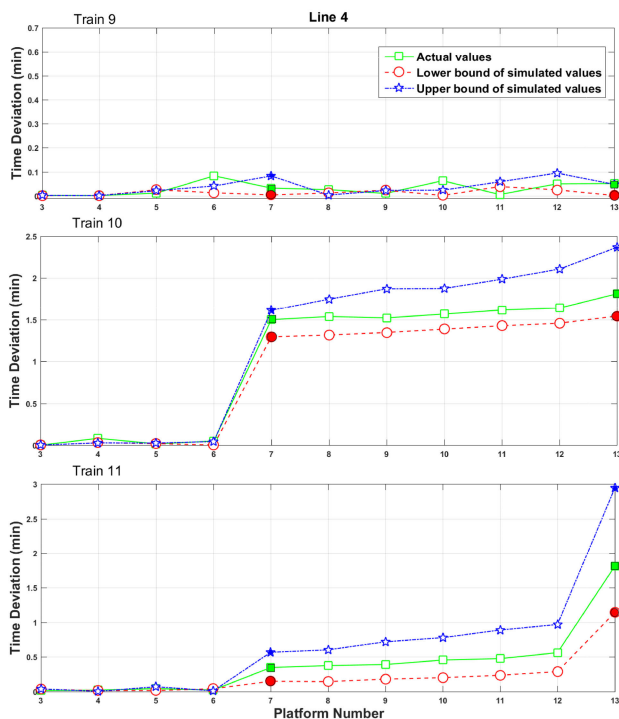

FIGURE 7. Actual and simulated values time of the train departure times for trains 9, 10, 11, and 12 on line 2. Platforms 11 and 19 are transfer stations and their responses are marked in the form of filled geometric shapes.

been used in which train 10 on line 2 departs with a 4 minutes delay at both platforms 10 and 11 of line 2, $w_{10}^{2,10} = 4$ min and $w_{11}^{2,10} = 4$ min. These values have been received from the control traffic center of Tehran metro network. The actual departure times of trains on lines 2 and 4 of the Tehran metro have been recorded to evaluate the model.

Tables 2 and 3 show the nominal and actual departure times from 16 : 54 to 17 : 30 on a weekday for lines 2 and 4, respectively. In this scenario, the automatic control system that compensates the delays in the control traffic center was disabled for about 20 minutes. Obviously, disabling the delay compensator system in Tehran metro network has been done to obtain the actual data in the open-loop conditions.

Figs. 7 and 8 show the actual and simulated values of the trains departure time deviations for a number of trains in Tehran metro lines 2 and 4, respectively. Please note that the upper and lower bounds of the simulation results have been shown in Figs. 7 and 8 based on maximum and minimum

delay rates, λ_{k+1}^{L} shown in Fig. 4. It can be seen that the simulation results include the actual data, which confirms the accuracy of the model. The length of a delay on line 2

TABLE 3. Nominal and actual departure time values for several trains in Tehran metro line 4 in January 2020, The effect of the transferred delay has occurred in 17 : 08 : 30 at Shemiran platform.

Line 4										
Train number	**Paltform name**	**EbnSina**	**Shohada**	**Shemiran**	**Dolat**	**Ferdosi**	**Shahr**	**Enghelab**	**Towhid**	**Shadman**
	Platform number	5	6	7	8	9	10	11	12	13
10	**Nominal Departure Time**	17 : 02 : 00	17 : 04 : 00	17 : 07 : 00	17 : 10 : 00	17 : 12 : 00	17 : 15 : 00	17 : 18 : 00	17 : 20 : 00	17 : 22 : 00
	Actual Departure Time	17 : 02 : 02	17 : 04 : 03	17 : 08 : 30	17 : 11 : 33	17 : 13 : 31	17 : 16 : 35	17 : 19 : 37	17 : 21 : 39	17 : 26 : 42
11	**Nominal Departure Time**	17 : 06 : 00	17 : 08 : 00	17 : 11 : 00	17 : 14 : 00	17 : 16 : 00	17 : 19 : 00	17 : 22 : 00	17 : 24 : 00	17 : 26 : 00
	Actual Departure Time	17 : 06 : 03	17 : 08 : 02	17 : 11 : 21	17 : 14 : 25	17 : 16 : 27	17 : 19 : 29	17 : 22 : 35	17 : 24 : 40	17 : 27 : 50

FIGURE 8. Actual and simulated values time of the train departure times for trains 9, 10 and 11 on line 4. Platforms 7 and 13 are transfer stations and their responses are marked in the form of filled geometric shapes.

and the actual data from Table 2 indicate that this ocurred delay in line 2 has been transmitted from platform 11 at the transfer station to train 10 on line 4, Table 3. After the first delay for train 10 at platform 10, this train arrives to the platform 11 with more passengers compared to the normal condition. Hence, the accumulation of transferred passengers at platform 7 of the transfer station on line 4 caused a delay for train 10 on this line.

IV. CONCLUSION

In this paper, a nonlinear discrete event model for the metro traffic systems of two intersecting lines with regard to the transfer stations has been introduced. The introduced model considers the buffer time and effect of transfer stations in the dynamic equations. By studying the effect of transfer stations on the metro traffic system, it was shown that passengers play a major role in the delay transmission between the intersecting metro lines.

The proposed model was validated by employing the actual data from the Tehran metro network. The introduced model has been simulated by considering the effect of uncertainty of delay rates (λ_{k+1}^L) and a delay scenario in Tehran metro lines 2 and 4. Comparing the simulation results and actual departure times confirmed the accuracy of the proposed model.

Further research can be done by considering more than two intersecting metro lines in modelling and designing the control system.

REFERENCES

[1] S. Derrible, "The properties and effects of metro network designs," Ph.D. dissertation, Dept. Civil Eng., Univ. Toronto, Toronto, ON, Canada, 2010.

[2] G. Campion, V. Van Breusegem, P. Pinson, and G. Bastin, "Traffic regulation of an underground railway transportation system by state feedback," *Optim. Control Appl. Methods*, vol. 6, no. 4, pp. 385–402, 1985.

[3] V. Van Breusegem, G. Campion, and G. Bastin, "Traffic modeling and state feedback control for metro lines," *IEEE Trans. Autom. Control*, vol. 36, no. 7, pp. 770–784, Jul. 1991.

[4] S. Murata and C. J. Goodman, "An optimal traffic regulation method for metro type railways based on passenger oriented traffic evaluation," *WIT Trans. Built Environ.*, vol. 34, pp. 573–583, Aug. 1998.

[5] C. Goodman and S. Murata, "Metro traffic regulation from the passenger perspective," *Proc. Inst. Mech. Eng. F, J. Rail Rapid Transit*, vol. 215, no. 2, pp. 137–147, 2001.

[6] A. Fernandez, A. P. Cucala, B. Vitoriano, and F. de Cuadra, "Predictive traffic regulation for metro loop lines based on quadratic programming," *Proc. Inst. Mech. Eng. F, J. Rail Rapid Transit*, vol. 220, no. 2, pp. 79–89, 2006.

[7] A. Berbey, R. Galán, P. S. Segundo, and J. Sanz-Bobi, "Lyapunov based stability analysis for metro lines," in *Proc. 14th Int. Conf. Urban Transp. Environ.*, Malta, Sep. 2008, pp. 111–119.

[8] Y. Gao, L. Kroon, M. Schmidt, and L. Yang, "Rescheduling a metro line in an over-crowded situation after disruptions," *Transp. Res. B, Methodol.*, vol. 93, pp. 425–449, Nov. 2016.

[9] W.-S. Lin and J.-W. Sheu, "Metro traffic regulation by adaptive optimal control," *IEEE Trans. Intell. Transp. Syst.*, vol. 12, no. 4, pp. 1064–1073, Dec. 2011.

[10] J.-W. Sheu and W.-S. Lin, "Adaptive optimal control for designing automatic train regulation for metro line," *IEEE Trans. Control Syst. Technol.*, vol. 20, no. 5, pp. 1319–1327, Sep. 2012.

[11] S. Li, B. de Schutter, L. Yang, and Z. Gao, "Robust model predictive control for train regulation in underground railway transportation," *IEEE Trans. Control Syst. Technol.*, vol. 24, no. 3, pp. 1075–1083, May 2015.

[12] B. Moaveni and M. Karimi, "Subway traffic regulation using model-based predictive control by considering the passengers dynamic effect," *Arabian J. Sci. Eng.*, vol. 42, no. 7, pp. 3021–3031, 2017.

[13] S. Najafi and B. Moaveni, "Modeling and real-time traffic regulation in metro loop lines using nonlinear model predictive control," *J. Control*, vol. 9, no. 2, pp. 3021–3031, 2015.

[14] B. Moaveni and S. Najafi, "Metro traffic modeling and regulation in loop lines using a robust model predictive controller to improve passenger satisfaction," *IEEE Trans. Control Syst. Technol.*, vol. 26, no. 5, pp. 1541–1551, Sep. 2018.

[15] F. Khosrosereshki and B. Moaveni, "Metro traffic regulation by considering the effect of transfer station," presented at the 6th Int. Conf. Control, Instrum., Autom. (ICCIA), Kurdistan, Iran, Oct. 2019.

[16] R. Goverde, "Optimal scheduling of connections in railway systems," in *Proc. 8th WTCR*, Antwerp, Belgium, Jul. 1998, pp. 1–17.

[17] B. De Schutter, T. van den Boom, and A. Hegyi, "Model predictive control approach for recovery from delays in railway systems," *Transp. Res. Rec., J. Transp. Res. Board*, vol. 1793, no. 1, pp. 15–20, Jan. 2002.

[18] T. van den Boom and B. De Schutter, "Modeling and control of railway networks," in *Proc. Amer. Control Conf.*, Boston, MA, USA, Jun./Jul. 2004, pp. 5728–5733.

[19] S. Li, X. Zhou, L. Yang, and Z. Gao, "Automatic train regulation of complex metro networks with transfer coordination constraints: A distributed optimal control framework," *Transp. Res. B, Methodol.*, vol. 117, pp. 228–253, Nov. 2018.

[20] F. Yuan, H. Sun, L. Kang, and J. Wu, "Passenger flow control strategies for urban rail transit networks," *Appl. Math. Model.*, vol. 82, pp. 168–188, Jun. 2020.

[21] X. Wang, J. Wu, X. Yang, X. Guo, H. Yin, and H. Sun, "Multistation coordinated and dynamic passenger inflow control for a metro line," *IET Intell. Transp. Syst.*, vol. 14, no. 9, pp. 1068–1078, Sep. 2020.

FATEMEH KHOSROSERESHKI received the M.Sc. degree in electrical engineering from the Amirkabir University of Technology, Tehran, Iran, in 2014. She is currently pursuing the Ph.D. degree in control and signaling engineering with the Control and Signaling Department, School of Railway Engineering, Iran University of Science and Technology, Tehran. Her current research interests include automatic train control systems, intelligent transportations systems, train rescheduling, modeling, and control.

BIJAN MOAVENI received the B.S. degree in control engineering from the Isfahan University of Technology, Isfahan, Iran, in 2000, and the M.S. and Ph.D. degrees in control engineering from the K. N. Toosi University of Technology, Tehran, Iran, in 2002 and 2007, respectively.

From 2009 to 2015, he was an Assistant Professor and from 2015 to 2018, he was an Associate Professor with the Department of Control and Signaling, School of Railway Engineering, Iran University of Science and Technology, Tehran. Since 2018, he has been an Associate Professor with the Systems and Control Engineering Group, K. N. Toosi University of Technology. He is also a member of the Center of Excellence for Modelling and Control of Complex Systems. He has authored or coauthored more than two books and more than 80 articles. His current research interests include large-scale control systems, control configuration selection, robust control systems, estimation theory, and automatic traffic control systems.

• • •