Received November 21, 2021, accepted December 14, 2021, date of publication December 20, 2021,
date of current version January 4, 2022.

Digital Object Identifier 10.1109/ACCESS.2021.3136892

Adaptive Event-Triggered Consensus of Fractional-Order Nonlinear Multi-Agent Systems

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This work was supported in part by the Fundamental Research Funds for the Central Universities under Grant 2019CDXYZDH0014, and in part by the Basic and Frontier Research Project of Chongqing under Grant cstc2017jcyjAX0037.

ABSTRACT The leader-following consensus of fractional-order nonlinear multi-agent systems is studied in this paper. An adaptive event-triggered control protocol is proposed to achieve the leader-following consensus scheme. By applying Lyapunov stability theory of fractional-order systems and some effective inequality techniques, some sufficient conditions for ensuring the consensus are derived intensively, and the proposed control method can reduce the communication between the agents greatly. Moreover, the Zeno behavior of event-triggered algorithm for multi-agent systems is excluded. Finally, a simulation example is presented to validate the effectiveness of the proposed consensus protocol.

INDEX TERMS Leader-following consensus, multi-agent systems, adaptive control, event-triggered control.

I. INTRODUCTION

Over the last decades, the coordination problem of multi-agent systems (MASs) has become one of the research hotspots in the automation field and drawn much attention to researchers owing to the extensive applications in such wide fields as robotic group, unmanned aerial vehicles formation flight and so on [1]–[3]. A lot of useful conclusions on cooperative control problem have hitherto been obtained on account of different application occasion. For instance, the consensus cases with impulsive control [4]–[8], adaptive control [9], [10], fuzzy control [11], [12] and event-triggered control [13], [14] are widely studied in the challenge of consistency. A significant topic is the leader-following coordination of MASs, in which the leader guides all the other agents to reach the same dynamic process.

A particular practical direction is to describe MASs with fractional-order dynamics, where the fractional calculus can reflect the essence and performance of a complicated system better than integral calculus. In [15], the consensus of fractional-order multi-agent systems (FOMASs) under a directed connection network topology and the characteristic relations between the agents and fractional calculus was studied. The cooperativity problem of FOMASs under switching topology with double-integrator was solved by employing Laplace transformation, dwell-time technology and Mittag-Leffler function in [16]. The system stabilization with fractional-order dynamic performance can be analyzed by using Lyapunov method under the control protocol [17], [18]. As developing theories and applications of fractional dynamical systems, the coordination problem of FOMASs with many different topics such as leader-following consensus [17], [19], [20], unknown system parameters [21]–[23], fixed-time consensus [24], input and distributed delays [25], has attracted more and more attention.

Moreover, adaptive control is an excellent control strategy for FOMASs with uncertain parameters, and the controller parameters can be automatically updated during control process. Over the past few decades, the increasing works applied adaptive control theory to discuss the dynamic response characteristics of distributed network systems, and the consensus of MASs is one of them. Combining the superiority of adaptive control laws in nonlinear system and the energy-saving of using event-triggered control method, the adaptive event-triggered controller was studied to reduce information exchange and resolve consensus problem

The associate editor coordinating the review of this manuscript and approving it for publication was Chao-Yang Chen.

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in [26], [27]. In [28], An adaptive control strategy is given to realize consensus of MASs with uncertain nonlinearity. The adaptive neural-network consensus problems of MASs with nonlinearity were investigated in [29]–[31]. The event trigger-based adaptive control method is proposed to solve consensus problems of linear MASs in [32].

Furthermore, the event-triggered protocol is one of the effective control strategies to reduce unnecessary control cost and can improve availability of system resources and computing capabilities. Note that the control variables are changed only when event-triggered condition is matched, which is completely different from the previous control strategies based on time driving [33]–[35]. Distributed and centralized event-triggered protocols have been applied to consider the consensus challenge of the first order MASs in [36]. In [37], the event-triggered case by sampled data states in traditional integer-order system was discussed. From the obtained results on event-triggered consensus problem of MASs, one of triggering conditions is the norm state of boundary function, and the control mechanism is triggered on any occasion that a defined error exceed limit value.

Enlightened by the previous discussions and knowledges, the consensus of leader-following FOMASs is studied in this paper. By designing an adaptive event-triggered controller, some new sufficient consensus conditions are derived, and the adaptive event-triggered protocol and event-triggered timing function are given for each agent. As far as the authors know, this is the first time to explore the combination of adaptive control and event-triggered method for achieving the coordination of FOMASs with nonlinear dynamics. The obtained results are not only functional and novel for consensus problem, but also consistent with the actual systems.

The main contributions of this paper are summed up as follows:

(1) For the fractional-order nonlinear dynamics, the proposed adaptive event-triggered protocol can achieve the consensus of leader-following FOMASs effectively.
(2) At the last event-triggering time instants, the state update of each agent only needs the local and neighboring state values.
(3) Note that the state updates only happen at event-triggering instants, and the FOMASs can greatly decrease the information exchange between agents.

The structure of the rest of this paper is presented as follows. Section 2 briefly reviews some concepts and notations of graph theory, summarize the properties of Caputo fractional derivative, and describe the nonlinear MASs. In Section 3, the sufficient consensus conditions of leader-following FOMASs with nonlinearity are analyzed intensively, and the Zeno phenomenon of the corresponding MASs can be excluded. In Section 4, a numerical simulation is provided to illustrate the feasibility of the proposed method. Finally, a conclusion is stated briefly in Section 5.

Notation: Throughout the paper, $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denotes the minimal and maximal eigenvalues of matrix A. $R^{m\times n}$, $\mathbb{N}$ and $\otimes$ stands for the set of all $m \times n$ real matrices, positive integers and the Kronecker product respectively. A^{-1} and A^T denote the inverse and the transpose of matrix A.

II. PRELIMINARIES

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a undirected graph which means the communication between the nodes, $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ and $\mathcal{E} \in (\mathcal{V} \times \mathcal{V})$ denotes the group of N nodes and the set of edges. $N_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}\}$ denotes the neighbors of node i. $\mathcal{A} = (a_{ij})_{N\times N}$ is the adjacency associate matrix of nodes, where a_{ij} represents the weight of edge (j, i). $a_{ii} = 0 (i \in \mathbb{N})$ denotes that corresponding node does not exist self-loops and $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$. $d_i = \sum_{j=1}^{N} a_{ij}$, $D = \mathrm{diag}\{d_i\} \in R^{N\times N}$ and $L = D - \mathcal{A}$ are the in-degree of node i, the related in-degree matrix and the Laplacian matrix respectively. To represent whether the agents exist information exchange with the leader, we define the adjacency matrix of the leader as $\mathcal{B} = \mathrm{diag}\{b_1, \ldots, b_N\}$ related to $\mathcal{G}$, where $b_i > 0$ if the leader exist communication with agent i and otherwise $b_i = 0$. Moreover let $Q = L + \mathcal{B}$. Due to the undirected topology graph, only when there exists a spanning tree in the undirected topology $\mathcal{G}$, which the leader is root node, all eigenvalues of matrix Q are positive [38].

In last several decades, there are few forms for the fractional calculus. Due to its initial value has actual physical meaning in many system models, the Caputo fractional operator have huge impact on fractional-order calculus. Therefore, we use Caputo fractional calculus to study the system dynamics. The differential equation of the Caputo fractional calculus is presented subsequently:

$$ {}_{t_0}^{C}D_t^{\alpha} x(t) = \frac{1}{\Gamma(r-\alpha)} \int_{t_0}^{t} \frac{x^{(r)}(\tau)}{(t-\tau)^{\alpha-r+1}} d\tau, \tag{1} $$

where $r - 1 < \alpha < r$, $r \in \mathbb{N}$. α means the differential order, $t \geq 0$. $\Gamma(\cdot)$ denotes the Gamma function and $\Gamma(\mathrm{n}) = \int_0^{\infty} e^{-t} t^{n-1} dt$. In order to simplify the analysis, ${}_{t_0}^{C}D_t^{\alpha} x(t)$ is expressed by $x^{(\alpha)}(t)$ in this paper. $\overrightarrow{1}$ denotes all elements of the column vector are 1.

Next, the Mittag-Leffler function is introduced, which has high-frequency application in judging the stability of the FOMASs with nonlinear dynamics and the solutions of fractional-order derivative.

$$ E_{\alpha,\beta}(z) = \sum_{l=0}^{\infty} \frac{z^l}{\Gamma(l\alpha + \beta)}, \tag{2} $$

when $\beta = 1, \alpha > 0$, a special form is obtained as follows:

$$ E_{\alpha}(z) = \sum_{l=0}^{\infty} \frac{z^l}{\Gamma(l\alpha + 1)}. \tag{3} $$

Lemma 1 [39]: Let $0 < \alpha < 1$, $\gamma \in R$ and $g(t)$ is a known differentiable function. The form of fractional derivative is

$$ D^{\alpha} f(t) = \gamma f(t) + g(t), \tag{4} $$

which can be solved as

$$f(t) = f(t_0)E_\alpha(\gamma(t-t_0)^\alpha) + \alpha\int_{t_0}^{t}(t-\tau)^{\alpha-1}E_{\alpha,\alpha}(\gamma(t-\tau)^\alpha)g(\tau)d\tau. \quad (5)$$

Lemma 2 [40]: Define $x(t) \in R^N$ and $H \in R^{N\times N}$ as a continuously derivable vector function of time and a real matrix. it can be guaranteed for $t \geq t_0$ that

$$\frac{1}{2}D^\alpha(x^T(t)Hx(t)) \leq x^T(t)HD^\alpha x(t), \quad (6)$$

where $\forall\alpha \in (0, 1]$ and $\forall t \geq t_0$.

III. MAIN RESULTS

The leader-following FOMASs consist of N agents with a leader is defined by

$$\begin{cases} x_0^\alpha(t) = Ax_0(t) + g(t, x_0(t)), & t \geq t_0 \\ x_i^\alpha(t) = Ax_i(t) + Bu_i(t) + g(t, x_i(t)), & i = 1, 2, \cdots, \mathbb{N}, \end{cases} \quad (7)$$

where $0 < \alpha < 1$, $A \in R^{n\times n}$, $B \in R^{n\times m}$. $x_i \in R^n$ and $u_i \in R^m$ are the state variables and input variables of agent i, respectively. $x_0 \in R^n$ is the state variable of the leader. $g(t, x_i(t))$ is a differentiable function with the local Lipschitz constant $\varpi > 0$ on $x_i(t)$, and the function satisfies

$$\|g(t, x_i(t)) - g(t, x_j(t))\| \leq \varpi \|x_i(t) - x_j(t)\|, \quad t > 0,\ i \neq j. \quad (8)$$

Definition 1: For any initial value, the consensus for FOMASs is reached if all state variables of followers satisfy

$$\lim_{t\to\infty} \|x_i(t) - x_0(t)\| = 0, \quad i = 1, 2, \cdots, N. \quad (9)$$

Assumption 1: In fact, due to the stabilization of (A, B), the following inequality holds

$$TA + A^TT - TBB^TT + \vartheta I_N \leq 0, \quad (10)$$

where $\vartheta > 0$ and T is a positive definite symmetric matrix.

Consider the latest state information at event instant t_k, the adaptive event-triggered distributed control controller for each agent is given as follows:

$$\begin{cases} u_i(t) = -Jw_i(t)\Delta_i(t_k^i), \\ w_i^\alpha(t) = \Delta_i^T(t_k^i)K\Delta_i(t_k^i) - \dfrac{\vartheta w_i(t)}{2}, \quad t \in [t_k^i, t_{k+1}^i), \end{cases} \quad (11)$$

where

$$\Delta_i(t) = \sum\nolimits_j^N a_{ij}(x_i(t) - x_j(t)) + b_i(x_i(t) - x_0(t)),$$

$J \in R^{m\times n}$ and $K \in R^{n\times n}$ are control gain matrixes. $w_i(t)$ denotes the adaptive control function related to the agent i and $w_i(t_0) > 0$. The triggering time $\{t_k^i\}$ for each agent is determined by

$$t_{k+1} = \inf\{t > t_k \text{ and } \xi_i(t) \geq 0\}, \quad (12)$$

where

$$\xi_i(t) = w_i(t)\varphi_i^T(t)K\varphi_i(t) - w_i(t)\Delta_i^T(t)K\Delta_i(t) - \delta\exp(-\psi(t-t_0)), \quad (13)$$

is defined as the triggering condition for parameter as $\psi > 0$, $\delta > 0$ and $\varphi_i(t) = \Delta_i(t_k^i) - \Delta_i(t)$.

Let $e_i(t) = x_i(t) - x_0(t)$, $e(t) = (e_1^T(t), e_2^T(t), \cdots, e_N^T(t))^T$, $\varphi(t) = (\varphi_1^T(t), \varphi_2^T(t), \cdots, \varphi_N^T(t))^T$ and $W(t) = \text{diag}\{w_1(t), w_2(t), \cdots, w_N(t)\}$, by connecting (7) and (11), one gets

$$\begin{cases} e^{(\alpha)}(t) = (I\otimes A - W(t)Q\otimes BJ)e(t) - (W(t)\otimes BJ)\varphi(t) \\ \qquad + G(t, x(t)) - 1_N \otimes g(t, x_0(t)), \\ w_i^{(\alpha)}(t) = \varphi_i^T(t)K\varphi_i(t) + 2\Delta_i^T(t)K\varphi_i(t) \\ \qquad + \Delta_i^T(t)K\Delta_i(t) - \dfrac{\vartheta w_i(t)}{2}. \end{cases} \quad (14)$$

It follows from (14) that the adaptive event-triggered function for each follower only needs the neighboring state variable at the latest triggering timing, which means the controller can reduce the exchange and computing cost between neighboring agents.

Theorem 1: The protocol (11) solves the cooperative problem of leader-following nonlinear FOMASs (7) if the information flow graph include a spanning tree, where the leader is the root node, $J = B^TT$, $K = TBB^TT$, $\lambda_{max}(T) \geq 1$, $\delta > 0$, and $0 < \psi < \vartheta/\lambda_{max}(T) - \lambda_{max}(BB^TT) - 2\varpi$, the event-triggered timing are defined by (12).

Proof: Q is a symmetric positive definite matrix (SPDM). By choosing a SPDM T, $Q\otimes T$ is also a SPDM. Therefore, the Lyapunov function is selected as below

$$V(t) = e^T(t)(Q\otimes T)e(t) + \sum_{i=1}^{N}\frac{w_i^2(t)}{2}. \quad (15)$$

From (1), one gets the derivative of $V(t)$

$$\begin{aligned} &D_t^\alpha V(t) \\ &= 2e^T(t)(Q\otimes T)e^{(\alpha)}(t) + \sum_{i=1}^{N} w_i(t)w_i^{(\alpha)}(t) \\ &= 2e^T(t)(Q\otimes TA)e(t) - 2e^T(t)(QW(t)Q\otimes TBJ)e(t) \\ &\quad -2e^T(t)(QW(t)\otimes TBJ)\varphi(t) + e^T(t)(QW(t)Q\otimes K)e(t) \\ &\quad +2e^T(t)(Q\otimes T)(G(t, x(t)) - 1_N\otimes g(t, x_0(t))) \\ &\quad -\sum_{i=1}^{N}\frac{\vartheta w_i^2(t)}{2} \\ &\quad +2e^T(t)(QW(t)\otimes K)\varphi(t) + \varphi^T(t)(W(t)\otimes K)\varphi(t) \\ &= 2e^T(t)(Q\otimes TA)e(t) - e^T(t)(QW(t)Q\otimes TBJ)e(t) \\ &\quad +2e^T(t)(Q\otimes T)(G(t, x(t)) - 1_N\otimes g(t, x_0(t))) \\ &\quad +\varphi^T(t)(W(t)\otimes K)\varphi(t) - \sum_{i=1}^{N}\frac{\vartheta w_i^2(t)}{2}. \end{aligned} \quad (16)$$

According to the triggering timing $\{t_k^i\}$ and triggering condition (12), one has

$$w_i(t)\varphi_i^T(t)K\varphi_i(t) \leq w_i(t)\Delta_i^T(t)K\Delta_i(t)+\delta\exp(-\psi(t-t_0)), \quad (17)$$

which implies that

$$\sum_{i=1}^{N} w_i(t)\varphi_i^T(t)K\varphi_i(t) \leq \sum_{i=1}^{N} w_i(t)\Delta_i^T(t)K\Delta_i(t)+\sum_{i=1}^{N}\delta\exp(-\psi(t-t_0)). \quad (18)$$

then, one gets

$$\varphi^T(t)(W(t)\otimes K)\varphi(t) \leq e^T(t)(QW(t)Q\otimes K)e(t)+N\delta\exp(-\psi(t-t_0)). \quad (19)$$

From (16) and (19), it derives that

$$\begin{aligned} D_t^\alpha V(t) &\leq e^T(t)(Q\otimes(TA+A^TT))e(t)+N\delta\exp(-\psi(t-t_0)) \\ &\quad +2e^T(t)(Q\otimes T)(G(t,x(t))-1_N\otimes g(t,x_0(t))) -\sum_{i=1}^{N}\frac{\vartheta w_i^2(t)}{2} \\ &\leq e^T(t)(Q\otimes(TA+A^TT))e(t)+N\delta\exp(-\psi(t-t_0)) \\ &\quad +2\varpi e^T(t)(Q\otimes T)e(t)-\sum_{i=1}^{N}\frac{\vartheta w_i^2(t)}{2} \\ &\leq -\vartheta e^T(t)(Q\otimes TT^{-1})e(t)+e^T(t)(Q\otimes TBB^TT)e(t) \\ &\quad +N\delta\exp(-\psi(t-t_0))+2\varpi e^T(t)(Q\otimes T)e(t) -\sum_{i=1}^{N}\frac{\vartheta w_i^2(t)}{2} \\ &\leq (-\vartheta\lambda_{min}(T^{-1})+\lambda_{max}(BB^TT)+2\varpi)e^T(t)(Q\otimes T)e(t) \\ &\quad -\sum_{i=1}^{N}\frac{\vartheta w_i^2(t)}{2}+N\delta\exp(-\psi(t-t_0)). \end{aligned} \quad (20)$$

which means that

$$D_t^\alpha V(t)\leq -\mu V(t)+N\delta\exp(-\psi(t-t_0)), \quad (21)$$

where $\mu=\vartheta\lambda_{min}(T^{-1})-\lambda_{max}(BB^TT)-2\varpi$, and one can make $\mu<1$ by choosing suitable T, B and ϖ.

Then, from Lemma 1 and (21), one gets

$$\begin{aligned} V(t) &\leq V(t_0)E_\alpha(-\mu(t-t_0)^\alpha) \\ &\quad +\alpha\int_{t_0}^{t}(t-\tau)^{\alpha-1}E_{\alpha,\alpha}(-\mu(t-\tau)^\alpha)N\delta\exp(-\psi(\tau-t_0))d\tau \\ &\leq V(t_0)E_\alpha(-\mu(t-t_0)^\alpha) \\ &\quad +\alpha N\delta\exp(-\psi(t-t_0))*(t^{\alpha-1}E_{\alpha,\alpha}(-\mu t^\alpha), \end{aligned} \quad (22)$$

where $*$ denotes the convolution operation, then one gets

$$\begin{aligned} &N\delta\exp(-\psi(t-t_0))*(t^{\alpha-1}E_{\alpha,\alpha}(-\mu t^\alpha)) \\ &=\int_0^\infty(t-\tau)^{\alpha-1}E_{\alpha,\alpha}(-\mu(t-\tau)^\alpha)N\delta\exp(-\psi(\tau-t_0))d\tau \\ &=\int_0^\infty N\delta\exp(-\psi(\tau-t_0))\cdot\exp(t-\tau) \\ &\quad\cdot\exp(\tau-t)(t-\tau)^{\alpha-1} \\ &E_{\alpha,\alpha}(-\mu(t-\tau)^\alpha)d\tau \\ &=\exp(t)\cdot(\int_0^\infty N\delta\exp(-\psi(\tau-t_0)-\tau) \\ &\quad\times\exp(\tau-t)(t-\tau)^{\alpha-1}E_{\alpha,\alpha}(-\mu(t-\tau)^\alpha)d\tau) \\ &=\exp(t)\cdot(N\delta\exp(-\psi(t-t_0)-t) \\ &\quad *\exp(-t)t^{\alpha-1}E_{\alpha,\alpha}(-\mu t^\alpha)\Big), \end{aligned} \quad (23)$$

and

$$\begin{aligned} &\int_0^\infty\exp(-t)t^{\alpha-1}E_{\alpha,\alpha}(-\mu t^\alpha)dt \\ &\quad=\int_0^\infty e^{-t}t^{\alpha-1}\frac{(-\mu t^\alpha)^l}{\Gamma(l\alpha+\alpha)}dt \\ &\quad=\sum_{l=0}^{\infty}\frac{(-\mu)^l}{\Gamma(l\alpha+\alpha)}\cdot\int_0^\infty e^{-t}t^{l\alpha+\alpha-1}dt \\ &\quad=\sum_{l=0}^{\infty}\frac{(-\mu)^l}{\Gamma(l\alpha+\alpha)}\cdot\Gamma(l\alpha+\alpha) \\ &\quad=\sum_{l=0}^{\infty}(-\mu)^l \\ &\quad=\frac{1}{1+\mu}, \quad 0<\mu<1, \end{aligned} \quad (24)$$

then, one further gets

$$\begin{aligned} V(t) &\leq V(t_0)E_\alpha(-\mu(t-t_0)^\alpha) \\ &\qquad +\alpha\int_{t_0}^{t}(t-\tau)^{\alpha-1}E_{\alpha,\alpha}(-\mu(t-\tau)^\alpha)N\delta \\ &\qquad \times\exp(-\psi(\tau-t_0))d\tau \\ &\leq V(t_0)E_\alpha(-\mu(t-t_0)^\alpha) \\ &\qquad +\alpha\cdot N\delta\exp(-\psi(t-t_0)) \\ &\qquad \times\int_0^\infty\exp(-t)t^{\alpha-1}E_{\alpha,\alpha}(-\mu t^\alpha)dt \\ &\leq V(t_0)E_\alpha(-\mu(t-t_0)^\alpha)+\Theta\cdot\exp(-\psi(t-t_0)), \end{aligned} \quad (25)$$

where $\Theta=\alpha N\delta/(1+\mu)$. Obviously, we can obtain from (25) and (2)

$$\lim_{t\to\infty}\|V(t)\|=0, \quad 0<\alpha<1. \quad (26)$$

Note that

$$\begin{aligned} \lambda_{min}(Q\otimes T)\|e(t)\|^2 &\leq V(t) \\ &= e^T(t)(Q\otimes T)e(t)+\sum_{i=1}^{N}\frac{w_i^2(t)}{2} \\ &\leq V(t_0)E_\alpha(-\mu(t-t_0)^\alpha)+\Theta\exp(-\psi(t-t_0)). \end{aligned} \quad (27)$$

therefore, one gets

$$\begin{aligned}&\lim_{t\to\infty}\|e(t)\|\\ &=\lim_{t\to\infty}\|x(t)-1_N\otimes x_0(t)\|\\ &\le\lim_{t\to\infty}\left\|\sqrt{V(t)/\lambda_{min}(Q\otimes T)}\right\|\\ &\le\lim_{t\to\infty}\frac{1}{\lambda_{min}(Q\otimes T)}\\ &\quad\times\left\|\sqrt{V(t_0)E_\alpha(-\mu(t-t_0)^\alpha)+\Theta\cdot\exp(-\psi(t-t_0))}\right\|\\ &=0 \qquad (28)\end{aligned}$$

which means for any agent i, one gets

$$\lim_{t\to\infty}\|x(t)-1_N\otimes x_0(t)\|=0, \qquad (29)$$

and all agents obviously reach consensus according to Definition 1. This completes the proof. □

Note that there are some control parameters to be chosen, the design procedure is given as follows for better clarity.

(1) From the inequality (10), one gets $\vartheta>0$ and T.

(2) From the conditions in Theorem 1, i.e., $J=B^TT$, $K=TBB^TT$, one gets control gains J and K.

(3) From the condition in Theorem 1, i.e., $\psi=\vartheta/\lambda_{max}(T)-\lambda_{max}(BB^TT)-2\varpi$, one gets ψ.

Theorem 2: The concerned leader-following FOMASs avoid Zeno phenomenon under the same conditions as Theorem 1, which means that the lower bound of minimum triggering time interval is a positive number.

Proof: Zeno behavior has a strict mathematical definition but can be described informally as the system making an infinite number of jumps in a finite amount of time. The adaptive event-triggered protocol is used to avoid Zeno-behavior problem, it is necessary to prove the lower bound of minimum triggering time interval is positive, i.e.,

$$\begin{aligned}t_{k+1}&=\inf\{t:t>t_k \text{ and } \xi_i(t)\ge 0\},\\ \xi_i(t)&=w_i(t)\varphi_i^T(t)K\varphi_i(t)-w_i(t)\Delta_i^T(t)K\Delta_i(t)\\ &\quad-\delta\exp(-\psi(t-t_0)).\end{aligned}$$

thus, the next time when the agents change its control variables will not update until triggering condition $\theta_i(t)=0$, and one has

$$\begin{aligned}&w_i(t_{k+1})\varphi_i^T(t_{k+1})K\varphi_i(t_{k+1})\\ &\quad=w_i(t_{k+1})\Delta_i^T(t_{k+1})K\Delta_i(t_{k+1})+\delta\exp(-\psi(t_{k+1}-t_0))\\ &\quad\le|w_i(t_{k+1})|\,\|K\|\,\|\varphi_i(t_{k+1})\|^2. \qquad (30)\end{aligned}$$

Since B is a real matrix and T is a SPDM, BB^T is positive semi-definite matrix and K is a positive semi-definite matrix. By using the second equality of (11) and Lemma 1, one gets

$$\begin{aligned}w_i(t)&=w_i(t_0)E_\alpha(-\frac{\vartheta}{2}(t-t_0)^\alpha)\\ &\quad+\alpha\int_{t_0}^{t}(t-\tau)^{\alpha-1}E_{\alpha,\alpha}(-\frac{\vartheta}{2}(t-t_0)^\alpha)\\ &\quad\times\Delta_i^T(t_k^i)K\Delta_i(t_k^i)d\tau. \qquad (31)\end{aligned}$$

Obviously, $w_i(t)$ is positive if $w_i(t_0)>0$. Then one derives that (32), as shown at the bottom of the page, where $d_1=\sqrt{\frac{\delta}{w_i(t_{k+1})\|K\|}}$. The fractional derivative of $\|\varphi\|$ over interval $[t_k^i,t_{k+1}^i)$ is derived as

$$\begin{aligned}&D_{t_k^+}^\alpha\|\varphi_i(t)\|\\ &\le\left\|D_{t_k}^\alpha\varphi_i(t)\right\|=\left\|D_{t_k}^\alpha\Delta_i(t)\right\|\\ &=\left\|D_{t_k}^\alpha(\sum_{j=1}^{N}a_{ij}(x_i(t)-x_j(t))+b_i(x_i(t)-x_0(t)))\right\|\\ &=\left\|\sum_{j=1}^{N}a_{ij}(A(e_i(t)-e_j(t))+B(u_i(t)-u_j(t))+g_i(t)\right.\\ &\qquad\left.+g_i(t)-g_j(t))+b_i(e_i(t)+Bu_i(t)+g_i(t))\right\|\\ &\le\|Q\otimes A\|\,\|e(t)\|+\|Q\otimes B\|\,\|u(t)\|+\varpi\,\|Q\|\,\|e(t)\|\\ &\le\|Q\otimes A\|\,\|e(t)\|+\|Q\otimes B\|\,\|W(t)Q\otimes J\|\,\|e(t_k)\|\\ &\qquad+\varpi\,\|Q\|\,\|e(t)\|\\ &\le(\|Q\otimes A\|+\varpi\,\|Q\|)\,\|e(t)\|\\ &\qquad+\|Q\otimes B\|\,\|W(t)Q\otimes J\|\,\|e(t_k)\|\\ &\le c_1\,\|e(t)\|+c_2\,\|e(t_k)\|. \qquad (33)\end{aligned}$$

Since $\exp(t)$ and $E_\alpha(t)$ are bounded on $[t_k,t_{k+1})$, that means, for $\forall t\in[t_k,t_{k+1})$, $\exists\sigma>0$

$$\sigma\exp(-\psi(t-t_0))\ge E_\alpha(-\mu(t-t_0)^\alpha), \qquad (34)$$

which implies that

$$\begin{aligned}\|e(t)\|&\le\sqrt{\frac{1}{\lambda_{min}(Q\otimes T)}(V(t_0)\sigma+\Theta)\exp(-\psi(t-t_0))}\\ &=\kappa\exp(-\frac{\psi}{2}(t-t_0)), \qquad (35)\end{aligned}$$

where $\kappa=\sqrt{\frac{1}{\lambda_{min}(Q\otimes T)}(V(t_0)\sigma+\Theta)}$.

$$\begin{aligned}\|\varphi_i(t_{k+1})\|&\ge\sqrt{\frac{1}{w_i(t_{k+1})\,\|K\|}(w_i(t_{k+1})\Delta_i^T(t_{k+1})K\Delta_i(t_{k+1})+\delta\exp(-\psi(t_{k+1}-t_0)))}\\ &\ge\sqrt{\frac{\delta}{w_i(t_{k+1})\,\|K\|}\exp(-\psi(t_{k+1}-t_0))}\\ &\ge d_1\exp(-\frac{\psi}{2}(t_{k+1}-t_0)), \qquad (32)\end{aligned}$$

Invoking (30), one gets

$$D_{t_k}^{\alpha}\|\varphi_i(t)\| \le c_1\|e(t)\| + c_2\|e(t_k)\| \le c_1\kappa\exp(-\frac{\psi}{2}(t-t_0)) + c_2\kappa\exp(-\frac{\psi}{2}(t_k-t_0)). \tag{36}$$

According to (1), one can get

$$D_t^{-\alpha}1 = \frac{1}{\Gamma(\alpha)}\int_{t_k}^{t}\frac{1}{(t-\tau)^{-\alpha+1}}d\tau = \frac{1}{\alpha\Gamma(\alpha)}(t-\tau)^{\alpha}|_{t_k}^{t} = \frac{1}{\alpha\Gamma(\alpha)}(t-t_k)^{\alpha} \tag{37}$$

where $-1 < -\alpha < 0$, which means the integral of constant 1.

From (36) and $\varphi_i(t_k^i) = 0$, we derive

$$\|\varphi_i(t)\| \le c_1\kappa(-\frac{2}{\psi})^{\alpha}(\exp(-\frac{\psi}{2}(t-t_0)) - \exp(-\frac{\psi}{2}(t_k-t_0)) + c_2\kappa\exp(-\frac{\psi}{2}(t_k-t_0))\frac{1}{\alpha\cdot\Gamma(\alpha)}(t-t_k)^{\alpha}. \tag{38}$$

Note that $d_1\exp(-\frac{\psi}{2}(t_{k+1}-t_0)) \le \|\varphi_i(t_{k+1})\|$, one gets

$$d_1\exp(-\frac{\psi}{2}(t_{k+1}-t_0)) \le c_1\kappa(-\frac{2}{\psi})^{\alpha}(\exp(-\frac{\psi}{2}(t_{k+1}-t_0)) - \exp(-\frac{\psi}{2}(t_k-t_0)) + c_2\kappa\exp(-\frac{\psi}{2}(t_k-t_0))\frac{1}{\alpha\cdot\Gamma(\alpha)}(t_{k+1}-t_k)^{\alpha}, \tag{39}$$

then, one gets

$$d_1\exp(-\frac{\psi}{2}(t_{k+1}-t_k)) \le d_2(\exp(-\frac{\psi}{2}(t_{k+1}-t_k)) - 1) + d_3(t_{k+1}-t_k)^{\alpha}, \tag{40}$$

where $d_2 = c_1\kappa(-\frac{2}{\psi})^{\alpha}$, $d_3 = \frac{c_2\kappa}{\alpha\Gamma(\alpha)}$.

Denoting $t^* = t_{k+1} - t_k$, $\varepsilon = \psi/2$, by (40), it yields

$$d_1\exp(-\varepsilon t^*) \le d_2(\exp(-\varepsilon t^*) - 1) + d_3 t^{*\alpha}. \tag{41}$$

By (41), one gets $t^* \neq 0$ for any triggering time and agents, which shows that the Zeno-behavior will not exhibit for the whole leader-following FOMASs. This completes the proof. □

IV. NUMERICAL EXAMPLES

A specific numerical example is presented to verify the proposed method in this section. Choose the MASs with a leader and five agents, where

$$x_i(t) = \left[x_i^1(t),\ x_i^2(t),\ x_i^3(t),\ x_i^4(t)\right]^T, \quad i = 0, 1, 2, 3, 4, 5.$$

Assume that the connection graph is defined as in Fig. 1. One gets the following relevant matrices,

$$\mathcal{A} = \begin{bmatrix} 0&0&0&0&1\\ 0&0&1&1&0\\ 0&1&0&0&0\\ 0&1&0&0&0\\ 1&0&0&0&0 \end{bmatrix}, \quad D = \begin{bmatrix} 1&0&0&0&0\\ 0&2&0&0&0\\ 0&0&1&0&0\\ 0&0&0&1&0\\ 0&0&0&0&1 \end{bmatrix},$$

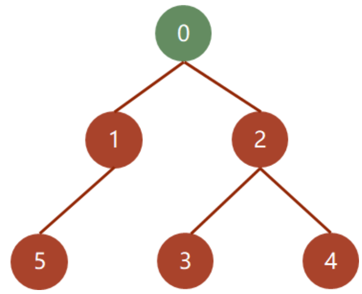

FIGURE 1. The undirected topology graph of leader-following FOMASs.

$$L = \begin{bmatrix} 1&0&0&0&-1\\ 0&2&-1&-1&0\\ 0&-1&1&0&0\\ 0&-1&0&1&0\\ -1&0&0&0&1 \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} 1&0&0&0&0\\ 0&1&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0 \end{bmatrix}.$$

Let

$$A = \begin{bmatrix} -0.5180 & -0.1858 & 0.3069 & -0.2435\\ 0.4372 & -0.4768 & 0.5276 & -0.1787\\ 0.4382 & -0.903 & -1.5114 & 0.4436\\ -0.3123 & -0.0476 & 0.1370 & -2.299 \end{bmatrix},$$

$$B = \begin{bmatrix} -0.1707\\ 0\\ 0.423\\ 0.325 \end{bmatrix},$$

and $\vartheta = 1.0714$, then one gets from Theorem 1 and (10)

$$T = \begin{bmatrix} 1.4403 & -0.0404 & 0.0821 & -0.0115\\ -0.0404 & 1.2354 & 0.1651 & 0.0117\\ 0.0821 & 0.1651 & 1.3216 & 0.0114\\ -0.0115 & 0.0117 & 0.0114 & 1.4079 \end{bmatrix},$$

$$J = \begin{bmatrix} -0.2177\\ 0.1671\\ 0.5603\\ 0.4652 \end{bmatrix}^T,$$

$$K = \begin{bmatrix} 0.0474 & -0.0364 & 0.1220 & -0.1013\\ -0.0364 & 0.0279 & 0.0936 & 0.0777\\ 0.1220 & 0.0936 & 0.3139 & 0.2606\\ -0.1013 & 0.0777 & 0.2606 & 0.2164 \end{bmatrix}.$$

the nonlinear part of FOMASs is $g(t, x_i(t)) = 0.1\sin(\frac{x_i(t)}{15})$.

Thus, one gets that $\varpi = 0.1$, $\lambda_{max}(T) = 1.4940 > 1$, and $\vartheta/\lambda_{max}(T) - \lambda_{max}(BB^T T) - 2\varpi = 0.0484$. Then we choose $\psi = 0.04$. Furthermore, it follows from Fig. 1 that the leader agent is the root node of spanning tree in connection graph. Therefore, the leader-following consensus will be reached for $\delta > 0$ based on Theorem 1. According to Theorem 2, the Zeno-behavior of the related leader-following FOMASs will be avoided.

Note that the above design parameters J, K, ϑ, ψ, δ are not the unique solutions to achieve the consensus goal. The different parameter combination can obtain different control performance. The single parameter has not positive correlation with the performance.

Figs. 2-4 show the tracking dynamic process, control variable and event-triggered time instants, respectively, which mean that MASs can achieve consensus while the Zeno-behavior does not exhibit.

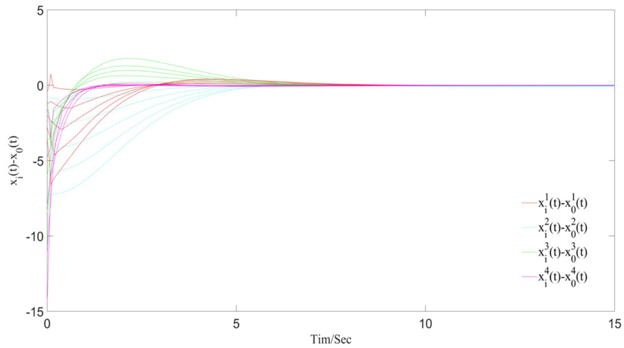

FIGURE 2. Tracking error between the leader and other agents.

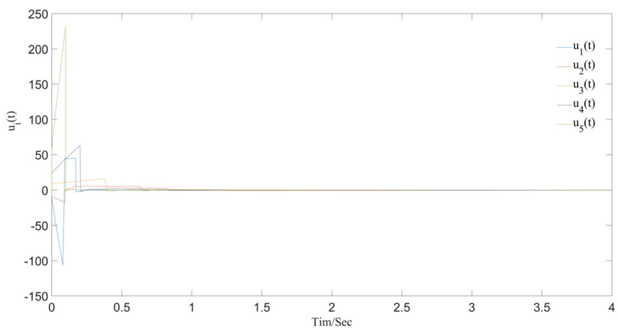

FIGURE 3. Controller variables update of followers.

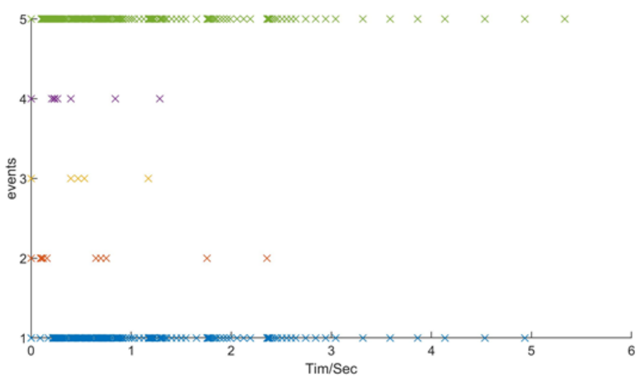

FIGURE 4. Event-trigger timing for each agent.

Let the initial conditions be $x_0(t) = 0.1 \times [1,\ 2,\ 3,\ 4]^T$, $x_i(t_0) = (0.5 - 0.8 \times i) \times [1,\ 2,\ 3,\ 4]^T$, $w_i(t_0) = i$, $\delta = 0.0134$, $i = 1,\ 2,\ 3,\ 4,\ 5$.

V. CONCLUSION

In this article, the adaptive event-triggered consensus of multi-agent systems described by fractional calculus with nonlinearity is investigated. At the latest triggering timing for each agent, adaptive protocols only use the neighboring state variable. By choosing the appropriate Lyapunov functions, the above proofs have revealed the obtained control protocol are effective to reach coordination. Simulations have been carried out to show the practicability of the conclusions. Moreover, the Zero-behavior is excluded. Note that the consensus scheme in this paper is just under ideal conditions, and some practical effects such as unknown false data-injection and replay cyber-attacks [41] are very worth exploring. More in-depth research will be developed in our future work.

ACKNOWLEDGMENT

The authors would like to thank the editor and the anonymous reviewers for the insightful comments and suggestions.

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