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Robust H∞ NLOS-Tolerant Localization Filter and NLOS-Tolerant Remote Reference Tracking Control of Mobile Robot in Wireless Sensor Networks

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ABSTRACT The need for accurate localization and robust reference tracking control of mobile robot using wireless sensor networks (WSNs) under the non-line-of-sight (NLOS) situations have been widely needed in diverse areas of industry. In order to overcome NLOS situations, a smoothing signal model is employed to embed bias signals due to NLOS in the dynamic system of the mobile robot to avoid the effect of NLOS on the positioning and remote control of the mobile robot using WSNs. In the study, a robust H∞ fuzzy localization filter is developed to efficiently estimate the mobile robot pose (position and orientation) and bias signals due to NLOS using the measurement of WSNs under NLOS situation, external disturbance and measurement noise. Further, a robust H∞ fuzzy NLOS-tolerant localization filter-based remote control design is also proposed for mobile robot system to track the desired trajectory for some purpose in the cluttered and noisy indoor environment in WSN. The robust H∞ NLOS-tolerant fuzzy localization filter-based tracking control design problems of mobile robot in WSN can be transformed to a corresponding linear matrix inequalities (LMIs)-constrained optimization problem, which could be easily solved with the LMI toolbox in Matlab. Finally, a simulation design example is provided to illustrate the design procedure and confirm the performance of the proposed methods for the localization estimation and reference tracking control of the mobile robot in WSN in comparison with the other method in an intelligent building.

INDEX TERMS Wireless sensor network (WSN), mobile robot, smoothing signal model, non-line-of-sight (NLOS), NLOS-tolerant localization filter, remote control of mobile robot, H∞ fuzzy estimator-based tracking control.

I. INTRODUTCTION

Indoor localization systems [1]–[7] for mobile robot localization and reference tracking control are the problems of estimating the mobile robot's pose (location and orientation) and then controlling the mobile robot at remote site [3] to track a desired reference pose for a variety of purposes in museums, factories or intelligent buildings. The conventional indoor mobile robot localization systems have typically taken advantage of state estimators for accurate localization in cluttered and noisy indoor environments [1]–[4]. Particularly, in order to overcome the impairment introduced by non-line-of-sight (NLOS) situations, more effort on the accurate state estimation of the mobile robot is necessary because NLOS-contaminated measures are less informative at the position calculation if no prior information is available [6], [8], [9].

Since the stochastic filter for the localization of the mobile robot is an inherent nonlinear state estimator of the nonlinear stochastic mobile robot system, nonlinear filters such as extended kalman filter and the particle filter (PF) have been employed for localization in wireless sensor networks (WSNs) [5]–[7]. Even the algorithms of the particle filters are more transparent and simple than the extended kalman filters, particle filters still fail in state estimation due to the loss of diversity among the samples [13]–[15]. Therefore, several

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improved algorithms, such as mixture Monte Carlo localization (MCL) [12], Markov Chain Monte Carlo (MCMC) inference [16], regularized particle filter (RPF) [17], combined PF/KF [18] have been proposed by means of mitigating the loss of sample diversity to overcome the drawback of sample impoverishment problem. At present, there have been a large number of preventative methods against sample impoverishment and PF failure. However, we still need to propose an effective and general remedy to cure a completely diverging or failed PF.

In some indoor localization systems, the statistical characteristics of external disturbances and measurement noises are rarely known. In the last decades, H∞ robust filter has been used to address the issue of unavailable external disturbances and measurement noises, which their worst-case effects on the filtering error to be efficiently attenuated. Therefore, the H∞ robust filter has been widely used to estimate the signal in the wireless communication systems [8], [9].

Recently, real-time indoor location systems (RTLs) of the mobile robot using WSN have attracted users and designers in the industry [7]. However, when using a PF to RTLs to improve the accuracy of real-time localization, the computation time of PF becomes an issue. If a small number of particles is used by PF to save computation time, it will accelerate sample impoverishment. To improve the reliability of particle filter-based localization in WSNs, a hybrid particle/FIR filter was proposed to recover the RPF from failure [7]. However, it is still not easy to overcome the impairment introduced by the NLOS situation.

In this study, a T-S fuzzy dynamic system in [24], [25] is proposed to interpolate a set of local linear state dynamic and output measurement systems to approximate the nonlinear state dynamic and nonlinear output measurement system of mobile robot in the wireless sensor network. Further, a smoothing signal model is employed to describe the bias signal of NLOS and is embedded in the state vector of the mobile robot system to avoid the impairment due to the NLOS situation in the localization process of the mobile robot in WSNs. Then a robust H∞ fuzzy localization filter (estimator) is proposed to precisely estimate both the mobile robot pose (position and orientation) and the NLOS signal under the corruption of external disturbance and measurement noise. We need to solve a set of linear matrix inequalities (LMIs) with the help of the LMI toolbox in Matlab for the design of a robust H∞ fuzzy NLOS-tolerant localization filter of the mobile robot in WSNs under NLOS situations.

In general, a mobile robot is employed for a variety of purposes in museums, factories, etc. Recently, how to control these mobile robots to track some desired trajectories by remote controllers through the wireless network has become an important topic of networked control in future smart cities [3], [19], [20]. Based on the estimated pose of a mobile robot by the robust H∞ fuzzy NLOS-tolerant localization filters, it is appealing that a robust H∞ fuzzy NLOS-tolerant estimator-based control design can be also proposed for the mobile robot in WSN to track a desired trajectory through a remote networked controller via the wireless channel for more practical applications to intelligent building in future smart cities. In the indoor cluttered and noisy environment in WSN, indoor wireless transmission of the control signal from remote localization filter-based controller to the actuators of the mobile robot is easy to lead to multiple paths and NLOS. It is noted that when the signal transmits via the wireless channel, the signal will be interfered indoor to induce with some physical phenomena such as reflection, refraction or shadowing which will result in multipath. In order to model multiple paths, a dynamic event-triggered system with a type-2 fuzzy variable at receiver in [33] and a randomly varying local nonlinear model in [32] are carried out. However, in this study, multiple paths are regarded as random interference and modeled as external random disturbances. Thus, the impact of multiple paths can be merged in random external disturbances. The NLOS is considered as the control bias signal on the actuator and can be also described by a smoothing signal model, which can be also embedded in the system model of mobile robot. Therefore, a robust H∞ fuzzy NLOS-tolerant localization filter (estimator) -based control design is proposed for the mobile robot in WSNs to track the desired trajectory for a variety of purposes in future smart cities. At the same time, the pose of the mobile robot can be estimated by localization filter to check whether the tracking control purpose is achieved despite external disturbance, NLOS and measurement noise in the intelligent buildings, museums or factories. We need to solve a set of filter and control gain-coupled LMIs by a two-step design procedure for the fuzzy localization filter gains and control gains of robust H∞ fuzzy estimator-based remote controller of mobile robot in WSNs under NLOS, external disturbance and measurement noise.

The contributions of this paper are described as follows:

(I) A smoothing signal model is employed to model the bias signal of NLOS and is embedded in the state vector of the mobile robot to avoid the corruption of NLOS in the positing process of the mobile robot in WSNs. Therefore, the impairment induced by the NLOS situation in the WSN-based localization system of the mobile robot can be overcome by the proposed robust H∞ NLOS-tolerant fuzzy localization filter design.

(II) For more practical applications of WSNs, a robust remote H∞ NLOS-tolerant fuzzy estimator-based reference tracking control design is also proposed for guiding the mobile robot to robustly track the desired trajectory for some tasks in WSNs so that designers and users not only can take advantage of robust state estimators for accurate localization but also can achieve an accurate desired trajectory tracking of mobile robot in WSNs for a variety of purposes inside intelligent buildings, museums or factories in the cluttered and noisy indoor environment, especially in NLOS situations.

(III) The robust H∞ NLOS-tolerant fuzzy estimator-based reference tracking control design problem is transformed to a set of control and estimator gains-coupled bilinear matrix inequalities (BMIs). A two-step design procedure is also

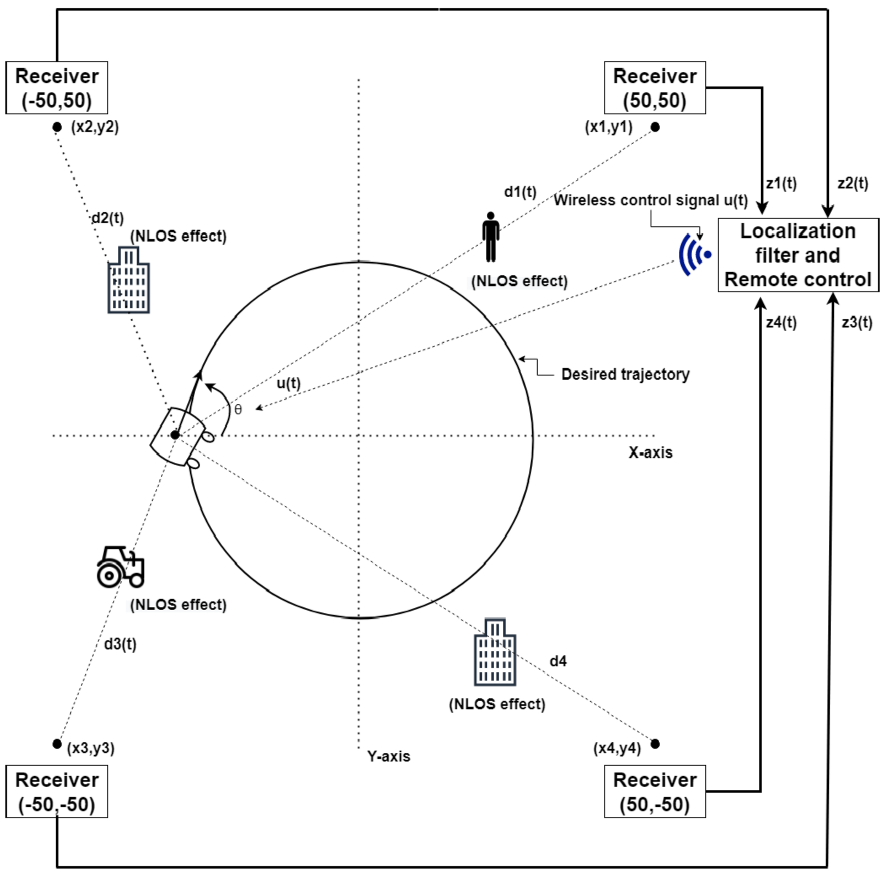

FIGURE 1. **2-D schematic diagram of mobile robot in the WSN-based indoor localization system. In the WSN-based indoor localization system, the four receivers can receive the wireless signal from the tag attached to the mobile robot. A localization filter is employed to estimate the pose (position and orientation) of mobile robot under external disturbance, measurement noise and NLOS situation. The remote controller can transmit control signal *u*(*t*) to the actuator of mobile robot for reference tracking control through wireless channel.**

proposed for efficiently solving these fuzzy filter gains and control gains by a corresponding LMIs-constrained optimization method to achieve the optimal H∞ NLOS-tolerant fuzzy estimator-based reference tracking control design of the mobile robot in WSNs. This will significantly simplify the design procedure of robust H∞ NLOS-tolerant fuzzy estimator-based reference tracking control of the mobile robot through WSN, which will be useful for practical applications through the localization facilities of WSN technology inside an intelligent building in future smart cities.

The remainder of the paper is organized as follows: The indoor localization system of the mobile robot using WSN under NLOS situations is described in Section II. A smoothing signal model is also introduced to describe bias signal of NLOS to be embedded in the localization system of the mobile robot. In Section III, an H∞ NLOS-tolerant fuzzy localization filter is designed for the mobile robot in the wireless sensor network. In Section IV, a robust H∞ NLOS-tolerant fuzzy remote reference tracking control of the mobile robot in the wireless sensor network is designed by using a Luenberger-type observer-based controller. In Section V, a simulation example of the localization filter-based reference tracking control of the mobile robot using the wireless sensor network inside an intelligent building is given to validate the efficiency of position and tracking by the proposed method. The conclusion is summarized in Section VI.

Notation: A^T: the transpose of matrix A; $A \geq 0 (A > 0)$: symmetric positive semi-definite (definite) matrix; I_n: the n by n identity matrix; $l_2[0, t_T] = \{v(t) : \mathbb{R}^T \rightarrow \mathbb{R}^n \parallel (\Sigma_{t=0}^{t_T} v^T(t)v(t) < \infty)\}$; $eig(A)$ denotes the set of eigenvalues of A; $E(\cdot)$ denotes the expectation of $\cdot$.

II. INDOOR LOCALIZATION SYSTEM USING WSN UNDER NLOS SITUATIONS

In this section, an indoor localization system using a WSN is introduced for positioning a mobile robot inside a building like a factory or a museum. As shown in Fig.1, the indoor localization system is connected by four receivers, a wireless tag with a transmitter and a server computer. In the 2-D schematic diagram of the WSN-based indoor localization system in Fig.1 [7], a wireless tag attached to a mobile robot could transmit wireless signals to four receivers installed at fixed positions with exactly known coordinates. In the WSN-based indoor localization system, the four receivers can receive the wireless signal from the tag attached to the mobile robot. We suppose the receivers' clocks are synchronized via the line of clock synchronization. The time-of-arrival (TOA) is measured by receivers of the WSN-based indoor localization system to give the traveling time of wireless signals from the transmitter to each of the receivers. Then the TOA measurements are sent to a server computer of mobile robot localization system to calculate the time-different-of-arrival (TDOA) as follows [7]:

$$\begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \end{bmatrix} = \frac{1}{c}\begin{bmatrix} d_1(t)-d_2(t) \\ d_1(t)-d_3(t) \\ d_1(t)-d_4(t) \end{bmatrix} \tag{1}$$

where $z_1(t)$, $z_2(t)$ and $z_3(t)$ denote the TDOA measurement (input parameters which calculate the relative position of the mobile robot by examining the difference in time from three or more receivers) at the discrete-time t (in unit of nanosecond); c is the speed of light; $d_i(t)$, $i = 1, 2, 3$ are the distances between the mobile robot at time t and the receivers as shown in Fig.1 and are defined as follows

$$d_i(t) = \sqrt{(x(t)-x_i)^2 + (y(t)-y_i)^2} \tag{2}$$

where $(x(t), y(t))$ is the coordinate of the mobile robot localization at time t and $(x_i, y_i,\ i = 1, 2, 3, 4)$ are the fixed coordinates of receivers.

The mobile robot pose could be described by the state vector $X(t) = [x(t), y(t), \theta(t)]^T$, where $x(t)$ and $y(t)$ are the coordinate on a 2-D plane in the WSN-based indoor localization system and $\theta(t)$ is the heading angle of mobile robot (i.e., $\theta(t) = \tan^{-1}(\frac{v_y(t)}{v_x(t)})$) as shown in Fig.1 [7]. The discrete-time motion of mobile robot with velocity $v(t)$, acceleration $a(t)$, angular velocity $w(t)$ and angular acceleration $\alpha(t)$ is controlled for a variety of purposes through control command $u(t) = [\Delta d(t)\ \Delta\theta(t)]^T$, where $\Delta d(t) = v(t) \times \Delta t$ is the incremental distance (in meters) in sampling time Δt and $\Delta\theta(t) = w(t) \times \Delta t$ is the incremental heading angle (in degrees) in sampling time Δt. Therefore, the robot motion in WSN-based indoor localization system can be described as the following discrete-time dynamical model with external disturbance [7]

$$\begin{aligned} x(t+1) &= x(t) + \Delta d(t) \times \cos\theta(t) + v_1(t) \\ y(t+1) &= y(t) + \Delta d(t) \times \sin\theta(t) + v_2(t) \\ \theta(t+1) &= \theta(t) + \Delta\theta(t) + v_3(t) \end{aligned} \tag{3}$$

where $v_1(t) = (\Delta t)^2 \times \alpha(t) \times \cos\theta(t)$, $v_2(t) = (\Delta t)^2 \times \alpha(t) \times \sin\theta(t)$, and $v_3(t) = (\Delta t)^2 \times \alpha(t)$ are incremental velocity due to the acceleration $\alpha(t)$ of mobile robot in the x-axis, y-axis and heading angle, respectively, which are always unavailable, to be considered as an equivalent external disturbance in WSN-based localization system. In general, the indoor localization accuracy can be improved by equipping the mobile robot with a fiber obit gyroscope (FOG) to directly measure the heading $\theta(t)$ [7]. Thus, after 3 TDOA measurements in (1), the fourth measurement is adopted as follows:

$$z_4(t) = \theta(t) \tag{4}$$

Therefore, combining 3 TDOA measurements in (1) with the heading angle in (4), the measurement vector of the WSN-based indoor localization system is contracted as $Z(t) = [z_1(t)\ z_2(t)\ z_3(t)\ z_4(t)]^T$. Then the state dynamic equation and output measurement equation of WSN-based indoor localization system of mobile robot in the cluttered and noisy indoor environment is given as [7]

$$\begin{aligned} x(t+1) &= x(t) + \Delta d(t) \times \cos\theta(t) + v_1(t) \\ y(t+1) &= y(t) + \Delta d(t) \times \sin\theta(t) + v_2(t) \\ \theta(t+1) &= \theta(t) + \Delta\theta(t) + v_3(t) \\ z_1(t) &= \frac{1}{c}\Big(\sqrt{(x(t)-x_1)^2 + (y(t)-y_1)^2} \\ &\quad -\sqrt{(x(t)-x_2)^2 + (y(t)-y_2)^2}\Big) + n_1(t) + s_1(t) \\ z_2(t) &= \frac{1}{c}\Big(\sqrt{(x(t)-x_1)^2 + (y(t)-y_1)^2} \\ &\quad -\sqrt{(x(t)-x_3)^2 + (y(t)-y_3)^2}\Big) + n_2(t) + s_2(t) \\ z_3(t) &= \frac{1}{c}\Big(\sqrt{(x(t)-x_1)^2 + (y(t)-y_1)^2} \\ &\quad -\sqrt{(x(t)-x_4)^2 + (y(t)-y_4)^2}\Big) + n_3(t) + s_3(t) \\ z_4(t) &= \theta(t) + n_4(t) \end{aligned} \tag{5}$$

where $n_1(t)$, $n_2(t)$, $n_3(t)$ and $n_4(t)$ denote the measurement noises in the output measurements $z_1(t)$, $z_2(t)$, $z_3(t)$ and $z_4(t)$, respectively. $s_1(t)$, $s_2(t)$ and $s_3(t)$ represent the sensor bias signals due to the NLOS of 4 sensors of the indoor localization system. Now, let us define the state vector $X(t)$, control input $u(t)$ and output measurement $Z(t)$ as follows:

$$X(t) = \begin{bmatrix} x(t) \\ y(t) \\ \theta(t) \end{bmatrix}, \quad Z(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \\ z_4(t) \end{bmatrix}$$

$$u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} \Delta d(t) \\ \Delta\theta(t) \end{bmatrix}$$

Then the state dynamic equation and output measure equation of WSN-based indoor localization system of the mobile robot in (5) can be represented by the following nonlinear dynamic state space system

$$\begin{aligned} X(t+1) &= X(t) + B(X(t))u(t) + v(t) \\ Z(t) &= C(X(t)) + n(t) + Ds(t) \end{aligned} \tag{6}$$

where

$$B(x(t)) = \begin{bmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$v(t)=\begin{bmatrix} v_1(t)\\ v_2(t)\\ v_3(t)\end{bmatrix},\quad s(t)=\begin{bmatrix} s_1(t)\\ s_2(t)\\ s_3(t)\end{bmatrix},\quad n(t)=\begin{bmatrix} n_1(t)\\ n_2(t)\\ n_3(t)\\ n_4(t)\end{bmatrix}$$

$$C(X(t))=\begin{bmatrix} \frac{1}{c}(\sqrt{(x(t)-x_1)^2+(y(t)-y_1)^2} \\ -\sqrt{(x(t)-x_2)^2+(y(t)-y_2)^2}) \\ \frac{1}{c}(\sqrt{(x(t)-x_1)^2+(y(t)-y_1)^2} \\ -\sqrt{(x(t)-x_3)^2+(y(t)-y_3)^2}) \\ \frac{1}{c}(\sqrt{(x(t)-x_1)^2+(y(t)-y_1)^2} \\ -\sqrt{(x(t)-x_4)^2+(y(t)-y_4)^2}) \\ \theta(t) \end{bmatrix}$$

In this study, we will propose a NLOS-tolerant fuzzy localization filter design based on TDOA and the heading angle (i.e. the measurement of $Z(t)$ in (6)) to precisely estimate the state $X(t)$ (including position and heading angle) of mobile robot in WSN under external disturbance, measurement noise and NLOS situation in indoor cluttered and noisy environment. After a precise state estimation is achieved, a robust NLOS-tolerant remote fuzzy control $u(t)$ is also developed at a remote site for the mobile robot via the wireless channel to track the desired reference pose (i.e, position and heading angle) for some purpose. However, it is well known that when the NLOS condition presents, it will introduce a bias signal $s(t)$ to induce the greatest impairment to the state estimation of mobile robot in (6) from output measurement $Z(t)$. Unlike the conventional methods [1]–[10] to mitigate the NLOS impact on positioning, this study will introduce a smoothing model for NLOS and then embed it in the system state model in (6). In this situation, the impact of NLOS on the positioning can be avoided to achieve precise localization of the mobile robot in the wireless sensor network. In general, the traditional filter can not estimate the sensor bias signal $s(t)$ directly. In order to estimate $X(t)$ and $s(t)$ in the WSN-based localization system of the mobile robot in (6) via wireless sensor network, the following smoothing signal model of sensor bias signal $s(t)$ due to NLOS is proposed as follows:

$$\begin{bmatrix} s(t+1)\\ s(t)\\ \vdots \\ s(t-d+1)\end{bmatrix}=\begin{bmatrix} a_0I_3 & \cdots & \cdots & \cdots & a_dI_3\\ I_3 & 0_{3\times3} & & & \vdots\\ 0_{3\times3} & I_3 & \ddots & & \vdots\\ \vdots & & \ddots & \ddots & \vdots\\ 0_{3\times3} & \cdots & 0_{3\times3} & I_3 & 0_{3\times3}\end{bmatrix} \times\begin{bmatrix} s(t)\\ s(t-1)\\ \vdots\\ \vdots\\ s(t-d)\end{bmatrix}+\begin{bmatrix} I_3\\ 0_{3\times3}\\ \vdots\\ \vdots\\ 0_{3\times3}\end{bmatrix} \times(s(t+1)-\sum_{i=0}^{d} a_is(t-i)) \tag{7}$$

which can be represent by

$$S(t+1)=A_sS(t)+M_s\tilde{s}(t+1) \tag{8}$$

where

$$S(t)=\begin{bmatrix} s(t)\\ s(t-1)\\ \vdots\\ s(t-d)\end{bmatrix},\quad M_s=\begin{bmatrix} I_3\\ 0_{3\times3}\\ \vdots\\ 0_{3\times3}\end{bmatrix}$$

$$A_s=\begin{bmatrix} a_0I_3 & \cdots & \cdots & \cdots & a_dI_3\\ I_3 & 0_{3\times3} & & & \vdots\\ 0_{3\times3} & I_3 & \ddots & & \vdots\\ \vdots & & \ddots & \ddots & \vdots\\ 0_{3\times3} & \cdots & 0_{3\times3} & I_3 & 0_{3\times3}\end{bmatrix}$$

and $\tilde{s}(t+1) = s(t+1) - \sum_{i=0}^{d} a_is(t-i)$ denotes the extrapolation error of $s(t+1)$, where $\hat{s}(t+1)=\sum_{i=0}^{d} a_is(t-i)$ is the extrapolation from $s(t-d),\ldots,s(t)$ to $s(t+1)$ with the extrapolation coefficients$\{a_i\}_{i=0}^{d}$. Some notations about the smoothing model in (8) are given as follows:

(I) Since the bias signal $s(t+1)$ is more related to $s(t)$, the extrapolation coefficients should be satisfied with $a_i \geqslant a_{i-1} \geqslant 0$ for $i=1,\ldots,d$. Furthermore, in order to avoid over-extrapolation, the condition $\sum_{i=0}^{d} a_i = 1$ should be satisfied.

(II) The smoothing signal model in (7) or (8) can be regarded as an improved kalman fixed-lag smoothing model for estimating the sensor bias signal $s(t)$ and its delays $s(t-1),\ldots,s(t-d)$. However, in the traditional kalman fixed-lag smoothing model [21], $a_0 = a_1 = \ldots = a_d = 0$ and $\tilde{s}(t+1)=s(t+1)$ in (8), which will significantly deteriorate the estimation performance. In the proposed smoothing signal model in (7) and (8), we have considered the extrapolation of $s(t+1)=\sum_{i=0}^{d} a_is(t-i)+\tilde{s}(t+1)$ to improve the observability of signal model and hence the estimation accuracy of sensor bias signal.

III. H∞ NLOS-TOLERANT FUZZY LOCALIZATION FILTER DESIGN FOR MOBILE ROBOT IN WIRELESS SENSOR NETWORK SYSTEM

In this section, based on the nonlinear dynamic system of mobile robot in WSN in (6) and the smoothing signal model in (8), we want to estimate the state (pose) of the mobile robot by the measurement $Z(t)$ via wireless sensor network. However, the dynamic state equation and measurement equations in (6) are highly nonlinear. For the convenience of localization filter design, the nonlinear localization system of the mobile robot in WSN in (6) can be described by the T-S fuzzy system [24]- [26]. The ith rule of T-S model for nonlinear localization system of mobile robot in (6) is of the following form [26]

System Rule i :

If $\varepsilon_1(t)$ *is* F_{i1}, *and*, $\ldots$, $\varepsilon_g(t)$ *is* F_{ig}, $i=1,\ldots,J$

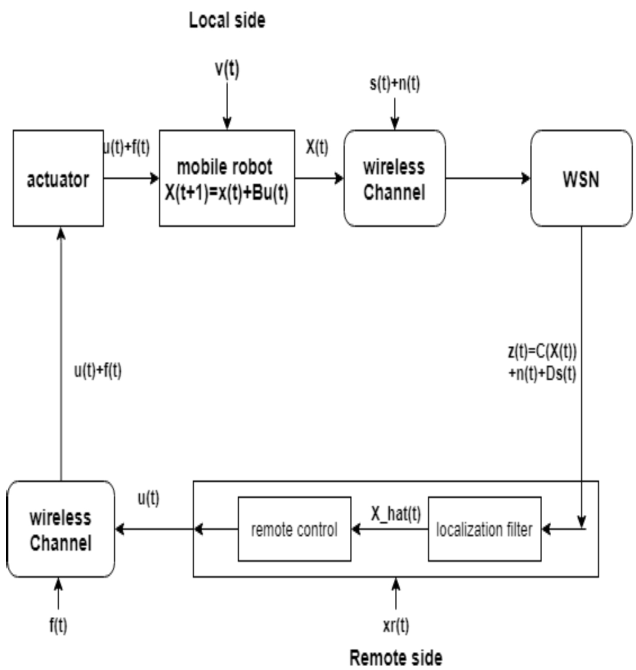

FIGURE 2. Block diagram of NLOS-tolerant localization and reference tracking control of indoor mobile robot in WSN.

Then

$$X(t+1) = X(t) + B_i u(t) + v(t)$$
$$Z(t) = C_i X(t) + n(t) + Ds(t) \tag{9}$$

where the matrices B_i and C_i are local system matrices with appropriate dimension, $\varepsilon(t) = [\varepsilon_1(t), \ldots, \varepsilon_g(t)]$ are the premise variables assumed to be measurable. $F_{i1}, \ldots, F_{ig}$ are fuzzy sets, g is the number of premise variables and J is the number of system rules. We assume (I_3, C_i) are observable for $i = 1, \ldots, J$. Then the T-S fuzzy system in (9) for mobile robot localization via wireless sensor network can be inferred as follows [26]:

$$X(t+1) = \sum_{i=1}^{J} \alpha_i(\varepsilon(t))[X(t) + B_i u(t) + v(t)]$$
$$Z(t) = \sum_{i=1}^{J} \alpha_i(\varepsilon(t))[C_i X(t) + n(t) + Ds(t)] \tag{10}$$

where $\varepsilon(t) = [\varepsilon_1(t), \ldots, \varepsilon_g(t)]$, $\alpha_i(\varepsilon(t)) = \Pi_{j=1}^{g} F_{ij}(\varepsilon(t))$, and $\alpha_i(\varepsilon_j(t)) = \dfrac{\alpha_i(\varepsilon(t))}{\sum_{i=1}^{J} \alpha_i(\varepsilon(t))}$, with $0 \leq \alpha_i(\varepsilon_j(t)) \leq 1$ and $\sum_{i=1}^{J} \alpha_i(\varepsilon(t)) = 1$.

The physical meaning of T-S fuzzy WSN-based indoor localization system of mobile robot in (10) is that the nonlinear WSN-based indoor localization system of the mobile robot in (6) can be represented by the interpolation of J linearized systems through J fuzzy smoothing functions $\alpha_i(\varepsilon(t))$, $i = 1, \ldots, J$.

For the convenience of estimating the state vector $X(t)$ and sensor bias signal $s(t)$ due to NLOS in wireless sensor network simultaneously, the smoothing signal model (8) of $S(t)$ is embedded as an internal model of T-S fuzzy mobile robot localization system in (10) as the following fuzzy augmented localization system

$$\bar{X}(t+1) = \sum_{i=1}^{J} \alpha_i(\varepsilon(t))[\bar{A}\bar{X}(t) + \bar{B}_i u(t) + \bar{H}\ \bar{v}(t)]$$
$$Z(t) = \sum_{i=1}^{J} \alpha_i(\varepsilon(t))[\bar{C}_i \bar{X}(t) + n(t)] \tag{11}$$

where

$$\bar{X}(t) = \begin{bmatrix} S(t) \\ X(t) \end{bmatrix}, \ \bar{v}(t) = \begin{bmatrix} \tilde{s}(t+1) \\ v(t) \end{bmatrix}, \ \bar{H} = \begin{bmatrix} M_s & 0 \\ 0 & I_3 \end{bmatrix}$$
$$\bar{A} = \begin{bmatrix} A_s & 0 \\ 0 & I_3 \end{bmatrix}, \ \bar{B}_i = \begin{bmatrix} 0 \\ B_i \end{bmatrix}, \ \bar{C}_i = \begin{bmatrix} DM_s^T & C_i \end{bmatrix}$$

Since the bias signal $S(t)$ due to NLOS is embedded in $\bar{X}(t)$, when estimating $\bar{X}(t)$ by localization filter, we can not only accurately estimate both mobile robot pose $X(t)$ and the bias signal $S(t)$ due to NLOS but also avoid the corruption of bias signal $S(t)$ due to NLOS in the estimation process. After the sensor bias signal of NLOS $S(t)$ is embedded in the augmented T-S fuzzy mobile robot localization system in (11), a T-S fuzzy estimator will be designed to estimate $\bar{X}(t)$ for estimating mobile robot state $X(t)$ and sensor bias signal $S(t)$ from the measurement $Z(t)$ via wireless sensor network. Before the design of the localization filter (estimator), the observability of augmented T-S fuzzy mobile robot localization system in (11) is discussed as follows:

Lemma 1: For T-S fuzzy mobile robot localization system, if J local fuzzy systems in (9) or (10) are observable, i.e.

$$rank\begin{bmatrix} zI_3 - I_3 \\ C_i \end{bmatrix} = 3, \ \forall z \in eig(I_3), \quad i = 1, \ldots, J \quad (12)$$

and the following conditions hold

$$\begin{aligned} &eig(A_s) \cap eig(I_3) = \phi \\ &rank\begin{bmatrix} zI_{3(d+1)} - A_s \\ DM_s^T \end{bmatrix} = 3(d+1), \ \forall z \in eig(A_s) \end{aligned} \quad (13)$$

then T-S fuzzy augmented mobile robot localization system in (11) is observable

Proof: see Appendix A □

The rank conditions in (12) and (13) imply the sensor bias signal $s(t)$ in (10) is observable from the output measurement $Z(t)$ in wireless sensor network. In the augmented T-S fuzzy WSN-based localization system of the mobile robot in (11), the sensor bias signal $s(t)$ is embedded in the augmented state $\bar{X}(t)$ and therefore its corruption to state $X(t)$ is avoided. i.e. the effect of bias signal $s(t)$ of NLOS on the state estimate can be avoided in the mobile robot localization process in WSN. After the observability of T-S fuzzy augmented WSN-based localization system of mobile robot in (11) is guaranteed by Lemma 1, the following fuzzy estimator (filter) is employed for state (pose) estimation of mobile robot in wireless sensor network:

$$\begin{aligned} &\textit{Localization filter (estimator) Rule } i: \\ &\textit{If } \varepsilon_i(t) \textit{ is } F_{i1}, \textit{and}, \ldots, \ \varepsilon_g(t) \textit{ is } F_{ig} \\ &\textit{Then} \\ &\hat{\bar{X}}(t+1) = \bar{A}\,\hat{\bar{X}}(t) + \bar{B}_i u(t) + \bar{L}_i(Z(t) - \hat{Z}(t)) \\ &\hat{Z}(t) = \bar{C}_i \hat{\bar{X}}(t) \end{aligned} \quad (14)$$

where $\hat{\bar{X}}(t)$ is the estimation of $\bar{X}(t)$ in (11) and $\{\bar{L}_i\}_{i=1}^J$ are the localization filter gains to be designed. Then the fuzzy Luenberger-type localization filter in (14) to estimate $\bar{X}(t)$ from output measurement in wireless sensor network in (11) is inferred as follows [25], [26]:

$$\begin{aligned} &\hat{\bar{X}}(t+1) \\ &\quad = \sum_{i=1}^J \alpha_i(\varepsilon(t)) \\ &\quad\quad \times \sum_{j=1}^J \alpha_j(\varepsilon(t))(\bar{A}\,\hat{\bar{X}}(t) + \bar{B}_i u(t) + \bar{L}_i(Z(t) - \bar{C}_j \hat{\bar{X}}(t))) \end{aligned} \quad (15)$$

In the conventional localization filters [1]- [10] without using the smoothing signal model of sensor bias signal in (8), it is not easy to estimate sensor bias signal of NLOS in WSN. Therefore, the sensor bias signal will deteriorate the state (pose) estimation of the mobile robot. The proposed smoothing signal model of sensor bias signal $S(t)$ due to NLOS is embedded in the augmented state in (11) to avoid its influence on the estimation of $X(t)$, i.e. $\hat{\bar{X}}(t)$ by the fuzzy localization filter in (15) is NLOS-tolerant localization. In this way, we could exactly reconstruct the pose $X(t)$ and bias signal $S(t)$ by the conventional T-S fuzzy Luenberger-type localization filter in (15) easily and efficiently for NLOS-tolerant control of mobile robot via wireless sensor network in the following section.

In the T-S fuzzy augmented system in (11), since $\bar{v}(t)$ and $n(t)$ are still unavailable, their effect on the estimation of state and sensor bias signal (i.e., $\bar{X}(t)$) by T-S fuzzy localization filter in (15) should be considered from the worst-case perspective, i.e., for all possible finite-energy $\bar{v}(t)$ and $n(t)$. Therefore, the following robust H∞ estimation strategy is employed for the fuzzy localization filter in (15), i.e. to specify the fuzzy filter gains $\{\bar{L}_i\}_{i=1}^J$ in (15) to achieve the following robust H∞ NLOS-tolerant estimation performance for some prescribed ρ.

$$\frac{E\sum_{t=0}^{t_T}(\bar{X}(t) - \hat{\bar{X}}(t))^T Q(\bar{X}(t) - \hat{\bar{X}}(t))}{E\sum_{t=0}^{t_T}\left[\bar{v}^T(t)\ n^T(t)\right] \times \begin{bmatrix} \bar{v}(t) \\ n(t) \end{bmatrix}} \le \rho\ , \quad \forall \bar{v}(t),\ n(t) \in l_2[0, t_T] \quad (16)$$

where $\rho > 0$ denotes the attenuation level of $\bar{v}(t)$ and $n(t)$, t_T denotes the terminal time, Q is the weighting matrix on estimation error and $E(\cdot)$ denotes the expectation of $\cdot$.

The physical meaning of H∞ NLOS-tolerant localization filtering strategy in (16) is that the effect of all possible finite-energy external disturbance $\bar{v}(t)$ and measurement noise $n(t)$ on the estimation error $\bar{X}(t) - \hat{\bar{X}}(t)$ must be less than or equal to a prescribed attenuation level ρ from the expected energy perspective. In general, ρ is selected to be less than 1 in order to efficiently attenuate the effect of $\bar{v}(t)$ and $n(t)$ on the estimation performance.

Let us denote the estimation error of the fuzzy localization filter as

$$\bar{e}(t) = \bar{X}(t) - \hat{\bar{X}}(t) \quad (17)$$

then from (11) and (15) we get the following estimation error equation

$$\begin{aligned} \bar{e}(t+1) = &\sum_{i=1}^J \alpha_i(\varepsilon(t)) \sum_{j=1}^J \alpha_j(\varepsilon(t))(\bar{A}_i - \bar{L}_i\bar{C}_j)\bar{e}(t) \\ &+ [\,\bar{H}\ \ -\bar{L}_i\,]\begin{bmatrix} \bar{v}(t) \\ n(t) \end{bmatrix} \end{aligned} \quad (18)$$

According to the estimation error equation in (18), the H∞ NLOS-tolerant estimation (localization) strategy in (16) can be represented by

$$\frac{E\sum_{t=0}^{t_T}\bar{e}^T(t) Q\,\bar{e}(t)}{E\sum_{t=0}^{t_T}\left[\bar{v}^T(t)\ n^T(t)\right]\begin{bmatrix} \bar{v}(t) \\ n(t) \end{bmatrix}} \le \rho, \quad \forall \begin{bmatrix} \bar{v}(t) \\ n(t) \end{bmatrix} \in l_2[0,\ t_T] \quad (19)$$

The H∞ NLOS-tolerant filtering strategy in (19) is assumed with the zero initial condition of the estimation error equation in (18),i.e., $\bar{e}(0) = 0$. If the initial condition $\bar{e}(0) \neq 0$, then the effect of the initial condition should be extracted from the H∞ NLOS-tolerant estimation strategy in (16). In this case, the robust H∞ NLOS-tolerant estimation strategy in (19) should be modified in the following [23][27]

$$\frac{E\sum_{t=0}^{t_T}\bar{e}^T(t)Q\,\bar{e}(t)-\bar{e}^T(0)P\,\bar{e}(0)}{E\sum_{t=0}^{t_T}\left[\bar{v}^T(t)\ n^T(t)\right]\begin{bmatrix}\bar{v}(t)\\ n(t)\end{bmatrix}} \leq \rho,$$
$$\forall \begin{bmatrix}\bar{v}(t)\\ n(t)\end{bmatrix} \in l_2[0,\ t_T] \tag{20}$$

for some positive symmetric definite matrix $P = P^T > 0$.

Before we continue the further analysis and design of the robust H∞ NLOS-tolerant localization filter for mobile robot in wireless sensor network, the following Schur complement is necessary.

Lemma 2: (Schur complement [29]): For a symmetric matrix A, a matrix B and an invertible symmetric C, the following statements are equivalent

$$\begin{bmatrix} A & B^T \\ B & C \end{bmatrix} < 0 <=> C < 0 \ \ and\ A - B^T C^{-1} B < 0 \tag{21}$$

Then the H∞ fuzzy NLOS-tolerant localization filter in (15) for mobile robot in wireless sensor network is designed as follows:

Theorem 1: If we can specify a positive matrix $P = P^T > 0$ and the matrices $\{\bar{Y}_i\}_{i=1}^{J}$ such that the following LMIs hold

$$\begin{bmatrix} Q-P & 0 & 0 & \bar{A}_i^T P - \bar{C}_j^T \bar{Y}_i^T \\ 0 & -\rho I & 0 & \bar{H}^T P \\ 0 & 0 & -\rho I & -\bar{Y}_i \\ P\bar{A}_i - \bar{Y}_i\bar{C}_j & P\bar{H} & -\bar{Y}_i & -P \end{bmatrix} \leq 0,$$
$$i, j = 1, \ldots, J \tag{22}$$

then the fuzzy localization filter with $\{\bar{L}_i = P^{-1}\bar{Y}_i\}_{i=1}^{J}$ in (15) can achieve the robust H∞ NLOS-tolerant estimation performance in (19) or (20).

Proof: : see Appendix B. □

Remark 1: The computational complexity of the proposed H∞ NLOS-tolerant fuzzy localization filter design is proportional to $O(\xi^6 J)$, where $\xi = 2(3 + 3(d + 1))$ is the order of the Lyapunov function matrix P in (22) [22], J is the number of fuzzy local linearized models of mobile robot localization system in (11). The computational complexity of the proposed method is dependent on the number of fuzzy rules. Besides, the computational complexity of the particle filter design is proportional to $O(\zeta N^2)$ [38], where ζ is the dimension of the state vector and N is the number of particles. However, when the number of particles decreases, it will cause sample impoverishment and fail of achieving precise estimation. Nevertheless, adopting fewer fuzzy rules will reduce the computational time and reach a lower computational complexity of H∞ fuzzy localization filter but only with a moderate decay of estimation performance because these fuzzy approximation errors can be considered as one kind of external disturbance and be efficient attenuated by the proposed H∞ fuzzy localization filter.

According to Theorem 1, we could solve the LMIs in (22) for P and $\bar{Y}_i$ with a prescribed attenuation ρ for the H∞ NLOS-tolerant estimation strategy in (19) and then obtain the filter gains $\{\bar{L}_i = P^{-1}\bar{Y}_i\}_{i=1}^{J}$ for the fuzzy localization filter in (15). Furthermore, if we want to design the optimal H∞ NLOS-tolerant fuzzy localization filter with a minimum attenuation level ρ^* in (19) to optimally attenuate the effect of $\bar{v}(t)$ and $n(t)$ on the localization accuracy, then we need to solve the following LMI-constrained optimization problem

$$\rho^* = \min_{P>0,\bar{Y}_1\ldots\bar{Y}_J} \rho$$
$$\textit{subject to LMIs in } (22) \tag{23}$$

which could be solved by decreasing ρ until there exists no solution $P > 0$ of LMIs, in (22).

Remark 2: The LMIs-constrained optimization problem in (23) could be easily solved by decreasing ρ until no $P > 0$ exists in LMIs in (22). The solutions of $P > 0, \bar{Y}_1 \ldots \bar{Y}_J$ in LMIs in (22) can be obtained with help of the LMI toolbox in Matlab.

Based on the above analysis, the designs procedure of robust H∞ NLOS-tolerant fuzzy localization filter design of mobile robot in WSN is given as follows:

(I) Construct state dynamic equation and output measurement equation of WSN-based localization system of mobile robot with NLOS in WSN in (6).

(II) Construct a smoothing signal model in (8) for NLOS information $s(t)$.

(III) Construct fuzzy augmented WSN-based localization system in (11) and its fuzzy localization filter in (15) of mobile robot in WSN.

(IV) Solve $\rho^*$and $\bar{Y}_1^*, \ldots \bar{Y}_J^*$ from the LMIs-constrained optimization problem problem in (23) for the optimal H∞ fuzzy localization filter gains $\{\bar{L}_i^* = P^{*-1}\bar{Y}_i^*\}_{i=1}^{J}$ in (15) to estimate $\hat{\bar{X}}(t) = [\hat{S}^T(t)\ \ \hat{X}^T(t)]^T$.

IV. ROBUST H∞ NLOS-TOLERANT REMOTE FUZZY REFERENCE TRACKING CONTROL DESIGN OF MOBILE ROBOT IN WIRELESS SENSOR NETWORK

After the robust localization detection of mobile robot in WSN is finished by the proposed H∞ localization filter in the above section, we still need to design H∞ NLOS-tolerant fuzzy estimator-based remote control for mobile robot in WSN through wireless communication to track the desired trajectory $X_r(t)$ for some task as shown in Fig.1. When transmitting a control signal from the remote controller through the wireless channel to control actuator of indoor mobile robot, there also exists NLOS at the down-link sensor of mobile robot. In this situation, the nonlinear dynamic state space system of mobile robot in WSN in (6) should be modified as

$$X(t+1) = X(t) + B(X(t))(u(t) + f(t)) + v(t)$$
$$Z(t) = C(X(t)) + n(t) + Ds(t) \tag{24}$$

where $f(t) \in R^{2\times1}$denotes actuator bias signal of mobile robot due to NLOS in wireless transmission of control signals from remote controller. Since $f(t)$ will deteriorate the state estimation and control of mobile robot, a smoothing signal model like (8) is employed as follows

$$F(t+1) = A_f F(t) + M_f \tilde{f}(t) \tag{25}$$

where

$$A_f = \begin{bmatrix} b_0 I_2 & \cdots & \cdots & \cdots & b_d I_2 \\ I_2 & \ddots & & & 0_{2\times2} \\ 0_{2\times2} & I_2 & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & \vdots \\ 0_{2\times2} & \cdots & 0_{2\times2} & I_2 & 0_{2\times2} \end{bmatrix}$$
$$F(t) = \begin{bmatrix} f(t) \\ f(t-1) \\ \vdots \\ f(t-d) \end{bmatrix} \quad M_f = \begin{bmatrix} I_2 \\ 0_{2\times2} \\ \vdots \\ 0_{2\times2} \end{bmatrix}$$

and $\tilde{f}(t) = f(t+1) - \sum_{i=0}^{d} b_i f(t-i)$ denotes the extrapolation error with the extrapolation coefficients$\{b_i \geq 0\}_{i=0}^{d}$.

Based on the T-S fuzzy model of (24) and the augmented T-S fuzzy system in (11), we get the following T-S fuzzy augmented system of mobile robot with the embedded smoothing signal models of $F(t)$ and $S(t)$ due to NLOS situations at sensor and actuator of (24) in WSN,

$$\bar{X}(t+1) = \sum_{i=1}^{J} \alpha_i(\varepsilon(t))(\bar{A}_i \bar{X}(t) + \bar{B}_i u(t) + \bar{H}\,\bar{v}(t))$$
$$Z(t) = \sum_{i=1}^{J} \alpha_i(\varepsilon(t))(\bar{C}_i \bar{X}(t) + \bar{N}\,\bar{v}(t)) \tag{26}$$

where

$$\bar{X}(t) = \begin{bmatrix} F(t) \\ S(t) \\ X(t) \end{bmatrix}, \bar{v}(t) = \begin{bmatrix} \tilde{f}(t+1) \\ \tilde{s}(t+1) \\ v(t) \\ n(t) \end{bmatrix}, \bar{B}_i = \begin{bmatrix} 0 \\ 0 \\ B_i \end{bmatrix}$$
$$\bar{C}_i = [\,0 \;\; DM_s^T \;\; C_i\,], \quad \bar{N} = [\,0\;0\;0\;I\,]$$
$$\bar{A}_i = \begin{bmatrix} A_f & 0 & 0 \\ 0 & A_s & 0 \\ B_i M_f^T & 0 & I_3 \end{bmatrix}, \quad \bar{H} = \begin{bmatrix} M_f & 0 & 0 & 0 \\ 0 & M_s & 0 & 0 \\ 0 & 0 & I_3 & 0 \end{bmatrix}$$

Before the design of T-S fuzzy estimator-based remote control of mobile robot system in wireless sensor network in Fig. 1, the observability of the fuzzy augmented system in (26) is discussed as follows:

Lemma 3: For the T-S fuzzy augmented system in (26), if the local fuzzy system matrices (I_3, C_i), $i = 1, \ldots, J$ are observable, i.e.

$$rank \begin{bmatrix} zI_3 - I_3 \\ C_i \end{bmatrix} = 3,\ \forall z \in eig(I_3), i = 1, \ldots, J \tag{27}$$

and

$$eig(I_3) \cap eig(A_f) = \phi,\ \ eig(A_f) \cap (A_s) = \phi,$$
$$eig(I_3) \cap eig(A_s) = \phi \tag{28}$$
$$col \begin{bmatrix} -B_i M_f^T \\ 0 \end{bmatrix} \cap \begin{bmatrix} zI_3 - I_3 \\ C_i \end{bmatrix} = \phi,$$
$$\forall z \in eig(A_f),\ i = 1, \ldots, J \tag{29}$$

and

$$rank \begin{bmatrix} zI_{2(d+1)} - A_f \\ -B_i M_f^T \end{bmatrix} = 2(d+1),$$
$$\forall z \in eig(A_f), i = 1, \ldots, J \tag{30}$$
$$rank \begin{bmatrix} zI_{3(d+1)} - A_s \\ DM_s^T \end{bmatrix} = 3(d+1), \quad \forall z \in eig(A_s) \tag{31}$$

then T-S fuzzy augmented system in (26) is observable, i.e. $(\bar{A}_i, \bar{C}_i)$ $i = 1, \ldots, J$ are observable.

Proof: Similar to the proof in Lemma 1. □

Based on the fuzzy estimator in (15), the following remote T-S fuzzy estimator-based reference tracking control law is employed for the augmented fuzzy system of mobile robot in (26) to track the desired reference $\bar{X}_r(t)$ via wireless communication,

$$\hat{\bar{X}}(t+1) = \sum_{i=1}^{J} \alpha_i(\varepsilon(t))(\bar{A}_i \hat{\bar{X}}(t) + \bar{B}_i u(t)$$
$$+ \bar{L}_i(Z(t) - \hat{Z}(t)))$$
$$u(t) = \sum_{j=1}^{J} \alpha_j(\varepsilon(t))\bar{K}_j(\hat{\bar{X}}(t) - \bar{X}_r(t)) \tag{32}$$

where $\hat{Z}(t) = \sum_{i=1}^{J} \alpha_i(\varepsilon(t))\bar{C}_i\hat{\bar{X}}$ and $\bar{X}_r(t) = [0^T\ 0^T\ X_r^T(t)]^T$ is the desired trajectory of $\bar{X}(t) = [F^T(t)\ S^T(t)\ X^T(t)]^T$, i.e., the desired trajectory of $F(t)$ and $S(t)$ are both 0 and the desired trajectory of $X(t)$ is $X_r(t)$.

Based on the estimated state $\hat{\bar{X}}(t)$ and $\bar{X}_r(t)$ in (32), the T-S fuzzy remote control $u(t)$ is employed to control the state $X(t)$ of the mobile robot to track the desired trajectory $X_r(t)$ for some task through wireless communication.

In this study, the T-S fuzzy estimator-based control $u(t)$ in (32) is employed for the state $X(t)$ of the mobile system in (24) to track the desired trajectory $X_r(t)$ which is specified by the following reference model

$$X_r(t+1) = A_r X_r(t) + B_r r(t) \tag{33}$$

where A_r is specified to characterize the transient state of $X_r(t)$ and B_r can scale the magnitude of $X_r(t)$.

As $X_r(t)$ in (33) approaches steady state, i.e. $X_r(t+1) = X_r(t)$ as t increases, we get

$$X_r(t) = (I - A_r)^{-1} B_r r(t) \tag{34}$$

If we specify $B_r = I - A_r$ in (33), then we get $X_r(t) = r(t)$ at the steady state. In this situation, if the desired trajectory of mobile robot is specified as $r(t)$ in (33) beforehand, then $X_r(t)$ will approach the desired trajectory $r(t)$ of mobile robot after a transient time. Therefore, based on the reference model in (33) with $B_r = I - A_r$, we only need to specify the desired trajectory of mobile robot as follows

$$r(t) = \begin{bmatrix} x_d(t) \\ y_d(t) \\ \theta_d(t) \end{bmatrix} \tag{35}$$

where $x_d(t)$, $y_d(t)$ and $\theta_d(t)$ are the desired trajectory specified by the designer for a mobile robot for some task in the wireless sensor network.

In addition, since the augmented state $\bar{X}(t)$ in (26) includes $X(t)$, $F(t)$ and $S(t)$, in order to be consistent in the dimension of $\bar{X}_r(t)$ in $u(t)$ in (32), the reference tracking control in (33) should be augmented as follows:

$$\bar{X}_r(t+1) = \bar{A}_r\bar{X}_r(t) + \bar{B}_r\bar{r}(t) \tag{36}$$

where $\bar{X}_r(t) = [0^T\ 0^T\ X_r^T(t)]^T$, $\bar{A}_r = diag(0\,0\,A_r)$ and $\bar{r}(t) = [0^T\ 0^T\ r^T(t)]^T$

Based on the above analysis, the following H∞ NLOS-tolerant fuzzy estimator-based reference tracking control strategy is employed for NLOS-tolerant remote reference tracking control of mobile robot system in wireless sensor network

$$\frac{E\sum_{t=0}^{t_T}(\bar{X}(t)-\bar{X}_r(t))^T\bar{Q}_1(\bar{X}(t)-\bar{X}_r(t)) + (\bar{X}(t)-\hat{\bar{X}}(t))^T\bar{Q}_2(\bar{X}(t)-\hat{\bar{X}}(t)) + u(t)^TRu(t)}{E\sum_{t=0}^{t_T}(\bar{v}^T(t)\bar{v}(t)+\bar{r}(t)^T\bar{r}(t))} \le \rho$$
$$\forall \bar{v}(t), \bar{r}(t) \in l_2[0, t_T] \tag{37}$$

where $\bar{Q}_1 \ge 0$ denotes the weighting matrix on the reference tracking error, $\bar{Q}_2 \ge 0$ denotes the weighting matrix on the estimator error of localization filter and $R \ge 0$ denotes the weighting matrix on control effort. In general, the weighting matrix $\bar{Q}_2$ must be larger than $\bar{Q}_1$ because a precise state estimation will lead to a precise estimator-based control. Since $\bar{v}(t)$ and $\bar{r}(t)$ are unavailable for the control designer, their worst-cast effect on estimation error and tracking error should be considered in the H∞ observer-based tracking control strategy in (37).

The physical meaning of H∞ NLOS-tolerant fuzzy estimator-based tracking control strategy of mobile robot in WSN in (37) is that the effect of $\bar{v}(t)$ and $\bar{r}(t)$in (37) on the quadratic reference tracking error $\bar{X}(t) - \bar{X}_r(t)$ of the desired trajectory $\bar{X}_r(t)$ in (36) and the state estimation error $\bar{e}(t) = \bar{X}(t) - \hat{\bar{X}}(t)$ of the augmented state $\bar{X}(t)$ in (26) must be less than ρ from the mean energy perspective.

Let us denote

$$\bar{\bar{Q}} = \begin{bmatrix} \bar{Q}_1 & -\bar{Q}_1 & 0 \\ -\bar{Q}_1 & \bar{Q}_1 & 0 \\ 0 & 0 & \bar{Q}_2 \end{bmatrix}, \bar{\bar{X}}(t) = \begin{bmatrix} \bar{X}(t) \\ \bar{X}_r(t) \\ \bar{e}(t) \end{bmatrix} \tag{38}$$

where the state estimation error $\bar{e}(t) = \bar{X}(t) - \hat{\bar{X}}(t)$ is obtained by

$$\bar{e}(t+1) = \sum_{i=1}^{J}\sum_{j=1}^{J}\alpha_i(\varepsilon(t))\alpha_j(\varepsilon(t))(\bar{A}_i - \bar{L}_i\bar{C}_j)\bar{e}(t) + (\bar{H} - \bar{L}_i\bar{N})\bar{v}(t) \tag{39}$$

Then, the H∞ NLOS-tolerant fuzzy estimator-based reference tracking control strategy of mobile robot in WSN in (37) could be simply represented by

$$\frac{E\sum_{t=0}^{t_T}\bar{\bar{X}}^T(t)\bar{\bar{Q}}\,\bar{\bar{X}}(t) + u(t)^TRu(t)}{E\sum_{t=0}^{t_T}\bar{\bar{v}}^T(t)\bar{\bar{v}}(t)} \le \rho,$$
$$\forall \bar{\bar{v}}(t) \le l_2[0, t_T] \tag{40}$$

where $u(t) = \Sigma_{j=1}^{J}\alpha_j(\varepsilon(t))\bar{K}_j[I\ \text{-}I\ \text{-}I]\bar{\bar{X}}(t)$, $\bar{\bar{X}}(t)$ and $\bar{\bar{v}}(t)$ are the state vector and external disturbance of the following T-S fuzzy augmented system

$$\bar{\bar{X}}(t+1) = \sum_{i=1}^{J}\sum_{j=1}^{J}\alpha_i(\varepsilon(t))\alpha_j(\varepsilon(t))(\bar{\bar{A}}_{ij}\bar{\bar{X}}(t) + \bar{\bar{H}}_{ij}\bar{\bar{v}}(t)) \tag{41}$$

where $\bar{\bar{A}}_{ij}$ and $\bar{\bar{H}}_{ij}$ are defined as

$$\bar{\bar{A}}_{ij} = \begin{bmatrix} \bar{A}_i + \bar{B}_i\bar{K}_j & -\bar{B}_i\bar{K}_j & -\bar{B}_i\bar{K}_j \\ 0 & A_r & 0 \\ 0 & 0 & \bar{A}_i - \bar{L}_i\bar{C}_j \end{bmatrix},$$
$$\bar{\bar{H}}_{ij} = \begin{bmatrix} \bar{H} & 0 \\ 0 & \bar{B}_r \\ \bar{H} - \bar{L}_i\bar{N} & 0 \end{bmatrix}, \quad \bar{\bar{v}}(t) = \begin{bmatrix} \bar{v}(t) \\ r(t) \end{bmatrix}$$

i.e. the H∞ NLOS-tolerant fuzzy estimator-based reference tracking control design problem in (37) of the mobile robot in (26) and (32) in WSN is reduced to an equivalent simplified H∞ stabilization design problem in (40) of the augmented T-S fuzzy system in (41). This will significantly simplify the design procedure of the H∞ NLOS-tolerant fuzzy estimator-based reference tracking control design problem of the mobile robot in WSN.

In the H∞ NLOS-tolerant fuzzy estimator-based H∞ reference tracking control strategy in (40), we assume $\bar{\bar{X}}(0) = 0$. If the initial condition $\bar{\bar{X}}(0) \ne 0$, the effect of initial condition should be extracted from H∞ estimator-based reference tracking control strategy as follows[24][27]:

$$\frac{E\sum_{t=0}^{t_T}\bar{\bar{X}}^T(t)\bar{\bar{Q}}\,\bar{\bar{X}}(t) + u(t)^TRu(t) - \bar{\bar{X}}^T(0)\bar{\bar{P}}\bar{\bar{X}}(0)}{E\sum_{t=0}^{t_T}\bar{\bar{v}}^T(t)\bar{\bar{v}}(t)} \le \rho \tag{42}$$

For some $\bar{\bar{P}}^T = \bar{\bar{P}} > 0$.

Theorem 2: If we can specify a positive symmetric matrix $\bar{\bar{P}} > 0$, *fuzzy control gains* $\{\bar{K}_j\}_{j=1}^J$ *and observer gains* $\{\bar{L}_i\}_{i=1}^J$ *in (32) such that the following matrix inequalities hold*

$$\begin{bmatrix} \bar{\bar{A}}_{ij}^T\bar{\bar{P}}\bar{\bar{A}}_{ij} - \bar{\bar{P}} + \bar{\bar{Q}} \\ +[I \text{ -}I \text{ -}I]^T\bar{K}_j^TR\bar{K}_j[I \text{ -}I \text{ -}I] & \bar{\bar{A}}_{ij}^T\bar{\bar{P}}\bar{\bar{H}}_{ij} \\ \bar{\bar{H}}_{ij}^T\bar{\bar{P}}\bar{\bar{A}}_{ij} & \bar{\bar{H}}_{ij}^T\bar{\bar{P}}\bar{\bar{H}}_{ij} - \rho I \end{bmatrix} \le 0, \quad i,j = 1,\ldots,J \tag{43}$$

then the H∞ NLOS-tolerant fuzzy estimator-based reference tracking control strategy in (37) or (40) for mobile robot system in WSN in (24) can be achieved.

Proof: see Appendix C □

Let us denote the Lyapunov (energy) function of 3 dynamic systems in (26), (36) and (39) as the following quadratic functions:

$$\begin{aligned} V(\bar{\bar{x}}(t)) &= V_1(\bar{X}(t)) + V_2(\bar{X}_r(t)) + V_3(\bar{e}(t)) \\ &= \bar{X}^T(t)P_1\bar{X}(t) + \bar{X}_r^T(t)P_2\bar{X}_r(t) + \bar{e}^T(t)P_3\bar{e}(t) \\ &= \begin{bmatrix} \bar{X}^T(t) & \bar{X}_r^T(t) & \bar{e}^T(t) \end{bmatrix} \begin{bmatrix} P_1 & 0 & 0 \\ 0 & P_2 & 0 \\ 0 & 0 & P_3 \end{bmatrix} \begin{bmatrix} \bar{X}(t) \\ \bar{X}_r(t) \\ \bar{e}(t) \end{bmatrix} \\ &= \bar{\bar{X}}^T(t)\bar{\bar{P}}\bar{\bar{X}}(t) \end{aligned} \tag{44}$$

i.e., $\bar{\bar{P}} = diag[P_1\ P_2\ P_3]$, where $P_1 > 0$, $P_2 > 0$ and $P_3 > 0$.

Substituting $\bar{\bar{A}}_{ij}$, $\bar{\bar{H}}_{ij}$ in (41) and $\bar{\bar{P}}$ in (44) into (43), we obtain the following bilinear matrix inequalities (BMIs)

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & 0 \\ * & \Pi_{22} & \Pi_{23} & \Pi_{24} & \Pi_{25} \\ * & * & \Pi_{33} & \Pi_{34} & 0 \\ * & * & * & \Pi_{44} & 0 \\ * & * & * & * & \Pi_{55} \end{bmatrix} \le 0, \quad i,j = 1,\ldots,J \tag{45}$$

in which $\Pi_{11} = (\bar{A}_i + \bar{B}_i\bar{K}_j)^TP_1(\bar{A}_i + \bar{B}_i\bar{K}_j) - P_1 + \bar{Q}_1 + \bar{K}_j^TR\bar{K}_j$, $\Pi_{12} = -(\bar{A}_i + \bar{B}_i\bar{K}_j)^TP_1\bar{B}_i\bar{K}_j - \bar{Q}_1 - \bar{K}_j^TR\bar{K}_j$, $\Pi_{13} = -(\bar{A}_i + \bar{B}_i\bar{K}_j)^TP_1\bar{B}_i\bar{K}_j - \bar{K}_j^TR\bar{K}_j$, $\Pi_{14} = (\bar{A}_i + \bar{B}_i\bar{K}_j)^TP_1\bar{H}$, $\Pi_{22} = (\bar{B}_i\bar{K}_j)^TP_1(\bar{B}_i\bar{K}_j) + \bar{A}_r^TP_2\bar{A}_r - P_2 + \bar{Q}_1 + \bar{K}_j^TR\bar{K}_j$, $\Pi_{23} = (\bar{B}_i\bar{K}_j)^TP_1(\bar{B}_i\bar{K}_j) + \bar{K}_j^TR\bar{K}_j$, $\Pi_{24} = -(\bar{B}_i\bar{K}_j)^TP_1\bar{H}$, $\Pi_{25} = \bar{A}_r^TP_2\bar{B}_r$, $\Pi_{33} = (\bar{B}_i\bar{K}_j)^TP_1(\bar{B}_i\bar{K}_j) - P_3 + \bar{Q}_2 + (\bar{A}_i - \bar{L}_i\bar{C}_j)^TP_3(\bar{A}_i - \bar{L}_i\bar{C}_j) + \bar{K}_j^TR\bar{K}_j$, $\Pi_{34} = (\bar{A}_i - \bar{L}_i\bar{C}_j)^TP_3(\bar{H} - \bar{L}_i\bar{N}) - (\bar{B}_i\bar{K}_j)^TP_1\bar{H}$, $\Pi_{44} = \bar{H}^TP_1\bar{H} + (\bar{H}^T - \bar{N}^T\bar{L}_i^T)P_3(\bar{H} - \bar{L}_i\bar{N}) - \rho I$, $\Pi_{55} = \bar{B}_r^TP_2\bar{B}_r - \rho I$

In general, it is still not easy to solve the above BMIs for fuzzy control gains $\{\bar{K}_j\}_{j=1}^J$ and estimator gains $\{\bar{L}_i\}_{i=1}^J$ in (32) for the T-S fuzzy estimator-based reference tracking control of mobile robot system in WSN simultaneously. A two-step procedure is proposed to solve P_1, P_2, P_3, $\{\bar{K}_j\}_{j=1}^J$ and $\{\bar{L}_i\}_{i=1}^J$ from the above matrix inequalities.

Two-step design procedure of robust H∞ fuzzy estimator-based tracking remote control of the mobile robot in wireless sensor network:

Step 1: Since the necessary condition for the matrix inequalities in (45) is that all diagonal terms must be less than or equal to 0, we consider the first diagonal term to solve $\{\bar{K}_j\}_{j=1}^J$ and $P_1 > 0$ first, i.e.,

$$(\bar{A}_i + \bar{B}_i\bar{K}_j)^TP_1(\bar{A}_i + \bar{B}_i\bar{K}_j) - P_1 + \bar{Q}_1 + \bar{K}_j^TR\bar{K}_j \le 0 \tag{46}$$

Let $W_1^{-1} = P_1$ and employ Schur complement in Lemma 2, then (46) is equivalent to

$$\begin{bmatrix} -W_1^{-1} + \bar{Q}_1 + \bar{K}_j^TR\bar{K}_j & \bar{A}_i^T + \bar{K}_j^T\bar{B}_i^T \\ \bar{A}_i + \bar{B}_i\bar{K}_j & -W_1 \end{bmatrix} \le 0 \tag{47}$$

Perform $\begin{bmatrix} W_1 & 0 \\ 0 & I \end{bmatrix}$ to both sides of (47) and let $\bar{Y}_j = \bar{K}_jW_1$, then we get

$$\begin{bmatrix} -W_1 + W_1\bar{Q}_1W_1 + \bar{Y}_j^TR\bar{Y}_j & W_1\bar{A}_i^T + \bar{Y}_j^T\bar{B}_i^T \\ \bar{A}_iW_1 + \bar{B}_i\bar{Y}_j & -W_1 \end{bmatrix} \le 0, \quad i,j = 1,\ldots,J \tag{48}$$

By Schur complement in Lemma 2 again, BMIs in (48) are equivalent to the following LMIs,

$$\begin{bmatrix} -W_1 & \bar{Y}_j^T & W_1Q_1^{1/2} & (\bar{A}_iW_1 + \bar{B}_i\bar{Y}_j)^T \\ \bar{Y}_j & -R^{-1} & 0 & 0 \\ Q_1^{1/2}W_1 & 0 & -I & 0 \\ \bar{A}_iW_1 + \bar{B}_i\bar{Y}_j & 0 & 0 & -W_1 \end{bmatrix} \le 0, \quad i,j = 1,\ldots,J \tag{49}$$

After solving W_1 and $\{\bar{Y}_j\}_{j=1}^J$ from LMIs in (49), we can obtain fuzzy control gains $\{\bar{K}_j = \bar{Y}_jW_1^{-1}\}_{j=1}^J$.

Step 2: Further, substituting these solutions $P_1 = W_1^{-1}$ and $\{\bar{K}_j\}_{j=1}^J$ into matrix inequalities in (45), it is still not easy to transform BMIs in (45) to LMIs because coupling terms $(\bar{A}_i - \bar{L}_i\bar{C}_j)^TP_3(\bar{H} - \bar{L}_i\bar{N})$ and $(\bar{H}^T - \bar{N}^T\bar{L}_i^T)P_3(\bar{A}_i - \bar{L}_i\bar{C}_j)$ in the off-diagonal lines. Under the concept of completing of squares method, i.e., $A^TB + B^TA \le \alpha A^TA + \alpha^{-1}B^TB$ for $\alpha > 0$ [27], the following inequalities hold

$$\begin{aligned} &\bar{e}^T(t)(\bar{A}_i - \bar{L}_i\bar{C}_j)^TP_3(\bar{H} - \bar{L}_i\bar{N})\bar{v}(t) \\ &\quad + \bar{v}^T(t)(\bar{H} - \bar{L}_i\bar{N})P_3(\bar{A}_i - \bar{L}_i\bar{C}_j)\bar{e}(t) \\ &\le \bar{e}^T(t)(\bar{A}_i - \bar{L}_i\bar{C}_j)^TP_3(\bar{A}_i - \bar{L}_i\bar{C}_j)e(t) \\ &\quad + \bar{v}^T(t)(\bar{H} - \bar{L}_i\bar{N})P_3(\bar{H} - \bar{L}_i\bar{N})^T\bar{v}(t) \end{aligned} \tag{50}$$

for any signal vectors $\bar{e}(t)$ and $\bar{v}(t)$.

Then, to decouple the terms $(\bar{A}_i - \bar{L}_i\bar{C}_j)^TP_3(\bar{A}_i - \bar{L}_i\bar{C}_j)$ and $(\bar{H} - \bar{L}_i\bar{N})^TP_3(\bar{H} - \bar{L}_i\bar{N})$ in the third and forth terms in (50), respectively, the following matrix inequalities are proposed by introducing stack variable $\{\bar{R}_{ij}, \bar{S}_{ij}\}_{i,j=1}^J$ as follows:

$$(\bar{A}_i - \bar{L}_i\bar{C}_j)^TP_3(\bar{A}_i - \bar{L}_i\bar{C}_j) \le \bar{R}_{ij} \tag{51}$$

$$(\bar{H} - \bar{L}_i\bar{N})^TP_3(\bar{H} - \bar{L}_i\bar{N}) \le \bar{S}_{ij} \tag{52}$$

which are equivalent to

$$\begin{bmatrix} -\bar{R}_{ij} & \bar{A}_i^TP_3 - \bar{C}_j\bar{M}_i^T \\ P_3\bar{A}_i - \bar{M}_i\bar{C}_j & -P_3 \end{bmatrix} \le 0 \tag{53}$$

$$\begin{bmatrix} -\bar{S}_{ij} & H^TP_3 - \bar{N}^T\bar{M}_i^T \\ P_3\bar{H} - \bar{M}_i\bar{N} & -P_3 \end{bmatrix} \le 0 \tag{54}$$

respectively, where $\bar{M}_i = P_3\bar{L}_i$ or $\bar{L}_i = P_3^{-1}\bar{M}_i$.

By (50),(51) and (52), the BMIs in (45) become the followings

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & 0 \\ * & \Pi_{22} & \Pi_{23} & \Pi_{24} & \Pi_{25} \\ * & * & \Pi_{33} & \Pi_{34} & 0 \\ * & * & * & \Pi_{44} & 0 \\ * & * & * & * & \Pi_{55} \end{bmatrix} \leq 0, \ i,j = 1,\ldots,J \quad (55)$$

in which $\Pi_{11} = (\bar{A}_i + \bar{B}_i\bar{K}_j)^T P_1(\bar{A}_i + \bar{B}_i\bar{K}_j) - P_1 + \bar{Q}_1 + \bar{K}_j^T R\bar{K}_j$, $\Pi_{12} = -(\bar{A}_i + \bar{B}_i\bar{K}_j)^T P_1\bar{B}_i\bar{K}_j - \bar{Q}_1 - \bar{K}_j^T R\bar{K}_j$, $\Pi_{13} = -(\bar{A}_i + \bar{B}_i\bar{K}_j)^T P_1\bar{B}_i\bar{K}_j - \bar{K}_j^T R\bar{K}_j$, $\Pi_{14} = (\bar{A}_i + \bar{B}_i\bar{K}_j)^T P_1\bar{H}$, $\Pi_{22} = (\bar{B}_i\bar{K}_j)^T P_1(\bar{B}_i\bar{K}_j) + \bar{A}_r^T P_2\bar{A}_r - P_2 + \bar{Q}_1 + \bar{K}_j^T R\bar{K}_j$, $\Pi_{23} = (\bar{B}_i\bar{K}_j)^T P_1(\bar{B}_i\bar{K}_j) + \bar{K}_j^T R\bar{K}_j$, $\Pi_{24} = -(\bar{B}_i\bar{K}_j)^T P_1\bar{H}$, $\Pi_{25} = \bar{A}_r^T P_2\bar{B}_r$, $\Pi_{33} = (\bar{B}_i\bar{K}_j)^T P_1(\bar{B}_i\bar{K}_j) - P_3 + \bar{Q}_2 + 2\bar{R}_{ij} + \bar{K}_j^T R\bar{K}_j$, $\Pi_{34} = -(\bar{B}_i\bar{K}_j)^T P_1\bar{H}$, $\Pi_{44} = \bar{H}^T P_1\bar{H} + 2\bar{S}_{ij} - \rho I$, $\Pi_{55} = \bar{B}_r^T P_2\bar{B}_r - \rho I$.

By solving (53), (54) and (55) for P_2, P_3 and $\bar{M}_i$, simultaneously, the fuzzy observer gains $\{\bar{L}_i = P_3^{-1}\bar{M}_i\}_{i=1}^J$ could be obtained. Therefore, in step 1, we could obtain $\{\bar{K}_j\}_{j=1}^J$ and in step 2 we could obtain fuzzy estimator gains $\{\bar{L}_i\}_{i=1}^J$ in (32) to achieve the H∞ estimator-based reference tracking control strategy in (37) of mobile robot system in wireless sensor network. In order to solve BMIs in (45) for H∞ fuzzy estimator-based controller in (32), the above two-step design procedure is proposed. In step 1, we propose to solve equivalent LMIs in (49) for the first diagonal term of BMIs in (45) to obtain fuzzy control gains $\{\bar{K}_j\}_{j=1}^J$. In the second step, we employed the decoupling technique (50), the relaxing technique in (51) and (52) to transform BMIs in (45) to LMIs in (53), (54) and (55) to solve fuzzy estimator gain $\{\bar{L}_i\}_{i=1}^J$.

Remark 3: For the fuzzy-model-based (FMB) control, the membership-function-independent (MFI) strategy is employed when analyzing the stability conditions. In this case, we drop the membership functions in the design procedure during the calculation of the stability analysis. However, it will increase the conservatism because fewer number of stability conditions and decision variables (information) are taken into consideration. Recently, several researchers have focused on the membership-function-dependent (MFD) strategy when faced with stability analysis problems. It has been known that the MFD strategy will bring more information through the bound of slack matrices or approximation errors of the membership function during the stability analysis, resulting in more relaxed stability conditions. However, the MFD method needs more experiences for choosing the information of membership functions with regard to the stability conditions in the system. Once the information does not in the interval of membership functions, the computational demand on finding a feasible solution to the stability conditions will increase. Although the MFI method is more conservative, it could guarantee the asymptotical stability and find feasible solutions easily.

Remark 4: To reduce the conservatism of the membership-function-independent (MFI) strategy, the selection of operation points and fuzzy If-Then rules model in (9) becomes an important issue. Obviously, if a lot of local linearized systems are used to interpolate the mobile robot system, the number of matrix inequalities in (54) will increase and the corresponding feasibility will be reduced. Therefore, there is a trade-off between the number of If-Then rules for the T-S fuzzy model in (9) and the feasible solvability of matrix inequalities in (54).

Similar to the optimal H∞ fuzzy localization filter design problem in (23), the optimal H∞ fuzzy estimator-based reference tracking control of mobile robot system in wireless sensor network based on the proposed two-step design can be formulated as the following LMIs-constrained optimization problem.

$$\begin{aligned} \rho^* &= \min_{W_1>0,\, \bar{Y}_j,\, P_2>0,\, P_3>0,\, \bar{M}_j} \rho \\ &\textit{subject to } (49), (53), (54) \textit{and} (55) \end{aligned} \quad (56)$$

The LMIs-constrained optimization problem in (56) could be easily solved by decreasing ρ until there exists no $W_1 > 0$, $P_2 > 0, P_3 > 0$ in LMIs in (49), (53), (54) and (55) with the help of LMI toolbox in Matlab.

Therefore we summarize the design procedure of H∞ NLOS-tolerant fuzzy estimator-based reference tracking control of the mobile robot in wireless sensor network under NLOS situation as follows:

(1) Construct the augmented T-S fuzzy system in (26) of mobile robot with embedded smoothing signal model (8) and (25) of bias signal $S(t)$ and $F(t)$ of NLOS in the transmission of output measurement and control signal, respectively.

(2) Construct the T-S fuzzy estimator-based tracking control in (32).

(3) Specify weighting matrices $\bar{Q}_1$, $\bar{Q}_2$ and R of the H∞ NLOS-tolerant fuzzy estimator-based reference tracking control strategy in (37).

(4) Solve the LMIs-constrained optimal problem in (56) by two-step design procedure for W_1^*, $\{\bar{Y}_j^*\}_{j=1}^J$, P_3^*, $\{\bar{M}_i^*\}_{i=1}^J$. Then, the optimal fuzzy localization filter (estimator) gains and fuzzy control gains in (32) are obtained as $\{\bar{L}_i^* = P_3^{*-1}\bar{M}_i^*\}_{i=1}^J$ and $\{\bar{K}_j^* = \bar{Y}_j^* W_1^{*-1}\}_{j=1}^J$, respectively.

Remark 5: The optimization problems with bilinear matrix inequality (BMI) constraints in (45) have been known as nonconvex and NP-hard problems, which are hard to solve. Several researchers have developed some approaches to solve these problems by using a sum-of-squares approach to the fixed order H∞ control synthesis [34], local and global methods based on techniques of the global optimization for control design [35] and sequential semidefinite programming (SDP) method for control design [36]. Also, the Matlab toolbox of BMI-solver has been proposed [37] to approximate the feasible set of the nonconvex problems in control by a sequence of inner positive semidefinite convex approximation sets. These methods can solve the BMI constraints problem to obtain control gains and observer gains simultaneously and effectively.

V. SIMULATION EXAMPLE

In this section, the proposed robust H∞ NLOS-tolerant localization filter and robust H∞ NLOS-tolerant fuzzy estimator-based reference tracking control design are applied to the localization and desired reference pose tracking control of the mobile robot at the remote site in wireless sensor network under NLOS situation inside an intelligent building as shown in Fig.1. In the simulation example, we assume four receivers are installed at points (-50,-50), (-50,50), (50,-50) and (50,50), respectively, all in meters. Further, the robust H∞ localization filter-based remote control is implemented by a computer (Core i7-2700K, 3.5 GHz, 16GB RAM) to run all the algorithms and analyze the data namely. All the simulation data are analyzed and calculated by MATLAB R2019B. Suppose the order of smoothing model of $S(t)$ in (8) is 3 (i.e., $d = 3$) and $F(t)$ in (25) is 3 (i.e., $d = 3$). The extrapolation coefficients of sensor bias signal in A_s are chosen $a_0 = 0.4, a_1 = 0.3, a_2 = 0.2, a_3 = 0.1$ in (8) and the extrapolation coefficients of actuator bias signal in A_f are chosen as $b_0 = 0.5, b_1 = 0.2, b_2 = 0.2, b_3 = 0.1$ in (25). Suppose the external disturbances $v_1(t)$, $v_2(t)$ and $v_3(t)$ and measurement noises $n_1(t)$, $n_2(t)$, $n_3(t)$ and $n_4(t)$ in (5) are all Gaussian white noises with zero mean and standard deviation $\sigma = 1$.

For the reference model in (36), we choose

$$\bar{A}_r = diag(0\;0\;A_r),\ \bar{B}_r = I_{11} - \bar{A}_r \tag{57}$$

where $A_r = 0.3I_3$.

In the wireless sensor network system, actuator bias signal $f(t)$ due to NLOS in wireless channels will be transmitted into the actuator with remote control command to influence the pose of the mobile robot. Four sensors of indoor localization system will suffer the interference from sensor bias signal $s(t)$ due to NLOS impact in wireless channels. Sensor bias signal $s(t) = [s_1(t)\ s_2(t)\ s_3(t)]^T$ and the actuator bias signal $f(t) = [f_1(t)\ f_2(t)]^T$ are displayed in Fig, 3.

The initial state of the mobile robot and its estimation in WSN are assumed to be $X(0) = [-40, 5, \pi/2]^T$ and $\hat{X}(0) = [-40, 5, 0]$, respectively, and the initial state $\bar{X}(0)$ of the augmented system in (26) and its estimation are given by

$$\begin{aligned}\bar{X}(0) &= [0, 0, 0, 0, 0, 0, 0, 0, -40, 5, \pi/2]^T\\ \hat{\bar{X}}(0) &= [0, 0, 0, 0, 0, 0, 0, 0, -40, 5, 0]^T\end{aligned} \tag{58}$$

respectively. The weighting matrices of the robust H∞ NLOS-tolerant fuzzy estimator-based reference tracking control strategy in (37) are selected as

$$\bar{Q}_1 = 10^{-3} \times I_{11},\ \ \bar{Q}_2 = 3 \times 10^{-3} \times I_{11},\ \ R = 0.5 \times I_3 \tag{59}$$

To construct the T-S fuzzy system of the mobile robot system in (9), the premise variables $\varepsilon_1(t)$, $\varepsilon_2(t)$ and $\varepsilon_3(t)$ are chosen as $x(t), y(t),$ *and* $\theta(t)$ respectively and the operation points are given as

$$\begin{aligned} x_{11} &= -50, x_{21} = 10, x_{31} = 50\\ y_{12} &= -50, y_{22} = 10, y_{32} = 50\\ \theta_{13} &= 275, \theta_{23} = 375, \theta_{33} = 450\end{aligned} \tag{60}$$

After solving the LMIs-constrained optimization problem in (56) by two-step design procedure with the optimal attenuation level $\rho^* = 0.8$, we can obtain the fuzzy control gain $\{\bar{L}_i^* = P_3^{*-1}\bar{M}_i^*\}_{i=1}^{27}$ and fuzzy observer gains $\{\bar{K}_j^* = \bar{Y}_j^*\bar{W}_1^{*-1}\}_{j=1}^{27}$, respectively.

The bias signals due to NLOS at actuator and output signal sensors and their estimates by T-S fuzzy estimator in (32) via the proposed H∞ NLOS-tolerant fuzzy estimator-based reference tracking control strategy in (37) are shown in Fig. 3. Fig. 3(a) shows the actuator bias signal $f(t) = [f_1(t)\ f_2(t)]^T$ and its estimation $\hat{f}(t)$. Fig. 3(b) shows the sensor bias signal $s(t) = [s_1(t)\ s_2(t)\ s_3(t)]^T$ and its estimation $\hat{s}(t)$ by the proposed H∞ NLOS-tolerant estimator-based reference tracking control strategy of the mobile robot in WSNs inside the intelligent building. At first, the estimation of $f(t)$ and $s(t)$ have large transient responses due to the initial condition. However, the proposed H∞ NLOS-tolerant fuzzy Luenberger observer can estimate both actuator and sensor bias signal as well as the pose of mobile robot precisely during a short transient time. In Fig. 4, the moving trajectory and the desired trajectory of the mobile robot in the wireless sensor network by the proposed H∞ NLOS-tolerant fuzzy estimator-based reference tracking control strategy are displayed. In addition, the trajectory of mobile robot and the corresponding desired reference trajectory are also shown in Fig. 5. From Figs. 4-5, while the mobile robot moves across the different Quadrants, the trajectory of the mobile robot will suffer from a huge interference because of the dramatic change of angle $\theta(t) = \tan^{-1}(\frac{v_y(t)}{v_x(t)})$ when $v_x(t) \simeq 0$. For example, once the mobile robot moves across the fourth quadrant to the first quadrant, the trajectory of the mobile robot will move a little bit away from the desired trajectory. Even there are some floats with the trajectory due to dramatic changes of angle $\theta(t)$, the result shows that the proposed robust H∞ NLOS-tolerant estimator-based fuzzy reference tracking control strategy can efficiently estimate the state of the mobile robot and bias signal due to NLOS in WSN as well as can make mobile robot to robustly track the desired reference quite well in the cluttered and noisy indoor environment inside the intelligent building, especially in NLOS situations.

The control signals $u^*(t) = [\Delta d^*(t), \Delta\theta^*(t)]$ for the mobile robot in the wireless sensor network are shown in Fig. 6. The control distance signal $\Delta d^*(t)$ can almost cancel the actuator signal $f(t)$ when it causes an effect on the control actuator of the mobile robot in the wireless system. In addition, in Fig. 6, the control angle signal $\Delta\theta^*(t)$ gives a large control signal in the 150s and 300s to cancel the bias signal of rapid change of the angle $\theta(t)$ from the first to fourth quadrant and the third to second quadrant. The proposed robust H∞ NLOS-tolerant observer-based reference tracking controller can eliminate the effect of actuator bias signal $f(t)$ and sensor bias signal $s(t)$ due to NLOS in WSN during the observer-based reference tracking control process of the

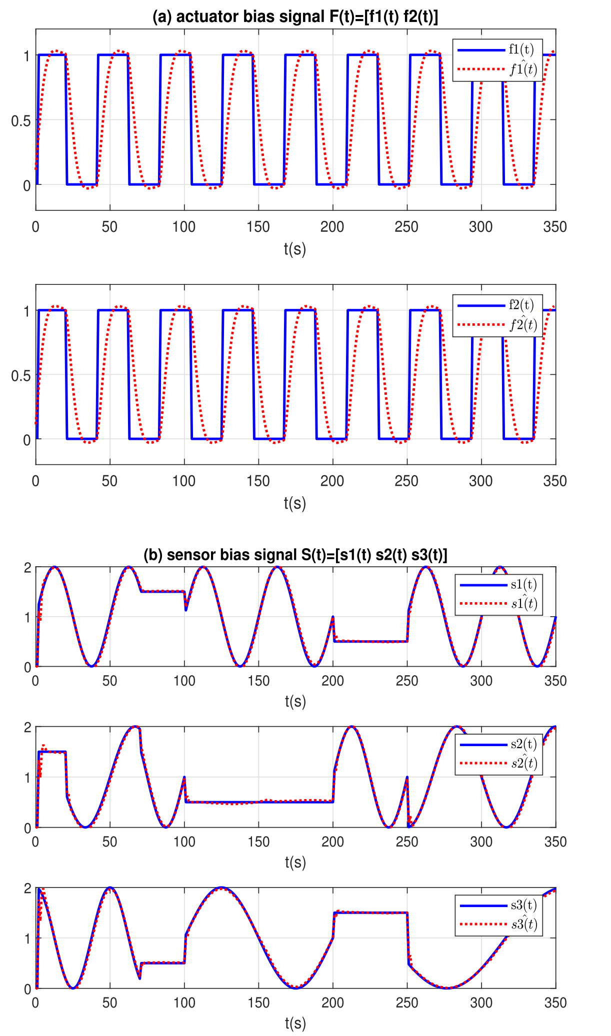

FIGURE 3. Bias signals $s(t) = [s_1(t)\ s_2(t)\ s_3(t)]^T$ and $f(t) = [f_1(t)\ f_2(t)]^T$ due to NLOS in sensor and actuator and their estimates $\hat{s}(t) = [\hat{s}_1(t)\ \hat{s}_2(t)\ \hat{s}_3(t)]^T$ and $\hat{f}(t) = [\hat{f}_1(t)\ \hat{f}_2(t)]^T$ by T-S fuzzy estimator in (32) based on the optimal H∞ NLOS-tolerant estimator-based reference tracking control strategy in (37).

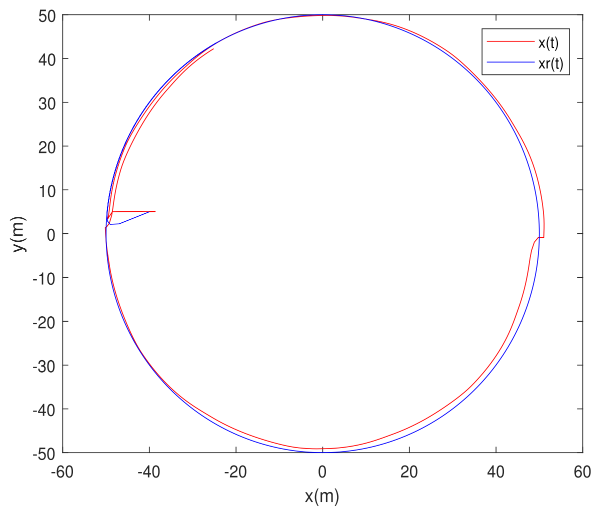

FIGURE 4. The trajectory and the desired reference trajectory of mobile robot under the proposed H∞ NLOS-tolerant fuzzy estimator-based reference tracking control. Since $\theta(t) = \tan^{-1}(\frac{v_y(t)}{v_x(t)})$, while $v_x(t) \simeq 0$, there exists a large change in $\theta(t)$.

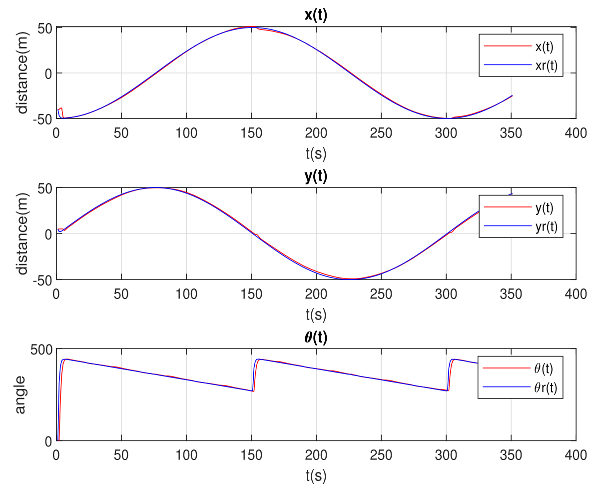

FIGURE 5. The desired reference trajectory $X_r(t)$ and the real trajectory $X(t)$ of mobile robot in WSN in Fig. 1 by the proposed H∞ NLOS-tolerant fuzzy estimator-based reference tracking control. Since $\theta(t) = \tan^{-1}(\frac{v_y(t)}{v_x(t)})$, $\theta(t)$ has a large change when $v_x(t) \simeq 0$.

mobile robot inside the intelligent building. The simulation performance of the proposed optimal H∞ NLOS-tolerant fuzzy estimator-based reference tracking control strategy in (37) is calculated as follows

$$\frac{\begin{array}{l}\sum_{t=0}^{350}(\bar{X}(t)-\bar{X}_r(t))^T\bar{Q}_1(\bar{X}(t)-\bar{X}_r(t)) \\ +(\bar{X}(t)-\hat{\bar{X}}(t))^T\bar{Q}_2(\bar{X}(t)-\hat{\bar{X}}(t))+u(t)^TRu(t) \\ -[\bar{X}(t)\ \bar{X}_r(t)\ \bar{e}(t)]^TP^*\begin{bmatrix}\bar{X}(t)\\ \bar{X}_r(t)\\ \bar{e}(t)\end{bmatrix}-\bar{\bar{X}}^T(0)\bar{P}\bar{\bar{X}}(0)\end{array}}{\sum_{t=0}^{350}(\bar{v}^T(t)\bar{v}(t)+\bar{r}(t)^T\bar{r}(t))} \approx 0.7, \tag{61}$$

Obviously, the optimal H∞ NLOS-tolerant fuzzy estimator-based reference tracking performance can be achieved by the result of simulation example with smoothing signal model of NLOS. In order to validate the effectiveness of our design scheme, the trajectory of the same H∞ estimator-based reference tracking control of the mobile robot in NLOS without using the smoothing signal model is carried out. From Fig. 7, once the bias signals (actuator bias signal and sensor bias signal) occur due to NLOS situations in the mobile robot in WSN inside the intelligent building, the mobile robot under the H∞ estimator-based reference tracking control without using the smoothing model can not track the desired reference well because of the corruption of bias signals due to NLOS in WSN. Obviously, the bias signals due to NLOS will deteriorate the pose estimate and reference tracking control of the mobile robot so that the reference tracking performance is severely degraded even the H∞ estimator-based reference tracking control has a robust

TABLE 1. Comparison of localization estimates between the proposed localization filter and the particle filter in [7].

Location time (sec)	Actual position	Estimated position by particle filter	Estimations by the proposed filter	localization error with particle filter (in meter)	localization with the proposed filter (in meter)
10	$(-48.93, 7.93)$	$(-49.86, 8.74)$	$(-48.88, 7.85)$	0.81	0.093
110	$(32.37, 38.81)$	$(32.26, 37.4)$	$(31.94, 39.33)$	1.41	0.68
220	$(6.99, -48.48)$	$(5.60, -48.2)$	$(7.07, -48.5)$	1.40	0.08
300	$(-49.98, -1.7)$	$(-50.96, -1.82)$	$(-49.84, -1.63)$	0.98	0.15

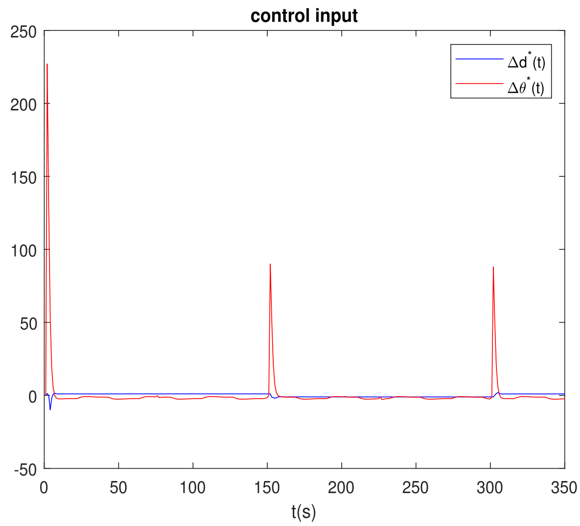

FIGURE 6. Control signal $u^*(t) = [\Delta d^*(t), \theta^*(t)]^T$ of the proposed H∞ NLOS-tolerant fuzzy estimator-based reference tracking control strategy. There exists a large control signal to overcome the large changes of $\theta(t)$ of the mobile robot in the reference tracking process.

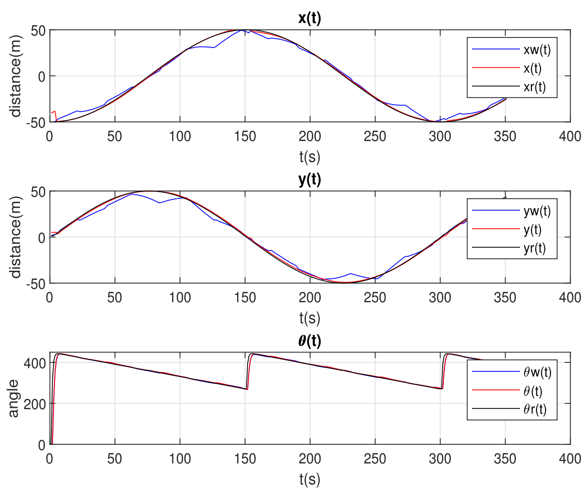

FIGURE 7. Comparison between H∞ NLOS-tolerant fuzzy estimator-based reference tracking control and conventional H∞ estimator-based reference tracking control without smoothing model in NLOS. The trajectory $[xw(t)\ yw(t)\ \theta w(t)]^T$ denotes the conventional H∞ estimator-based reference tracking results of the same method but without considering the smoothing signal model and the trajectory $[x(t)\ y(t)\ \theta(t)]^T$ denotes the tracking results of our method.

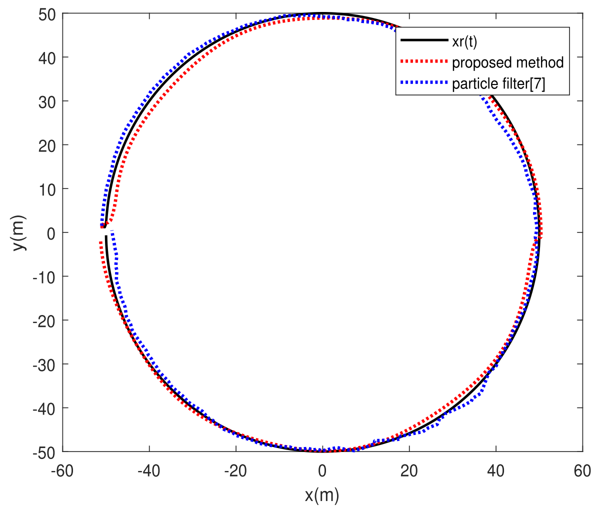

FIGURE 8. Localization estimation by the proposed H∞ NLOS-tolerant filter and particle filter.

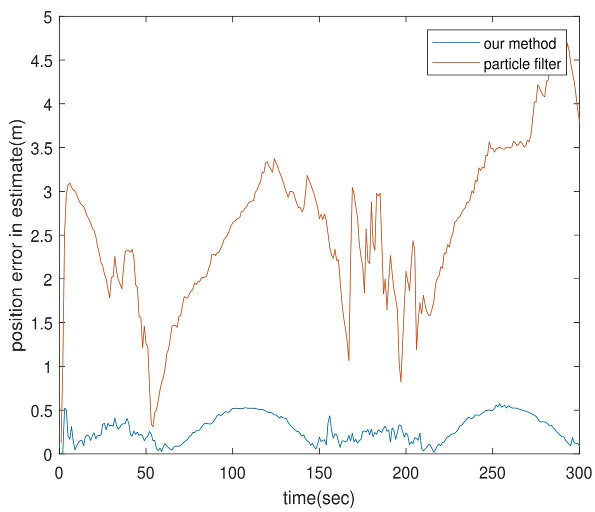

FIGURE 9. Position error by the proposed method and particle filter method are compared by simulation.

state estimation and control performance under external disturbance and measurement noise simultaneously.

The NLOS-tolerant localization filter estimation performance of the proposed H∞ estimator-based reference tracking control method is shown in Fig. 8. It seems that when the mobile robot moves across different quadrants, some dramatic change of angle $\theta(t)$ will cause an inaccurate estimation. To validate the effectiveness of our method, the trajectory estimation of the particle filter in [7] is carried out based on the proposed H∞ NLOS-tolerant control scheme. It is means that, once the mobile robot moves across the second quadrant to the third quadrant, the proposed H∞ NLOS-tolerant localization filter can estimate precisely than the traditional particle filter.

In order to evaluate the performance of the proposed method, obviously, we need to calculate the localization filtering error and the average localization error. The localization filtering error is computed by

$$E_r(t) = \sqrt{(x(t) - \hat{x}(t))^2 + (y(t) - \hat{y}(t))^2} \tag{62}$$

and the average localization filtering error is computed as $\frac{\Sigma_{t=1}^{t_T} E_r(t)}{t_T}$, where $(x(t), y(t))$ is the true position of a mobile robot and $(\hat{x}(t), \hat{y}(t))$ is its estimated position. In addition, in Table 1, the localization estimates with the proposed scheme for a few sample locations are given to clearly confirm the performance of the proposed method for the localization estimation of indoor mobile robot in WSN. In Fig. 9, the position filtering error by the proposed method and particle filter in [7] are carried out. Obviously, it is found that the proposed H∞ estimator-based reference tracking control method demonstrates a better location estimation performance than the particle filter in [7]. The result in Fig. 9 shows that the proposed method can be effectively against the dramatic change of angle $\theta(t)$ while the mobile robot moves across the different Quadrants. It is noted that the average localization error of our method is reduced by about 90% as compared to particle filter according to the average localization error of our method (0.2874) and particle filter (3.0207), respectively. From the simulation experiments, it can be concluded that the estimation accuracy (trajectory and localization error) of the proposed method is better than the traditional particle filter under NLOS situations, external disturbance and measurement noise.

VI. CONCLUSION

In this study, the robust H∞ NLOS-tolerant fuzzy localization filter and H∞ NLOS-tolerant localization filter-based reference tracking control of the mobile robot are proposed for the mobile robot to track a desired reference trajectory in the cluttered and noisy indoor environment with NLOS in the wireless sensor network. A discrete-time smoothing signal model is proposed to efficiently estimate the bias signals due to NLOS in the wireless sensor network. By using T-S fuzzy interpolation method, the nonlinear WSN-based indoor localization system of the mobile robot can be interpolated by a set of local discrete-time linear dynamic systems via fuzzy basis functions to simplify the design procedure. Further, by embedding the smoothing signal models of bias signals due to NLOS in the mobile robot system to avoid the corruption of bias signals due to NLOS, a fuzzy Luenberger observer can be used to estimate state and bias signals due to NLOS of mobile robot localization system in WSN. Then, both the robust H∞ NLOS-tolerant localization filter design problem and NLOS-tolerant observer-based reference tracking control design problem of the indoor mobile robot reference tracking control design in WSN could be transformed into a corresponding LMI-constrained optimization problem which could be efficiently solved by the proposed two-step design procedure. A simulation of robust H∞ NLOS-tolerant fuzzy estimator-based reference tracking control design of indoor mobile robot in an intelligent building via wireless sensor network is provided to validate the effectiveness of the proposed method. Because of the corruption avoidance of NLOS and precise estimation of NLOS via our proposed discrete smoothing model, the proposed H∞ NLOS-tolerant fuzzy estimator-based reference tracking control can eliminate the influence of bias signals due to NLOS in wireless channels to improve the reference tracking performance of the indoor mobile robot in WSN inside the intelligent building. Since the accurate localization and robust tracking control of the mobile robot using wireless networks under NLOS situations can be guaranteed well, the proposed NLOS-tolerant localization filter and the NLOS-tolerant tracking control design method will be much potential for more practical applications in future smart cities.

APPENDIX A
PROOF OF LEMMA 1

By utilizing the rank test in [28], the augmented T-S fuzzy system in (11) is observable if the following rank condition holds

$$rank \begin{bmatrix} zI_{3(d+2)} - \bar{A} \\ \bar{C}_i \end{bmatrix} = 3(d+2), \quad \forall z \in Z \tag{63}$$

where Z denotes the set in complex domain.

The proof is divided into two parts:
(i) $z \notin eig(I_3)$ *and* $eig(A_s)$ and (ii) $z \in eig(I_3)$ *and* $eig(A_s)$.

(i) When $z \notin eig(I_3)$ *and* $eig(A_s)$

$$\begin{aligned} rank \begin{bmatrix} zI_{3(d+1)} - \bar{A} \\ \bar{C}_i \end{bmatrix} &= rank \begin{bmatrix} zI - A_s & 0 \\ 0 & zI - I_3 \\ DM_s^T & C_i \end{bmatrix} \\ &= rank \begin{bmatrix} zI - A_s \\ DM_s^T \end{bmatrix} + rank \begin{bmatrix} zI - I_3 \\ C_i \end{bmatrix} \end{aligned} \tag{64}$$

By (12) and (13), We get

$$rank \begin{bmatrix} zI_{3(d+1)} - \bar{A} \\ \bar{C}_i \end{bmatrix} = 3 + 3(d+1) = 3(d+2) \tag{65}$$

(ii) When $z \in eig(I_3)$ *and* $eig(A_s)$

$$rank \begin{bmatrix} zI_{3(d+2)} - \bar{A} \\ \bar{C}_i \end{bmatrix} = rank \begin{bmatrix} zI - A_s & 0 \\ 0 & zI - I_3 \\ D & C_i \end{bmatrix} \tag{66}$$

Since we assume $eig(A_s) \cap eig(I_3) = 0$,by assumption on (12) and (13)

$$\begin{aligned} & rank \begin{bmatrix} zI_{3(d+2)} - \bar{A} \\ \bar{C}_i \end{bmatrix} \\ &= rank \begin{bmatrix} zI - A_s \\ DM_s^T \end{bmatrix} + rank \begin{bmatrix} zI - I_3 \\ C_i \end{bmatrix} \\ &= 3 + 3(d+1) = 3(d+2) \end{aligned} \tag{67}$$

Under conditions in (12) and (13), and from (i) and (ii) we can prove $rank \begin{bmatrix} zI_3 - \bar{A} \\ \bar{C}_i \end{bmatrix} = 3(d+2)$ for all fuzzy local systems in all z-complex domain, i.e. T-S Fuzzy augmented mobile robot localization system in (11) is observable. Q.E.D

APPENDIX B
PROOF OF THEOREM 1

For the estimation error equation in (18), we choose the Lyapunov function $V(e(t)) = \bar{e}^T(t)P\bar{e}(t)$ as its energy function with $P = P^T > 0$. Then the numerator of (20) becomes

$$
\begin{aligned}
&E\left\{\sum_{t=0}^{t_T}[\bar{e}^T(t)Q\,\bar{e}(t)] - \bar{e}^T(0)P\bar{e}(0)\right\} \\
&= E\{V(\bar{e}(0))\} - E\{V(\bar{e}(t_T+1))\} + E\left\{\sum_{t=0}^{t_T}[\bar{e}^T(t)Q\bar{e}(t)\right. \\
&\quad \left. + V(\bar{e}(t+1)) - V(\bar{e}(t))] - \bar{e}^T(0)P\bar{e}(0)\right\} \\
&= -E\{\bar{e}^T(t_f+1)P\bar{e}(t_f+1)\} + E\left\{\sum_{t=0}^{t_T}[\bar{e}^T(t)Q\bar{e}(t)\right. \\
&\quad \left. + \bar{e}^T(t+1)P\bar{e}(t+1) - \bar{e}^T(t)P\bar{e}(t)]\right\}
\end{aligned} \tag{68}
$$

By substituting (18) into (68), we get

$$
\begin{aligned}
&E\left\{\sum_{t=0}^{t_T}[\bar{e}^T(t)Q\bar{e}(t)] - \bar{e}^T(0)P\bar{e}(0)\right\} \\
&= E\left\{\sum_{t=0}^{t_T}\sum_{i=1}^{J}\sum_{j=1}^{J}\alpha_i(\varepsilon(t))\alpha_j(\varepsilon(t))\left\{\bar{e}^T(t)Q\,\bar{e}(t)\right.\right. \\
&\quad -\bar{e}^T(t)P\,\bar{e}(t) + \left\{(\bar{A}_i - \bar{L}_i\bar{C}_j)\bar{e}(t) + \left[\bar{H}\ -\bar{L}_i\right]\right. \\
&\quad \times \left.\begin{bmatrix}\bar{v}(t)\\ n(t)\end{bmatrix}\right\}^T \times P\left\{(\bar{A}_i - \bar{L}_i\bar{C}_j)\bar{e}(t) + \left[\bar{H}\ -\bar{L}_i\right]\right. \\
&\quad \times \left.\begin{bmatrix}\bar{v}(t)\\ n(t)\end{bmatrix}\right\} - \rho\left[\bar{v}^T(t)\ n^T(t)\right]\begin{bmatrix}\bar{v}(t)\\ n(t)\end{bmatrix} \\
&\quad \left.\left. + \rho\left[\bar{v}^T(t)\ n^T(t)\right]\begin{bmatrix}\bar{v}(t)\\ n(t)\end{bmatrix}\right\}\right\} \\
&= E\left\{\sum_{t=0}^{t_T}\sum_{i=1}^{J}\sum_{j=1}^{J}\alpha_i(\varepsilon(t))\alpha_j(\varepsilon(t))\right. \\
&\quad \times \left[\bar{e}^T(t)\ \bar{v}^T(t)\ \bar{n}^T(t)\right] \\
&\quad \times \left\{\begin{bmatrix}Q-P & 0 & 0\\ 0 & -\rho I & 0\\ 0 & 0 & -\rho I\end{bmatrix} + \begin{bmatrix}(\bar{A}_i - \bar{L}_i\bar{C}_j)^T\\ \bar{H}^T\\ -\bar{L}_i^T\end{bmatrix}\right. \\
&\quad \left. \times P\left[\bar{A}_i - \bar{L}_i\bar{C}_j\ \bar{H}\ -\bar{L}_i\right]\right\} \\
&\quad \left. \times \begin{bmatrix}\bar{e}(t)\\ \bar{v}(t)\\ \bar{n}(t)\end{bmatrix} + \rho\left[\bar{v}^T(t)\ n^T(t)\right]\begin{bmatrix}\bar{v}(t)\\ n(t)\end{bmatrix}\right\}
\end{aligned} \tag{69}
$$

Therefore, if the following inequalities hold

$$
\begin{bmatrix}Q-P & 0 & 0\\ 0 & -\rho I & 0\\ 0 & 0 & -\rho I\end{bmatrix} + \begin{bmatrix}(\bar{A}_i - \bar{L}_i\bar{C}_j)^T\\ \bar{H}^T\\ -\bar{L}_i^T\end{bmatrix} \times P\left[(\bar{A}_i - \bar{L}_i\bar{C}_j)\ \bar{H}\ -\bar{L}_i\right] \le 0, \quad i,j = 1,\ldots,J \tag{70}
$$

then we have

$$
\begin{aligned}
&E\left\{\sum_{t=0}^{t_T}[\bar{e}^T(t)Q\,\bar{e}(t)] - \bar{e}^T(0)P\bar{e}(0)\right\} \\
&\qquad \le \rho E\left\{\sum_{t=0}^{t_T}\left[\bar{v}(t)^T\ n(t)^T\right]\begin{bmatrix}\bar{v}(t)\\ n(t)\end{bmatrix}\right\}
\end{aligned} \tag{71}
$$

i.e. the H∞ estimation performance in (20) holds.

The inequalities in (70) are equivalent to the following inequalities

$$
\begin{aligned}
&\begin{bmatrix}Q-P & 0 & 0\\ 0 & -\rho I & 0\\ 0 & 0 & -\rho I\end{bmatrix} + \begin{bmatrix}(\bar{A}_i - \bar{L}_i\bar{C}_j)^T\\ \bar{H}^T\\ -\bar{L}_i^T\end{bmatrix} P \\
&\quad \times P^{-1}P\left[(\bar{A}_i - \bar{L}_i\bar{C}_j)\ \bar{H}\ -\bar{L}_i\right] \le 0, \\
&\qquad i,j = 1,\ldots,J
\end{aligned} \tag{72}
$$

or

$$
\begin{aligned}
&\begin{bmatrix}Q-P & 0 & 0\\ 0 & -\rho I & 0\\ 0 & 0 & -\rho I\end{bmatrix} + \begin{bmatrix}\bar{A}_i^TP - \bar{C}_j^T\bar{L}_i^TP\\ \bar{H}^TP\\ -\bar{L}_i^TP\end{bmatrix} \\
&\quad \times P^{-1}\left[(P\bar{A}_i - P\bar{L}_i\bar{C}_j)\ P\bar{H}\ -P\bar{L}_i\right] \le 0, \\
&\qquad i,j = 1,\ldots,J
\end{aligned} \tag{73}
$$

By the fact $\bar{Y}_i = P\bar{L}_i$, we get

$$
\begin{aligned}
&\begin{bmatrix}Q-P & 0 & 0\\ 0 & -\rho I & 0\\ 0 & 0 & -\rho I\end{bmatrix} + \begin{bmatrix}\bar{A}_i^TP - \bar{C}_j^T\bar{Y}_i^T\\ \bar{H}^TP\\ -\bar{Y}_i^T\end{bmatrix} \\
&\quad \times P^{-1}\left[(P\bar{A}_i - \bar{Y}_i\bar{C}_j)\ P\bar{H}\ -\bar{Y}_i\right] \le 0, \\
&\qquad i,j = 1,\ldots,J
\end{aligned} \tag{74}
$$

By the Schur complement in Lemma 1, the inequalities in (74) are equivalent to the following LMIs

$$
\begin{bmatrix}Q-P & 0 & 0 & \bar{A}_i^TP - \bar{C}_j^T\bar{Y}_i^T\\ 0 & -\rho I & 0 & \bar{H}^TP\\ 0 & 0 & -\rho I & -\bar{Y}_i^T\\ P\bar{A}_i - \bar{Y}_i\bar{C}_j & P\bar{H} & -\bar{Y}_i & -P\end{bmatrix} \le 0, \quad i,j = 1,\ldots,J \tag{75}
$$

which are the LMIs in (22). After solving $P > 0$ and Y_i from LMIs in (22), we could obtain fuzzy filter gains $\bar{L}_i = P^{-1}Y_i, i = 1,\ldots,J$ Q.E.D.

APPENDIX C
PROOF OF THEOREM 2

The numerator of (42) becomes

$$E\left\{\sum_{t=0}^{t_T}[\bar{\bar{X}}^T(t)\bar{\bar{Q}}\,\bar{\bar{X}}(t)+u^T(t)Ru^T(t)]-\bar{\bar{X}}^T(0)\bar{\bar{P}}\bar{\bar{X}}(0)\right\}$$
$$=E\{\bar{\bar{X}}^T(t_f+1)\bar{\bar{P}}\bar{\bar{X}}(t_f+1)+\sum_{t=1}^{t_T}\bar{\bar{X}}^T(t)\bar{\bar{Q}}\bar{\bar{X}}(t)$$
$$+u^T(t)Ru^T(t)+\bar{\bar{X}}^T(t+1)\bar{\bar{P}}\bar{\bar{X}}(t+1)-\bar{\bar{X}}^T\bar{\bar{P}}\bar{\bar{X}}(t)\} \tag{76}$$

Substituting T-S fuzzy aygmented system in (41) into (76), we get

$$E\left\{\sum_{t=0}^{t_T}[\bar{\bar{X}}^T(t)\bar{\bar{Q}}\,\bar{\bar{X}}(t)+u^T(t)Ru^T(t)]-\bar{\bar{X}}^T(0)\bar{\bar{P}}\bar{\bar{X}}(0)\right\}$$
$$=E\{\bar{\bar{X}}^T(t_T+1)\bar{\bar{P}}\bar{\bar{X}}(t_T+1)\}+E\left\{\sum_{t=0}^{t_T}\sum_{i=1}^{J}\sum_{j=1}^{J}\right.$$
$$\alpha_i(\varepsilon(t))\alpha_j(\varepsilon(t))\{\bar{\bar{X}}^T(t)\bar{\bar{Q}}\bar{\bar{X}}(t)+\bar{\bar{X}}^T[I\text{ -}I\text{ -}I]^TR[I\text{ -}I\text{ -}I]\bar{\bar{X}}$$
$$+(\bar{\bar{X}}^T\bar{\bar{A}}_{ij}^T+\bar{\bar{v}}^T(t)\bar{\bar{H}}_{ij})\times\bar{\bar{P}}(\bar{\bar{A}}_{ij}\bar{\bar{X}}(t)+\bar{\bar{H}}_{ij}\bar{\bar{v}}(t))$$
$$\left.-\bar{\bar{X}}^T(t)\bar{\bar{P}}\bar{\bar{X}}(t)\right\}$$
$$\le E\left\{\sum_{t=0}^{t_T}\sum_{i=1}^{J}\sum_{j=1}^{J}\left[\bar{\bar{X}}^T(t)\ \bar{\bar{v}}^T(t)\right]\right.$$
$$\times\begin{bmatrix}\bar{\bar{Q}}+\bar{\bar{A}}_{ij}^T\bar{\bar{P}}\bar{\bar{A}}_{ij}-\bar{\bar{P}}\\+[I\text{ -}I\text{ -}I]^TK_j^TRK_j[I\text{ -}I\text{ -}I] & \bar{\bar{A}}_{ij}^T\bar{\bar{P}}\bar{\bar{H}}_{ij}\\ \bar{\bar{H}}_{ij}^T\bar{\bar{P}}\bar{\bar{A}}_{ij} & \bar{\bar{H}}_{ij}^T\bar{\bar{P}}\bar{\bar{H}}_{ij}-\rho I\end{bmatrix}$$
$$\left.\times\begin{bmatrix}\bar{\bar{X}}(t)\\ \bar{\bar{v}}(t)\end{bmatrix}+\rho\bar{\bar{v}}^T(t)\bar{\bar{v}}(t)\right\} \tag{77}$$

By the fact of matrix inequalities in (43), we get $E\{\sum_{t=0}^{t_T}[\bar{\bar{X}}^T(t)\bar{\bar{Q}}\ \bar{\bar{X}}(t)+u^T(t)Ru^T(t)]-\bar{\bar{X}}^T(0)\bar{\bar{P}}\bar{\bar{X}}(0)\}\le\rho E\{\sum_{t=0}^{t_T}[\bar{\bar{v}}^T(t)\bar{\bar{v}}(t)]\}$ which is the H∞ estimator-based tracking control performance in (42). Q.E.D

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