IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received November 24, 2021, accepted December 5, 2021, date of publication December 8, 2021, date of current version December 17, 2021.

*Digital Object Identifier 10.1109/ACCESS.2021.3134009*

# Model Predictive Controller for Quadcopter Trajectory Tracking Based on Feedback Linearization

**ZHENHUAN CAI[1], SUOHUAI ZHANG[1], AND XUEDONG JING[2]**
[1]School of Mechanical Engineering, Shanghai Institute of Technology, Shanghai 201416, China
[2]School of Electrical and Electronic Engineering, Shanghai Institute of Technology, Shanghai 201416, China

Corresponding author: Xuedong Jing (jingkd2003@aliyun.com)

This work was supported in part by Science and Technology Commission of Shanghai Municipality under Grant 16090503700.

**ABSTRACT** In this study, a feedback linearization model predictive control algorithm is designed for quadcopter trajectory tracking. By applying feedback linearization to the quadcopter nonlinear model, the quadcopter nonlinear model is transformed into a linear system as the foundation for further controller design. By feedback linearization, the linear control schemes can be used to control the quadcopter. A model predictive controller is then designed for the linearized quadcopter model without considering the external disturbance. A disturbance observer is designed to estimate the external disturbance, keeping the estimation error BIBO stable to compensate for the external disturbance. Numerical simulations are performed to evaluate the proposed control algorithm. The simulations are performed in two test scenarios to examine the tracking performance. The quadcopter is commended for reaching a waypoint (scenario. I) and tracking a helical trajectory (scenario. II) in the simulations, and the root means square errors are calculated to demonstrate the tracking effectiveness. The simulation results show that the designed control algorithm can effectively ensure the quadcopter tracks a given trajectory under constant or continuous disturbances.

**INDEX TERMS** Quadcopter, model predictive control, feedback linearization.

## I. INTRODUCTION

Quadcopters are widely used UAVs (Unmanned Aerial Vehicles) in practical applications. Quadcopters are popular consumer products for individual users for entertainment. The quadcopters' application is expanding quickly from personal users to multiple fields like industries, agriculture, filming, etc. For example, the quadcopters can monitor the status of the growing agricultural products and do the pesticide spraying, watering, and fertilizing. The employment of the quadcopters improves agriculture's intelligence level, making it more profitable and effective [1]–[3]. Most quadcopters' autonomous tasks require the quadcopters to follow a given trajectory like mapping, surveillance, spraying, etc. Therefore, trajectory tracking control is an essential research subject for quadcopters.

The quadcopters can perform 6 DOFs (Degrees of Freedom) movements by the thrust and moments generated by four rotors. The quadcopters are economically efficient and can be easily assembled. However, the control of the quadcopters is still challenging for reliability, precision, safety, and efficiency. The quadcopter is a MIMO system with four rotors' rotation rates as inputs to control multiple system states of position and rotation. The system states are coupled and nonlinear. Thus, the quadcopter system is an underactuated coupled nonlinear system—the fluence of external disturbances like winds, temperature changes, solid particles, etc., also affects tracking performance. To overcome the nonlinearity of the quadcopter systems, linearization methods based on the Taylor series are often applied. However, the round-off errors of these kinds of linearization approaches are inevitable and may decrease the controllers' performance. The controllers are usually designed with multi-loops to avoid the coupling between the position states and rotation states. The inner-loop stabilize the rotation states and the outer-loop for the position, where the outer-loop generates the desired trajectory for the inner-loop. The external disturbances need to be estimated by appropriate approaches to be compensated by the quadcopter controller.

The associate editor coordinating the review of this manuscript and approving it for publication was Xujie Li.

### A. RELATED WORKS

Several linear controllers are first deployed to control the quadcopters for trajectory tracking, like PID(Proportional-Integral-Differential) controllers and LQR (linear quadratic regulator) controllers [4]–[8]. PID controllers are commonly designed around the quadcopter system's equilibrium point, the hovering point for quadcopters. The setting of the controller parameters for PID controllers is the main challenge for these control schemes. A set of constant parameters can not satisfy different flight circumstances and external disturbances.

Linear Quadratic Regulator controllers are based on the quadcopters' state-space model. The central concept of LQR is to optimize a defined cost function to find an optimal control inputs vector. However, the LQR control schemes require linear models. The linearization of the quadcopter systems ignores multiple nonlinear parts, leading to deficient performance of such controllers when the internal or external parameters are variable. According to [7], the LQR control can only stabilize the quadcopter system for trajectory tracking when the quadratic sum of pitch and roll angle is less than $48^{\circ 2}$.

The linear control schemes like PID and LQR can stabilize the quadcopter system when the states of the quadcopter are around the equilibrium point, which is caused by the nonlinear characteristic of the quadcopters. The commonly used linearization approaches will ignore the nonlinear parts, which will reduce the controllers' performance. Thus, nonlinear control schemes are applied to control the quadcopter systems.

Feedback linearization approaches like input-output linearization and state-space linearization transform the nonlinear quadcopter system into a linear one [9], [10]. The transformed mathematical model and the original system are diffeomorphisms; therefore, the nonlinear parts of the system are still concerned for further control algorithm design. Sliding Mode Control is well known for its robustness and ability to reject internal disturbances and eliminate external disturbances [11]. In [12] and [13], second-order SMC controllers are designed for a quadcopter to track a trajectory, and the Lyapunov theory proves the stability of the proposed SMC controllers. Backstepping control schemes are focused on the dynamic models of the quadcopter without linearization [14], [24], [25]. Reference [15] proposed a dynamic backstepping control scheme that only requires first and second-order derivatives of the desired trajectory, thus decreasing the controller's sensibility to the disturbances. Robust control approaches are employed to control the quadcopter systems to overcome the uncertainties of the system parameters and external disturbances.

In [16], a robust attitude tracking controller based on a nonlinear disturbance observer is designed for a quadcopter to track the desired attitude trajectory with internal model uncertainties. Reference [17] proposed an H infinity controller with two degrees of freedom to improve the quadcopter's hovering performance under external disturbances. In [18] and [19], by establishing multiple linearized models around different operating points of the quadcopter, various controllers are designed based on Linear Matrix Inequality to ensure the overall stability of the quadcopter's attitude control. Model Predictive Control (MPC) is a kind of optimization control scheme. The central concept is to predict the system's future response and optimize control inputs on a receding horizon. In [20], a switching model predictive controller for quadcopter is designed, different predictive models are presented for different quadcopter states to stabilize the quadcopter subject to atmospheric disturbances.

Besides the above control schemes, learning strategies are becoming popular to enhance the quadcopters' control performance. Combining machine learning and MPC, reference [21] acquired the quadcopter's dynamics by learning algorithms and improved the performance of the MPC controller. Reinforcement learning is used for a quadcopter to learn the control law itself by failure and reward functions [22].

### B. MAIN CONTRIBUTIONS

In this study, a model predictive controller for quadcopter is designed based on feedback linearization. The main motivation of this study is to obtain a more accurate linearized predictive model for applying model predictive control and decrease the calculate complexity of optimizing the control inputs for applications. Model predictive control is highly reliant on the accuracy of the predictive model. Therefore, approximative linear methods based on the Taylor series will decrease the controller performance. To reduce the errors caused by approximative linearization, feedback linearization is performed. The original nonlinear model is transformed into a linear one and converts the original 4-inputs-6-outputs system into a 4-inputs-4-outputs system, which eliminates the coupling of the horizontal movements and the Euler angles. By feedback linearization, the system can be controlled by a linear model predictive controller, and the linearized model is a better approximation to the original model. In the stage of receding horizon control, a linear model predictive controller solves a convex optimization problem, where a nonlinear model predictive controller solves a nonconvex optimization problem, which consumes much more computational power and reduces the controller's instantaneity. Therefore, the proposed feedback linearization model predictive controller increases both accuracy and efficiency for quadcopter trajectory control compared to traditional model predictive controllers. To compensate for external disturbances, a disturbance observer is designed to estimate constant or variable wind disturbances. Numerical simulations examine the trajectory tracking performance of the designed controller, and the results show the validity of the proposed controller. The quadcopter can track the given trajectories with low RMSE and can eliminate the external wind disturbances.

The article is structured as follows: in Section II, the dynamic model of the quadcopter is proposed. The controller design with feedback linearization, MPC design, and disturbance observer design is presented in Section III. Simulations and results are presented in Section IV with discussion. The conclusion of this study is presented in Section V.

## II. DYNAMIC MODEL OF QUADCOPTER

The modeling of the quadcopter based on Newton-Euler equations is widely used by studies on quadcopter control schemes. The establishment of the quadcopter mathematic model is briefly discussed in this section. First, as shown in Fig. 1, the inertial frame of reference $I$ and the body frame $B$ are defined. The position of the quadcopter in the inertial frame is determined by $\zeta$ along the axis $x_I, y_I, z_I$. The rotation of the quadcopter is defined by Euler angles $\eta$. The Roll angle $\phi$ denotes the rotation around the axis $x_I$, the Pitch angle $\theta$ denotes the rotation around the axis $y_I$ and the Yaw angle $\psi$ denotes the rotation around the axis $z_I$.

$$\zeta = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \eta = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} \tag{1}$$

The transform matrix transforming the body frame to the inertial frame using the X-Y-Z rotation sequence can be written as (2):

$$R = \begin{bmatrix} c_\theta c_\psi & c_\theta & -s_\theta \\ s_\phi s_\theta c_\psi - c_\phi s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & s_\phi c_\theta \\ c_\phi s_\theta c_\psi + s_\phi s_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi & c_\phi c_\theta \end{bmatrix} \tag{2}$$

where $s_{(\cdot)}$ denotes $sin(\cdot)$ and $c_{(\cdot)}$ denotes $cos(\cdot)$. We define four quadcopter control inputs $U_1, U_2, U_3, U_4$ to represent four kinds of movements of a quadcopter where $U_1$ is the total thrust generated by four propellers, $U_2$ is the roll moment, $U_3$ is the pitch moment and $U_4$ is the yaw moment. Thus, the translation dynamics of the quadcopter can be obtained by Newton equation and the transform matrix (2) as:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} -s_\theta \dfrac{U_1}{m} \\ s_\phi c_\theta \dfrac{U_1}{m} \\ c_\phi c_\theta \dfrac{U_1}{m} - g \end{bmatrix} \tag{3}$$

where $\ddot{x}, \ddot{y}, \ddot{z}$ are the acceleration along the axis $x_I, y_I, z_I$, and $m$ is the mass of the quadcopter.

Define the angular rate in the body frame as $p, q, r$ around the axis $x_B, y_B, z_B$ respectively. Taking the transform matrix (2) into consideration, we can obtain the relationship between the body angular rates and the Euler rates $\dot{\phi}, \dot{\theta}, \dot{\psi}$:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \dfrac{c_\psi}{c_\theta} & -\dfrac{s_\psi}{c_\theta} & 0 \\ s_\psi & c_\psi & 0 \\ -c_\psi t_\theta & s_\psi t_\theta & 1 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{4}$$

Assume the rotation of the roll angle, and the pitch angle is slight, equation (4) can linearize as:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} c_\psi & -s_\psi & 0 \\ s_\psi & c_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{5}$$

By the Euler equation, we can obtain the rotation dynamics:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{I_x}\left[\left(I_y - I_z\right) qr - I_p q \omega_T + U_2\right] \\ \dfrac{1}{I_y}\left[\left(I_z - I_x\right) pr - I_p p \omega_T + U_3\right] \\ \dfrac{1}{I_z}\left[\left(I_x - I_y\right) pq + U_4\right] \end{bmatrix} \tag{6}$$

where $I_x, I_y, I_z$ are the moments of inertia along the body axes $x_B, y_B, z_B, I_p$ is the moment of inertia of the propeller along its rotation axis, $\omega_T$ is the sum of the four propellers' rotational speed.

The quadcopter is X-type as shown in Figure 1, the relationship between the control inputs $\boldsymbol{U} = \left[ U_1 \; U_2 \; U_3 \; U_4 \right]^T$ and the rotational speeds of four propellers $\boldsymbol{\omega}^2 = \left[ \omega_1^2 \; \omega_2^2 \; \omega_3^2 \; \omega_4^2 \right]^T$ is:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} b & b & b & b \\ \dfrac{\sqrt{2}}{2} lb & -\dfrac{\sqrt{2}}{2} lb & -\dfrac{\sqrt{2}}{2} lb & \dfrac{\sqrt{2}}{2} lb \\ \dfrac{\sqrt{2}}{2} lb & \dfrac{\sqrt{2}}{2} lb & -\dfrac{\sqrt{2}}{2} lb & -\dfrac{\sqrt{2}}{2} lb \\ d & -d & d & -d \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \tag{7}$$

where $b$ denotes the thrust coefficient, and $d$ denotes the drag coefficient. $l$ is the length from the propeller's rotation axis to the quadcopter's rotation center.

Combine equations (3) to (6) and consider wind disturbance, the nonlinear state-space model of the quadcopter can be written as follows:

$$\dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \\ \dot{x}_9 \\ \dot{x}_{10} \\ \dot{x}_{11} \\ \dot{x}_{12} \end{bmatrix} = f(X) = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ -s_\theta \dfrac{U_1}{m} + d_{wx} \\ s_\phi c_\theta \dfrac{U_1}{m} + d_{wy} \\ c_\phi c_\theta \dfrac{U_1}{m} - g + d_{wz} \\ c_\psi p - s_\psi q \\ s_\psi p + c_\psi q \\ r \\ \dfrac{1}{I_x}\left[\left(I_y - I_z\right) qr - I_p q \omega_T + U_2\right] \\ \dfrac{1}{I_y}\left[\left(I_z - I_x\right) pr - I_p p \omega_T + U_3\right] \\ \dfrac{1}{I_z}\left[\left(I_x - I_y\right) pq + U_4\right] \end{bmatrix} \tag{8}$$

the state vector is $X = [x \; y \; z \; \dot{x} \; \dot{y} \; \dot{z} \; \phi \; \theta \; \psi \; p \; q \; r]^T$. $d_{wx}, d_{wy}, d_{wz}$ are the acceleration caused by wind disturbance along $x_I, y_I, z_I$ respectively.

## III. CONTROLLER DESIGN

In this section, a feedback linearization model predictive controller for quadcopter is proposed in two main steps.

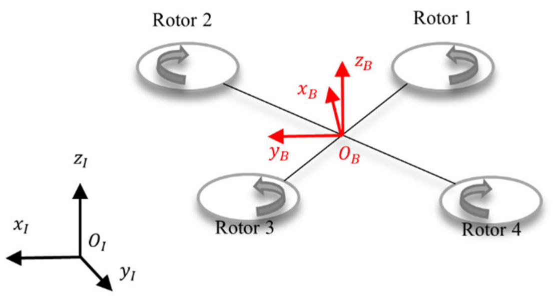


**FIGURE 1. Frame definition of the quadcopter.**

Firstly, the quadcopter's nonlinear state-space model is linearized with a feedback-linearization-based control law. Then a model predictive control is established based on the linearized model to follow the desired trajectory.

### A. FEEDBACK LINEARIZATION

The goal of feedback linearization is to convert the nonlinear quadcopter model into a linear one. In this subsection, the nonlinear quadcopter state-space model is linearized by feedback linearization. The first step to feedback-linearize the quadcopter system is to define the outputs of the linearized system. The main task of the proposed controller is to let the quadcopter track a given translation trajectory and follow a given yaw angle. Therefore, the outputs are chosen as:

$$Y = \begin{bmatrix} x & y & z & \psi \end{bmatrix}^T \tag{9}$$

After defining the outputs of the system, a control law $V \in \mathbb{R}^{4\times 1}$ is designed to linearize the nonlinear model, where it is intended to be:

$$V = \alpha(X) + \beta(X)U \tag{10}$$

where:

$$\alpha(X) = \begin{bmatrix} \alpha_1(X) \\ \alpha_2(X) \\ \alpha_3(X) \\ \alpha_4(X) \end{bmatrix} \tag{11}$$

is a $4 \times 1$ vector of scalar functions $\alpha_i(X)$, $i = 1, 2, 3, 4$. And

$$\beta(X) = \begin{bmatrix} \beta_{11}(X) & \beta_{12}(X) & \beta_{13}(X) & \beta_{14}(X) \\ \beta_{21}(X) & \beta_{22}(X) & \beta_{23}(X) & \beta_{24}(X) \\ \beta_{31}(X) & \beta_{32}(X) & \beta_{33}(X) & \beta_{34}(X) \\ \beta_{41}(X) & \beta_{42}(X) & \beta_{43}(X) & \beta_{44}(X) \end{bmatrix}$$

is a $4 \times 4$ matrix of scalar functions $\beta_{ij}(X)$, $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$. To determine the control law $V$, a new state vector $W$ is defined as:

$$W = h(X) \tag{12}$$

where $h(X)$ is a diffeomorphism from $X$ to $W$ that ensures the new state vector $W$ describe the same system as $X$. The $h(X)$ is determined by taking Lie derivatives of $i \in \{x, y, z, \psi\}$ respect to $f(X)$ until former control inputs $\mathbf{U}$ appear in the $n^{th}$ Lie derivative, then take the 0th to the $n - 1^{th}$ Lie derivatives as the diffeomorphism $h(X)$. In the state-space model (8), the nonlinear parts: $\frac{(I_y - I_z)qr - I_p q \omega_T}{I_x}$, $\frac{(I_z - I_x)pr - I_p q \omega_T}{I_y}$, $\frac{(I_x - I_y)qr - I_p q \omega_T}{I_z}$ are ignored because the value of these parts is relatively small compared to control inputs, and the inertia is small due to the light-weighted propellers. Therefore, the $h(X)$ is determined as:

$$h(X) = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \\ w_8 \\ w_9 \\ w_{10} \\ w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \end{bmatrix} = \begin{bmatrix} L_f^0 x \\ L_f^1 x \\ L_f^2 x \\ L_f^3 x \\ L_f^0 y \\ L_f^1 y \\ L_f^2 y \\ L_f^3 y \\ L_f^0 z \\ L_f^1 z \\ L_f^2 z \\ L_f^3 z \\ L_f^0 \psi \\ L_f^1 \psi \end{bmatrix}$$

$$= \begin{bmatrix} x \\ \dot{x} \\ -s_\theta \frac{U_1}{m} \\ -\dot{\theta} c_\theta \frac{U_1}{m} - s_\theta \frac{\dot{U}_1}{m} \\ y \\ \dot{y} \\ s_\phi c_\theta \frac{U_1}{m} \\ \dot{\phi} c_\phi c_\theta \frac{U_1}{m} - \dot{\theta} s_\phi s_\theta \frac{U_1}{m} + s_\phi c_\theta \frac{\dot{U}_1}{m} \\ z \\ \dot{z} \\ c_\phi c_\theta \frac{U_1}{m} - g \\ -\dot{\phi} s_\phi c_\theta \frac{U_1}{m} - \dot{\theta} c_\phi s_\theta \frac{U_1}{m} + c_\phi c_\theta \frac{\dot{U}_1}{m} \\ \psi \\ r \end{bmatrix} \tag{13}$$

Notice that $\ddot{U}_1$ appears in $L_f^4 x$, $L_f^4 y$, $L_f^4 z$, Therefore $U$ and $\dot{U}$ are added into the original state-space model as $x_{13}$ and $x_{14}$ respectively. Therefore, the linearized state-space model can be written as:

$$\dot{W} = \begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \\ \dot{w}_4 \\ \dot{w}_5 \\ \dot{w}_6 \\ \dot{w}_7 \\ \dot{w}_8 \\ \dot{w}_9 \\ \dot{w}_{10} \\ \dot{w}_{11} \\ \dot{w}_{12} \\ \dot{w}_{13} \\ \dot{w}_{14} \end{bmatrix} = \begin{bmatrix} w_2 \\ w_3 \\ w_4 \\ \alpha_1(X) + \beta_{11}\ddot{U}_1 + \beta_{12}U_2 + \beta_{13}U_3 + \beta_{14}U_4 \\ w_6 \\ w_7 \\ w_8 \\ \alpha_2(X) + \beta_{21}\ddot{U}_1 + \beta_{22}U_2 + \beta_{23}U_3 + \beta_{24}U_4 \\ w_{10} \\ w_{11} \\ w_{12} \\ \alpha_3(X) + \beta_{31}\ddot{U}_1 + \beta_{32}U_2 + \beta_{33}U_3 + \beta_{34}U_4 \\ w_{14} \\ \alpha_4(X) + \beta_{41}\ddot{U}_1 + \beta_{42}U_2 + \beta_{43}U_3 + \beta_{44}U_4 \end{bmatrix} \tag{14}$$

where:

$$\alpha_1 = \dot{\theta}^2 s_\theta \frac{U_1}{m} - 2\dot{\theta} c_\theta \frac{\dot{U}_1}{m},$$
$$\alpha_2 = -\left(\dot{\phi}^2 + \dot{\theta}^2\right) s_\phi c_\theta \frac{U_1}{m} - 2\dot{\phi}\dot{\theta} c_\phi s_\theta \frac{U_1}{m} + 2(\dot{\phi} c_\phi c_\theta \frac{\dot{U}_1}{m} - \dot{\theta} s_\phi s_\theta \frac{\dot{U}_1}{m}),$$
$$\alpha_3 = -\left(\dot{\phi}^2 + \dot{\theta}^2\right) c_\phi c_\theta \frac{U_1}{m} + 2\dot{\phi}\dot{\theta} s_\phi s_\theta \frac{U_1}{m} - 2(\dot{\phi} s_\phi c_\theta \frac{\dot{U}_1}{m} + \dot{\theta} c_\phi s_\theta \frac{\dot{U}_1}{m}),$$
$$\alpha_4 = 0.$$
$$\beta_{11} = -\frac{s_\theta}{m}, \quad \beta_{12} = -\frac{c_\theta s_\psi U_1}{m I_x}, \quad \beta_{13} = -\frac{c_\theta c_\psi U_1}{m I_y},$$
$$\beta_{14} = 0, \quad \beta_{21} = \frac{s_\phi c_\theta}{m},$$
$$\beta_{22} = \frac{(c_\phi c_\theta c_\psi - s_\phi s_\theta s_\psi) U_1}{m I_x},$$
$$\beta_{23} = -\frac{(c_\phi c_\theta s_\psi + s_\phi s_\theta c_\psi) U_1}{m I_y}, \quad \beta_{24} = 0, \ \beta_{31} = \frac{c_\phi c_\theta}{m},$$
$$\beta_{32} = -\frac{(s_\phi c_\theta c_\psi + c_\phi s_\theta s_\psi) U_1}{m I_x}, \ \beta_{33} = \frac{(s_\phi c_\theta s_\psi - c_\phi s_\theta c_\psi) U_1}{m I_y},$$
$$\beta_{34} = 0, \quad \beta_{41} = \beta_{42} = \beta_{43} = 0, \ \beta_{44} = \frac{1}{I_z}.$$

Thus, the quadcopter system is linearized into the linear state-space model bellow:

$$\begin{aligned} \dot{W} &= AW + BV \\ Y &= CW \end{aligned} \tag{15}$$

where

$$A = \begin{bmatrix} A_1 & 0^{4\times4} & 0^{4\times4} & 0^{4\times2} \\ 0^{4\times4} & A_1 & 0^{4\times4} & 0^{4\times2} \\ 0^{4\times4} & 0^{4\times4} & A_1 & 0^{4\times2} \\ 0^{2\times4} & 0^{2\times4} & 0^{2\times4} & A_2 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 1&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&1&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&1&0 \\ 0&0&0&0 \\ 0&0&0&1 \end{bmatrix}, \quad C^T = \begin{bmatrix} 1&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&1&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&1&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&1 \\ 0&0&0&1 \end{bmatrix}$$

### B. MODEL PREDICTIVE CONTROLLER

In this subsection, a model predictive controller is designed for the quadcopter system to track a given trajectory. Model predictive control is achieved by two main steps: prediction based on a predictive model and receding horizon optimization. The predictive model is the feedback-linearized quadcopter model (15). Model predictive control is commonly used in digital control systems, so the quadcopter model needs to be discretized before designing the model predictive controller. Define predictive period $T$, which is the step length of the model predictive controller. Then the discrete form of the quadcopter model is:

$$W(k+1|k) = (I + TA)\, W(k) + TBV(k) \tag{16}$$

where $k$ denotes the current time step, and $I$ is an identity matrix. Notation $k|k+1$ means the state is the prediction of time step $k+1$ at time step $k$. Therefore, the discrete model of the quadcopter system is:

$$W(k+1) = \bar{A}W(k) + \bar{B}V(k) \tag{17}$$

where $\bar{A} = (I + TA)$ and $\bar{B} = TB$. Denote the predicted state vector in predictive horizon $p$ as:

$$W_p = \left[W(k+1\,|\,k)^T W(k+2\,|\,k)^T \cdots W(k+p\,|\,k)^T\right]^T \tag{18}$$

and the control inputs are

$$V_k = \left[V(k\,|\,k)^T V(k+1\,|\,k)^T \cdots V(k+p-1\,|\,k)^T\right]^T \tag{19}$$

Apply the model (17) and (19) into (18), and the predicted states can be written as:

$$W_p = \Psi W(k) + \Theta V_p \tag{20}$$

where:

$$\Psi = \begin{bmatrix} \bar{A}^1 \\ \bar{A}^2 \\ \vdots \\ \bar{A}^p \end{bmatrix}, \quad \Theta = \begin{bmatrix} \bar{A}^{1-1}\bar{B} & \cdots & 0 & 0 \\ \bar{A}^{2-1}\bar{B} & \bar{A}^{2-2}\bar{B} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \bar{A}^{p-1}\bar{B} & \bar{A}^{p-2}\bar{B} & \cdots & \bar{A}^{p-p}\bar{B} \end{bmatrix} \tag{21}$$

The goal of the design of the quadcopter controller is to track a given trajectory. The desired trajectory in predictive horizon $p$ is:

$$R_k = [R(k+1)^T R(k+2)^T \cdots R(k+p)^T] \tag{22}$$

To optimize the trajectory tracking performance, define the cost function as:

$$J = \left(W_p - R_k\right)^T Q \left(W_p - R_k\right) + V_k^T F V_k \tag{23}$$

where $Q$ and $F$ are weight matrices to weight the tracking error and the control inputs, respectively, by substituting equation (21) into (23), the cost function can be written as:

$$J = E(k+1)^T G E(k+1) + 2E(k)^T M V_k + V_k^T H V_k \tag{24}$$

where $G = \Psi^T Q\Psi$, $E = \Psi^T Q\Theta$, $H = \Theta Q\Theta + F$ and $E(k+1) = W(k+1) - R(k+1)$. To control the quadcopter, the cost function should be minimized. Therefore, the problem is transformed into a quadratic programming problem:

$$\min_{V_k} \frac{1}{2} V_k^T H V_k + E(k)^T M V_k$$
$$s.b.\ V_{kmin} \leq V_k \leq V_{kmax} \tag{25}$$

By solving the quadratic programming problem, we can acquire $V_k$ with $p$ control inputs vectors. Then choose the first inputs vector $V(k)$ as the control inputs for current time step $k$. According to the equation (10), $U$ can be acquired as:

$$U = \beta^{-1}(X)[V_k - \alpha(X)] \tag{26}$$

The matrix $\beta(X)$ should be invertible to calculate the control input $U$. The determinant of matrix $\beta(X)$ is calculated to be:

$$|\beta(X)| = \frac{c_\theta U_1^2}{m^3 I_x I_y I_z} \tag{27}$$

The pitch angle $\theta$ is constrained to be $-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$, hence, $c_\theta > 0$. $U_1$ is the total thrust generated by four propellers, which is constantly greater than zero during the flight. Therefore, $|\beta(X)| > 0$ and the control inputs $U$ can be determined by equation (26).

### C. DISTURBANCE OBSERVER

In actual flight circumstances, the wind disturbance is not negligible. Therefore, a simple disturbance observer is designed in this subsection. Since the quadcopter mathematical model is linearized by feedback linearization, it is possible to develop a linear disturbance observer for the controller.

Taking disturbance into consideration, system (15) can be extended into:

$$\dot{W} = AW + BV + B_d d$$
$$Y = CW \tag{28}$$

where $B_d$ is the coefficient matrix of disturbance vector $d$. Given by [23], the time-domain disturbance observer can be employed to estimate the disturbances in the linearized quadcopter system (15):

$$\begin{cases} \dot{\gamma} = -LB_d(\gamma + LW) - L(AW + BV) \\ \hat{d} = \gamma + LW \end{cases} \tag{29}$$

where $\gamma$ is the internal variable vector with the initial value of zero, $L$ is the gain matrix of the disturbance observer to be designed later.

Define the error of the estimation as:

$$e = d - \hat{d} \tag{30}$$

Take the derivative of $e$:

$$\dot{e} = -LB_d e + \dot{d} \tag{31}$$

Therefore, by designing an appropriate observer gain $L$ to ensure the matrix $-LB_d$ Hurwitz, the estimation error is then BIBO stable. In addition, if the disturbance derivative $\dot{d}$ is tend to zero when $t \to \infty$, the error system is asymptotically stable.

To compensate for the estimated disturbances of the quadcopter system, the disturbance observer is transformed into the discrete form:

$$\begin{cases} \dot{\gamma}(k) = -LB_d(\gamma(k-1) + LW(k)) - L(AW(k) + BV(k)) \\ \hat{d} = \gamma(k-1) + T\dot{\gamma}(k) + LW \end{cases} \tag{32}$$

Therefore, the estimated disturbances $\hat{d}$ can be used to extract the disturbances from the measured states of the quadcopter system.

## IV. SIMULATIONS AND RESULTS

This section performs numerical simulations to evaluate the proposed feedback linearization model predictive controller (FL-MPC). The simulations are performed in two scenarios. In the first scenario, the desired trajectory is a point in a 3D space, which evaluates the controllers' response to a set waypoint. The performance of the disturbance observer is also examined. The second scenario is to track a helical trajectory in a 3D space, which evaluates the overall performance of the controllers for trajectory tracking by RMSE.

### A. ASSUMPTIONS

The simulations are based on certain assumptions: the attitude and position of the quadcopter are measured by inertial measurement units (IMUs) placed at the center of the quadcopter, the measurements are without errors, and the actuator errors are ignored. The real quadcopter is represented by the nonlinear model (8), the wind disturbance is bounded and continuous. The parameters of the quadcopter are selected to be: $m = 2kg$, $g = 9.81m/s^2$, $I_x = 1.25kg \cdot m^2$, $I_y = 1.25kg \cdot m^2$, $I_z = 2.5kg \cdot m^2$, $I_p = 5\times10^{-5}kg{\cdot}m^2$, $b = 2.5\times10^{-5}N \cdot s^2$, $l = 0.25m$, $d = 0.5\times10^{-6}N \cdot m{\cdot}s^2$. The initial condition of the quadcopter's states are all zeros. Assume that the quadcopter is taken off and already hovering around the initial position. The parameters of the FL-MPC are shown in Table. 1.

The disturbance observer gain $L$ should satisfy that $-LB_d$ is Hurwitz. The coefficient matrix $B_d$ is:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T.$$

The disturbance observer gain $L$ is then chosen to be:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

which ensures that the observer has sufficient convergence speed and stability.

**TABLE 1. Parameters for FL-MPC.**

| Symbol | Quantity | Value |
|---|---|---|
| $T$ | controller time step | $0.1\ s$ |
| $p$ | control horizon | $10\ steps$ |
| $Q$ | weight for tracking error | $10 \cdot I^{14\times p}$ |
| $F$ | weight for control inputs | $0.01 \cdot I^{4\times p}$ |
| $umax$ | maximum control constraint | $[1\ 0.05\ 0.05\ 0.05]^T$ |
| $umin$ | minimum control constraint | $[-1 - 0.05 - 0.05 - 0.05]^T$ |

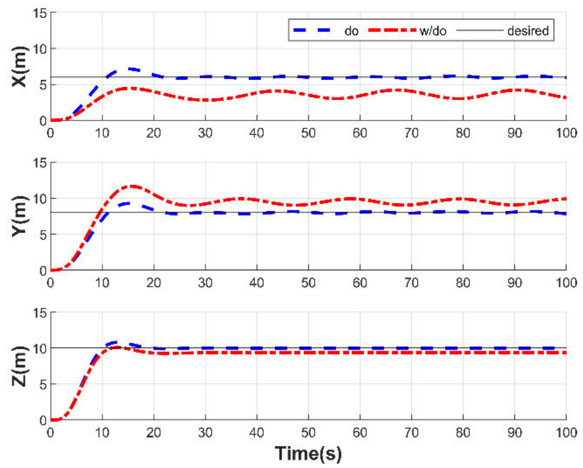


**FIGURE 2. Simulation No.1: Position trajectories with constant wind disturbance.**

### B. TEST SCENARIO I: WAYPOINT TRACKING

In test scenario I, the desired waypoint is given by: $\boldsymbol{x_d = 6m, y_d = 8m, z_d = 10m}$ and the desired yaw angle is set to $\boldsymbol{\psi_d = 0}$. The simulation lasts 100 seconds. Two kinds of wind disturbance are added to the quadcopter, respectively: in simulation No.1, a constant wind disturbance is added to the system, where $\boldsymbol{d_{wx} = 0.12m/s^2, d_{wy} = -0.08m/s^2}$ and $\boldsymbol{d_{wy} = 0.05m/s^2}$.

Fig. 2 and Fig. 3 presents the quadcopter position trajectories with constant wind disturbance with or without the disturbance observer designed to compensate for the external disturbance. The external disturbance results in obvious steady-state error and oscillations when the disturbance observer is not employed to the FL-MPC controller. The disturbance observer reduced the steady-state error tremendously, and the oscillations were alleviated. However, small offsets still exist compared to the desired trajectory due to the ignored small nonlinear dynamics like the gyroscopic effect of the propellers and the quadcopter itself.

The actual disturbance and the estimated disturbance are presented in Fig. 4. The result shows that the designed linear disturbance observer can estimate the constant disturbance with acceptable speed and accuracy.

In the simulation I, the wind disturbance varies over time. The disturbance is given as superposed sine waves. The

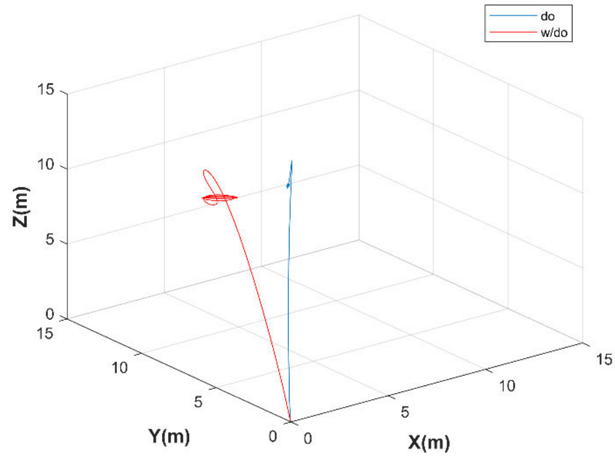


**FIGURE 3. Simulation No.1: Position trajectories with constant wind disturbance in 3D space.**

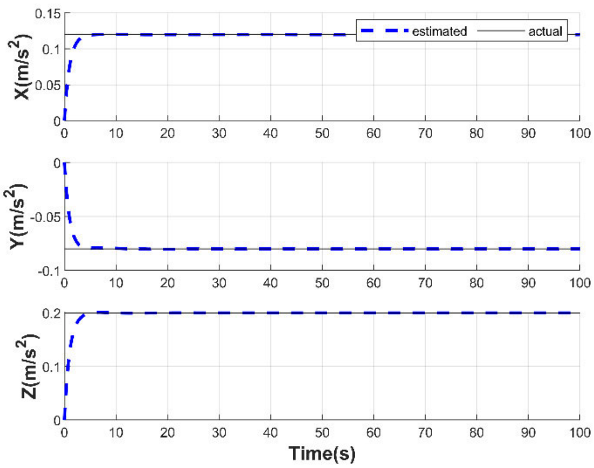


**FIGURE 4. Actual and estimated disturbance (Simulation No.1).**

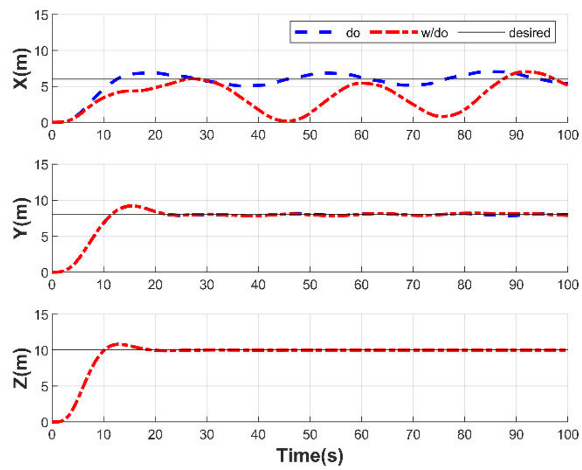


**FIGURE 5. Simulation No.2: Position trajectories with variational wind disturbance.**

disturbance is added to the system as:

$$d_{wx} = 0.15 \sin\left(\frac{\pi t}{100}\right) + 0.1 \sin(0.2t) + 0.03 sint \quad (33)$$

and $d_{wy} = 0m/s^2$ and $d_{wy} = 0m/s^2$.

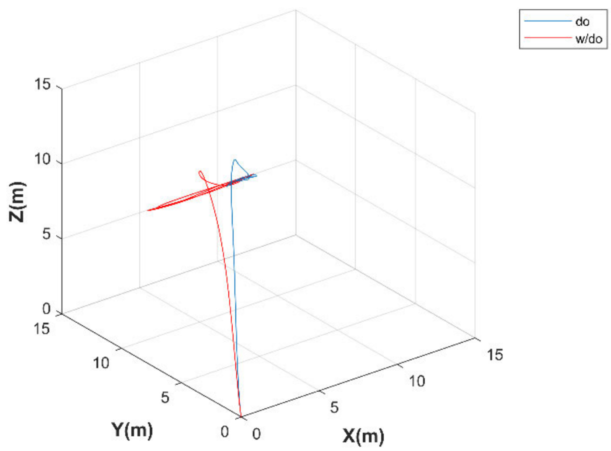


**FIGURE 6. Simulation No.2: Position trajectories with variational wind disturbance in 3D space.**

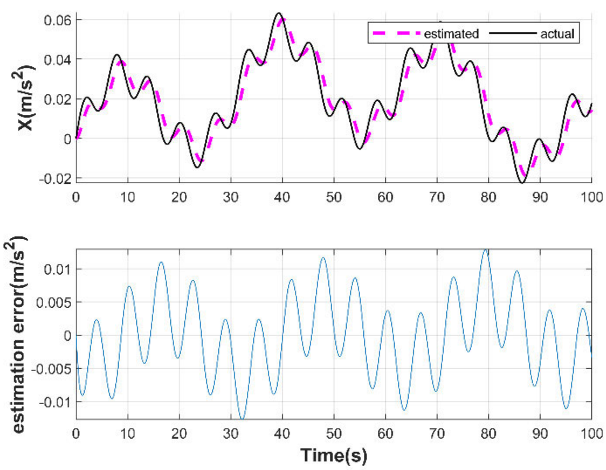


**FIGURE 7. Actual and estimated disturbance (Simulation No.2).**

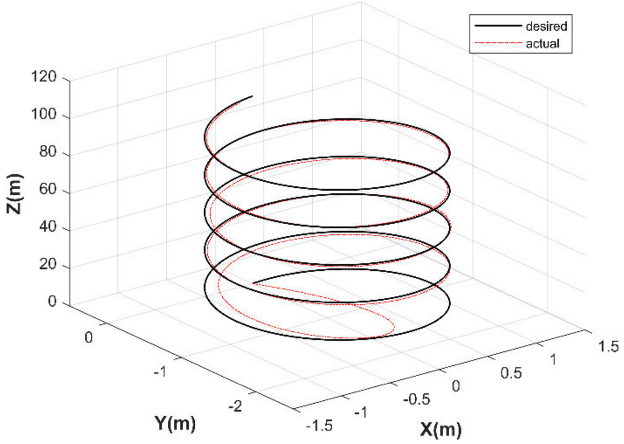


**FIGURE 8. Actual and desired trajectory in 3D space (Simulation No.3).**

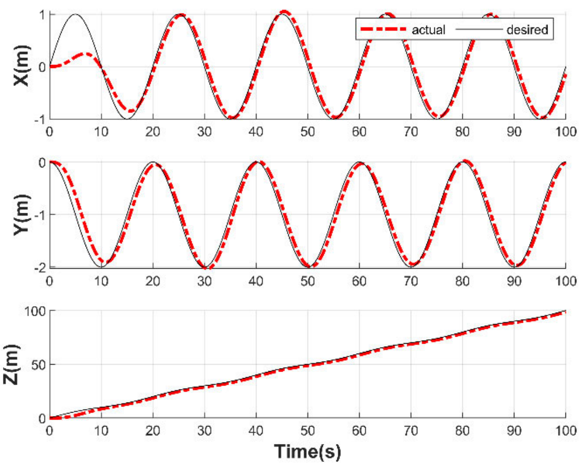


**FIGURE 9. Actual and desired trajectory (Simulation No.3).**

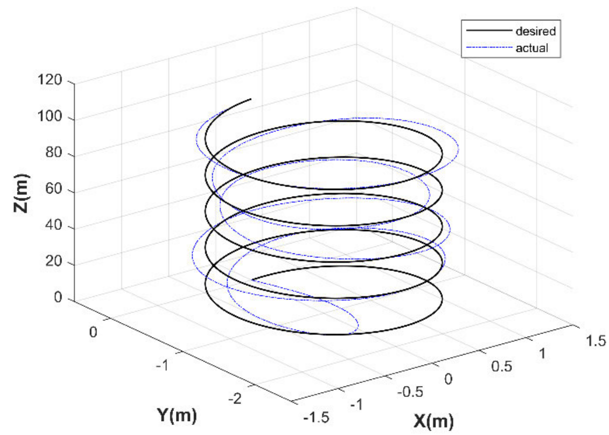


**FIGURE 10. Actual and desired trajectory in 3D space (Simulation No.4).**

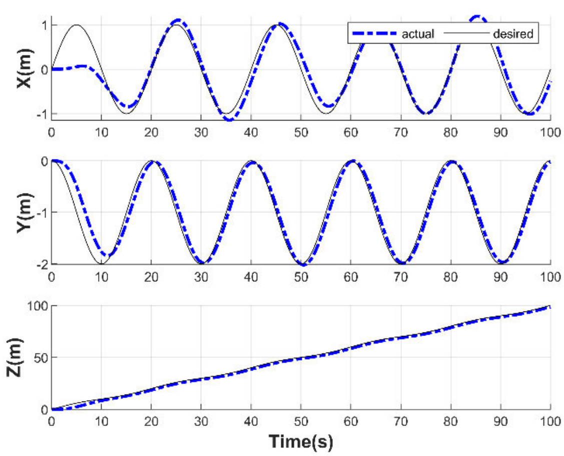


**FIGURE 11. Actual and desired trajectory (Simulation No.4).**

As shown in Fig. 5 and Fig. 6, the variational disturbance makes the quadcopter unable to be stable around the designated position without the disturbance observer. The tracking error is reduced when the disturbance observer is employed, but the quadcopter still swings around the desired position. The phenomenon's cause is the error between the actual disturbance and the estimated disturbance, as shown in Fig. 7.

**TABLE 2.** RMSE of the tracking error.

| | X(m) | Y(m) | Z(m) | 3D(m) |
|---|---|---|---|---|
| *Without disturbance* | 0.028 | 0.016 | 1.453 | 1.551 |
| *With disturbance* | 0.031 | 0.023 | 1.453 | 1.561 |

Notice that the estimated disturbance is delayed, and the estimation error is BIBO stable yet not asymptotically stable when the disturbance is not constant. However, the estimated disturbance can reflect the variation of the actual disturbance and keep the estimation error bounded. The designed FL-MPC with disturbance observer is effective under disturbing circumstances for waypoint tracking.

### C. TEST SCENARIO II: TRAJECTORY TRACKING

In test scenario II, the desired trajectory is given by:

$$\begin{aligned} x_d &= \sin(\frac{\pi}{10}t) \\ y_d &= -1 + \cos\left(\frac{\pi}{10}t\right) \\ z_d &= t + \sin\left(\frac{\pi}{10}t\right) \end{aligned} \tag{34}$$

In simulation No.3, the quadcopter is commanded to track the desired trajectory without external disturbance. As shown in Fig. 8 and Fig. 9, the designed FL-MPC enables the quadcopter to track the desired helical trajectory. Notice that there is an obvious tracking error in axis $X$ in the first 10 seconds, which is caused by the difference between the system initial condition $\dot{x}$ and the desired trajectory initial condition $\dot{x}_d$, where $\dot{x} = 0$ and $\dot{x}_d = \frac{\pi}{10}$.

In simulation No.4, the disturbance is added as:

$$\begin{aligned} d_{wx} &= 0.03\sin\left(\frac{\pi t}{100}\right) + 0.025\sin(0.2t) + 0.01 sint \\ d_{wy} &= -0.08 \\ d_{wz} &= 0 \end{aligned}$$

As shown in Fig. 10 and Fig. 11, the FL-MPC can still ensure the quadcopter track the given trajectory.

Furthermore, the tracking error's 2D and 3D RMSE (Root Mean Square Error) are calculated with or without the disturbance as Table. 2.

The FL-MPC for quadcopter has a superior performance in axis $X$ and axis $Y$. The controller can compensate for the disturbance and reduce the final tracking error with the disturbance observer.

## V. CONCLUSION

In this study, a feedback-linearization model predictive controller is designed for a quadcopter. By feedback linearization, the original nonlinear quadcopter state-space model is linearized in a precise way without creating round-off errors like linearization based on the Taylor series. With the linearized model, a model predictive controller is designed for the quadcopter to track a given trajectory, and a disturbance observer is added to compensate for the external disturbance. The simulation results show that the proposed FL-MPC enables the quadcopter to track the desired trajectory with or without external disturbance effectively. The experiments on actual quadcopters and the improvements of disturbance compensating will be the future subjects of this study.

## REFERENCES


[1] V. Puri, A. Nayyar, and L. Raja, "Agriculture drones: A modern breakthrough in precision agriculture," *J. Statist. Manage. Syst.*, vol. 20, no. 4, pp. 507–518, 2017, doi: 10.1080/09720510.2017.1395171.

[2] U. R. Mogili and B. B. V. L. Deepak, "Review on application of drone systems in precision agriculture," *Proc. Comput. Sci.*, vol. 133, pp. 502–509, Jul. 2018, doi: 10.1016/j.procs.2018.07.063.

[3] J. Navia, I. Mondragon, D. Patino, and J. Colorado, "Multispectral mapping in agriculture: Terrain mosaic using an autonomous quadcopter UAV," in *Proc. Int. Conf. Unmanned Aircr. Syst. (ICUAS)*, Jun. 2016, pp. 1351–1358.

[4] G. Hoffmann, S. Waslander, and C. Tomlin, "Quadrotor helicopter trajectory tracking control," in *Proc. AIAA Guid., Navigat. Control Conf. Exhib.*, Aug. 2008, p. 7410.

[5] S. Bouabdallah, A. Noth, and R. Siegwart, "PID vs LQ control techniques applied to an indoor micro quadrotor," in *Proc. IEEE/RSJ Int. Conf. Intell. Robots Syst. (IROS)*, vol. 3, Sep. 2004, pp. 2451–2456.

[6] P. Pounds, R. Mahony, and P. Corke, "Modelling and control of a quadrotor robot," in *Proc. Australas. Conf. Robot. Automat.*, 2006, pp. 1–10.

[7] I. D. Cowling, O. A. Yakimenko, J. F. Whidborne, and A. K. Cooke, "Direct method based control system for an autonomous quadrotor," *J. Intell. Robotic Syst.*, vol. 60, no. 2, pp. 285–316, Nov. 2010, doi: 10.1007/s10846-010-9416-9.

[8] M. Valenti, B. Bethke, G. Fiore, J. How, and E. Feron, "Indoor multivehicle flight testbed for fault detection, isolation, and recovery," in *Proc. AIAA Guid., Navigat., Control Conf. Exhib.*, Aug. 2006, p. 6200.

[9] D. Lee, H. J. Kim, and S. Sastry, "Feedback linearization vs. adaptive sliding mode control for a quadrotor helicopter," *Int. J. Control, Autom., Syst.*, vol. 7, no. 3, pp. 419–428, 2009, doi: 10.1007/s12555-009-0311-8.

[10] A. Mokhtari, N. K. M'Sirdi, K. Meghriche, and A. Belaidi, "Feedback linearization and linear observer for a quadrotor unmanned aerial vehicle," *Adv. Robot.*, vol. 20, no. 1, pp. 71–91, Jan. 2006, doi: 10.1163/156855306775275495.

[11] H. L. N. N. Thanh and S. K. Hong, "Quadcopter robust adaptive second order sliding mode control based on PID sliding surface," *IEEE Access*, vol. 6, pp. 66850–66860, 2018, doi: 10.1109/ACCESS.2018.2877795.

[12] E.-H. Zhang, J.-J. Xiong, and J.-L. Luo, "Second order sliding mode control for a quadrotor UAV," *ISA Trans.*, vol. 53, no. 4, pp. 1350–1356, Jul. 2014, doi: 10.1016/j.isatra.2014.03.010.

[13] S. Nadda and A. Swamp, "Second order sliding mode control for quadrotor," in *Proc. IEEE 1st Int. Conf. Control, Meas. Instrum. (CMI)*, Jan. 2016, pp. 92–96.

[14] O. Härkegård and S. T. Glad, "Flight control design using backstepping," *IFAC Proc. Volumes*, vol. 34, no. 6, pp. 283–288, Jul. 2001, doi: 10.1016/s1474-6670(17)35187-x.

[15] S. Skogestad and I. Postlethwaite, *Multivariable Feedback Control: Analysis and Design*, vol. 2. New York, NY, USA: Wiley, 2007.

[16] K. Lee, J. Back, and I. Choy, "Nonlinear disturbance observer based robust attitude tracking controller for quadrotor UAVs," *Int. J. Control, Autom. Syst.*, vol. 12, no. 6, pp. 1266–1275, 2014, doi: 10.1007/s12555-014-0145-x.

[17] M. Chen and M. Huzmezan, "A combined MBPC/2 DOF H infinity controller for a quad rotor UAV," in *Proc. AIAA Guid., Navigat., Control Conf. Exhib.*, Aug. 2003, p. 5520.

[18] F. Yacef, "Takagi-sugeno model for quadrotor modelling and control using nonlinear state feedback controller," *Int. J. Control Theory Comput. Model.*, vol. 2, no. 3, pp. 9–24, May 2012, doi: 10.5121/ijctcm.2012.2302.

[19] H. Lee and H. J. Kim, "Robust control of a quadrotor using Takagi-Sugeno fuzzy model and an LMI approach," in *Proc. 14th Int. Conf. Control, Autom. Syst. (ICCAS)*, Oct. 2014, pp. 370–374.

[20] K. Alexis, G. Nikolakopoulos, Y. Koveos, and A. Tzes, "Switching model predictive control for a quadrotor helicopter under severe environmental flight conditions," *IFAC Proc. Volumes*, vol. 44, no. 1, pp. 11913–11918, Jan. 2011, doi: 10.3182/20110828-6-IT-1002.03010.

[21] P. Bouffard, A. Aswani, and C. Tomlin, "Learning-based model predictive control on a quadrotor: Onboard implementation and experimental results," in *Proc. IEEE Int. Conf. Robot. Autom.*, May 2012, pp. 279–284.
[22] W. Koch, R. Mancuso, R. West, and A. Bestavros, "Reinforcement learning for UAV attitude control," *ACM Trans. Cyber-Phys. Syst.*, vol. 3, no. 2, pp. 1–21, Feb. 2019, doi: 10.1145/3301273.
[23] J. Yang, W.-H. Chen, and S. Li, "Robust autopilot design of uncertain bank-to-turn missiles using state-space disturbance observers," *Proc. Inst. Mech. Eng., G, J. Aerosp. Eng.*, vol. 226, no. 1, pp. 97–107, Jan. 2012, doi: 10.1177/0954410011404741.
[24] X. Zheng and X. Yang, "Command filter and universal approximator based backstepping control design for strict-feedback nonlinear systems with uncertainty," *IEEE Trans. Autom. Control*, vol. 65, no. 3, pp. 1310–1317, Mar. 2020, doi: 10.1109/TAC.2019.2929067.
[25] X. Yang, X. Zheng, and H. Gao, "SGD-based adaptive NN control design for uncertain nonlinear systems," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 10, pp. 5071–5083, Oct. 2018, doi: 10.1109/TNNLS.2018.2790479.

**ZHENHUAN CAI** was born in Shanghai, China, in 1996. He received the B.S. degree in mechanical engineering from the University of Shanghai for Science and Technology, in 2019. He is currently pursuing the M.S. degree with the School of Mechanical Engineering, Shanghai Institute of Technology, Shanghai, China. His research interests include nonlinear control, robotics, and quadcopter control.

**SUOHUAI ZHANG** received the B.S. and M.S. degrees in mechanical engineering from the Shaanxi University of Science & Technology, Shaanxi, China, in 1984 and 1987, respectively, and the Ph.D. degree from Xi'an Jiaotong University, Shaanxi, China, in 2000. From 2001 to 2003, he worked as Postdoctoral Fellow with the Shaanxi University of Science & Technology. He is currently a Professor and the Vice President of the Shanghai Institute of Technology, Shanghai, China. His research interests include mechanical system dynamics and computer aided engineering.

**XUEDONG JING** received the B.S. and M.S. degrees in mechanical engineering from the Shaanxi University of Science & Technology, Shaanxi, China, in 1991 and 1999, respectively, and the Ph.D. degree from Shanghai Jiao Tong University, Shanghai, China, in 2000. He is currently a Professor and the Dean of the School of Electrical and Electronic Engineering, Shanghai Institute of Technology, Shanghai. His research interests include robotics and virtual instrument technology.