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Northern Goshawk Optimization: A New Swarm-Based Algorithm for Solving Optimization Problems

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ABSTRACT Optimization algorithms are one of the effective stochastic methods in solving optimization problems. In this paper, a new swarm-based algorithm called Northern Goshawk Optimization (NGO) algorithm is presented that simulates the behavior of northern goshawk during prey hunting. This hunting strategy includes two phases of prey identification and the tail and chase process. The various steps of the proposed NGO algorithm are described and then its mathematical modeling is presented for use in solving optimization problems. The ability of NGO to solve optimization problems is evaluated on sixty-eight different objective functions. To analyze the quality of the results, the proposed NGO algorithm is compared with eight well-known algorithms, particle swarm optimization, genetic algorithm, teaching-learning based optimization, gravitational search algorithm, grey wolf optimizer, whale optimization algorithm, tunicate swarm algorithm, and marine predators algorithm. In addition, for further analysis, the proposed algorithm is also employed to solve four engineering design problems. The results of simulations and experiments show that the proposed NGO algorithm, by creating a proper balance between exploration and exploitation, has an effective performance in solving optimization problems and is much more competitive than similar algorithms.

INDEX TERMS Exploitation, exploration, northern goshawk, optimization, optimization problem.

I. INTRODUCTION

Optimization means choosing the best solution out of all available candidate solutions for an optimization problem. An optimization problem consists of three main parts: decision variables, constraints (equality and inequality), and objective functions [1]. From the general point of view, optimization problem solving methods can be grouped into deterministic methods and stochastic methods. Deterministic methods implement the optimization problem-solving process based on the use of information about the derivatives of objective functions or based on information in the form of the first-order and the second-order derivatives. This information enables deterministic methods to effectively find the exact optimal for linear or convex nonlinear problems. However, these methods fail to solve more complex problems, especially those with many local optimizations. The time-consuming process of solving complex problems, high-dimensional problems, non-convex problems, problems for non-differentiable objective functions, problems with random or unknown search space are other issues that challenge deterministic methods [2]. Challenges and inability of deterministic methods led to the introduction of stochastic methods and optimization algorithms. Stochastic-based optimization algorithms are efficient tools in solving optimization problems that are able to provide suitable solutions to optimization problems without using information about the derivatives of the objective function and relying only on random scanning of the search space and random operators [3]. The process of solving the optimization problem in optimization algorithms is such that at first, a certain number of solvable solutions are generated randomly as candidate solutions. Then in an iteration-based process and based on the steps of the algorithm, these candidate solutions are

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improved. After the full implementation of the algorithm, the best candidate solution is selected as the solution to the problem. The solution obtained from the optimization algorithm is at best equal to the global optimal, otherwise it must be very close to it. For this reason, the solutions obtained from the optimization algorithms are called quasi-optimal [4]. The desire to achieve better quasi-optimal solutions and closer to the global optimal has led to the design of numerous optimization algorithms by researchers.

Optimization algorithms can be divided according to the type of their inspiration in nature or society into four groups: evolutionary-based, swarm-based, physics-based, and game-based optimization algorithms.

Evolutionary-based optimization algorithms rely on the simulation of biological sciences, genetics, and the use of evolutionary operators such as natural selection. Genetic Algorithm (GA) is one of the oldest evolutionary algorithms developed based on the modeling of the reproductive process and the use of selection, crossover, and mutation sequence operators [5]. Differential Evolution (DE) algorithm is another popular evolutionary optimization algorithm that has a good ability to optimize non-differentiable nonlinear functions, which has been introduced as a powerful and fast way to optimize problems in continuous spaces [6].

Swarm-based optimization algorithms are introduced based on modeling the natural behaviors of animals, insects, aquatic animals, plants, and other living things. Particle Swarm Optimization (PSO) is one of the most widely used swarm-based algorithms, which is inspired by the intelligent behavior of birds and fish [7]. Modeling ant swarm behavior in finding the shortest path between the food source and the nest has inspired the design of the Ant Colony Optimization (ACO) [8]. Hierarchical leadership behavior modeling as well as the strategy of gray wolves during hunting have been used in the design of the Grey Wolf Optimization (GWO) [9]. In the design of the Whale Optimization Algorithm (WOA) is inspired by the bubble net hunting method performed by humpback whales [10]. Some other swarm-based algorithms are Raccoon Optimization Algorithm (ROA) [11], Teaching-Learning Based Optimization (TLBO) [12], Crow Search Algorithm (CSA) [13], Grasshopper Optimization Algorithm (GOA) [14], Tunicate Swarm Algorithm (TSA) [15], and Marine Predators Algorithm (MPA) [16].

Physics-based optimization algorithms have been developed based on the simulation of various laws and phenomena in physics. One of the oldest algorithms in this group is Simulated Annealing (SA), which is inspired by the simulation of the annealing process by melting and cooling operations in metallurgy [17], [18]. Simulation of the gravitational force that objects exert on each other at different distances has led to the design of a Gravitational Search Algorithm (GSA) [19]. Water Cycle Algorithm (WCA) is inspired by the water cycle in nature by modeling the evaporation of water from the ocean, cloud formation, rainfall, and river formation, as well as modeling the overflow of water from pits [20]. Some other physics-based algorithms are Artificial Chemical Reaction Optimization Algorithm (ACROA) [21], Multi-Verse Optimizer (MVO) [22], Electromagnetic Field Optimization (EFO) [23], Nuclear Reaction Optimization (NRO) [24], Optics Inspired Optimization (OIO) [25], Atom Search Optimization (ASO) [26], and Equilibrium Optimizer (EO) [27].

Game-based optimization algorithms are based on modeling the behavior of players in different games and the rules of these games. Simulation of competition and interactions between teams in the game of volleyball, the coaching process during the game, is employed in the design of the Volleyball Premier League (VPL) algorithm [28]. Mathematical modeling of players' behavior in tug-of-war game led to the Tug of War algorithm Optimization (TWO) [29].

With the advancement of science and technology, engineering problems become more complex, which require effective and efficient optimization methods. Therefore, this issue is resolved by improving existing methods or introducing newer optimization algorithms. An important issue in improving the capability of optimization algorithms is to increase the exploration power to global search the problem-solving space and to increase the exploitation power to local search the optimal area discovered, while a proper balance must be struck between these two indicators [30].

A major question that arises in the study of optimization algorithms is that given the existing optimization algorithms, is there still a need to design new optimization algorithms? The answer to this question lies in the No Free Lunch (NFL) Theorem [31]. The NFL states that an algorithm that provides effective performance in solving one or more optimization problems has no guarantee that it will perform effectively in solving other optimization problems and may even fail. This means it cannot be claimed that a particular optimization algorithm is the best optimizer for all problems. It is always possible to design new algorithms that solve optimization problems better than existing algorithms. The NFL encourages researchers to be motivated to design newer optimization algorithms that can solve optimization problems more effectively. The concepts expressed in the NFL theorem have also motivated the authors of this paper to develop a new optimizer.

Northern goshawk is a bird of prey whose hunting strategy represents an optimization process. In this strategy, the northern goshawk first selects the prey and attacks it, then hunts the selected prey in a chase process. However, to the best of our knowledge of the literature, no optimization algorithm has been developed based on northern goshawk behavior. This research gap motivated the authors to develop a new optimization algorithm by mathematically modeling the northern goshawk strategy while hunting.

The novelty of this paper is in designing a new swarm-based optimization algorithm called Northern Goshawk Optimization (NGO) that mimics the behavior of northern goshawks while hunting. The various steps of the proposed NGO algorithm are expressed and then mathematically modeled. Sixty-eight objective functions are employed to evaluate the capability of NGO. The performance of the proposed NGO algorithm in optimization is compared with the performance of eight well-known algorithms. In order to analyze the NGO for solving real-world problems, this algorithm has also been implemented on four design optimization problems.

The structure of the paper is created in such a way that the proposed NGO algorithm is introduced and modeled in Section II. Simulation studies are presented in Section III. The performance of NGO in solving engineering design problems is evaluated in Section IV. Conclusions and suggestions for further study of this paper are provided in Section V.

II. NORTHERN GOSHAWK OPTIMIZATION

In this section, the proposed Northern Goshawk Optimization (NGO) algorithm is introduced and then its mathematical modeling is presented.

A. INPIRATION AND BEHAVIOR OF NORTHERN GOSHAWK

The northern goshawk is a medium-large hunter in the family Accipitridae, which was first described by the current scientific name, i.e., Accipiter gentilis by Linnaeus in his Systema naturae in 1758 [32]. Northern goshawk is a member of the Accipiter genus that hunts on a variety of prey, including small and large birds and possibly other birds of prey, small mammals such as mice, rabbits, squirrels, and even animals such as foxes and raccoons. Northern goshawk is the only member of this genus which is distributed in Eurasia and North America [33]. The male is slightly larger than the female. The male body length is 46 to 61cm, the distance between the two wings is 89 to 105 cm and weighs about 780 grams. However, the female species is 58 to 69 cm long with a weight of 1220 grams and the distance between the two wings is estimated at 108 to 127 cm [34], [35]. A photo of the northern goshawk is shown in Figure 1. The northern goshawk hunting strategy consists of two stages, so that in the first stage, after identifying the prey, it moves towards it at a high speed, and in the second stage, it hunts the prey in a short tail-chase process [36].

FIGURE 1. Northern goshawk (take from Wikimedia Commons – Northern Goshawk juv).

Northern goshawk behavior when hunting and catching prey is an intelligent process. Mathematical modeling of the mentioned strategy is the main inspiration in designing the proposed NGO algorithm.

B. ALGORITHM INITIALIZATION PROCESS

The proposed NGO is a population-based algorithm that northern goshawks are searcher members of this algorithm. In NGO, each population member means a proposed solution to the problem that determines the values of the variables. From a mathematical point of view, each population member is a vector, and these vectors together form the population of the algorithm as a matrix. At the beginning of the algorithm, population members are randomly initialized in the search space. The population matrix in the proposed NGO algorithm is determined using (1).

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$$X = \begin{bmatrix} X_1 \\ \vdots \\ X_i \\ \vdots \\ X_N \end{bmatrix}_{N\times m} = \begin{bmatrix} x_{1,1} & \cdots & x_{1,j} & \cdots & x_{1,m} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ x_{i,1} & \cdots & x_{i,j} & \cdots & x_{i,m} \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ x_{N,1} & \cdots & x_{N,j} & \cdots & x_{N,m} \end{bmatrix}_{N\times m}, \tag{1}$$

where X is the population of northern goshawks, X_i is the ith proposed solution, $x_{i,j}$ xi,j is the value of the jth variable specified by the ith proposed solution, N is the number of population members, and m is the number of problem variables.

As stated, each population member is a proposed solution to the problem. Therefore, the objective function of the problem can be evaluated based on each population member. These values obtained for the objective function can be represented as a vector using (2).

$$F(X) = \begin{bmatrix} F_1 = F(X_1) \\ \vdots \\ F_i = F(X_i) \\ \vdots \\ F_N = F(X_N) \end{bmatrix}_{N\times 1} \tag{2}$$

where F is the vector of obtained objective function values and F_i is the objective function value obtained by ith proposed solution.

The criterion for deciding which solution is best is the value of the objective function. In minimization problems, the smaller the value of the objective function, and in maximization problems, the larger the value of the objective function, the better the proposed solution. Given that in each iteration new values are obtained for the objective function, the best proposed solution should be updated in each iteration.

C. MATHEMATICAL MODELLING OF PROPOSED NGO

In designing the proposed NGO algorithm to update the population members, the simulation of northern goshawk strategy during hunting has been employed. The two main behaviors of northern goshawk in this strategy, including

(i) prey identification and attack and

(ii) chase and escape operation

are simulated in two phases.

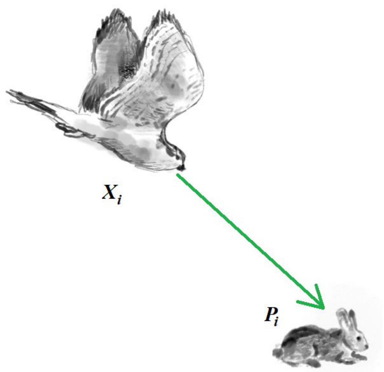

FIGURE 2. **Scheme of prey selection and attacking it by northern goshawk.**

1) PHASE 1: PREY IDENTIFICATION (EXPLORATION)

Northern goshawk in the first phase of hunting, randomly selects a prey and then quickly attacks it. This phase increases the exploration power of the NGO due to the random selection of prey in the search space. This phase leads to a global search of the search space with the aim of identifying the optimal area. A schematic of northern goshawk behavior in this phase involving prey selection and attack is shown in Figure 2. The concepts expressed in the first phase are mathematically modeled using (3) to (5).

$$P_i = X_k,\ i=1, 2, \ldots, N,\ k=1, 2, \ldots, i-1, i+1, \ldots, N, \tag{3}$$

$$x_{i,j}^{new,P1} = \begin{cases} x_{i,j} + r\left(p_{i,j} - I\ x_{i,j}\right), & F_{P_i} < F_i, \\ x_{i,j} + r\left(x_{i,j} - p_{i,j}\right), & F_{P_i} \geq F_i, \end{cases} \tag{4}$$

$$X_i = \begin{cases} X_i^{new,P1}, & F_i^{new,P1} < F_i, \\ X_i, & F_i^{new,P1} \geq F_i, \end{cases} \tag{5}$$

where P_i is the position of prey for the ith northern goshawk, F_{P_i} is its objective function value, k is a random natural number in interval $[1, N]$, $X_i^{new,P1}$ is the new status for the ith proposed solution, $x_{i,j}^{new,P1}$ is its jth dimension, $F_i^{new,P1}$ is its objective function value based on first phase of NGO, r is a random number in interval [0, 1], and I is a random number that can be 1 or 2. Parameters r and I are random numbers used to generate random NGO behavior in search and update.

2) PHASE 2: CHASE AND ESCAPE OPERATION (EXPLOITATION)

After the northern goshawk attacks the prey, the prey tries to escape. Therefore, in a tail and chase process, the northern goshawk continues to chase prey. Due to the high speed of the northern goshawks, they can chase their prey in almost any situation and eventually hunt. Simulation of this behavior increases the exploitation power of the algorithm to local search of the search space. In the proposed NGO algorithm, it is assumed that this hunting is closed to an attack position with radius R. The chase process between the northern goshawk and prey is shown in Figure 3. The concepts expressed in the second phase are mathematically modeled using (6) to (8).

$$x_{i,j}^{new,P2} = x_{i,j} + R\,(2r - 1\,)\,x_{i,j}, \tag{6}$$

$$R = 0.02\left(1 - \frac{t}{T}\right), \tag{7}$$

$$X_i = \begin{cases} X_i^{new,P2}, & F_i^{new,P2} < F_i, \\ X_i, & F_i^{new,P2} \geq F_i. \end{cases} \tag{8}$$

where t is the iteration counter, T is the maximum number of iterations, $X_i^{new,P2}$ is the new status for ith proposed solution, $x_{i,j}^{new,P2}$ is its jth dimension, $F_i^{nex,P2}$ is its objective function value based on second phase of NGO.

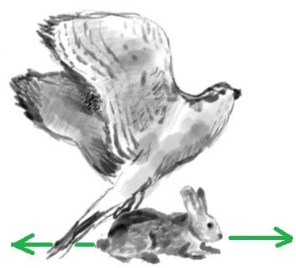

FIGURE 3. **Scheme of the chase between northern goshawk and prey.**

3) REPETITION PROCESS, PSEUDO-CODE, AND FLOWCHART OF NGO

After all members of the population have been updated based on the first and second phases of the proposed NGO algorithm, an iteration of the algorithm is completed and the new values of the population members, the objective function, and the best proposed solution are determined. The algorithm then enters the next iteration and the population members update continues based on Equations (3) to (8) until the last iteration of the algorithm is reached. At the end and after the complete implementation of NGO, the best proposed solution obtained during the iterations of the algorithm is introduced as a quasi-optimal solution for the given optimization problem. The various stages of the proposed NGO algorithm are specified as pseudo-code in Algorithm 1 and its flowchart is shown in Figure 4.

D. COMPYTIONAL COMPLEXITY

In this subsection, the computational complexity of the proposed NGO algorithm is analyzed. The computational complexity of the initialization of the NGO algorithm is equal to $O(N)$ where N is the number of population members of

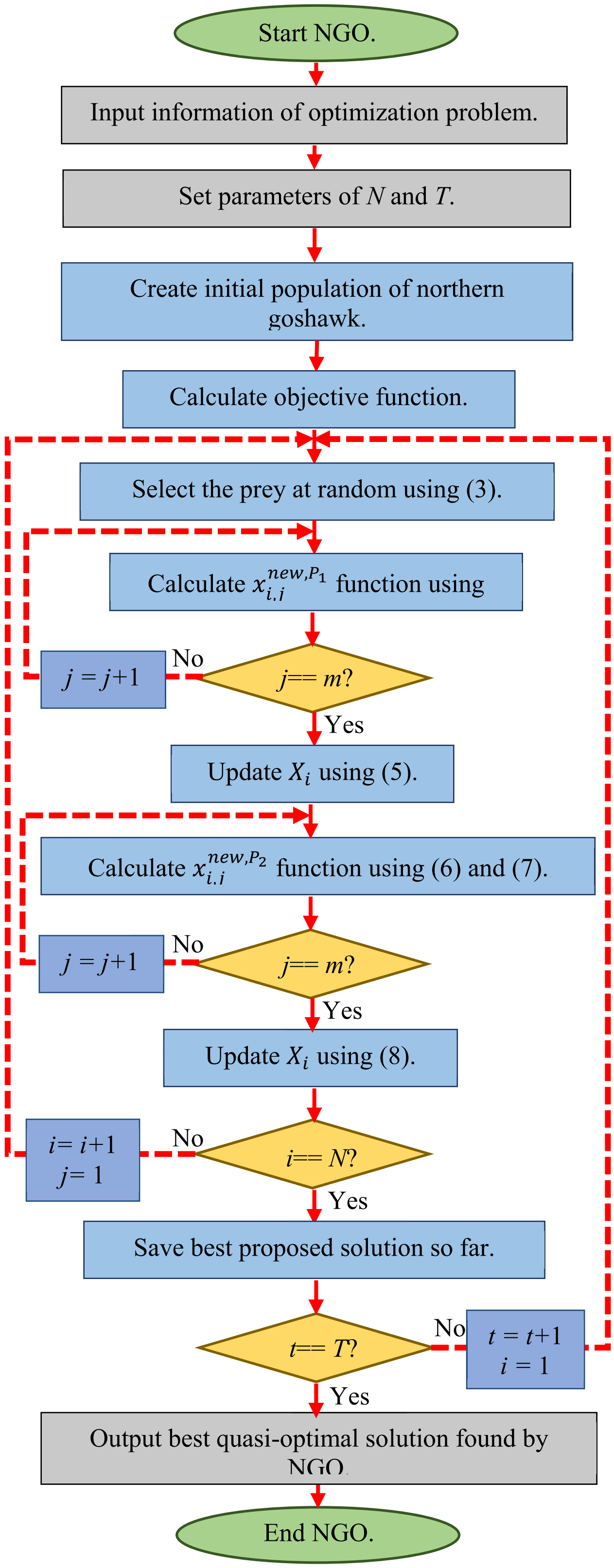

FIGURE 4. **Flowchart of proposed NGO algorithm.**

northern goshawks. Given that in the NGO, in each iteration, each member of the population is updated in two phases and its objective function is evaluated, the computational complexity of the update process is equal to $O(2T \cdot N \cdot m)$ where T is the maximum number of iterations, and m is the number of problem variables. Therefore, the computational complexity of the proposed NGO algorithm is equal to $O(N \cdot (1+2T \cdot m))$.

Algorithm 1 Pseudo-Code of Proposed NGO Algorithm

Start NGO.
1. Input the optimization problem information.
2. Set the number of iterations (T) and the number of members of the population (N).
3. Initialization of the position of northern goshawks and evaluation of the objective function.
4. For $t = 1: T$
6. For $i = 1: N$
7. Phase 1: prey identification (exploration phase)
8. Select the prey at random using (3).
9. For $j = 1: m$
10. Calculate new status of jth dimension using (4).
11. end $j = 1: m$
12. Update ith population member using (5).
13. Phase 2: tail and chase operation (exploitation phase)
14. Update R using (6)
15. For $j = 1: m$
16. Calculate new status of jth dimension using (7).
17. end for $j = 1: m$
18. Update ith population member using (8).
19. end for $i = 1: N$
20. Save best proposed solution so far.
21. end for $t = 1: T$
22. Output best quasi-optimal solution obtained by NGO for given optimization problem.

End NGO.

III. SIMULATION STUDIES AND DISCUSSION

In this section, the performance of the proposed NGO algorithm in solving optimization problems is tested. For this purpose, NGO is implemented on sixty-eight different objective functions including unimodal, high-dimensional multimodal, fixed-dimensional multimodal [37], CEC2015 [38], and CEC2017 [39]. The performance of the proposed NGO algorithm is compared with eight well-known algorithms PSO, GA, GSA, TLBO, GWO, WOA, MPA, and TSA. The values set for the control parameters of these algorithms are specified in Table 1. The proposed NGO algorithm and each of the competing algorithms are implemented in twenty independent executions on every objective function, while each execution contains 1000 iterations. The optimization results are reported using two indicators

(i) the average of the best proposed solutions and

(ii) the standard deviation of the best proposed solutions.

The experimentation has been done on Matlab R2020a version using 64 bit Core i7 processor with 3.20 GHz and 16 GB main memory.

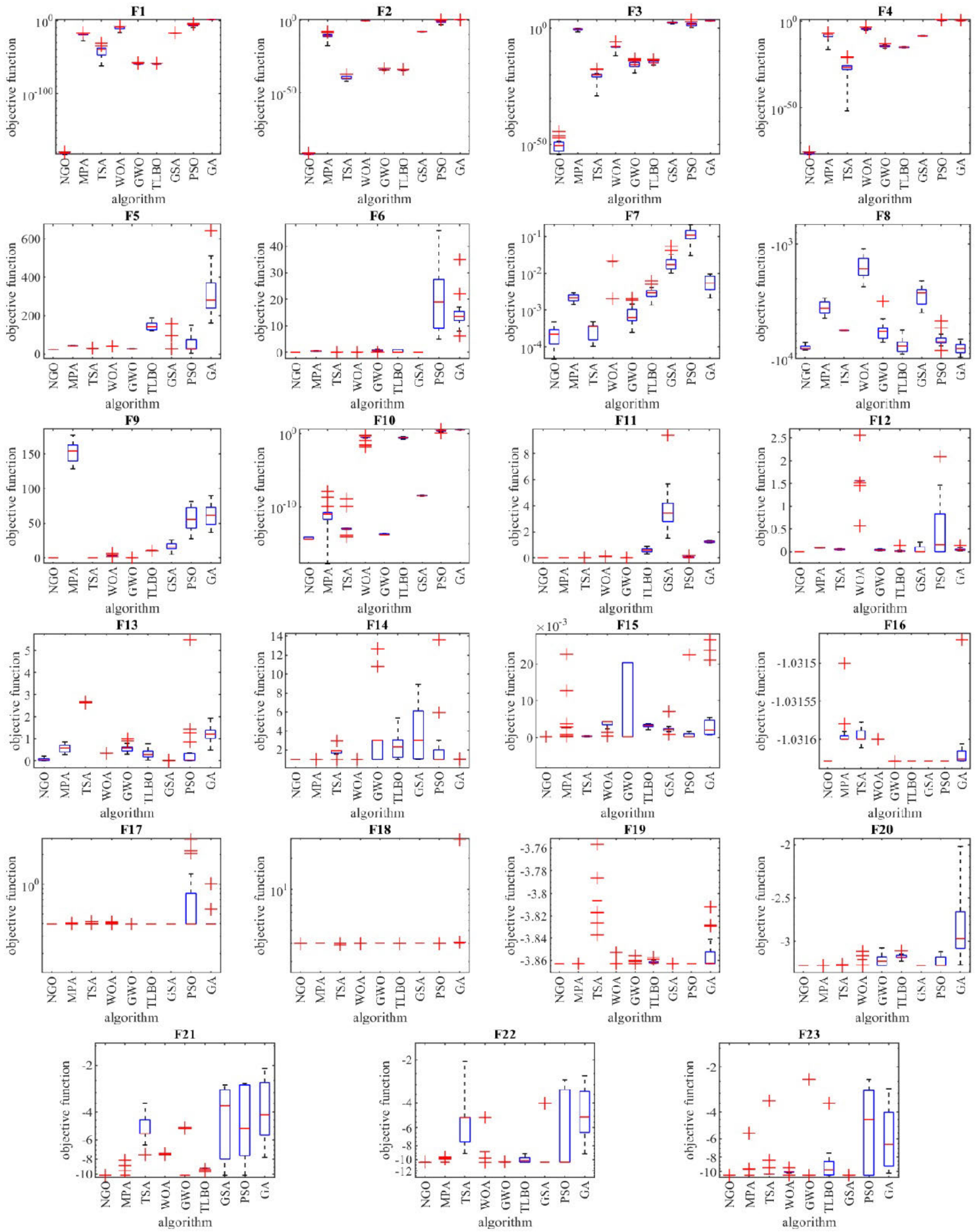

FIGURE 5. Boxplot of performance of optimization algorithms on F1 to F23 test functions.

TABLE 1. Parameter values for the competitor algorithms.

Algorithm	Parameter	Value
MPA		
	Constant number	$P = 0.5$
	Random vector	R is a vector of uniform random numbers from interval [0,1].
	Fish aggregating devices (FADs)	FADs = 0.2
	Binary vector	$U = 0$ or 1
TSA		
	P_{min} and P_{max}	1, 4
	c_1, c_2, c_3	random numbers lying in the interval [0,1].
WOA		
	Convergence parameter (a)	a: Linear reduction from 2 to 0.
	r is a random vector in [0,1].	
	l is a random number in [-1,1]. [-1,1].	
GWO		
	Convergence parameter (a)	a: Linear reduction from 2 to 0.
TLBO		
	T_F: teaching factor	T_F = round[1 + rand] [(1 + rand)]
	random number	rand is a random number from [0,1].
GSA		
	Alpha, G_0, R_{norm}, R_{power}	20, 100, 2, 1
PSO		
	Topology	Fully connected
	Cognitive and social constant	$c_1 = c_2 = 2$.
	Inertia weight	Linear reduction from 0.9 to 0.1
	Velocity limit	10% of dimension range
GA		
	Type	Real coded
	Selection	Roulette wheel (Proportionate)
	Crossover	Whole arithmetic (Probability = 0.8, $\alpha \in$ [-0.5, 1.5]).
	Mutation	Gaussian (Probability = 0.05)

A. EVALUATION OF UNIMODAL OBJECTIVE FUNCTION (F1-F7)

The optimization results of F1 to F7 functions using the proposed NGO algorithm and eight competitor algorithms are reported in Table 2. The simulation results show that NGO has been able to provide the optimal global for F6. The NGO algorithm is the first best optimizer in solving F1, F2, F3, F4, F5, and F7 functions. What can be deduced from the analysis of the simulation results is that the proposed NGO algorithm has a superior and much more competitive performance than the eight compared algorithms.

B. EVALUATION OF HIGH-DIMENSIONAL MULTIMODAL OBJECTIVE FUNCTION (F8-F13)

The implementation results of the proposed NGO algorithm and eight compared algorithms on the objective functions of F8 to F13 are presented in Table 3. The NGO with its high exploration power has been able to achieve the optimal global value for F9 and F11. In the F8 function optimizer, GA is the first best optimizer while NGO is the second best optimizer for this function. GSA is the first best optimizer and NGO is the second best optimizer for the F13 function. The proposed NGO algorithm is the first best optimizer for solving F10 and F12 functions. The simulation results show that the proposed NGO algorithm has an acceptable ability to solve high-dimensional multimodal optimization problems.

C. EVALUATION OF FIXED-DIMENSIONAL MULTIMODAL OBJECTIVE FUNCTION (F14-F23)

The solving results of the objective functions F14 to F23 using the NGO and eight competitor algorithms are presented in Table 4. The proposed NGO algorithm has been able to converge to the global optimum for F14 and F17. The NGO is the first best optimizer in solving F15 and F20 functions. In optimizing the functions of F16, F18, F19, F21, F22, and F23, the proposed NGO algorithm has the same performance in the *avg* index as some competing algorithms. However, in these functions, the proposed NGO algorithm has better conditions in the *std* index. Analysis of the simulation results shows that the proposed NGO algorithm has a high capability in solving F14 to F23 functions and is much more competitive than the eight compared algorithms.

The performance of NGO and eight competitor algorithms in optimizing F1 to F23 functions is shown in the form of a boxplot in Figure 5. The analysis of this boxplot shows that the NGO has less width and a more efficient center than competitor algorithms in optimizing most F1 to F23 functions. This means that the NGO has offered close and almost similar solutions in different implementations. Therefore, NGO is able to provide more efficient solutions to optimal problems.

D. STATISTICAL ANALYSIS

Comparison of optimization algorithms based on avg and std criteria provides valuable information about their capabilities. However, it may be a chance that one algorithm is superior to another, even after twenty independent executions with the least probability. Therefore, in this subsection, a statistical analysis is presented to further analyze the performance of the proposed algorithm in effectively solving optimization problems than the eight competitor algorithms. For this purpose, Wilcoxon rank sum test is used to show whether the superiority of the proposed algorithm over the competing algorithms is significant or not. In this test, a p-value is used to show the superiority of one algorithm over another algorithm.

TABLE 2. Optimization results of NGO and other algorithms on unimodal test function.

		NGO	TSA	MPA	WOA	GWO	GSA	TLBO	GA	PSO
F_1	Ave	6.65E-181	7.71E-38	3.2715E-21	2.1741E-09	1.09E-58	2.0255E-17	8.3373E-60	13.2405	1.7740E-05
	std	0	7.00E-21	4.6153E-21	7.3985E-25	5.1413E-74	1.1369E-32	4.9436E-76	4.7664E-15	6.4396E-21
F_2	Ave	4.04E-93	8.48E-39	1.57E-12	0.5462	1.2952E-34	2.3702E-08	7.1704E-35	2.4794	0.3411
	std	4.20E-93	5.92E-41	1.42E-12	1.7377E-16	1.9127E-50	5.1789E-24	6.6936E-50	2.2342E-15	7.4476E-17
F_3	Ave	1.36E-46	1.15E-21	0.0864	1.7634E-08	7.4091E-15	279.3439	2.7531E-15	1536.8963	589.4920
	std	5.93E-46	6.70E-21	0.1444	1.0357E-23	5.6446E-30	1.2075E-13	2.6459E-31	6.6095E-13	7.1179E-13
F_4	Ave	8.18E-77	1.33E-23	2.6E-08	2.9009E-05	1.2599E-14	3.2547E-09	9.4199E-15	2.0942	3.9634
	std	8.89E-77	1.15E-22	9.25E-09	1.2121E-20	1.0583E-29	2.0346E-24	2.1167E-30	2.2342E-15	1.9860E-16
F_5	Ave	22.9681	28.8615	46.049	41.7767	26.8607	36.10695	146.4564	310.4273	50.26245
	std	4.6701E-15	4.76E-03	0.4219	2.5421E-14	0	3.0982E-14	1.9065E-14	2.0972E-13	1.5888E-14
F_6	Ave	0	7.10E-21	0.398	1.6085E-09	0.6423	0	0.4435	14.55	20.25
	std	0	1.12E-25	0.1914	4.6240E-25	6.2063E-17	0	4.2203E-16	3.1776E-15	7.5612E-04
F_7	Ave	2.1716E-04	3.72E-04	0.0018	0.0205	0.0008	0.0206	0.0017	5.6799E-03	0.1134
	std	1.05E-21	5.09E-05	0.0010	1.5515E-18	7.2730E-20	2.7152E-18	3.87896E-19	7.7579E-19	4.3444E-17

TABLE 3. Optimization results of GMBO and other algorithms on high dimensional test function.

		NGO	TSA	MPA	WOA	GWO	GSA	TLBO	GA	PSO
F_8	Ave	-7994.3973	-5740.3388	-3594.16321	-1663.9782	-5885.1172	-2849.0724	-7408.6107	-8184.4142	-6908.6558
	std	370.6706	41.5	811.32651	716.3492	467.5138	264.3516	513.5784	833.2165	625.6248
F_9	Ave	0	5.70E-03	140.1238	4.2011	8.5265E-15	16.2675	10.2485	62.4114	57.0613
	std	0	1.46E-03	26.3124	4.3692E-15	5.6446E-30	3.1776E-15	5.5608E-15	2.5421E-14	6.3552E-15
F_{10}	Ave	5.68E-15	9.80E-14	9.6987E-12	0.3293	1.7053E-14	3.5673E-09	0.2757	3.2218	2.1546
	std	1.74E-15	4.51E-12	6.1325E-12	1.9860E-16	2.7517E-29	3.6992E-25	2.5641E-15	5.1636E-15	7.9441E-16
F_{11}	Ave	0	1.00E-07	0	0.1189	0.0037	3.7375	0.6082	1.2302	0.0462
	std	0	7.46E-07	0	8.9991E-17	1.2606E-18	2.7804E-15	1.9860E-16	8.4406E-16	3.1031E-18
F_{12}	Ave	1.27E-10	0.0368	0.0851	1.7414	0.0372	0.0362	0.0203	0.0470	0.4806
	std	6.84E-11	1.5461E-02	0.0052	8.1347E-12	4.3444E-17	6.2063E-17	7.7579E-16	4.6547E-17	1.8619E-16
F_{13}	Ave	0.0649	2.9575	0.4901	0.3456	0.5763	0.0020	0.3293	1.2085	0.5084
	std	1.0673E-15	1.5682E-12	0.1932	3.25391E-12	2.4825E-15	4.2617E-14	2.1101E-14	3.2272E-14	4.9650E-15

TABLE 4. Optimization results of GMBO and other algorithms on fixed dimensional test function.

		NGO	TSA	MPA	WOA	GWO	GSA	TLBO	GA	PSO
F_{14}	Ave	0.9980	1.9923	0.9980	0.9980	3.7408	3.5913	2.2721	0.9986	2.1735
	std	0	2.6548E-07	4.2735E-16	9.4336E-16	6.4545E-15	7.9441E-16	1.9860E-16	1.5640E-15	7.9441E-16
F_{15}	Ave	0.0003	0.0004	0.0030	0.0049	0.0063	0.0024	0.0033	5.3952E-02	0.0535
	std	4.44E-16	9.0125E−04	4.0951E-15	3.4910E-18	1.1636E-18	2.9092E-18	1.2218E-17	7.0791E-18	3.8789E-16
F_{16}	Ave	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316
	std	2.28E-16	5.6514E-16	4.4652E-16	9.9301E-16	3.9720E-16	5.9580E-16	1.4398E-15	7.9441E-16	3.4755E-16
F_{17}	Ave	0.3978	0.3991	0.3979	0.4047	0.3978	0.3978	0.3978	0.4369	0.7854
	std	0	2.1596E-16	9.1235E-15	2.4825E-14	8.6888E-16	9.9301E-16	7.4476E-16	4.9650E-14	4.9650E-15
F_{18}	Ave	3	3	3	3	3.0000	3	3.0009	4.3592	3
	std	5.09E-16	2.6528E-15	1.9584E-15	5.6984E-15	2.0853E-15	6.9511E-16	1.5888E-15	5.9580E-16	3.6741E-15
F_{19}	Ave	-3.86278	-3.8066	-3.8627	-3.8627	-3.8621	-3.8627	-3.8609	-3.85434	-3.8627
	std	2.28E-15	2.6357E-15	4.2428E-15	3.1916E-15	2.4825E-15	8.3413E-15	7.3483E-15	9.9301E-14	8.9371E-15
F_{20}	Ave	-3.322	-3.3206	-3.3211	-3.2424	-3.2523	-3.0396	-3.2014	-2.8239	-3.2619
	std	4.56E-16	5.6918E-15	1.1421E-11	7.9441E-16	2.1846E-15	2.1846E-14	1.7874E-15	3.97205E-11	2.9790E-12
F_{21}	Ave	-10.1532	-5.5021	-10.1532	-7.4016	-9.6452	-5.1486	-9.1746	-4.3040	-5.3891
	std	4.07E-15	5.4615E-13	2.5361E-11	2.3819E-11	6.5538E-15	2.9790E-14	8.5399E-15	1.5888E-12	1.4895E-13
F_{22}	Ave	-10.4029	-5.0625	-10.4029	-8.8165	-10.4025	-9.0239	-10.0389	-5.1174	-7.6323
	std	3.51E-16	8.4637E-14	2.8154E-11	6.7524E-15	1.9860E-15	1.6484E-12	1.5292E-14	6.2909E-15	7.5888E-15
F_{23}	Ave	-10.5364	-10.3613	-10.5364	-10.0003	-10.1302	-8.9045	-9.2905	-6.5621	-6.1648
	std	3.43E-16	7.6492E-12	3.9861E-11	9.1357E-15	4.5678E-15	7.1497E-14	6.1916E-15	3.8727E-15	2.7804E-15

The results of statistical analysis of the proposed NGO algorithm against eight competitor algorithms are presented in Table 5. According to the results of the Wilcoxon rank sum test, in cases where a p-value is less than 0.05, the proposed NGO algorithm is significantly better than all competitor algorithms. According to Table 5, the NGO has a significantly

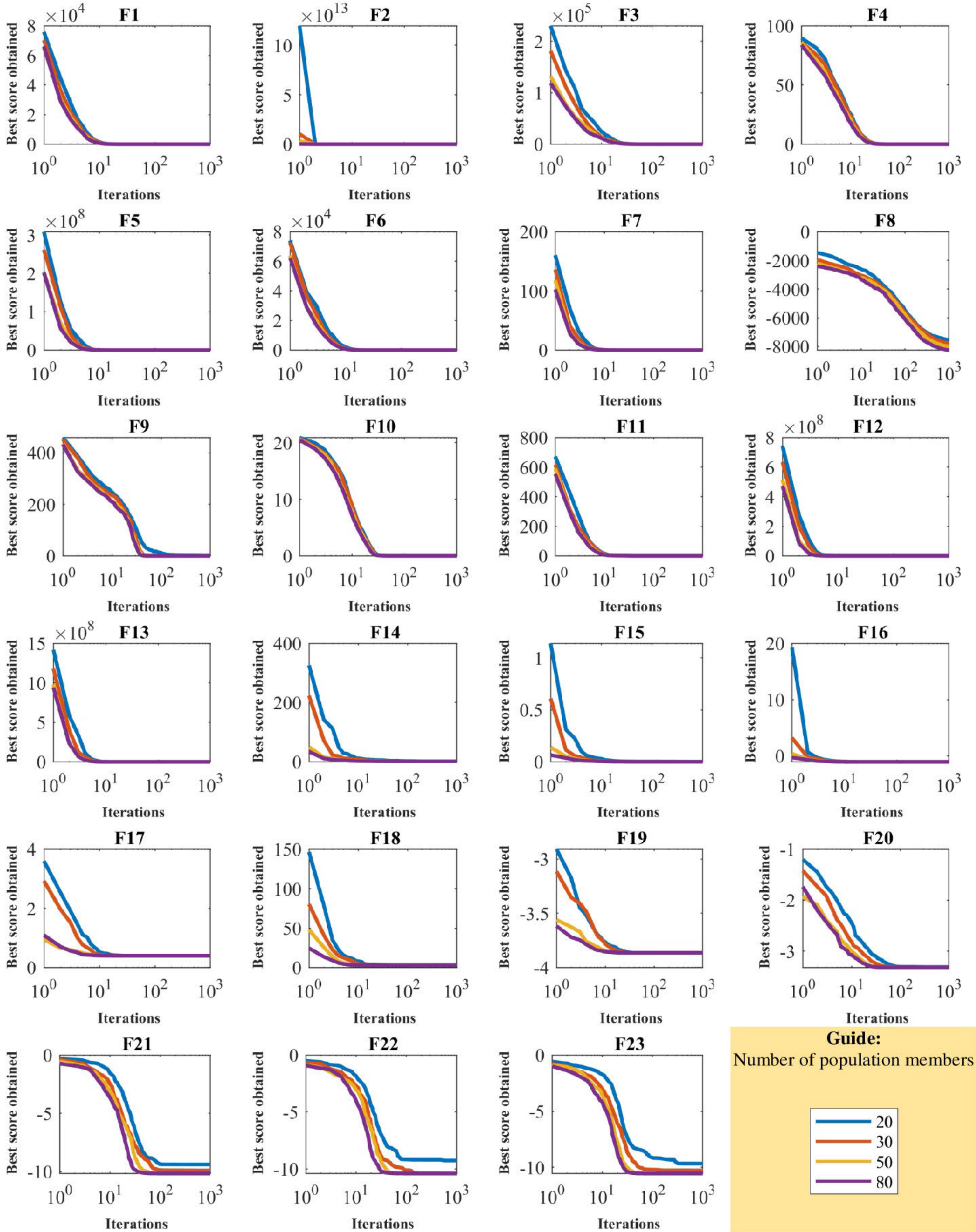

FIGURE 6. **Sensitivity analysis of the NGO for the number of population members.**

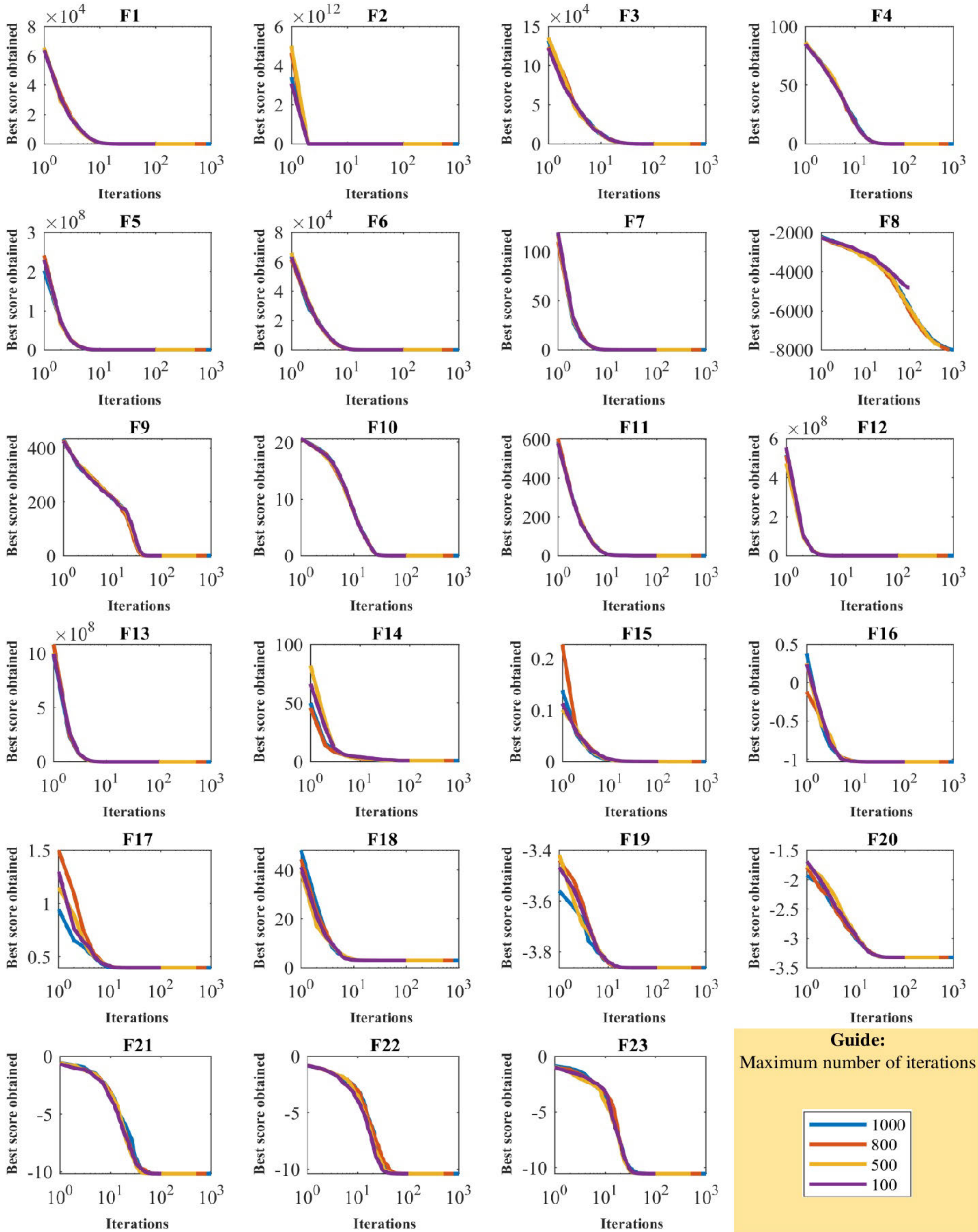

FIGURE 7. Sensitivity analysis of the NGO for the maximum number of iterations.

TABLE 5. ***p*-values obtained from Wilcoxon rank sum test.**

Compared Algorithms	Unimodal	High-Multimodal	Fixed-Multimodal
NGO vs. MPA	0.015625	0.0625	0.019531
NGO vs. TSA	0.015625	0.03125	0.003906
NGO vs. WOA	0.015625	0.03125	0.007813
NGO vs. GWO	0.015625	0.03125	0.011719
NGO vs. TLBO	0.015625	0.03125	0.005859
NGO vs. GSA	0.03125	0.15625	0.019531
NGO vs. PSO	0.015625	0.03125	0.003906
NGO vs. GA	0.015625	0.4375	0.001953

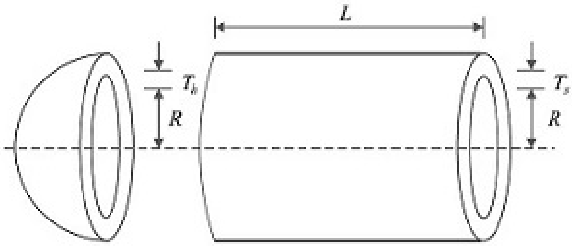

FIGURE 8. Schematic view of the pressure vessel problem.

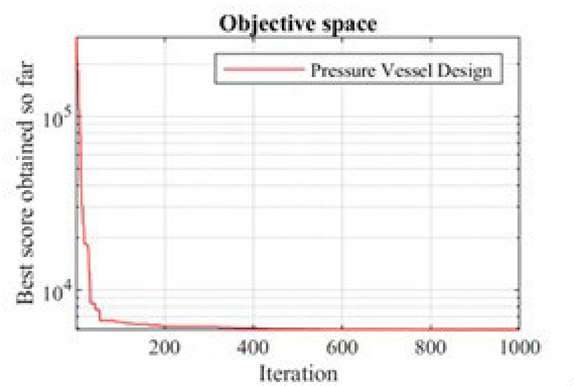

FIGURE 9. Convergence analysis of the NGO for the pressure vessel design optimization problem.

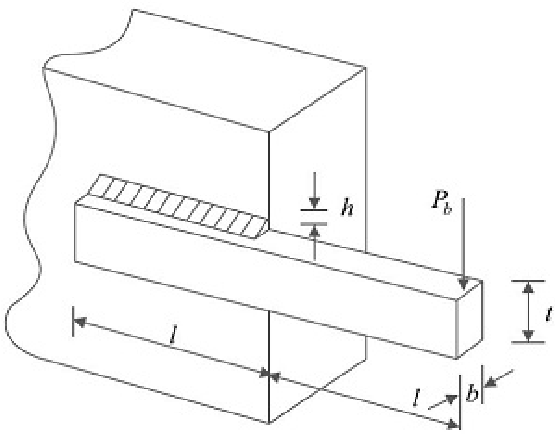

FIGURE 10. Schematic view of the welded beam problem.

superiority over each of the competitor algorithms in optimizing unimodal and fixed-dimensional multimodal functions. Also, NGO has a significant superiority in optimizing high-dimensional multimodal functions compared to MPA, TSA, WOA, GWO, TLBO, and PSO.

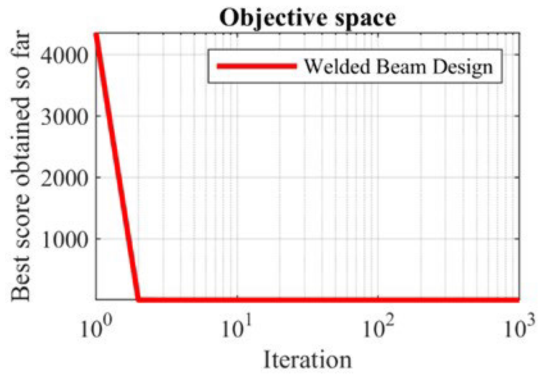

FIGURE 11. Convergence analysis of the NGO for the welded beam design optimization problem.

FIGURE 12. Schematic view of the tension/compression spring problem.

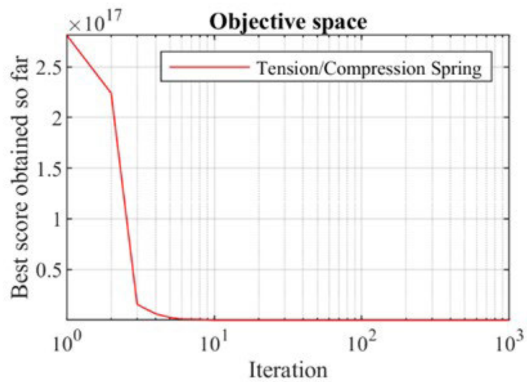

FIGURE 13. Convergence analysis of the NGO for the tension/compression spring optimization problem.

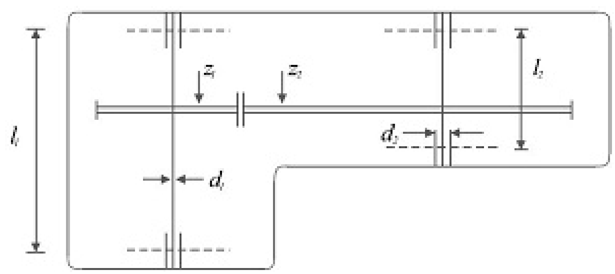

FIGURE 14. Schematic view of the speed reducer design problem.

E. SENSITIVITY ANALYSIS

The proposed NGO algorithm is a population-based algorithm that solves optimization problems in a repetition-based process. Therefore, the two parameters of the population,

TABLE 6. Sensitivity analysis of the NGO for the number of population members.

Objective Function	Number of Population Members			
	20	30	50	80
F_1	4.6E-169	1.2E-176	6.6E-181	1.4E-182
F_2	1.33E-89	4.23E-92	4.04E-93	3.68E-93
F_3	1.18E-36	8.03E-42	1.36E-46	1.11E-46
F_4	8E-74	2.08E-75	8.18E-77	2.05E-77
F_5	25.58091	24.44535	22.96812	22.45099
F_6	0	0	0	0
F_7	0.000521	0.000297	0.000217	0.000177
F_8	-7540.92	-7774.39	-7994.4	-8249.67
F_9	0.147311	0	0	0
F_{10}	7.11E-15	5.68E-15	5.68E-15	5.15E-15
F_{11}	0	0	0	0
F_{12}	0.005314	3.04E-07	1.27E-10	1.56E-11
F_{13}	1.162429	0.220698	0.064956	0.019496
F_{14}	1.784076	0.998004	0.998004	0.998004
F_{15}	0.001356	0.000307	0.000307	0.000307
F_{16}	-1.03163	-1.03163	-1.03163	-1.03163
F_{17}	0.397887	0.397887	0.397887	0.397887
F_{18}	3	3	3	3
F_{19}	-3.86278	-3.86278	-3.86278	-3.86278
F_{20}	-3.31011	-3.322	-3.322	-3.322
F_{21}	-9.3885	-9.8983	-10.1532	-10.1532
F_{22}	-9.27172	-10.4029	-10.4029	-10.4029
F_{23}	-9.6605	-10.2660	-10.5364	-10.5364

TABLE 7. Sensitivity analysis of the NGO for the maximum number of iterations.

Objective Function	Maximum Number of Iterations			
	100	500	800	1000
F_1	2.56E-14	1.86E-88	9.1E-144	6.6E-181
F_2	4.06E-08	8.44E-46	3.53E-74	4.04E-93
F_3	0.103703	9.24E-22	1.05E-36	1.36E-46
F_4	1.58E-06	1.86E-37	2.78E-61	8.18E-77
F_5	28.27281	25.49215	23.86167	22.96812
F_6	0	0	0	0
F_7	0.002592	0.000518	0.000272	0.000217
F_8	-4827.65	-7774.62	-7984.27	-7994.4
F_9	6.75E-10	0	0	0
F_{10}	3.33E-08	5.68E-12	6.39E-15	5.68E-15
F_{11}	6.81E-14	0	0	0
F_{12}	0.058359	1.98E-07	3.8E-09	1.27E-10
F_{13}	1.012204	0.071563	0.057661	0.064956
F_{14}	0.998004	0.998004	0.998004	0.998004
F_{15}	0.000449	0.000307	0.000307	0.000307
F_{16}	-1.03163	-1.03163	-1.03163	-1.03163
F_{17}	0.397887	0.397887	0.397887	0.397887
F_{18}	3	3	3	3
F_{19}	-3.86278	-3.86278	-3.86278	-3.86278
F_{20}	-3.32199	-3.322	-3.322	-3.322
F_{21}	-10.1532	-10.1532	-10.1532	-10.1532
F_{22}	-10.4029	-10.4029	-10.4029	-10.4029
F_{23}	-10.5364	-10.5364	-10.5364	-10.5364

number of northern goshawks (N) and the maximum number of iterations (T) affect the performance of the proposed NGO algorithm. Therefore, in this subsection, the sensitivity analysis of the NGO to the two parameters N and T is presented.

To evaluate the sensitivity analysis to parameter N, the proposed NGO algorithm for different values of population members equal to 20, 30, 50, and 80 has been implemented on the functions F1 to F23. The results of the sensitivity analysis of the NGO with respect to parameter N are reported in Table 6. The simulation results show that increasing the number of population members has improved the performance of the NGO and the values of the objective functions have decreased. The behavior of the convergence curves of the NGO in the study of this analysis is shown in Figure 6. These convergence curves show that increasing the number of population members leads to an increase in the exploratory power of NGO in identifying the optimal area more quickly and thus converging to more appropriate solutions.

In order to evaluate the sensitivity analysis for the T parameter, the NGO for different values of the maximum

TABLE 8. Evaluation results of CEC2015 objective functions.

		PSO	GA	GSA	TLBO	GWO	WOA	TSA	MPA	**NGO**
CEC1	Ave	1.50E+05	3.20E+07	7.65E+06	6.06E+05	1.47E+06	4.37E+05	2.02E+06	2.28E+06	1.23E+05
	std	1.21E+06	8.37E+06	3.07E+06	5.02E+05	2.63E+06	4.73E+05	2.08E+06	2.18E+06	1.12E+06
CEC2	Ave	6.70E+06	4.58E+03	7.33E+08	1.43E+04	1.97E+04	9.41E+03	5.65E+06	3.13E+05	5.54E+05
	std	1.34E+08	1.09E+03	2.33E+08	1.03E+04	1.46E+04	1.08E+04	6.03E+06	4.19E+05	1.00E+06
CEC3	Ave	3.20E+02	3.20E+02	3.20E+02	3.20E+02	3.20E+02	3.20E+02	3.20E+02	3.20E+02	3.20E+02
	std	1.16E-03	1.11E-05	7.53E-02	3.19E-02	9.14E-02	8.61E-02	7.08E-02	3.76E-02	2.35E-03
CEC4	Ave	4.10E+02	4.39E+02	4.42E+02	4.18E+02	4.26E+02	4.09E+02	4.16E+02	4.11E+02	5.69E+02
	std	5.61E+01	7.25E+00	7.72E+00	1.03E+01	1.17E+01	3.96E+00	1.03E+01	1.71E+01	3.28E+01
CEC5	Ave	9.81E+02	1.75E+03	1.76E+03	1.09E+03	1.33E+03	8.65E+02	9.20E+02	9.13E+02	8.74E+02
	std	2.06E+02	2.79E+02	2.30E+02	2.81E+02	3.45E+02	2.16E+02	1.78E+02	1.85E+02	3.18E+02
CEC6	Ave	2.05E+03	3.91E+06	2.30E+04	3.82E+03	7.35E+03	1.86E+03	2.26E+04	1.29E+04	2.09E+03
	std	1.05E+04	2.70E+06	2.41E+04	2.44E+03	3.82E+03	1.93E+03	2.45E+04	1.15E+04	1.27E+04
CEC7	Ave	7.02E+02	7.08E+02	7.06E+02	7.02E+02	7.02E+02	7.02E+02	7.02E+02	7.02E+02	7.02E+02
	std	5.50E-01	1.32E+00	9.07E-01	9.40E-01	1.10E+00	7.75E-01	7.07E-01	6.76E-01	3.16E-02
CEC8	Ave	1.47E+03	6.07E+05	6.73E+03	2.58E+03	9.93E+03	3.43E+03	3.49E+03	1.86E+03	1.40E+03
	std	2.34E+03	4.81E+05	3.36E+03	1.61E+03	8.74E+03	2.77E+03	2.04E+03	1.98E+03	1.01E+03
CEC9	Ave	1.00E+03	1.00E+03	1.00E+03	1.00E+03	1.00E+03	1.00E+03	1.00E+03	1.00E+03	1.00E+03
	std	1.51E+01	5.33E+00	9.79E-01	5.29E-02	2.20E-01	7.23E-02	1.28E-01	1.43E-01	1.64E+01
CEC10	Ave	1.23E+03	3.42E+05	9.91E+03	2.62E+03	8.39E+03	3.27E+03	4.00E+03	2.00E+03	1.18E+03
	std	2.51E+04	1.74E+05	8.83E+03	1.78E+03	1.12E+04	1.84E+03	2.82E+03	2.73E+03	4.41E+04
CEC11	Ave	1.35E+03	1.41E+03	1.35E+03	1.39E+03	1.37E+03	1.35E+03	1.40E+03	1.38E+03	1.34E+03
	std	1.41E+01	7.73E+01	1.11E+02	5.42E+01	8.97E+01	1.12E+02	5.81E+01	2.42E+01	1.20E+01
CEC12	Ave	1.30E+03	1.31E+03	1.31E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03
	std	7.50E+00	2.05E+00	1.54E+00	8.07E-01	9.14E-01	6.94E-01	6.69E-01	7.89E-01	5.34E+00
CEC13	Ave	1.30E+03	1.35E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03
	std	6.43E-05	4.70E+01	3.78E-03	2.43E-04	1.04E-03	5.44E-03	1.92E-04	2.76E-04	4.39E-07
CEC14	Ave	3.22E+03	9.30E+03	7.51E+03	7.34E+03	7.60E+03	7.10E+03	7.29E+03	4.25E+03	3.15E+03
	std	2.12E+03	4.04E+02	1.52E+03	2.47E+03	1.29E+03	3.12E+03	2.45E+03	1.73E+03	1.76E+03
CEC15	Ave	1.60E+03	1.64E+03	1.62E+03	1.60E+03	1.61E+03	1.60E+03	1.61E+03	1.60E+03	1.60E+03
	std	5.69E+01	1.12E+01	3.64E+00	1.80E-02	1.13E+01	2.66E-07	4.94E+00	3.76E+00	3.44E+01

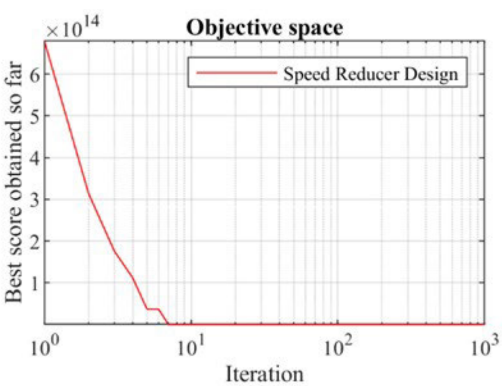

FIGURE 15. Convergence analysis of the NGO for the speed reducer design problem.

number of iterations equal to 100, 500, 800, and 1000 is employed to solve the functions of F1 to F23. The results of this analysis are presented in Table 7. Sensitivity analysis of the NGO to parameter T shows that as the maximum number of iterations increases, the value of all objective functions decreases. The behavior of the convergence curves of the proposed NGO algorithm under the influence of different values of the T parameter is presented in Figure 7. These convergence curves show that increasing the value of T gives the NGO more opportunity to converge towards better solutions.

F. EVALUATION OF IEEE CEC2015 (CEC1-CEC15)

The results of optimization of CEC2015 functions using the NGO and eight competitor algorithms are presented in Table 8. The simulation results show that the NGO has better results than the eight competitor algorithms in CEC1, CEC3, CEC7, CEC8, CEC9, CEC10, CEC11, CEC12, CEC13, CEC14, and CEC15 functions. In optimizing CEC5 and CEC6 the WOA performed better. However, the proposed NGO is the second best optimizer to solve these functions.

G. EVALUATION OF IEEE CEC2017 (C1-C30)

The performance results of the proposed NGO and eight competitor algorithms on the CEC2017 objective functions

TABLE 9. Evaluation results of CEC2017 objective functions.

		PSO	GA	GSA	TLBO	GWO	WOA	TSA	MPA	**NGO**
C1	Ave	0.00E+00	7.75E+05	3.30E+06	6.16E+04	1.57E+05	2.12E+05	4.47E+04	2.38E+05	2.70E+04
	std	0.00E+00	3.17E+07	8.47E+07	5.12E+06	2.73E+07	2.18E+07	4.83E+06	2.28E+07	1.21E+07
C2	Ave	0.00E+00	7.43E+07	4.68E+03	1.53E+04	1.07E+03	5.75E+05	9.51E+03	3.23E+04	5.82E+05
	std	0.00E+00	2.43E+09	1.19E+04	1.13E+05	1.56E+05	6.13E+07	1.18E+05	4.29E+06	3.41E+09
C3	Ave	0.00E+00	3.30E+02	3.30E+02	3.30E+02	3.30E+02	3.30E+02	3.30E+02	3.30E+02	3.30E+02
	std	0.00E+00	7.63E-03	1.21E-06	3.29E-03	9.24E-03	7.18E-03	8.71E-03	3.86E-03	2.38E-05
C4	Ave	5.62E+01	4.52E+02	4.49E+02	4.28E+02	4.36E+02	4.26E+02	4.19E+02	4.21E+02	3.40E+01
	std	4.88E+01	7.82E+02	7.35E+02	1.13E+02	1.27E+02	1.13E+02	3.06E+01	1.81E+02	4.62E+02
C5	Ave	1.64E+01	1.86E+03	1.85E+03	1.19E+04	1.43E+04	9.30E+02	8.75E+02	9.23E+02	1.40E+01
	std	3.46E+00	2.40E+03	2.89E+03	2.91E+03	3.55E+04	1.88E+03	2.26E+03	1.95E+03	2.19E+03
C6	Ave	1.09E-06	2.40E+04	3.01E+05	3.92E+04	7.45E+03	2.36E+03	1.96E+03	1.39E+04	2.17E+04
	std	2.62E-06	2.51E+05	2.80E+07	2.54E+04	3.92E+04	2.55E+05	1.03E+04	1.25E+05	2.13E+05
C7	Ave	6.65E+01	7.16E+02	7.18E+02	7.12E+03	7.12E+02	7.12E+02	7.12E+03	7.12E+02	7.12E+02
	std	3.47E+00	9.17E-02	1.42E+01	9.50E-02	1.20E+01	7.17E-02	7.85E-02	6.86E-02	4.49E-02
C8	Ave	1.70E+01	6.83E+03	6.17E+04	2.68E+04	9.03E+03	3.59E+04	3.53E+03	1.96E+03	1.56E+01
	std	3.14E+00	3.46E+04	4.91E+08	1.71E+04	8.84E+05	2.14E+04	2.87E+04	1.08E+04	3.00E+04
C9	Ave	0.00E+00	1.10E+03	1.10E+04	1.10E+03	1.10E+04	1.10E+04	1.10E+03	1.10E+04	1.10E+03
	std	0.00E+00	9.89E-02	5.43E+01	5.39E-03	2.30E-01	1.38E-02	7.33E-02	1.53E-02	1.50E+02
C10	Ave	3.14E+03	9.01E+04	3.52E+04	2.72E+03	8.49E+04	4.10E+04	3.37E+03	2.10E+03	1.32E+03
	std	3.67E+02	8.93E+04	1.84E+06	1.88E+04	1.22E+05	2.92E+04	1.94E+04	2.83E+04	1.50E+04
C11	Ave	2.79E+01	1.45E+04	1.51E+03	1.49E+04	1.47E+04	1.50E+04	1.45E+03	1.48E+03	1.45E+01
	std	3.33E+00	1.21E+03	7.83E+02	5.52E+02	8.07E+02	5.91E+02	1.22E+03	2.52E+02	2.76E+01
C12	Ave	1.68E+03	1.41E+03	1.41E+06	1.40E+05	1.40E+03	1.40E+03	1.40E+04	1.40E+04	1.40E+03
	std	5.23E+02	1.64E+01	2.15E+01	8.17E-02	9.24E-02	6.79E-02	6.04E-02	7.99E-02	6.79E+00
C13	Ave	3.06E+01	1.40E+04	1.45E+02	1.40E+03	1.40E+04	1.40E+06	1.40E+03	1.40E+02	1.40E+01
	std	2.12E+01	3.88E-04	4.80E+02	2.53E-05	1.14E-04	1.02E-05	5.54E-04	2.86E-05	5.49E-06
C14	Ave	2.50E+01	7.61E+03	9.40E+03	7.44E+04	7.70E+04	7.39E+03	7.20E+03	4.35E+04	3.33E+03
	std	1.87E+00	1.62E+04	4.14E+03	2.57E+04	1.39E+04	2.55E+04	3.22E+04	1.83E+04	2.00E+03
C15	Ave	2.39E+01	1.72E+06	1.74E+06	1.70E+03	1.71E+06	1.71E+04	1.70E+03	1.70E+03	1.70E+03
	std	2.49E+00	3.74E+01	1.22E+01	1.90E-03	1.23E+02	4.04E+01	2.76E-05	3.86E+01	9.86E+01
C16	Ave	4.51E+02	8.65E+05	4.20E+06	7.06E+05	2.47E+05	3.02E+06	5.37E+05	3.28E+05	2.46E+04
	std	1.38E+02	4.07E+09	9.37E+09	6.02E+09	3.63E+09	3.08E+09	5.73E+08	3.18E+09	3.20E+08
C17	Ave	2.83E+02	8.33E+06	5.58E+03	2.43E+05	2.97E+04	6.65E+06	8.41E+04	4.13E+04	8.48E+05
	std	8.61E+01	3.33E+07	2.09E+03	2.03E+03	2.46E+04	7.03E+05	2.08E+04	5.19E+04	7.03E+06
C18	Ave	2.43E+01	4.20E+02	4.20E+03	4.20E+03	4.20E+02	4.20E+02	4.20E+02	4.20E+02	4.20E+02
	std	2.02E+00	8.53E-04	2.11E-07	4.19E-04	8.14E-04	8.08E-04	9.61E-04	4.76E-04	7.47E-05
C19	Ave	1.41E+01	5.42E+02	5.39E+03	5.18E+03	5.26E+02	5.16E+02	1.09E+01	5.11E+03	4.05E+02
	std	2.26E+00	8.72E+02	8.25E+02	2.03E+02	2.17E+02	2.03E+02	4.90E-01	2.71E+02	5.60E+02
C20	Ave	1.40E+02	2.76E+04	2.75E+03	2.09E+03	2.33E+03	8.20E+02	7.65E+02	8.13E+03	1.11E+02
	std	7.74E+01	3.30E+02	3.79E+01	3.81E+01	4.45E+02	2.78E+00	3.16E+01	2.85E+01	4.07E+01
C21	Ave	2.19E+02	3.30E+04	4.91E+06	4.82E+04	8.35E+04	3.26E+04	2.86E+03	2.29E+03	3.02E+03
	std	3.77E+00	3.41E+04	3.70E+07	3.44E+04	4.82E+03	3.45E+05	2.93E+03	2.15E+04	2.16E+04
C22	Ave	1.49E+03	8.06E+03	8.08E+02	8.02E+03	8.02E+02	8.02E+03	8.02E+02	8.02E+02	8.02E+02
	std	1.75E+03	8.07E-02	2.32E+00	8.40E-01	2.10E+01	8.07E-01	8.75E-01	7.76E-01	5.47E-02
C23	Ave	4.30E+02	7.73E+03	7.07E+04	3.58E+03	8.93E+04	4.49E+04	4.43E+04	2.86E+03	3.35E+03
	std	6.24E+00	4.36E+04	5.81E+06	2.61E+04	9.74E+04	3.04E+04	3.77E+04	2.98E+04	1.31E+03
C24	Ave	5.07E+02	2.00E+04	2.00E+04	2.00E+04	2.00E+04	2.00E+04	2.00E+04	2.00E+04	2.00E+04
	std	4.13E+00	8.79E-02	6.33E+01	6.29E-03	3.20E-02	2.28E-02	7.23E-03	2.43E-02	6.76E+01
C25	Ave	4.81E+02	8.91E+04	4.42E+06	3.62E+04	9.39E+04	5.00E+04	4.27E+04	3.00E+04	2.22E+02
	std	2.80E+00	6.83E+04	2.74E+06	2.78E+04	2.12E+05	3.82E+04	2.84E+04	3.73E+04	6.46E+05
C26	Ave	1.13E+03	2.35E+04	2.41E+04	2.39E+04	2.37E+04	2.40E+04	2.35E+05	2.38E+04	1.00E+03
	std	5.62E+01	2.11E+03	8.73E+02	6.42E+02	9.97E+02	6.81E+02	2.12E+03	3.42E+02	2.26E+01
C27	Ave	5.11E+02	2.31E+04	2.31E+04	2.30E+04	2.30E+04	2.30E+04	2.30E+04	2.30E+04	2.30E+04
	std	1.11E+01	3.54E+01	3.05E+00	9.07E-02	8.14E-02	7.69E-02	7.94E-02	8.89E-02	1.40E+00
C78	Ave	4.60E+02	5.30E+04	5.35E+04	5.30E+04	5.30E+04	5.30E+04	5.30E+04	5.30E+04	5.30E+04
	std	6.84E+00	4.78E-04	4.70E+02	3.43E-05	2.04E+04	3.92E-05	6.44E-04	3.76E-05	6.11E-06
C29	Ave	3.63E+02	8.51E+03	8.30E+04	8.34E+04	8.60E+03	8.29E+03	8.10E+04	5.25E+04	3.20E+02
	std	1.32E+01	2.52E+04	5.04E+03	3.47E+05	2.29E+05	3.45E+04	4.12E+04	2.73E+04	4.66E+03
C30	Ave	6.01E+05	2.62E+04	2.64E+04	2.60E+04	2.61E+04	2.61E+04	2.60E+04	2.60E+04	2.60E+04
	std	2.99E+04	4.64E+01	2.12E+02	2.80E-03	2.13E+02	5.94E+01	3.66E-04	4.76E+01	5.55E+01

TABLE 10. Comparison results for the pressure vessel design problem.

Algorithm	Optimum Variables				Optimum Cost
	T_s	T_h	R	L	
NGO	0.7781779	0.3846819	40.31963	200	5885.4958
TSA	0.8303737	0.4162057	42.75127	169.3454	6048.7844
MPA	0.779035	0.384660	40.327793	199.65029	5889.3689
WOA	0.778961	0.384683	40.320913	200.00000	5891.3879
GWO	0.845719	0.418564	43.816270	156.38164	6011.5148
TLBO	0.817577	0.417932	41.74939	183.57270	6137.3724
GSA	1.085800	0.949614	49.345231	169.48741	11550.2976
PSO	0.752362	0.399540	40.452514	198.00268	5890.3279
GA	1.099523	0.906579	44.456397	179.65887	6550.0230

TABLE 11. Statistical results for the pressure vessel design problem.

Algorithm	Best	Mean	Worst	SD	Median
NGO	5885.4958	5888.0206	5890.1952	1.0215	5886.9142
TSA	6048.7844	6052.6241	6071.2496	2.893	6050.2282
MPA	5889.3689	5891.5247	5894.6238	13.910	5890.6497
WOA	5891.3879	6531.5032	7394.5879	534.119	6416.1138
GWO	6011.5148	6477.3050	7250.9170	327.007	6397.4805
TLBO	6137.3724	6326.7606	6512.3541	126.609	6318.3179
GSA	11550.2976	23342.2909	33226.2526	5790.625	24010.0415
PSO	5890.3279	6264.0053	7005.7500	496.128	6112.6899
GA	6550.0230	6643.9870	8005.4397	657.523	7586.0085

TABLE 12. Comparison results for the welded beam design problem.

Algorithm	Optimum Variables				Optimum Cost
	h	l	t	b	
NGO	0.20576	3.471	9.0361	0.20577	1.725202
TSA	0.205563	3.474846	9.035799	0.205811	1.725661
MPA	0.205678	3.475403	9.036964	0.206229	1.726995
WOA	0.197411	3.315061	10.00000	0.201395	1.820395
GWO	0.205611	3.472103	9.040931	0.205709	1.725472
TLBO	0.204695	3.536291	9.004290	0.210025	1.759173
GSA	0.147098	5.490744	10.00000	0.217725	2.172858
PSO	0.164171	4.032541	10.00000	0.223647	1.873971
GA	0.206487	3.635872	10.00000	0.203249	1.836250

are presented in Table 9. What is clear from the analysis of the results is that the proposed NGO algorithm offers better quasi-optimal solution for objective functions C4, C5, C8, C10, C11, C12, C13, C20, C22, C25, C26, C29, and C30.

IV. NGO APPLICATION FOR ENGINEERING DESIGN PROBLEMS

In this section, the performance of the NGO in solving problems in real-world applications is evaluated. For this purpose, the NGO is implemented on four optimization problems, namely pressure vessel design, welded beam design, tension/compression spring, and speed reducer design.

A. PRESSURE VESSEL DESIGN OPTIMIZATION PROBLEM

The mathematical model used was adapted from [40]. Figure 8 shows the schematic view of the pressure vessel problem. In this design, T_s is the thickness of the shell, T_h is the thickness of the head, R is the inner radius, and L is the length of the cylindrical section without considering the head. Tables 10 and 11 report the performance of the NGO and other algorithms. The NGO provides an optimal solution at (0.7781779, 0.3846819, 40.31963, 200.00000) with a corresponding fitness value of 5885.4958.

Figure 9 presents the convergence analysis of the NGO for the pressure vessel design optimization problem.

B. WELDED BEEM DESIGN OPTIMIZATION PROBLEM

The mathematical model of a welded beam design was adapted from [10]. Figure 10 displays the schematic view of the welded beam problem. In this design, h is the thickness of weld, l is the length of the clamped bar, t is the height of the bar, and b is the thickness of the bar. The results to this optimization problem are presented in Tables 12 and 13.

TABLE 13. Statistical results for the welded beam design problem.

Algorithm	Best	Mean	Worst	SD	Median
NGO	1.725202	1.725312	1.725496	0.0000106	1.725284
TSA	1.725661	1.725828	1.726064	0.000287	1.725787
MPA	1.726995	1.727128	1.727564	0.001157	1.727087
WOA	1.820395	2.230310	3.048231	0.324525	2.244663
GWO	1.725472	1.729680	1.741651	0.004866	1.727420
TLBO	1.759173	1.817657	1.873408	0.027543	1.820128
GSA	2.172858	2.544239	3.003657	0.255859	2.495114
PSO	1.873971	2.119240	2.320125	0.034820	2.097048
GA	1.836250	1.363527	2.035247	0.139485	1.9357485

TABLE 14. Comparison results for the tension/compression spring design problem.

Algorithm	Optimum Variables			Optimum cost
	d	*D*	*P*	
NGO	0.0523593	0.372854	10.4093	0.012672000
TSA	0.051144	0.343751	12.0955	0.012674000
MPA	0.050178	0.341541	12.07349	0.012678321
WOA	0.05000	0.310414	15.0000	0.013192580
GWO	0.05000	0.315956	14.22623	0.012816930
TLBO	0.050780	0.334779	12.72269	0.012709667
GSA	0.05000	0.317312	14.22867	0.012873881
PSO	0.05010	0.310111	14.0000	0.013036251
GA	0.05025	0.316351	15.23960	0.012776352

TABLE 15. Statistical results for the tension/compression spring design problem.

Algorithm	Best	Mean	Worst	SD	Median
NGO	0.012672000	0.012682410	0.012702561	0.0000204	0.012678261
TSA	0.012674000	0.012684106	0.012715185	0.000027	0.012687293
MPA	0.012678321	0.012697116	0.012720757	0.000041	0.012699686
WOA	0.013192580	0.014817181	0.017862507	0.002272	0.013192580
GWO	0.012816930	0.014464372	0.017839737	0.001622	0.014021237
TLBO	0.012709667	0.012839637	0.012998448	0.000078	0.012844664
GSA	0.012873881	0.013438871	0.014211731	0.000287	0.013367888
PSO	0.013036251	0.014036254	0.016251423	0.002073	0.013002365
GA	0.012776352	0.013069872	0.015214230	0.000375	0.012952142

The NGO provides an optimal solution at (0.20576, 3.471, 9.0361, 0.20577) with a corresponding fitness value equal: 1.725202. Figure 11 presents the convergence analysis of the NGO for the welded beam design optimization problem.

C. TENSION/COMPRESSION SPRING DESIGN OPTIMIZATION PROBLEM

The mathematical model of this problem was adapted from [10]. Figure 12 displays the schematic view of the tension/compression spring problem. in this design, d is the wire diameter, D is the mean coil diameter, and P is the number of active coils. The results to this optimization problem are displayed in Tables 14 and 15. The AMBOA provides the optimal solution at (0.0523593, 0.372854, 10.4093) with a corresponding fitness value of 0.012672. Figure 13 shows the convergence analysis of the NGO for the tension/compression spring optimization problem.

D. SPEED REDUCER DESIGN OPTIMIZATION PROBLEM

This problem is modeled mathematically in [41], [42]. Figure 14 displays the schematic view of the speed reducer design problem. In this design, b is the face width, m is the module of teeth, p is the number of teeth in the pinion, l_1 is the length of the first shaft between bearings, l_2 is the length of the second shaft between bearings, d_1 is the diameter of first shafts, and d_2 is the diameter of second shafts. The results of the optimization problem are presented in Table 16 and 17. The optimal solution was provided by the

TABLE 16. Comparison results for speed reducer design problem.

Algorithms	Optimum variables							Optimum cost
	b	m	p	l_1	l_2	d_1	d_2	
NGO	3.50122	0.7	17	7.3	7.8	3.334208	5.26535	**2994.2471**
TSA	3.50159	0.7	17	7.3	7.8	3.35127	5.28874	2998.5507
MPA	3.506690	0.7	17	7.380933	7.815726	3.357847	5.286768	3001.288
WOA	3.500019	0.7	17	8.3	7.8	3.352412	5.286715	3005.763
GWO	3.508502	0.7	17	7.392843	7.816034	3.358073	5.286777	3002.928
TLBO	3.508755	0.7	17	7.3	7.8	3.461020	5.289213	3030.563
GSA	3.600000	0.7	17	8.3	7.8	3.369658	5.289224	3051.120
PSO	3.510253	0.7	17	8.35	7.8	3.362201	5.287723	3067.561
GA	3.520124	0.7	17	8.37	7.8	3.366970	5.288719	3029.002

TABLE 17. Statistical results for speed reducer design problem.

Algorithms	**Best**	**Mean**	**Worst**	**SD**	**Median**
NGO	2994.2471	2997.481	2999.091	1.78090	**2996.317**
TSA	2998.5507	2999.640	3003.889	1.93193	2999.187
MPA	3001.288	3005.845	3008.752	5.83794	3004.519
WOA	3005.763	3105.252	3211.174	79.6381	3105.252
GWO	3002.928	3028.841	3060.958	13.0186	3027.031
TLBO	3030.563	3065.917	3104.779	18.0742	3065.609
GSA	3051.120	3170.334	3363.873	92.5726	3156.752
PSO	3067.561	3186.523	3313.199	17.1186	3198.187
GA	3029.002	3295.329	3619.465	57.0235	3288.657

TABLE 18. Unimodal objective functions.

Objective Function	**Range**	***Dim***	F_{min}
$F_1(X)=\sum_{i=1}^{m} x_i^2$	$[-100,100]$	*30*	*0*
$F_2(X)=\sum_{i=1}^{m}\lvert x_i\rvert+\prod_{i=1}^{m}\lvert x_i\rvert$	$[-10,10]$	30	0
$F_3(X)=\sum_{i=1}^{m}\left(\sum_{j=1}^{i} x_i\right)^2$	$[-100,100]$	30	0
$F_4(X)=max\{\lvert x_i\rvert\},\quad 1\le i\le m$	$[-100,100]$	30	0
$F_5(X)=\sum_{i=1}^{m-1}\left[100\left(x_{i+1}-x_i^2\right)^2+(x_i-1)^2\right]$	$[-30,30]$	30	0
$F_6(X)=\sum_{i=1}^{m}([x_i+0.5])^2$	$[-100,100]$	30	0
$F_7(X)=\sum_{i=1}^{m} i x_i^4+r,$ *where r is a random real number in the range* 0 *to* 1	$[-1.28,1.28]$	30	0

NGO at (3.50122, 0.7, 17, 7.3, 7.8, 3.334208, 5.26535) with a corresponding fitness value equal to 2994.2471. Figure 15 presents the convergence analysis of the NGO for the speed reducer design optimization problem.

V. CONCLUSION AND FUTURE WORKS

In this paper, a new intelligence swarm-based algorithm called Northern Goshawk Optimization (NGO) was designed, which its main inspiration is to simulate the

TABLE 19. High-dimensional multimodal objective functions.

Objective Function	Range	*Dim*	F_{min}
$F_8(X) = \sum_{i=1}^{m} -x_i \sin(\sqrt{\lvert x_i \rvert})$	[−500,500]	30	−12569
$F_9(X) = \sum_{i=1}^{m} [\, x_i^2 - 10\cos(2\pi x_i) + 10]$	[−5.12,5.12]	30	0
$F_{10}(X) = -20\exp\left(-0.2\sqrt{\frac{1}{m}\sum_{i=1}^{m} x_i^2}\right) - \exp\left(\frac{1}{m}\sum_{i=1}^{m} cos(2\pi x_i)\right) + 20 + e$	[−32,32]	30	0
$F_{11}(X) = \frac{1}{4000}\sum_{i=1}^{m} x_i^2 - \prod_{i=1}^{m} cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	[−600,600]	30	0
$F_{12}(X) = \frac{\pi}{m}\{10\sin(\pi y_1) + \sum_{i=1}^{m}(y_i - 1)^2[1 + 10\sin^2(\pi y_{i+1})] + (y_n - 1)^2\} + \sum_{i=1}^{m} u(x_i, 10,100,4)$, where $y_i = 1 + \frac{x_i+1}{4}, u(x_i, a, i, n) = \begin{cases} k(x_i - a)^n, & x_i > a; \\ 0, & -a \le x_i \le a; \\ k(-x_i - a)^n, & x_i < -a, \end{cases}$	[−50,50]	30	0
$F_{13}(X) = 0.1\{\sin^2(3\pi x_1) + \sum_{i=1}^{m}(x_i - 1)^2[1 + \sin^2(3\pi x_i + 1)] + (x_n - 1)^2[1 + \sin^2(2\pi x_m)]\} + \sum_{i=1}^{m} u(x_i, 5,100,4)$, where $u(x_i, a, i, n) = \begin{cases} k(x_i - a)^n, & x_i > a; \\ 0, & -a \le x_i \le a; \\ k(-x_i - a)^n, & x_i < -a. \end{cases}$	[−50,50]	30	0

behavior and strategy of northern goshawk while hunting. Mathematical modeling of the proposed NGO algorithm was presented and then its performance in optimization was tested on sixty-eight objective functions. The optimization results indicate the ability of NGO to provide desired quasi-optimal solutions for optimization problems. The performance of NGO in optimization was compared with eight well-known algorithms including PSO, GA, GSA, TLBO, GWO, WOA, MPA, and TSA. The analysis of the simulation results showed the obvious superiority of the proposed NGO algorithm over the eight competitor algorithms. In addition, the implementation of NGO on four design problems showed that the proposed algorithm was highly capable of solving real-world problems.

The authors make several suggestions for future studies of this paper. Attempts to design the binary as well as the multi-objective version of the proposed NGO algorithm are among the main study potentials for the future. In addition, the application of NGO in solving optimization problems in different sciences and comparing it with other existing algorithms are other suggestions for further studies in line with this paper.

APPENDIX A

See Tables 18–22.

APPENDIX B
PRESSURE VESSEL DESIGN PROBLEM

Consider $X = [x_1, x_2, x_3, x_4] = [T_s, T_h, R, L]$.

Minimize $f(x) = 0.6224x_1x_3x_4 + 1.778x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3$.

Subject to:

$$\begin{aligned} g_1(x) &= -x_1 + 0.0193x_3 \le 0, \\ g_2(x) &= -x_2 + 0.00954x_3 \le 0, \\ g_3(x) &= -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \le 0, \\ g_4(x) &= x_4 - 240 \le 0. \end{aligned}$$

With

$$0 \le x_1, x_2 \le 100, \textit{and}\, 10 \le x_3, x_4 \le 200.$$

APPENDIX C
WELDED BEAM DESIGN PROBLEM

Consider $X = [x_1, x_2, x_3, x_4] = [h, l, t, b]$.

Minimize $f(x) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14.0 + x_2)$.

Subject to:

$$\begin{aligned} g_1(x) &= \tau(x) - 13600 \le 0, \\ g_2(x) &= \sigma(x) - 30000 \le 0, \\ g_3(x) &= x_1 - x_4 \le 0, \end{aligned}$$

TABLE 20. Fixed-dimensional multimodal objective functions.

Objective Function	Range	Dim	F_{min}
$F_{14}(X)=\left(\frac{1}{500}+\sum_{j=1}^{25}\frac{1}{j+\sum_{i=1}^{2}(x_i-a_{ij})^6}\right)^{-1}$	[−65.53,65.53]	2	0.998
$F_{15}(X)=\sum_{i=1}^{11}\left[a_i-\frac{x_1(b_i^2+b_ix_2)}{b_i^2+b_ix_3+x_4}\right]^2$	[−5,5]	4	0.00030
$F_{16}(X)=4x_1^2-2.1\cdot x_1^4+\frac{1}{3}x_1^6+x_1x_2-4x_2^2+4x_2^4$	[−5,5]	2	−1.0316
$F_{17}(X)=\left(x_2-\frac{5.1}{4\pi^2}x_1^2+\frac{5}{\pi}x_1-6\right)^2+10\left(1-\frac{1}{8\pi}\right)cosx_1+10$	[-5,10] × [0,15]	2	0.398
$F_{18}(X)=[1+(x_1+x_2+1)^2(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2)]$ $[30+(2x_1-3x_2)^2(18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2)]$	[−5,5]	2	3
$F_{19}(X)=-\sum_{i=1}^{4}c_i\exp(-\sum_{j=1}^{3}a_{ij}(x_j-p_{ij})^2)$	[0,1]	3	−3.86
$F_{20}(X)=-\sum_{i=1}^{4}c_i\exp(-\sum_{j=1}^{6}a_{ij}(x_j-p_{ij})^2)$	[0,1]	6	−3.22
$F_{21}(X)=-\sum_{i=1}^{5}[(X-a_i)\cdot(X-a_i)^T+6c_i]^{-1}$	[0,10]	4	−10.1532
$F_{22}(X)=-\sum_{i=1}^{7}[(X-a_i)\cdot(X-a_i)^T+6c_i]^{-1}$	[0,10]	4	−10.4029
$F_{23}(X)=-\sum_{i=1}^{10}[(X-a_i)\cdot(X-a_i)^T+6c_i]^{-1}$	[0,10]	4	−10.5364

$$g_4(x)=0.10471x_1^2+0.04811x_3x_4(14+x_2)-5.0\le 0,$$
$$g_5(x)=0.125-x_1\le 0,$$
$$g_6(x)=\delta(x)-0.25\le 0,$$
$$g_7(x)=6000-p_c(x)\le 0.$$

where

$$\tau(x)=\sqrt{\tau'+(2\tau\tau')\frac{x_2}{2R}+(\tau'')^2},$$
$$\tau'=\frac{6000}{\sqrt{2}x_1x_2},$$
$$\tau''=\frac{MR}{J},$$
$$M=6000\left(14+\frac{x_2}{2}\right),$$
$$R=\sqrt{\frac{x_2^2}{4}+\left(\frac{x_1+x_3}{2}\right)^2},$$
$$J=2\left\{x_1x_2\sqrt{2}\left[\frac{x_2^2}{12}+\left(\frac{x_1+x_3}{2}\right)^2\right]\right\},$$
$$\sigma(x)=\frac{504000}{x_4x_3^2}$$
$$\delta(x)=\frac{65856000}{(30\cdot 10^6)\,x_4x_3^3},$$
$$p_c(x)=\frac{4.013\,(30\cdot 10^6)\sqrt{\frac{x_3^2x_4^6}{36}}}{196}\left(1-\frac{x_3}{28}\sqrt{\frac{30\cdot 10^6}{4(12\cdot 10^6)}}\right).$$

With

$$0.1\le x_1,x_4\le 2 and 0.1\le x_2,x_3\le 10.$$

APPENDIX D
TENSION/COMPRESSION SPRING DESIGN PROBLEM

Consider $X=[x_1,x_2,x_3]=[d,D,P]$.

Minimize $f(x)=(x_3+2)\,x_2x_1^2$.

TABLE 21. IEEE CEC-2015 benchmark test functions.

	Functions	Related basic functions	Dim	*Fmin*
CEC1	Rotated Bent Cigar Function	Bent Cigar Function	30	100
CEC2	Rotated Discus Function	Discus Function	30	200
CEC3	Shifted and Rotated Weierstrass Function	Weierstrass Function	30	300
CEC4	Shifted and Rotated Schwefel's Function	Schwefel's Function	30	400
CEC5	Rotated Katsuura	Katsuura Function	30	500
CEC6	Shifted and Rotated HappyCat Function	HappyCat Function	30	600
CEC7	Shifted and Rotated HGBat Function	HGBat Function	30	700
CEC8	Shifted and Rotated Expanded Griewank's plus Rosenbrock's Function	Griewank's Function Rosenbrock's Function	30	800
CEC9	Shifted and Rotated Expanded Scaffer's F6 Function	Expanded Scaffer's F6 Function	30	900
CEC10	Hybrid Function 1 ($N = 3$)	Schwefel's Function Rastrigin's Function High Conditioned Elliptic Function	30	1000
CEC11	Hybrid Function 2 ($N = 4$)	Griewank's Function Weierstrass Function Rosenbrock's Function Scaffer's F6 Function	30	1100
CEC12	Hybrid Function 3 ($N = 5$)	Katsuura Function HappyCat Function Expanded Griewank's plus Rosenbrock's Function Schwefel's Function Ackley's Function	30	1200
CEC13	Composition Function 1 ($N = 5$)	Rosenbrock's Function High Conditioned Elliptic Function Bent Cigar Function Discus Function High Conditioned Elliptic Function	30	1300
CEC14	Composition Function 2 ($N = 3$)	Schwefel's Function Rastrigin's Function High Conditioned Elliptic Function	30	1400
CEC15	Composition Function 3 ($N = 5$)	HGBat Function Rastrigin's Function Schwefel's Function Weierstrass Function High Conditioned Elliptic Function	30	1500

Subject to:

$$g_1(x) = 1 - \frac{x_2^3 x_3}{71785 x_1^4} \leq 0,$$

$$g_2(x) = \frac{4x_2^2 - x_1 x_2}{12566(x_2 x_1^3)} + \frac{1}{5108 x_1^2} - 1 \leq 0,$$

$$g_3(x) = 1 - \frac{140.45 x_1}{x_2^2 x_3} \leq 0,$$

$$g_4(x) = \frac{x_1 + x_2}{1.5} - 1 \leq 0.$$

With

$$0.05 \leq x_1 \leq 2, 0.25 \leq x_2 \leq 1.3 and 2 \leq x_3 \leq 15.$$

APPENDIX E
SPEED REDUCER DESIGN PROBLEM

Consider

$$X = [x_1, x_2, x_3, x_4, x_5, x_6, x_7] = [b, m, p, l_1, l_2, d_1, d_2].$$

Minimize

$$f(x) = 0.7854 x_1 x_2^2 \left(3.3333 x_3^2 + 14.9334 x_3 - 43.0934\right) - 1.508 x_1 \left(x_6^2 + x_7^2\right) + 7.4777 \left(x_6^3 + x_7^3\right) + 0.7854(x_4 x_6^2 + x_5 x_7^2).$$

Subject to:

$$g_1(x) = \frac{27}{x_1 x_2^2 x_3} - 1 \leq 0,$$

TABLE 22. IEEE CEC-2017 benchmark test functions.

	functions	fmin
C1	Shifted and Rotated Bent Cigar Function	100
C2	Shifted and Rotated Sum of Different Power Function	200
C3	Shifted and Rotated Zakharov Function	300
C4	Shifted and Rotated Rosenbrock's Function	400
C5	Shifted and Rotated Rastrigin's Function	500
C6	Shifted and Rotated Expanded Scaffer's Function	600
C7	Shifted and Rotated Lunacek Bi_Rastrigin Function	700
C8	Shifted and Rotated Non-Continuous Rastrigin's Function	800
C9	Shifted and Rotated Levy Function	900
C10	Shifted and Rotated Schwefel's Function	1000
C11	Hybrid Function 1 ($N = 3$)	1100
C12	Hybrid Function 2 ($N = 3$)	1200
C13	Hybrid Function 3 ($N = 3$)	1300
C14	Hybrid Function 4 ($N = 4$)	1400
C15	Hybrid Function 5 ($N = 4$)	1500
C16	Hybrid Function 6 ($N = 4$)	1600
C17	Hybrid Function 6 ($N = 5$)	1700
C18	Hybrid Function 6 ($N = 5$)	1800
C19	Hybrid Function 6 ($N = 5$)	1900
C20	Hybrid Function 6 ($N = 6$)	2000
C21	Composition Function 1 ($N = 3$)	2100
C22	Composition Function 2 ($N = 3$)	2200
C23	Composition Function 3 ($N = 4$)	2300
C24	Composition Function 4 ($N = 4$)	2400
C25	Composition Function 5 ($N = 5$)	2500
C26	Composition Function 6 ($N = 5$)	2600
C27	Composition Function 7 ($N = 6$)	2700
C28	Composition Function 8 ($N = 6$)	2800
C29	Composition Function 9 ($N = 3$)	2900
C30	Composition Function 10 ($N = 3$)	3000

$$g_2(x) = \frac{397.5}{x_1 x_2^2 x_3} - 1 \le 0,$$

$$g_3(x) = \frac{1.93 x_4^3}{x_2 x_3 x_6^4} - 1 \le 0,$$

$$g_4(x) = \frac{1.93 x_5^3}{x_2 x_3 x_7^4} - 1 \le 0,$$

$$g_5(x) = \frac{1}{110 x_6^3}\sqrt{\left(\frac{745 x_4}{x_2 x_3}\right)^2 + 16.9 \times 10^6} - 1 \le 0,$$

$$g_6(x) = \frac{1}{85 x_7^3}\sqrt{\left(\frac{745 x_5}{x_2 x_3}\right)^2 + 157.5 \times 10^6} - 1 \le 0,$$

$$g_7(x) = \frac{x_2 x_3}{40} - 1 \le 0,$$

$$g_8(x) = \frac{5 x_2}{x_1} - 1 \le 0,$$

$$g_9(x) = \frac{x_1}{12 x_2} - 1 \le 0,$$

$$g_{10}(x) = \frac{1.5 x_6 + 1.9}{x_4} - 1 \le 0,$$

$$g_{11}(x) = \frac{1.1 x_7 + 1.9}{x_5} - 1 \le 0.$$

With

$$2.6 \le x_1 \le 3.6,\ 0.7 \le x_2 \le 0.8,\ 17 \le x_3 \le 28,$$
$$7.3 \le x_4 \le 8.3,\ 7.8 \le x_5 \le 8.3,\ 2.9 \le x_6 \le 3.9,\ \text{and}$$
$$5 \le x_7 \le 5.5.$$

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REFERENCES

[1] T. Ray and K. M. Liew, "Society and civilization: An optimization algorithm based on the simulation of social behavior," *IEEE Trans. Evol. Comput.*, vol. 7, no. 4, pp. 386–396, Aug. 2003.

[2] S. Mirjalili, "The ant lion optimizer," *Adv. Eng. Softw.*, vol. 83, pp. 80–98, May 2015.

[3] R. G. Rakotonirainy and J. H. van Vuuren, "Improved metaheuristics for the two-dimensional strip packing problem," *Appl. Soft Comput.*, vol. 92, Jul. 2020, Art. no. 106268.

[4] K. Iba, "Reactive power optimization by genetic algorithm," *IEEE Trans. Power Syst.*, vol. 9, no. 2, pp. 685–692, May 1994.

[5] D. E. Goldberg and J. H. Holland, "Genetic algorithms and machine learning," *Mach. Learn.*, vol. 3, nos. 2–3, pp. 95–99, 1988.

[6] K. Price, R. M. Storn, and J. A. Lampinen, *Differential Evolution: A Practical Approach to Global Optimization*. Berlin, Germany: Springer, 2006.
[7] J. Kennedy and R. Eberhart, "Particle swarm optimization," in *Proc. Int. Conf. Neural Netw.*, vol. 4, 1994, pp. 1942–1948.
[8] M. Dorigo, V. Maniezzo, and A. Colorni, "Ant system: Optimization by a colony of cooperating agents," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 26, no. 1, pp. 29–41, Feb. 1996.
[9] S. Mirjalili, S. M. Mirjalili, and A. Lewis, "Grey wolf optimizer," *Adv. Eng. Softw.*, vol. 69, pp. 46–61, Mar. 2014.
[10] S. Mirjalili and A. Lewis, "The whale optimization algorithm," *Adv. Eng. Softw.*, vol. 95, pp. 51–67, Feb. 2016.
[11] S. Z. Koohi, N. A. W. A. Hamid, M. Othman, and G. Ibragimov, "Raccoon optimization algorithm," *IEEE Access*, vol. 7, pp. 5383–5399, 2019.
[12] R. V. Rao, V. J. Savsani, and D. P. Vakharia, "Teaching-learning-based optimization: A novel method for constrained mechanical design optimization problems," *Comput.-Aided Des.*, vol. 43, no. 3, pp. 303–315, Mar. 2011.
[13] A. G. Hussien, M. Amin, M. Wang, G. Liang, A. Alsanad, A. Gumaei, and H. Chen, "Crow search algorithm: Theory, recent advances, and applications," *IEEE Access*, vol. 8, pp. 173548–173565, 2020.
[14] Y. Meraihi, A. B. Gabis, S. Mirjalili, and A. Ramdane-Cherif, "Grasshopper optimization algorithm: Theory, variants, and applications," *IEEE Access*, vol. 9, pp. 50001–50024, 2021.
[15] S. Kaur, L. K. Awasthi, A. L. Sangal, and G. Dhiman, "Tunicate swarm algorithm: A new bio-inspired based metaheuristic paradigm for global optimization," *Eng. Appl. Artif. Intell.*, vol. 90, Apr. 2020, Art. no. 103541.
[16] A. Faramarzi, M. Heidarinejad, S. Mirjalili, and A. H. Gandomi, "Marine predators algorithm: A nature-inspired metaheuristic," *Expert Syst. Appl.*, vol. 152, Aug. 2020, Art. no. 113377.
[17] Z. Huang, Z. Lin, Z. Zhu, and J. Chen, "An improved simulated annealing algorithm with excessive length penalty for fixed-outline floorplanning," *IEEE Access*, vol. 8, pp. 50911–50920, 2020.
[18] S. Kirkpatrick, C. D. Gelatt, and M. P. Vecchi, "Optimization by simulated annealing," *Science*, vol. 220, no. 4598, pp. 671–680, 1983.
[19] E. Rashedi, H. Nezamabadi-Pour, and S. Saryazdi, "GSA: A gravitational search algorithm," *J. Inf. Sci.*, vol. 179, no. 13, pp. 2232–2248, 2009.
[20] H. Eskandar, A. Sadollah, A. Bahreininejad, and M. Hamdi, "Water cycle algorithm—A novel metaheuristic optimization method for solving constrained engineering optimization problems," *Comput. Struct.*, vols. 110–111, pp. 151–166, Nov. 2012.
[21] B. Alatas, "ACROA: Artificial chemical reaction optimization algorithm for global optimization," *Expert Syst. Appl.*, vol. 38, no. 10, pp. 13170–13180, 2011.
[22] S. Mirjalili, S. M. Mirjalili, and A. Hatamlou, "Multi-verse optimizer: A nature-inspired algorithm for global optimization," *Neural Comput. Appl.*, vol. 27, no. 2, pp. 495–513, 2016.
[23] H. Abedinpourshotorban, S. M. Shamsuddin, Z. Beheshti, and D. N. A. Jawawi, "Electromagnetic field optimization: A physics-inspired metaheuristic optimization algorithm," *Swarm Evol. Comput.*, vol. 26, pp. 8–22, Feb. 2016.
[24] Z. Wei, C. Huang, X. Wang, T. Han, and Y. Li, "Nuclear reaction optimization: A novel and powerful physics-based algorithm for global optimization," *IEEE Access*, vol. 7, pp. 66084–66109, 2019.
[25] A. H. Kashan, "A new metaheuristic for optimization: Optics inspired optimization (OIO)," *Comput. Oper. Res.*, vol. 55, pp. 99–125, Mar. 2015.
[26] W. Zhao, L. Wang, and Z. Zhang, "Atom search optimization and its application to solve a hydrogeologic parameter estimation problem," *Knowl.-Based Syst.*, vol. 163, pp. 283–304, Jan. 2019.
[27] A. Faramarzi, M. Heidarinejad, B. Stephens, and S. Mirjalili, "Equilibrium optimizer: A novel optimization algorithm," *Knowl.-Based Syst.*, vol. 191, Mar. 2020, Art. no. 105190.
[28] R. Moghdani and K. Salimifard, "Volleyball premier league algorithm," *Appl. Soft. Comput.*, vol. 64, pp. 161–185, Mar. 2018.
[29] A. Kaveh and A. Zolghadr, "A novel meta-heuristic algorithm: Tug of war optimization," *Iran Univ. Sci. Technol.*, vol. 6, no. 4, pp. 469–492, 2016.
[30] A. Sharma, A. Sharma, A. Dasgotra, V. Jately, M. Ram, S. Rajput, M. Averbukh, and B. Azzopardi, "Opposition-based tunicate swarm algorithm for parameter optimization of solar cells," *IEEE Access*, vol. 9, pp. 125590–125602, 2021.
[31] D. H. Wolper and W. G. Macready, "No free lunch theorems for optimization," *IEEE Trans. Evol. Comput.*, vol. 1, no. 1, pp. 67–82, Apr. 1997.
[32] C. V. Linnaeus, "Systema Naturae per regna tria naturae. Secundum classes, ordines, genera, species, cum characteribus, differentiis, synonymis, locis," *Editio*, vol. 1, no. 10, p. 823, 1758.
[33] J. Ferguson-Lees and D. A. Christie, *Raptors of the World*. Boston, MA, USA: Houghton Mifflin Harcourt, 2001.
[34] J. Del Hoyo, J. Del Hoyo, A. Elliott, and J. Sargatal, *Handbook of the Birds of the World*. Barcelona, Spain: Lynx edicions Barcelona, 1992.
[35] H. C. Mueller and G. Allez, "The identification of North American accipiters," *Amer. Birds*, vol. 33, no. 3, pp. 236–240, May 1979.
[36] R. E. Kenward, "Goshawk hunting behaviour, and range size as a function of food and habitat availability," *J. Animal Ecol.*, vol. 51, no. 1, pp. 69–80, Feb. 1982.
[37] X. Yao, Y. Liu, and G. Lin, "Evolutionary programming made faster," *IEEE Trans. Evol. Comput.*, vol. 3, no. 2, pp. 82–102, Jul. 1999.
[38] Q. Chen, B. Liu, Q. Zhang, J. Liang, P. Suganthan, and B. Qu, "Problem definitions and evaluation criteria for CEC 2015 special session on bound constrained single-objective computationally expensive numerical optimization," Comput. Intell. Lab., Zhengzhou Univ., China Nanyang Technol. Univ., Singapore, Tech. Rep. 178, 2014.
[39] N. Awad, M. Ali, J. Liang, B. Qu, P. Suganthan, and P. Definitions, "Evaluation criteria for the CEC 2017 special session and competition on single objective real-parameter numerical optimization," Technol. Rep., 2016.
[40] B. K. Kannan and S. N. Kramer, "An augmented Lagrange multiplier based method for mixed integer discrete continuous optimization and its applications to mechanical design," *J. Mech. Des.*, vol. 116, no. 2, pp. 405–411, 2008.
[41] A. H. Gandomi and X.-S. Yang, "Benchmark problems in structural optimization," in *Computational Optimization, Methods and Algorithms*. Berlin, Germany: Springer, 2011, pp. 259–281.
[42] E. Mezura-Montes and C. A. C. Coello, "Useful infeasible solutions in engineering optimization with evolutionary algorithms," in *Proc. Mex. Int. Conf. Artif. Intell.*, 2005, pp. 652-662.

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