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THOR 2 Method: An Efficient Instrument in Situations Where There Is Uncertainty or Lack of Data

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ABSTRACT The present study aims to propose an axiomatic evolution of the method, called THOR 2, based on the analysis of the original algorithm. It was proposed, in the evolution, the distinction in the attribution of weights in the sum of scores as well as the multiplication of the value of the criterion weight by the fuzzy-rough index in all preference relations. This functionality allows that, in the absence of data to fill in the classification of alternatives and weights in the decision matrix, it is possible to estimate the data and assign a low pertinence value for attributing that data, thus avoiding the elimination of the alternative or criterion due to the absence of the data. In order to validate the pertinence function proposed for THOR 2, an analysis of the ranking of alternatives in three different scenarios was carried out. In this way, the scenarios were simulated in which there was an absence of data in the original decision matrix. The analysis aimed to compare the result of the ranking of the alternatives when there is no data with the situation that the decision matrix is complete (all data are available), observing the impact on the ranking of the alternatives. In all scenarios that used data estimated in conjunction with the pertinence function, the ranking was kept in line with the ranking in the initial situation. However, when it was decided to exclude the criteria, the ordering was different from the ordering in the situation of origin.

INDEX TERMS Brazilian navy, decision support systems, multiple criteria decision analysis, THOR 2.

I. INTRODUCTION

Research to deal with inaccurate information in multicriteria decision analysis has been ongoing over the past few decades [1]. According to Xiao [2], there are a variety of methods to deal with uncertainty, being applied in several areas such as selection, workflow scheduling, prediction, failure mode analysis and effects analysis (FMEA), fusion, evidence reasoning, medical diagnosis, decision making and classification.

In many of the approaches, data/preferences on the values of attributes and /or compensation weights are required to be the most accurate or representative of the decision makers' preferences.

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However, providing such accurate data is not always an easy task for decision makers since there may be the inclusion of unattainable attributes to reflect social and environmental impacts [3]. Since not all data are always available, to apply a multi-criteria decision analysis (MCDA), it is essential to eliminate variants without data or complete the data [4]. The exclusion of criteria or alternatives, however, often causes important data to be disregarded, impacting the quality of the ordering generated by the decision algorithms, as occurs in traditional multi-criteria decision support methods such as Analytic Hierarchy Process (AHP) [5], ÉLimination et Choix Traduisant la REalité (ELECTRE) [6] and Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE) [7], for example.

The following research problem arises: "If you do not have all the necessary data, is it a best practice to eliminate the alternative or the criterion or to assign an estimate conditional on a pertinence value?''. The use of data estimated in THOR 2 proved to be an efficient instrument in situations where there is uncertainty or lack of data, since it is not necessary to eliminate the criterion or alternative in question. Thus, the use of the THOR 2 method appears as an alternative for decision problems of this type.

This study uses the problem presented by Tenório *et al.* [8] in which the THOR method is applied to select a ship for purchase, among eight ships used by navies around the world, in order to satisfies the replenishment of escorts. In the study, the description of the problem, the alternatives and the criteria were presented in greater detail.

This work aims to fill the gap in decision problems where data are missing from the decision matrix, since it is not necessary to eliminate the alternative or criterion. For this, the THOR 2 method will be used.

II. THOR METHOD

The THOR method uses Preference Modeling (approaching the French School), Utility Theory and Multiattribute Theory (approaching the American School) to quantify the attractiveness of an alternative through the creation of a non-transitive aggregation function [9]. Thus, it considers the non-determinism of the weight assignment process and quantifies the non-determinism, reapplying it in the process of ordering the alternatives [10].

Through the use of Fuzzy Set Theory, it is possible to quantify the uncertainty of the process. The method also allows, through the Rough Set Theory, to eliminate the criteria that do not impact the ordering of the process [11]. In this way, it is possible to infer whether a given criterion is sufficient or irrelevant regarding the ordering of alternatives. It is understood as irrelevant, the criterion or set of criteria that, if removed, does not alter the original ordering of the alternatives, in which all the criteria are used. THOR is able to aggregate the weights assigned by different decision makers through the sum of these normalized values.

For the application of THOR it is necessary [12]

1) Represent the relative importance to the criteria in the form of a weight.

2) Establish a preference threshold (p_j) and an indifference threshold (q_j) for each criterion j.

3) Establish a discordance.

4) Determine the pertinence of the weight values assigned to each criterion.

5) Determine the pertinence of the classification of the alternative in the criterion.

In a scenario of lack of security or uncertainty in judging the weights and classification of alternatives, a value that quantifies the inaccuracy is assigned. In this way, the decision maker expresses the levels of certainty, for the weights of the criteria and for the classification of alternatives, through the pertinence indexes. A value within a real scale of [0,1] is assigned, in which the closer to 1, the greater the certainty in relation to the assigned item and the closer to 0, the greater the uncertainty [11].

Given two alternatives a and b, three situations should be considered when using THOR: S_1, S_2 and S_3. In using the S_1 context, the alternatives have their attractiveness punctuated in situations where a strong preference (aP_jb) occurs. Thus, comparing alternative with the other alternatives, we can identify the criteria in which aP_jb, considering the preference threshold, indifference and discordance, checking whether the imposed condition is satisfied. If satisfied, we know that dominates [12]. If the intra-criterion difference is greater than the preference threshold (p), the comparison is configured as a strong preference (aP_jb). When the intra-criterion difference module is between the values of the indifference threshold (q) and the preference threshold (p), the comparison is configured as a weak preference (aQ_jb). If the intra-criterion difference module is between the negative value of the indifference limit ($-q$) and the positive value of the indifference limit (q), the comparison is configured as an indifference (aI_jb). The relations (strict preference), (indifference) and (weak preference) are expressed in (1), (2) and (3) respectively.

$$aPb \leftrightarrow g(a) - g(b) > p \tag{1}$$

$$aIb \leftrightarrow -q \leq |g(a) - g(b)| \leq q \tag{2}$$

$$aQb \leftrightarrow q < |g(a) - g(b)| \leq p \tag{3}$$

The Equations (4), (5) and (6) reflect the three situations for an alternative to be ranked better than the other [13]

$$\begin{aligned} \mathrm{S1} : &\sum\nolimits_{j=1}^{n} (\mathrm{w_j}|\mathrm{aP_jb}) \\ &> \sum\nolimits_{j=1}^{n} (\mathrm{w_j}|\mathrm{aQ_jb} + \mathrm{aI_jb} + \mathrm{aR_jb} + \mathrm{bQ_ja} + \mathrm{bP_ja}) \end{aligned} \tag{4}$$

$$\begin{aligned} \mathrm{S2} : &\sum\nolimits_{j=1}^{n} (w_j|aP_j + aQ_jb) \\ &> \sum\nolimits_{j=1}^{n} (w_j|\mathrm{aI_jb} + aR_jb + bQ_ja + bP_ja) \end{aligned} \tag{5}$$

$$\begin{aligned} S3 : &\sum\nolimits_{j=1}^{n} (w_j|aP_jb + aQ_jb + aI_jb) \\ &> \sum\nolimits_{j=1}^{n} (w_j|aR_jb + bQ_ja + bP_ja) \end{aligned} \tag{6}$$

In the S2 context, the attractiveness of the alternatives is scored in situations of strong (aP_jb) and weak (aQ_jb) preference. In the S3 context, the attractiveness in situations of strong preference (aP_jb), weak preference (aQ_jb) and indifference (aI_jb) are pointed out. S1 is considered the strictest scenario, while S3 is the most flexible [14].

III. THOR 2 METHOD

The THOR 2 method contemplates an axiomatic evolution of THOR, presenting, as a first contribution in relation to THOR, a distinction in the attribution of weights in situations of indifference aI_jb and weak preference aQ_jb in situations S_1, S_2 and S_3. The situations that occur aI_jb, bring with them half the weight value of the respective criterion and the comparisons in which aQ_jb occur, carry a proportion between half the value of the criterion weight and the total value of the weight [15]. The equation is described in (7):

$$weight * (((ai - qi)/(pi - qi) * 0.5 + 0.5)) \tag{7}$$

As a second contribution in relation to THOR, it is agreed, in situations where aP_jb, aQ_jb and aI_jb occur, that the weight value of the criterion is multiplied by the fuzzy-rough index, thus downgrading the comparison in function the degree of security of the data. The original THOR only considers the multiplication by the index in the situation aQ_jb, downgrading the gain only in that case. THOR 2, however, will also include the downgrading of the score in situations of strong preference and indifference. In this way, THOR 2 represents a significant contribution in this respect, since all the uncertainty present in the attribution of the classification of alternatives and weights is quantified. This functionality allows that, in the lack of data to fill in the classification of alternatives and weights in the decision matrix, it is possible to estimate the data and assign a low pertinence value, degree of security for attributing that data. In this way, the elimination of the alternative or the criterion is avoided due to the lack of the data. Thus, even if there is a score gain, the weight is downgraded by the degree of existing inaccuracy.

IV. THOR WEB

The THOR Web system was developed at the Military Engineering Institute (IME) located in Rio de Janeiro, Brazil (for more information, see the website www.thor-web.com) [16]. The system was implemented in Html, Java Script, Python and Flash.

THOR Web contemplates the use of the THOR and THOR 2 methods, helping to compare the results of the methods and contributing to the application in more complex problems, in which a greater number of criteria and alternatives are used. The system aims to expand and disseminate the use of methods by the scientific community. In the initial screen, the user must choose one of the two methods to perform his analysis. In case a comparison between the methods is desired, the user must run the problem using both algorithms (THOR and THOR 2) in order to compare their results. Fig. 1 presents the initial screen of the website.

THOR Web allows the use of multiple decision-makers. In this way, it is possible to aggregate the weights of different decision makers, allowing them to express their judgment (s) of value (s) through direct assignments and in ratio scales and interval scales (Fig. 2). Normalization consists of dividing the weights by the highest value of the set of weights assigned by the decision maker. Therefore, the most important weight for each decision maker will be 1, and the rest of the weights will be less than 1. In the direct assignment and on the interval scale, the normalized values of each decision maker are added together to determine the weights. While on the ratio scale, the geometric average of the normalized weights of each decision maker is calculated. The decision matrix, together with the weights and preference limits, indifference and discordance limits are shown in Fig. 3. It is possible

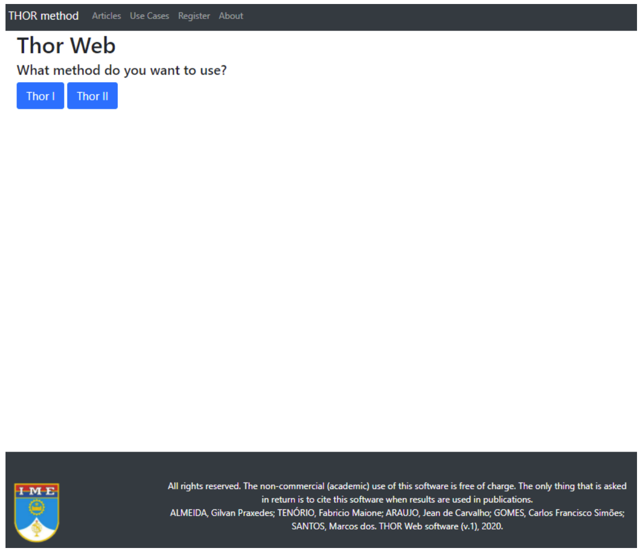

FIGURE 1. Initial screen.

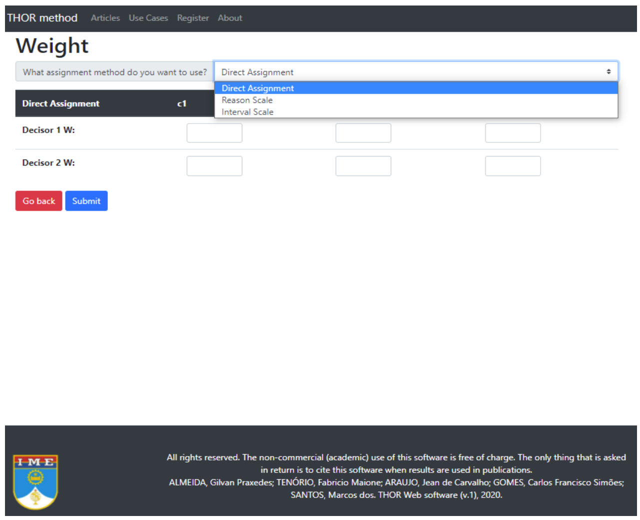

FIGURE 2. Types of assignment.

to select whether to use pertinences and Rough Set Theory (RST). If the DM chooses to use pertinences, the pertinence matrix will be available for completion (Fig. 4).

Through RST, it is possible to infer whether a given criterion is sufficient or irrelevant with regard to the ordering of alternatives. It is understood as irrelevant, the criterion or set of criteria that, if removed, does not alter the original ordering of the alternatives, in which all the criteria are used. This results from a close classification for these criteria, associated with the weights attributed to the criteria by the decision maker, which shows the irrelevance of these criteria in the process [10]. THOR Web performs the calculation in S_1, S_2 and S_3 informing the ordering of the alternatives. In Fig. 5, an example of ordering in S_1 is found. Figure 6 illustrates the RST analysis. In the example, the original (complete) ordering is compared with the ordering in which criterion 1 was

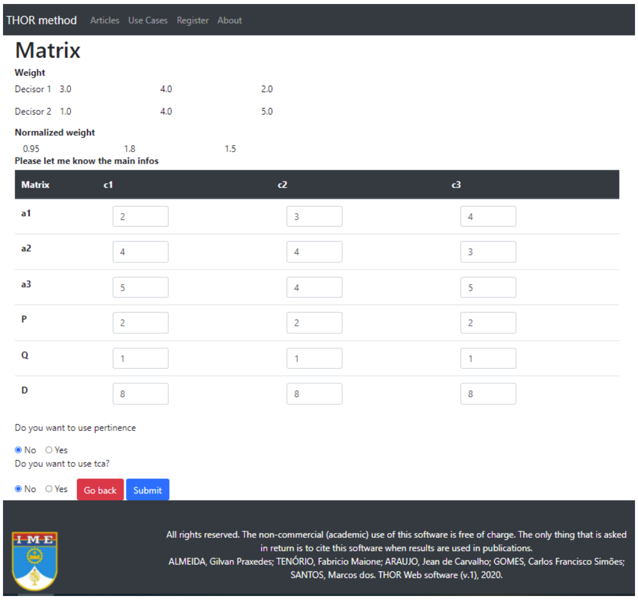

FIGURE 3. Example of decision matrix.

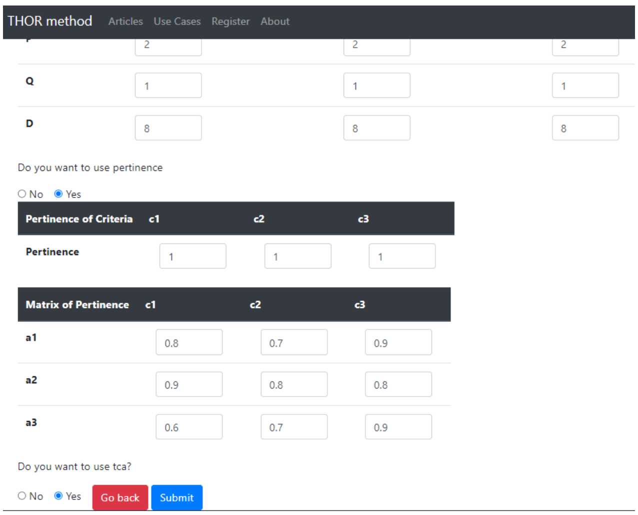

FIGURE 4. Pertinence matrix.

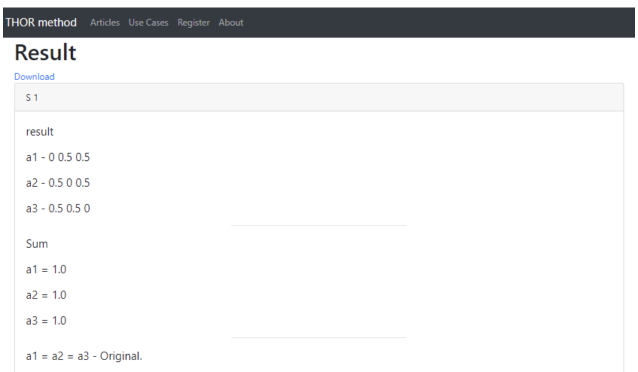

FIGURE 5. S_1 sorting example.

removed from the decision matrix. Despite the removal of criterion 1, the order of alternatives remained the same in the

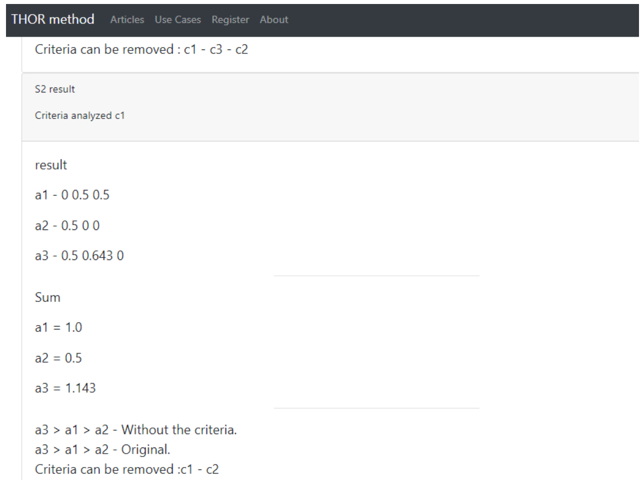

FIGURE 6. RST example.

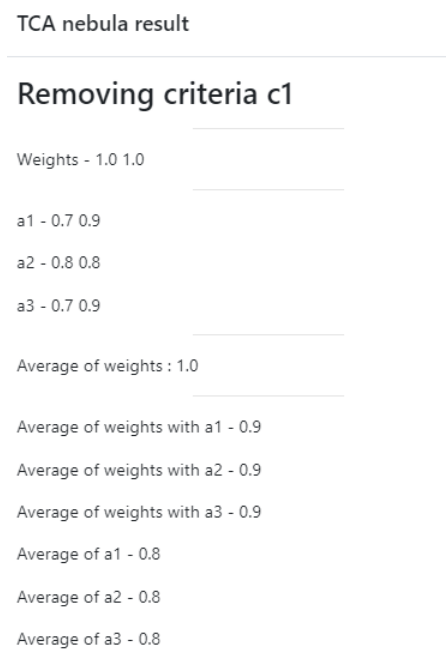

FIGURE 7. FRST example.

two ordinations. Thus, criterion 1 is considered irrelevant and can be removed in S2.

In the fuzzy rough set theory (FRST), the RST is combined with the Fuzzy Set Theory (FST), in order to verify if there is an increase in the inaccuracy of the decision process. The values of the FRST are composed by the average of the pertinences of the alternatives. The alternatives are also analyzed together with the criteria. In this way, it constitutes the average of the average of the values of the pertinences of the alternatives with the average of the set of criteria (average of the relevance of the weights) (Fig. 7).

V. METHODOLOGY

In order to validate the pertinence function proposed for THOR 2, an analysis of the ranking of alternatives in different

TABLE 1. Comparative table of the parameters of modern frigates and destroyers, adapted from Vogt (2018).

Typical Data	F-124	LCF	F-100	FREMM	HORIZON	T-45 DARING	DDG-51	F-125
L (m)	143,0	144,2	146,7	137,0	153,0	152,4	153,8	149,5
LWL (m)	132,2	-	133,2	-	141,7	143,5	142,0	-
T (m)	5,0	5,2	4,9	5,0	5,1	5,7	6,3	5,0
DISPL. FL. (tons)	5.600	6.050	5.800	5.500	6.700	7.350	8.300	7.200
Max Sped (kts)	29	30	29	27	29	29	32	26
Radius of action	4.000 / 18	5.000 / 18	4.500 / 18	6.000 / 15	7.000 / 18	7.000 / 18	8.150 / 20	4.000 /?
Crew	243	230	250	145	230	235	380	190
Main Artillery (mm)	1x76 mm SupRapid	1x127 mm OTO54LW	1x127/54 Mk 45-2	1x76 mm SupRapid	1x76 mm SupRapid	1x114 mm Mk 8-1	1x127 mm Mk 45-1	1x127 mm OTO62LW
Secondary Artillery (mm)	2 x 27 mm	2 x CIWS 30 mm	2 x CIWS 20 mm	2 x KBA 25 mm	2 x KBA 25 mm	2 x CIWS 20 mm	2 x CIWS 20 mm	2 x 27 mm 7 x 12,7 mm
AAeW	ESSM SM-2 MR	ESSM SM-2 MR	ESSM SM-2 MR	ASTER-15 /30	ASTER-15 /30	ASTER-15 /30	ESSM SM-2 MR	2 x RAM
ASuW	EXOCET	HARPOON	HARPOON	MM-40	MM-40 III	HARPOON	HARPOON	HARPOON
ASW	MU-90	Mk-46	Mk-46	MU-90	MU-90	STGRAY	ASROC	SUB ROV
He	2 x LYNX	1x LYNX	1x SH60B	1x NH-90	1x NH-90	LYNX 300	1x SH60B	2x NH-90

TABLE 2. Comparative table of the parameters of modern frigates and destroyers.

Typical Data	Max Speed	Radius of action	Crew	Main Artillery	Secondary Artillery	ASuW	ASW	He	Cost ($)
F-124	29	4000	243	1	1	1	1	4	800
LCF	30	5000	230	3	5	4	1	1	500
F-100	29	4500	250	3	4	4	1	3	600
FREMM	27	6000	145	1	3	2	1	3	745
HORIZON	29	7000	230	1	3	3	1	3	1030
T-45 DARING	29	7000	235	2	4	4	1	2	1500
DDG-51	32	8150	380	3	4	4	3	3	1650
F-125	26	4000	190	3	2	4	2	5	740
Weights	2	2	1	3	3	3	3	4	6
p	3	400	28	1	1	1	1	1	200
q	1	40	14	0,5	0,5	0,5	0,5	0,5	50
Discordance	6,32	4231,5	238,8	2,03	4,05	3,04	2,03	4,05	1166,5

scenarios was carried out. In this way, three different scenarios were simulated in which there was an absence of data in the original decision matrix. The analysis aimed to compare the result of the ranking of the alternatives when there is no data with the situation that the decision matrix is complete (all data are available), observing the impact on the ranking of the alternatives.

In each scenario, two data from the original decision matrix were randomly disregarded. For this purpose, a function of generating random numbers was used to determine the positioning of the row (alternative) and column (criterion) of the data to be discarded.

In each scenario, two values from the original decision matrix were discarded and new values were estimated using a random number generation function. The values were estimated within a range considered feasible by the specialists, with pertinence being attributed according to the security in the allocation of the values of the respective range. Later, in each of the scenarios, the difference in the ordering of the alternatives in S_1, S_2 and S_3 was compared.

VI. CASE STUDY

Table 1, from the reference approached in Vogt [17], summarizes the main technical-operational characteristics of the main ships available in the world for sale.

Table 2 presents the alternatives and criteria used in the study; each cell corresponds to an alternative ship classified in its respective criteria. The assignment of the classification of the alternatives of the Main Artillery, Secondary Artillery, AsuW, ASW and He criteria was made through an interval scale, where the distance between the values of the alternatives was considered. The classification of alternatives, weights, preference limits, limits of indifference and discordance for each criterion were attributed through a joint analysis with experts in the field. Weights were assigned on a

TABLE 3. Pertinences.

	Max Speed	Radius of action	Crew	Main Artillery	Secondary Artillery	ASuW	ASW	He	Cost ($)
F-124	1	1	1	0,9	0,9	0,9	0,9	0,9	0,8
LCF	1	1	1	0,9	0,9	0,9	0,9	0,9	0,8
F-100	1	1	1	0,9	0,9	0,9	0,9	0,9	0,7
FREMM	1	1	1	0,9	0,9	0,9	0,9	0,9	0,5
HORIZON	1	1	1	0,9	0,9	0,9	0,9	0,9	0,75
T-45 DARING	1	1	1	0,9	0,9	0,9	0,9	0,9	0,8
DDG-51	1	1	1	0,9	0,9	0,9	0,9	0,9	0,8
F-125	1	1	1	0,9	0,9	0,9	0,9	0,9	0,5
Weights	0,95	0,95	0,95	0,95	0,95	0,95	0,95	0,95	0,95

scale of 1 to 6, using a scale ranging from 1 (least important) to 6 (most important).

Table 3 shows the pertinence attributed to each weight and classification of the alternative in each criterion. For the pertinence of the criteria Maximum Speed, Radius of Action (at cruising speed) and Crew, since these are real data extracted from the technical specifications of the ships, the value 1 was assigned, indicating that there is no doubt as to the attribution of the data. As for the Main Artillery, Secondary Artillery, ASuW and He criteria, a value of 0.9 was assigned, since it is a qualitative analysis carried out by specialists with extensive experience in the area, however, even so, subject to some inaccuracy. Regarding the cost criterion, the values were attributed due to the degree of reliability of the sources from which the costs of the ships were extracted. The AAeW (Anti-Air Warfare) criterion, although important, was not considered because the ships under study have similar operational characteristics. Regarding the cost criterion, the cost of acquiring ships in Brazil was not determined, therefore, the cost of construction (in millions of dollars) of the ship in the country of origin was considered. However, it can be inferred that the acquisition cost will be proportional.

A. APPLICATION OF THE THOR 2 METHOD

In the result generated by THOR 2, the existence of three groups was found (Table 4). The first group consists of the alternatives LCF, DDG-51 and F-100 present in THOR 2, the three frigates alternate in the first position in the types of ordering S_1, S_2 and S_3. The second group consists of the alternatives T-45, HORIZON, F-125 and FREMM, which exchange between the fourth, fifth, sixth and seventh positions in S_1, S_2 and S_3. The third group consists of the alternative F-124, which remains in the last position in all types of ordering.

A more detailed analysis was carried out, by evaluating an additional scenario composed only of the alternatives that make up the first group (LCF, DDG-51 and F-100). For this, the THOR 2 algorithm was used. The result can be seen in Table 5.

In the additional scenario, the alternatives LCF, F-100 and DDG-51 obtained the same score in S_1 and S_2, and it is not possible to differentiate them. The use of S_1 and S_2 alone does not indicate to the decision maker an alternative to be chosen, demonstrating that there is no full agreement that one alternative is dominant over the others. On the other hand, the situation S_3, being less rigorous, allows the ordering of alternatives. However, the difference in scores for each alternative is remarkably close, demonstrating, by using the method, that the difference in attractiveness is small. It is concluded that the LCF alternative should be chosen, as it is slightly more suitable than the others.

TABLE 4. Ordering of alternatives using the THOR 2 method.

S_1		S2		S3	
LCF	3,787	F-100	4,134	F-100	5,190
DDG-51	3,573	DDG-51	4,053	LCF	4,990
F-100	3,500	LCF	4,002	DDG-51	4,689
T-45 DARING	3,500	F-125	3,806	F-125	4,115
HORIZON	3,500	T-45 DARING	1,526	HORIZON	1,869
F-125	3,000	HORIZON	1,500	T-45 DARING	1,768
FREMM	2,500	FREMM	1,077	FREMM	1,372
F-124	2,500	F-124	1,000	F-124	1,165

TABLE 5. Ordering of alternatives using the THOR 2 method.

S_1		S2		S3	
LCF	1,000	LCF	1,000	LCF	1,330
F-100	1,000	F-100	1,000	DDG-51	1,270
DDG-51	1,000	DDG-51	1,000	F-100	1,142

B. VALIDATION OF THE THOR 2 METHOD PERTINENCE FUNCTION

Scenario 01: The Maximum Speed data for the alternative F-124 and the crew of the alternative FREMM were disregarded. For Maximum Speed, an interval between 27 to 31 kts was assigned by specialists (the knot is measured in nautical miles per hour), with a relevance of 0.8. For the crew, on the other hand, an interval between 140 to 200 crew members was

TABLE 6. Alternatives and criteria used in scenario 01.

	Max Speed	Radius of action	Crew	Main Artillery	Secondary Artillery	ASuW	ASW	He	Cost ($)
F-124	30	4000	243	1	1	1	1	4	800
LCF	30	5000	230	3	5	4	1	1	500
F-100	29	4500	250	3	4	4	1	3	600
FREMM	27	6000	198	1	3	2	1	3	745
HORIZON	29	7000	230	1	3	3	1	3	1030
T-45 DARING	29	7000	235	2	4	4	1	2	1500
DDG-51	32	8150	380	3	4	4	3	3	1650
F-125	26	4000	190	3	2	4	2	5	740
Weights	2	2	1	3	3	3	3	4	6
p	3	400	28	1	1	1	1	1	200
q	1	40	14	0,5	0,5	0,5	0,5	0,5	50
Discordance	6,32	4231,5	238,8	2,03	4,05	3,04	2,03	4,05	1166,5

TABLE 7. Pertinence of alternatives and criteria in scenario 01.

	Max Speed	Radius of action	Crew	Main Artillery	Secondary Artillery	ASuW	ASW	He	Cost ($)
F-124	0,8	1	1	0,9	0,9	0,9	0,9	0,9	0,8
LCF	1	1	1	0,9	0,9	0,9	0,9	0,9	0,8
F-100	1	1	1	0,9	0,9	0,9	0,9	0,9	0,7
FREMM	1	1	0,8	0,9	0,9	0,9	0,9	0,9	0,5
HORIZON	1	1	1	0,9	0,9	0,9	0,9	0,9	0,75
T-45 DARING	1	1	1	0,9	0,9	0,9	0,9	0,9	0,8
DDG-51	1	1	1	0,9	0,9	0,9	0,9	0,9	0,8
F-125	1	1	1	0,9	0,9	0,9	0,9	0,9	0,5
Weights	0,95	0,95	0,95	0,95	0,95	0,95	0,95	0,95	0,95

assigned using a 0.8 relevance. The random number generator resulted in the Maximum Speed values of 30 kts for the F-124 ship and a crew of 198 crew members on the FREMM ship. The changes can be seen in Tables 6 and 7.

Scenario 02: The Radius of Action data for the alternative HORIZON and the crew of the alternative F-124 were disregarded. For the Radius of Action, an interval between 6500 to 7500 miles was assigned by the specialists, with a pertinence of 0.75. For the crew, on the other hand, an interval between 200 and 250 crew members was assigned using a 0.75 relevance. The random number generator resulted in the Radius of Action values of 7281 miles for the HORIZON ship and a crew of 237 crew members on the F-124 ship. The changes can be seen in tables 8 and 9.

Scenario 03: The data for the Maximum Speed of the alternative F-125 and the Radius of Action of the alternative LCF were disregarded. For Maximum Speed, an interval between 23 and 26 kts was assigned by the specialists, with a pertinence of 0.7. For the Radius of Action, on the other hand, an interval between 4000 to 7000 miles was assigned using a 0.7 relevance. The random number generator resulted in the Maximum Speed values of 26 kts for the F-125 vessel and a Radius of Action of 5513 miles in the LCF vessel. The changes can be seen in tables 10 and 11.

The results of the THOR 2 method for the S_1, S_2 and S_3 rankings in the original situation and in each of the scenarios are found in tables 12, 13 and 14.

From the result of the rankings in S_1, S_2 and S_3 in the three different scenarios, it was found that, despite the change in the score, the ranking of the alternatives remained unchanged in all simulated scenarios.

A new analysis is proposed in which the initial situation is compared with the situation of eliminating the criteria. From there, the following research problem arises: "In the event that you do not have all the necessary data, it is a best practice to eliminate the alternative or criterion or to assign a conditional estimate to a value of relevance?"

As shown in Table 15, there were changes, in S_1, in the ordering of the alternatives F-100 and T-45 and the alternatives HORIZON, F-125 and F-124 when comparing the initial situation with the situation of elimination of the Maximum Speed and Crew criteria. In the original situation, the alternatives F-100 and T-45 have a tie, however, after the elimination of the criteria, F-100 presents a higher score, changing the ordering of the alternatives. The alternatives HORIZON and F-125, after eliminating the criteria, changed positions. The alternative F-124 is tied with the alternative FREMM in the initial situation, whereas in the situation of elimination of criteria it remains isolated in the last position.

TABLE 8. Alternatives and criteria in scenario 02.

	Max Speed	Radius of action	Crew	Main Artillery	Secondary Artillery	ASuW	ASW	He	Cost ($)
F-124	29	4000	237	1	1	1	1	4	800
LCF	30	5000	230	3	5	4	1	1	500
F-100	29	4500	250	3	4	4	1	3	600
FREMM	27	6000	145	1	3	2	1	3	745
HORIZON	29	7281	230	1	3	3	1	3	1030
T-45 DARING	29	7000	235	2	4	4	1	2	1500
DDG-51	32	8150	380	3	4	4	3	3	1650
F-125	26	4000	190	3	2	4	2	5	740
Weights	2	2	1	3	3	3	3	4	6
p	3	400	28	1	1	1	1	1	200
q	1	40	14	0,5	0,5	0,5	0,5	0,5	50
Discordance	6,32	4231,5	238,8	2,03	4,05	3,04	2,03	4,05	1166,5

TABLE 9. Pertinence of alternatives and criteria in scenario 02.

	Max Speed	Radius of action	Crew	Main Artillery	Secondary Artillery	ASuW	ASW	He	Cost ($)
F-124	1	1	0,75	0,9	0,9	0,9	0,9	0,9	0,8
LCF	1	1	1	0,9	0,9	0,9	0,9	0,9	0,8
F-100	1	1	1	0,9	0,9	0,9	0,9	0,9	0,7
FREMM	1	1	1	0,9	0,9	0,9	0,9	0,9	0,5
HORIZON	1	0,75	1	0,9	0,9	0,9	0,9	0,9	0,75
T-45 DARING	1	1	1	0,9	0,9	0,9	0,9	0,9	0,8
DDG-51	1	1	1	0,9	0,9	0,9	0,9	0,9	0,8
F-125	1	1	1	0,9	0,9	0,9	0,9	0,9	0,5
Weights	0,95	0,95	0,95	0,95	0,95	0,95	0,95	0,95	0,95

TABLE 10. Alternatives and criteria in scenario 03.

	Max Speed	Radius of action	Crew	Main Artillery	Secondary Artillery	ASuW	ASW	He	Cost ($)
F-124	29	4000	243	1	1	1	1	4	800
LCF	30	5513	230	3	5	4	1	1	500
F-100	29	4500	250	3	4	4	1	3	600
FREMM	27	6000	145	1	3	2	1	3	745
HORIZON	29	7000	230	1	3	3	1	3	1030
T-45 DARING	29	7000	235	2	4	4	1	2	1500
DDG-51	32	8150	380	3	4	4	3	3	1650
F-125	26	4000	190	3	2	4	2	5	740
Weights	2	2	1	3	3	3	3	4	6
p	3	400	28	1	1	1	1	1	200
q	1	40	14	0,5	0,5	0,5	0,5	0,5	50
Discordance	6,32	4231,5	238,8	2,03	4,05	3,04	2,03	4,05	1166,5

Ordering of S_1 (Initial Situation): LCF> DDG-51> (F-100, T-45 DARING and HORIZON)> F-125> (FREMM and F-124).

Ordering of S_1 (After eliminating the Maximum Speed and Crew criteria): LCF> DDG-51> F-100> T-45 DARING> F-125> (HORIZON and FREMM)> F-124.

In S_2, there was a change in the order of the first four alternatives (F-100, DDG-51, LCF and F-125) regarding the situation in which the criteria were eliminated. In S_3, the alternatives of the first and the second position (LCF and F-100) had their order changed in the same situation.

There were changes in the ordering of the alternatives, in S_1, compared to the initial situation when the Radius of Action and Crew criteria were eliminated (Table 16). The HORIZON and F-125 alternatives have their positions switched. The alternatives FREMM and F-124 tie in the original situation, however, in the situation of elimination of criteria, the alternative F-124 presents a higher score.

TABLE 11. Pertinence of alternatives and criteria in scenario 03.

	Max Speed	Radius of action	Crew	Main Artillery	Secondary Artillery	ASuW	ASW	He	Cost ($)
F-124	1	1	1	0,9	0,9	0,9	0,9	0,9	0,8
LCF	1	0,7	1	0,9	0,9	0,9	0,9	0,9	0,8
F-100	1	1	1	0,9	0,9	0,9	0,9	0,9	0,7
FREMM	1	1	1	0,9	0,9	0,9	0,9	0,9	0,5
HORIZON	1	1	1	0,9	0,9	0,9	0,9	0,9	0,75
T-45 DARING	1	1	1	0,9	0,9	0,9	0,9	0,9	0,8
DDG-51	1	1	1	0,9	0,9	0,9	0,9	0,9	0,8
F-125	0,7	1	1	0,9	0,9	0,9	0,9	0,9	0,5
Weights	0,95	0,95	0,95	0,95	0,95	0,95	0,95	0,95	0,95

TABLE 12. THOR 2: S_1 sorting.

Inicial Situation		Scenario 1		Scenario 2		Scenario 3	
LCF	3,787	LCF	3,791	LCF	3,788	LCF	3,780
DDG-51	3,573	DDG-51	3,587	DDG-51	3,575	DDG-51	3,573
F-100	3,500	F-100	3,500	F-100	3,500	F-100	3,500
T-45 DARING	3,500	T-45 DARING	3,500	T-45 DARING	3,500	T-45 DARING	3,500
HORIZON	3,500	HORIZON	3,500	HORIZON	3,500	HORIZON	3,500
F-125	3,000	F-125	3,000	F-125	3,000	F-125	3,000
FREMM	2,500	FREMM	2,500	FREMM	2,500	FREMM	2,500
F-124	2,500	F-124	2,500	F-124	2,500	F-124	2,500

TABLE 13. THOR 2: S_2 sorting.

Inicial Situation		Scenario 1		Scenario 2		Scenario 3	
F-100	4,134	F-100	4,139	F-100	4,141	F-100	4,134
DDG-51	4,053	DDG-51	4,045	DDG-51	4,053	DDG-51	4,053
LCF	4,002	LCF	4,006	LCF	4,007	LCF	4,006
F-125	3,806	F-125	3,826	F-125	3,810	F-125	3,829
T-45 DARING	1,526	T-45 DARING	1,528	T-45 DARING	1,526	T-45 DARING	1,526
HORIZON	1,500	HORIZON	1,500	HORIZON	1,500	HORIZON	1,500
FREMM	1,077	FREMM	1,065	FREMM	1,075	FREMM	1,077
F-124	1,000	F-124	1,000	F-124	1,000	F-124	1,000

Ordering of S_1 (Initial Situation): LCF> DDG-51> (F-100, T-45 DARING and HORIZON)> F-125> (FREMM and F-124).

Ordering of S_1 (After eliminating the Radius of Action and Crew criteria): LCF> DDG-51> (F-100 and T-45 DARING)> F-125> (HORIZON and F-124)> FREMM.

In S_2, there was a change in the ordering of the second, third, fourth, seventh and eighth alternative (DDG-51, LCF, F-125, FREMM and F-124) regarding the situation in which the criteria were eliminated. In S_3, the alternatives of the last three positions (T-45 DARING, FREMM and F-124) had their order changed in the same situation.

In table 17, when the Maximum Speed and Radius of Action criteria were removed, there were changes in the ordering of the second, fifth and sixth positions (DDG-51, HORIZON and F-125) compared to the initial situation in S1.

Ordering of S_1 (Initial Situation): LCF> DDG-51> (F-100, T-45 DARING and HORIZON)> F-125> (FREMM and F-124).

Ordering of S_1 (After eliminating the Maximum Speed and Radius of Action): LCF> F-125> (F-100 and T-45 DARING)> DDG-51> (HORIZON, FREMM and F-124).

In S_2, there was a change in the first, second, fourth, fifth and seventh position (F-100, DDG-51, F-125, T-45 DARING

TABLE 14. **THOR 2: S_3 sorting.**

Inicial Situation		Scenario 1		Scenario 2		Scenario 3	
F-100	5,190	F-100	5,192	F-100	5,196	F-100	5,191
LCF	4,990	LCF	4,992	LCF	4,996	LCF	5,005
DDG-51	4,689	DDG-51	4,681	DDG-51	4,689	DDG-51	4,681
F-125	4,115	F-125	4,162	F-125	4,120	F-125	4,144
HORIZON	1,869	HORIZON	1,871	HORIZON	1,872	HORIZON	1,869
T-45 DARING	1,768	T-45 DARING	1,769	T-45 DARING	1,708	T-45 DARING	1,768
FREMM	1,372	FREMM	1,356	FREMM	1,376	FREMM	1,372
F-124	1,165	F-124	1,163	F-124	1,169	F-124	1,165

TABLE 15. **THOR 2: Ordering after eliminating the maximum speed and crew criteria.**

S1				S2				S3			
Inicial Situation		Elimination of criteria		Inicial Situation		Elimination of criteria		Inicial Situation		Elimination of criteria	
LCF	3,787	LCF	3,916	**F-100**	**4,134**	**LCF**	**4,116**	**F-100**	**5,190**	**LCF**	**4,937**
DDG-51	3,573	DDG-51	3,626	**DDG-51**	**4,053**	**F-125**	**4,055**	**LCF**	**4,990**	**F-100**	**4,808**
F-100	**3,500**	**F-100**	**3,507**	**LCF**	**4,002**	**DDG-51**	**4,038**	DDG-51	4,689	DDG-51	4,704
T-45 DARING	**3,500**	**T-45 DARING**	**3,500**	**F-125**	**3,806**	**F-100**	**3,810**	F-125	4,115	F-125	4,406
HORIZON	**3,500**	**F-125**	**3,014**	T-45 DARING	1,526	T-45 DARING	1,519	HORIZON	1,869	HORIZON	1,821
F-125	**3,000**	**HORIZON**	**2,500**	HORIZON	1,500	HORIZON	1,500	T-45 DARING	1,768	T-45 DARING	1,706
FREMM	**2,500**	**FREMM**	**2,500**	FREMM	1,077	FREMM	1,104	FREMM	1,372	FREMM	1,461
F-124	**2,500**	**F-124**	**2,000**	F-124	1,000	F-124	0,500	F-124	1,165	F-124	0,605

TABLE 16. **THOR 2: Ordering after eliminating the radius of action and crew criteria.**

S1				S2				S3			
Inicial Situation		Elimination of criteria		Inicial Situation		Elimination of criteria		Inicial Situation		Elimination of criteria	
LCF	3,787	LCF	3,880	F-100	4,134	F-100	4,437	F-100	5,190	F-100	5,729
DDG-51	3,573	DDG-51	3,535	**DDG-51**	**4,053**	**LCF**	**4,211**	LCF	4,990	LCF	5,207
F-100	3,500	F-100	3,500	**LCF**	**4,002**	**F-125**	**4,042**	DDG-51	4,689	DDG-51	4,693
T-45 DARING	3,500	T-45 DARING	3,500	**F-125**	**3,806**	**DDG-51**	**4,038**	F-125	4,115	F-125	4,401
HORIZON	**3,500**	**F-125**	**3,035**	T-45 DARING	1,526	T-45 DARING	1,505	HORIZON	1,869	HORIZON	1,786
F-125	**3,000**	**HORIZON**	**2,500**	HORIZON	1,500	HORIZON	1,500	**T-45 DARING**	**1,768**	**F-124**	**1,769**
FREMM	**2,500**	**F-124**	**2,500**	**FREMM**	**1,077**	**F-124**	**1,500**	**FREMM**	**1,372**	**T-45 DARING**	**1,674**
F-124	**2,500**	**FREMM**	**2,000**	**F-124**	**1,000**	**FREMM**	**1,000**	**F-124**	**1,165**	**FREMM**	**1,383**

and FREMM). In S_3, there was a change in the ordering from the second to the seventh position (LCF, DDG-51, F-125, HORIZON, T-45 DARING and FREMM).

In all scenarios that used data estimated in conjunction with the pertinence function, the ranking was kept in line with the ranking in the initial situation. However, when it was decided to exclude the criteria, the ordering was different from the ordering in the situation of origin.

The use of data estimated in THOR 2 proved to be an efficient instrument in situations where there is uncertainty or lack of data, since it is not necessary to eliminate the criterion or alternative in question. The study, in this way, fills a gap

TABLE 17. THOR 2: Ordering after eliminating the maximum speed and radius of action.

S1				S2				S3			
Inicial Situation		Elimination of criteria		Inicial Situation		Elimination of criteria		Inicial Situation		Elimination of criteria	
LCF	3,787	LCF	3,945	**F-100**	**4,134**	**F-125**	**4,846**	F-100	5,190	F-100	5,564
DDG-51	**3,573**	**F-125**	**3,636**	**DDG-51**	**4,053**	**F-100**	**4,419**	**LCF**	**4,990**	**F-125**	**5,393**
F-100	3,500	F-100	3,500	LCF	4,002	LCF	4,196	**DDG-51**	**4,689**	**LCF**	**5,120**
T-45 DARING	3,500	T-45 DARING	3,500	**F-125**	**3,806**	**DDG-51**	**3,250**	**F-125**	**4,115**	**DDG-51**	**3,714**
HORIZON	**3,500**	**DDG-51**	**3,030**	**T-45 DARING**	**1,526**	**FREMM**	**1,579**	**HORIZON**	**1,869**	**FREMM**	**2,125**
F-125	**3,000**	**HORIZON**	**2,500**	HORIZON	1,500	HORIZON	1,500	**T-45 DARING**	**1,768**	**HORIZON**	**1,681**
FREMM	2,500	FREMM	2,500	**FREMM**	**1,077**	**T-45**	**1,500**	**FREMM**	**1,372**	**T-45**	**1,588**
F-124	2,500	F-124	2,500	F-124	1,000	F-124	1,000	F-124	1,165	F-124	1,247

present in decision problems in which data are missing in the decision matrix, since it is not necessary to eliminate the alternative or criterion.

In THOR 2, the use of estimated data generated the same ranking, in S_1, S_2 and S_3, as the original situation in all scenarios. The fact that in the THOR 2 the fuzzy-rough index is multiplied by the weight value in all situations (aP_jb, aQ_jb e aI_jb) causes all the uncertainty of the model to be considered.

VII. CONCLUSION

This article aimed to propose an evolution of the THOR multicriteria decision support method. The evolution presented an axiomatic variation of the THOR method, called THOR 2. After an analysis of the THOR method, modifications were made in the distinction of the weights in the sum of the scores for the scenarios of weak preference and indifference. In addition, the weight of the criterion was multiplied by the fuzzy-rough index in situations of strong preference and indifference, in order to downgrade the comparison according to the data security degree. These modifications provided a better distribution of the weights in the function of non-transitive aggregation and allowed to fully contemplate the model's uncertainty, since all weights are multiplied by their respective fuzzy-approximate index.

In order to expand and disseminate the use of the methods by the scientific community, a decision support system called Thor Web was developed, which contemplates the use of the THOR and THOR 2 methods. Thor Web helped in the comparison of the results of the methods, also contributing, for application in more complex problems, in which a greater number of criteria and alternatives are used.

It was also proposed to validate the relevance function for THOR 2. In the event that not all the necessary data are available, assigning an estimate conditional on the relevance value presented a better result than the elimination of the alternative or criterion. In all simulated situations, the ordering of the alternatives remained in line with the original ordering. However, when it was decided to exclude the criteria, the sorting obtained a different result. In this way, THOR 2 proved to be an effective tool to deal with situations of uncertainty and lack of data, with no need to eliminate an alternative or criterion.

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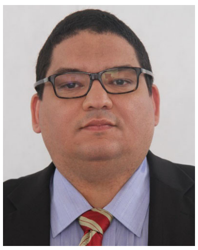

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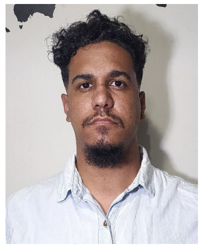

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