IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received November 17, 2021, accepted November 30, 2021, date of publication December 6, 2021, date of current version December 15, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3132948

Improved Stabilization Criteria for Sampled-Data Control Systems via a Less Conservative Looped-Functional Method

KHANH HIEU NGUYEN AND SUNG HYUN KIM

Department of Electrical, Electronic and Computer Engineering, University of Ulsan (UOU), Nam-Gu, Ulsan 680-749, South Korea

Corresponding author: Sung Hyun Kim (shnkim@ulsan.ac.kr)

This work was supported by the National Research Foundation of Korea funded by the Government of Korea under Grant NRF-2018R1D1A1B07041456.

ABSTRACT This paper aims to propose an improved method capable of designing a sampled-data control for linear systems. To this end, a refined two-sided looped functional method is proposed such that the chosen Lyapunov-Krasovskii functional can contain more input-delay-dependent state information based on the two-sided sampling interval. Furthermore, two novel zero equality constraints are introduced to strengthen the relationship between the input-delay-dependent states and the current states. Finally, through two illustrative examples, the effectiveness of the proposed method is verified by comparing the maximum allowable sampling interval and computational complexity with other existing methods.

INDEX TERMS Sampled-data systems, looped-functional, input-delay-dependent state.

I. INTRODUCTION

In the past decade, the sampled-data control method has been widely studied and applied in many fields such as automotive control systems, embedded control systems, manufacturing machine control systems, and power grid control systems (refer to [1]–[3] and references therein). The reason is that compared to analog control systems, the sampled-data control systems have the following advantages: (i) the reliability and accuracy, (ii) the ease of changing the implemented control algorithm, and (iii) the extensibility to networked and event-triggered control systems. In particular, the sampled-data control scheme can provide a criterion for obtaining the maximum allowable sampling interval required to reduce the computational load of digital controllers. Thus, as the development of digital technology accelerates, the importance of sampled-data control theory continues to be emphasized.

Following this trend, various attempts have been made to develop effective sampled-data control techniques that can increase the maximum allowable sampling interval. To be specific, three main approaches have been proposed in the literature: (i) the input delay approach that incorporates the time delay resulting from the sampling process into the control input, (ii) the discrete-time approach that transforms the sampled-data system into a discrete-time parameter varying system, and (iii) the impulsive model approach that utilizes the impulsive modeling of sampled-data systems (see [4]–[6]). Among them, the input delay approach has been recognized as the most popular approach, which can be specifically classified according to Lyapunov-Krasovskii functional methods such as the time-dependent Lyapunov functional method [7], the discontinuous Lyapunov functional method [8], [9], and the looped-functional-based method [10], [11]. In detail, the time-dependent Lyapunov functional method is given to capture information about the sawtooth structure of sampling pattern. Furthermore, the discontinuous Lyapunov functional method is devised to efficiently deal with the discontinuity of control signals in sampled-data control systems. Recently, the looped-functional method is used to obtain less conservative stability criteria by relaxing the positive condition of the functional during the sampling intervals. As a representative study, [12] has utilized the looped-functional method to address the aperiodic sampled-data control for fuzzy systems. Furthermore, [13] has presented the two-sided looped-functional method to take full consideration of the information about the sampling interval. Meanwhile, to deal with the integral terms derived from the time-derivative of the Lyapunov-Krasovskii functional, the following inequality

The associate editor coordinating the review of this manuscript and approving it for publication was Feiqi Deng.

approaches have been proposed: the Jensen's inequality [14], the Wirtinger's inequality [15], and the free-matrix-based integral inequality [16]. Recently, [17]–[19] have proposed the input-delay-dependent vector approach to fully exploit the information about the actual sampling pattern. However, it should be noted that there is still room for further improvement in [17]–[19], because (i) the used Lyapunov-Krasovskii functional can be generalized in such a way that it contains more input-delay-dependent state information on sampling interval and (ii) the relationship between the input-delay-dependent states and the current states can be further enhanced.

Motivated by the aforementioned discussion, this paper focuses on dealing with the stability analysis and control design problem of sampled-data control systems. In particular, considerable efforts are made to develop an effective method that can provide the stabilization conditions in a less conservative manner. Overall, the main contributions of this paper can be summarized as follows:

- Compared to [13], [17], [25], this paper proposes a refined two-sided looped functional method such that the chosen Lyapunov-Krasovskii functional can contain more input-delay-dependent state information based on the two-sided sampling interval.
- To achieve the less conservative stability and stabilization conditions, different from [13], [17], [25], this paper proposes two novel zero equality constraints that can strengthen the relationship between the input-delay-dependent states and the current states through two time-varying weighting factors.
- To obtain a set of linear matrix inequality (LMI)-based conditions from the time derivative of the proposed Lyapunov-Krasovskii functional, a proper free-matrix-based integral inequality and a relaxation process for the time-varying weighting factors are presented in this paper.

Finally, two illustrative examples are provided to verify the effectiveness of the proposed method. The rest of the paper is organized as follows. Section 2 presents the system model and the preliminary results. Section 3 proposes the stability and stabilization conditions for the closed-loop system. Section 4 shows the illustrative examples. Finally, the paper is concluded in Section 5.

Notations: The notations $\mathbb{N}$ and $\mathbb{R}$ represent sets of natural numbers (including zero) and real numbers, respectively. For any matrix $X \in \mathbb{R}^{n\times n}$, the notations $X > 0$ (or $X \geq 0$) mean that X is positive definite (or positive semi-definite), the notations X^{-1} and X^T signify the inverse and the transpose of X, respectively. In symmetric block matrices, $(*)$ is used as an ellipsis for terms induced by symmetry. The operator $\otimes$ denotes the Kronecker product, $\mathbf{He}\{\mathcal{Q}\}$ is used to represent $\mathcal{Q}+\mathcal{Q}^T$ for any square matrix $\mathcal{Q}$, $\mathbf{diag}(\cdot)$ stands for a block-diagonal matrix, and $col(v_1, v_2, \ldots, v_n) = \begin{bmatrix} v_1^T & v_2^T & \cdots & v_n^T \end{bmatrix}^T$ for scalars or vectors v_i. The notations $0_{n\times m}$ is the $n \times m$-dimensional zero matrix, and I_n is the $n \times n$-dimensional identity matrix.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

This paper aims to propose an improved method for designing a sampled-data controller that stabilizes the following linear continuous-time system:

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ denote the state and the control input, respectively; A and B represent real constant system matrices with appropriate dimensions. To this end, this paper considers the following sampled-data control law:

$$u(t) = Fx(t_k),\ t \in [t_k, t_{k+1}),\ \forall k \in \mathbb{N}, \tag{2}$$

where $F \in \mathbb{R}^{m\times n}$ denotes the control gain to be designed later; and $x(t_k)$ denotes the sampled state at a specified time t_k such that $0 = t_0 < t_1 < \cdots < t_k < \cdots < t_\infty$. Furthermore, in (2), the sampling interval between two consecutive sampling instants t_k and t_{k+1} is defined to be expressed in a periodic ($\psi_1 = \psi_2$) or aperiodic ($\psi_1 < \psi_2$) form:

$$h_k = t_{k+1} - t_k, \quad 0 < \psi_1 \leq h_k \leq \psi_2, \quad \forall k \in \mathbb{N}, \tag{3}$$

where ψ_1 and ψ_2 indicate the lower and upper bounds of sampling interval, respectively. As a result, the closed-loop system with (1) and (2) is described as follows:

$$\dot{x}(t) = Ax(t) + BFx(t_k). \tag{4}$$

Throughout this paper, the following lemmas will be used.

Lemma 2.1 ([20]): Let x be a differentiable function: $[a,\ b] \to \mathbb{R}^n$. Then, for any $\eta(t) \in \mathbb{R}^p$, $0 < R = R^T \in \mathbb{R}^{n\times n}$, $M \in \mathbb{R}^{n\times p}$, and $N \in \mathbb{R}^{n\times p}$, the following inequality holds:

$$\begin{aligned} -\int_a^b \dot{x}^T(\tau)R\dot{x}(\tau)d\tau &\leq (b-a)\big(\eta^T(t)\Phi\eta(t) + 2\eta^T(t)N^T(x(b)+x(a))\big) \\ &\quad +2\eta^T(t)\Big(M^T(x(b)-x(a)) - 2N^T\int_a^b x(\tau)d\tau\Big), \end{aligned}$$

where $\Phi = M^TR^{-1}M + \frac{(b-a)^2}{3}N^TR^{-1}N \in \mathbb{R}^{p\times p}$.

Lemma 2.2 ([17]): Let x be a differentiable function: $[a,\ b] \to \mathbb{R}^n$. Then, for any $\eta(t) \in \mathbb{R}^p$, $0 < R = R^T \in \mathbb{R}^{n\times n}$, and $M \in \mathbb{R}^{n\times p}$, the following inequality holds:

$$\begin{aligned} -\int_a^b \dot{x}^T(\tau)R\dot{x}(\tau)d\tau &\leq (b-a)\eta^T(t)M^TR^{-1}M\eta(t) \\ &\quad +2\eta^T(t)M^T\big(x(b)-x(a)\big). \end{aligned}$$

III. CONTROL SYNTHESIS

For the sake of technical simplicity, let us define

$$\alpha_1(t) = t - t_k,\ \alpha_2(t) = t_{k+1} - t,$$

which lead to $\dot{\alpha}_1(t) = 1$ and $\dot{\alpha}_2(t) = -1$. In addition, let us establish the following augmented states and the block entry matrices:

- $$\eta(t) = col\Big\{x(t),\ \alpha_1(t)x(t),\ \alpha_2(t)x(t),\ x(t_k),\ x(t_{k+1}),\ \int_{t_k}^{t} x(\tau)d\tau,\ \int_{t}^{t_{k+1}} x(\tau)d\tau, \dot{x}(t),\ \alpha_1(t)\dot{x}(t),\ \alpha_2(t)\dot{x}(t)\Big\} \in \mathbb{R}^{10n}, \quad (5)$$
- $$e_q = \left[\, 0_{n\times(q-1)n} \middle| \, I_n \middle| \, 0_{n\times(10-q)n} \right] \in \mathbb{R}^{n\times 10n}, \quad \forall q = 1, 2, \ldots, 10.$$

Then, letting

- $$\eta_1(t) = \mathbf{col}\left\{x(t) - x(t_k),\ \alpha_1(t)x(t),\ \int_{t_k}^{t} x(\tau)d\tau\right\} = \Xi_1\eta(t) \in \mathbb{R}^{3n},$$
- $$\dot{\eta}_1(t) = \bar{\Xi}_1\eta(t) \in \mathbb{R}^{3n},$$
- $$\eta_2(t) = \mathbf{col}\left\{x(t_{k+1}) - x(t),\ \alpha_2(t)x(t),\ \int_{t}^{t_{k+1}} x(\tau)d\tau\right\} = \Xi_2\eta(t) \in \mathbb{R}^{3n},$$
- $$\dot{\eta}_2(t) = \bar{\Xi}_2\eta(t) \in \mathbb{R}^{3n},$$

where

$$\begin{aligned}
\Xi_1^T &= \left[\, e_1^T - e_4^T \mid e_2^T \mid e_6^T \,\right],\\
\bar{\Xi}_1^T &= \left[\, e_8^T \mid e_1^T + e_9^T \mid e_1^T \,\right],\\
\Xi_2^T &= \left[\, e_5^T - e_1^T \mid e_3^T \mid e_7^T \,\right],\\
\bar{\Xi}_2^T &= \left[\, -e_8^T \middle| -e_1^T + e_{10}^T \middle| -e_1^T \,\right],
\end{aligned}$$

we can choose the following Lyapunov-Krasovskii functional candidate:

$$V(t) = V_1(t) + V_2(t) + V_3(t), \text{ for } t \in [t_k, t_{k+1}), \quad (6)$$

where

$$\begin{aligned}
V_1(t) &= x^T(t)Px(t),\\
V_2(t) &= \alpha_2(t)\eta_1^T(t)Q_1\eta_1(t) + \alpha_1(t)\eta_2^T(t)Q_2\eta_2(t)\\
&\quad +2\eta_1^T(t)Q_3\eta_2(t),\\
V_3(t) &= \alpha_2(t)V_{31}(t) - \alpha_1(t)V_{32}(t),\\
V_{31}(t) &= \int_{t_k}^{t} \dot{x}^T(\tau)R_1\dot{x}(\tau)d\tau, \quad (7)\\
V_{32}(t) &= \int_{t}^{t_{k+1}} \dot{x}^T(\tau)R_2\dot{x}(\tau)d\tau, \quad (8)
\end{aligned}$$

in which $0 < P = P^T \in \mathbb{R}^{n\times n}$, $Q_1 = Q_1^T$, $Q_2 = Q_2^T$, $Q_3 \in \mathbb{R}^{3n\times 3n}$, and $R_1 = R_1^T$, $R_2 = R_2^T \in \mathbb{R}^{n\times n}$.

Remark 1: The functions $V_2(t)$ and $V_3(t)$ satisfy the looping conditions, i.e., $\lim_{t\to t_{k+1}^-} V_i(t) = 0$ and $V_i(t_{k+1}) = 0$, for $i = 2, 3$. Thus, according to the looped-functional-based approach [21], [22], the positive definiteness of $V_2(t)$ and $V_3(t)$ can be relaxed on the sampling interval.

Remark 2: Similar to [17]–[19], our approach is developed based on the looped-functional framework and the input-delay-dependent vector approach. However, to obtain less conservative stabilization conditions of (4), this paper proposes a refined two-sided looped-functional method by incorporating more information about the following input-delay-dependent states into the Lyapunov-Krasovskii functional: $(t-t_k)x(t)$, $(t_{k+1}-t)x(t)$, $(t-t_k)\dot{x}(t)$, and $(t_{k+1}-t)\dot{x}(t)$, for $t \in [t_k, t_{k+1})$.

The following theorem provides the stability criterion of closed-loop system (4).

Theorem 3.1: Let the scalar values ψ_1 and ψ_2 be prescribed such that $0 < \psi_1 \le \psi_2$. Suppose that there exist matrices $0 < P = P^T \in \mathbb{R}^{n\times n}$, $Q_1 = Q_1^T$, $Q_2 = Q_2^T$, $Q_3 \in \mathbb{R}^{3n\times 3n}$, $R_1 = R_1^T$, $R_2 = R_2^T \in \mathbb{R}^{n\times n}$, $M_1, M_2 \in \mathbb{R}^{n\times 10n}$, $N_1, N_2 \in \mathbb{R}^{n\times 10n}$, $S_1, S_2, S_3, S_4 \in \mathbb{R}^{10n\times n}$, $G_1, G_2, G_3, G_4, G_5 \in \mathbb{R}^{n\times n}$, and $G \in \mathbb{R}^{n\times n}$ such that for all $p, \nu \in \{1, 2\}$, the following conditions hold:

$$0 > \begin{bmatrix} \sum_{i=1}^{9} \mathbf{T}_i + \psi_\nu\Psi_p & (*) & (*)\\ \psi_\nu M_p & -\psi_\nu R_p & 0\\ \psi_\nu\psi_2 N_p & 0 & -3\psi_\nu R_p \end{bmatrix}, \quad (9)$$

where

$$\begin{aligned}
\mathbf{T}_1 &= He\left\{e_1^T P e_8\right\},\\
\mathbf{T}_2 &= -\Xi_1^T Q_1\Xi_1 + \Xi_2^T Q_2\Xi_2 + He\Big\{\Xi_1^T Q_3\bar{\Xi}_2 + \bar{\Xi}_1^T Q_3\Xi_2\Big\},\\
\mathbf{T}_3 &= He\left\{(e_1^T - e_4^T)M_1 - 2e_6^T N_1\right\},\\
\mathbf{T}_4 &= He\left\{(e_5^T - e_1^T)M_2 - 2e_7^T N_2\right\},\\
\mathbf{T}_5 &= He\{S_1e_2 + S_2e_9\}, \quad \mathbf{T}_6 = He\{S_3e_3 + S_4e_{10}\},\\
\mathbf{T}_7 &= He\big\{\big(e_1^T G_1^T + e_8^T G_2^T\big)\big(-e_8 + Ae_1 + BFe_4\big)\big\},\\
\mathbf{T}_8 &= He\big\{\big(e_2^T G_3^T + e_9^T G_4^T\big)\big(-e_9 + Ae_2\big)\big\},\\
\mathbf{T}_9 &= He\big\{\big(e_3^T G_5^T + e_{10}^T G^T\big)\big(-e_{10} + Ae_3\big)\big\},\\
\Psi_1 &= He\left\{\Xi_2^T Q_2\bar{\Xi}_2 + (e_1^T + e_4^T)N_1 - S_1e_1 - S_2e_8\right\}\\
&\quad + He\big\{\big(e_2^T G_3^T + e_9^T G_4^T\big)BFe_4\big\} + e_8^T R_2 e_8,\\
\Psi_2 &= He\left\{\Xi_1^T Q_1\bar{\Xi}_1 + (e_1^T + e_5^T)N_2 - S_3e_1 - S_4e_8\right\}\\
&\quad + He\big\{\big(e_3^T G_5^T + e_{10}^T G^T\big)BFe_4\big\} + e_8^T R_1 e_8.
\end{aligned}$$

Then, closed-loop system (4) is asymptotically stable.

Proof: The time derivatives of $V_1(t)$, $V_2(t)$, and $V_3(t)$ are given as follows:

$$\begin{aligned}
\dot{V}_1(t) &= \eta^T(t)\mathbf{T}_1\eta(t), \quad (10)\\
\dot{V}_2(t) &= \eta^T(t)\big(\mathbf{T}_2 + \alpha_2(t)He\big\{\Xi_1^T Q_1\bar{\Xi}_1\big\}\big)\eta(t)\\
&\quad + \eta^T(t)\big(\alpha_1(t)\mathbf{He}\big\{\Xi_2^T Q_2\bar{\Xi}_2\big\}\big)\eta(t), \quad (11)\\
\dot{V}_3(t) &= -V_{31}(t) - V_{32}(t)\\
&\quad + \eta^T(t)\big(\alpha_2(t)e_8^T R_1 e_8 + \alpha_1(t)e_8^T R_2 e_8\big)\eta(t). \quad (12)
\end{aligned}$$

Furthermore, based on $\alpha_1(t) \le \psi_2$ and $\alpha_2(t) \le \psi_2$, Lemma 2.1 offers

$$-V_{31}(t) \le \eta^T(t) \times\Big(\alpha_1(t)\big(\Phi_1 + He\left\{(e_1^T + e_4^T)N_1\right\}\big) + \mathbf{T}_3\Big)\eta(t), \quad (13)$$

$$-V_{32}(t) \leq \eta^T(t) \times\Big(\alpha_2(t)\big(\Phi_2 + He\left\{(e_1^T + e_5^T)N_2\right\}\big) + \mathbf{T}_4\Big)\eta(t), \tag{14}$$

where $\Phi_p = M_p^T R_p^{-1} M_p + \frac{\psi_2^2}{3} N_p^T R_p^{-1} N_p$, for $p = 1, 2$.

In addition, it is worth noticing that from (5), the following zero equalities hold:

$$\begin{aligned} 0 &= 2\eta^T(t)\begin{bmatrix} S_1 & S_2 \end{bmatrix}\begin{bmatrix} e_2 - \alpha_1(t)e_1 \\ e_9 - \alpha_1(t)e_8 \end{bmatrix}\eta(t) \\ &= \eta^T(t)\left(\mathbf{T}_5 - \alpha_1(t)He\left\{S_1e_1 + S_2e_8\right\}\right)\eta(t), \end{aligned} \tag{15}$$

$$\begin{aligned} 0 &= 2\eta^T(t)\begin{bmatrix} S_3 & S_4 \end{bmatrix}\begin{bmatrix} e_3 - \alpha_2(t)e_1 \\ e_{10} - \alpha_2(t)e_8 \end{bmatrix}\eta(t) \\ &= \eta^T(t)\left(\mathbf{T}_6 - \alpha_2(t)He\left\{S_3e_1 + S_4e_8\right\}\right)\eta(t). \end{aligned} \tag{16}$$

Subsequently, from (4), it also holds that

$$\bullet\; e_8\eta(t) = \big(Ae_1 + BFe_4\big)\eta(t), \tag{17}$$

$$\bullet\; e_9\eta(t) = \big(Ae_2 + \alpha_1(t)BFe_4\big)\eta(t), \tag{18}$$

$$\bullet\; e_{10}\eta(t) = \big(Ae_3 + \alpha_2(t)BFe_4\big)\eta(t), \tag{19}$$

which leads to

$$\begin{aligned} 0 &= 2\eta^T(t)\big(e_1^T G_1^T + e_8^T G_2^T\big)\big(-e_8 + Ae_1 + BFe_4\big)\eta(t) \\ &= \eta^T(t)\mathbf{T}_7\eta(t), \end{aligned} \tag{20}$$

$$\begin{aligned} 0 &= 2\eta^T(t)\big(e_2^T G_3^T + e_9^T G_4^T\big)\big(-e_9 + Ae_2 + \alpha_1(t)BFe_4\big)\eta(t) \\ &= \eta^T(t)\big(\mathbf{T}_8 + \alpha_1(t)BFe_2\big)\eta(t), \end{aligned} \tag{21}$$

$$\begin{aligned} 0 &= 2\eta^T(t)\big(e_3^T G_5^T + e_{10}^T G^T\big)\big(-e_{10} + Ae_3 + \alpha_2(t)BFe_4\big)\eta(t) \\ &= \eta^T(t)\big(\mathbf{T}_9 + \alpha_2(t)BFe_2\big)\eta(t). \end{aligned} \tag{22}$$

As a result, by combining (10)–(12), (15), (16), (20)–(22), and by applying (13) and (14), we can obtain

$$\dot{V}(t) \leq \eta^T(t)\Psi\eta(t), \tag{23}$$

where

$$\Psi = \textstyle\sum_{i=1}^{9}\mathbf{T}_i + \frac{\alpha_1(t)}{h_k}h_k\Big(\Psi_1 + \Phi_1\Big) + \frac{\alpha_2(t)}{h_k}h_k\Big(\Psi_2 + \Phi_2\Big). \tag{24}$$

Accordingly, from (26), it can be seen that the stability condition $\dot{V}(t) < 0$ is ensured by $\Psi < 0$. Finally, since $\frac{\alpha_1(t)}{h_k}$ and $\frac{\alpha_2(t)}{h_k}$ belong to the unit simplex and $h_k \in [\psi_1, \psi_2]$ holds, the condition $\Psi < 0$ can be expressed as a linear convex combination of $\sum_{i=1}^{9}\mathbf{T}_i + \psi_\nu\big(\Psi_p + \Phi_p\big)$, for $p, \nu = 1, 2$, which can be transformed into (9) by Schur complement. ■

Remark 3: In this paper, we have introduced two additional zero equality constraints (21) and (22) that can strengthen the relationship between the input-delay-dependent states and the current states using the weighting factors $\alpha_1(t)$ and $\alpha_2(t)$, which plays an important role in deriving less conservative stabilization conditions.

The following theorem provides the stabilization conditions of closed-loop system (4).

Theorem 3.2: Let the scalar values ψ_1, ψ_2, λ_1, λ_2, λ_3, λ_4 and λ_5 be prescribed such that $0 < \psi_1 \leq \psi_2$. Suppose that there exist matrices $0 < \bar{P} = \bar{P}^T \in \mathbb{R}^{n\times n}$, $\bar{Q}_1 = \bar{Q}_1^T$, $\bar{Q}_2 = \bar{Q}_2^T$, $\bar{Q}_3 \in \mathbb{R}^{3n\times 3n}$, $\bar{R}_1 = \bar{R}_1^T$, $\bar{R}_2 = \bar{R}_2^T \in \mathbb{R}^{n\times n}$, $\bar{M}_1, \bar{M}_2 \in \mathbb{R}^{n\times 10n}$, $\bar{N}_1, \bar{N}_2 \in \mathbb{R}^{n\times 10n}$, $\bar{S}_1, \bar{S}_2, \bar{S}_3, \bar{S}_4 \in \mathbb{R}^{10n\times n}$, $\bar{G} \in \mathbb{R}^{n\times n}$, and $\bar{F} \in \mathbb{R}^{m\times n}$ such that for all $p, \nu \in \{1, 2\}$, the following conditions hold:

$$0 > \begin{bmatrix} \sum_{i=1}^{9}\bar{\mathbf{T}}_i + \psi_\nu\bar{\Psi}_p & (*) & (*) \\ \psi_\nu\bar{M}_p & -\psi_\nu\bar{R}_p & 0 \\ \psi_\nu\psi_2\bar{N}_p & 0 & -3\psi_\nu\bar{R}_p \end{bmatrix}, \tag{25}$$

where

$$\begin{aligned}
\bar{\mathbf{T}}_1 &= He\left\{e_1^T\bar{P}e_8\right\}, \\
\bar{\mathbf{T}}_2 &= -\Xi_1^T\bar{Q}_1\Xi_1 + \Xi_2^T\bar{Q}_2\Xi_2 + He\left\{\Xi_1^T\bar{Q}_3\bar{\Xi}_2 + \bar{\Xi}_1^T\bar{Q}_3\Xi_2\right\}, \\
\bar{\mathbf{T}}_4 &= He\left\{(e_1^T - e_4^T)\bar{M}_1 - 2e_6^T\bar{N}_1\right\}, \\
\bar{\mathbf{T}}_3 &= He\left\{(e_5^T - e_1^T)\bar{M}_2 - 2e_7^T\bar{N}_2\right\}, \\
\bar{\mathbf{T}}_5 &= He\{\bar{S}_1e_2 + \bar{S}_2e_9\},\; \bar{\mathbf{T}}_6 = He\{\bar{S}_3e_3 + \bar{S}_4e_{10}\}, \\
\bar{\mathbf{T}}_7 &= He\{\big(\lambda_1e_1^T + \lambda_2e_8^T\big)\big(-\bar{G}e_8 + A\bar{G}e_1 + B\bar{F}e_4\big)\}, \\
\bar{\mathbf{T}}_8 &= He\{\big(\lambda_3e_2^T + \lambda_4e_9^T\big)\big(-\bar{G}e_9 + A\bar{G}e_2\big)\}, \\
\bar{\mathbf{T}}_9 &= He\{\big(\lambda_5e_3^T + e_{10}^T\big)\big(-\bar{G}e_{10} + A\bar{G}e_3\big)\}, \\
\bar{\Psi}_1 &= He\left\{\Xi_2^T\bar{Q}_2\bar{\Xi}_2 + (e_1^T + e_4^T)\bar{N}_1 - \bar{S}_1e_1 - \bar{S}_2e_8\right\} \\
&\quad + He\{\big(\lambda_3e_2^T + \lambda_4e_9^T\big)B\bar{F}e_4\} + e_8^T\bar{R}_2e_8, \\
\bar{\Psi}_2 &= He\left\{\Xi_1^T\bar{Q}_1\bar{\Xi}_1 + (e_1^T + e_5^T)\bar{N}_2 - \bar{S}_3e_1 - \bar{S}_4e_8\right\} \\
&\quad + He\{\big(\lambda_5e_3^T + e_{10}^T\big)B\bar{F}e_4\} + e_8^T\bar{R}_1e_8.
\end{aligned}$$

Then, closed-loop system (4) is asymptotically stable, and the control gain is designed as follows: $F = \bar{F}\bar{G}^{-1}$.

Proof: Since the (10, 10)th block matrix of (25) is negative definite, it holds that $He\{-\bar{G}\} < 0$, which means $\bar{G}$ is nonsingular. Thus, based on $\bar{G}$, we can construct the following congruent transformation matrices:

$$\bar{G}_3 = I_3 \otimes \bar{G}, \quad \bar{G}_{10} = I_{10} \otimes \bar{G},$$

which lead to

$$\begin{aligned} &\Xi_1\bar{G}_{10} = \bar{G}_3\Xi_1,\;\; \Xi_2\bar{G}_{10} = \bar{G}_3\Xi_2,\;\; \bar{\Xi}_1\bar{G}_{10} = \bar{G}_3\bar{\Xi}_1, \\ &\bar{\Xi}_2\bar{G}_{10} = \bar{G}_3\bar{\Xi}_2,\;\; e_q\bar{G}_{10} = \bar{G}e_q, \quad \forall q = 1, 2, \ldots, 10. \end{aligned}$$

Furthermore, letting $G = \bar{G}^{-1}$, and $G_i = \lambda_i\bar{G}^{-1}$, for $i = 1, 2, \ldots, 5$, and using the following replacement variables:

$$\begin{aligned} &\bar{Q}_1 = \bar{G}_3^TQ_1\bar{G}_3,\;\; \bar{Q}_2 = \bar{G}_3^TQ_2\bar{G}_3,\;\; \bar{Q}_3 = \bar{G}_3^TQ_3\bar{G}_3, \\ &\bar{R}_1 = \bar{G}^TR_1\bar{G},\;\; \bar{R}_1 = \bar{G}^TR_1\bar{G},\;\; \bar{M}_1 = \bar{G}^TM_1\bar{G}_{10}, \\ &\bar{M}_2 = \bar{G}^TM_2\bar{G}_{10},\;\; \bar{N}_1 = \bar{G}^TN_1\bar{G}_{10},\;\; \bar{N}_2 = \bar{G}^TN_2\bar{G}_{10}, \\ &\bar{S}_1 = \bar{G}_{10}^TS_1\bar{G},\;\; \bar{S}_2 = \bar{G}_{10}^TS_2\bar{G},\;\; \bar{S}_3 = \bar{G}_{10}^TS_3\bar{G}, \\ &\bar{S}_4 = \bar{G}_{10}^TS_4\bar{G},\;\; \bar{P} = \bar{G}^TP\bar{G},\;\; \bar{F} = F\bar{G}, \end{aligned}$$

we can obtain

$$\begin{aligned} \bar{G}_{10}^T\mathbf{T}_i\bar{G}_{10} &= \bar{\mathbf{T}}_i, \quad \forall i = 1, 2, \ldots, 9, \\ \bar{G}_{10}^T\Psi_p\bar{G}_{10} &= \bar{\Psi}_p, \quad \forall p = 1, 2. \end{aligned}$$

Therefore, pre- and post-multiply (9) by $\mathbf{diag}(\bar{G}_{10}^T, \bar{G}^T, \bar{G}^T)$ and its transpose, we can transform (9) into (25). ■

Remark 4: The conditions of Theorem 3.1 can be used to analyze the stability of the closed-loop system (4) under a given control gain, and the conditions of Theorem 3.2 can be used to design a control gain that stabilizes the closed-loop system (4). Furthermore, to derive the LMI-based stabilization conditions of Theorem 3.1 from Theorem 3.2, the decision variable G_i is set to $G_i = \lambda_i G$, where λ_i plays a key role in changing GBF and G_iBF into $B\bar{F}$ and $\lambda_i B\bar{F}$ through the congruent transformation and variable replacement methods.

As a by-product, the following corollary provides the stabilization conditions of closed-loop system (4), obtained from Lemma 2.2.

Corollary 3.1: Let the scalar values $\psi_1, \psi_2, \lambda_1, \lambda_2, \lambda_3, \lambda_4$ and λ_5 be prescribed such that $0 < \psi_1 \leq \psi_2$. Suppose that there exist matrices $0 < \bar{P} = \bar{P}^T \in \mathbb{R}^{n\times n}$, $\bar{Q}_1 = \bar{Q}_1^T, \bar{Q}_2 = \bar{Q}_2^T, \bar{Q}_3 \in \mathbb{R}^{3n\times 3n}$, $\bar{R}_1 = \bar{R}_1^T, \bar{R}_2 = \bar{R}_2^T \in \mathbb{R}^{n\times n}$, $\bar{M}_1, \bar{M}_2 \in \mathbb{R}^{n\times 10n}$, $\bar{S}_1, \bar{S}_2, \bar{S}_3, \bar{S}_4 \in \mathbb{R}^{10n\times n}$, $\bar{G} \in \mathbb{R}^{n\times n}$, and $\bar{F} \in \mathbb{R}^{m\times n}$ such that for all $p, v \in \{1, 2\}$, the following conditions hold:

$$0 > \begin{bmatrix} \sum_{i=1}^{9} \bar{\mathbf{T}}_i + \psi_v \bar{\Psi}_p & (*) \\ \psi_v \bar{M}_p & -\psi_v \bar{R}_p \end{bmatrix}, \tag{26}$$

where

$$\begin{aligned}
\bar{\mathbf{T}}_1 &= He\left\{e_1^T \bar{P} e_8\right\},\\
\bar{\mathbf{T}}_2 &= -\Xi_1^T \bar{Q}_1 \Xi_1 + \Xi_2^T \bar{Q}_2 \Xi_2 + He\left\{\Xi_1^T \bar{Q}_3 \bar{\Xi}_2 + \bar{\Xi}_1^T \bar{Q}_3 \Xi_2\right\},\\
\bar{\mathbf{T}}_4 &= He\left\{(e_1^T - e_4^T)\bar{M}_1\right\},\\
\bar{\mathbf{T}}_3 &= He\left\{(e_5^T - e_1^T)\bar{M}_2\right\},\\
\bar{\mathbf{T}}_5 &= He\{\bar{S}_1 e_2 + \bar{S}_2 e_9\},\ \bar{\mathbf{T}}_6 = He\{\bar{S}_3 e_3 + \bar{S}_4 e_{10}\},\\
\bar{\mathbf{T}}_7 &= He\{(\lambda_1 e_1^T + \lambda_2 e_8^T)(-\bar{G}e_8 + A\bar{G}e_1 + B\bar{F}e_4)\},\\
\bar{\mathbf{T}}_8 &= He\{(\lambda_3 e_2^T + \lambda_4 e_9^T)(-\bar{G}e_9 + A\bar{G}e_2)\},\\
\bar{\mathbf{T}}_9 &= He\{(\lambda_5 e_3^T + e_{10}^T)(-\bar{G}e_{10} + A\bar{G}e_3)\},\\
\bar{\Psi}_1 &= He\left\{\Xi_2^T \bar{Q}_2 \bar{\Xi}_2 - \bar{S}_1 e_1 - \bar{S}_2 e_8\right\}\\
&\quad + He\left\{(\lambda_3 e_2^T + \lambda_4 e_9^T)B\bar{F}e_4\right\} + e_8^T \bar{R}_2 e_8,\\
\bar{\Psi}_2 &= He\left\{\Xi_1^T \bar{Q}_1 \bar{\Xi}_1 - \bar{S}_3 e_1 - \bar{S}_4 e_8\right\}\\
&\quad + He\left\{(\lambda_5 e_3^T + e_{10}^T)B\bar{F}e_4\right\} + e_8^T \bar{R}_1 e_8.
\end{aligned}$$

Then, closed-loop system (4) is asymptotically stable, and the control gain is designed as follows: $F = \bar{F}\bar{G}^{-1}$.

Proof: In the same derivation process in Theorem 3.1 and Theorem 3.2, stabilization conditions (26) can be obtained by using Lemma 2.2 instead of Lemma 2.1, i.e., by setting $N = 0$. ■

Remark 5: Since the free-matrix-based integral inequality used in [25] can be converted into Lemma 2.2 by setting $N_1 = M$, $N_2 = 0$, and $N_3 = 0$, it can be seen that Lemma 2.2 corresponds to a conservative case of [25] and Lemma 2.1. Hence, Corollary 3.1 can be used to show that our performance improvement is not achieved solely through the use of free-matrix-based integral inequality.

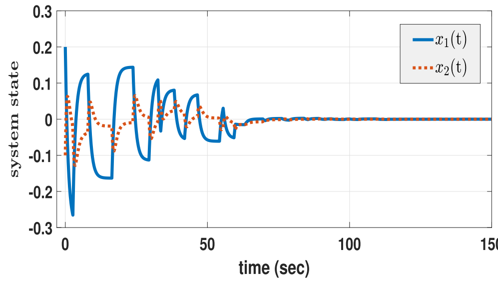

FIGURE 1. State response of closed-loop system.

Remark 6: The number of scalar variables (NSVs) for the LMI-based conditions in Theorem 3.1, Theorem 3.2, and Corollary 3.1 are given as follows:

- Theorem 3.1 : NSVs $= 105.5n^2 + 4.5n$,
- Theorem 3.2 : NSVs $= 100.5n^2 + 4.5n + mn$,
- Corollary 3.1 : NSVs $= 80.5n^2 + 4.5n + mn$.

IV. ILLUSTRATIVE EXAMPLES

Example 1 (For Stability Analysis): Let us consider the following linear sampled-data system adopted in [24], [25]:

$$A = \begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix}, \quad BF = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}. \tag{27}$$

Then, as a simulation result for (27), Table 1 shows the comparison of the maximum allowable sampling intervals obtained by [15, Theorem 7], [23, Theorem 2], [24, Theorem 4], [25, Theorem 1], and Theorem 3.1. As can be seen in Table 1, the proposed method achieves a less conservative result than those of [15], [23]–[25]. Especially, from Table 1, it can be found that the proposed method guarantees an improved sampling interval despite using fewer NSVs than that of [25]. Moreover, for $\psi_1 = 10^{-5}$ and $\psi_2 = 8.7530$, Fig. 1 shows the state response of closed-loop system with $x(0) = \begin{bmatrix} 0.2 & -0.1 \end{bmatrix}^T$. As can be seen in Fig. 1, the state converges to zero as time increases, which clearly verifies our result listed in Table 1.

Example 2 (For Control Design): Let us consider the following inverted pendulum system used in [25]:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{gm_1}{m_2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{g}{l} & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \frac{1}{m_2} \\ 0 \\ -\frac{1}{m_2 l} \end{bmatrix} Fx(t_k), \tag{28}$$

where $x(t) = \begin{bmatrix} x_1(t) & x_2(t) & x_3(t) & x_4(t) \end{bmatrix}^T$ indicates the state in which $x_1(t)$, $x_2(t)$, $x_3(t)$, and $x_4(t)$ represent the cart position (m), the cart velocity (m/s), the pendulum angle (rad), and pendulum angle velocity (rad/s), respectively. In detail, m_1, m_2, l, and g denote the pendulum mass (kg), the cart mass (kg), the pendulum length (m), and the gravitational acceleration (m/s^2), respectively. Then, for $m_1 = 1$, $m_2 = 10$, $l = 3$, and $g = 10$, Table 2 shows the maximum sampling intervals obtained by [25, Theorem 1], Corollary 3.1, and

TABLE 1. Maximum allowable sampling interval ψ_2 for $\psi_1 = 10^{-5}$.

Methods	ψ_2	NSVs
[15, Theorem 7]	2.0948	54
[23, Theorem 2]	2.2236	156
[24, Theorem 4]	3.9306	263
[25, Theorem 1]	5.3040	475
Theorem 3.1	8.7530	431

TABLE 2. Maximum allowable sampling interval ψ_2 for $\psi_1 = 10^{-5}$.

Methods	ψ_2	NSVs
[25, Theorem 1]	0.3595	1882
Corollary 3.1	0.5070	1310
Theorem 3.2	0.5290	1630

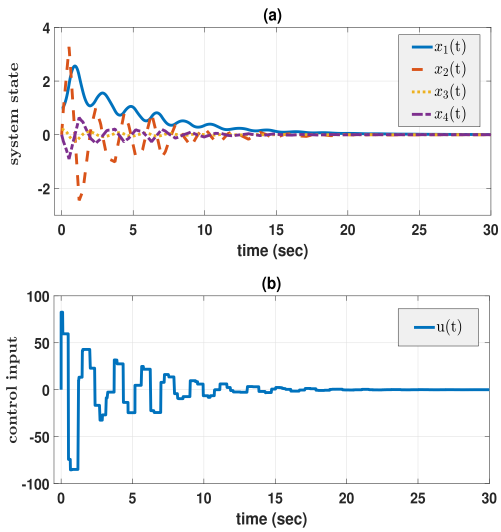

FIGURE 2. Simulation result: (a) state response and (b) control input.

Theorem 3.2 with $\lambda_1 = 8000$, $\lambda_2 = 50$, $\lambda_3 = 2$, $\lambda_4 = 6$, and $\lambda_5 = 1$. As can be seen in Table 2, Corollary 3.1 and Theorem 3.2 achieve both less conservative results and lower computational complexity than [25]. In particular, it is worth noting that Corollary 3.1 also provides an improved result compared to [25], despite using a conservative free-matrix-based integral inequality. Accordingly, as mentioned in Remark 5, this comparison reveals that our performance improvement is not dependent only on the use of the free-matrix-based integral inequality. Meanwhile, for $\psi_1 = 10^{-5}$ and $\psi_2 = 0.5290$, Theorem 3.2 provides the following control gain:

$$F = \begin{bmatrix} 5.8264 & 35.1537 & 384.2357 & 206.6447 \end{bmatrix}.$$

In addition, for this solution, Figs. 2-(a) and (b) show the state response of closed-loop system and the control input, respectively, where $x(0) = \begin{bmatrix} 0.98 & 0 & 0.2 & 0 \end{bmatrix}^T$. As can be seen in Fig. 2, the state converges to zero as time increases, which demonstrates the availability of the above control gain as well as our result listed in Table 2.

Remark 7: Since Theorem 3.1 and Theorem 3.2 serve different purposes as mentioned in Remark 4, Table 1 (for the stability analysis) and Table 2 (for the control synthesis) do not provide comparisons between Theorem 3.1 and Theorem 3.2, but provide comparisons with other studies for their respective purposes.

V. CONCLUDING REMARKS

This paper has investigated the stability and stabilization problem of sampled-data control systems. In particular, to obtain less conservative stability and stabilization conditions, a refined Lyapunov-Krasovskii functional and two new zero equalities have been proposed. Then, the effectiveness of the proposed method has been verified through two examples. In future research, the proposed method will be extended to variable systems such as nonlinear systems, time-delay systems, and multi-agent systems.

REFERENCES

[1] J. Lian, C. Li, and B. Xia, "Sampled-data control of switched linear systems with application to an F-18 aircraft," *IEEE Trans. Ind. Electron.*, vol. 64, no. 2, pp. 1332–1340, Oct. 2017.
[2] Q. Meng, C. Qian, and P. Wang, "Lateral motion stability control via sampled-data output feedback of a high-speed electric vehicle driven by four in-wheel motors," *J. Dyn. Syst., Meas., Control*, vol. 140, no. 1, Jan. 2018, Art. no. 011002.
[3] Y. Wu, H. R. Karimi, and R. Lu, "Sampled-data control of network systems in industrial manufacturing," *IEEE Trans. Ind. Electron.*, vol. 65, no. 11, pp. 9016–9024, Nov. 2018.
[4] E. Fridman, A. Seuret, and J.-P. Richard, "Robust sampled-data stabilization of linear systems: An input delay approach," *Automatica*, vol. 40, no. 8, pp. 1441–1446, 2004.
[5] Y. S. Suh, "Stability and stabilization of nonuniform sampling systems," *Automatica*, vol. 44, no. 12, pp. 3222–3226, 2008.
[6] C. Briat, "Convex conditions for robust stability analysis and stabilization of linear aperiodic impulsive and sampled-data systems under dwell-time constraints," *Automatica*, vol. 49, no. 11, pp. 2768–2772, 2013.
[7] E. Fridman, "A refined input delay approach to sampled-data control," *Automatica*, vol. 46, no. 2, pp. 421–427, Feb. 2010.
[8] K. Liu and E. Fridman, "Wirtinger's inequality and Lyapunov-based sampled-data stabilization," *Automatica*, vol. 48, no. 1, pp. 102–108, Jan. 2012.
[9] T. B. Nguyen and S. H. Kim, "Improved discontinuous Lyapunov functionals for stability analysis of sampled-data systems," *IEEE Access*, vol. 9, pp. 38620–38625, 2021.
[10] A. Seuret, "A novel stability analysis of linear systems under asynchronous samplings," *Automatica*, vol. 48, no. 1, pp. 177–182, 2012.
[11] C. Guan, Z. Fei, and P. Park, "Modified looped functional for sampled-data control of T–S fuzzy Markovian jump systems," *IEEE Trans. Fuzzy Syst.*, vol. 29, no. 9, pp. 2543–2552, Sep. 2021.
[12] C. Ge, H. Wang, Y. Liu, and J. H. Park, "Improved stabilization criteria for fuzzy systems under variable sampling," *J. Franklin Inst.*, vol. 354, no. 14, pp. 5839–5853, Sep. 2017.
[13] H.-B. Zeng, K. L. Teo, Y. He, and W. Wang, "Sampled-data-based dissipative control of T–S fuzzy systems," *Appl. Math. Model.*, vol. 65, pp. 415–427, Jan. 2019.
[14] A. Seuret and F. Gouaisbaut, "Jensen's and Wirtinger's inequalities for time-delay systems," *IFAC Proc. Volumes*, vol. 46, pp. 343–348, Feb. 2013.
[15] A. Seuret and F. Gouaisbaut, "Wirtinger-based integral inequality: Application to time-delay systems," *Automatica*, vol. 49, no. 9, pp. 2860–2866, Sep. 2013.
[16] H.-B. Zeng, Y. He, M. Wu, and J. She, "Free-matrix-based integral inequality for stability analysis of systems with time-varying delay," *IEEE Trans. Autom. Control*, vol. 60, no. 10, pp. 2768–2772, Oct. 2015.
[17] R. Zhang, X. Liu, D. Zeng, S. Zhong, and K. Shi, "A novel approach to stability and stabilization of fuzzy sampled-data Markovian chaotic systems," *Fuzzy Sets Syst.*, vol. 344, pp. 108–128, Aug. 2018.

[18] D. Zeng, R. Zhang, X. Liu, S. Zhong, and K. Shi, "Improved results on synchronisation of delayed complex dynamical networks via sampled-data control," *Int. J. Syst. Sci.*, vol. 49, no. 6, pp. 1242–1255, Apr. 2018.

[19] T. Wu, J. Cao, L. Xiong, and H. Zhang, "New stabilization results for semi-Markov chaotic systems with fuzzy sampled-data control," *Complexity*, vol. 2019, pp. 1–15, Nov. 2019, doi: 10.1155/2019/7875305.

[20] H.-B. Zeng, K. L. Teo, Y. He, H. Xu, and W. Wang, "Sampled-data synchronization control for chaotic neural networks subject to actuator saturation," *Neurocomputing*, vol. 260, pp. 25–31, Oct. 2017.

[21] T. H. Lee and J. H. Park, "New methods of fuzzy sampled-data control for stabilization of chaotic systems," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 48, no. 12, pp. 2026–2034, Dec. 2018.

[22] J. Park and P. Park, "H_∞ sampled-state feedback control for synchronization of chaotic Lur'e systems with time delays," *J. Franklin Inst.*, vol. 355, no. 16, pp. 8005–8026, Nov. 2018.

[23] T. Lee and J. H. Park, "Stability analysis of sampled-data systems via free-matrix-based time-dependent discontinuous Lyapunov approach," *IEEE Trans. Autom. Control*, vol. 62, no. 7, pp. 3653–3657, Jul. 2017.

[24] H.-B. Zeng, K. L. Teo, and Y. He, "A new looped-functional for stability analysis of sampled-data systems," *Automatica*, vol. 82, pp. 328–331, Aug. 2017.

[25] X. Wang, J. Sun, and L. Dou, "Improved results on stability analysis of sampled-data systems," *Int. J. Robust Nonlinear Control*, vol. 31, no. 14, pp. 6549–6561, 2021.

KHANH HIEU NGUYEN received the B.S. degree in electrical engineering from the Da Nang University of Technology, Da Nang, Vietnam, in 2018. He is currently pursuing the Ph.D. degree with the Department of Electrical, Electronic and Computer Engineering, University of Ulsan, Ulsan, South Korea. His research interests include control system design for stochastic hybrid systems, fuzzy-based nonlinear systems, and sampled-data systems.

SUNG HYUN KIM received the M.S. and Ph.D. degrees in electrical and electronic engineering from the Pohang University of Science and Technology (POSTECH), South Korea, in 2003 and 2008, respectively. Since 2011, he has been affiliated with the School of Electrical Engineering, University of Ulsan (UOU), South Korea. He is currently a Professor and the Vice Dean of the College of Engineering. His research interests include control system design and signal processing related to networked-embedded control systems, multi-agent systems, fuzzy-based nonlinear systems, stochastic hybrid systems, and radio frequency identification tag protocol. Moreover, he is also working as an Editorial Board Member of international journals, such as KIEE and *IJCAS*.