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Study of a Memory Type Shrinkage Estimator of Population Mean in Quality Control Process

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ABSTRACT Controlling and improving quality is a major business strategy for various organizations. A company gains an advantage over its competitors by maintaining a high level of quality for products or services and pleasing its customers or clients. Control charts are a tool used in Statistical Process Control for the purpose of reducing variability in a process as well as estimating certain parameters. In the current manuscript, auxiliary information has been utilized to propose an estimator for process mean using hybrid exponentially moving average statistic. The proposed memory-type estimator utilizes prior information to provide better estimates from data which is time-scale based than conventional estimators which were designed for data collected at a single point of time. The properties of the estimator have been studied under Simple Random Sampling as well as Stratified Sampling structure. Simulation study has been conducted to illustrate its efficiency over contemporary estimators. Application to real data on datasets "Machine", "Automobile" and "Auto MPG" is introduced to illustrate the method.

INDEX TERMS Control chart, estimation, hybrid exponentially weighted moving average, population mean, quality control, sample survey, simulation.

I. INTRODUCTION

Controlling and improving quality is a major business strategy for various organizations, including government agencies, as well as manufacturers, distributors, companies involved in transportation, financial services, health care, etc. Quality improvement methods have applications in various domains within a company or organization, such as engineering design, manufacturing, process development, marketing, finance and accounting, distribution and logistics, customer service, and field service of products (See [13]). Some products where quality control proves vital are manufactured goods like automobiles, computers, clothing, as well as services such as the generation and distribution of electrical energy, public transportation, retailing, banking, health care, etc. ([13]). A company gains an advantage over its competitors by maintaining a high level of quality for products or services and pleasing its customers or clients. Before introducing the proposed estimator, a brief discussion of the technical tools available in the literature is presented in the subsequent paragraphs.

Quality characteristic is the term given to the parameters that define the thoughts of the consumer on quality of a product ([13]). Quality engineering consists of the collection of operational, managerial and engineering activities used to affirm that the quality characteristics of a product conform to the product specifications. The main goal is to reduce the variability in the chief quality characteristics of the product. Variability may be due to natural or inherent variability or a system, or there may be assignable causes of variation.

Statistical Process Control (SPC) ([13]) is the set of problem-solving tools used to make a process stable and improve its capability by reducing the variability. Some statistical tools employed for this purpose are histogram, stem-and-leaf plot, check sheet, Pareto chart, cause-and-effect diagram, scatter diagram, and control charts.

Control chart ([8], [13]) is an on-line technique used for monitoring a process. It is one of the fundamental methods used in SPC and produces a graphical representation of a quality characteristic. The average of measurements of some

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quality characteristic in the samples taken from the process is plotted against either time or sample number. This is used to keep the process output under monitoring and also for estimation of the parameters of a production process, such as mean, standard deviation, fraction non-conforming or fallout, etc. Such estimates are helpful in determining how capable a process is in producing acceptable products. Some recent works on control chart have been done by [1], [9], [10], [14], [21], [25], among others.

Time-weighted control charts are used to detect small shifts in process. Commonly used charts of this kind are Cumulative Sum (CUSUM) charts and Exponentially Weighted Moving Average (EWMA) charts.

Cumulative Sum or CUSUM was first developed by [18] and is utilized for efficient and accurate detection of small and continual shifts taking place in the process mean. Samples of size n are taken during the process, and the sample means are observed. Usually, $n = 1$.

The cumulative sum up to and including the k^{th} sample is defined as $C_k = \sum_{j=1}^{k}(\bar{x}_j - \mu_0)$, where $\bar{x}_j$ is the average of the j^{th} sample and μ_0 is the target for the process mean. If the mean shifts to a value μ_1, then an upward (positive) or downward (negative) drift will develop in C_k, depending on whether $\mu_1 > \mu_0$ or $\mu_1 < \mu_0$.

In a Tabular CUSUM, deviations from μ_0 which are above and below target respectively are accumulated by one-sided upper CUSUM (C^+) and one-sided lower CUSUM(C^-). Mathematically, they are defined as

$$\begin{aligned} C_h{}^+ &= \max[0, x_h - (\mu_0 + k) + C_{h-1}{}^+] \\ C_h{}^- &= \max[0, (\mu_0 - k) - x_h + C_{h-1}{}^-] \end{aligned} \tag{1}$$

where k is the reference value or slack value, taken as $k = \frac{|\mu_1 - \mu_0|}{2}$. The starting values are assumed to be $C_0{}^+ = C_0{}^- = 0$. Further details about CUSUM charts can be found in [8].

The exponentially weighted moving average (EWMA), first introduced by [20], is more efficient at detecting smaller shifts in comparison to Shewhart control charts. It is also known as Geometric Moving Average (GMA), since the values decline geometrically when a smooth curve connects them. It is defined for a variable X as

$$E_t = \lambda \bar{X}_t + (1 - \lambda)E_{t-1} \tag{2}$$

where λ is a smoothing constant whose value ranges between 0 and 1. The starting value is usually taken as $E_0 = \mu_0$ or $E_0 = \bar{x}$.

[22], [23], [26], among others, have worked on Doubly Exponentially Weighted Moving Average (DEWMA) based on current and past observations, obtained by modifying EWMA. The mathematical expression for DEWMA is:

$$\begin{aligned} E_t &= \lambda \bar{X}_t + (1 - \lambda)E_{t-1}, \quad 0 < \lambda \le 1 \\ HE_t &= \lambda E_t + (1 - \lambda)HE_{t-1}, \quad 0 < \lambda \le 1 \end{aligned} \tag{3}$$

Hybrid exponentially weighted moving average statistic (HEWMA), first introduced by [7], is also obtained by modifying the EWMA statistic. However, it differs from DEWMA because of its use of two smoothing parameters λ_1 and λ_2, instead of the single parameter λ in DEWMA. Mathematically, it is defined as:

$$\begin{aligned} E_t &= \lambda_2 \bar{X}_t + (1 - \lambda_2)E_{t-1}, \quad 0 < \lambda_2 \le 1 \\ HE_t &= \lambda_1 E_t + (1 - \lambda_1)HE_{t-1}, \quad 0 < \lambda_1 \le 1 \end{aligned} \tag{4}$$

The initial values are either estimated from a pilot survey, or taken as $HE_0 = E_{t0} = 0$. Some interesting work involving EWMA and HEWMA statistic have been done by [2], [3], [11], [16], [17], among others. [4] has developed estimators under stratified sampling scheme.

The usual estimators for population mean, such as mean estimator, ratio estimator, product estimator, etc. were designed for data collected on a single point of time, and may not be equally competent for situations in which data is time-scale based ([15]). The solution offered by [15] involves developing memory type estimators for time series data. Such estimators have the advantage that they utilize the prior data, including the most recent measurement, and assign weightage depending on how far removed in time the data is. Less weight is given to data that are further removed in time. The use of current as well as previous information increases the efficiency of the estimators, in comparison with the conventional estimators mentioned above.

In this manuscript, a shrinkage type estimator involving HEWMA statistic is developed for estimation of population mean. The structure of the estimator has been motivated by the estimator due to [24]. Under conventional sampling designs such as SRSWOR, in situations that do not involve time-series data, such estimator is seen to take advantage of both mean estimator and ratio estimator by utilizing a linear combination of the two. Weightage is assigned to the two estimators depending on the value of the constant in the linear combination, and the estimator is seen to perform better than both its constituent mean and ratio estimators. Hence, it is reasonable to believe that the same may be true for memory-type estimators, and explore the same. This is the motivation behind the structure of the proposed estimator. Information collected from previous surveys has been incorporated as auxiliary information during the design of the proposed estimator, for the purpose of improving the efficiency of the estimator. By auxiliary information, one means information on a variable that is related to the variable of interest. Such information may be readily available from preceding surveys or may be collected by utilizing minimal survey funds. It is to be noted that limited attempts have been made to utilize auxiliary information for developing estimation strategies involving memory type estimators. Based on the discussions so far, it is reasonable to believe that such estimators would be efficient for estimation of population mean in quality control processes. Details of some applications has been provided at the end of this manuscript.

The remainder of the manuscript is structured in the following way: Section II gives a brief idea about the sampling schemes used in the manuscript. Sections III and IV introduce the proposed estimators under Simple Random Sampling Without Replacement (SRSWOR) and Stratified Sampling schemes respectively, and discuss their properties in terms of bias and Mean Square Errors (MSE). Section V presents the simulation study comparing the proposed estimator with the ratio and the product type estimators proposed by ([17]). The interpretations of the results are briefly discussed in Section VII, whereas Section VIII discusses some possible practical applications and future scope of work.

II. SAMPLING STRATEGY AND NOTATIONS

Two sampling schemes have been used for the current work, namely, Simple Random Sampling Without Replacement (SRSWOR) and Stratified Sampling.

In SRSWOR, every unit in the population has an equal chance of being selected in the sample. However, each unit can be included only once in the sample, i.e. repetition of units is prohibited. The chief advantage of the sampling scheme lies in its simplicity. It is useful for populations which are approximately homogeneous in nature with respect to the characteristic under study.

Let a sample of size n be taken from a population of size N. Let Y be the variable under study and X be the auxiliary variable which is positively correlated to Y and information on which is assumed to be known. The following standard notations are used henceforth:

- μ_y, μ_x: Population means of Y and X respectively
- $\bar{y}, \bar{x}$: Sample means of Y and X respectively
- C_Y, C_X: Coefficient of variation of Y and X respectively
- ρ: Correlation coefficient between Y and X

In real world situations, SRSWOR may not always be the suitable method for handling data. When the population is heterogeneous in terms of the character of interest, it is beneficial to divide the population into several groups known as strata such that the units within each stratum are relatively homogeneous, whereas there remains relative heterogeneity between any two strata. Samples are then drawn appropriately from each stratum to produce a sample that is a true representative of the population.

In this connection, we consider a population that is divided into L strata, with each stratum consisting of $N_i, i = 1, \cdots, L$ units, from which $n_i, i = 1, \cdots, L$ units are included in the sample.

The following standard notations for stratified sampling scheme are used henceforth:

- $f_k = \frac{1}{n_k} - \frac{1}{N_k}, k = 1, \cdots, L$
- $W_k = \frac{N_k}{N}, k = 1, \cdots, L$: Strata weight of stratum $k, k = 1, \cdots, L$, where $N = \sum_{i=1}^{L} N_i$
- μ_y, μ_x: Population means of Y and X respectively
- $\bar{y}_k, \bar{x}_k$: Sample means of Y and X respectively in stratum $k, k = 1, \cdots, L$
- $\bar{y}_{st} = \sum_{k=1}^{L} W_k \bar{y}_k, \bar{x}_{st} = \sum_{k=1}^{L} W_k \bar{x}_k$: Sample mean for Y and X respectively
- S_{yk}, S_{xk}: Sample mean square of Y and X respectively in stratum $k, k = 1, \cdots, L$

III. THE CONSTRUCTION OF THE ESTIMATOR UNDER SRSWOR SAMPLING SCHEME

Motivated by the work of [4] under stratified sampling, the estimator in the current manuscript is developed. Let $t, t-1$ in the subscript denote the sample number. The HEWMA statistic for Y and X respectively are given by

$$\begin{aligned} Z_t &= \lambda_1 E_{ty} + (1-\lambda_1) Z_{t-1} \\ Q_t &= \lambda_1 E_{tx} + (1-\lambda_1) Q_{t-1} \end{aligned} \tag{5}$$

which utilizes the corresponding EWMA statistics

$$\begin{aligned} E_{ty} &= \lambda_2 \bar{y}_t + (1-\lambda_2) E_{ty-1} \\ E_{tx} &= \lambda_2 \bar{x}_t + (1-\lambda_2) E_{tx-1} \end{aligned} \tag{6}$$

with the two smoothing parameters λ_1 and λ_2.

Utilizing (5), a shrinkage type of estimator for population mean is proposed by taking a linear combination of the HEWMA statistic and the ratio type estimator. Mathematically, the proposed estimator takes the following form:

$$\tau_P = \alpha Z_t + (1-\alpha) \frac{Z_t \mu_x}{Q_t} \tag{7}$$

The constant α is to be determined such that it minimizes the Mean Square Error (MSE) of the proposed estimator. It is to be noted that when $\alpha = 1$, $\tau_P = Z_t$, i.e. the proposed estimator reduces to the HEWMA statistic for Y. When $\alpha = 0$, $\tau_P = \frac{Z_t \mu_x}{Q_t}$, i.e. the proposed estimator assumes the form of the ratio-type estimator.

A. PROPERTIES OF THE PROPOSED ESTIMATOR UNDER SRSWOR

Several criteria are available in survey literature for defining a "good" estimator. Two such properties are bias and efficiency of an estimator. Bias measures whether an estimator, on average, overestimates or underestimates the true value of a parameter. An estimator is said to be unbiased if its bias equals zero. In a given class of estimators, the one with the minimum variance or mean square error (MSE) is said to be the most efficient.

In this section, we strive to determine the mathematical expression for bias and MSE of the proposed estimator. To this end, the following two transformations involving error terms are taken:

$$Z_t = \mu_y (1 + e_0), \quad Q_t = \mu_x (1 + e_1) \tag{8}$$

where $|e_i| < 1, i = 1, 2$, assumed under large sample approximations.

Let $f = \frac{1}{n} - \frac{1}{N}$, $a_1 = 1 - \lambda_1, a_2 = 1 - \lambda_2$, $\gamma = \left\{ \frac{a_1^2}{1-a_1^2} + \frac{a_2^2}{1-a_2^2} - \frac{2a_1 a_2}{1-a_1 a_2} \right\}$, and $A = \frac{(\lambda_1 \lambda_2)^2}{(\lambda_2 - \lambda_1)^2} \gamma$

Under the above defined transformations, and using the notations defined above, the expectations of the error terms are as follows:

$E(e_0) = 0 = E(e_1), E(e_0^2) = Af\frac{\sigma_{Yt}^2}{\mu_y^2}, E(e_1^2) = Af\frac{\sigma_X^2}{\mu_x^2}, E(e_0e_1) = Af\rho C_Y C_X$

Utilizing the transformations defined in Equation (8), the estimator introduced in Equation (7) assumes the following form:

$$\tau_P = \mu_y\left[1 + e_0 - e_1 + \alpha e_1 + e_1^2 - \alpha e_1^2 - e_0e_1 + \alpha e_0e_1\right] \tag{9}$$

Rearranging the terms appropriately and taking expectations on both sides provides the mathematical expressions for bias (B(.)) and MSE (M(.)) as mentioned subsequently:

$$B(\tau_P) = E(\tau_P - \mu_y) = Af\mu_y(1-\alpha)\left[\frac{\sigma_X^2}{\mu_x^2} - \rho C_Y C_X\right] \tag{10}$$

$$M(\tau_P) = E(\tau_P - \mu_y)^2 = Af\mu_y^2\left[\frac{\sigma_{Yt}^2}{\mu_y^2} + (1-\alpha)^2\frac{\sigma_X^2}{\mu_x^2} - 2(1-\alpha)\rho C_Y C_X\right] \tag{11}$$

For the purpose of determining the value of α which minimizes the MSE given in equation (11), one sets $\frac{\partial M}{\partial \alpha} = 0$. Subsequent calculations yield the optimal value of α as:

$$\alpha_{\text{opt}} = 1 - \rho\frac{C_Y}{C_X} \tag{12}$$

The expression for the optimal MSE is obtained by substituting the optimal value of α from Equation (12) in the expression for MSE in Equation (11). It is presented in Equation (13) below.

$$M(\tau_P)_{\text{opt}} = Af\sigma_{Yt}^2(1-\rho^2) \tag{13}$$

Remark: Following [17], it is to be noted that for time varying variance, the γ will be replaced by γ_t, where

$$\gamma_t = \frac{(1-\lambda_1)^2(1-(1-\lambda_1)^{2t})}{1-(1-\lambda_1)^2} + \frac{(1-\lambda_2)^2(1-(1-\lambda_2)^{2t})}{1-(1-\lambda_2)^2} - \frac{2(1-\lambda_1)(1-\lambda_2)(1-(1-\lambda_1)^t(1-\lambda_2)^t)}{1-(1-\lambda_1)(1-\lambda_2)}$$

IV. THE CONSTRUCTION OF THE ESTIMATOR UNDER STRATIFIED SAMPLING SCHEME

The HEWMA statistic for Y and X respectively are given by

$$\begin{aligned} Z_{st} &= \lambda_1 E_{sty-1} + (1-\lambda_1)Z_{s(t-1)} \\ Q_{st} &= \lambda_1 E_{stx-1} + (1-\lambda_1)Q_{s(t-1)} \end{aligned} \tag{14}$$

where

$$\begin{aligned} E_{sty} &= \lambda_2\bar{y}_{st} + (1-\lambda_2)E_{sty-1} \\ E_{stx} &= \lambda_2\bar{x}_{st} + (1-\lambda_2)E_{stx-1} \end{aligned} \tag{15}$$

Analogous to Section III, the estimator under stratified sampling structure is designed as follows:

$$\tau_{Pst} = \alpha_{st}Z_{st} + (1-\alpha_{st})\frac{Z_{st}}{\mu_x}Q_{st} \tag{16}$$

where the constant α_{st} is to be determined such that it minimizes the MSE of the proposed estimator.

A. PROPERTIES OF THE PROPOSED ESTIMATOR UNDER STRATIFIED SAMPLING

For the purpose of obtaining the expressions for bias and MSE, the following two transformations involving error terms are taken:

$$\begin{aligned} Z_{st} &= \mu_y(1 + e_{0st}) \\ Q_{st} &= \mu_x(1 + e_{1st}) \end{aligned} \tag{17}$$

where $|e_{ist}| < 1, i = 1, 2$, assumed under large sample approximations.

$$V(\bar{y}_{st}) = \sum_{k=1}^{L} f_k W_k^2 S_{yk}^2,$$

$$V(\bar{x}_{st}) = \sum_{k=1}^{L} f_k W_k^2 S_{xk}^2,$$

$$Cov(\bar{y}_{st}, \bar{x}_{st}) = \sum_{k=1}^{L} f_k W_k^2 S_{yxk}$$

Under the above defined transformations, the expectations of the error terms areas follows:

$E(e_{0st}) = 0 = E(e_{1st}), E(e_{0st}^2) = \gamma\frac{V(\bar{y}_{st})}{\mu_y^2}, E(e_{1st}^2) = \gamma\frac{V(\bar{x}_{st})}{\mu_x^2}, E(e_{0st}e_{1st}) = \gamma\frac{Cov(\bar{y}_{st},\bar{x}_{st})}{\mu_x\mu_y}$

Utilizing the transformations defined in Equation (17), the estimator introduced in Equation (16) assumes the following form:

$$\tau_{Pst} = \mu_y[1 + e_{0st} - e_{1st}(1-\alpha_{st}) + e_{1st}^2(1-\alpha_{st}) - e_{0st}e_{1st}(1-\alpha_{st})] \tag{18}$$

Computations analogous to the ones described in Subsection (III-A) yield the following expressions for the bias and the MSE of the proposed estimator when stratified sampling is utilized:

$$M(\tau_{Pst}) = \mu_y^2\gamma\left[\begin{array}{c}\frac{V(\bar{y}_{st})}{\mu_y^2} + (1-\alpha_{st})^2\frac{V(\bar{x}_{st})}{\mu_x^2} \\ -2(1-\alpha_{st})\frac{Cov(\bar{y}_{st},\bar{x}_{st})}{\mu_x\mu_y}\end{array}\right] \tag{19}$$

For the purpose of determining the value of α_{st} which minimizes the MSE given in equation (19), one sets $\frac{\partial M}{\partial \alpha_{st}} = 0$. The optimal value of α_{st} is subsequently determined to be the following:

$$\alpha_{st\text{opt}} = 1 - \frac{\mu_x}{\mu_y}\frac{Cov(\bar{y}_{st}, \bar{x}_{st})}{V(\bar{x}_{st})} \tag{20}$$

The expression for the optimal MSE is obtained by substituting the optimal value of α_{st} from Equation (20) in the expression for MSE in Equation (19). It is presented in Equation (21) below.

$$M(\tau_P)_{\text{opt}} = \gamma \left[V(\bar{y}_{st}) - \frac{\{Cov(\bar{y}_{st}, \bar{x}_{st})\}^2}{V(\bar{x}_{st})} \right] \tag{21}$$

V. SIMULATION

In this section, a simulation study has been conducted to study the performance of the proposed estimator in comparison with some contemporary estimators. The statistical software R([19]) has been used for the purpose. The proposed estimator has been compared with the memory type ratio and product estimators proposed by [4]. The structure of the respective estimators are given in equations (22) and (23) below.

$$T_{rat} = \frac{Z_{st}}{Q_{st}} \mu_x \tag{22}$$

$$T_{pdt} = \frac{Z_{st}}{\mu_x} Q_{st} \tag{23}$$

The expressions for the mean square errors of the memory type ratio and product estimators, as given in [4], are as follows:

$$M(T_{rat}) = \frac{\lambda}{2-\lambda} \sum_{k=1}^{L} f_k W_k^2 \left[S_{yk}^2 + R^2 S_{xk}^2 - 2RS_{yxk} \right] \tag{24}$$

$$M(T_{pdt}) = \frac{\lambda}{2-\lambda} \sum_{k=1}^{L} f_k W_k^2 \left[S_{yk}^2 + R^2 S_{xk}^2 + 2RS_{yxk} \right] \tag{25}$$

The comparison between two or more estimators under a chosen sampling scheme is done in terms of Percentage Relative Efficiencies (PREs). The focus of the simulation study is the comparison of the performance of the proposed estimator with that of a contemporary estimator. Hence PRE has been chosen as the evaluation metric. The PREs of the proposed estimator w.r.t. the ratio and the product estimators mentioned above are defined as follows:

$$P_1 = \frac{M(T_{rat})}{M(\tau_P)_{\text{opt}}} \times 100, \quad P_2 = \frac{M(T_{pdt})}{M(\tau_P)_{\text{opt}}} \times 100$$

To conduct the simulation study, the first step consists of generating data. The outline of the algorithm for this purpose is given in brief below.

1) Define the number of strata L in the population. For the current study, $L = 5$.
2) For each strata, the data is generated randomly from a normal distribution as follows:
    a) Set the correlation coefficient between X and Y to a given number.
    b) Generate two random numbers A and B from uniform distribution with parameters $(0.1, 0.9)$. In `R` ([19]), this is done using the line of code `runif(1,0.1,0.9)`.
    c) Set $\mu_x = 100 * A$, $\mu_y = 100 * B$.
    d) Generate the data. In `R` ([19]), the `genCorGen` function from the package `simstudy` ([6]) is used for the purpose.

TABLE 1. PRE of the proposed estimator w.r.t. contemporary estimator when $\lambda \in (0.1, 0.2, 0.3)$, $\lambda_1 \in (0.2, 0.3)$.

		$\lambda = 0.1$		$\lambda = 0.2$		$\lambda = 0.3$	
λ_1	ρ	P_1	P_2	P_1	P_2	P_1	P_2
0.2	0.1	53.35	61.97	160.57	190.91	292.52	344.85
	0.2	78.12	105.97	81.09	117.55	169.68	253.07
	0.3	60.54	98.97	197.44	300.98	238.90	417.11
	0.4	44.80	82.22	78.15	174.99	249.93	422.57
	0.5	62.45	158.55	118.45	306.08	180.20	474.26
	0.6	29.87	111.50	73.46	237.79	232.10	655.65
	0.7	35.87	182.36	64.43	338.12	103.17	468.58
	0.8	48.27	274.96	60.89	475.61	116.07	938.72
	0.9	39.99	450.73	95.75	1218.65	106.95	1796.74
0.3	0.1	124.84	150.92	256.62	303.77	905.92	1068.48
	0.2	304.24	406.87	172.26	223.44	560.22	817.30
	0.3	210.79	337.38	1069.86	1449.23	1135.11	1591.52
	0.4	155.81	283.30	220.10	484.63	965.56	1829.80
	0.5	132.16	343.48	237.80	601.30	467.35	1258.79
	0.6	118.65	407.40	184.80	729.43	377.41	1164.65
	0.7	162.40	707.86	182.94	928.89	241.61	774.61
	0.8	70.78	441.58	443.22	2269.02	325.53	2516.27
	0.9	70.61	952.54	157.80	2626.03	239.60	3244.95

TABLE 2. PRE of the proposed estimator w.r.t. contemporary estimator when $\lambda \in (0.4, 0.5, 0.6)$, $\lambda_1 \in (0.2, 0.3)$.

		$\lambda = 0.4$		$\lambda = 0.5$		$\lambda = 0.6$	
λ_1	ρ	P_1	P_2	P_1	P_2	P_1	P_2
0.2	0.1	206.94	251.39	258.26	297.63	490.77	580.92
	0.2	174.36	251.73	229.12	303.77	511.55	711.65
	0.3	241.62	440.57	321.77	563.33	570.43	921.38
	0.4	172.10	372.54	223.84	446.02	249.13	442.33
	0.5	188.01	475.46	449.66	1127.51	296.51	750.49
	0.6	156.20	578.53	206.81	709.33	225.77	708.57
	0.7	145.49	727.74	208.20	1059.14	303.17	1547.53
	0.8	130.77	973.34	202.04	1561.64	326.28	2394.23
	0.9	138.09	2456.80	198.94	3524.53	222.49	3317.97
0.3	0.1	601.24	725.96	1347.55	1606.29	719.40	835.02
	0.2	1094.16	1425.15	586.95	840.17	578.72	710.54
	0.3	1637.90	2515.55	836.03	1415.17	572.58	762.27
	0.4	842.61	1634.94	1092.27	2244.17	604.44	957.63
	0.5	444.51	1255.90	676.93	1972.54	753.46	2132.31
	0.6	346.82	1153.84	648.21	2059.42	671.53	2119.56
	0.7	342.86	1241.79	561.39	2887.63	571.45	2129.46
	0.8	360.13	2846.31	447.42	3414.42	582.01	4428.98
	0.9	396.82	3834.99	882.65	6635.08	622.19	7554.83

For the purpose of the current study, the correlation coefficient of the various strata are assumed to take a common value ρ. Simulation studies are conducted for various values of ρ, as well as various values of the parameter in the [4] estimators, namely, λ, where $0 < \lambda \leq 1$, and the various values of parameters of the proposed estimator, namely, λ_1, λ_2. Here $\lambda_1 \neq \lambda_2$, so various combinations of λ_1, λ_2 may be taken. For simplicity and to keep the paper brief and free of unnecessary tabular results of the same type, we present only the results for $\lambda_2 = \lambda_1 + 0.1$. Readers are free to follow the steps of the simulation presented values for other pairs of values of λ_1 and λ_2. The results of the simulation study have been presented in Tables (1)-(10) below.

VI. APPLICATIONS OF THE PROPOSED WORK

Secondary data has been used for the purpose of demonstrating the utilization of the proposed estimator under SRSWOR

TABLE 3. PRE of the proposed estimator w.r.t. contemporary estimator when $\lambda \in (0.7, 0.8, 0.9)$, $\lambda_1 \in (0.2, 0.3)$.

		$\lambda = 0.7$		$\lambda = 0.8$		$\lambda = 0.9$	
λ_1	ρ	P_1	P_2	P_1	P_2	P_1	P_2
0.2	0.1	635.60	734.34	905.94	1072.36	1194.10	1424.66
	0.2	634.13	879.06	457.49	621.61	1289.10	1713.24
	0.3	595.04	1058.16	501.28	846.03	720.48	1329.56
	0.4	355.91	797.15	456.99	1057.37	765.48	1691.74
	0.5	475.38	1126.88	457.85	1154.96	513.03	1310.88
	0.6	368.23	1229.44	449.19	1692.84	671.22	2127.05
	0.7	284.80	958.43	632.49	2985.19	508.92	2584.82
	0.8	328.87	2389.39	1114.25	5641.30	563.44	3903.00
	0.9	282.22	4437.95	400.85	6179.82	541.85	7406.38
0.3	0.1	925.80	1093.80	3863.43	4515.20	3266.97	3904.21
	0.2	825.29	1115.29	4715.81	6258.18	1716.57	2406.59
	0.3	836.11	1397.28	2734.54	4252.96	2011.13	3356.40
	0.4	855.75	1923.01	3586.62	6489.19	2028.05	3951.19
	0.5	862.19	2159.07	1494.27	3459.06	1494.43	3656.39
	0.6	721.61	2154.22	3219.85	8850.73	1309.01	4766.59
	0.7	836.30	4583.31	2845.75	10045.12	1280.43	6106.53
	0.8	862.14	6543.65	938.90	6702.33	1192.97	7643.74
	0.9	1064.93	13023.78	2164.37	19078.47	1189.02	15246.42

TABLE 4. PRE of the proposed estimator w.r.t. contemporary estimator when $\lambda \in (0.1, 0.2, 0.3)$, $\lambda_1 \in (0.4, 0.5)$.

		$\lambda = 0.1$		$\lambda = 0.2$		$\lambda = 0.3$	
λ_1	ρ	P_1	P_2	P_1	P_2	P_1	P_2
0.4	0.1	265.73	317.47	1134.16	1287.77	1067.06	1288.68
	0.2	165.98	225.27	961.65	1265.73	664.41	854.27
	0.3	195.32	345.58	369.38	584.20	509.49	853.62
	0.4	330.42	643.36	517.86	1097.07	1698.02	2806.01
	0.5	170.39	464.11	614.04	1576.05	596.88	1547.05
	0.6	264.54	830.20	342.43	1038.11	674.47	2102.27
	0.7	195.67	934.62	388.76	1936.67	551.05	2582.58
	0.8	150.15	1118.47	288.84	2008.22	640.82	4811.65
	0.9	139.18	2079.24	292.72	5285.47	524.92	7557.61
0.5	0.1	300.30	356.27	615.96	733.98	1618.96	1934.30
	0.2	259.85	338.41	541.18	680.61	1702.06	2380.66
	0.3	320.89	575.19	712.87	1314.90	2075.46	3419.67
	0.4	221.84	351.61	628.46	1357.55	1879.84	3264.78
	0.5	273.97	642.41	625.09	1739.25	1544.32	3836.23
	0.6	223.61	616.15	614.38	2171.99	800.02	2690.26
	0.7	232.79	1163.30	1040.19	4379.13	1824.19	7341.00
	0.8	247.84	1908.04	608.35	3961.76	814.45	4985.00
	0.9	221.06	2814.74	480.14	7777.22	795.86	10898.22

sampling scheme. The datasets have been chosen to illustrate the use of the proposed estimator in real world scenarios for estimating population mean. Three datasets have been obtained from UCI Machine Learning Repository ([5]), a brief description of which follows:

A. DATASET I: "MACHINE"

This dataset consists of data on Relative CPU Performance. The dataset contains 209 instances of 10 attributes. For the purpose of this study, we have concentrated on products from vender named "nas". The attribute named "MMAX", denoting the maximum main memory in kilobytes is taken as study variable Y, while the attribute named "MMIN", denoting minimum main memory in kilobytes is taken as auxiliary variable X. Here, $N = 19$, $\mu_x = 5368.421$, $\mu_y = 17894.74$, $S_X = 4494.166$, $S_Y = 10100.73$, $C_X = 0.5644524$, $C_Y = 0.8371486$, $\rho = 0.8148075$. A total of

TABLE 5. PRE of the proposed estimator w.r.t. contemporary estimator when $\lambda \in (0.4, 0.5, 0.6)$, $\lambda_1 \in (0.4, 0.5)$.

		$\lambda = 0.4$		$\lambda = 0.5$		$\lambda = 0.6$	
λ_1	ρ	P_1	P_2	P_1	P_2	P_1	P_2
0.4	0.1	1700.20	1971.61	2069.84	2440.43	1923.08	2254.95
	0.2	1300.91	1890.28	2439.92	3206.85	3564.96	4787.21
	0.3	2781.91	4156.57	2185.22	3551.22	2656.51	4599.26
	0.4	952.49	1898.59	998.55	2057.42	2433.57	5199.28
	0.5	792.67	2021.92	958.30	1935.28	2414.35	5594.61
	0.6	1460.10	4696.48	1810.58	5956.72	1227.04	4631.81
	0.7	774.76	3638.71	892.52	3936.68	2109.41	7594.73
	0.8	760.98	5733.00	951.61	6516.06	1678.79	10534.30
	0.9	742.89	12332.83	961.02	16045.87	5319.58	38277.51
0.5	0.1	1724.69	2033.15	3256.70	3865.74	2311.81	2647.30
	0.2	1153.38	1443.31	1975.66	2798.57	2151.34	2954.26
	0.3	1219.96	1864.64	2257.38	3732.92	2864.19	5227.31
	0.4	2701.61	5340.45	3559.16	7177.23	3101.86	6762.73
	0.5	4167.47	8980.51	2278.68	6021.41	2093.11	5076.55
	0.6	1869.35	6580.85	2743.75	7759.47	1911.11	6110.26
	0.7	1466.49	6922.48	2410.04	10105.84	1811.63	8682.59
	0.8	1340.68	10324.65	1416.18	7875.87	1833.09	11586.23
	0.9	1053.27	10542.93	2058.02	32444.30	1989.07	28949.82

TABLE 6. PRE of the proposed estimator w.r.t. contemporary estimator when $\lambda \in (0.7, 0.8, 0.9)$, $\lambda_1 \in (0.4, 0.5)$.

		$\lambda = 0.7$		$\lambda = 0.8$		$\lambda = 0.9$	
λ_1	ρ	P_1	P_2	P_1	P_2	P_1	P_2
0.4	0.1	1841.44	2209.68	3824.56	4628.94	5141.18	6168.26
	0.2	1584.32	2149.16	2759.68	3657.80	4710.72	6710.15
	0.3	1816.18	2746.67	2565.39	4645.37	8614.37	13712.77
	0.4	1992.96	4061.77	3191.88	5676.26	2187.30	4158.21
	0.5	1472.85	3931.29	1853.02	5079.95	3627.01	10206.33
	0.6	2520.48	8711.90	2918.81	9378.35	3265.80	10979.95
	0.7	1392.09	5721.61	2518.08	11842.16	2173.86	7338.52
	0.8	2301.76	15978.20	2003.69	11470.15	2404.13	18616.25
	0.9	1455.64	19653.69	2410.12	29492.82	2432.31	32500.82
0.5	0.1	20609.33	22484.45	5430.78	6333.32	9633.97	11574.80
	0.2	3926.45	5790.34	6000.87	8827.78	6623.32	8890.81
	0.3	2753.55	4820.36	8800.33	14125.84	3707.94	5781.95
	0.4	3387.91	7439.71	5450.98	11807.58	4354.88	9858.59
	0.5	3813.67	8975.77	4949.30	11613.80	3525.01	8620.98
	0.6	2844.45	9448.40	3212.75	10494.07	3904.82	15023.62
	0.7	4700.07	21593.56	2893.60	12945.17	3495.65	16694.08
	0.8	2722.12	19302.69	3198.48	26546.53	5155.62	38936.42
	0.9	3572.14	52129.62	3804.14	54348.45	5377.52	76924.89

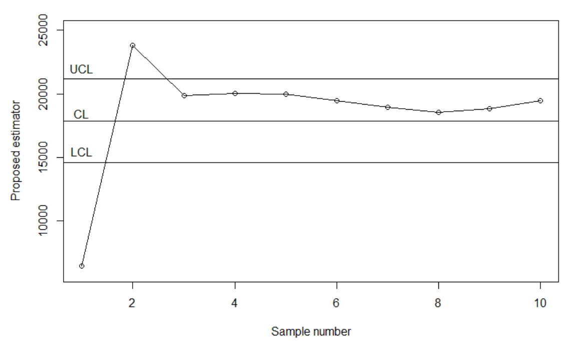

FIGURE 1. Control chart using Dataset I.

10 samples of size $n = 3$ have been taken. The sample data as well as the values of the proposed estimator have been presented in Table (11). Additionally, control chart has been plotted and shown in Figure (1).

TABLE 7. PRE of the proposed estimator w.r.t. contemporary estimator when $\lambda \in (0.1, 0.2, 0.3)$, $\lambda_1 \in (0.6, 0.7)$.

		$\lambda = 0.1$		$\lambda = 0.2$		$\lambda = 0.3$	
λ_1	ρ	P_1	P_2	P_1	P_2	P_1	P_2
	0.1	345.63	396.85	943.85	1053.17	1150.45	1293.66
	0.2	437.96	579.35	792.01	1090.72	1458.35	2016.06
	0.3	595.03	1004.81	841.72	1370.19	1091.96	1678.13
	0.4	340.79	692.61	828.31	1729.64	1172.11	2508.80
0.6	0.5	345.91	834.78	1024.38	2404.86	1431.10	4128.32
	0.6	345.69	1242.62	949.64	2926.26	1407.04	5276.58
	0.7	470.31	2032.19	820.29	3943.08	2329.50	7836.11
	0.8	440.04	2701.33	1439.67	7182.55	1132.21	6330.36
	0.9	432.78	5654.03	895.72	11812.42	1065.55	16888.23
	0.1	723.01	854.70	1491.72	1781.21	2056.29	2389.85
	0.2	700.51	992.38	1231.68	1713.17	2057.45	2780.05
	0.3	1467.37	2248.43	1459.05	2237.23	3990.66	6649.33
	0.4	1018.58	1984.97	898.77	1830.86	2128.25	4692.57
0.7	0.5	859.92	2229.70	1188.18	2137.33	1588.65	4160.34
	0.6	555.68	2092.69	997.52	3598.98	2331.02	8001.79
	0.7	573.50	2955.53	1130.19	5816.26	2245.87	7842.80
	0.8	502.85	2668.60	894.82	6015.69	1502.88	11315.22
	0.9	425.97	6732.22	1095.32	14466.54	1738.15	29604.97

TABLE 8. PRE of the proposed estimator w.r.t. contemporary estimator when $\lambda \in (0.4, 0.5, 0.6)$, $\lambda_1 \in (0.6, 0.7)$.

		$\lambda = 0.4$		$\lambda = 0.5$		$\lambda = 0.6$	
λ_1	ρ	P_1	P_2	P_1	P_2	P_1	P_2
	0.1	7641.56	8686.59	3856.37	4597.07	5775.80	6862.56
	0.2	1883.55	2668.17	4983.41	6939.87	4469.99	6353.82
	0.3	2049.34	3637.08	3112.64	5743.57	3785.61	6713.72
	0.4	2348.06	4932.54	3744.54	8420.31	5562.23	11016.51
0.6	0.5	2107.17	5938.15	2546.65	6455.40	3337.61	9046.09
	0.6	1822.04	6555.96	3415.94	11337.93	2864.42	10297.78
	0.7	1751.59	8710.19	2029.67	7941.66	4840.14	20766.36
	0.8	2285.22	15572.38	2464.86	12556.99	2685.11	14100.37
	0.9	2766.92	34986.38	2490.02	39579.36	3460.81	56218.84
	0.1	4322.71	5235.28	6003.61	7011.16	8065.81	9752.46
	0.2	2557.52	3526.43	4800.97	6616.95	4227.84	5986.15
	0.3	5003.43	8428.82	6375.64	10184.97	7008.70	10134.84
	0.4	2193.99	4228.73	3728.76	7619.60	6998.47	13005.29
0.7	0.5	2503.49	6254.06	3038.62	7295.84	4646.80	12803.64
	0.6	1943.63	5882.89	6588.94	19989.44	5572.37	17266.70
	0.7	2167.37	11666.74	3776.41	16750.86	6382.49	29299.88
	0.8	1958.90	15243.43	2618.65	20347.38	7550.13	50217.58
	0.9	2059.32	24405.53	3718.13	42193.10	9578.59	71678.32

B. DATASET II: "AUTOMOBILE"

This dataset consists of data on Automobiles of different make and models. The dataset contains 205 instances of 26 attributes. For the purpose of this study, we have concentrated on automobiles with the make "mazda". The attribute "height" is taken as study variable Y, while the attribute named "width" is taken as auxiliary variable X. Here, $N = 17, \mu_x = 65.58824, \mu_y = 53.35882, S_X = 1.004599, S_Y = 2.304903, C_X = 0.04319629, C_Y = 0.01531675, \rho = 0.07662348$. A total of 10 samples of size $n = 3$ have been taken. Table (12) presents the sample data as well as the values of the proposed estimator.

C. DATASET III: "AUTO MPG"

This dataset is concerned with the city-cycle fuel consumption in miles per gallon. The dataset contains 398 instances of 8 attributes. For the purpose of this study, we have concentrated on the model "datsun". The attribute "displacement" is taken as study variable Y, while the attribute named "horsepower" is taken as auxiliary variable X. Here, $N = 23, \mu_x = 83.82609, \mu_y = 103.2609, S_X = 19.68617, S_Y = 25.46729, C_X = 0.2466306, C_Y = 0.2348454, \rho = 0.9319278$. A total of 10 samples of size $n = 5$ have been taken. Table (13) presents the sample data as well as the values of the proposed estimator.

TABLE 9. PRE of the proposed estimator w.r.t. contemporary estimator when $\lambda \in (0.7, 0.8, 0.9)$, $\lambda_1 \in (0.6, 0.7)$.

		$\lambda = 0.7$		$\lambda = 0.8$		$\lambda = 0.9$	
λ_1	ρ	P_1	P_2	P_1	P_2	P_1	P_2
	0.1	4391.34	5112.14	5768.09	6941.14	7077.32	8414.27
	0.2	5694.60	8233.23	6318.49	9052.13	13741.99	18248.82
	0.3	3359.87	5034.87	16496.53	26702.01	6307.26	10305.84
	0.4	3567.43	7401.81	12578.86	19705.84	5421.51	9996.53
0.6	0.5	3970.37	10403.50	5270.05	13468.43	5757.17	13840.20
	0.6	3654.45	12892.54	5253.23	18499.48	4971.33	16020.75
	0.7	3768.47	15779.19	5273.11	27114.92	5083.50	24989.00
	0.8	3234.26	19314.40	7404.59	41082.28	5112.90	25167.28
	0.9	3242.04	54076.55	5117.62	86208.38	5007.03	37577.76
	0.1	4956.59	5567.68	8216.97	9509.85	10223.55	12133.63
	0.2	4616.10	6172.22	6311.90	8952.29	8080.89	11303.36
	0.3	5708.28	9439.22	10748.33	18386.23	8964.96	14851.22
	0.4	4961.83	10164.76	8078.96	17908.11	9036.26	15948.36
0.7	0.5	5922.36	17043.99	7768.46	21944.01	6443.82	14651.06
	0.6	13328.67	38975.68	6604.90	21897.12	7039.44	23135.32
	0.7	4787.66	25264.26	5463.93	20118.58	6378.78	31419.36
	0.8	4505.09	29580.01	5372.75	40255.05	7751.20	49312.17
	0.9	4173.84	51052.32	5570.32	83404.98	6650.65	82477.22

VII. INTERPRETATIONS OF THE RESULTS

This section is concerned with discussing the various numerical and graphical results presented in the previous sections.

The following conclusions may be drawn from Tables (1)-(10):

1) The proposed estimator is more efficient than the [4] estimator under optimal conditions. From the Tables, it is seen that the values of the PRE are much greater than 100 in most of the cases, indicating the dominance of the proposed estimator over the [4] estimator.
2) The proposed estimator retains its efficiency for values of ρ in the range $(0.1, 0.9)$. Thus, the proposed estimator can be effectively utilized in various practical scenarios involving data on variable of interest and additional data on a variable with various levels of correlation between them. Data in various projects are seen to consist of variables which are sometimes strongly correlated, sometimes weakly. The results in the Tables show that the proposed estimator is suitable for application in all such situations of positive correlation among the variables.
3) The proposed estimator is seen to be more efficient than the [4] estimator for a wide range of values of the respective parameters of the estimators. Hence, the parameters of the HEWMA statistic, namely, λ_1, λ_2, may be suitably chosen to assign suitable weightage to

TABLE 10. PRE of the proposed estimator w.r.t. contemporary estimator for various values of the parameters when $\lambda_1 = 0.8$.

λ	ρ	P_1	P_2	λ	ρ	P_1	P_2	λ	ρ	P_1	P_2
	0.1	526.47	584.03		0.1	6479.40	7653.71		0.1	8457.74	10119.38
	0.2	798.51	1151.47		0.2	2920.14	4109.35		0.2	15327.47	21758.84
	0.3	659.65	1170.52		0.3	4224.98	7414.22		0.3	8657.33	14959.48
	0.4	813.27	1751.87		0.4	3327.94	6767.37		0.4	8295.44	18129.17
0.1	0.5	639.45	1829.76	0.4	0.5	2833.35	6311.90	0.7	0.5	8001.21	20362.59
	0.6	1403.50	4118.83		0.6	2847.03	8710.27		0.6	5148.43	17256.07
	0.7	660.30	2870.27		0.7	2314.96	10320.76		0.7	6069.11	31625.96
	0.8	491.61	3150.18		0.8	2342.89	17941.05		0.8	5186.88	42373.84
	0.9	562.92	9395.41		0.9	2283.91	35304.83		0.9	4972.28	71973.23
	0.1	1853.30	2185.91		0.1	5205.51	6107.36		0.1	35520.16	40028.26
	0.2	2305.60	3196.19		0.2	5842.68	8328.02		0.2	8427.23	12272.06
	0.3	1868.40	3132.57		0.3	4867.38	8545.76		0.3	12805.30	21539.71
	0.4	1208.53	1752.38		0.4	4202.80	8876.27		0.4	14793.99	27646.03
0.2	0.5	1846.70	5003.34	0.5	0.5	6439.05	13889.05	0.8	0.5	11757.03	24447.89
	0.6	2459.11	8210.02		0.6	3729.14	14840.38		0.6	15629.70	43715.07
	0.7	1427.68	6276.27		0.7	3942.09	17380.38		0.7	12791.92	53534.35
	0.8	1512.33	11832.88		0.8	3628.20	26926.17		0.8	6994.45	59928.22
	0.9	1227.76	21475.31		0.9	3103.67	46713.89		0.9	6201.45	104685.83
	0.1	2159.74	2515.19		0.1	5847.99	6784.79		0.1	18913.73	22598.76
	0.2	2537.64	3642.28		0.2	9095.46	12602.66		0.2	12000.23	16771.82
	0.3	1713.90	2677.88		0.3	5847.79	10438.18		0.3	10959.79	19347.31
	0.4	1841.02	3080.77		0.4	6633.46	14168.25		0.4	9506.88	21219.77
0.3	0.5	2410.84	5704.26	0.6	0.5	7943.60	20712.36	0.9	0.5	18357.73	42292.85
	0.6	2561.23	8596.24		0.6	5248.24	16623.42		0.6	7493.83	22541.86
	0.7	1617.98	8044.73		0.7	4357.40	21722.99		0.7	11360.58	55114.22
	0.8	5517.25	26226.11		0.8	9114.45	50093.26		0.8	8710.98	70914.34
	0.9	1884.56	21716.24		0.9	4496.40	55294.99		0.9	8136.89	141780.52

TABLE 11. Computation of the proposed estimator for Dataset I: "Machine".

Sample values of X			$\bar{x}$	Sample values of Y			$\bar{y}$	Z_t	Q_t	τ_P
4000	2000	4000	3333.33	16000	8000	16000	13333.33	1.00	1.00	6487.28
2000	4000	8000	4666.67	4000	16000	32000	17333.33	1040.80	280.80	23829.27
16000	4000	4000	8000.00	32000	16000	16000	21333.33	2840.64	900.64	19869.56
2000	2000	8000	4000.00	8000	8000	32000	16000.00	4638.11	1433.71	20020.44
4000	2000	8000	4666.67	16000	16000	16000	16000.00	6326.41	1926.21	19988.64
8000	2000	4000	4666.67	16000	8000	16000	13333.33	7692.27	2366.44	19484.58
4000	8000	4000	5333.33	16000	16000	16000	16000.00	8955.62	2790.98	18950.07
8000	4000	8000	6666.67	32000	16000	16000	21333.33	10405.76	3261.26	18530.62
2000	2000	8000	4000.00	16000	8000	32000	18666.67	11713.49	3568.94	18850.61
4000	4000	2000	3333.33	24000	16000	8000	16000.00	12703.01	3727.11	19463.14

TABLE 12. Computation of the proposed estimator for Dataset II: "Automobile".

Sample values of X			$\bar{x}$	Sample values of Y			$\bar{y}$	Z_t	Q_t	τ_P
65.7	64.2	65.7	65.20	49.6	54.1	49.60	51.10	1.00	1.00	2.75
64.2	65.7	66.1	65.33	54.1	49.6	54.40	52.70	3.96	4.72	5.35
66.5	65.7	65.7	65.97	55.5	49.6	49.60	51.57	8.48	10.48	9.69
64.2	66.5	66.5	65.73	54.1	53.7	55.50	54.43	13.76	17.02	14.83
64.2	65.7	66.5	65.47	54.1	49.6	53.70	52.47	19.05	23.59	19.97
66.5	64.2	65.7	65.47	53.7	54.1	49.60	52.47	24.01	29.78	24.79
65.7	66.5	65.7	65.97	49.6	53.7	49.60	50.97	28.41	35.42	29.06
66.5	65.7	66.5	66.23	55.5	49.6	53.70	52.93	32.34	40.42	32.89
65.7	64.2	66.5	65.47	49.6	54.1	53.70	52.47	35.75	44.73	36.20
66.5	65.7	64.2	65.47	55.5	49.6	54.10	53.07	38.70	48.39	39.07

current data and past data, depending on the requirement of the particular application.

Thus, utilization of the proposed estimator for various practical scenarios may be recommended. To this end, Tables (11)-(13) illustrate the computation of the proposed estimator using real data from the 3 datasets introduced previously. Ten points of time have been considered in each case, and samples of appropriate sizes have been taken for each point of time. Values of the sample means $\bar{x}$, $\bar{y}$ are calculated at first, followed by the values of the estimators Z_t and Q_t. Finally, the value of the proposed estimator is calculated by taking the linear combination of functions of Z_t and Q_t. All the values have been tabulated so that readers can suitably replicate the results and understand the use of the proposed estimator.

Figure (1) illustrates how the proposed chart works. It presents the Control Chart based on the proposed estimator, using Dataset I. The UCL, CL and LCL have been indicated in the figure as horizontal lines. The UCL and the LCL are indicators of whether a process is in-control or out-of-control. The values of the proposed estimator τ_P (that have been tabulated in Table 11) have been plotted on the Y-axis, with time on the X-axis. A line of best fit has also been provided. Most of the values of the estimates are seen to lie within the control limits. However, one value is seen to be below the

TABLE 13. Computation of the proposed estimator for Dataset III: "Auto MPG".

Sample values of X					$\bar{x}$	Sample values of Y					$\bar{y}$	Z_t	Q_t	τ_P
65	65	92	100	67	77.80	86	85	119	119	91.00	100.00	1.00	1.00	74.50
92	67	65	92	65	76.20	119	79	86	97	85.00	93.20	6.39	5.37	89.23
97	70	65	65	88	77.00	119	85	85	86	97.00	94.40	14.69	12.12	91.84
97	65	65	70	61	71.60	146	85	86	85	83.00	97.00	24.28	19.46	95.52
94	97	92	97	65	89.00	108	146	119	119	86.00	115.60	35.13	27.75	98.11
65	120	70	92	65	82.40	85	146	85	97	85.00	99.60	45.07	35.67	99.06
92	70	65	70	61	71.60	119	85	86	85	83.00	91.60	53.43	42.26	100.06
70	65	61	70	97	72.60	85	85	83	85	119.00	91.40	60.39	47.77	100.83
65	70	120	70	92	83.40	86	85	146	85	119.00	104.20	66.92	53.00	101.46
92	67	70	65	65	71.80	119	79	85	85	85.00	90.60	71.99	57.05	101.98

LCL, while another value is seen to be above the UCL. This suggests that the process may have some assignable causes of variation and inputs may need to be modified. The proposed control chart thus shows indications that the process may be out-of-control.

VIII. RECOMMENDATIONS AND FUTURE SCOPE OF WORK

The proposed estimator can be utilized to estimate population mean. This has wide-spread applications in the various industries, such as industries which manufacture household items like appliances, bulbs, etc., automobile manufacturing industries, industries which manufacture machine parts such as screws, bolts, etc., industries which manufacture parts of computers, such as CPU, chips, etc. Control charts based on the proposed estimators may be subsequently generated to visually analyze whether a process in out of control (See [3]). Hence, utilization of the proposed estimator may be recommended to the various industries.

Future scope of the work includes designing various estimators under different sampling schemes. Auxiliary information can be effectively utilized to increase the efficiency of the estimators. New estimators may also be designed by utilizing auxiliary information on more than one additional variable.

DECLARATIONS OF INTEREST

None

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