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Nonlinear Fault-Tolerant Control Design for Singular Stochastic Systems With Fractional Stochastic Noise and Time-Delay

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ABSTRACT This work focuses on the stabilization issue for a class of singular stochastic systems against fractional Gaussian noise driven by fractional Brownian motion. In particular, the system is formulated with time-delay, nonlinear actuator faults and randomly occurring parameter uncertainties. Primarily, a fractional-infinitesimal operator is incorporated to deal with the fractional Ito stochastic systems in the derivation part of Lyapunov-based stability analysis. Further, the considered system is subjected to both linear and nonlinear actuator faults and the stabilization will be achieved by the consideration of a nonlinear resilient fault-tolerant proportional-retarded controller. By incorporating the fractional-infinitesimal operator and with the choice of a relevant Lyapunov-Krasovskii functional candidate, a new adequate criterion is deduced by means of linear matrix inequalities. Then the established inequalities are then solved for obtaining the controller gain matrices. Thereafter, an example illustrating the effectiveness and applicability of the proposed results is provided.

INDEX TERMS Singular system, fractional Brownian motion, time-delay, proportional retarded controller, nonlinear actuator faults, randomly occurring parameter uncertainties.

I. INTRODUCTION

During the past several decades, singular systems, also known as descriptor systems are widely implemented when compared with regular systems because of their generalized nature and vast applicability in real processes [5]–[7]. These systems are fundamentally different from regular systems because of their infinite dynamic modes and impulse-free characteristics. Especially, they received much attention owing to their application in many practical models, for instance, chemical systems, electric power systems, aircraft control systems, robotic manipulator systems, economic mineral industries, mechanic engineering systems and telecommunication systems, etc. [1]–[4]. The nature of many practical systems is also speculated by stochastic processes thus it plays an important role in the various real process and system modelling. By taking advantage of both the singular and stochastic nature, a singular system with stochastic noise need to be taken into consideration.

Above all, the dynamical systems may be affected by various phenomena such as stochastic processes namely Brownian motion. The classical Brownian motion can be described by a stochastic process namely Wiener process which is continuous-time real-valued together with independent increments and the mean value is considered to be zero. Therefore, it is widely implemented in the modelling of the stochastic perturbation occurring in dynamical control systems. Recently, much attention has been received by fractional Brownian motion (fBm), which is an extension of the classical Brownian motion because of its long-range memory hierarchy and its broad application in many fields such as economics, hydrology, biology, telecommunication, network traffic, financial mathematics, wind speed and so on [8]. A fBm is a continuous-time Gaussian process represented by $B^H = \{B_t^H, t \geq 0\}$ with Hurst parameter $H \in (0, 1)$ and it has zero mean. Herein, the value of this Hurst exponent

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determines the classical and fractional sense of Brownian motion. If the Hurst parameter value $H = \frac{1}{2}$, the process is pertinent to classical Brownian motion and if the Hurst parameter value H is greater or less than $\frac{1}{2}$, then the process is fBm [9]. Obviously, this parameter servers the purpose of carrying long-term memory of time series and its increments are positive or negatively correlated [10]. The introduction of fBm in the stability analysis and control synthesis of dynamical systems poses a difficulty that cannot be solved by the traditional stochastic calculus involving the conventional Ito's formula. On this account, a fractional infinitesimal operator is adopted from [9] which could be used for the derivation analysis of the fractional stochastic process. By virtue of this aspect, the problem of stabilization involving fBm has been demanding attention among the researchers and the corresponding results have been addressed in a few of the works [8]–[11].

Further, the nature of the dynamical systems are affected by delay in time which persistently happen everywhere in real-time processes for instance in chemical processes, communication systems, population dynamics models and biological systems and so on. These time-delay cannot be ignored and it is unavoidable in real-time since it induces the source of instability, oscillation and ultimately the poor performance of the system [12]–[14]. In this regard, it is of prime importance to focus on the stabilization criteria for time-delay systems. On the other side, the time delay can be implemented in the control design for better performance. To be precise, the delay-based controllers rather in turn utilizes the delay term with the updated dynamical characteristics of the signal [15], [16]. In particular, a proportional-retarded controller is one such type of delay-based controller that avoids the use of time differencing action. Consequently, it helps in fast dynamics and also mitigates the noise amplification thus facilitating a better performance rather than conventional ones. The major benefit of this controller is that it is easy to implement because it has only a proportional term and its corresponding retarded delay term. To this end, some noteworthy works have been reported in the literature [17]–[20]. Specifically, an analytical tuning technique for the second-order systems involving the proportional retarded controller is experimentally verified in [17].

In practice, the role of the actuators seems to be crucial throughout the control processes but there is a possibility of the occurrence of frequent temporary flops leading to the unsuccessful dispatch of information in the control signals. Moreover, the failure may not always be linear in nature. With regard to this circumstance, the performance of the control system will be affected and hence it will be very complex to preserve it by utilizing the classical control scheme. For this purpose, a nonlinear reliable control can be implemented from a protection viewpoint. These nonlinear reliable fault controllers have the ability to endure the actuator failures without affecting the performance of the desired system [21]–[23]. Thus, the authors in [24] designed a reliable controller for singular Markov jump systems under nonlinear actuator fault to guarantee the system's stochastic stability with $L_2 - L_\infty$ performance level. From this perspective, a nonlinear fault-tolerant proportional retarded controller is designed such that the system can be stabilized irrespective of the existence of nonlinear actuator faults. Moreover, the randomly occurring parameter uncertainty is another major complexity in the system model [25]–[31], which occurs in a probabilistic way due to the random changes in environmental circumstances.

It should be mentioned that the control systems involving fBm have been attracting researchers because of their long-memory property in contrast with Brownian noises. Henceforth, this work focuses mainly on the problem of stability of the singular system with fractional Brownian noises. On the whole, a class of singular systems driven by fractional Gaussian noise along with time-delay, randomly occurring uncertainty and nonlinear actuator faults is formulated. Moreover, the highlights of this article are articulated as

- Predominantly, a framework of a singular system governed by fBm is set up with time-delay, randomly occurring uncertainties and nonlinear actuator faults.
- Successively, a fractional infinitesimal operator has been incorporated to deal with the stabilization analysis of the fractional Ito process present in the given system.
- The impact of the occurrence of both linear and nonlinear actuator faults are counterbalanced by introducing a proportional-retarded controller. Further, the controller is designed in such a way that it is reliable to faults.
- A suitable Lyapunov-Krasovskii functional has been imposed such that it aids in the subsequent Lyapunov-based derivation analysis for the considered system lead by fBm.
- Then, the derived criteria ascertains that the consider system is regular, impulse free and stochastically stable which are then validated by a numerical example.

Notations: Throughout this paper, $\mathbb{R}^n$ denotes the n dimensional Euclidean space; I denotes the identity matrix with compatible dimensions. All the matrices are assumed to have compatible dimensions if not explicitly stated; $\mathbb{E}[\cdot]$ stands for the mathematical expectation operator; $C^{2,1}(S_h \times \mathbb{R}_+; \mathbb{R}_+)$ denote the family of all continuous functions from $S_h \times \mathbb{R}_+$ to $\mathbb{R}_+$ with first and second continuous partial derivatives with respect to x and first partial derivative with respect to t. Further, we use an asterisk $(*)$ to represent a term that is induced by symmetry.

II. PROBLEM FORMULATION AND PRELIMINARIES

In this work, we deal with the dynamical systems encountered by fractional Gaussian noise (fGn) that is an incremental process of fBm. These noises are non-Markovian stochastic processes that are widely present in many real systems. Let us consider a class of singular systems driven by fractional stochastic noise along with time-delay, randomly occurring uncertainties, nonlinear actuator faults in the following form:

$$E\dot{x}(t) = \bar{A}x(t) + \bar{A}_\tau x(t-\tau) + Bu^f(t) + Cx(t)W^H(t), \quad (1)$$

where $\mathsf{x}(t) \in \mathbb{R}^n$ and $u^f(t) \in \mathbb{R}^m$ represents the trajectories of the system state and control input which comprises of nonlinear actuator faults, respectively; H is the Hurst parameter corresponding to the fGn $W^H(t)$; τ is the constant time delay. The matrix $E \in \mathbb{R}^{n\times n}$ is a singular matrix with the assumption that $rank(E) = p < n$. The matrices $\bar{A}$ and $\bar{A}_\tau$ stands for $\bar{A} = A + \delta_1(t)\Delta A(t)$ and $\bar{A}_\tau = A_\tau + \delta_2(t)\Delta A_\tau(t)$ respectively, wherein the time dependent functions $\Delta A(t)$ and $\Delta A_\tau(t)$ denote the uncertainties satisfying the following conditions: $\left[\Delta A(t)\ \Delta A_\tau(t)\right] = M\zeta(t)\left[N_1\ N_2\right]$, here the matrices M, N_1 and N_2 are suitably dimensioned constant matrices and $\zeta(t)$ is the unknown time-varying matrix function satisfying $\zeta^T(t)\zeta(t) \leq I$. The terms A, A_τ, B and C represents the appropriately dimensioned matrices of the system states. The randomly occurring uncertainties are represented by the variables $\delta_1(t)$ and $\delta_2(t)$ which are of stochastic nature obeying the Bernoulli distribution as follows

$$\begin{aligned}
&Prob\{\delta_1(t) = 1\} = \mathbb{E}\{\delta_1(t)\} = \delta_1,\\
&Prob\{\delta_1(t) = 0\} = 1 - \mathbb{E}\{\delta_1(t)\} = 1 - \delta_1,\\
&Prob\{\delta_2(t) = 1\} = \mathbb{E}\{\delta_2(t)\} = \delta_2,\\
&Prob\{\delta_2(t) = 0\} = 1 - \mathbb{E}\{\delta_2(t)\} = 1 - \delta_2,
\end{aligned}$$

where the constants δ_1 and $\delta_2 \in [0, 1]$ are known that describe the randomness in the system modeling.

In practice, the actuators may be prone to changes which may affect the system performance. The abrupt changes in the actuators are modelled as faults which may not always be linear. Owing to this, it is significant to consider the nonlinear effect in the actuator fault model. In general, the nonlinearities in the fault model can be tolerated by the resilient control input signal $u^f(t)$ expressed as follows:

$$u^f(t) = \mathsf{G}u(t) + \Lambda(u(t)), \tag{2}$$

where G and $\Lambda(.)$ represents the linear actuator fault matrix and nonlinear vector-valued function. The control signal $u(t)$ is chosen as a proportional retarded controller given in the following form:

$$u(t) = K_p\mathsf{x}(t) + K_r\mathsf{x}(t-h), \tag{3}$$

where K_p and K_r are respectively the proportional and retarded gains. Also, the retarded term is represented by h which is assumed to be a constant. The matrix G denoting the fault is specified by $\mathsf{G} = diag\{\mathsf{g}_1, \mathsf{g}_2, \ldots, \mathsf{g}_m\}$, $\mathsf{g}_i \in [\underline{\mathsf{g}}_i, \bar{\mathsf{g}}_i]$ $0 \leq \underline{\mathsf{g}}_i \leq \mathsf{g}_i \leq \bar{\mathsf{g}}_i \leq 1$, $i = 1, 2, \ldots, m$. Let $\underline{\mathsf{G}} = diag\{\underline{\mathsf{g}}_1, \underline{\mathsf{g}}_2, \ldots, \underline{\mathsf{g}}_m\}$, $\mathsf{G} = diag\{\mathsf{g}_1, \mathsf{g}_2, \ldots, \mathsf{g}_m\}$ and $\bar{\mathsf{G}} = diag\{\bar{\mathsf{g}}_1, \bar{\mathsf{g}}_2, \ldots, \bar{\mathsf{g}}_m\}$. Here, the variables g_i, $i = 1, \ldots, m$ indicates the actuator failures. Now, considering $\mathsf{G}_0 = \frac{\bar{\mathsf{G}}+\underline{\mathsf{G}}}{2}$ and $\mathsf{G}_1 = \frac{\bar{\mathsf{G}}-\underline{\mathsf{G}}}{2}$, the matrix G can be signified as $\mathsf{G} = \mathsf{G}_0 + \mathsf{G}_1\mathfrak{E}$ where $\mathfrak{E} = diag\{e_1, e_2, \ldots, e_m\} \in \mathbb{R}^{m\times m}$, $-1 \leq e_i \leq 1$, $i = 1, 2, \ldots, m$. Let $\Lambda(u(t)) = \left[\Lambda_1(u(t)), \Lambda_2(u(t)), \ldots, \Lambda_m(u(t))\right]^T$ satisfying $|\Lambda_i(u(t))| \leq \sqrt{\nu_i}|u(t)|$, $i = 1, 2, \ldots, m$, $\nu > 0$ and hence

$$\Lambda^T(u(t))\Lambda(u(t)) \leq u^T(t)G_2u(t), \tag{4}$$

where $G_2 = \text{diag}\{\nu_1, \nu_2, \ldots \nu_m\}$. Subsequently, the fractional Ito form of the system (1) can be written as

$$Ed\mathsf{x}(t) = [\bar{A}\mathsf{x}(t) + \bar{A}_\tau\mathsf{x}(t-\tau) + Bu^f(t)]dt + C\mathsf{x}(t)dB^H(t), \tag{5}$$

where H indicates the Hurst parameter corresponding to the fBm $B^H(t)$. Substituting (2) and (3) in (5) and the subsequent closed-loop system can be obtained in the following form:

$$Ed\mathsf{x}(t) = f(t)dt + C\mathsf{x}(t)dB^H(t), \tag{6}$$

where $f(t) = (\bar{A} + B\mathsf{G}K_p)\mathsf{x}(t) + \bar{A}_\tau\mathsf{x}(t-\tau) + B\mathsf{G}K_r\mathsf{x}(t-h) + B\Lambda(u(t)))$.

III. FRACTIONAL INFINITESIMAL OPERATOR

The primary motive of this work is to obtain a control law so that it will stabilize the resultant closed-loop system (6). Owing to which, in this section, a fractional infinitesimal operator is employed and its usefulness on the derivation part of the stabilization process for the fractional-order stochastic systems is discussed. Accordingly, we consider the following fractional stochastic system $d\mathsf{x}(t) = u(t, \mathsf{x}(t))dt + v(t, \mathsf{x}(t))dB^H(t)$, where u and v are chosen to be real-valued functions and H denotes the Hurst exponent with the value ranging from (0.5, 1) corresponding to the fBm $B^H(t)$. The differential of Lyapunov candidate $\mathcal{V}(\mathsf{x}(t), t)$ will be obtained by the fractional Ito's formula as follows:

$$\begin{aligned}
d\mathcal{V}(\mathsf{x}(t), t) = &\left[\frac{\partial\mathcal{V}(\mathsf{x}(t), t)}{\partial t} + \frac{\partial\mathcal{V}(\mathsf{x}(t), t)}{\partial \mathsf{x}}u(t, \mathsf{x}(t))\right]dt\\
&+ \left[v^T(t, \mathsf{x}(t))\frac{\partial^2\mathcal{V}(\mathsf{x}(t), t)}{\partial \mathsf{x}^2}D_r^\Theta\mathsf{x}(t)\right]dt\\
&+ \frac{\partial\mathcal{V}(\mathsf{x}(t), t)}{\partial \mathsf{x}}v(t, \mathsf{x}(t))dB^H(t),
\end{aligned}$$

where $D_r^\Theta\mathsf{x}(t)$ signifies the Malliavin derivative of $\mathsf{x}(t)$ and $\Theta(t, s)$ is a function signified by $\Theta(t, s) = H(2H-1)|s-t|^{2H-2,}$. If the linear functions $u(t, \mathsf{x}(t))$ and $v(t, \mathsf{x}(t))$ fulfils the conditions $u(t, \mathsf{x}(t)) = O(t)\mathsf{x}(t)$ and $v(t, \mathsf{x}(t)) = U(t)\mathsf{x}(t)$, then it can be directly seen from [11] that the Malliavin derivative is demonstrated as $D_r^\Theta\mathsf{x}(t) = \mathsf{x}(t)\int_0^t\Theta(t, s)U(s)ds$. Now, let us consider the following linear fractional stochastic differential equation

$$d\mathsf{x}(t) = O(t)\mathsf{x}(t)dt + U(t)\mathsf{x}(t)dB^H(t). \tag{7}$$

Further, the term L^H denotes the fractional infinitesimal operator with Hurst parameter ranging from (0.5,1) is given by:

$$\begin{aligned}
L^H = &\frac{\partial}{\partial t} + O(t)\mathsf{x}(t)\frac{\partial}{\partial \mathsf{x}}\\
&+ \mathsf{x}^T(t)U^T(t)\frac{\partial^2}{\partial \mathsf{x}^2}\mathsf{x}(t)\int_0^t\Theta(s, t)U(s)ds.
\end{aligned} \tag{8}$$

This operator is specified to tackle the effect of stochastic process driven by fBm in the place of classical Brownian motion. Consequently, it is straightforward to imply this as the altered form of the standard infinitesimal operator.

Now, we provide some of the basic definitions and lemmas which play a vital role in the derivation of the main results. The subsequent lemma aids in the stabilization analysis for the linear stochastic systems guided by fBm mentioned in (7).

Lemma 1 [9]: Let $\mathcal{L}(0,t)$ be a family of stochastic processes defined in Theorem 3.7 in [32]. In general, let us assume a stochastic system with the fBm mentioned as in equation (7). Then the solution of equation (7) is said to be stochastically stable if

$$L^H\mathcal{V}(\mathsf{x},t) \leq 0, \tag{9}$$

if there exists a function $\mathcal{V}(\mathsf{x},t) \in C^{2,1}$ such that it satisfies the following conditions

$$\mathcal{V}(0,t) = 0, \quad \mathcal{V}(\mathsf{x},t) \geq \kappa(|\mathsf{x}|), \tag{10}$$

$$\mathcal{V}_{\mathsf{x}}(\mathsf{x},t)C(t)\mathsf{x}(t) \in \mathcal{L}(0,t), \quad \forall(\mathsf{x},t) \in S_h \times \mathbb{R}_+. \tag{11}$$

Moreover, when the value of the Hurst parameter is 0.5 and the inequality $L\mathcal{V}(\mathsf{x},t) \leq 0$ is satisfied, then the trivial solution of (7) is stochastically stable, where L is the conventional infinitesimal operator.

IV. MAIN RESULTS

In this section, the stochastic stability conditions are derived with the aid of fractional infinitesimal operator and suitable Lyapunov-Krasovskii functional candidate. To be precise, the derived results are summarized in the subsequent theorems. Mainly, Theorem 1 deals with the derivation of some adequate conditions to guarantee the stochastic stability by considering the proportional-retarded controller gain matrices to be known. Next, in Theorem 2, the controller gain matrices are chosen to be unknown and subsequent results are established for guaranteeing the stochastic stability.

Theorem 1: For given positive scalars $h, \tau, \gamma, \omega, \rho$, scalars $\delta_1, \delta_2 \in [0,1]$, known actuator fault matrix G and controller gain matrices K_p and K_r, the closed-loop delayed stochastic fractional system (6) is stochastically stable, if there exist symmetric positive definite matrices $\mathcal{P}, \mathcal{Q}, \mathcal{R}$, appropriately dimensioned matrices J, H, L, G_2 and scalars ϵ_1 and ϵ_2, such that the following LMI holds:

$$[\widehat{\Psi}]_{10\times 10} < 0, \tag{12}$$

where, $\widehat{\Psi}_{(1,1)} = \mathcal{Q} + \mathcal{R} + J^TA + A^TJ + J^TB\mathsf{G}_0K_p + K_p^T\mathsf{G}_0^TB^TJ$, $\widehat{\Psi}_{(1,2)} = E^T\mathcal{P} + \gamma A^TJ + \gamma K_p^T\mathsf{G}_0^TB^TJ - J^T$, $\widehat{\Psi}_{(1,3)} = J^TA_\tau$, $\widehat{\Psi}_{(1,4)} = \omega A^TJ + \omega K_p^T\mathsf{G}_0^TB^TJ + J^TB\mathsf{G}_0K_r$, $\widehat{\Psi}_{(1,5)} = J^TB$, $\widehat{\Psi}_{(1,6)} = \sqrt{\rho}K_P^T$, $\widehat{\Psi}_{(1,7)} = \epsilon_1J^TM$, $\widehat{\Psi}_{(1,8)} = \delta_1N_1^T$, $\widehat{\Psi}_{(1,9)} = \epsilon_2J^TB\mathsf{G}_1$, $\widehat{\Psi}_{(1,10)} = K_p^T$, $\widehat{\Psi}_{(2,2)} = -\gamma J^T - \gamma J$, $\widehat{\Psi}_{(2,3)} = \gamma J^TA_\tau$, $\widehat{\Psi}_{(2,4)} = \gamma J^TB\mathsf{G}_0K_r - \omega J$, $\widehat{\Psi}_{(2,5)} = \gamma J^TB$, $\widehat{\Psi}_{(2,7)} = \epsilon_1\gamma J^TM$, $\widehat{\Psi}_{(2,9)} = \epsilon_2\gamma J^TB\mathsf{G}_1$, $\widehat{\Psi}_{(3,3)} = -\mathcal{Q}$, $\widehat{\Psi}_{(3,4)} = \omega A_\tau^TJ$, $\widehat{\Psi}_{(3,8)} = \delta_2N_2^T$, $\widehat{\Psi}_{(4,4)} = -\mathcal{R} + \omega J^TB\mathsf{G}_0K_r + \omega K_r^T\mathsf{G}_0^TB^TJ$, $\widehat{\Psi}_{(4,5)} = \omega J^TB$, $\widehat{\Psi}_{(4,6)} = \sqrt{\rho}K_r^T$, $\widehat{\Psi}_{(4,7)} = \epsilon_1\omega J^TM$, $\widehat{\Psi}_{(4,9)} = \epsilon_2\omega J^TB\mathsf{G}_1$, $\widehat{\Psi}_{(4,10)} = K_r^T$, $\widehat{\Psi}_{(5,5)} = -\rho$, $\widehat{\Psi}_{(6,6)} = -G_2^{-1}$, $\widehat{\Psi}_{(7,7)} = \widehat{\Psi}_{(8,8)} = -\epsilon_1$, $\widehat{\Psi}_{(9,9)} = \widehat{\Psi}_{(10,10)} = -\epsilon_2$.

Proof: Primarily, to prove the stochastic stability for the nominal system (6), the regular and impulse free nature of the considered system (6) is established. With regard to the condition in (12), it follows that

$$\begin{bmatrix} \widehat{\Upsilon}_{(1,1)} & \widehat{\Upsilon}_{(1,2)} \\ * & \widehat{\Upsilon}_{(2,2)} \end{bmatrix} < 0, \tag{13}$$

where $\widehat{\Upsilon}_{(1,1)} = A^TJ + J^TA$, $\widehat{\Upsilon}_{(1,2)} = E^T\mathcal{P}$, $\widehat{\Upsilon}_{(2,2)} = -\gamma(J + J^T)$. Let $S = \begin{bmatrix} I & A^T \\ 0 & A_\tau^T \end{bmatrix}$ and pre-multiplying and post-multiplying (13) by S and S^T respectively, we get $\Phi = \begin{bmatrix} \Phi_1 & \Phi_2 \\ * & \Phi_3 \end{bmatrix}$, where $\Phi_1 = \widehat{\Upsilon}_{(1,1)} + A^T\widehat{\Upsilon}_{(1,2)}^T + \widehat{\Upsilon}_{(1,2)}A + A^T\widehat{\Upsilon}_{(2,2)}A$, $\Phi_2 = \widehat{\Upsilon}_{(1,2)}A_\tau + A^T\widehat{\Upsilon}_{(2,2)}A_\tau$, $\Phi_3 = A_\tau^T\widehat{\Upsilon}_{(2,2)}A_\tau$. It can be seen that $rank(E) = p < n$, hence it is easy to choose the invertible matrices W and $K \in \mathbb{R}^{n\times n}$ in order that

$$\tilde{E} = WEK = \begin{bmatrix} I_p & 0 \\ 0 & 0 \end{bmatrix}, \tag{14}$$

Next, the matrix $\mathfrak{U}$ can be parametrized by considering a non-singular matrix $\varphi \in \mathbb{R}^{(n-p)\times(n-p)}$ in a way that $\mathfrak{U} = W^T\begin{bmatrix} 0 \\ \varphi \end{bmatrix}$. Now, we define $\tilde{A} = WAK = \begin{bmatrix} A_{11} & A_{12} \\ A_{13} & A_{14} \end{bmatrix}$, $\tilde{\mathcal{P}} = W^{-T}\mathcal{P}W^{-1} = \begin{bmatrix} \mathcal{P}_{11} & \mathcal{P}_{12} \\ \mathcal{P}_{13} & \mathcal{P}_{14} \end{bmatrix}$ and $\tilde{F} = K^TF = \begin{bmatrix} F_{11} \\ F_{12} \end{bmatrix}$ analogous to (14). Then, by applying congruence transformation on both sides of Φ by K^T and K, we obtain $\hat{\Phi} = \begin{bmatrix} \hat{\Phi}_{11} & \hat{\Phi}_{12} \\ \hat{\Phi}_{21} & A_{14}^T\Phi F_{12}^T + F_{12}\varphi^TA_{14} \end{bmatrix}$. Then, from (6), it is easy to see that $A_{14}^T\Phi F_{12}^T + F_{12}\varphi^TA_{14} < 0$ which directly implies that A_{14} is non-singular. Thus, by Definition 2.1 in [2], the nominal system (6) is regular and impulse free.

Further, a Lyapunov-Krasovskii functional candidate is chosen to prove the stochastically stability of nominal system (6) which is presented as follows:

$$\mathcal{V}(\mathsf{x}(t),t) = [\mathsf{x}^T(t)E^T\mathcal{P}\mathsf{x}(t) + \int_{t-\tau}^{t}\mathsf{x}^T(s)\mathcal{Q}\mathsf{x}(s)ds + \int_{t-h}^{t}\mathsf{x}^T(s)\mathcal{R}\mathsf{x}(s)ds]e^{-\lambda\int_0^t\int_0^s\Theta(s,\tau)d\tau ds}, \tag{15}$$

which satisfies the conditions (10) and (11) in Lemma 1. Then, by using the fractional infinitesimal operator (8), we find that

$$\begin{aligned}
&\mathbb{E}\{L^H\mathcal{V}(\mathsf{x}(t),t)\} \\
&\quad = \mathbb{E}\{[\mathsf{x}^T(t)(\mathcal{Q}+\mathcal{R})\mathsf{x}(t) \\
&\qquad - \mathsf{x}^T(t-\tau)\mathcal{Q}\mathsf{x}(t-\tau) \\
&\qquad - \mathsf{x}^T(t-h)\mathcal{R}\mathsf{x}(t-h)]e^{-\lambda\int_0^t\int_0^s\Theta(s,\tau)d\tau ds} \\
&\qquad + 2\mathsf{x}^T(t)E^T\mathcal{P}f(t)e^{-\lambda\int_0^t\int_0^s\Theta(s,\tau)d\tau ds} \\
&\qquad + [2\mathsf{x}^T(t)C^TE^T\mathcal{P}C\mathsf{x}(t) - \lambda\mathsf{x}^T(t)E^T\mathcal{P}\mathsf{x}(t) \\
&\qquad - \lambda\int_{t-\tau}^{t}\mathsf{x}^T(s)\mathcal{Q}\mathsf{x}(s)ds \\
&\qquad - \lambda\int_{t-h}^{t}\mathsf{x}^T(s)\mathcal{R}\mathsf{x}(s)ds] \\
&\qquad \times \int_0^t\Theta(t,\tau)d\tau e^{-\lambda\int_0^t\int_0^s\Theta(s,\tau)d\tau ds}\}.
\end{aligned} \tag{16}$$

By choosing λ such that $\lambda \leq \lambda_{max}(2C^TE^T\mathcal{P}C)/\lambda_{min}(E^T\mathcal{P})$, then from Weyl's inequality, it follows that $2C^TE^T\mathcal{P}C - \lambda E^T\mathcal{P} \leq 0$. In accordance with this, the last term in (16) will vanish, then the equation (16) can be written as

$$\begin{aligned}\mathbb{E}\{L^H\mathcal{V}(\mathsf{x}(t),t)\} \leq \mathbb{E}\{[&\mathsf{x}^T(t)(\mathcal{Q}+\mathcal{R})\mathsf{x}(t)\\ &-\mathsf{x}^T(t-\tau)\mathcal{Q}\mathsf{x}(t-\tau)\\ &-\mathsf{x}^T(t-h)\mathcal{R}\mathsf{x}(t-h)+2\mathsf{x}^T(t)E^T\mathcal{P}f(t)]\\ &\times e^{-\lambda\int_0^t\int_0^s\Theta(s,\tau)d\tau ds}\}. \end{aligned} \quad (17)$$

However, by utilizing the condition in (4) for given positive scalar ρ, it is easy to obtain

$$\begin{aligned}\rho\{[K_p\mathsf{x}(t)+K_r\mathsf{x}(t-h)]^TG_2[K_p\mathsf{x}(t)+K_r\mathsf{x}(t-h)]\\ -\Lambda^T(u(t))\Lambda(u(t))\}e^{-\lambda\int_0^t\int_0^s\Theta(s,\tau)d\tau ds} \geq 0. \end{aligned}\quad (18)$$

For any matrices J, H, L of appropriate dimension with $H=\omega J$ and $L=\gamma J$, the following equality holds

$$\begin{aligned}\mathbb{E}\{2[\mathsf{x}^T(t)J^T+\mathsf{x}^T(t-h)H^T+f^T(t)L^T][-f(t)\\ +(\bar{A}+B\mathsf{G}K_p)\mathsf{x}(t)+\bar{A}_\tau\mathsf{x}(t-\tau)+B\mathsf{G}K_r\mathsf{x}(t-h)\\ +B\Lambda(u(t))]e^{-\lambda\int_0^t\int_0^s\Theta(s,\tau)d\tau ds}\}=0. \end{aligned}\quad (19)$$

Combining the equations from (17) to (19), we get

$$\mathbb{E}\{L^H\mathcal{V}(\mathsf{x}(t),t)\} \leq \mathbb{E}\{\eta^T(t)\Omega\eta(t)e^{-\lambda\int_0^t\int_0^s\Theta(s,\tau)d\tau ds}\}, \quad (20)$$

where $\eta^T(t) = \left[\mathsf{x}^T(t)f^T(t)\mathsf{x}^T(t-\tau)\mathsf{x}^T(t-h)\Lambda^T(u(t))\right]$ and the elements of the matrix $[\Omega]_{5\times 5}$ is given as follows: $\Omega_{(1,1)} = \mathcal{Q}+\mathcal{R}+\rho K_p^T\mathsf{G}_2K_p + J^T(\bar{A}+B\mathsf{G}K_p) + (\bar{A}^T+K_p^T\mathsf{G}^TB^T)J$, $\Omega_{(1,2)} = E^T\mathcal{P}+(\bar{A}^T+K_p^T\mathsf{G}^TB^T)\gamma J - J^T$, $\Omega_{(1,3)} = J^T\bar{A}_\tau$, $\Omega_{(1,4)} = (\bar{A}^T+K_p^T\mathsf{G}^TB^T)J\omega + J^TB\mathsf{G}K_r + \rho K_p^TG_2K_r$, $\Omega_{(1,5)} = J^TB$, $\Omega_{(2,2)} = -\gamma J^T - \gamma J$, $\Omega_{(2,3)} = \gamma J^T\bar{A}_\tau$, $\Omega_{(2,4)} = \gamma J^TB\mathsf{G}K_r - \omega J$, $\Omega_{(2,5)} = \gamma J^TB$, $\Omega_{(3,3)} = -\mathcal{Q}$, $\Omega_{(3,4)} = \bar{A}_\tau^TJ\omega$, $\Omega_{(4,4)} = -\mathcal{R} + \omega J^TB\mathsf{G}K_r + K_r^T\mathsf{G}^TB^TJ\omega + \rho K_r^TG_2K_r$, $\Omega_{(4,5)} = \omega J^TB$, $\Omega_{(5,5)} = -\rho$.

Further, by substituting for $\bar{A} = A+\delta_1(t)M\zeta(t)N_1$, $\bar{A}_\tau = A_\tau+\delta_2(t)M\zeta(t)N_2$ and $\mathsf{G} = \mathsf{G}_0+\mathsf{G}_1\mathfrak{E}$ in (20) together with the followup of s-procedure and Schur complement Lemma, it is obvious that

$$\mathbb{E}\{L^H\mathcal{V}(\mathsf{x}(t),t)\} \leq \eta^T(t)\widehat{\Psi}\eta(t)e^{-\lambda\int_0^t\int_0^s\Theta(s,\tau)d\tau ds}, \quad (21)$$

Then from the inequality (12) we obtain $\widehat{\Psi} < 0$, which is then applied to (21) for obtaining the condition $\mathbb{E}\{L^H\mathcal{V}(\mathsf{x}(t),t)\} \leq 0$. Thus, with the aid of Lemma 1, the stochastic stability of the closed-loop fractional Ito form of the system (6) is ensured.

In the sequel theorem, the proportional-retarded controller is considered as in (3) that will establish the stochastic stability of the system (6).

Theorem 2: Given positive scalars $h, \tau, \gamma, \omega, \rho$, scalars $\delta_1, \delta_2 \in [0,1]$ and the actuator fault matrix G is known, the delayed fractional stochastic system (6) is stochastically stabilized via the controller (2), if there exist symmetric positive definite matrices $\mathcal{P}, \mathcal{Q}, \mathcal{R}$, matrices $J, H, L, G_2, Z, Y_p, Y_r$ of appropriate dimension and scalars ϵ_1, ϵ_2, such that the following LMI holds:

$$[\Xi]_{10\times 10} < 0, \quad (22)$$

where $\Xi_{(1,1)} = \mathcal{Q}+\mathcal{R}+J^TA^T+AJ+Y_p^T\mathsf{G}_0^TB^T+B\mathsf{G}_0Y_p$, $\Xi_{(1,2)} = E\mathcal{P}+\gamma AJ+\gamma B\mathsf{G}_0Y_p - J^T$, $\Xi_{(1,3)} = J^TA_\tau^T$, $\Xi_{(1,4)} = \omega AJ+\omega B\mathsf{G}_0Y_p+Y_r^T\mathsf{G}_0^TB^T$, $\Xi_{(1,5)} = J^TB^T$, $\Xi_{(1,6)} = \sqrt{\rho}Y_p$, $\Xi_{(1,7)} = \epsilon_1 M$, $\Xi_{(1,8)} = \delta_1 J^TN_1^T$, $\Xi_{(1,9)} = \epsilon_2 B\mathsf{G}_1$, $\Xi_{(1,10)} = Y_p^T$, $\Xi_{(2,2)} = -\gamma J^T - \gamma J$, $\Xi_{(2,3)} = \gamma J^TA_\tau^T$, $\Xi_{(2,4)} = \gamma Y_r^T\mathsf{G}_0^TB^T - \omega J$, $\Xi_{(2,5)} = \gamma J^TB^T$, $\Xi_{(2,7)} = \epsilon_1\gamma M$, $\Xi_{(2,9)} = \epsilon_2\gamma B\mathsf{G}_1$, $\Xi_{(3,3)} = -\mathcal{Q}$, $\Xi_{(3,4)} = \omega A_\tau J$, $\Xi_{(3,8)} = \delta_2 J^TN_2^T$, $\Xi_{(4,4)} = -\mathcal{R}+\omega Y_r^T\mathsf{G}_0^TB^T+\omega B\mathsf{G}_0Y_r$, $\Xi_{(4,5)} = \omega J^TB^T$, $\Xi_{(4,6)} = \sqrt{\rho}Y_r$, $\Xi_{(4,7)} = \epsilon_1\omega M$, $\Xi_{(4,9)} = \epsilon_2\omega B\mathsf{G}_1$, $\Xi_{(4,10)} = Y_r^T$, $\Xi_{(5,5)} = -\rho$, $\Xi_{(6,6)} = -Z$, $\Xi_{(7,7)} = \Xi_{(8,8)} = -\epsilon_1$, $\Xi_{(9,9)} = \Xi_{(10,10)} = -\epsilon_2$. *Moreover, the controller gain can be obtained by $K_p = Y_pJ^{-1}$ and $K_r = Y_rJ^{-1}$.*

Proof: Since $det(sE-(\bar{A}+B\mathsf{G}K_p)) = det(sE^T-(\bar{A}+B\mathsf{G}K_p)^T)$, the pairs $(E,(\bar{A}+B\mathsf{G}K_p))$ are regular and impulse free. Furthermore, since the solution of $det(sE-(\bar{A}+B\mathsf{G}K_p)-e^{-\tau s}\bar{A}_\tau - e^{-hs}B\mathsf{G}K_r) = 0$ is the same as $det(sE^T-(\bar{A}+B\mathsf{G}K_p)^T-e^{-\tau s}\bar{A}_\tau^T-e^{-hs}(B\mathsf{G}K_r)^T) = 0$. Subsequently, following the procedure in [2], we replace $E, (\bar{A}+B\mathsf{G}K_p), \bar{A}_\tau, B\mathsf{G}K_r$ with $E^T, (\bar{A}+B\mathsf{G}K_p)^T, \bar{A}_\tau^T, K_r^T\mathsf{G}^TB^T$ respectively in (12). Further, applying congruence transformation to the resultant LMI by $diag\{I,I,I,I,J,I,I,I,I\}$ and then by letting $Z = J^TG_2^{-1}J$, $Y_p = K_pJ$ and $Y_r = K_rJ$, we obtain the matrix Ξ in (22). Thus, when the inequality (22) holds, $\mathbb{E}\{L^H\mathcal{V}(\mathsf{x}(t),t)\} \leq 0$ ensures the stochastic stabilization of the system (6) with the controller (2). This completes the proof. □

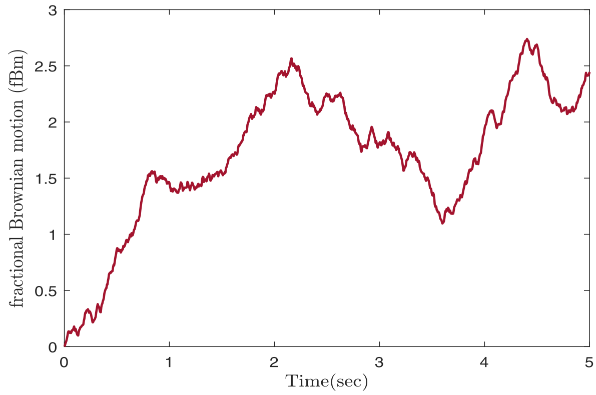
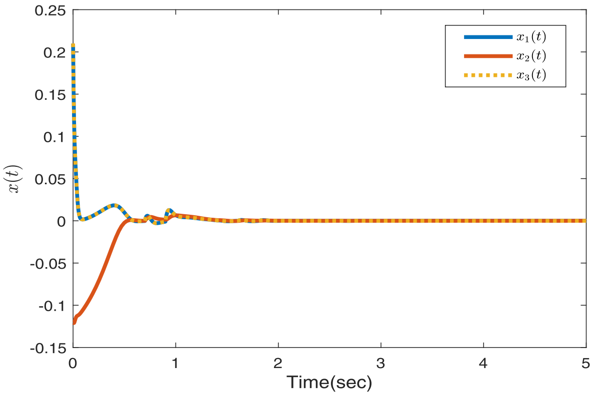


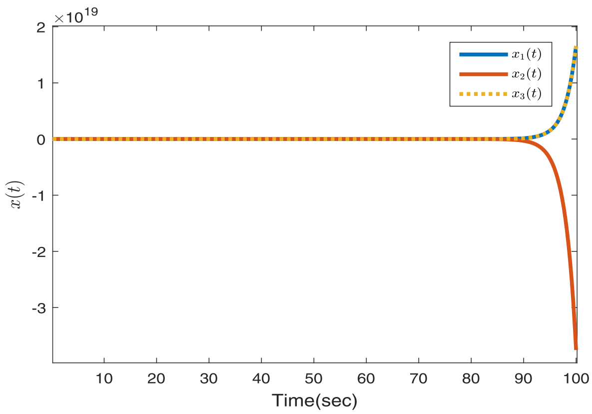
FIGURE 1. Fractional Brownian motion under Hurst parameter H = 0.75.

V. VALIDATION

In this part, we verify the potential and capability of the proposed fault-tolerant proportional-retarded controller design by presenting the graphical representation with an example. Let us consider a singular system with fractional Gaussian

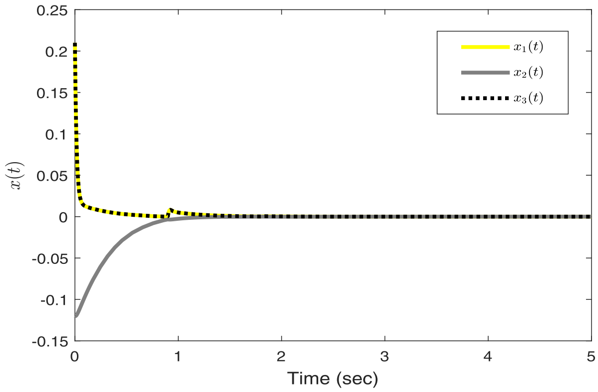


(a) With the controller (2)

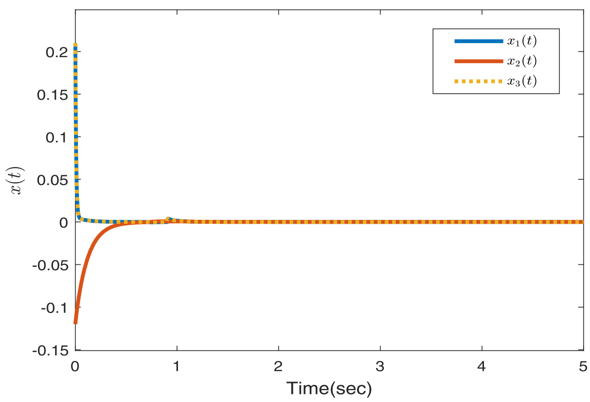


(b) Without the controller $(u(t) = 0)$

FIGURE 2. State responses of system states (a) with the controller and (b) without the control input.



(a) Linear fault $(\Lambda(u(t)) = 0)$



(b) Without fault implementation

FIGURE 3. State response of system (1) (a) under linear fault and (b) in the absence of fault.

noise (1) with the following parameters:

$$E = \begin{bmatrix} 0.9 & 0 & 1 \\ 0 & 0.1 & 0 \\ 0.9 & 0 & 1 \end{bmatrix},$$

$$A = \begin{bmatrix} -4.2 & -0.9 & 0.5 \\ 1 & 0.9 & 1 \\ 1 & 0.7 & -0.8 \end{bmatrix},$$

$$A_\tau = \begin{bmatrix} 1 & -5.1 & 2 \\ -0.3 & 0.1 & -2 \\ 0.5 & 1.5 & 0.2 \end{bmatrix},$$

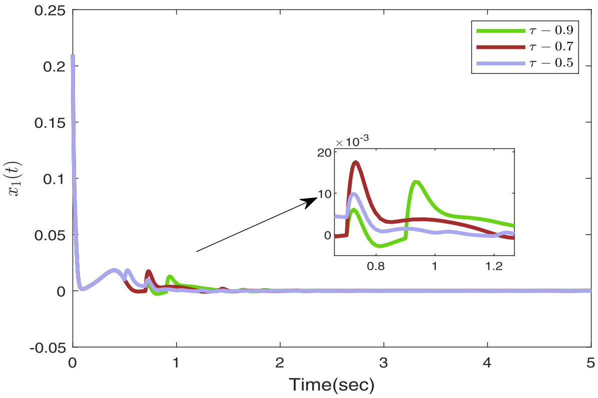
$$B = \begin{bmatrix} -0.36 & -0.44 & 0.1 \\ -1.5 & 0.25 & 0.5 \\ 0.6 & -0.5 & 0.2 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.1 & 0.3 & 0.5 \\ 3 & 0.2 & 0.7 \\ 0.5 & 0.3 & 1.5 \end{bmatrix}.$$

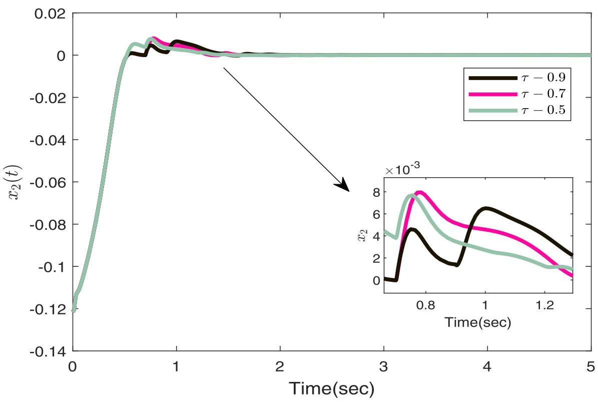
Further, the parameters associated with the system uncertainties are selected as $M = \begin{bmatrix}0.1 \ 0.3 \ 0.2\end{bmatrix}^T$, $\zeta(t) = 0.5 * sin(t)$, $N_1 = \begin{bmatrix}0.1 \ 0.7 \ 0.5\end{bmatrix}$ and $N_2 = \begin{bmatrix}0.3 \ 0.6 \ 0.4\end{bmatrix}$. In addition, the value of Hurst parameter of fractional Brownian motion is chosen as $H = 0.75$. The time-delay in the system and the retarded term in the controller are respectively chosen as $\tau = 0.9$ and $h = 0.7$. The random variables pertaining to uncertainties are taken as $\delta_1 = 0.65$ and $\delta_2 = 0.75$ and the remaining parameters are considered as $\rho = 3, \gamma = 0.071$ and $\omega = 0.091$. Moreover, the values of the parameters corresponding to actuator faults can be chosen as $\mathsf{G}_0 = \text{diag}\{0.25, 0.4, 0.55\}$, $\mathsf{G}_1 = \text{diag}\{0.15, 0.2, 0.25\}$ and $\mathfrak{E} = sin(t) * I_3$. Besides, the function representing the nonlinear nature of the fault is chosen as $\Lambda(u(t)) = 3.2 * sin(u(t))$. Using the above parameter values under the initial conditions $\mathsf{x}(0) = [-0.1; -1.2; 0.3]$ and by means of MATLAB LMI control tool box, we can determine the feasible solution by computing the LMIs in Theorem 2. In this connection, the proportional-retarded gain matrices are fetched as

$$K_p = \begin{bmatrix} 1.2030 & 36.0123 & -42.8978 \\ 18.4438 & -4.6364 & 56.8716 \\ -26.0212 & -57.2330 & -79.9950 \end{bmatrix} \text{ and}$$

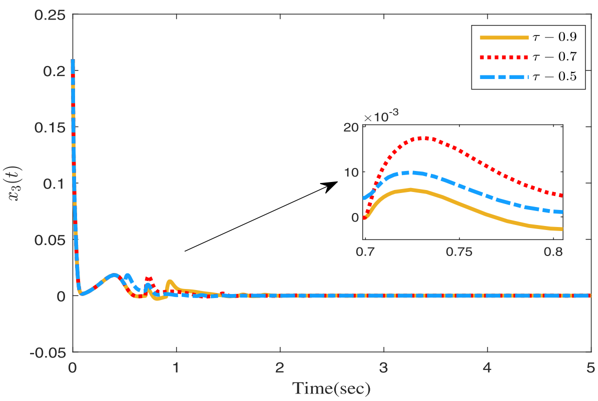
$$K_r = \begin{bmatrix} -0.1536 & 0.693 & -3.1616 \\ -0.2412 & -0.8494 & -2.1588 \\ 0.0522 & -0.6569 & -1.8074 \end{bmatrix}.$$



(a)



(b)



(c)

FIGURE 4. State response for different time-delays. (a) Evolution of $x_1(t)$. (b) Evolution of $x_2(t)$. (c) Evolution of $x_3(t)$.

In accordance with the aforementioned control design parameters, the simulated outcomes are graphically provided in Figs.1-5. Primarily, the simulation results have been carried out by incorporating the fractional Brownian motion signal under the Hurst parameter value $H = 0.75$. Thus, the graphical representation of the fractional Brownian motion is depicted in Fig.1. Moreover, the choice of the value of E clearly shows that it is of $rank = 2$, thus making the

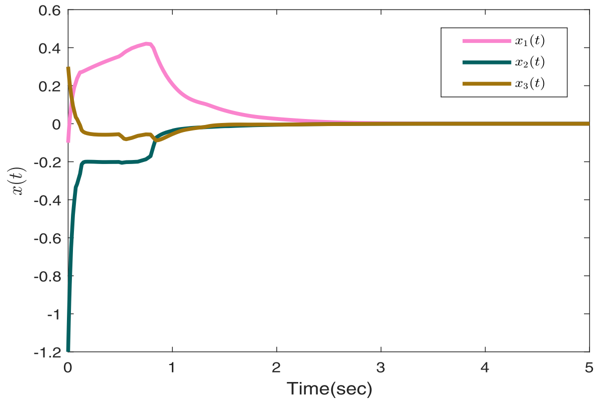


FIGURE 5. State response of the system when $E = I$.

system to be singular. In this accordance, the response of the state trajectories of the considered singular system is demonstrated in Fig.2 (a). From this figure, it can be seen that the trajectories are stabilized regardless of the existence of fractional Gaussian noise, nonlinear actuator faults and randomly occurring uncertainties. On the contrary, in the absence of controller, the considered system seems to diverge and as a result the stability is not attained, which is clearly shown in Fig. 2 (b). Subsequently, to illustrate the reliability of the proposed technique, the responses of the state trajectory of the system (1) under the presence of linear fault is presented in Fig.3 (a) and the system dynamics under the absence of faults is plotted in Fig.3 (b). It can be seen from these figures that although the presence of both linear and nonlinear faults in Fig.2 (a) affects the system performance to a greater extent when compared to the trajectories in Fig.3, the stabilization is achieved by the proposed controller design. From these comparison analysis, it can be concluded that the designed controller can surmount the occurence of faults in both linear and nonlinear form. In addition, the state response of the system (1) for different time-delays are presented in Fig.4. It can be seen that the deterioration of the system dynamics due to the impact of the presence of time-delay is explicitly depicted in this figure. Next, if we consider the system (1) is regular, that is when $E = I$, the conventional proportional-retarded gain matrices can be obtained as $K_p = \begin{bmatrix} 8.6594 & 40.8247 & 4.3562 \\ 4.7328 & -0.1595 & 20.2480 \\ -12.1272 & -29.6964 & -28.0133 \end{bmatrix}$ and $K_r = \begin{bmatrix} 0.1697 & 0.6214 & 0.2911 \\ 0.0688 & -0.0890 & 0.4075 \\ -0.0726 & -0.3546 & -0.0435 \end{bmatrix}$. Under the above gain matrices, simulation results for state trajectories with nonlinear actuator fault is depicted in Fig.5. As a result, we can say that the system can attain its stability even when the system is regular. Hence, from these simulation results, it is worth noting that the uncertain singular system driven by fBm along with time-delay, actuator faults and parametric uncertainties is stabilized by means of the designed fault-tolerant proportional-retarded controller.

VI. CONCLUSION

In this work, we have addressed the uncertain singular systems represented by fractional Ito model under time-delay and actuator faults. In this connection, a nonlinear fault-tolerant proportional retarded controller is designed such that the stabilization is achieved regardless to the existence of both linear and nonlinear faults. Besides, a fractional infinitesimal operator is implemented to study the stability analysis of the addressed system. Further, in accordance with the Lyapunov theory and free weighting matrices, a new set of conditions is derived in the LMI framework for assuring the stochastical stability of the considered system. Later, the obtained results has been verified by presenting a simulation. Moreover, in this direction the neutral singular stochastic differential systems with actuator saturation, input time varying delay and mismatched disturbances will be topic of our future research work.

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