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Multivariate Nonlinear Sparse Mode Decomposition and Its Application in Gear Fault Diagnosis

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ABSTRACT Multi-channel signal has more abundant and accurate state characteristic information than single channel signal. How to separate fault characteristic information from the multi-channel signal is the key of fault diagnosis. As two typical multi-channel signal decomposition methods, multivariate empirical mode decomposition (MEMD) and multivariate variational mode decomposition (MVMD) are widely used in multi-channel signal analysis. However, MEMD and MVMD use cyclic iteration to complete the analysis of multi-channel signals, and it is difficult to overcome their inherent defects. In view of this, based on nonlinear sparse mode decomposition (NSMD), this paper proposes a multivariate nonlinear sparse mode decomposition (MNSMD) by constraining singular local linear operators to separate the natural oscillation modes in multi-channel signal. By constraining singular local linear operators into signal decomposition, MNSMD has obvious advantages in restraining mode aliasing and robustness. In addition, the local narrow-band component is used as the basis function for iteration, and the component signal is obtained by approaching the original signal. Through the simulation signal and gear fault signal analysis, the results show that, compared with MEMD and MVMD methods, MNSMD method can effectively complete gear fault diagnosis.

INDEX TERMS Multivariate nonlinear sparse mode decomposition, singular local linear operator, gear, fault diagnosis.

I. INTRODUCTION

Gear is the most vulnerable part of rotating machinery and equipment, and its state will affect the healthy operation of the entire machinery [1]. Due to the complexity and diversity of rotating machinery system, the vibration signals measured on site inevitably contain a lot of noise, and the fault signals are often submerged in the noisy signal. Therefore, it is necessary to extract gear fault characteristic from the noisy signal [2], [3].

The vibration signals generated in the gear transmission process are usually non-stationary and nonlinear, time-frequency analysis is widely used in gear vibration signal analysis because it can provide local information in both time domain and frequency domain [4], [5]. For example, Yang, *et al.* proposed an adaptive chirp mode decomposition method, which can adaptively decompose a complex original signal into several components, and it realizes the fault diagnosis of circuit-breaker [6]. Cheng, *et al.* proposed an adaptive weighted symplectic geometry decomposition method, which is an effective method to reduce the noise of early gear fault signal [7]. Xun *et al.* proposed a median ensemble empirical mode decomposition (MEEMD) method, and the median operator is used instead of the average operator to reduce the additional mode splitting problem [8]. Mojtaba *et al.* proposed a successive variational mode decomposition method, which has lower computational complexity and is more robust against the initialization compared with VMD [9]. Zhao *et al.* proposed a modified variational mode decomposition method based on envelope nesting and multi-criteria evaluation, which can adaptively decompose a signal into a series of quasi-orthogonal natural

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modes [10]. Unfortunately, although the above methods have a good decomposition effect on single channel signal, it is difficult to obtain fault information in the case of weak fault signal or large noise [11]. With the development of multi-sensor measurement technology, the evaluation of the dynamic relationship within and between multi-channel data series observed by one or more sensors has become an effective data analysis method, which has been paid more and more attention by researchers [12], [13].

Multivariate empirical mode decomposition (MEMD) is a typical multi-channel signal processing method, which can complete multi-channel fusion and separation of information [14], [15]. The principle of MEMD is to separate fast and slow multivariable oscillations, and the local mean value of multiple signals is estimated directly by the uniform projection method in the multi-dimensional space. At present, MEMD and its improved algorithms have been widely used in image processing [16], biological signal processing [17], fault detection [18] and other fields. For example, Suman *et al.* proposed a fault classification method-based brain-computer interface using phase space features in multivariate empirical mode decomposition, which completes the classification of non-motor cognitive task in EEG [19]. Adarsh *et al.* proposed a scale dependent prediction of reference evapotranspiration based on multi-variate empirical mode decomposition, which has achieved a good prediction result [20]. However, MEMD is sensitive to sampling and noise, which have important influence on decomposition performance.

Recently, multivariate variational mode decomposition (MVMD) is proposed as a multi variable extended signal processing method based on variational mode decomposition (VMD) [21]. Under the constraint of the existence of joint frequency components in all signal channels, a multi-signal representation is defined. MVMD has the characteristics of mode alignment and quasi-orthogonal, which is the most promising development direction in multi-signal decomposition. For example, Cao et.al proposed a multi-channel signal denoising method based on MVMD [22], which uses the subspace projection of multivariate variational decomposition to complete the noise reduction of multi-channel signal. Gavas *et al.* proposed a multi-variable extension method based on VMD, which can remove flicker-related eye artifacts without manual intervention automatically. However, like VMD, MVMD also needs to determine the second penalty parameters and the number of decomposition modes in advance [23].

Aiming at the limitation of MEMD and MVMD methods in multi-channel signal analysis, this paper proposes a multivariable nonlinear sparse mode decomposition (MNSMD) based on nonlinear sparse mode decomposition (NSMD). In MNSMD, by constraining singular local linear operators into signal decomposition, MNSMD can adaptively decompose a complex signal into several local narrowband components with physical significance of instantaneous frequency, and has obvious advantages in restraining mode aliasing and robustness [24]. Furthermore, MNSMD is defined by inputting instantaneous frequency information among several channels, which makes MNSMD algorithm robust. Then, MNSMD method is different from the method of cyclic screening decomposition, and the intrinsic oscillation modes contained in multiple input signals are separated by constraining singular local linear operators. Meanwhile, the local narrow-band component is used as the basis function for iteration, and the component signal is obtained by approaching the original signal, which can reduce the complexity of the model and improve the running speed of the algorithm. Through the analysis of simulation signals and experimental signals, MNSMD algorithm can accurately and effectively separate multi-modal coupled signals and has good anti-noise capability.

MNSMD method, like MEMD and MVMD methods, can decompose the complex multi-channel signal into the sum of components adaptively. The main highlights of this paper are as follows:

(1) The singular local linear operator is used in the signal decomposition of MNSMD, which can restrain mode aliasing and enhance robustness.

(2) The component signal is obtained by approaching the original signal in MNSMD, which can reduce the complexity of the model and improve the running speed of the algorithm.

(3) Two experimental datasets, including the gear fault data of gear comprehensive fault test bed and planetary gearbox fault test bed, are applied to fully evaluate the proposed MNSMD method. Experimental results show that the proposed method can successfully achieve fault diagnosis of different gear types.

The rest of this paper is organized as follows: In Section II, the proposed MNSMD method is introduced, and simulation signals are used to verify the decomposition ability of the proposed method. In Section III, MNSMD method is applied to gear experimental data. Finally, we draw conclusions and discuss future work in Section IV.

II. MULTIVARIATE NONLINEAR SPARSE MODE DECOMPOSITION

In this paper, the core idea of MNSMD is to separate the low-frequency oscillation signals coupled with multi-modes into multiple natural modes by constructing a variational optimization problem. Under the constraints of the variational model, the bandwidth corresponding to each mode is iteratively updated.

A. NONLINEAR SPARSE MODE DECOMPOSITION

In NSMD method, the problem of signal decomposition is transformed into the nonlinear constrained optimization problem, a complex non-stationary signal is decomposed into several sparse components adaptively. Meanwhile, each sparse component signal as a basic signal is demodulated to approximate the original component signal. The steps are as follows:

Suppose S is a single channel signal, and its expression is as follows:

$$S = \sum_{i-1}^{k} V_i + U_k \tag{1}$$

where, V is the component signal, U is the residual useless signal and k is the number of components.

By introducing the local singular linear operator N into the signal decomposition, $N(V(t)) = 0$ can be obtained, that is, $V(t)$ is mapped to the null space by the local linear operator N, where $N(V(t)) = U(t)$. Therefore, the expression can be further expressed as:

$$U(t) = \min[\|N(S-U)\|^2 + \lambda\,\|D(U)\|^2] \tag{2}$$

where, D is a diagonal matrix with U as the main diagonal and λ is Lagrange multiplier.

Because N is usually a differential operator, it is difficult to decompose the component signal effectively. Therefore, by introducing the leakage factor, the expression can be changed to

$$U(t) = \min[\|N(V-U)\|^2 + \lambda_1(\|D(U)\|^2 + \gamma\,\|S-U\|^2) + \lambda_2\,\|D\alpha(t)\|^2] \tag{3}$$

where D is the leakage factor and $\alpha(t)$ is the differential operator constant.

Therefore, a number of sparse components (SCs) are obtained by minimizing Eq. (3). Each component is demodulated to obtain its instantaneous amplitude and frequency, and then the complete time-frequency distribution of the original signal is obtained.

B. THE PRINCIPLE OF MNSMD

MNSMD extends the traditional NSMD algorithm from one-dimensional to multi-dimensional, which provides great convenience for processing multivariable or multi-channel data. The MNSMD method is different from the method of cyclic screening decomposition, and the intrinsic oscillation modes contained in multiple input signals are separated by constraining singular local linear operators. Meanwhile, the local narrow-band component is used as the basis function for iteration, and the component signal is obtained by approaching the original signal. Suppose a multichannel data set S, the MNSMD algorithm transforms the k intrinsic mode functions of multiple signals into a set of sparse component signals $s(t)$, and the signal can be expressed as

$$S(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_p(t) \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{k} v_{1,i} + u_{1,k} \\ \sum_{i=1}^{k} v_{2,i} + u_{2,k} \\ \vdots \\ \sum_{i=1}^{k} v_{p,i} + u_{p,k} \end{bmatrix} \tag{4}$$

where, $v_{p,i}$ is the i-th component of the p channel and $u_{p,i}$ is the i-th residual useless signal of the p channel.

By using the linear singular linear operator to solve equation Eq.(4), and Eq.(5) can be obtained.

$$T(s_1, s_2, \cdots, s_p) = [u_1, u_2, \cdots, u_p] \tag{5}$$

According to the basic idea of NSMD, the local narrow-band component is used as the basis function for iteration, so as to approximate the original signal to complete the signal decomposition.

$$\min_{u_i} \left\{ \sum_{i=1}^{p} \|T(s_i - u_i)\|^2 + \lambda\,\|D(u_i)\|^2 \right\} \tag{6}$$

where, D is a diagonal matrix with u_i and λ is a Lagrange coefficient. $T(s_i - u_i)$ indicates that $s_i - u_i$ is in the zero space of operator T.

When the differential operator is used for calculation, it is difficult to separate multiple modes at one time. The null space pursuit (NSP) algorithm and leakage factor γ are used to solve this problem, and Eq.(6) becomes

$$\min_{u_i} \sum_{i=1}^{p} \|T(s_i - u_i)\|^2 + \lambda_1(\|u_i\|^2 + \gamma\,\|s_i - u_i\|^2) + \lambda_2(\|D_2 Q\|^2 + \|P\|^2) \tag{7}$$

where $T = D_2 + PD_1 + Q$, D_1 and D_2 are first-order and second-order differential operators respectively, and the value of leakage factor γ determines the amount of information in $s_i - u_i$.

To facilitate the solution, Eq.(7) is transformed into

$$F = \sum_{i=1}^{p} \|T(s_i - u_i)\|^2 + \lambda_1(\|u_i\|^2 + \gamma\,\|s_i - u_i\|^2) + \lambda_2(\|D_2 Q\|^2 + \|P\|^2) \tag{8}$$

Let $\theta = [P^{\mathrm{T}}, Q^{\mathrm{T}}]$ and $M = [D_2 0, 0E]$,

$$F = \sum_{i=1}^{p} \|T(s_i - u_i)\|^2 + \lambda_1(\|u_i\|^2 + \gamma\,\|s_i - u_i\|^2) + \lambda_2\,\|M\theta\|^2 \tag{9}$$

The partial derivative of θ and u for F is obtained. Let $\frac{\partial F}{\partial \theta}$ and $\frac{\partial F}{\partial u}$ be equal to zero, we can get Eq.(10)

$$\begin{cases} \tilde{\theta}_i = -(A^{\mathrm{T}}A + \lambda_2 M^{\mathrm{T}}M)^{-1}A^{\mathrm{T}}D_2(s_i - u_i) \\ \tilde{u}_i = (T^{\mathrm{T}}T + (1+\gamma)\lambda_1 E)^{-1}(T^{\mathrm{T}}Ts_i + \lambda_1\gamma s_i) \end{cases} \tag{10}$$

where $A = [A_p A_q]$, A_p and A_q are diagonal matrices of P and Q, respectively.

In Eq.(10), λ and γ are two important parameters, which will be updated according to the literature [25].

$$\lambda = \frac{1}{1+\tilde{\gamma}} \times \frac{s^{\mathrm{T}}[(T_s^{\mathrm{T}}T_s + (1+\gamma)\lambda_1 E)^{-1}]^{\mathrm{T}} s}{s^{\mathrm{T}}[(T_s^{\mathrm{T}}T_s + (1+\gamma)\lambda_1 E)^{-1}]^{\mathrm{T}}(T_s^{\mathrm{T}}T_s + (1+\gamma)\lambda_1 E)^{-1} s} \tag{11}$$

$$\gamma = \frac{(s-\tilde{u})^{\mathrm{T}}s}{\|s-\tilde{u}\|^2} - 1 \tag{12}$$

The termination conditions of the p modes are independent of each other, that is, $\left\|u_i^{t+1} - u_i^t\right\| < \varepsilon \|s_i\| (\varepsilon = 0.0001)$. When the termination conditions are satisfied, the corresponding modes are output according to Eq.(13).

$$\tilde{v}_i = (s_i - \tilde{u}_i)(1+\gamma) \tag{13}$$

C. SIMULATION ANALYSIS

To verify the effectiveness of the proposed algorithm, a three-channel simulation signal is decomposed as shown in Eq. (14). Meanwhile, the dominant mode with frequency of 20Hz is set for three signals to verify the effectiveness of MNSMD algorithm for identifying the same mode of multi-channel signal.

$$\begin{cases} f_1 = 2(1+0.5\sin(5\pi t)\cos(50\pi t)) + 2.5\sin(20\pi t) + n(t) \\ f_2 = 2.5(1+0.5\sin(3\pi t)\cos(60\pi t) + 2.5\sin(20\pi t) + n(t) \\ f_3 = 2(1+0.5\sin(2\pi t)\sin(60\pi t) + 2.5\sin(20\pi t) + n(t) \end{cases} \tag{14}$$

where $n(t)$ is Gaussian white noise with 5dB, and the time domain waveform of the signal and its components is shown in Figure.1.

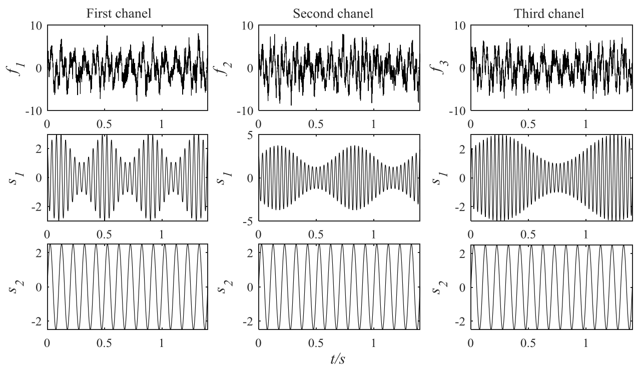

FIGURE 1. The time domain waveform of the simulated signal.

In the process of experimental verification, MEMD and MVMD methods are used for comparison, and the decomposition results of the three decomposition methods are shown in Figures.2-4. As can be seen from Figure.2, MNSMD is used to decompose the three-channel simulation signal, and the multiple sparse components and noise are obtained. Meanwhile, the components decomposed by MNSMD are smooth and have no mode aliasing, and the error is small compared with the actual data, which verifies the effectiveness of the proposed method.

Figure.3 shows the decomposition results of MVMD, and the simulation signal is decomposed into 3 layers. As shown in Figure.3, the time-domain waveform of the first component obtained by MVMD decomposition contains less noise, and the noise is not completely decomposed. Meanwhile, the time domain waveform of the component obtained is not smooth, which cannot accurately reflect the information of the original signal.

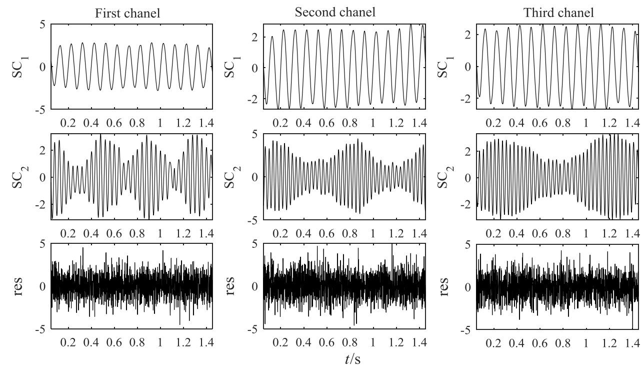

FIGURE 2. The decomposition result of MNSMD.

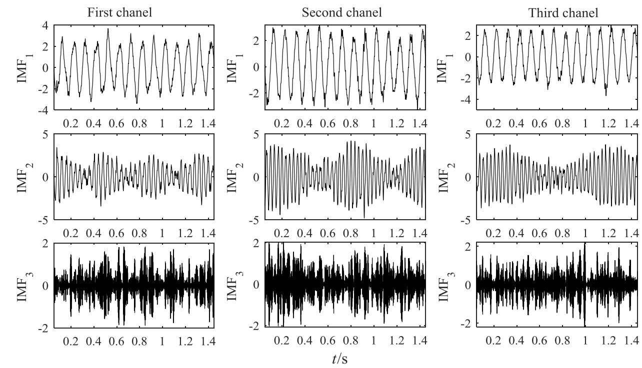

FIGURE 3. The decomposition result of MVMD.

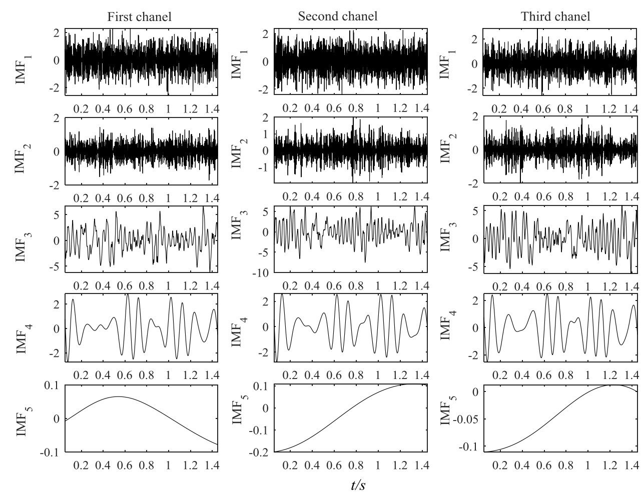

FIGURE 4. The decomposition result of MEMD.

Figure.4 shows the decomposition results of MEMD, and each channel is decomposed into five components. As shown in Figure.4, the third component and the fourth component of three-channel simulation signal show mode aliasing phenomenon, which is quite different from the corresponding real component and the decomposed components

TABLE 1. Comparison of three decomposition methods under three evaluation indexes.

Signal	Indicator	Decomposition methods (s_1/s_2)		
		MNSMD	MVMD	MEMD
f_1	RMSE	0.122/0.113	0.856/0.925	1.254/1.651
	CC	0.988/0.989	0.925/0.935	0.346/0.425
	SNR	22.42/23.21	8.42/9.42	3.21/2.48
f_2	RMSE	0.112/0.102	0.869/0.874	1.542/1.657
	CC	0.987/0.988	0.934/0.842	0.325/0.254
	SNR	22.71/25.24	9.23/8.25	3.54/4.32
f_3	RMSE	0.112/0.124	0.687/0.746	1.754/1.358
	CC	0.984/0.991	0.942/0.921	0.536/0.425
	SNR	26.25/24.26	9.46/8.63	4.32/2.35

are seriously distorted. Therefore, it is proved that the proposed MNSMD method has obvious advantages in multi-channel signal analysis.

Therefore, MNSMD method can effectively separate the three-channel simulation signal with noise, and it has good noise robustness. The reason is that constrained optimization is used to obtain the instantaneous frequency, which has physical significance and is close to the original signal as the constraint condition. Therefore, MNSMD has better decomposition performance than MEMD and MVMD.

The advantages of the proposed method are verified from the time-domain components, and quantitative comparison is carried out by using indicators, such as root mean square error (RMSE), correlation coefficient (CC) and signal to noise ratio (SNR). TABLE 1. shows the comparison results of three decomposition methods under three indexes. It can be seen from TABLE 1. that the proposed MNSMD method has obvious advantages and all indicators are the best.

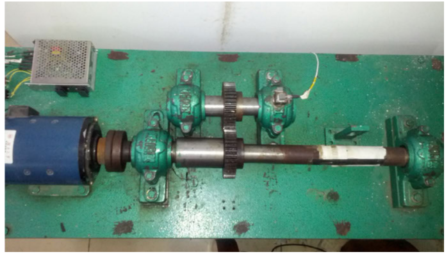

FIGURE 5. Gear fault simulation test bed.

FIGURE 6. The gear with a cracked tooth.

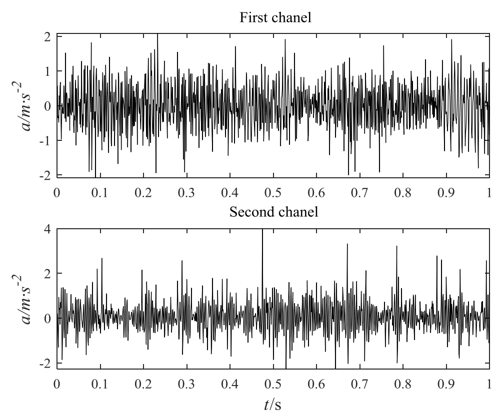

FIGURE 7. Time-domain waveform of measured signal.

III. EXPERIMENTAL ANALYSIS

A. CASE#1

To verify the feasibility and practicability of the proposed MNSMD method in this paper, it is applied to the signal analysis of conventional gear simulation fault. The experimental data is from the gear fault simulation test bed of Hunan University, as shown in Figure.5. In the process of experiment, 40 tooth driving gear and 80 tooth driven gear are selected as the analysis objects. Meanwhile, the fault width and depth of the fault gear are 0.15mm and 0.10mm, respectively, as shown in Figure.6. In addition, the gear speed is 420 r/min (The fault frequency is $f_r = 7$ Hz), and the sampling frequency is 1024Hz.

In the process of experiment, the dual-channel original signal is obtained, and the time-domain waveform is shown in Figure.7. However, due to the large background noise of the extracted vibration signal, the periodic amplitude modulation characteristics of the gear cracked cannot be observed only from the time domain waveform. Meanwhile, envelope spectrum analysis is performed on the fault signal of the gear cracked, as shown in Figure.8. As can be seen from Figure.8, although the characteristic frequency can be found, it is interfered by the background noise, so that the peak value is not obvious. Therefore, it is necessary to use multi-channel signal analysis method for vibration signal decomposition and feature extraction.

Similarly, to verify the superiority of the proposed MNSMD method for dual-channel gear fault signal, the MNSMD method is compared with the MEMD and MVMD methods. MNSMD method is used to decompose the dual-channel fault signal, and the decomposition results are shown in Figure.9. It can be seen from Figure.9 that the components obtained have certain modulation and impulse characteristics, but it is not possible to determine whether the gear is faulty only from the time domain waveform. The envelope spectrum of each component is further obtained, as shown in Figure.10.

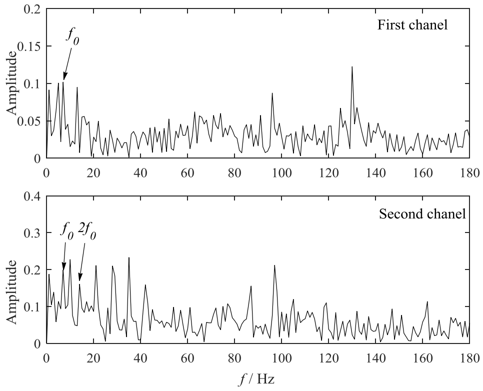

FIGURE 8. The envelope spectrum of measured signal.

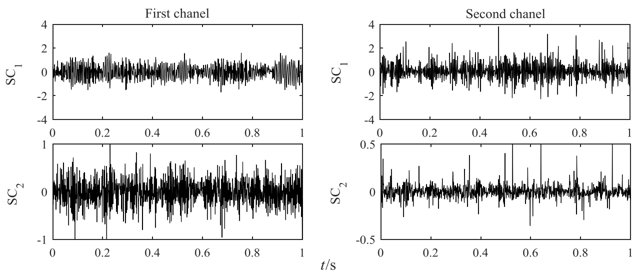

FIGURE 9. The decomposition result of MNSMD.

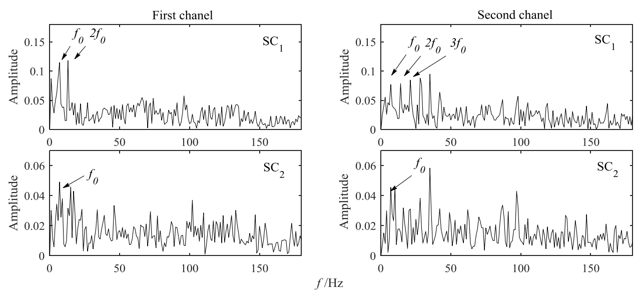

FIGURE 10. The component envelope spectrum of MNSMD.

From the envelope spectrum, it can be observed that the peak value at the fault frequency is obvious, and the amplitude of each order multiple frequency is prominent. Therefore, the frequency band extracted by the proposed method contains rich gear fault feature information, which can obviously extract and highlight the gear fault feature information.

As a comparison, the same group of multi-channel signals are decomposed by MVMD method, and the decomposition results are shown in Figure.11(mode number is 3, penalty parameter is 2500, the gear signal is decomposed into three layers). Meanwhile, the envelope spectra of the three components of the dual-channel are given, as shown in Figure.12. It can be seen from Figure.12 that the envelope spectrum of MVMD decomposition component has obvious peak line at the fault frequency, which can determine that the gear has fault. However, only the first-order fault frequency of the envelope spectrum of the dual-channel signal is

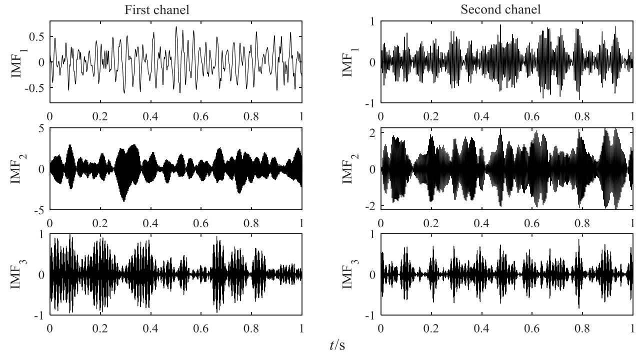

FIGURE 11. The decomposition result of MVMD.

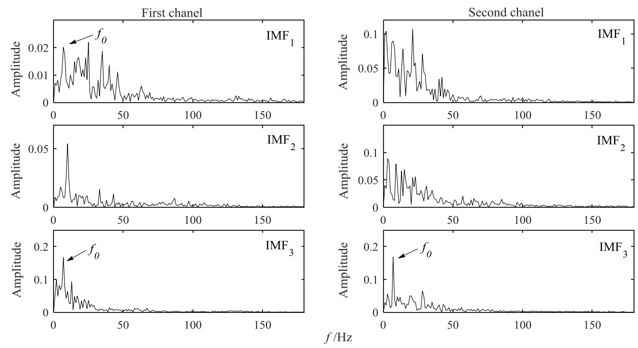

FIGURE 12. The component envelope spectrum of MVMD.

obvious, so it is difficult to observe the frequency doubling. Therefore, compared with MNSMD method, MVMD method has information omission after decomposition.

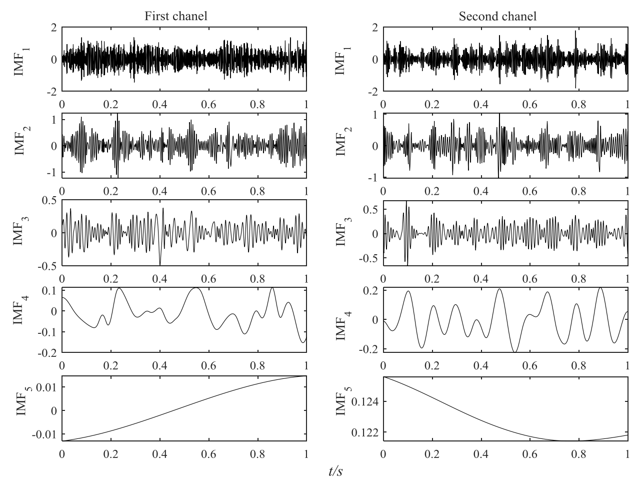

FIGURE 13. The decomposition result of MEMD.

Further, MEMD method is used to decompose the multi-channel signal, and the decomposition results are shown in Figure.13. Meanwhile, the envelope spectra of the first three components are obtained, as shown in Figure.14. As can be seen from Figure.14, the decomposition results of MEMD are greatly affected by noise, and a large amount of interference information is contained in the envelope spectrum, so it is impossible to judge whether the gear has fault. Therefore, compared with MNSMD and MVMD, the decomposition performance of MEMD is the worst.

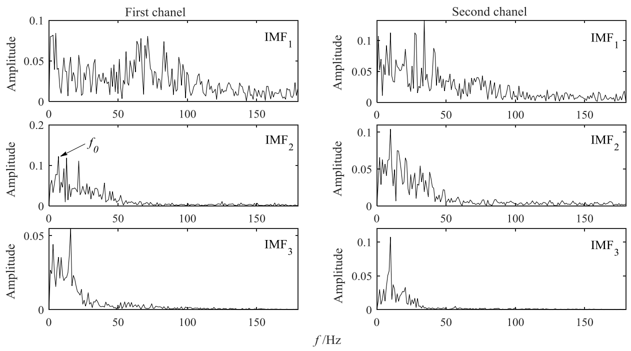

FIGURE 14. The component envelope spectrum of MEMD.

FIGURE 15. Planetary gearbox fault simulation test-bed.

FIGURE 16. Cracked solar wheel.

B. CASE # 2

The effectiveness of the MNSMD method is verified by dual-channel signal, and three-channel signal is selected for further analysis and verification. The experimental data is from the planetary gearbox fault simulation platform of Anhui University of Technology, as shown in Figure.15. The fault of solar wheel is set by wire-electrode cutting, as shown in Figure.16. The experimental parameters and conditions are shown in TABLE 2 and TABLE 3.

During the experiment, three-channel vibration signal is obtained through the three-way acceleration sensor, and the time-domain waveform is shown in Figure.17. However, due to the large amount of background noise in the collected vibration signal, the periodic amplitude modulation characteristics of cracked solar wheel cannot be observed only from the time domain waveform. Furthermore, the envelope spectrum analysis of the cracked solar wheel vibration signal is carried out, as shown in Figure.18. It can be seen from Figure.18 that although the envelope spectrum of three-channel vibration signals has peak lines at the fault frequency (31.25Hz), the interference noise component is too prominent to completely cover up the fault frequency, and it is unable to accurately determine the state type of the planetary gearbox. Therefore, it is necessary to decompose and extract features of the three-channel signal, so as to judge the true state of the planetary gearbox.

TABLE 2. Tooth parameters of planetary gearbox.

Gear	Solar wheel	Planet wheel	Inner ring gear
Tooth number	28	36	100

TABLE 3. Characteristic frequency of planetary gearbox (Hz).

Meshing frequency f_m	Characteristic frequency f_s	Rotation frequency of planet carrier f_c	Rotation frequency of solar wheel f_{sr}
218.75	31.25	2.1875	10

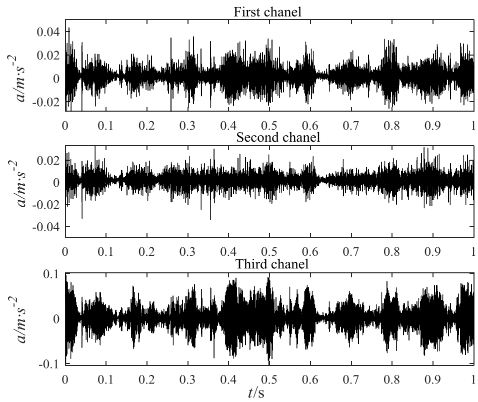

FIGURE 17. Time-domain waveform of measured signal.

Herein, MNSMD, MVMD and MEMD methods are used to decompose the same group of multi-channel planetary gearbox fault signals to further verify the effectiveness of the proposed method.

Figure.19 shows the decomposition results of MNSMD method. It can be seen from Figure. 19 that the obtained components have some modulation and pulse characteristics, but it is impossible to determine whether the gear is faulty only from the time domain waveform. As shown in Figure. 20, the envelope spectrum of each component is further plotted. It can be seen from the envelope spectrum that the peak value of cracked solar wheel fault frequency is obvious, and the frequency doubling of each order is large.

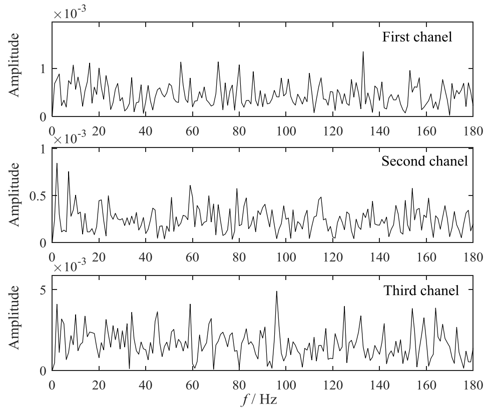

FIGURE 18. The envelope spectrum of measured signal.

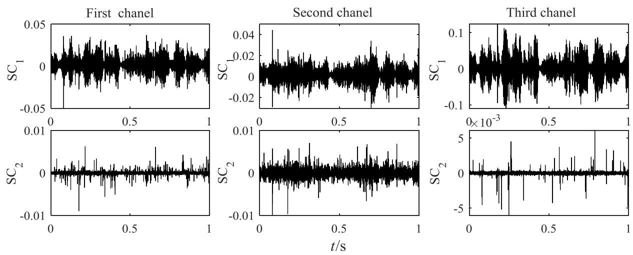

FIGURE 19. The decomposition result of MNSMD.

Therefore, it can be determined that the solar wheel has a fault.

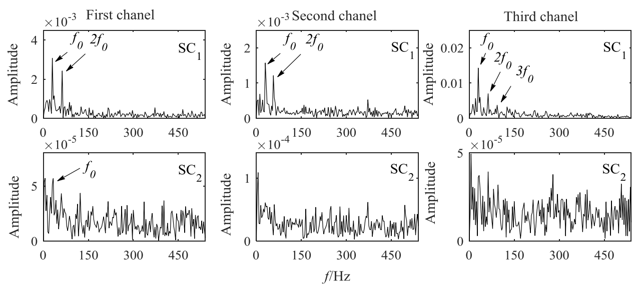

FIGURE 20. The component envelope spectrum of MNSMD.

MVMD is used to decompose the above planetary gearbox fault signal, and the decomposition results are shown in Figure.21 (mode number is 3). It can be seen from Figure.21 that the time domain waveform has tiny modulation characteristics, which cannot be used as the basis for judging the fault of planetary gearbox. Further, the envelope spectrum is obtained, as shown in Figure. 22. It can be seen from Figure.22 that the envelope spectrum of the first two components of the three-channel signal has no obvious peak line at the fault frequency of the cracked solar wheel, and the fault information is submerged by the interference noise. However, the envelope spectrum of the third component of the three-channel signal has obvious fault frequency information, so it can be judged that the solar

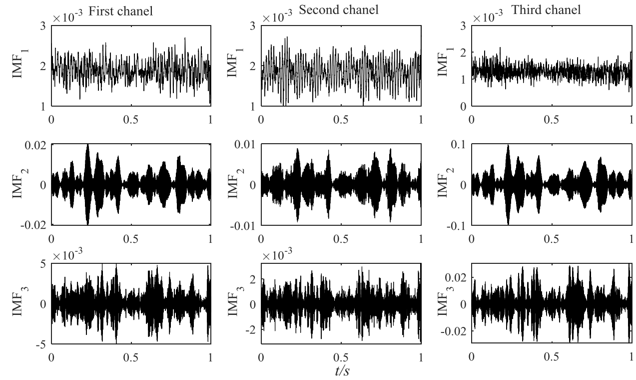

FIGURE 21. The decomposition result of MVMD.

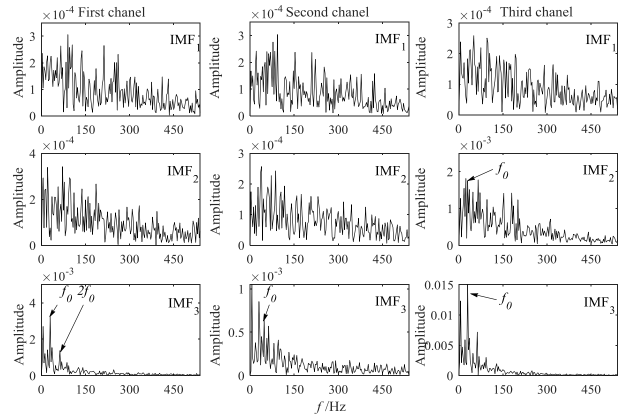

FIGURE 22. The component envelope spectrum of MVMD.

wheel has fault, but it lacks frequency doubling information. Therefore, compared with MNSMD method, MVMD has a slightly worse decomposition effect on multi-channel signals.

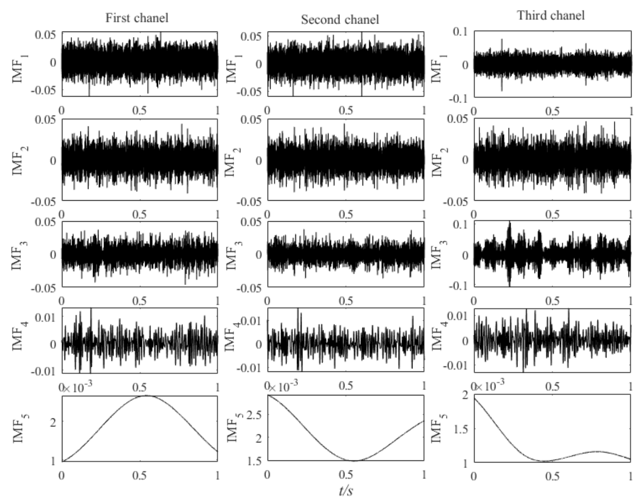

FIGURE 23. The decomposition result of MEMD.

Figure. 23 and Figure. 24 are the analysis results of the MEMD (The envelope spectra of the first three components of each channel are obtained). From Figure.23,

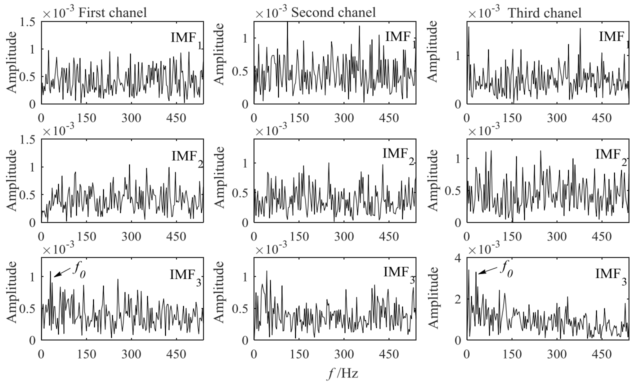

FIGURE 24. The component envelope spectrum of MEMD.

the modulation characteristics of each component are not obvious. From Figure.24 that only the third component of the first channel and the third component of the third channel have obvious peaks at the fault frequency of solar wheel, but other interference noise information is still very large.

In conclusion, the decomposition effects of MEMD, MVMD and MNSMD are compared by decomposing dual-channel common gear fault signal and three-channel planetary gear fault signal. From the modulation characteristics of the time domain waveform and the fault frequency of the envelope spectrum, the MNSMD method adaptively decomposes a multi-channel complex signal into several local narrow-band components with physical meaning of the instantaneous frequency by using singular local linear operators. Therefore, compared with MEMD and MVMD, MNSMD is an effective multi-channel signal decomposition method, which provides a reference for gear fault diagnosis.

IV. CONCLUSION

In view of the shortcomings of the existing multi-channel analysis methods, A multivariate nonlinear sparse mode decomposition (MNSMD) method is proposed by constraining singular local linear operators. In the MNSMD method, MNSMD transforms the multi-channel signal decomposition into a nonlinear constrained optimization problem, which avoids the shortcomings of MEMD and MVMD methods in multi-channel signal decomposition through cyclic iteration. Meanwhile, MNSMD uses the local narrow-band component as the iterative basis function, and obtains the local narrow-band component signal by approximating the original signal, which has obvious advantages in suppressing mode aliasing and robustness. By analyzing the simulation signal and actual gear fault signal, the experimental results show that MNSMD has better decomposition performance than MEMD and MVMD in multi-channel analysis.

Although the proposed multi-channel analysis method can effectively diagnose gear fault and has certain advantages compared with MEMD and MVMD methods, the proposed MNSMD method still has some shortcomings, such as the parameter initialization problem needs further research and improvement.

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