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Matrix Measure Strategies for Stabilization of Delayed Inertial Neural Networks via Intermittent Control

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ABSTRACT In this paper, stabilization control of a class of delayed inertial neural networks (INN) is investigated. Employing matrix measure method and two Halanay-type inequalities, some succinct stabilization criteria in terms of algebraic inequalities are derived for the INN with time-varying delays under periodically intermittent control (PIMC) strategy. Moreover, more precise results are obtained to stabilize the INN with time-invariant delays by using comparison principle. Specifically, the criteria of matrix measure form proposed in this paper can be converted into LMI-type condition for the case of 2-norm, which provides a bridge between the matrix-measure method and the Lyapunov function method. Finally, two numerical examples validate the efficacy of the derived results. The comparative research shows that the proposed methods generalize and develop some known results.

INDEX TERMS Stabilization, inertial neural networks, intermittent control, matrix measure, time delay.

I. INTRODUCTION

In recent years, artificial neural networks (NN) have clearly taken a prominent place as important mathematical/engineering tools and are often used to solve many real-world complex problems, see, e.g., pattern recognition [1], fault diagnosis [2], communication secure [3] and industrial applications [4]. In these practical applications, stability or stabilization of NN is critical. Kinds of control techniques are extensively developed to ensure the stability of NN. For instance, asymptotical region stabilization of switched NN with multiple modes and multiple equilibria was investigated in [5] by distributed state feedback controllers based on the pole assignment approach. The continuous/periodic event-based control strategy was introduced to achieve the stabilization for memristor-based NN [6]. The global μ-stabilization was addressed in [7] by impulsive control for NN with any time-varying delays. Specifically, intermittent control (IMC) technique is considered as a favorable manner for stabilization of NN recently. IMC is also named 'Act & Wait' control [8], which is activating (on) during some nonzero time-interval, and is waiting (off) during other time-interval. As an effective non-continuous feedback control, IMC plugs this gap between continuous control and impulsive control [9]. Comparing these two control methods, IMC has certain advantages in saving resources efficiently and implementing in engineering practice easily. Therefore, it has been applied for various purposes including manufacturing, signal processing, communication and transportation etc.. According to the activation mode of the controller, IMC can be divided into event-driven IMC in response to a pre-given event [10] and clock-driven IMC in response to a series of finite time intervals. In the clock-driven IMC type, if the control time is periodic, and the time in each control period is determined by the fixed control width and rest width, this control strategy is called periodically intermittent control (PIMC). Recent decades have witnessed an increasing concern on the stability and synchronization of NN by PIMC. For example, combining with Lyapunov method and free-matrix-based integral inequality, exponential stabilization of NN was addressed in [11] by PIMC. The synchronization criteria for fractional-order NN with chaotic dynamics were derived in [12] by PIMC and piecewise Lyapunov function method. In [13], by using matrix measure method, the

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intermittent controllers were designed to realize global exponential synchronization for switched NN with interval parameters uncertainty.

Inertial neural networks (INN) were first put forward by Babcock and Westervelt in [14]. Taking a physical perspective, an INN is a hybrid resistor-capacitor-inductor variety circuit. In [15]–[18], some biological and engineering examples are presented to highlight the important finding that adding an inductance (phenomenological inductance) to the traditional NN circuit can more clearly mimic the complicated dynamical behaviors of biological neurons. And so, of course, INN has great potentials in the nervous system and has received plenty of concern. Two particular classes of global stabilization problem for a class of delayed switched INN were addressed in [19] by state-feedback control strategy and Lyapunov functional method, in which a non-reduced order approach was proposed to analyze the second-order INN directly. In [20], some algebraic stabilization criteria for memristive INN with mixed delays are obtained via nonsmooth analysis and continuous feedback control. Then, the finite-time stabilization of delayed memristive INN was further investigated under discontinuous feedback controller with sign function [21]. Global exponential stabilization about complex-valued INN with time-varying delays under impulsive control was investigated in [22] by impulsive differential inequality and matrix measure method. A periodical intermittent controller was devised in [23] to ensure that the INN with mixed delay can be exponential synchronization via non-reduced order transform and Lyapunov functional method.

From the perspective of switched network, the NN under PIMC strategy can be regarded as a clock-driven switched network consisting of a stable-activating subnetwork and an unstable-waiting subnetwork. The stability analysis of NN/INN with PIMC is focused on collective Lyapunov function/functional (LFN) method, where two different subnetworks share the same LFN [9], [11], [23], [25]. In order to reduce the conservatism caused by the shared LFN, the piecewise LFN methodology is proposed for stabilizing or synchronizing of intermittently controlled NN [12], [26]. However, constructing the appropriate LFN for switched NN is not an easy task. Recently, matrix measure technique has become a valid tool for analyzing switched NN because it relaxes from the design of complicated multiple LFN [13]. Meanwhile, matrix measure method removes the constraints of non-negative constants of algebraic approach and norm approach. In this paper, we consider the stability of INN under PIMC by employing matrix measure method which is not yet investigated in the literature. First, the addressed INN is converted into the first-order dynamic model via the reduced-order variable transformation. Then by utilizing matrix measure method and two different Halanay-type inequalities, a PIMC scheme is presented to stabilize the INN with time-varying delays. Beside, more precise results are obtained for the stabilization of INN under the assumption that the delays of the INN are time-invariant.

Roughly stated, the main advantages of this paper include:

(i) Different from continuous control schemes, a non-continuous IMC strategy is proposed to stabilize the delayed INN by using matrix-measure method. With the matrix-measure method, Lyapunov functions are no longer necessary to establish the stability conditions for the switched INN.

(ii)Two types of delays are taken into account: time-varying and time-invariant delay. Criteria for delayed INN are obtained. It shows that the criterion can be more precise when the delays of the INN are time-invariant.

(iii)Different information of the activation functions have been used. It is illustrated that the full use of the network information helps obtain less conservative criteria, compared with just using rough network information [19]–[23], [29], [31].

(iv) Compared with most of the existing results [13], [22], [27]–[29], the criteria of matrix measure form proposed in this paper can be converted into linear matrix inequality (LMI) type condition for the case of 2-norm. The LMI conditions can be easily checked or solved, which are frequently used in Lyapunov method. It provides a bridge between the matrix-measure method and the Lyapunov method.

The structure of this paper is as follows. Section II describes stabilization problem of the addressed INN and gives some preliminaries. Section III establishes the stabilization conditions of the INN for time-varying delays and time-invariant delays, respectively. Section IV provides two numerical examples to verify the proposed methods. Section V gives some concluding remarks.

Notations. $\mathbb{R}$ is the real number set. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\mathbb{R}^{n\times n}$ represent $n\times n$ matrix. $\|.\|$ is the Euclidean norm in $\mathbb{R}^n$. $\mathcal{I}_n := \{1, 2, \cdots, n\}$. $W = [w_{ij}]_{n\times n}$ is a $n \times n$ matrix. $\mathbf{I}_n$ is the $n \times n$ identity matrix. Q^T denotes the transpose of matrix Q. $\text{diag}\{g_1, g_2, \cdots, g_n\}$ denotes the diagonal matrix with its diagonal elements $g_1, g_2, \cdots, g_n$. $\boldsymbol{\lambda}_{max}(Q)$ is the maximum eigenvalue of matrix Q. The upper-right Dini-derivative for $q(x)$ is defined as $D^+q(x) = lim_{\delta\to 0^+} \frac{q(x+\delta)-q(x)}{\delta}$, $\delta \to 0^+$ represents that δ approaches 0 from the right-hand side.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following INN

$$\ddot{v}_i(t) = -a_i^{\dagger}\dot{v}_i(t) - b_i^{\dagger}v_i(t) + \sum_{j=1}^{n} c_{ij}f_j(v_j(t)) + \sum_{j=1}^{n} d_{ij}f_j(v_j(t-\tau(t))) + p_i(t), \quad (1)$$

with its initial values

$$\begin{aligned} v_i(s) &= \phi_i(s),\ s \in [-\tau_0, 0], \\ \dot{v}_i(s) &= \psi_i(s),\ s \in [-\tau_0, 0], \end{aligned} \quad (2)$$

where $i, j \in \mathcal{I}_n$, n is the number of neurons, $v_i(t) \in \mathbb{R}$ is the state of neuron i, $a_i^{\dagger}$ and $b_i^{\dagger}$ are constants, c_{ij} and d_{ij} are connection weights, $f_j(\cdot)$ is the activation function with

$f_j(0) = 0$ for all $j \in \mathcal{I}_n$, $\tau(t)$ is the time-varying delay with $\tau_0 = \sup_t\{\tau(t)\} < +\infty$. $p_i(t)$ is the control protocol to be designed later.

Now given $\zeta_i > 0$, $i \in \mathcal{I}_n$, and let

$$w_i(t) = \dot{v}_i(t) + \zeta_i v_i(t), \tag{3}$$

INN (1) could be expressed as

$$\begin{cases} \dot{v}_i(t) = -\zeta_i v_i(t) + w_i(t), \\ \dot{w}_i(t) = -a_i v_i(t) - b_i w(t) + \sum_{j=1}^{n} c_{ij} f_j(v_j(t)) \\ + \sum_{j=1}^{n} d_{ij} f_j(v_j(t-\tau(t))) + p_i(t), \end{cases}$$

with initial values

$$v_i(s) = \phi_i(s),$$
$$w_i(s) = \psi_i(s) + \zeta_i \phi_i(s),$$

for any $s \in [-\tau_0, 0]$, where $a_i = b_i^{\dagger} + \zeta_i(\zeta_i - a_i^{\dagger})$, $b_i = a_i^{\dagger} - \zeta_i$. Let $v(t) = (v_1(t), v_2(t), \cdots, v_n(t))^T$, $w(t) = (w_1(t), w_2(t), \cdots, w_n(t))^T$, $\Xi = \text{diag}\{\zeta_1, \zeta_2, \cdots, \zeta_n\}$, $A = \text{diag}\{a_1, a_2, \cdots, a_n\}$, $B = \text{diag}\{b_1, b_2, \cdots, b_n\}$, $C = (c_{ij})_{n\times n}$, $D = (d_{ij})_{n\times n}$, $f(v(\cdot)) = (f_1(v_1(\cdot)), f_2(v_2(\cdot)), \cdots, f_n(v_n(\cdot)))^T$, $p(t) = (p_1(t), p_2(t), \cdots, p_n(t))^T$, then it can be further rewritten in the compact form

$$\begin{cases} \dot{v}(t) = -\Xi v(t) + w(t), \\ \dot{w}(t) = -Av(t) - Bw(t) + Cf(v(t)) \\ + Df(v(t-\tau(t))) + p(t), \end{cases} \tag{4}$$

for any $t \geq 0$.

IMC protocol is adopted in this paper to stabilize INN (1), or equivalently (4). The controller is in the form of

$$p_i(t) = \begin{cases} p_i(v_i, w_i), & t \in [m\mathcal{T}, m\mathcal{T} + \xi) \\ 0, & t \in [m\mathcal{T} + \xi, m\mathcal{T} + \mathcal{T}), \end{cases} \tag{5}$$

where $p_i(v_i, w_i) = -\eta_{1i} v_i(t) - \eta_{2i} w_i(t)$, $i \in \mathcal{I}_n$.

Substituting (5) into INN (4), the closed-loop NN is given by

$$\begin{cases} \dot{v}(t) = -\Xi v(t) + w(t), \\ \dot{w}(t) = -\tilde{A}v(t) - \tilde{B}w(t) + Cf(v(t)) \\ + Df(v(t-\tau(t))), \end{cases} \tag{6}$$

for any $t \in [m\mathcal{T}, m\mathcal{T} + \xi)$, and

$$\begin{cases} \dot{v}(t) = -\Xi v(t) + w(t), \\ \dot{w}(t) = -Av(t) - Bw(t) + Cf(v(t)) \\ + Df(v(t-\tau(t))), \end{cases} \tag{7}$$

for any $t \in [m\mathcal{T} + \xi, m\mathcal{T} + \mathcal{T})$, where

$$\tilde{A} = A + \Omega_1, \Omega_1 = \text{diag}\{\eta_{11}, \eta_{12}, \cdots, \eta_{1n}\},$$
$$\tilde{B} = B + \Omega_2, \Omega_2 = \text{diag}\{\eta_{21}, \eta_{22}, \cdots, \eta_{2n}\}.$$

Some assumption are made for the main results.

Assumption 1: $f_j(\cdot)$ is bounded and satisfies

$$|f_j(\vartheta_1) - f_j(\vartheta_2)| \leq r_j|\vartheta_1 - \vartheta_2|, \ j \in \mathcal{I}_n,$$

for $\forall \vartheta_1, \vartheta_2 \in \mathbb{R}$, where constant $r_j > 0$.

Assumption 2: The upper bound of time-delay satisfies $\tau_0 \leq \xi$ and $\tau_0 \leq \mathcal{T} - \xi$.

Definition 1: A real matrix $S = [s_{ij}]_{n\times n}$, and the matrix measure of S with respect to $\mathfrak{p}$ is defined as

$$\mu_{\mathfrak{p}}(S) = \lim_{\varepsilon \to 0^+} \frac{\|\mathbf{I}_n + \varepsilon S\|_{\mathfrak{p}} - 1}{\varepsilon},$$

for any $\mathfrak{p} = 1, 2, \infty$, where $\|\cdot\|_{\mathfrak{p}}$ is the corresponding matrix norm, i.e., $\|S\|_1 = \max_j \sum_{i=1}^n |s_{ij}|$, $\|S\|_2 = \sqrt{\boldsymbol{\lambda}_{max}(S^TS)}$ and $\|S\|_\infty = \max_i \sum_{j=1}^n |s_{ij}|$.

$\mu_{\mathfrak{p}}(S)$ can be computed as follows

$$\mu_1(S) = \max_j \left\{ s_{jj} + \sum_{i=1, i\neq j}^{n} |s_{ij}| \right\},$$
$$\mu_2(S) = \frac{1}{2}\boldsymbol{\lambda}_{max}(S + S^T),$$
$$\mu_\infty(S) = \max_i \left\{ s_{ii} + \sum_{j=1, j\neq i}^{n} |s_{ij}| \right\}.$$

An important property of the matrix measure is that, for any given $S^*, S^{**} \in \mathbb{R}^{n\times n}$,

$$\mu_{\mathfrak{p}}(S^* + S^{**}) \leq \mu_{\mathfrak{p}}(S^*) + \mu_{\mathfrak{p}}(S^{**}). \tag{8}$$

Lemma 1 [30]: Assume that $g(t)$, $t \in [t_0 - \tau_0, \infty)$ is a nonnegative continuous function, and constants l_1 and l_2 satisfy $l_1 > l_2 > 0$. If

$$D^+g(t) \leq -l_1 g(t) + l_2 \bar{g}(t),$$

for $t \geq t_0$, where $\bar{g}(t) = \sup_{s\in[t-\tau_0, t]}\{g(s)\}$, τ_0 is a nonnegative constant. Then we have

$$g(t) \leq \bar{g}(t_0)e^{-k(t-t_0)},$$

for $t \geq t_0$, where k is the unique positive root of $k = l_1 - l_2 e^{k\tau_0}$.

Lemma 2: Assume that $g(t), t \in [t_0 - \tau_0, \infty)$ is a nonnegative continuous function, and constants l_1 and l_2 satisfy $l_1 < l_2$, $l_2 > 0$. If

$$D^+g(t) \leq -l_1 g(t) + l_2 \bar{g}(t), \tag{9}$$

for $t \geq t_0$, where $\bar{g}(t) = \sup_{s\in[t-\tau_0, t]}\{g(s)\}$, τ_0 is a nonnegative constant. Then we have

$$g(t) \leq \bar{g}(t_0)e^{k(t-t_0)},$$

for $t \geq t_0$, where $k = l_2 - l_1$.

Proof: It is clear that $g(t_0) \leq \bar{g}(t_0) = \sup_{s\in[t_0-\tau_0, t_0]}\{g(s)\}$. For $t > t_0$, let

$$q(t) = \bar{g}(t_0)e^{k(t-t_0)}, \quad t \geq t_0, \quad k = l_2 - l_1 > 0, \quad l_2 > 0$$

and we claim that $g(t) \leq q(t)$, for $\forall t \geq t_0$. If not, $\exists\ t_1 > t_0$ such that

$$g(t) \leq q(t), \ \ t \in [t_0, t_1), \ \ g(t_1) = q(t_1)$$

and

$$D^+(g(t_1) - q(t_1)) > 0 \tag{10}$$

However, let $x(t) = g(t) - q(t), t \geq t_0$, and from (9), we have

$$\begin{aligned} D^+x(t_1) &= D^+g(t_1) - D^+q(t_1) \\ &\leq -l_1 g(t_1) + l_2\bar{g}(t_1) - kq(t_1) \\ &= -l_1 g(t_1) + l_2\bar{g}(t_1) - l_2 q(t_1) + l_1 q(t_1) \\ &= l_2(\bar{g}(t_1) - q(t_1)) \end{aligned} \tag{11}$$

Since $k = l_2 - l_1 > 0, x(t)$ is increasing function of t, then,

$$\bar{g}(t_1) = \sup_{s\in[t_1-\tau_0,t_1]}\{g(s)\} \leq \sup_{s\in[t_1-\tau_0,t_1]}\{q(s)\} = q(t_1)$$

Thus, $\bar{g}(t_1) - q(t_1) \leq 0$. Moreover, because $l_2 > 0$, (11) implies

$$D^+x(t_1) = D^+(g(t_1) - q(t_1)) \leq 0$$

which contradicts with (10), and completes the proof of Lemma 2. ■

Lemma 3: Suppose that $g(t), q(t)$ are differentiable functions for $t \in [t_0 - \tau, \infty)$ and satisfy

$$\begin{cases} \dot{g}(t) \leq -\theta_1 g(t) + \theta_2 g(t-\tau), \\ \dot{q}(t) = -\theta_1 q(t) + \theta_2 q(t-\tau), \end{cases}$$

for $t \geq t_0$, where $\theta_1, \theta_2 > 0$, then

$$g(t) \leq q(t), \quad \forall t \in [t_0 - \tau, t_0],$$

implies

$$g(t) \leq q(t), \quad \forall t \geq t_0.$$

Proof: Take $x(t) = g(t) - q(t),\ t \geq t_0 - \tau$, then we have $x(t) \leq 0,\ \forall t \in [t_0 - \tau, t_0]$. Let $t_1 \geq t_0$ satisfy $x(t) \leq 0, \forall t \leq t_1$, then in the right neighborhood of t_1, that is, $t \in [t_1, t_1 + \varepsilon]$, $\varepsilon > 0$, then we can get

$$\begin{aligned} \dot{x}(t) &= \dot{g}(t) - \dot{q}(t) \\ &\leq -\theta_1 x(t) + \theta_2 x(t-\tau) \\ &\leq -\theta_1 x(t) \end{aligned}$$

Then

$$x(t) \leq x(t)e^{-\theta_1(t-t_1)} \leq 0, \forall t \in [t_1, t_1+\varepsilon]$$

Therefore, if $g(t) \leq q(t), t \leq t_0$, then $g(t) \leq q(t)$, $t \geq t_0$. ■

Lemma 4: A real matrix H satisfies $\mu_2(H) < \sigma$, if and only if $H + H^T - 2\sigma\mathbf{I}_n < 0$.

Proof: Let $q_i, i \in \mathcal{I}_n$ be the eigenvalues of $H + H^T$, then there exists an orthogonal matrix U and satisfy

$$H + H^T = U^T \begin{bmatrix} q_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & q_n \end{bmatrix} U$$

To prove necessity,

$$\mu_2(H) < \sigma \Rightarrow \boldsymbol{\lambda}_{max}(H + H^T) < 2\sigma$$

Then

$$\begin{bmatrix} q_1 - 2\sigma & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & q_n - 2\sigma \end{bmatrix} < 0$$
$$\Rightarrow U^T \begin{bmatrix} q_1 - 2\sigma & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & q_n - 2\sigma \end{bmatrix} U < 0$$

Thus,

$$H + H^T - 2\sigma\mathbf{I}_n < 0$$

The sufficiency is obtained by inverse derivation of the above process. Here we omit it. ■

III. MAIN RESULTS

The stability conditions for the closed-loop system of the INN (4) under the intermittent control protocol (5) are addressed in this section. To this end, let

$$u(t) = \begin{bmatrix} v(t) \\ w(t) \end{bmatrix},$$
$$\mathbb{M} = \begin{bmatrix} -\Xi & \mathbf{I}_n \\ -A & -B \end{bmatrix},$$
$$\tilde{\mathbb{M}} = \begin{bmatrix} -\Xi & \mathbf{I}_n \\ -\tilde{A} & -\tilde{B} \end{bmatrix}.$$

A. STABILIZATION OF INN WITH TIME-VARYING DELAYS VIA PIMC

For notational convenience, denote

$$\begin{aligned} \alpha_{\{1,\mathfrak{p}\}} &= -(\mu_\mathfrak{p}(\mathbb{M}) + r\|C\|_\mathfrak{p}), \\ \tilde{\alpha}_{\{1,\mathfrak{p}\}} &= -(\mu_\mathfrak{p}(\tilde{\mathbb{M}}) + r\|C\|_\mathfrak{p}), \\ \beta_{\{1,\mathfrak{p}\}} &= r\|D\|_\mathfrak{p}, r = \max\{r_1, r_2, \cdots, r_n\}. \end{aligned} \tag{12}$$

Theorem 1: If Assumptions 1-2 hold, INN (4) is asymptotically stable under the intermittent control protocol (5) provided that

$$\tilde{\alpha}_{\{1,\mathfrak{p}\}} > \beta_{\{1,\mathfrak{p}\}}, \tag{13a}$$
$$-\tilde{k}_{\{1,\mathfrak{p}\}}(\xi - \tau_0) + k_{\{1,\mathfrak{p}\}}(\mathcal{T} - \xi) < 0, \tag{13b}$$

for any $\mathfrak{p} \in \{1, 2, \infty\}$, where $\tilde{k}_{\{1,\mathfrak{p}\}}$ is the unique positive solution of the equality $k = \tilde{\alpha}_{\{1,\mathfrak{p}\}} - \beta_{\{1,\mathfrak{p}\}}e^{k\tau_0}$ with respect to k, $k_{\{1,\mathfrak{p}\}} = \beta_{\{1,\mathfrak{p}\}} - \alpha_{\{1,\mathfrak{p}\}}$.

Proof: Calculate the following expression (14), as shown at the bottom of the next page.

Therefore,

$$D^+\|u(t)\|_\mathfrak{p} \leq -\tilde{\alpha}_{\{1,\mathfrak{p}\}}\|u(t)\|_\mathfrak{p} + \beta_{\{1,\mathfrak{p}\}} \sup_{s\in[t-\tau_0,t]}\{\|u(s)\|_\mathfrak{p}\},$$

for any $t \in [m\mathcal{T}, m\mathcal{T} + \xi)$.

By Lemma 1, we have

$$\|u(t)\|_\mathfrak{p} \leq \sup_{s\in[m\mathcal{T}-\tau_0,m\mathcal{T}]}\{\|u(s)\|_\mathfrak{p}\}e^{-\tilde{k}_{\{1,\mathfrak{p}\}}(t-m\mathcal{T})}, \tag{15}$$

for any $t \in [m\mathcal{T}, m\mathcal{T} + \xi)$.

Similarly, as calculated in (14),

$$D^+\|u(t)\|_{\mathfrak{p}} \le -\alpha_{\{1,p\}}\|u(t)\|_{\mathfrak{p}} + \beta_{\{1,\mathfrak{p}\}} \sup_{s\in[t-\tau_0,t]} \{\|u(s)\|_{\mathfrak{p}}\},$$

for any $t \in [m\mathcal{T}+\xi, m\mathcal{T}+\mathcal{T})$.

Using Lemma 2,

$$\|u(t)\|_{\mathfrak{p}} \le \sup_{s\in[m\mathcal{T}+\xi-\tau_0, m\mathcal{T}+\xi]} \{\|u(s)\|_{\mathfrak{p}}\} e^{k_{\{1,\mathfrak{p}\}}(t-m\mathcal{T}-\xi)}, \quad (16)$$

for any $t \in [m\mathcal{T}+\xi, m\mathcal{T}+\mathcal{T})$.

Additionally, by the continuity of $\|u(t)\|_{\mathfrak{p}}$, for any integer m, we have

$$\begin{cases} \|u(m\mathcal{T}+\xi)\|_{\mathfrak{p}} = \lim_{\epsilon\to 0^+} \|u(m\mathcal{T}+\xi-\epsilon)\|_{\mathfrak{p}}, \\ \|u(m\mathcal{T}+\mathcal{T})\|_{\mathfrak{p}} = \lim_{\epsilon\to 0^+} \|u((m\mathcal{T}+\mathcal{T}-\epsilon)\|_{\mathfrak{p}}. \end{cases} \quad (17)$$

Let $m = 0$, for any $t \in [0, \xi]$,

$$\|u(t)\|_{\mathfrak{p}} \le \sup_{s\in[-\tau_0,0]} \|u(s)\|_{\mathfrak{p}} e^{-\tilde{k}_{\{1,\mathfrak{p}\}}t},$$

and for any $t \in [\xi, \mathcal{T}]$,

$$\begin{aligned} \|u(t)\|_{\mathfrak{p}} &\le \sup_{s\in[\xi-\tau_0,\xi]} \|u(s)\|_{\mathfrak{p}} e^{k_{\{1,\mathfrak{p}\}}(t-\xi)} \\ &\le \sup_{s\in[-\tau_0,0]} \|u(s)\|_{\mathfrak{p}} e^{-\tilde{k}_{\{1,\mathfrak{p}\}}(\xi-\tau_0)+k_{\{1,\mathfrak{p}\}}(t-\xi)}, \end{aligned}$$

Then by (17)

$$\|u(T)\|_{\mathfrak{p}} \le \sup_{s\in[-\tau_0,0]} \|u(s)\|_{\mathfrak{p}} e^{-\tilde{k}_{\{1,\mathfrak{p}\}}(\xi-\tau_0)+k_{\{1,\mathfrak{p}\}}(\mathcal{T}-\xi)},$$

Let $m = 1$, for any $t \in [\mathcal{T}, \mathcal{T}+\xi]$,

$$\|u(t)\|_{\mathfrak{p}} \le \sup_{s\in[\mathcal{T}-\tau_0,\mathcal{T}]} \|u(s)\|_{\mathfrak{p}} e^{-\tilde{k}_{\{1,\mathfrak{p}\}}(t-\mathcal{T})},$$

and for any $t \in [\mathcal{T}+\xi, 2\mathcal{T}]$,

$$\begin{aligned} \|u(t)\|_{\mathfrak{p}} &\le \sup_{s\in[\mathcal{T}+\xi-\tau_0,\mathcal{T}+\xi]} \|u(s)\|_{\mathfrak{p}} e^{k_{\{1,\mathfrak{p}\}}(t-\mathcal{T}-\xi)} \\ &\le \sup_{s\in[\mathcal{T}-\tau_0,\mathcal{T}]} \|u(s)\|_{\mathfrak{p}} e^{-\tilde{k}_{\{1,\mathfrak{p}\}}(\xi-\tau_0)+k_{\{1,\mathfrak{p}\}}(t-\mathcal{T}-\xi)} \\ &\le \sup_{s\in[-\tau_0,0]} \|u(s)\|_{\mathfrak{p}} e^{-2\tilde{k}_{\{1,\mathfrak{p}\}}(\xi-\tau_0)+k_{\{1,\mathfrak{p}\}}(t-2\xi)}. \end{aligned}$$

Again, by (17)

$$\|u(2\mathcal{T})\|_{\mathfrak{p}} \le \sup_{s\in[-\tau_0,0]} \|u(s)\|_{\mathfrak{p}} e^{2(-\tilde{k}_{\{1,\mathfrak{p}\}}(\xi-\tau_0)+k_{\{1,\mathfrak{p}\}}(\mathcal{T}-\xi))}.$$

By induction, for any integer m

$$\|u(m\mathcal{T})\|_{\mathfrak{p}} \le \sup_{s\in[-\tau_0,0]} \|u(s)\|_{\mathfrak{p}} e^{m(-\tilde{k}_{\{1,\mathfrak{p}\}}(\xi-\tau_0)+k_{\{1,\mathfrak{p}\}}(\mathcal{T}-\xi))},$$

and for any $t \in [m\mathcal{T}+\xi, m\mathcal{T}+\mathcal{T}]$, we have (18), as shown at the bottom of the next page.

Therefore, combining with condition (13), we have

$$\|u(t)\|_{\mathfrak{p}} \to 0 \text{ as } t \to +\infty,$$

that is

$$v_i(t) \to 0 \text{ and } w_i(t) \to 0, \quad \forall i \in \mathcal{I}_n \text{ as } t \to +\infty,$$

the proof is completed. ■

Remark 1: Theorem 1 solves the stabilization problem of INN (1) by using matrix measure, where no Lyapunov function is involved. Moreover, compared with some previous works, we here employed the non-continuous intermittent control strategy, which is more economic and practical in real applications. From the proof of Theorem 1, it is quit clear that $\alpha_{\{1,\mathfrak{p}\}} > \beta_{\{1,\mathfrak{p}\}}$ implies that INN (1) is autonomously asymptotically stable. If this condition is failure, the intermittent control protocol (5) satisfying (13) will stabilize INN (1).

$$\begin{aligned} D^+\|u(t)\|_{\mathfrak{p}} &= \lim_{\delta\to 0^+} \frac{\|u(t+\delta)\|_{\mathfrak{p}} - \|u(t)\|_{\mathfrak{p}}}{h} \\ &= \lim_{\delta\to 0^+} \frac{\|u(t)+\delta\dot{u}(t)\|_{\mathfrak{p}} - \|u(t)\|_{\mathfrak{p}}}{\delta} \\ &= \lim_{\delta\to 0^+} \frac{\left\| u(t) + \delta \begin{bmatrix} -\Xi v(t) + w(t) \\ -Av(t) - Bw(t) + Cf(v(t)) + Df(v(t-\tau(t))) + p(t) \end{bmatrix} \right\|_{\mathfrak{p}} - \|u(t)\|_{\mathfrak{p}}}{\delta} \\ &\le \lim_{\delta\to 0^+} \frac{\left\| u(t) + \delta \begin{bmatrix} -\Xi & \mathbf{I}_n \\ -\tilde{A} & -\tilde{B} \end{bmatrix} u(t) \right\|_{\mathfrak{p}} - \|u(t)\|_{\mathfrak{p}}}{h} + r\,\|C\|_{\mathfrak{p}}\,\|v(t)\|_{\mathfrak{p}} + r\,\|D\|_{\mathfrak{p}}\,\|v(t-\tau(t))\|_{\mathfrak{p}} \\ &\le \lim_{\delta\to 0^+} \frac{\left\| \mathbf{I}_{2n} + \delta \begin{bmatrix} -\Xi & \mathbf{I}_n \\ -\tilde{A} & -\tilde{B} \end{bmatrix} \right\|_{\mathfrak{p}} \|u(t)\|_{\mathfrak{p}} - \|u(t)\|_{\mathfrak{p}}}{\delta} + r\,\|C\|_{\mathfrak{p}}\,\|u(t)\|_{\mathfrak{p}} + r\,\|D\|_{\mathfrak{p}}\,\|u(t-\tau(t))\|_{\mathfrak{p}} \\ &= \mu_{\mathfrak{p}}(\tilde{\mathbb{M}})\|u(t)\|_{\mathfrak{p}} + r\|C\|_{\mathfrak{p}}\|u(t)\|_p + r\|D\|_{\mathfrak{p}}\|u(t-\tau(t))\|_{\mathfrak{p}} \\ &\le (\mu_{\mathfrak{p}}(\tilde{\mathbb{M}}) + r\|C\|_{\mathfrak{p}})\|u(t)\|_{\mathfrak{p}} + r\|D\|_{\mathfrak{p}} \sup_{s\in[t-\tau_0,t]} \{\|u(s)\|_{\mathfrak{p}}\} \\ &= -\tilde{\alpha}_{\{1,\mathfrak{p}\}}\|u(t)\|_{\mathfrak{p}} + \beta_{\{1,\mathfrak{p}\}} \sup_{s\in[t-\tau_0,t]} \{\|u(s)\|_{\mathfrak{p}}\}. \end{aligned} \quad (14)$$

For the condition (13a) in Theorem 1, $\tilde{\alpha}_{\{1,\mathfrak{p}\}} > \beta_{\{1,\mathfrak{p}\}}$ can be replaced with $\mu_{\mathfrak{p}}(\tilde{\mathbb{M}}) < -r(\|C\|_{\mathfrak{p}}) + \|D\|_{\mathfrak{p}}) = \sigma$. For the case $\mathfrak{p} = 2$, it is not difficult to further transform it into linear matrix inequality (LMI) by applying Lemma 4. To clear this point, we have the following corollary.

Corollary 1: If Assumptions 1-2 hold, INN (4) is asymptotically stable under the intermittent control protocol (5) provided that

$$\tilde{\mathbb{M}} + \tilde{\mathbb{M}}^T - 2\sigma \mathbf{I}_n < 0, \tag{19a}$$

$$-\tilde{k}_{\{1,\mathfrak{p}\}}(\xi - \tau_0) + k_{\{1,\mathfrak{p}\}}(\mathcal{T} - \xi) < 0, \tag{19b}$$

for $\mathfrak{p} = 2$, where σ is a contant, $\tilde{k}_{\{1,\mathfrak{p}\}}$ is the unique positive solution of the equality $k = \tilde{\alpha}_{\{1,\mathfrak{p}\}} - \beta_{\{1,\mathfrak{p}\}}e^{k\tau_0}$ with respect to k, $k_{\{1,\mathfrak{p}\}} = \beta_{\{1,\mathfrak{p}\}} - \alpha_{\{1,\mathfrak{p}\}}$.

Remark 2: In Corollary 1, we convert the stabilization criterion with the matrix measure form (13a) into the LMI-type condition (19a) for the case $\mathfrak{p} = 2$. For continuous control strategy, we can remove the condition (19b), and in such case, the results here generalize some existing works [27]–[29]. Since (19a) is an LMI, the intermittent controller gain matrix can be directly derived by solving it with MATLAB LMI toolbox.

It is worthy of noting that Theorem 1 involves only the maximum value of r_i, $i \in \mathcal{I}_n$, which may result in conservativeness. For this reason, more detailed information of $r_i, i \in \mathcal{I}_n$ is used to reduce the conservativeness of Theorem 1. For convenience, we define

$$\begin{aligned}
\check{C} &= \begin{bmatrix} 0 & 0 \\ C & 0 \end{bmatrix} \in \mathbb{R}^{2n\times 2n}, \\
\Upsilon &= \mathrm{diag}\{r_1, r_2, \cdots, r_n, 1, 1, \cdots, 1\} \in \mathbb{R}^{2n\times 2n}, \\
\tilde{\alpha}_{\{2,\mathfrak{p}\}} &= -(\mu_{\mathfrak{p}}(\tilde{\mathbb{M}}) + \mu_{\mathfrak{p}}(\check{C}\Upsilon)), \\
\alpha_{\{2,\mathfrak{p}\}} &= -(\mu_{\mathfrak{p}}(\mathbb{M}) + \mu_{\mathfrak{p}}(\check{C}\Upsilon)).
\end{aligned} \tag{20}$$

Theorem 2: If Assumptions 1-2 hold, INN (4) is asymptotically stable under the intermittent control protocol (5) provided that

$$\begin{aligned}
&\tilde{\alpha}_{\{2,\mathfrak{p}\}} > \beta_{\{1,\mathfrak{p}\}}, \\
&-\tilde{k}_{\{2,\mathfrak{p}\}}(\xi - \tau_0) + k_{\{2,\mathfrak{p}\}}(T - \xi) < 0,
\end{aligned} \tag{21}$$

for any $\mathfrak{p} \in \{1, \infty\}$, where $\beta_{\{1,\mathfrak{p}\}}$ is defined in (12), $\tilde{k}_{\{2,\mathfrak{p}\}}$ is the unique positive solution of the equality $k = \tilde{\alpha}_{\{2,\mathfrak{p}\}} - \beta_{\{1,\mathfrak{p}\}}e^{k\tau_0}$ with respect to k, $k_{\{2,\mathfrak{p}\}} = \beta_{\{1,\mathfrak{p}\}} - \alpha_{\{2,\mathfrak{p}\}}$.

Proof: It follows from Theorem 1, for any $t \in [m\mathcal{T}, m\mathcal{T} + \xi)$, we have (22), as shown at the bottom of the next page.

If $v_k(t) = 0$ for some $k \in \mathcal{I}_n$ and $t \in \mathbb{R}$, then define $\frac{f(v_k(t))}{v_k(t)} = r_k$, let

$$\mathrm{F}(v(t)) = \mathrm{diag}\left\{\frac{f_1(v_1(t))}{v_1(t)}, \frac{f_2(v_2(t))}{v_2(t)}, \cdots, \frac{f_n(v_n(t))}{v_n(t)}\right\},$$

then,

$$\begin{aligned}
\begin{bmatrix} 0 \\ Cf(v(t)) \end{bmatrix} &= \begin{bmatrix} 0 & 0 \\ C & 0 \end{bmatrix} \\
&\quad \times \begin{bmatrix} \mathrm{F}(v(t)) & 0 \\ 0 & \mathbf{I}_n \end{bmatrix} \begin{bmatrix} v(t) \\ w(t) \end{bmatrix} \\
&= \check{C}\mathcal{F}(v(t))u(t),
\end{aligned}$$

where

$$\mathcal{F}(v(t)) = \begin{bmatrix} \mathrm{F}(v(t)) & 0 \\ 0 & \mathbf{I}_n \end{bmatrix}.$$

Therefore, we get (23), as shown at the bottom of the next page.

By property (8), we have

$$\mu_{\mathfrak{p}}(\tilde{\mathbb{M}} + \check{C}\mathcal{F}(v(t))) \leq \mu_{\mathfrak{p}}(\tilde{\mathbb{M}}) + \mu_{\mathfrak{p}}(\check{C}\mathcal{F}(v(t))).$$

By simple computation, when $\mathfrak{p} = 1, \infty$, we also have

$$\mu_{\mathfrak{p}}(\check{C}\mathcal{F}(v(t))) \leq \mu_{\mathfrak{p}}(\check{C}\Upsilon).$$

Then, we get from (23) that

$$\begin{aligned}
D^+\|u(t)\|_{\mathfrak{p}} &\leq (\mu_{\mathfrak{p}}(\tilde{\mathbb{M}}) + \mu_{\mathfrak{p}}(\check{C}\Upsilon))\|u(t)\|_{\mathfrak{p}} \\
&\quad + r\,\|D\|_{\mathfrak{p}}\,\|u(t - \tau(t))\|_{\mathfrak{p}} \\
&\leq -\tilde{\alpha}_{\{2,\mathfrak{p}\}}\|u(t)\|_{\mathfrak{p}} \\
&\quad + \beta_{\{1,\mathfrak{p}\}} \sup_{s\in[t-\tau_0,t]} \{\|u(s)\|_{\mathfrak{p}}\},
\end{aligned} \tag{24}$$

for any $t \in [m\mathcal{T}, m\mathcal{T} + \xi)$.

By the same analysis,

$$\begin{aligned}
D^+\|u(t)\|_{\mathfrak{p}} &\leq -\alpha_{\{2,\mathfrak{p}\}}\|u(t)\|_{\mathfrak{p}} \\
&\quad + \beta_{\{1,\mathfrak{p}\}} \sup_{s\in[t-\tau_0,t]} \{\|u(s)\|_{\mathfrak{p}}\},
\end{aligned} \tag{25}$$

for any $t \in [m\mathcal{T} + \xi, m\mathcal{T} + \mathcal{T})$.

The remainder of this proof is omitted since it is similar to that in Theorem 1. ■

Remark 3: In Theorem 2, each r_i, $i \in \mathcal{I}_n$ is involved, that is, comparing with Theorem 1, Theorem 2 enjoys more information of the activation functions. As is verified in Section IV., Theorem 2 gives a less conservative result than Theorem 1.

$$\begin{aligned}
\|u(t)\|_{\mathfrak{p}} &\leq \sup_{s\in[-\tau_0,0]} \|u(s)\|_{\mathfrak{p}} e^{-(m+1)\tilde{k}_{\{1,\mathfrak{p}\}}(\xi-\tau_0)+mk_{\{1,\mathfrak{p}\}}(\mathcal{T}-\xi)+k_{\{1,\mathfrak{p}\}}(t-m\mathcal{T}-\xi)}, \\
&= \sup_{s\in[-\tau_0,0]} \|u(s)\|_{\mathfrak{p}} e^{-(m+1)\tilde{k}_{\{1,\mathfrak{p}\}}(\xi-\tau_0)+k_{\{1,\mathfrak{p}\}}[t-(m+1)\xi]}.
\end{aligned} \tag{18}$$

B. STABILIZATION OF INN WITH A TIME-INVARIANT DELAY VIA PIMC

In previous subsection, time-varying delays in the INN is addressed. When the time delay reduces to time-invariant, it is possible to establish some more interesting and more precise results. Thus, in this subsection, the stabilization problem of INN with time-invariant delays is considered via intermittent control.

Theorem 3: If Assumptions 1-2 hold, INN (4) with $\tau(t) = \tau_0$ is asymptotically stable under the intermittent control protocol (5) provided that

$$\tilde{\alpha}_{\{1,\mathfrak{p}\}} > \beta_{\{1,\mathfrak{p}\}}, \quad -\tilde{k}_{\{1,\mathfrak{p}\}}(\xi - \tau_0) + k_{\{3,\mathfrak{p}\}}(\mathcal{T} - \xi) < 0, \tag{26}$$

for any $\mathfrak{p} \in \{1, 2, \infty\}$, where $\tilde{k}_{\{1,\mathfrak{p}\}}$ is given by (13), $k_{\{3,\mathfrak{p}\}}$ is the unique positive solution of the equality $(k + \alpha_{\{1,\mathfrak{p}\}})e^{k\tau_0} = \beta_{\{1,\mathfrak{p}\}}$ with respect to k, and $\alpha_{\{1,\mathfrak{p}\}}, \beta_{\{1,\mathfrak{p}\}}, \tilde{\alpha}_{\{1,\mathfrak{p}\}}$ are given by (12).

Proof: Given an arbitrary integer m, when $t \in [m\mathcal{T}, m\mathcal{T} + \xi)$, similar to the calculation of (14), we have

$$D^+\|u(t)\|_{\mathfrak{p}} \le -\tilde{\alpha}_{\{1,\mathfrak{p}\}}\|u(t)\|_{\mathfrak{p}} + \beta_{\{1,\mathfrak{p}\}}\|u(t-\tau_0)\|_{\mathfrak{p}}, \tag{27}$$

for any $t \in [m\mathcal{T}, m\mathcal{T} + \xi)$.

Let $\varpi_1(t) = \varpi_0 e^{-\tilde{k}_{\{1,\mathfrak{p}\}}(t-m\mathcal{T})}$ with $\varpi_0 = \sup_{s\in[m\mathcal{T}-\tau_0, m\mathcal{T}]}\{\|u(s)\|_{\mathfrak{p}}\}$, then

$$\begin{aligned}&\dot{\varpi}_1(t) + \tilde{\alpha}_{\{1,\mathfrak{p}\}}\varpi_1(t) - \beta_{\{1,\mathfrak{p}\}}\varpi_1(t-\tau_0)\\ &\quad = ((-\tilde{k}_{\{1,\mathfrak{p}\}} + \tilde{\alpha}_{\{1,\mathfrak{p}\}})e^{-\tilde{k}_{\{1,\mathfrak{p}\}}\tau_0} - \beta_{\{1,\mathfrak{p}\}})\varpi_1(t-\tau_0) = 0,\end{aligned}$$

that is,

$$\dot{\varpi}_1(t) = -\tilde{\alpha}_{\{1,\mathfrak{p}\}}\varpi_1(t) + \beta_{\{1,\mathfrak{p}\}}\varpi_1(t-\tau_0). \tag{28}$$

Together (27) with (28) and Lemma 3, we have

$$\begin{aligned}\|u(t)\|_{\mathfrak{p}} &\le \varpi_1(t)\\ &= \sup_{s\in[m\mathcal{T}-\tau_0, m\mathcal{T}]}\{\|u(s)\|_{\mathfrak{p}}\}e^{-\tilde{k}_{\{1,\mathfrak{p}\}}(t-m\mathcal{T})},\end{aligned} \tag{29}$$

for any $t \in [m\mathcal{T}, m\mathcal{T} + \xi)$.

Similar to (27),

$$D^+\|u(t)\|_{\mathfrak{p}} \le -\alpha_{\{1,\mathfrak{p}\}}\|u(t)\|_{\mathfrak{p}} + \beta_{\{1,\mathfrak{p}\}}\|u(t-\tau_0)\|_{\mathfrak{p}}, \tag{30}$$

for any $t \in [m\mathcal{T} + \xi, m\mathcal{T} + \mathcal{T})$.

Let $\varpi_2(t) = \varpi_0 e^{k_{\{3,\mathfrak{p}\}}(t-m\mathcal{T}-\xi)}$ with $\varpi_0 = \sup_{s\in[m\mathcal{T}+\xi-\tau_0, m\mathcal{T}+\xi]}\{\|u(s)\|_{\mathfrak{p}}\}$, then

$$\begin{aligned}&\dot{\varpi}_2(t) + \alpha_{\{1,\mathfrak{p}\}}\varpi_2(t) - \beta_{\{1,\mathfrak{p}\}}\varpi_2(t-\tau_0)\\ &\quad = ((k_{\{3,\mathfrak{p}\}} + \alpha_{\{1,\mathfrak{p}\}})e^{k_{\{3,\mathfrak{p}\}}\tau_0} - \beta_{\{1,\mathfrak{p}\}})\varpi_2(t-\tau_0) = 0,\end{aligned}$$

that is,

$$\dot{\varpi}_2(t) = -\alpha_{\{1,\mathfrak{p}\}}\varpi_2(t) + \beta_{\{1,\mathfrak{p}\}}\varpi_2(t-\tau_0). \tag{31}$$

Together (30) with (31), and Lemma 3, we have

$$\|u(t)\|_{\mathfrak{p}} \le \sup_{s\in[m\mathcal{T}+\xi-\tau_0, m\mathcal{T}+\xi]}\{\|u(s)\|_{\mathfrak{p}}\}e^{k_{\{3,\mathfrak{p}\}}(t-m\mathcal{T}-\xi)}, \tag{32}$$

for any $t \in [m\mathcal{T} + \xi, m\mathcal{T} + \mathcal{T})$.

By the same analysis as in Theorem 1, the proof of Theorem 3 is completed. ■

Remark 4: From the proof of Theorem 1 and Theorem 3, we find that the states of the closed-loop NN satisfy the following properties. For time-varying delays with $\tau_0 = \sup_t\{\tau(t)\} < +\infty$,

$$\|u(t)\|_{\mathfrak{p}} \le \sup_{s\in[m\mathcal{T}-\tau_0, m\mathcal{T}]}\{\|u(s)\|_{\mathfrak{p}}\}e^{-\tilde{k}_{\{1,\mathfrak{p}\}}(t-m\mathcal{T})}$$

for any $t \in [m\mathcal{T}, m\mathcal{T} + \xi)$; and

$$\|u(t)\|_{\mathfrak{p}} \le \sup_{s\in[m\mathcal{T}+\xi-\tau_0, m\mathcal{T}+\xi]}\{\|u(s)\|_{\mathfrak{p}}\}e^{k_{\{1,\mathfrak{p}\}}(t-m\mathcal{T}-\xi)}$$

for any $t \in [m\mathcal{T} + \xi, m\mathcal{T} + \mathcal{T})$. But, for time-invariant delays, i.e., $\tau(t) = \tau_0$,

$$\|u(t)\|_{\mathfrak{p}} \le \sup_{s\in[m\mathcal{T}-\tau_0, m\mathcal{T}]}\{\|u(s)\|_{\mathfrak{p}}\}e^{-\tilde{k}_{\{1,\mathfrak{p}\}}(t-m\mathcal{T})}$$

for any $t \in [m\mathcal{T}, m\mathcal{T} + \xi)$; and

$$\|u(t)\|_{\mathfrak{p}} \le \sup_{s\in[m\mathcal{T}+\xi-\tau_0, m\mathcal{T}+\xi]}\{\|u(s)\|_{\mathfrak{p}}\}e^{k_{\{3,\mathfrak{p}\}}(t-m\mathcal{T}-\xi)}$$

$$\begin{aligned}D^+\|u(t)\|_{\mathfrak{p}} &= \lim_{\delta\to 0^+}\frac{\left\|u(t) + \delta\begin{bmatrix}-\Xi v(t) + w(t)\\ -Av(t) - Bw(t) + Cf(v(t)) + Df(v(t-\tau(t))) + p(t)\end{bmatrix}\right\|_{\mathfrak{p}} - \|u(t)\|_{\mathfrak{p}}}{\delta}\\ &\le \lim_{\delta\to 0^+}\frac{\left\|u(t) + \delta\begin{bmatrix}-\Xi & \mathbf{I}_n\\ -\tilde{A} & -\tilde{B}\end{bmatrix}u(t) + \delta\begin{bmatrix}0\\ Cf(v(t))\end{bmatrix}\right\|_{\mathfrak{p}} - \|u(t)\|_{\mathfrak{p}}}{\delta} + r\|D\|_{\mathfrak{p}}\|v(t-\tau(t))\|_{\mathfrak{p}}.\end{aligned} \tag{22}$$

$$\begin{aligned}D^+\|u(t)\|_{\mathfrak{p}} &\le \lim_{\delta\to 0^+}\frac{\left\|u(t) + \delta\begin{bmatrix}-\Xi & \mathbf{I}_n\\ -\tilde{A} & -\tilde{B}\end{bmatrix}u(t) + \delta\check{C}\mathcal{F}(v(t))u(t)\right\|_{\mathfrak{p}} - \|u(t)\|_{\mathfrak{p}}}{\delta} + r\|D\|_{\mathfrak{p}}\|v(t-\tau(t))\|_{\mathfrak{p}}\\ &\le \mu_{\mathfrak{p}}(\tilde{\mathbb{M}} + \check{C}\mathcal{F}(v(t)))\|u(t)\|_{\mathfrak{p}} + r\|D\|_{\mathfrak{p}}\|u(t-\tau(t))\|_{\mathfrak{p}}.\end{aligned} \tag{23}$$

for any $t \in [m\mathcal{T}+\xi, m\mathcal{T}+\mathcal{T})$. Noting that

$$0 < k_{\{3,\mathfrak{p}\}} = \beta_{\{1,\mathfrak{p}\}} e^{-k_{\{3,\mathfrak{p}\}}\tau_0} - \alpha_{\{1,\mathfrak{p}\}} < \beta_{\{1,\mathfrak{p}\}} - \alpha_{\{1,\mathfrak{p}\}} = k_{\{1,\mathfrak{p}\}}.$$

That is to say, the estimation of divergent rate during the period of no control ($t \in [m\mathcal{T}+\xi, m\mathcal{T}+\mathcal{T})$) in the intermittent control protocol for time-varying delay case is larger than that for time-invariant delay case. Because the upper estimation of convergent rate $\tilde{k}_{\{1,\mathfrak{p}\}}$ during the period of control interval ($t \in [m\mathcal{T}, m\mathcal{T}+\xi)$) in the intermittent control protocol is identical in both cases, we can conclude that Theorem 3 is less conservative than Theorem 1.

Similarly, in Theorem 3, when $\mathfrak{p}=2$, $\tilde{\alpha}_{\{1,\mathfrak{p}\}} > \beta_{\{1,\mathfrak{p}\}}$ also can be written as $\tilde{\mathbb{M}}+\tilde{\mathbb{M}}^T+2r(\|C\|_{\mathfrak{p}}+\|D\|_{\mathfrak{p}})\mathbf{I}_n < 0$. Further, Theorem 3 also only involves in the maximum value of r_i, $i \in \mathcal{I}_n$. Following from Theorem 2, the next result is obtained to reduce the conservativeness for Theorem 3.

Corollary 2: If Assumptions 1-2 hold, INN (4) with $\tau(t)=\tau_0$ is asymptotically stable under the intermittent control protocol (5) provided that

$$\tilde{\alpha}_{\{2,\mathfrak{p}\}} > \beta_{\{1,\mathfrak{p}\}},$$
$$-\tilde{k}_{\{2,\mathfrak{p}\}}(\xi-\tau_0) + k_{\{4,\mathfrak{p}\}}(\mathcal{T}-\xi) < 0,$$

for any $\mathfrak{p} \in \{1,\infty\}$, where $\tilde{k}_{\{2,\mathfrak{p}\}}$ is given by (21), $k_{\{4,\mathfrak{p}\}}$ is the unique positive solution of the equality $(k+\alpha_{\{2,\mathfrak{p}\}})e^{k\tau_0} = \beta_{\{1,\mathfrak{p}\}}$ with respect to k, $\alpha_{\{2,\mathfrak{p}\}}$, $\tilde{\alpha}_{\{2,\mathfrak{p}\}}$ are given by (20), $\beta_{\{1,\mathfrak{p}\}}$ is given by (12).

Proof: This could be referred to Theorems 2 and 3, and omitted here for brevity. ■

Remark 5: Different from the continuous control schemes in most of the existing results on dynamic analysis of INN, a non-continuous IMC strategy is proposed in this paper. The authors in [24] and [31] investigated the stabilization or synchronization problem for INN by periodically intermittent controllers, in which the corresponding criteria were obtained by applying common LFN based techniques. As mentioned above, the periodically intermittently controlled NN can be considered as a switched NN. The requirement of distinct subnetworks to share the same LFN in [24] and [31] may be conservative. In this paper, with the matrix-measure method, LFN is no longer necessary. As far as I know, few literature if not none investigated the stabilization of INN by IMC and matrix-measure method, and this implies that our results are novel and meaningful.

IV. NUMERICAL EXAMPLES

Example 1: Consider the following INN with two neurons and time-varying delays, in which the plant rule is given by

$$\ddot{v}_i(t) = -a_i^{\dagger}\dot{v}_i(t) - b_i^{\dagger}v_i(t) + \sum_{j=1}^{n} c_{ij}f_j(v_j(t)) + \sum_{j=1}^{n} d_{ij}f_j(v_j(t-\tau(t))) + p_i(t), \quad (33)$$

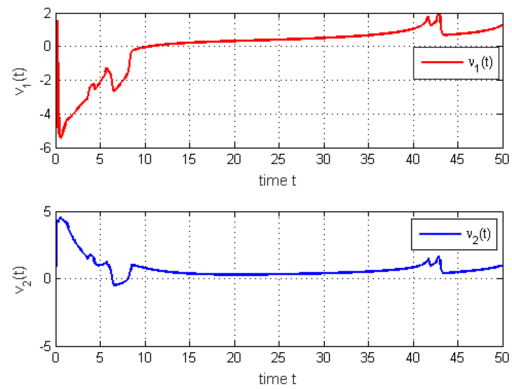

FIGURE 1. (Nonconvergent) Phase evolution of $v_1(t)$ and $v_2(t)$ for INN (33).

where parameters for (33) are given as follows. $\tau(t)=0.05+0.05\sin(t)$; $a_1^{\dagger}=6.2$, $a_2^{\dagger}=6.3$, $b_1^{\dagger}=1.4$, $b_2^{\dagger}=1.8$, $c_{11}=2.4$, $c_{12}=2.5$, $c_{21}=2.3$, $c_{22}=2.2$, $d_{11}=3.3$, $d_{12}=3.2$, $d_{21}=3.1$, $d_{22}=3.4$, and the activation function is specified as $f_1(v)=0.2\tanh(v)$ and $f_2(v)=0.1\tanh(v)$ for any $v \in \mathbb{R}$.

Let Ξ =diag{5, 5}, we can rewrite (33) into the form of (4), and the corresponding matrix as follows.

$$A = \begin{bmatrix} -4.6 & 0 \\ 0 & -4.7 \end{bmatrix}, \quad B = \begin{bmatrix} 1.2 & 0 \\ 0 & 1.3 \end{bmatrix},$$
$$C = \begin{bmatrix} 2.4 & 2.5 \\ 2.3 & 2.2 \end{bmatrix}, \quad D = \begin{bmatrix} 3.3 & 3.2 \\ 3.1 & 3.4 \end{bmatrix}.$$

At first, let $p_i(\cdot) \equiv 0$, thus,

$$\mathbb{M}_1 = \begin{bmatrix} -\Xi & \mathbf{I}_n \\ -A & -B \end{bmatrix} = \begin{bmatrix} -5 & 0 & 1 & 0 \\ 0 & -5 & 0 & 1 \\ 4.6 & 0 & -1.2 & 0 \\ 0 & 4.7 & 0 & -1.3 \end{bmatrix},$$

and

$$\alpha_{\{1,1\}} = -0.74, \quad \beta_{\{1,1\}} = 1.32, \quad k_{\{1,1\}} = 2.06,$$
$$\alpha_{\{1,2\}} = -1.22, \quad \beta_{\{1,2\}} = 1.30, \quad k_{\{1,2\}} = 2.52,$$
$$\alpha_{\{1,\infty\}} = -4.38, \quad \beta_{\{1,\infty\}} = 1.30, \quad k_{\{1,\infty\}} = 5.68.$$

The evolutions of $v_1(t)$ and $v_2(t)$ are shown in Figure 1, which shows that $v_1(t)$ and $v_2(t)$ are not convergent. Obviously, the INN (33) is not stable when no control is applied.

As such, intermittent control protocol (5) is used to stabilize (33). The parameters in (5) are given by $\eta_{11}=4.5$, $\eta_{12}=4.6$, $\eta_{21}=27.3$, $\eta_{22}=26.9$, which result in

$$\tilde{\mathbb{M}}_1 = \begin{bmatrix} -\Xi & \mathbf{I}_n \\ -\tilde{A} & -\tilde{B} \end{bmatrix} = \begin{bmatrix} -5 & 0 & 1 & 0 \\ 0 & -5 & 0 & 1 \\ 0.1 & 0 & -28.5 & 0 \\ 0 & 0.3 & 0 & -28.2 \end{bmatrix}.$$

and $\tilde{\alpha}_{\{1,1\}}=3.76$, $\tilde{\alpha}_{\{1,2\}}=4.04$, $\tilde{\alpha}_{\{1,\infty\}}=3.02$, $\tilde{k}_{\{1,1\}}=2.1272$, $\tilde{k}_{\{1,2\}}=2.3675$, $\tilde{k}_{\{1,\infty\}}=1.5084$. We have $\tau_0=0.1$ and let $\mathcal{T}=1$ as the control period, then any $\xi>0.5428$ suffices the condition (13) in Theorem 1 when using 1-norm, and any $\xi>0.5640$ and $\xi>0.8111$ also suffice

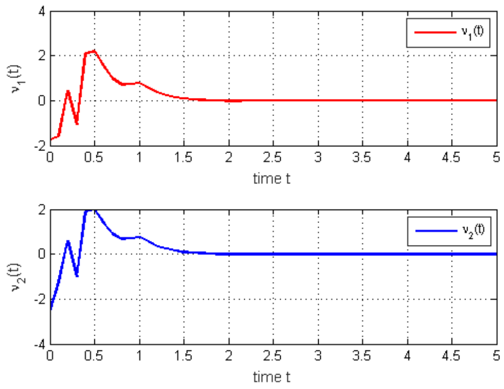

FIGURE 2. (Convergent) Phase evolution of $v_1(t)$ and $v_2(t)$ for INN (33).

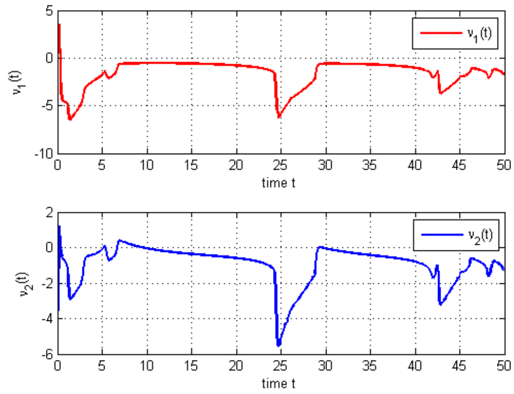

FIGURE 3. (Nonconvergent) The trajectories of state $v_1(t)$ and $v_2(t)$ for INN (33).

the condition (13) in Theorem 1 when using 2-norm and ∞-norm, respectively. Now take $\xi = 0.6$ and using 1-norm, the time responses of $v_1(t)$ and $v_2(t)$ are shown in Figure 2, which shows that the intermittent control scheme (5) designed above stabilizes the INN (33).

It is interesting to compare the results between Theorem 1 and Theorem 2. For the case of 1-norm, as the parameters obtained from Theorem 1 and Theorem 2 are identical, the minimum ξ in Theorem 2 is the same as the one in Theorem 1. However, for the case of ∞-norm, some calculations show that $\alpha_{\{2,\infty\}} = -4.13$, $\tilde{\alpha}_{\{2,\infty\}} = 3.27$, $\beta_{\{2,\infty\}} = 1.3$, then $k_{\{2,\infty\}} = 5.43$ and $\tilde{k}_{\{2,\infty\}} = 1.725$, and hence any $\xi > 0.7839$ suffices the condition (21) in Theorem 2 when using ∞-norm. Comparing with Theorem 1 ($\xi > 0.8111$), Theorem 2 ($\xi > 0.7839$) gives a less conservative criterion to stabilize INN (33). This indicates that the good use of the network information could reduce the conservatism of the stabilization criteria to some extent.

Next we verify the validity of Corollary 1 for the case $\mathfrak{p} = 2$. By some computation, $\sigma = -r(\|C\|_{\mathfrak{p}}) + \|D\|_{\mathfrak{p}}) = -2.24$. Employing the LMI Tool-box, the following feasible solution for LMIs (19a) is obtained.

$$\Omega_1 = \begin{bmatrix} 5.6000 & 0 \\ 0 & 5.7000 \end{bmatrix}, \quad \Omega_2 = \begin{bmatrix} 4.2978 & 0 \\ 0 & 4.1978 \end{bmatrix}.$$

Here, the intermittent controller gain matrix is directly derived. Further, let $\mathcal{T} = 1$ as the control period, then any $\xi > 0.6315$ suffices the condition (19) in Corollary 1 when using 2-norm.

Next, two comparative studies are given to further show the effectiveness of this work. Cao and Wan in [27] investigated stability problem of inertial BAM neural network with matrix measure strategies. To compare with [27], we remove the IMC condition (13b) in Theorem 1, and by Remark 1, the condition (13a) of Theorem 1 in this paper can be turned into $\alpha_{\{1,\mathfrak{p}\}} > \beta_{\{1,\mathfrak{p}\}}$. Then consider the example 1 of [27], we have $\alpha_{\{1,\mathfrak{p}\}} = 1.0224 > \beta_{\{1,\mathfrak{p}\}} = 0.1414$. Obviously, by Theorem 1, we demonstrate that the example 1 of [27] is globally asymptotically stable under our results.

In recent work [22], several exponential stabilization results of the complex-valued INN were presented by matrix measure method. If we restrict results of Theorem 1 in [22] to real-valued space, then the condition (13) of Theorem 1 in [22] can be converted to $k_1 = -(\mu_p(H) + 2l^R\|C\|_p) > k_2 = 2l^R\|D\|_p$. In light of Theorem 1 here, we have $\tilde{\alpha}_{\{1,\mathfrak{p}\}} > k_1$, $\tilde{\beta}_{\{1,\mathfrak{p}\}} < k_2$, which implies that the analytical results in this paper are less conservative than Theorem 1 in [22]. For the sake of comparison, considering Example 1 (33) again, let $\mathfrak{p} = 2$, we get $k_1 = 0.17 < k_2 = 2.60$ for [22]. Obviously, the condition (13) of Theorem 1 in [22] is not satisfied. That is to say, the stabilization cannot be verified by Theorem 1 of [22] under the same example.

Remark 6: Comparison studies show that our method developed in this paper can derive less conservative results than some previously reported work [22]. Note that if the IMC condition is disregarded in Theorem 1, then the results in this paper reduces to the ones in [27]. Moreover, according to Corollary 1, we can convert the stabilization criterion with the matrix measure form into the LMI-type condition for the case $\mathfrak{p} = 2$, which provides a bridge between the matrix- measure method and the Lyapunov method. Therefore, we can conclude that the theoretical results in this paper generalize and improve some existing results.

Example 2: Consider the INN (33) again with the same parameters, the only exception is that time delay in this example is constant, and $\tau(t) \equiv \tau_0 = 0.1$ for all $t \in \mathbb{R}$. Based on Example 1, if no control is applied, INN (33) is unstable, and the trajectories of state $v_1(t)$ and $v_2(t)$ are depicted in Figure 3.

Moreover, when control protocol is applied as similar to that in Example 1, INN (33) is stabilized provided that ξ is chosen appropriately. By some computation, we have $\tilde{k}_{\{1,1\}} = 2.1272$, $k_{\{3,1\}} = 1.838$, and hence any $\xi > 0.5272$ suffices the condition (26) in Theorem 3 when using 1-norm. It is obvious that $0 < k_{\{3,1\}} < k_{\{1,1\}}$, and as stated in Remark 4, we can conclude that Theorem 3 gives less conservative results than that given by Theorem 1. Now take $\xi = 0.53$, and then the state trajectories of INN (33) are shown in Figure 4.

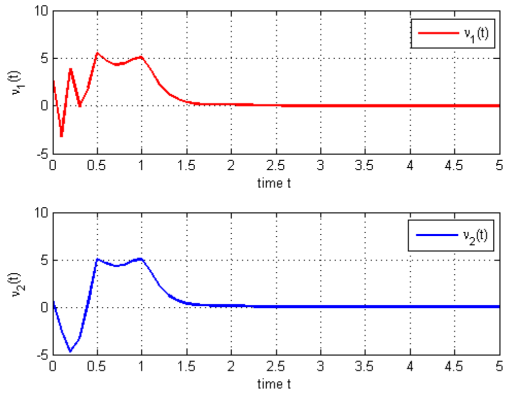

FIGURE 4. (Convergent) The trajectories of state $v_1(t)$ and $v_2(t)$ for INN (33).

Similar comparisons could also be done for the 2-norm and ∞-norm.

V. CONCLUSION

This paper addresses stabilization for a class of delayed INN by periodically intermittent control strategy. Combining with two different Halanay inequalities and matrix-measure method, novel algebraic criteria for the stabilization of the delayed INN are obtained for both time-varying delays and time-invariant delays. The obtained results show that the criterion can be more precise when the delays of the INN are time-invariant. In addition, different from the most works on dynamic analysis of INN [13], [22], [27]–[29], in which the matrix measure methods are applied, the criteria of matrix measure form proposed in this paper can be converted into LMI-type condition for the case of 2-norm for the first time, which provides a bridge between the matrix-measure method and the LFN method. The effectiveness of the proposed methods is illustrated with some illustrative examples. In particular, the obtained results generalize and develop some existing work.

More recently, investigates on distributed control method for networks have attracted widespread attention [32]–[36]. For instance, the distributed adaptive dual control approach in [32] was proposed to ensure the safe operation and reduce the energy consumption of energy Internet. Zhang *et al.* presented a distributed impulsive control strategy to cluster synchronization of delayed coupled neural networks [36]. Inspired by these well-designed works, our future research will concentrate on distributed control devise for INN.

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