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Chaos Addresses Energy in Networks of Electrical Oscillators

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ABSTRACT In this paper, a new application of active chaos is presented. It will be shown how chaotic behavior allows to address the energy transport in networks of linear oscillators. The paper is focused on RLC oscillators coupled in a network through links implemented by capacitors. The capacitances are subjected to an external input which leads to fluctuations in their values. This creates the conditions for the onset of decoherence, a situation in which a phenomenon called environment-assisted quantum transport occurs in quantum mechanics. From numerical and experimental observations, the use of active chaos to alter the value of the coupling capacitances reveals to be more effective than the use of random fluctuations. This implies that an energy control strategy for linear systems based on chaos can be outlined.

INDEX TERMS Chaos, nonlinear circuits, nonlinear dynamics.

I. INTRODUCTION

The topic of chaos has been on the last decades a dominant theme in nonlinear dynamics, as regards both the discovery of chaotic behavior in a wide class of systems, from biology to social dynamics, and the design and realization of analog electronic devices producing strange attractors, chaotic and hyperchaotic behavior [1]. Even if a wide literature exists on this topic, the active use of chaos in real applications has been mainly described in secure communication [2]–[5] and in the improvement of motion control performance [6]. The interest of researchers has been also dedicated to the use of chaos for the design of random number generators [7], especially in the discrete-time domain [8]. Within these applications, the problem of chaos synchronization achieved excellent results [9], as well as the problem of defining anti-chaos control strategies to elicit chaotic behavior for applications [10].

In this paper, a novel active application of chaotic systems is proposed. It covers the problem of electrical energy distribution and its flux control in a network, a problem that is timely also in view of microgrids, where a decentralized strategy often leads to improved performance [11].

Transport phenomena [12] observed in multiple scientific field, especially as concern quantum mechanics, have been

found to be efficiently driven in presence of dephasing and incoherence [13], a phenomenon called environment-assisted quantum transport (ENAQT). Evidences of energy transport driven by intentionally injected noise have been recently retrieved in different scenarios, not only in quantum environment, where it is at the basis of novel quantum computing paradigms [14], but also in classical mechanics, where it is used to design novel energy harvesting devices [15]. This idea of using noise for addressing energy has been successfully applied recently in [16], where coupled linear oscillators are considered. Numerical and experimental analyses unveiled the possibility to obtain an improvement of the energy transport efficiency in networks of electronic oscillators through a mechanisms similar to that happening in quantum systems.

More specifically, the experimental results discussed in [16] are related to a ring of three passive linear identical RLC oscillators coupled by capacitors, where a random time-dependent perturbation acts on one of the coupling capacitance by altering its nominal value. When a limited amount of energy is injected in the system, the effect of the random variation of the coupling capacitance is to control the directionality of the energy transfer among the oscillators. This effect depends on the topological structure of the network, i.e. which coupling capacitance is randomly varied, and on the intensity of the noise, i.e. the amplitude

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of the fluctuation around the nominal value of the coupling capacitance.

As seen in quantum systems, also in electronic oscillators the key ingredient to obtain the ENAQT effect is a decoherent environment. This can be associated to dephasing [17] or broadband oscillations. The stochastic approach considered in [16], therefore, allows to obtain the decoherent environment but lacks in controllability, as the stochastic flow cannot be adaptively fixed to gain specific transport effects. Here, instead, we focus on deterministic chaotic oscillations to alter the value of the coupling capacitance, leading to significant improvements in terms of energy transport efficiency.

The active use of chaos to induce the decoherent environment leads also to further innovative and promising results. Chaos shares with noise a broadband spectrum, yet being the outcome of a deterministic nonlinear process [18]. Even if the topic of chaos is of wide interest for scientist in several fields, the active use of chaos has limited practical applications [19], especially in cryptography [20], pseudo-random number generators [21] and evolutionary algorithms [22]. The active use of chaos for controlling the flux of energy in a network is therefore innovative and the numerical and experimental results which will be shown herein will encourage the research in this direction.

Moreover, by using chaotic signals generated by nonlinear electronic devices, it is possible to control their characteristics, as regards both their distribution and intensity. Therefore, in this paper, the importance of using chaos instead of random signals will be emphasized showing that the efficiency of the energy transport can be improved.

The paper is organized as follows: in Sec. II the mathematical model of the considered network of oscillators is introduced, thus allowing for both numerical and experimental analysis of the setup, in Sec. III a direct comparison with the results presented in [16] is given focusing, numerically and experimentally, on the case of a ring of three coupled oscillators and introducing the chaotic fluctuation in the coupling capacitance. The more general case of networks with a higher number of nodes or different topologies is dealt with in Sec. IV by means of numerical simulations. Conclusions and future directions are discussed in Sec. V.

II. THE MODEL

Let us consider the setup introduced in [16] and schematically reported in Fig. 1. It consists in N RLC oscillator coupled through the capacitors C_{nm} . The mathematical model of the network can be written as:

$$\begin{aligned} \frac{dV_n}{dt} &= -\frac{1}{C} \left[i_n + \frac{V_n}{R} + \sum_{n \neq m}^N C_{nm} \left(\frac{dV_n}{dt} - \frac{dV_m}{dt} \right) \right] \\ \frac{di_n}{dt} &= \frac{V_n}{L} \end{aligned} \quad (1)$$

where V_n is the voltage across the n -th RLC parallel, i_n is the current flowing through the inductor L of the n -th oscillator, and C_{nm} is the coupling capacitor between the n -th and the

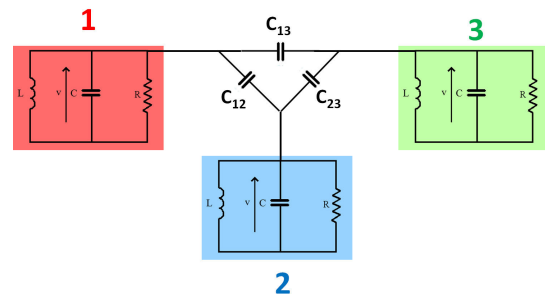


FIGURE 1. Schematic representation of the setup with $N = 3$.

m -th oscillator. Let us consider at first the case $N = 3$, thus recovering the conditions discussed in [16].

The coupling capacitance C_{12} is made time-varying as $C_{12}(t) = C_{12} [1 + \phi(t)]$. In [16], $\phi(t)$ is a random noise signal characterized by a normal distribution with zero mean, whose variance σ is the control parameter. In this paper, the signals derived from a Chua's circuit, whose implementation can be based on operational amplifiers [18], are used as $\phi(t)$.

The Chua's circuit can be represented by the following system of nonlinear differential equations:

$$\begin{aligned} \dot{x} &= \kappa [\alpha (y - h(x))] \\ \dot{y} &= \kappa [x - y + z] \\ \dot{z} &= \kappa [-\beta y] \end{aligned} \quad (2)$$

where $h(x) = m_1 x + \frac{1}{2}(m_0 - m_1)(|x + 1| - |x - 1|)$ is a piece-wise linear function and α , β , m_0 and m_1 are system parameters and κ is the time-scale of the system.

The chaotic signals produced by the Chua's circuit are zero-mean signals whose amplitude is scaled so that it corresponds to the range of a random signal with normal distribution and unitary variance. This allows to act on a single control parameter, namely the variance σ , which scales both the random signal and the chaotic signal, leading to a direct and coherent comparison of the results. Moreover, in the Sec. III, all the parameters of the system have been chosen consistently with [16] in order to allow for a direct comparison of the effect of the different choice for $\phi(t)$.

Following [16], we define the transport efficiency of the n -th RLC oscillator as

$$W_n = \frac{1}{\mathcal{N}} \int_0^\infty \left\langle \frac{V_n^2(t)}{R} \right\rangle dt \quad (3)$$

where \mathcal{N} is the total energy dissipated by all oscillators, $V_n(t)$ and R are the voltage across the capacitor and the value of the resistance of the n -th RLC oscillator.

III. NUMERICAL AND EXPERIMENTAL RESULTS

The numerical and experimental analysis, described in the following, is performed accordingly to [16], so that a single input pulse is provided to circuit 1 with a pulse duration of 200μ s. The numerical integration of system in Eqs. (1) is not straightforward, as the derivatives of the voltages appear

in the right hand side of the equations:

$$\begin{aligned}
 C \frac{dV_1}{dt} &= -i_1 - \frac{V_1}{R} - C_x [1 + \phi(t)] \left(\frac{dV_1}{dt} - \frac{dV_2}{dt} \right) \\
 &\quad - C_x \frac{dV_1}{dt} + C_x \frac{dV_3}{dt} \\
 L \frac{di_1}{dt} &= V_1 \\
 C \frac{dV_2}{dt} &= -i_2 - \frac{V_2}{R} + C_x [1 + \phi(t)] \left(\frac{dV_1}{dt} - \frac{dV_2}{dt} \right) \\
 &\quad - C_x \frac{dV_2}{dt} + C_x \frac{dV_3}{dt} \\
 L \frac{di_2}{dt} &= V_2 \\
 C \frac{dV_3}{dt} &= -i_3 - \frac{V_3}{R} - 2C_x \frac{dV_3}{dt} + C_x \frac{dV_1}{dt} + C_x \frac{dV_2}{dt} \\
 L \frac{di_3}{dt} &= V_3
 \end{aligned} \tag{4}$$

where $C_x = C_{12} = C_{13} = C_{23} = 39 \mu\text{F}$, $R = 1 \text{ k}\Omega$, $L = 1 \text{ mH}$.

Therefore, in order to numerically integrate the system and also to propose a reliable circuit implementation, the Eqs. (1) are rewritten isolating the derivatives by subsequent substitutions. Since the explicit equations are too long to be reported in the text, we refer the reader to the Appendix where an algorithm, based on symbolic computation, to determine them is described. The equations have been integrated by using the Runge-Kutta method (RK4) with fixed step size $\Delta t = 0.001 \text{ s}$.

As concern the experimental circuits, we followed the approach discussed in the Supplementary Information of [16]. The circuits, in fact, are not implemented as physical RLC oscillators, rather they are implemented using properly wired operational amplifiers TL084 and analog multipliers AD633 with voltage supply $V_+ = -V_- = 12 \text{ V}$. This solution allows to avoid the use of inductors in the circuit implementation. The conceived circuitry, even if based on active operational amplifiers, can be considered as an analogue realization of the ideal passive RLC circuits. The time-varying coupling induced by $C_{12}(t)$ in Eqs.(1) is, then, obtained through a coupling circuit based on an analog multiplier (see the Supplementary Information of [16]) that, rather than implementing a physical time-varying capacitor or a varactor, implements the coupling coefficient $C_{12}(t)$ as a voltage controlled gain. The Chua's circuit is implemented by following the scheme based on operational amplifiers reported in Fig. 2, which follows a design approach based on the state-controlled Cellular Nonlinear Networks (SC-CNN), as described in [18]. The chaotic variable is then injected in the circuits and through the coupling circuit modulates the gain implementing the value of the coupling coefficient $C_{12}(t)$.

In Fig. 3(a), the efficiencies W_n calculated by numerical integration of the system equations are reported for different values of the parameter σ , which accounts for the amplitude of the injected signal, either random (dashed lines) or chaotic (continuous lines). In particular, when using chaos we have

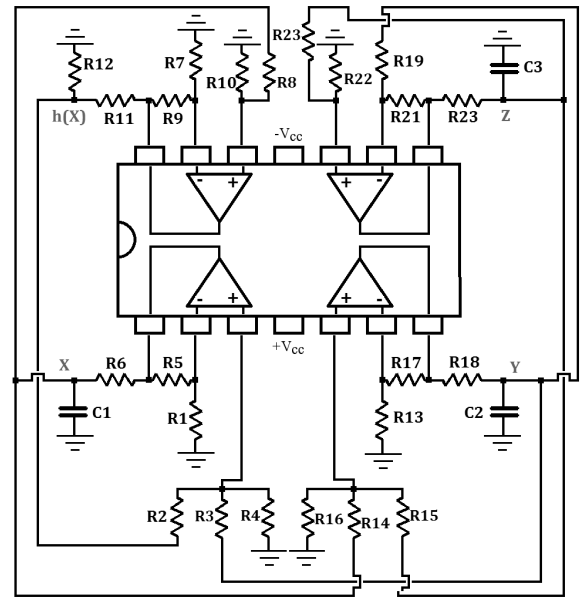


FIGURE 2. SC-CNN based circuit implementation of the Chua's equations (2). Components: $R_1 = 4 \text{ k}\Omega$, $R_2 = 13.3 \text{ k}\Omega$, $R_3 = 5.6 \text{ k}\Omega$, $R_4 = 20 \text{ k}\Omega$, $R_5 = 20 \text{ k}\Omega$, $R_6 = 380 \Omega$ (potentiometer), $R_7 = 112 \text{ k}\Omega$, $R_8 = 112 \text{ k}\Omega$, $R_9 = 1 \text{ M}\Omega$, $R_{10} = 1 \text{ M}\Omega$, $R_{11} = 8.2 \text{ k}\Omega$, $R_{12} = 1 \text{ k}\Omega$, $R_{13} = 51.1 \text{ k}\Omega$, $R_{14} = 100 \text{ k}\Omega$, $R_{15} = 100 \text{ k}\Omega$, $R_{16} = 100 \text{ k}\Omega$, $R_{17} = 100 \text{ k}\Omega$, $R_{18} = 1 \text{ k}\Omega$, $R_{19} = 8.2 \text{ k}\Omega$, $R_{20} = 100 \text{ k}\Omega$, $R_{21} = 100 \text{ k}\Omega$, $R_{22} = 7.8 \text{ k}\Omega$, $R_{23} = 1 \text{ k}\Omega$, $C_1 = C_2 = C_3 = 50 \text{ nF}$, $V_{cc} = 12 \text{ V}$.

that $\phi(t) = y(t)$ with $\alpha = 9$, $\beta = 14.286$, $m_0 = -\frac{1}{7}$ and $m_1 = \frac{2}{7}$ (i.e. the system is in the so-called double-scroll attractor [18]). To allow a direct comparison of the effects of the different choice of $\phi(t)$, the range of $y(t)$ has been scaled so that it is equal to that of a random signal taken from a gaussian distribution with unit variance (i.e. $\sigma = 1$) and the time-scale $\kappa = 1000$ has been adopted, so that the Chua's circuit operates around 1 kHz, consistently with [16]. The corresponding plots obtained from the realized experimental circuits are reported in Fig. 3(b).

It appears clear that the transport efficiency is impacted by selecting $\phi(t)$ either as a random or a chaotic signal. Moreover, the specific signal used to make time-varying the coupling capacitor $C_{12}(t)$ leads to different transport conditions. The improvement introduced by chaos, in fact, is evident noticing that a narrower fluctuation range for $C_{12}(t)$, i.e. lower values of σ , leads to a significant increase in the energy transport from oscillator 1 to oscillator 2.

This effect is arguably linked with the different distributions of the random signal and of the variable $y(t)$, reported in the histograms shown in Fig. 4(a). Another advantage of using chaos hinges on this property. Different variables of the Chua's circuit in the same operating conditions are associated with different distributions and, thus, produce different effects in terms of energy transport, as shown by the experimental results reported in Fig. 4(b). Moreover, the behavior of the Chua's circuit can be modified by acting on one of the two parameters α and β . In this way, the characteristics of the signal $\phi(t)$ can be suitably adapted to specific situations as its

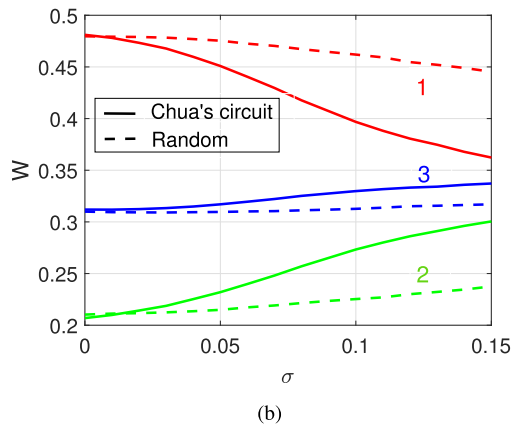
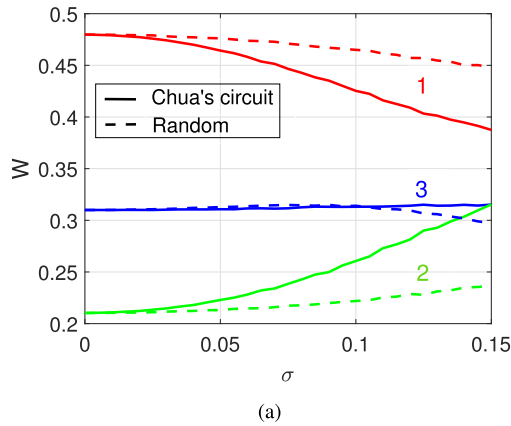


FIGURE 3. Efficiency W calculated for each RLC oscillator as a function of σ when $\phi(t)$ is a chaotic (continuous line) or a random (dashed line) signal: (a) numerical and (b) experimental results. Parameters as reported in the main text.

histogram and its amplitude can be shaped varying the two control parameters. This consideration leads us to reinforce the concept of using nonlinear deterministic analog circuits in order to control the flux of energy in the network.

With the aim of verifying the occurrence of a similar energy transport phenomenon as a byproduct of varying the RLC parameters, even in absence of fluctuations of the coupling capacitor C_{12} , we evaluated the efficiencies of the three oscillators when the quality factor of the RLC circuits is varied. In particular, we acted on the value of the resistor R to change the quality factor simultaneously in all the three oscillators. We then calculated the efficiencies when $\sigma = 0$ and $\sigma = 0.1$, with $\phi(t) = y(t)$ from the Chua's circuit. As shown in Fig. 5, a variation of R leads to a similar effect of addressing the energy transport between a pair of them. However, using chaos in modifying the value of the coupling capacitance leads to the significant capability of addressing energy transport over the whole network, including the third oscillator whose efficiency remain unaltered for $\sigma = 0$.

Moreover, we observed that the trend of the energy dissipated by resistor R in each circuit decays as a damped oscillation as shown in Fig. 6, where the transients of V_1 , V_2 , and V_3 for one of the experiments conducted are reported,

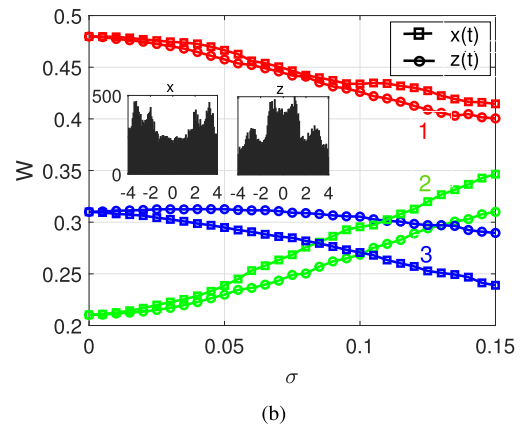
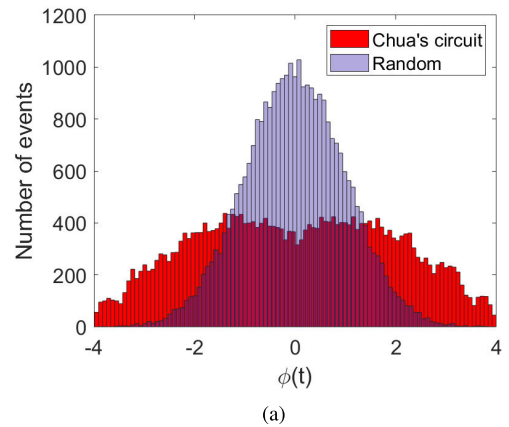


FIGURE 4. Effect of the different distributions of the signal $\phi(t)$. (a) Histograms of the scaled variable $y(t)$ (in red) and of the random noise with zero mean and unit variance (in blue). (b) Efficiency W calculated for each RLC oscillator when $\phi(t)$ is the $x(t)$ (square markers) or $z(t)$ (circle markers); in the insets the distributions of the two Chua's circuit variables $x(t)$ and $z(t)$ are reported.

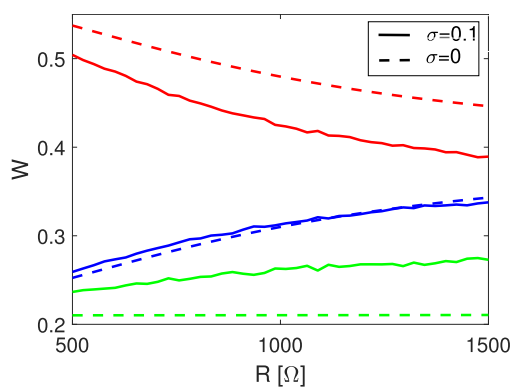


FIGURE 5. Efficiency W calculated for each RLC oscillator as a function of R when $\phi(t)$ is a chaotic signal with $\sigma = 0$ (continuous line), i.e. no fluctuations of the capacitance C_{12} and $\sigma = 0.1$ (dashed line). Parameters as reported in the main text.

together with the chaotic signal ϕ with $\sigma = 0.1$. In Fig. 7, the time-constant τ_E of the energy decay has been evaluated in the experiment as a function of σ for the three oscillators. It appears evident that, while the decay for oscillator 3 is

TABLE 1. Slope of the linear dependence of τ_E with respect to σ .

δ	$\alpha = 8.6, \beta = 12$	$\alpha = 8.6, \beta = 14.286$	$\alpha = 9, \beta = 12$	$\alpha = 9, \beta = 14.286$	random noise
$\delta^{(1)}$	-5.546	-8.406	-7.045	-6.318	-1.790
$\delta^{(2)}$	7.781	9.375	9.983	7.743	2.334

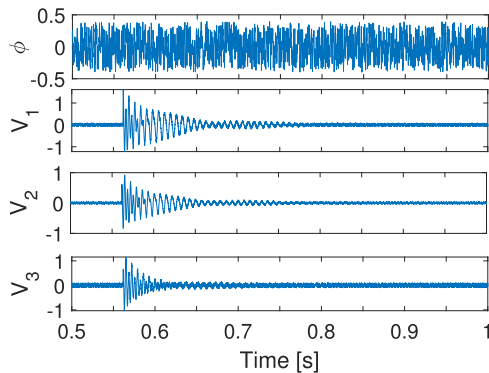


FIGURE 6. Transient of the three voltages V_1 , V_2 , and V_3 for a single experiment when the chaotic signal $\phi(t)$ is selected as the state variable $y(t)$ of the Chua's circuit with $\sigma = 0.1$.

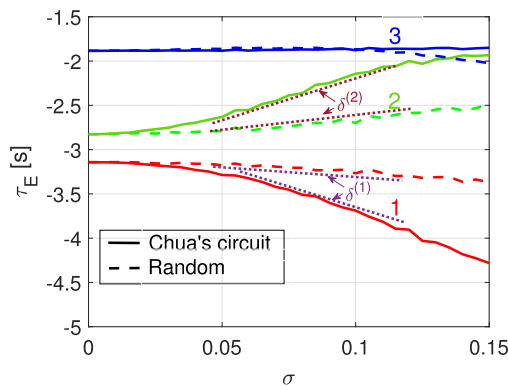


FIGURE 7. Time-constant τ_E of the energy decay in each RLC oscillator as a function of σ . The slope $\delta^{(i)}$ of the linear trend of τ_E with respect to σ is also graphically indicated.

not affected neither by the noise, nor by the chaotic signal driving $C_{12}(t)$, the decays of oscillators 1 and 2 are significantly modified. Moreover, chaos leads to a decrement of τ_E for oscillator 1, and a corresponding increment of τ_E for oscillator 2, thus showing that energy is actually transferring to oscillator 2 that need more time to damp it out.

Since the trend of the time-constant τ_E for oscillators 1 and 2 shows a region with a linear behavior with respect to σ , we can define the slope δ of this linear trend. The value of δ is therefore a measure of the increase of the energy transport in a given direction in presence of an increase of σ . We evaluated experimentally δ in the oscillators 1 and 2 for different values of α and β , choosing $\phi(t) = y(t)$, i.e. the second variable of the Chua's circuit, as reported in the Tab. 1. For the sake of comparison, we also report in the Table the value of δ obtained using as $\phi(t)$ a random signal [16].

It is, therefore, evident that the time-constant of the flux of energy can be effectively controlled by acting on the parameters of the Chua's circuit. It is also manifest that the slopes are sharper when using chaos with respect to the random noise as considered by [16]. These findings allow to determine that it is possible to aim at an adaptive control strategy for the energy dynamics of the network.

IV. ADDRESSING ENERGY IN NETWORKS OF RLC OSCILLATORS

The encouraging results obtained in addressing energy in a ring of three RLC oscillators by using chaos, lead us to attempt a generalization of the concept by considering either larger rings of RLC oscillator or more complex topologies. In this Section, we will report two sample cases, namely a $N = 4$ ring network and a $N = 4$ star topology.

A. RING OF $N = 4$ RLC OSCILLATORS

Let us consider the case of $N = 4$ RLC oscillators connected in a ring where the capacitor $C_{12}(t) = C_{12} [1 + \phi(t)]$, thus the capacitance fluctuations occur on the coupling between the oscillators 1 and 2, as in the case discussed in Sec. III. The equations of the model can be written in this case as:

$$\begin{aligned}
 C \frac{dV_1}{dt} &= -i_1 - \frac{V_1}{R} - C_x [1 + \phi(t)] \left(\frac{dV_1}{dt} - \frac{dV_2}{dt} \right) \\
 &\quad - C_x \frac{dV_1}{dt} + C_x \frac{dV_4}{dt} \\
 L \frac{di_1}{dt} &= V_1 \\
 C \frac{dV_2}{dt} &= -i_2 - \frac{V_2}{R} + C_x [1 + \phi(t)] \left(\frac{dV_1}{dt} - \frac{dV_2}{dt} \right) \\
 &\quad - C_x \frac{dV_2}{dt} + C_x \frac{dV_3}{dt} \\
 L \frac{di_2}{dt} &= V_2 \\
 C \frac{dV_3}{dt} &= -i_3 - \frac{V_3}{R} - 2C_x \frac{dV_3}{dt} + C_x \frac{dV_2}{dt} + C_x \frac{dV_4}{dt} \\
 L \frac{di_3}{dt} &= V_3 \\
 C \frac{dV_4}{dt} &= -i_4 - \frac{V_4}{R} - 2C_x \frac{dV_4}{dt} + C_x \frac{dV_1}{dt} + C_x \frac{dV_3}{dt} \\
 L \frac{di_4}{dt} &= V_4
 \end{aligned} \tag{5}$$

where $C_x = C_{12} = C_{14} = C_{23} = C_{34} = 39 \mu F$, $R = 1 \text{ k}\Omega$, $L = 1 \text{ mH}$.

The efficiencies W_n calculated by numerical integration of the system equations (5), rewritten according to the subsequent substitutions described in the Appendix, are reported

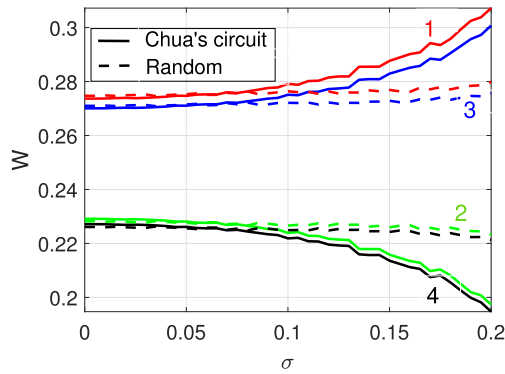


FIGURE 8. Ring network with $N = 4$ oscillators. Efficiency W calculated for each RLC oscillator as a function of σ when $\phi(t)$ is a chaotic (continuous line) or a random (dashed line) signal. Parameters as reported in the main text.

for different values of the parameter σ in Fig. 8, considering a random (dashed lines) or chaotic (continuous lines) fluctuation of $C_{12}(t)$. As in Sec. III, when using chaos we selected $\phi(t) = y(t)$ with $\alpha = 9$, $\beta = 14.286$, $m_0 = -\frac{1}{7}$ and $m_1 = \frac{2}{7}$.

It appears evident that the energy is transferred towards oscillator 1 and 3 using either chaos or random signals, but with a higher rate in the case of using the Chua's circuit chaotic behavior. Interestingly, the energy transport is controlled not only among the pair directly linked by the time-varying capacitor, but also among the oscillators symmetric to that link, i.e. between oscillator 3 and 4. This implies that the symmetry of the network plays a crucial role in the control of energy transport.

B. STAR NETWORK OF $N = 4$ RLC OSCILLATORS

Let us now investigate case in which a different, not-symmetric, topology is adopted to connect the RLC oscillators. In Fig. 9, a star network composed by $N = 4$ oscillators is shown. In this case, oscillator 1 is the hub of the network, as it is connected to all the other oscillators, while oscillators 2, 3, and 4 are the leaves, as no direct links are established among them.

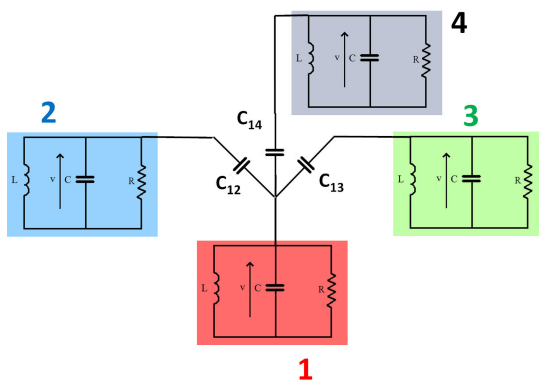


FIGURE 9. Schematic representation of the star network with $N = 4$ oscillators.

The equations of the star network can be written as:

$$\begin{aligned}
 C \frac{dV_1}{dt} &= -i_1 - \frac{V_1}{R} - C_x [1 + \phi(t)] \left(\frac{dV_1}{dt} - \frac{dV_2}{dt} \right) \\
 &\quad - 3C_x \frac{dV_1}{dt} + C_x \frac{dV_2}{dt} + C_x \frac{dV_3}{dt} + C_x \frac{dV_4}{dt} \\
 L \frac{di_1}{dt} &= V_1 \\
 C \frac{dV_2}{dt} &= -i_2 - \frac{V_2}{R} + C_x [1 + \phi(t)] \left(\frac{dV_1}{dt} - \frac{dV_2}{dt} \right) \\
 L \frac{di_2}{dt} &= V_2 \\
 C \frac{dV_3}{dt} &= -i_3 - \frac{V_3}{R} - C_x \frac{dV_3}{dt} + C_x \frac{dV_1}{dt} \\
 L \frac{di_3}{dt} &= V_3 \\
 C \frac{dV_4}{dt} &= -i_4 - \frac{V_4}{R} - C_x \frac{dV_4}{dt} + C_x \frac{dV_1}{dt} \\
 L \frac{di_4}{dt} &= V_4
 \end{aligned} \tag{6}$$

where $C_x = C_{12} = C_{13} = C_{14} = 39 \mu F$, $R = 1 k\Omega$, $L = 1 mH$.

The numerical integration of the equations (6), again rewritten according to the guidelines reported in the Appendix, allows to evaluate the efficiencies W_n for different values of the parameter σ , as reported in Fig. 10, considering a random (dashed lines) or chaotic (continuous lines) fluctuation of $C_{12}(t)$, other parameters are set as in Sec. III.

Besides confirming the higher performance linked to the use of chaos, the results reported in Fig. 10 show that the energy is transferred between the hub and the leaf actually connected by the time-varying capacitance, while keeping constant and identical the energy in the other two leaves, thus confirming the key role of the network topology. This result can be straightforwardly generalized to star networks with a single hub and a general number N of leaves, as the time-varying capacitance acts only between the hub and the given leaf.

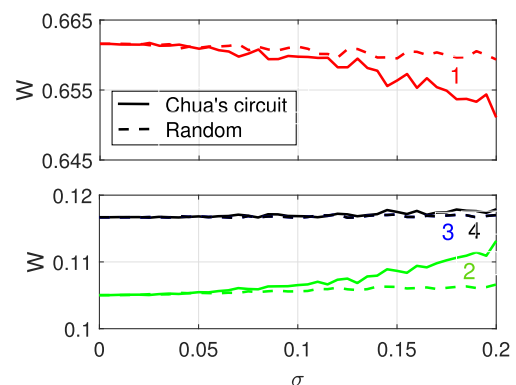


FIGURE 10. Star network with $N = 4$ oscillators. Efficiency W calculated for each RLC oscillator as a function of σ when $\phi(t)$ is a chaotic (continuous line) or a random (dashed line) signal. Parameters as reported in the main text.

V. CONCLUSION

Starting from the inspiring results shown in [16], it has been remarked that chaotic signals can be usefully adopted for particular applications, such as when energy addressing in electrical networks is efficiently required. Moreover, the main advantage of using such type of signals is the capability of controlling not only the directionality of the energy flux, but also the time-constant of the energy distribution, by acting on the parameters of the chaotic system adopted. This occurs thanks to the fact that the shape of the histograms of the chaotic signals can be directly tamed.

We remark that the system results in a time-varying system, with coupling capacitances varying according to a chaotic dynamics. While the RLC network remains globally stable, as all state variables of the RLC oscillators reach the steady state in a given time, since the chaotic dynamics controlling the value of the time-varying coupling capacitances is not affected by the network behavior, it evolves along a chaotic trajectory. For this reason, the global largest Lyapunov exponent is that of the chaotic system used for controlling the network.

It has been here reported the case of the chaotic Chua's circuit, moreover the findings are quite general and other chaotic circuits lead to analogue results. If we consider a ring consisting of more than three oscillators, the energy can be addressed also involving oscillators not directly coupled by the time-varying capacitance. In the perspective of complex interconnected networks, it is possible to use synchronized chaos generators acting on different coupling capacitance, thus leading to a generalization of the results which enforces the concept that chaos can contribute to energy management.

Even if energy is also partially addressed when the parameters of the oscillators are varied, the use chaos in modifying the value of one or more coupling capacitances allows to address the energy transport over the whole network.

The energy transport, in fact, occurs as the result of the interplay among the location of the time-varying capacitors, the way the time-varying capacitors actually vary and the topology of the network itself. Symmetries in the network allow to reflect energy transport control also between oscillators not connected by time-varying capacitors. This allows to control the energy among distant pairs in the network by acting on a single pair, thus reducing the cost of the control. As concerns not-symmetric parts of the network, two time-varying capacitors must be used, controlled by two synchronized chaotic circuits. This represents a further fundamental advantage of using chaos, as it can be synchronized while random signals cannot.

It has been shown that, using chaos, energy can be addressed with lower amplitude signals with respect to the use of a random noise, i.e. smaller fluctuations of the time-varying capacitance. This is in agreement with the concept of chaos control, where the amplitude of the control signal must be as small as possible, that is: controlling the behavior of the system without forcing it.

It is intriguing to note that the flux energy in linear electrical networks can be suitably controlled with a nonlinear network implementing a chaotic circuit, thus remarking that this application represents a novel type of active use of chaos.

APPENDIX DERIVING THE EXPLICIT FORMULATION OF NETWORK EQUATIONS

In order to determine an explicit formulation of the Eqs. (1), a MATLAB code, based on the use of the Symbolic Toolbox, can be adopted. Here, we report the code used to determine the explicit formulation of Eqs. (4). The code can be generalized by including successive substitutions.

```
syms C R C12 C13 C23 v1p v2p v3p ...
... v1 v2 v3 i1 i2 i3 phi real
vv1p=solve(C*v1p+i1+v1/R+C12*...
... (v1p-v2p)*(1+phi)-C13*v3p+C13*v1p,
v1p);
vv2p=solve(C*v2p+i2+v2/R-C12
*(vv1p-v2p)...
... *(1+phi))-C23*v3p+C23*v2p, v2p);
vv1p=subs(vv1p, v2p, vv2p);
vv2p=subs(vv2p, v1p, vv1p);
vv3p=solve(C*v3p+i3+v3/R-C13*v3p+...
... C23*v3p+C13*v3p-C23*vv2p-C13*vv2p,
v3p);
vv1p=subs(vv1p, v3p, vv3p);
vv2p=subs(vv2p, v3p, vv3p);
vv3p=subs(vv3p, v1p, vv1p);
vv3p=subs(vv3p, v2p, vv2p);
simplify(vv1p)
simplify(vv2p)
simplify(vv3p)
```

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