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Efficient Multi-Start With Path Relinking Search Strategy for Transmission System Expansion Planning

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ABSTRACT Transmission expansion planning is a complex problem that deals with the selection of new transmission lines that guarantee meeting future demand/generation and technical limits with the minimal investment cost. The transmission expansion planning problem has been solved through approaches and techniques aimed at reducing the computational effort required for its solution. Nevertheless, finding the optimal solution or even good-quality solutions for large-scale transmission systems is still challenging. In that context, an efficient multi-start with path relinking search strategy for the transmission expansion planning problem is proposed. The proposed strategy has two phases: constructive phase and local search. In the former, the multi-start applies a diversification process to guide the search along different regions to obtain good-quality solutions. Then, the local search phase executes an intensive search in the neighborhood of the best feasible solutions found in the constructive phase. The intensification process is performed in two steps: application of the Villasana-Garver-Salon algorithm in the best solutions after consecutive removal of transmission lines and path relinking using elite solution pairs. Tests performed using data from four systems show the efficiency of the proposed search strategy. Thus, the optimal solutions were obtained with a very low computational effort.

INDEX TERMS Multi-start metaheuristic (MSM), path relinking (PR), power system optimization, transmission expansion planning (TEP).

NOTATION

The notation used throughout this paper is reproduced below for quick reference.

Indices:

i	Index for buses.
ij	Index for corridors.

Sets:

Ω_b	Set of all network buses.
Ω_l	Set of all corridors.
Ω_l^0	Set of all existing transmission lines (lines in the initial configuration plus the lines added during the iterative process).

Function:

υ	Investment to build new transmission lines.

Constants:

k_1	Number of solutions obtained in the constructive phase.
k_2	Number of solutions employed in the local search phase I.
k_3	Number of solutions employed in the local search phase II (path relinking).
NoC	Number of corridors.
γ_{ij}	Susceptance for a line in corridor ij.
c_{ij}	Cost of a new line in corridor ij.
d_i	Load demand at bus i.

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$\bar{f}_{ij}$	Maximum power flow allowed for a line in corridor ij.
$\bar{g}_{ij}$	Maximum generation at bus i.
$\bar{n}_{ij}$	Maximum number of lines that can be built in corridor ij.
n_{ij}^0	Number of existing lines in corridor ij.
Δ	Maximum random percentage variation in line costs c_{ij}.
θ_{ref}	Angular reference.
g_i	Generation at bus i (without rescheduling).
Variables:	
θ_i	Voltage angle at bus i.
f_{ij}^0	Power flow in the existing lines of corridor ij.
f_{ij}	Power flow in candidate lines of corridor ij.
n_{ij}	Number of lines to be added in corridor ij.
g_i	Generation at bus i (with rescheduling).

I. INTRODUCTION

Transmission expansion planning (TEP) is a classic problem for the optimization of electrical transmission systems (ETS). TEP aims the minimization of investment costs related to the installation of new transmission lines (TL) and transformers in the ETS such it properly operates for the new load and generation defined in the planning horizon. New contributions related to the TEP problem can be made through research in mathematical modeling or in the solution technique.

In the TEP problem, the simplest mathematical modeling considers only constraints related to Kirchhoff's laws and the operating limits of the TL and the voltage level in the system buses. The network modeling related to Kirchhoff laws can be represented with different levels of precision, such as the transport model, the hybrid model, the DC model (or its linear disjunctive equivalent model), and the AC model. Currently, the AC model is considered the ideal mathematical model, but the DC model is still used when additional operational constraints are taken into account as in the proposals presented in [1]–[3]. Details related to those mathematical models can be found in [4]–[5]. Regarding the planning horizon, the TEP problem can be considered static (only one horizon) or multistage, in which the planning horizon is separated into several stages. Also, the TEP problem may include additional requirements, such as planning considering contingencies (N-1), additional reliability criteria, electric market requirements, and the integration of renewable generating sources, among others. Therefore, the TEP problem can present more complex modeling. Thus, in [6], a TEP model with electric market constraints is presented; in [7], a mathematical model is presented that incorporates HVAC/HVDC links, security constraints, and power losses and, in [8], a detailed disjunctive linear mathematical model of the multistage TEP problem is presented to reduce the search space and incorporating security constraints and, in [9], the proposal to use linear sensitivity factors in the TEP problem appears as a way to incorporate security constraints.

A new modeling proposal regarding the TEP problem suggests incorporating energy storage systems as an expansion element in addition to adding transmission lines. Thus, in [10], a TEP model is presented considering line losses and the allocation and sizing of energy storage systems, and the results show that expansion plans with lower costs can be found. Also, in [11], a modeling proposal to incorporate energy storage systems as an expansion element is presented, and the results found are promising. Thus, storage systems are consolidating as elements of expansion in the TEP problem, and this trend is expected to increase in the future as the costs of this type of device decrease. In [12], the mathematical modeling that considers multiple generation scenarios appears, and this model can be used for real cases or to solve reliability problems that represent generation uncertainties through generation scenarios.

In [13], an AC model of the TEP problem is presented that considers contingencies (N-1), and this model is solved in two stages. In the first stage, the DC model is used, as a way to circumvent the complexity of the solution process. In [14], the traditional AC model of the TEP problem is also used. In [15], the impact of incorporating uncontrolled generation in the TEP problem is analyzed, especially solar and wind generation, in addition to comparing the performance of DC and AC models. In [16], a model of the TEP problem is presented that considers probabilistic reliability criteria related to the problem's uncertainties. Thus, two probabilistic reliability criteria are used as constraints. In [17], an integrated disjunctive linear model is presented to solve the TEP problem and the natural gas network problem. This modeling considers the contingencies (N-1) for both systems, in addition to transforming the non-linear model into an equivalent linear model. In all these high complexity models, low complexity test systems are used, that is, systems with expansion plans that add a reduced number of transmission lines in the optimal solution.

To solve the TEP problem, many optimization techniques have been presented in the specialized literature. They can be categorized into heuristics, metaheuristics, and classical techniques that solve the complete mathematical model. Important heuristics can be found in [18]–[20]. Among applications that use metaheuristics, the proposals presented in [21] showed an efficient scatter search method with a genetic algorithm (GA) for the TEP problem. The proposed method was tested in Garver 6-bus and IEEE 24-bus test systems. The proposed method was validated through a comparison with conventional methods such as GA, simulated annealing (SA), and tabu search (TS) in terms of solution quality and computational time. The authors in [22] also presented a hybrid method with the application of an ant colony optimization search algorithm to solve the static TEP problem including security constraints. The proposed method presented a better performance in comparison with conventional GA and TS methods in computational effort, solution quality, and stable-convergence characteristics.

The TEP problem is characterized by the presence of a very large search space. Therefore, in many works, the first step aims to reduce the search space; then, a local search is performed in the reduced search space. In this context, [23] presented a constructive heuristic algorithm (CHA) to identify the most relevant routes to reduce the search space and increase the efficiency of the particle swarm optimization (PSO). Reference [24] followed the same line because it also used a CHA to reduce the search space and applied PSO in the reduced search space. This hybridization between CHA and PSO identified good-quality solutions in the tests presented. Following the same line again, in [25], a reduced set of routes was obtained through the heuristic's information to identify relevant paths for the TEP problem and reduce the search space. In the next step, firefly optimization used heuristic information to find the optimal solution.

Reference [26] presented an efficient GA to solve static and multistage TEP problems. Results using Garver 6-bus, IEEE 24-bus, and Southern Brazilian 46-bus test systems showed that the proposed GA solved a smaller number of linear programming (LP) problems to find the optimal solutions. The study in [27] showed a high-performance hybrid GA to solve static and multistage TEP problems. The proposed algorithm was able to find high-quality solutions for all test systems. An enhanced GA also was applied to solve the long-term TEP problem in [28]. Results using Southern Brazilian 46-bus and Colombian 93-bus systems showed that the proposed enhanced GA presented higher efficiency in solving the static and multistage TEP problems. Finally, [29] showed a methodology to solve the TEP problem considering active power losses. The problem was formulated as a mixed-integer nonlinear programming (MINLP) problem. The Chu-Beasley genetic algorithm (CBGA) was used to solve the TEP problem.

Nowadays, researchers use efficient general-purpose mathematical optimization solvers to solve the TEP problem. Proposals that use classical optimization are presented in [1]–[3], [30], and [31].

This paper's contribution is in the solution technique aspect. Thus, to demonstrate the performance of the solution technique, the DC model is adopted because it allows one to find the optimal solution in problems of reasonable size using the disjunctive linear model, whose optimal solution is the same as the DC model. Thus, the optimal solution, for comparison purposes, can be found by solving the disjunctive linear model. Thus, the multi-start metaheuristic (MSM) applied to the TEP problem is presented.

Regarding the first steps of the optimization process, metaheuristics can be separated into two groups. In the first group, there are metaheuristics such as TS, SA, and GA, where the optimization process is carried out based on a proposal for an initial solution or a set of initial solutions, and the search space is explored according to the logic of each metaheuristic until a stopping criterion is fulfilled. A critical flaw of those metaheuristics is that, in highly complex problems and with combinatorial search spaces, a final solution depends on the initial solution or the set of initial solutions. Moreover, there are also stagnation problems. In addition, in most of them, the initial solution or set of initial solutions are randomly generated. The second group includes the MSM, in which an optimization process is repeated many times and very swiftly, using different proposals for initial solutions. In this case, a good-quality initial solution is generated and improved using a local search strategy or more sophisticated forms of optimization. This process is repeated through a specified number of iterations with another starting solution. As each search process is very fast, this optimization proposal does not suffer from stagnation and has low dependency on the initial solution, as it is repeated many times. An obvious criticism is that very fast optimization processes can hardly find optimal or almost optimal solutions and that the same strategy used only with different initial solutions can hardly exploit the search space properly. Therefore, an efficient multi-start proposal is one that bypasses these criticisms. Thus, a multi-start proposal can be very efficient if each initial solution generated is significantly different from the other generated solution proposals, and all of these proposals must be of excellent quality. Finally, it should be mentioned that GRASP is the best-known MSM. Additional literature on MSM can be found in [32] and [33].

The bibliography related to the TEP problem is very extensive. Thus, the most active topics are related to reducing the problem search space and incorporating uncertainties into the TEP problem. Among the most important references in these topics in [34], a proposal for reducing the search space to more efficiently solve the TEP problem is presented. In [35], a robust adaptive optimization model is presented, separating the problem into two structures related to investment and operation decisions. In [36], a model is presented that considers the uncertainties in generation and demand, with the ability to consider the correlation between the sources of uncertainty, and, in [37], a robust technique is presented that considers the uncertainties, short-term and long-term, of the demand and the intermittent renewable generation. Additionally, in [38], the TEP problem is solved for the multistage AC model and considering contingencies (N-1). To get around the complexity of the model, the proposed solution has four stages, and in two of these stages, the DC model is used. In [39], a significantly different mathematical model is presented in which the possibility of reconductoring transmission lines with the possibility of changing the voltage level appears, in addition to considering the addition of series compensation devices as an expansion element. In relation to constructive type proposals, in [40], a two-stage constructive strategy is presented where the sensitivity indicator is found solving nonlinear programming problems, and in [41], the proposal to solve the TEP problem with security constraints appears and also considers load uncertainties. The modeling proposal is solved using a constructive metaheuristic, which is a genetic algorithm with specialized operators for the TEP problem.

In this work, an MSM with the capacity to generate initial good-quality solutions with great diversity among them to solve the TEP problem is presented. This proposal is used for the static planning problem and the DC model so that the optimality of the found solutions is verifiable by solving the same problem using the disjunctive linear model. In summary, the main contributions of this work are the following:

- The formulation of the specialized MSM metaheuristic for the TEP problem. To the authors' knowledge, the MSM has not yet been used to solve the MSM problem but has been used successfully in the field of operations research.
- The idealization of an efficient strategy that allows the generation of excellent quality solution proposals in the MSM construction phase. This proposal can be incorporated into other metaheuristics.
- The formulation of a generic MSM proposal that can be easily extended to more complex models of the TEP problem.

In the rest of this paper, Section 2 presents the DC model and the hybrid Villasana-Garver-Salon (VGS) model, as well as the way it is used via multi-start with path relinking – MSPR. The proposed methodology consists of three steps: the Constructive Phase, Local Search Phase I, and Local Search Phase II. In Section 3 the proposed method is applied in four standard test systems: 6-bus Garver, 24-bus IEEE, 46-bus South Brazilian, and Colombian 93-bus. After that, the quality of the solution is verified using the CPLEX solver, and the obtained results are compared with other works of the specialized literature. Finally, the conclusions of this paper are presented in Section 4.

II. MATHEMATICAL MODELING AND SOLUTION STRATEGY

The DC model is still widely used in TEP and is adopted in this work. However, the DC model is a highly complex MINLP mathematical model. For this reason, metaheuristics are still widely employed to solve the TEP problem when using the DC model.

This section presents the DC and the linear hybrid models for the TEP problem. The linear hybrid model (LHM) is integrated into the MSM as an auxiliary model within the generalized CHA and in a local search. The DC model is presented here to contextualize the problem. It should be noted that metaheuristics do not always employ the mathematical model to solve a complex problem. In the constructive phase, the modified hybrid model provides a sensitivity index. Thus, the constructive phase is a generalization of the CHA of the VGS algorithm [18].

After introducing the mathematical models, the MSM is presented to generate solution proposals with high diversity and quality as a strategy to explore the search space to obtain good-quality solutions. The path relinking (PR) metaheuristic is used in the intensification process to explore regions that connect pairs of elite solutions.

A. DC AND LINEAR HYBRID MODELS

The DC model takes the following form in TABLE 1–TABLE 2, according to [4]. The objective function (1) corresponds to the expansion cost, while the set of constraints (2) represents Kirchhoff's current law for each bus of the ETS; (3) is an approximation of Kirchhoff's voltage law (KVL) for each fundamental loop formed by a TL and the grounded loads. Constraint (4) limits the power flow through each TL, and (5) limits the generation capacity. Moreover, (6) limits the addition of TLs in each path, (7) sets the phase angle at the reference bus, and (8) states the integer nature of the decision variable for the number of TL, n_{ij}. The DC model is a difficult to solve MINLP problem for large-size and highly stressed systems.

The linear hybrid model (LHM) for the TEP problem is presented in (9)-(17). The fundamental difference in the model, regarding the DC formulation, is that, for each path, the flows on existing lines are separated from the candidate line flows. This separation is done because, in the LHM, the fundamental loops formed by existing lines in the base topology must comply with the KVL, whilst the candidate lines are not required to comply with the KVL. This change in the modeling turns the LHM into a mixed-integer linear programming problem (MILP) and; therefore, renders it much easier to solve.

$$\min\ v = \sum_{ij\in\Omega l} c_{ij} n_{ij} \tag{1}$$

$$\text{subject to:} \sum_{ji\in\Omega_l} f_{ji} - \sum_{ij\in\Omega_l} f_{ij} + g_i = d_i \quad \forall i \in \Omega_b \tag{2}$$

$$f_{ij} = \gamma_{ij}\left(n_{ij}^0 + n_{ij}\right)\left(\theta_i - \theta_j\right) \quad \forall ij \in \Omega_l \tag{3}$$

$$\left|f_{ij}\right| \le \left(n_{ij}^0 + n_{ij}\right)\overline{f}_{ij} \quad \forall ij \in \Omega_l \tag{4}$$

$$0 \le g_i \le \bar{g}_i \quad \forall i \in \Omega_b \tag{5}$$

$$0 \le n_{ij} \le \bar{n}_{ij} \quad \forall ij \in \Omega_l \tag{6}$$

$$\theta_{ref} = 0 \tag{7}$$

$$n_{ij}\ integer \quad \forall ij \in \Omega_l \tag{8}$$

It is worth mentioning that, if the optimal solution of the LHM is found, this optimal solution will generally be infeasible for the DC model. Therefore, this work does not intend to find the optimal solution for the hybrid model. The fundamental idea is to use a relaxed and modified version of the LHM to be incorporated into a constructive strategy to generate good-quality solutions for the DC model. This proposal consists of a generalization of the constructive heuristic proposal of the VGS algorithm.

The VGS heuristic algorithm employs a modified version of the LHM as a sensitivity index of the CHA. A CHA is an algorithm that adds, at each step, a line to the TEP problem. The line identified by the sensitivity index is added to the electrical system. Thus, the VGS algorithm solves the modified and relaxed LHM to quantify the sensitivity index and identifies the line that should be added to the system at each step.

$$\min\ v = \sum_{ij \in \Omega l} c_{ij} n_{ij} \tag{9}$$

$$\text{subject to: } \sum_{ji \in \Omega_l} f_{ji} - \sum_{ij \in \Omega_l} f_{ij} + \sum_{ji \in \Omega_l^0} f_{ji}^0 - \sum_{ij \in \Omega_l^0} f_{ij}^0 + g_i = d_i \quad \forall i \in \Omega_b \tag{10}$$

$$f_{ij}^0 = \gamma_{ij} n_{ij}^0 \left(\theta_i - \theta_j\right) \quad \forall ij \in \Omega_l^0 \tag{11}$$

$$\left|f_{ij}^0\right| \leq n_{ij}^0 \bar{f}_{ij}^0 \quad \forall ij \in \Omega_l^0 \tag{12}$$

$$\left|f_{ij}\right| \leq n_{ij} \bar{f}_{ij} \quad \forall ij \in \Omega_l \tag{13}$$

$$0 \leq g_i \leq \bar{g}_i \quad \forall i \in \Omega_b \tag{14}$$

$$0 \leq n_{ij} \leq \bar{n}_{ij} \quad \forall ij \in \Omega_l \tag{15}$$

$$\theta_{ref} = 0 \tag{16}$$

$$n_{ij}\ integer \quad \forall ij \in \Omega_l \tag{17}$$

The LHM (9)-(17) is transformed through the following strategies:

1. The integrality of the TL is relaxed; therefore, the variables n_{ij} can be continuous, and the model becomes an easy-to-solve LP problem, even for large-size systems.
2. The lines that must satisfy the KVL are those existing in the base topology, as well as those added throughout the iterative process. Therefore, a line added during the iterative process will obey the KVL in the following step.

After adding some TL, if the solved LP indicates $\upsilon = 0$, then the system does not need new TL additions, and most importantly, the proposed solution will also be feasible for the DC model. It is important to emphasize that the VGS strategy employs the modified LHM to find a good-quality solution for the DC model. In addition, VGS CHA is the most efficient heuristic when the DC model is used.

B. MULTI-START METAHEURISTIC WITH PATH RELINKING

An MSM performs a specified number of iterations. In each iteration, an initial solution proposal is generated in the constructive phase, and it is improved through a local improvement strategy. MSPR strategy, there are three consecutive steps:

1. A set of good-quality and diverse feasible solutions is generated in the construction phase using the generalized VGS algorithm;
2. A subset of the solutions found in the construction phase goes through a local search process;
3. Finally, a PR search process is applied to a subset of the best solutions found in the previous step.

After finishing the optimization process, the obtained solution is represented by the incumbent, i.e., the best solution found during the whole iterative process.

The proposal presented follows the suggestion of [32]–[33], in which every metaheuristic must have an appropriate balance between diversification and intensification throughout the search process. This balance is critical to generate high-quality solutions and find the overall optimal solution.

Diversification is important to explore the search space by reducing the possibility that the algorithm will stagnate in local optimal solutions. Therefore, the chance of finding the optimal solution of the problem under analysis increases. The intensification aims to improve the solutions obtained in the construction phase in search of the optimal solution. The MSM usually incorporates powerful forms of diversification to generate feasible solutions. Without the diversification strategy, this method can remain confined to small regions of the solution space [32].

In the local search, Phase I and Phase II, intensification methods involving the application of the VGS algorithm in the best solutions after consecutive removal of TL along with PR are used. The MSPR strategy proposed in this paper assumes three main phases:

1. ***Constructive Phase***: Application of the constructive phase of the MSM. A generalized CHA is used to generate k_1 feasible solutions from the step-by-step addition, of TL in the initial configuration.
2. ***Local Search Phase I***: The best k_2 solutions obtained in the construction phase are selected – those with the lowest cost. A local search is implemented to improve the quality of each solution proposal.
3. ***Local Search Phase II***: PR application. k_3 elite solutions from the previous step are selected. From this set, two solution proposals are formed to perform a local search. One proposal is called the initial solution, and the other is the guide solution. The process starts from the initial solution toward the guide solution and vice versa. Along this route, the objective is to find high-quality solutions.

These three phases of the proposed MSPR method are described in more detail.

1) CONSTRUCTIVE PHASE: THE GENERALIZED CONSTRUCTIVE HEURISTIC ALGORITHM OF VILLASANA-GARVER-SALON

The CHA is an optimization technique that, in a step-by-step process, generates a good-quality feasible solution for a complex problem. At each step, the CHA selects a component of the solution being built and, in the last step, it ends with a feasible solution. The element chosen in each CHA step is defined using a sensitivity index, which points out the most convenient TL to be incorporated into the solution under construction. The fundamental difference among the numerous CHA employed to solve the same problem is the adopted sensitivity index.

The VGS CHA is used to generate feasible solutions in the MSM constructive phase for the DC model. Thus, an LP is solved by relaxing the integer variables n_{ij} in (9)-(17) to find the optimal non-integer solution for the current configuration. The VGS algorithm generates only one solution proposal. To generate many good and diverse solution proposals, a simple but very efficient strategy is applied. Whenever the constructive phase is executed to generate an initial solution, a random perturbation in the costs of the TL is performed using the relation $c_{ij} \leftarrow c_{ij}\,(1 \pm \Delta \centerdot rand())$ where Δ is the maximum random percentage variation in line costs c_{ij}, and $rand()$ is a function for generating pseudo-random numbers in the interval [0, 1]. Each randomly generated $(1 \pm \Delta \centerdot rand())$ disrupts the cost of a TL by making the line more or less attractive in the generation of a solution proposal. After solving each LP in the construction phase, the variables n_{ij} are known, which allows one to find the power flow in each line added in the LP solution.

The sensitivity index chosen here corresponds to the flow through the circuits with $n_{ij} \neq 0$ in the LP solution. At each step of the CHA, the line to be added to the ETS is identified by the sensitivity index, $SI_{ij^{wp}}$, defined by II-B2:

$$SI_{ij^{wp}} = max\left\{n_{ij}.\bar{f}_{ij}\right\} \quad \forall n_{ij} \neq 0,\ ij \in \Omega_l \tag{18}$$

The winning path (wp) ij^{wp}, i.e., the path ij with the higher power flow is the most attractive one to add the new TL. At each CHA step, the current topology must be updated. The current topology is formed by the initial topology and the lines added during the iterative process.

The algorithm attempts to solve TEP and satisfy the constraints of the mathematical model only by using the existing lines in the current topology. Adding new lines is required only when the existing lines in the current topology are insufficient to solve the operation problem.

A contribution of the VGS CHA is the requirement that all lines of the current topology (with the lines added in the iterative process) must obey Kirchhoff's laws. With this strategy, it is possible to find a good and feasible topology for the DC model.

In summary, the CHA proposes to add a line to the current topology on the most attractive path. This strategy is repeated by adding, at each step, the most attractive line. The process ends when the LP solution for the current topology has $n_{ij} = 0$ and $\upsilon = 0$. At this point, TL additions are no longer required, and the set of additions made corresponds to a feasible solution for the DC model. The three steps of the VGS CHA are as follows:

1. Assume the initial configuration n_{ij}^0 as the current one.
2. An LP problem is solved by relaxing the integrality of the integer variables n_{ij} in (9)-(17) in the current topology. If the value of the objective function is zero ($\upsilon = 0$) stop; a feasible solution of good quality was found for the DC model.
3. Calculate the power flows in all-new lines. Through the sensitivity index (18), the winning path ij^{wp} with the higher sensitivity index $SI_{ij^{wp}}$ is identified. The current topology is updated by adding a line in that path. Go back to step 2.

Fig. 1 shows the flowchart of the MSM constructive phase with the steps of the VGS CHA.

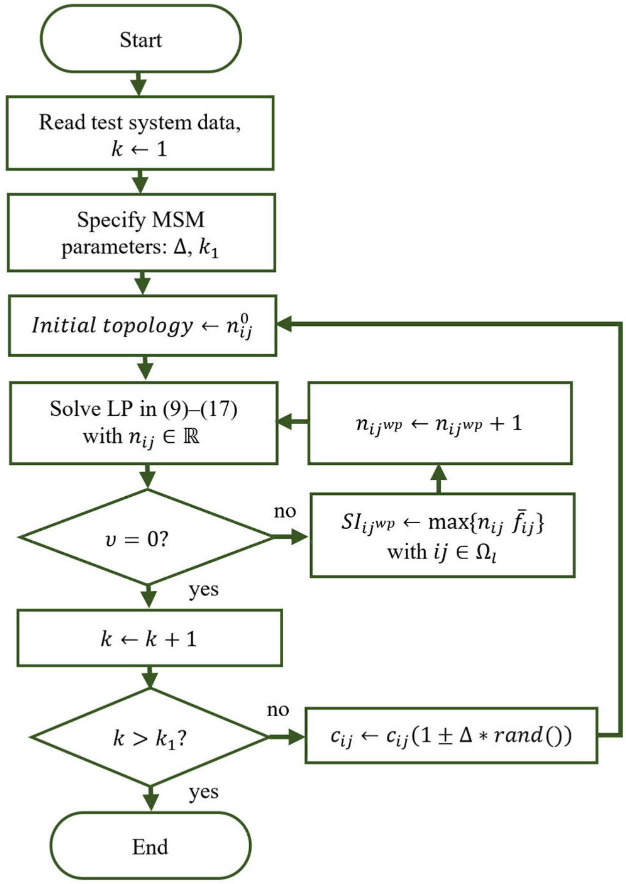

FIGURE 1. Flowchart of the MSM constructive phase.

At the end of the constructive phase, an additional line removal step is performed to remove the addition of redundant lines. Each of the added lines is removed alternately in each case. The LP in (9)-(17) is solved by relaxing the integrality of the integer variables n_{ij}. If $\upsilon = 0$, the line under analysis is removed from the current topology because it is an irrelevant line; otherwise, the line is preserved, and the next line added in the constructive phase is analyzed. The process is repeated until all lines added in the constructive phase are analyzed.

2) LOCAL SEARCH PHASE I: NEIGHBORHOOD SEARCH HEURISTIC

The Local Search Phase I is a neighborhood search heuristic applied to the k_2 best feasible solutions obtained in the constructive phase. It is an adaptation of the VGS CHA to explore the neighborhood of the best solutions in search of feasible solutions with lower costs.

The process removes added lines during the constructive phase one by one and applies the VGS CHA. It starts with a good-quality solution (n_{ij}^{gqs}) that is assigned to the current solution. A line is removed in the corridor n_{ij^k} for $n_{ij^k} \neq 0$. Otherwise, increment k and repeat the process for $k \leq NoC$, in which NoC is the number of corridors. The LP in (9)-(17)

is solved by relaxing the integrality of the integer variables n_{ij} from the obtained solution.

The winning path ij^{wp} is the one with the higher sensitivity index $SI_{ij^{wp}}$, defined by II-B2, such that $k \neq wp$. Then, the current solution is updated by adding a line in the winning path ij^{wp}. At the end of the VGS CHA, a feasible solution can be obtained after consecutive line additions.

The incumbent solution n_{ij}^{inc} is updated if a feasible and better-quality solution is obtained. If the incumbent solution is updated with $n_{ij^k} \neq 0$, then another TL is removed from that same corridor; otherwise, the k counter is incremented, a good-quality solution (n_{ij}^{gqs}) is assigned to the current solution again, and a line is removed from another corridor not evaluated with $n_{ij^k} \neq 0$.

The process ends after evaluating all corridors with lines added, i.e., when $k > NoC$.

The steps of the Local Search Phase I are summarized below. They are applied in a good-quality solution n_{ij}^{gqs}.

Step 1) Read test system data, $k \leftarrow 0$,

Step 2) The value of a good-quality solution (n_{ij}^{gqs}) is assigned to the current solution, $n_{ij} \leftarrow n_{ij}^{gqs}$,

Step 3) The counter is incremented, $k \leftarrow k+1$. If $k > NoC$ then stop,

Step 4) Are there lines added in the *k-th* corridor, i.e., $n_{ij}^{k} \neq 0$? If true, go to the next step; otherwise, go back to step 3,

Step 5) A line is removed from the *k-th* corridor, $n_{ij^k} \leftarrow n_{ij^k} - 1$,

Step 6) Solve the LP in (9)-(17) with $n_{ij} \in \mathbb{R}$,

Step 7) If the objective function is null $\upsilon = 0$ (feasible solution), go to the next step; otherwise, go to step 9,

Step 8) If the cost of the feasible solution obtained is less than the cost of the incumbent solution (n_{inc}), then update the incumbent solution $n_{inc} \leftarrow n_{ij^k}$ and go back to step 4; otherwise, go back to step 2,

Step 9) Solve (18) to find the most attractive path or winner path ij^{wp} for add lines,

Step 10) If $wp \neq k$ then increment a line in the winner path $w_{ij^{wp}} \leftarrow w_{ij^{wp}} + 1$ and go to step 6; otherwise, go to step 2.

3) LOCAL SEARCH PHASE II: PATH RELINKING

PR performs an intensive neighborhood search between two high-quality solutions to find better quality solutions along this route. A path is generated between the initial solution and the guide solution. During the trajectory between the initial solutions toward the guide solution, the neighborhood is carefully assessed to find lower-cost solutions [33]. The fundamental idea of PR is to try to find excellent quality solution proposals in the region that unites two considered elite solutions proposals. PR is a neighborhood search strategy starting from the base solution until reaching the guide's solution. In this process, few LP problems are solved with less computational effort, especially for larger ETS.

The local search phase II, implemented via PR, is applied to all pairs of elite solutions ($k_3 \geq 2$) formed by the best solutions with $C_2^{k_3} = k_3!/\{2!(k_3-2)!\}$, in which $C_2^{k_3}$ is the number of distinct pairs that can be formed. The steps of the PR metaheuristic are as follows [42]. The internal composition of each vector in Fig. 2 indicates the path where a TL was added. The steps of the Local Search Phase II are summarized below.

Step 1) One of the elite solution pairs consisting of the initial solution and the guide solution is selected.

Step 2) The initial solution value is assigned to the current solution.

Step 3) The values at each position of the current solution are added or subtracted by one unit. Move them toward the values of the corresponding positions of the guide solution one by one. Each change will represent a new candidate solution.

Step 4) The feasibility is verified. The LP model is solved by relaxing the integrality of the integer variables n_{ij} in (9)-(17) for each candidate topology.

Step 5) The better solution is updated if there is a feasible, better-quality solution with lower cost among the candidate solutions.

Step 6) If one of the candidate solutions is the same as the guide solution, stop; otherwise, the current new solution will be the better and feasible candidate solution.

Step 7) Repeat steps 3 through 6 until some candidate solution becomes identical to the guide solution verified in step 6.

Step 8) After the process in step 7 is completed, the permutation between the initial solution and the guide solution is performed: the initial solution becomes the guide solution and vice versa. Repeat steps 2 through 7.

Fig. 2 illustrates the execution steps of PR applied to the Garver 6-bus test system without generation rescheduling. After the execution of the previous phases, two elite solutions are selected: the initial solution and guide solution, step 1.

As shown in Fig. 2 (a), the initial solution costs MUS\$ 230 and there is the addition of lines in three paths: $n_{2-6} = 5$; $n_{3-5} = 1$; $n_{4-6} = 2$. The guide solution costs MUS\$ 238 and adds lines in four paths: $n_{2-6} = 3$; $n_{3-5} = 2$; $n_{3-6} = 1$; $n_{4-6} = 2$. The initial solution value is assigned to the current solution, step 2.

In step 3, the values at each position of the current solution are modified by one unit to approximate the corresponding values in the guide solution. Each substitution will represent a new candidate solution. Thus, the number of lines added in the first position of the guide solution ($\boldsymbol{n_{2-6} = 5}$) is subtracted by one unit to move it toward the value of that same path in the guide solution to produce the first candidate solution with a cost of MUS\$ 200, which adds lines in three paths: $\boldsymbol{n_{2-6} = 4}$; $\boldsymbol{n_{3-5} = 1}$; and $\boldsymbol{n_{4-6} = 2}$.

There are two lines added in the second position of the guide solution: $\boldsymbol{n_{3-5} = 2}$. This number of lines is attributed to this same path in the current solution. Thus, the second candidate solution is produced with a cost of MUS\$ 250 and additions of lines in paths: $\boldsymbol{n_{2-6} = 5}$; $\boldsymbol{n_{3-5} = 2}$; and $\boldsymbol{n_{4-6} = 2}$.

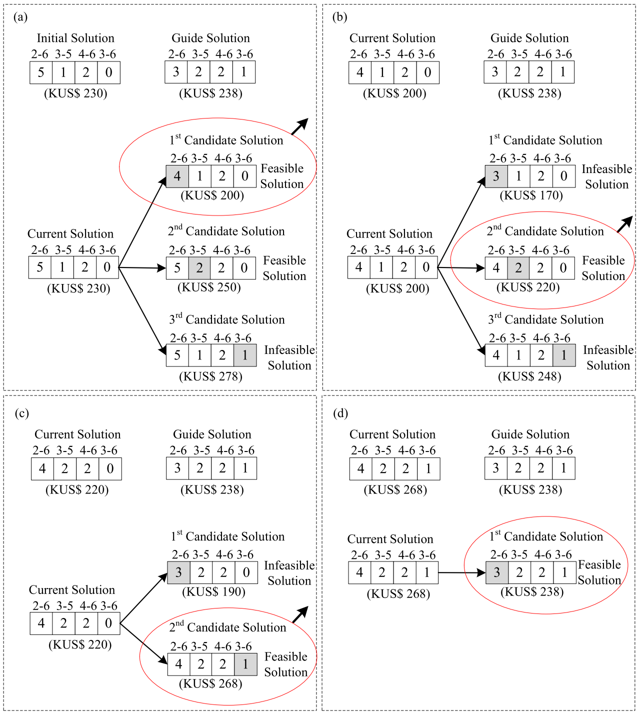
FIGURE 2. Example of the execution steps of path relinking.

Finally, the same number of lines existing in the third position of the guide solution ($\boldsymbol{n_{3-6} = 1}$) is assigned to the same path in the current solution. In this way, the third candidate solution is constructed with a cost of MUS$ 278 and additions of lines in paths: $\boldsymbol{n_{2-6} = 5}$; $\boldsymbol{n_{3-5} = 1}$; $\boldsymbol{n_{3-6} = 1}$; and $\boldsymbol{n_{4-6} = 2}$.

The fourth candidate solution cannot be built, because the fourth position of the guide solution ($n_{4-6} = 2$) has an equal number of lines added on the same path in the current solution.

The feasibility is verified. The LP model is solved by relaxing the integrality of the integer variables n_{ij} in (9)-(17) for each of the three candidate configurations, step 4.

The better solution is updated if there is a feasible solution with a lower cost among the candidate solutions, step 5. The better solution is updated because, among the candidate solutions, there is one feasible solution whose cost is lower than the cost of the initial and guide solution. According to step 6, the process continues, because no candidate solution is identical to the guide solution. Thus, the new current solution will be the feasible candidate solution with the lowest cost. Therefore, the first candidate solution will be the new current solution. Repeat steps 3 through 6 until all positions of the guide solution are equal to the current solution. This occurs in Fig. 2 (c).

It is observed that the example presented in Fig. 2 achieves the optimal solution for the Garver 6-bus test system without generation rescheduling, that is, the first candidate solution at Fig. 2 (a). It is worth mentioning that, in the MSPR, constants k_1, k_2, k_3, and Δ are empirically obtained through simulations.

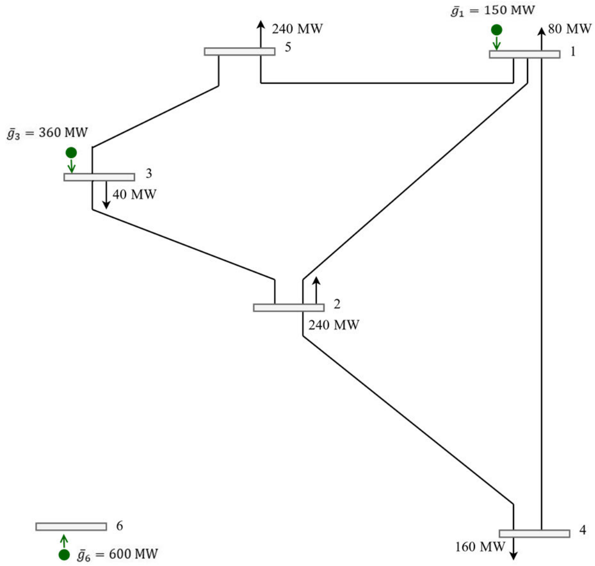

FIGURE 3. Initial configuration of the 6-bus garver test system.

Finally, it should be noted that one of the advantages of metaheuristics is that it does not solve the original model, and, in some applications, it may also completely dispense with the original mathematical modeling. Thus, the MSPR algorithm does not directly solve the model (1)–(8) and at all stages of the process, it solves only LP problems.

III. TESTS AND RESULTS

The results using the Garver 6-bus, the IEEE 24-bus, the Southern Brazilian 46-bus, and the Colombian 93-bus test systems are presented. The MSPR method was implemented in the MATLAB programming environment. Since no specialized toolbox was used, the algorithm could be implemented in any high-level programming language. MATLAB *linprog* (linear programming) function was used for the solution of the LP problems. Tests were carried out, considering the original generation values (without generation rescheduling) and with generation rescheduling, in which the generation can vary from 0 to $\bar{g}$. All simulations were executed in a personal computer with an Intel Core i7 4.2 GHz processor with 16 GB of RAM.

A. GARVER 6-BUS TEST SYSTEM

The Garver 6-bus test system has 15 paths, 3 generation buses, and loads in 5 buses with a total value of 760 MW, and up to 5 lines per path are allowed. The initial topology is shown in Fig. 3, and the electrical data of this system can be found in [4]. This system is too simple to be solved via MSPR. Thus, it was relatively easy to find the optimal solutions for this system for cases with and without generation rescheduling. The optimal solution for Garver's test system with generation rescheduling has a cost of MUS$ 110 and takes the following form:

- $n_{3-5} = 1$; and $n_{4-6} = 3$.

TABLE 1. Optimal solutions of the 6-bus garver test system for the DC model.

Parameters	With Generation Rescheduling	Without Generation Rescheduling
Optimal cost [MUS$]	110	200
Number of LP problems	6	13
Computational time [s]	0.05	0.11
Maximum percentage variation in the cost of the TL – Δ [%]	30	30
Constructive phase: k_1 solutions	1	1

The optimal solution for the case without rescheduling has a cost of MUS$ 200 and is defined by the following added lines:

- $n_{2-6} = 4$; $n_{3-5} = 1$; and $n_{4-6} = 2$.

Table 1 contains the results of the simulations performed for Garver's test system. The optimal solution, the number of LP problems, the number of constructive phase iterations, the constant Δ for maximum variation in TL costs during the constructive phase, and the computational time are presented. It was not necessary to perform the Local Search I and Local Search II phases to achieve the optimal solutions.

B. IEEE 24-BUS TEST SYSTEM

The 24-bus IEEE test system has 41 paths. There are 10 generation buses with a maximum generation capacity of 10,215 MW. There are 17 buses with a total demand of 8,550 MW. Up to three lines are allowed in each path. Fig. 4 shows the initial topology, whose electrical data can be found in [6], [12]. This system was solved using MSPR with and without generation rescheduling. Similar, to Garver's system, the optimal solutions were found for the DC model.

The optimal solution for the case with generation rescheduling has an investment cost of MUS$ 152, corresponding to the following lines:

- $n_{6-10} = 1$; $n_{7-8} = 2$; $n_{10-12} = 1$; and $n_{14-16} = 1$.

Without generation rescheduling, the G4 plan data indicated in [12] were used. The optimal solution has a cost of MUS$ 342 and the following added lines:

- $n_{3-24} = 1$; $n_{6-10} = 1$; $n_{7-8} = 2$; $n_{9-11} = 1$; $n_{10-12} = 1$; $n_{14-16} = 2$; and $n_{16-17} = 1$.

Table 2 presents a summary of the simulations performed for the 24-bus system with and without generation rescheduling. It was not necessary to perform the Local Search II phase to achieve the optimal solutions.

C. SOUTHERN BRAZILIAN 46-BUS SYSTEM

The Southern Brazilian 46-bus system is a real ETS with 46 buses and 79 paths. It has 12 active power generation buses, with a maximum generation capacity of 10,545 MW. There are 19 load buses, with a total demand of 6,880 MW. Up to 3 TL can be added per path. The initial topology is shown in Fig. 5, and the electrical data can be found in [4].

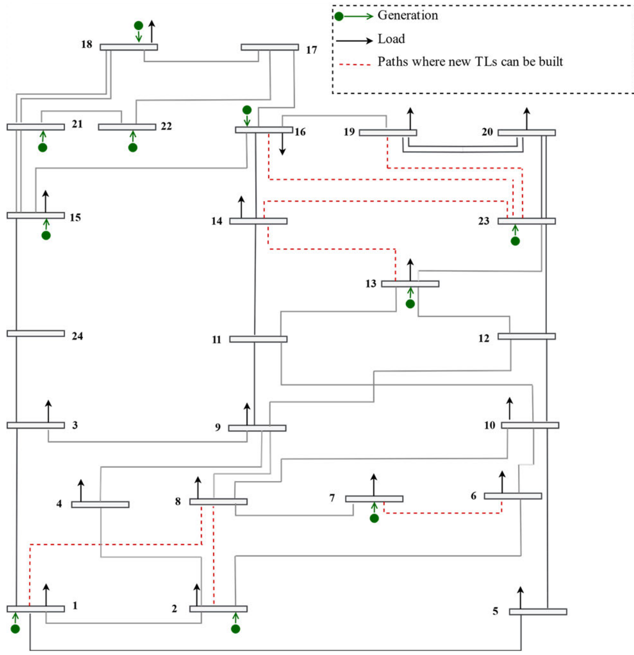

FIGURE 4. Initial configuration of the IEEE 24-bus test system.

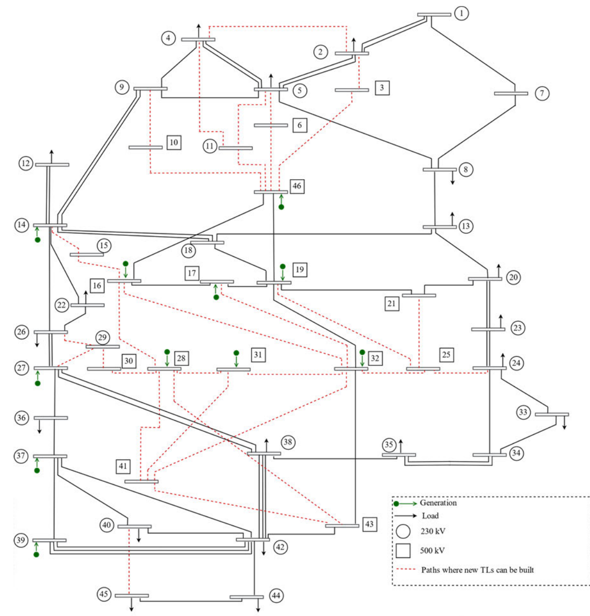

FIGURE 5. Initial configuration of the southern brazilian 46-bus system.

For the case of generation rescheduling, a solution with an investment of MUS$ 72.87 was found, which proposed the addition of the following lines:

- $n_{2-5} = 1$; $n_{13-20} = 1$; $n_{20-23} = 1$; $n_{20-21} = 2$; $n_{42-43} = 1$; $n_{46-6} = 1$; and $n_{5-6} = 2$.

For the case without generation rescheduling, a solution with an investment of MUS$ 154.42 was found, corresponding to the following added lines:

- $n_{20-21} = 1$; $n_{42-43} = 2$; $n_{46-6} = 1$; $n_{19-25} = 1$; $n_{31-32} = 1$; $n_{28-30} = 1$; $n_{26-29} = 3$; $n_{24-25} = 2$; $n_{29-30} = 2$; and $n_{5-6} = 2$.

TABLE 2. Optimal solutions of the IEEE 24-bus system for the DC model.

Parameters	With Generation Rescheduling	Without Generation Rescheduling
Optimal cost [MUS$]	152	342
Number of LP problems	26	192
Computational time [s]	0.24	1.86
Maximum percentage variation in the cost of the TL – Δ [%]	80	80
Constructive phase: k_1 solutions	3	10
Local search phase I: k_2 permutations	0	1

TABLE 3. Optimal solutions of the southern brazilian 46-bus test system for the DC model.

Parameters	With Rescheduling	Without Rescheduling
Optimal cost [MUS$]	72.87	154.42
Number of LP problems	232	289
Computational time [s]	2.65	3.31
Maximum percentage variation in the cost of the TL Δ [%]	80	80
Constructive phase: k_1 solutions	5	15
Local search phase I: k_2 permutations	1	1
Local search phase II: Path Relinking with k_3 elite solutions	2	2

Table 3 summarizes the relevant information regarding the simulations carried out with the 46-bus system.

D. COLOMBIAN 93-BUS SYSTEM

The Colombian 93-bus system is a real ETS with 93 buses, 193 existing circuits at the base topology, and 155 paths. It has a total active power generation and demand of 12,162 MW. Up to four TL can be added per path. The initial topology is shown in [42]. Colombian system data are only available without generation rescheduling with the P2 plan data indicated in [20], [23], and [30]. To solve the expansion planning problem under the mentioned conditions, a solution with MUS$ 443.494 was found with the following added lines:

- $n_{57-81} = 2$; $n_{55-57} = 1$; $n_{55-62} = 1$; $n_{27-29} = 1$; $n_{62-73} = 1$; $n_{45-81} = 1$; $n_{64-74} = 1$; $n_{19-82} = 10$; and $n_{82-85} = 1$.

Table 4 summarizes the relevant information regarding the simulations carried out with the 93-bus system.

TABLE 4. Optimal solution of the colombian 93-bus system for the DC model.

Parameters	Without Rescheduling
Optimal cost [MUS$]	443.494
Number of LP problems	319
Computational time [s]	5.82
Maximum percentage variation in the cost of the TL– Δ [%]	80
Constructive phase: k_1 solutions	10
Local search phase I: k_2 permutations	1
Local search phase II: Path Relinking with k_3 elite solutions	2

E. COMPARATIVE ANALYSIS OF THE RESULTS

A joint analysis of the four simulated systems is made in this section along with a comparison with other approaches of the specialized literature. Compared with other metaheuristics, the MSPR algorithm finds the optimal solution with fewer LP problems solved for the Garver system (compared with [21] and [30]), for the 24-bus system (compared with [29], and [30]), for the 46-bus system (compared to [22], [28] and [30]), and for the 93-bus system (compared with [30]), as shown in Table 5. No time comparisons were made because the tests were carried out under very different conditions. This difference can be very important when MSPR is used to solve more complex models like those used in [1]–[3].

Additionally, MSPR solves fewer LP problems than the disjunctive linear model solved with CPLEX [43]. For example, MSPR finds an optimal solution of the 46-bus system after solving 324.9 LP problems on average (see Table 6), while CPLEX solves the same system after generating 5,654 branch and bound nodes. However, CPLEX solves this system with less processing time due to the sophistication inherent to the solver. Nevertheless, for more complex models and more complex instances of the TEP problem, CPLEX converges with a very large gap, i.e., it finds only a local optimum very far from the global optimum [3].

Additionally, it is well known that a metaheuristic does not always converge to the same solution in each independent test, because most metaheuristics have a stochastic component. Thus, to assess the overall performance of the proposed MSPR strategy for the TEP, 100 executions for the four systems, for cases with and without generation rescheduling, are evaluated. The results are shown in Table 6. From that table, it can be noted that the MSPR found the optimal solution in all simulated instances. However, as expected, there was no convergence to the optimal solution in all performed simulations. Due to this, the average cost, the worst cost, the percentage difference between the worst and the best cost, and the standard deviation obtained on over 100 executions of the proposed strategy are presented.

It should be noted that, in the case of the Southern Brazilian system, the case without generation rescheduling is greatly restricted, and other metaheuristics need to solve a large number of LP problems to find the optimal solution. However, the MSPR solves this case quickly and with a computational

TABLE 5. Comparison among the optimal investment solutions for the garver 6-bus; IEEE 24-bus, southern brazilian 46-bus, and colombian 93-bus test systems with and without generation rescheduling.

Parameters		Garver 6-bus			IEEE 24-bus			Southern Brazilian 46-bus				Colombian 93-bus	
		This Work	[24]	[29]	This Work	[29]	[30]	This Work	[27]	[28]	[29]	This Work	[30]
Cost [MUS$]	R	110	110	110	152	152	152	72.87	72.87	72.87	72.87	—	—
	NR	200	200	NI	342	NI	NI	154.42	NI	NI	NI	443.494	443.494
LP Executed	R	6	2,300	33	26	41	169	232	400	600	248	—	—
	NR	13	3,100	NI	192	NI	NI	289	NI	NI	NI	319	14,430
Time [s]	R	0.05	30.30	2.58	0.24	2.70	2.40	2.65	1.20	1.60	6.78	—	—
	NR	0.11	52,00	NI	1.86	NI	NI	3.31	NI	NI	NI	5.82	8,717.96
Solution Method		MSPR	PSO	CBGA	MSPR	CBGA	HBCA	MSPR	CBGA	EGA	CBGA	MSPR	HBCA
Language		Mat.	Mat.	Mat.	Mat.	Mat.	Fortran	Mat.	C++	C++	Mat.	Mat.	Fortran

R: Rescheduling; NR: No Rescheduling; NI: Not informed; Mat.: MATLAB; CBGA: Chu-Beasley Genetic Algorithm; EGA: Enhanced Genetic Algorithm; HBCA: Heuristic based on the Branch and Cut Algorithm; MSPR: Multi-Start with Path Relinking; PSO: Particle Swarm Optimization.

TABLE 6. Summary of 100 executions of the proposed MSPR strategy for the test systems.

Parameters		Garver 6-bus	IEEE 24-bus	Southern Brazilian 46-bus	Colombia 93-bus
Best Cost [MUS$]	R	110.00	152.00	72.87	—
	NR	200.00	342.00	154.42	443.494
Average Cost [MUS$]	R	127.90	196.52	91.36	—
	NR	200.48	372.96	165.43	473.662
Worst Cost [MUS$]	R	140.00	340.00	107.29	—
	NR	248.00	376.00	180.22	627.029
Difference between best and worst cost [%]	R	27.27	123.68	47.23	—
	NR	24.00	9.94	16.70	41.38
Standard Deviation [MUS$]	R	10.32	49.44	9.07	—
	NR	4.78	8.52	4.86	39.15
Average LP Executed	R	8.0	43.3	166.8	—
	NR	16.0	262.9	324.9	640.9
Average Time[s]	R	0.07	0.42	1.87	—
	NR	0.14	2.50	3.85	12.02

R: Rescheduling. NR: No Rescheduling.

effort near to the one related to the case with generation rescheduling.

The best solution found for each instance shown in Table 6 is an optimal solution verified by performing tests with CPLEX and using the linear disjunctive model that has the same optimal solution as the DC model.

Finally, it should be noted that the main contribution of this work is the application of the MSPR metaheuristics for the TEP problem. Thus, initial solution proposals of excellent quality were generated using the VGS algorithm and, with high diversification, using disruptions in the TL's costs, bypassing the main difficulties of MSM. The static planning model that incorporates the DC model was chosen to facilitate the presentation of the strategy and show the performance of the MSPR algorithm and a comparative analysis with other existing methods. Thus, MSPR can be extended to more complex models of the TEP problem, such as those presented in [1]–[3], [13], [14].

IV. CONCLUSION

In this work, a specialized multi-start meta-heuristic for the transmission system expansion planning problem was presented. Multi-start strategies showed adequate performance in operational research problems but were not yet used to solve the TEP problem. The results presented show the efficient performance of the multi-start strategy when compared with other recent meta-heuristics applied to the TEP problem, especially in relation to the number of LPs solved. All metaheuristics applied to the DC model solve LP problems to assess the quality of a proposed solution. In the constructive phase, a generalization of one of the best CHAs proposed for the DC model of the TEP problem was used. This proposal generates initial solutions of excellent quality and with great diversity, which can explain the performance of the multi-start strategy. In the local search phase, a path relinking strategy was implemented, which also presents an efficient performance. The tests presented show that the MSPR meta-heuristic finds the optimal solutions of four systems widely used in the literature, solving a reduced number of LP problems. These optimal solutions were confirmed using the CPLEX solver.

Future works should introduce additional improvements in the MSPR meta-heuristic and perform tests for the DC model of the TEP problem in highly complex systems where the

CPLEX solver does not have the ability to find the optimal solution and ends the process with a very high gap. A natural extension is also to use the MSPR metaheuristic for the AC model and in more complex models, such as the TEP problem models that use reliability (N-1), and multistage planning, among others. For these more complex models, meta-heuristics need small changes when compared with other optimization strategies.

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