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Event-Triggered H-Infinity Controller Design of Nonlinear Networked Cascade Control System

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ABSTRACT Nonlinearity is ubiquitous in practical industrial production, so this paper investigates the problems of modeling, H_∞ control and stabilization for a class of nonlinear networked cascade control systems (NCCSs) under an event-triggered scheme, which makes the system have more practical application value. Firstly, aiming at a kind of typical nonlinear system in industrial production, a new mathematical model is established by adding networked cascade control system and event-triggered scheme. The collaborative design method of primary and secondary controller parameters and event-triggered parameters is completed utilizing linear matrix inequality (LMI). Secondly, based on ensuring its stability, combined with H_∞ control, the system's disturbance is suppressed, making the system more universal. Finally, the feasibility and effectiveness of the proposed method in the nonlinear networked cascade control system has verified by a marine boiler liquid level cascade control system.

INDEX TERMS Networked cascade control systems, nonlinearity, event-triggered scheme, stability.

I. INTRODUCTION

Due to the sustained development and maturity of networked control systems (NCSs) [1] in recent decades, it has received widespread attention from scholars. It uses the network signal to realize the control. Modularization, real-time control, and the low cost of the system are its advantages. Therefore, networked control systems have been applied widely in actual industrial production. However, many papers ignore some nonlinear terms when modeling based on the actual system, such as: [1], [2]. The linear system constructed in this way cannot be applied to most practical systems, and has a narrow application field. Therefore, when designing the control system, it will be more practical to add the possible nonlinear system in production.

Nonlinear factors are inevitable in industrial process control. Therefore, the research on nonlinear networked Control systems has important practical significance. Many outstanding scholars have made many achievements in theory. On the one hand, the system model and stability are studied for the delay of network control. For example, in [2], the network delay is defined as nonlinear to establish the model. In contrast, in [3], the network delay is described by the probability density function. In addition, some scholars have studied the generalized systems [4]. On the other hand, in order to deal with different control objects, different control methods are added, such as PID control [5], [6], sliding mode control [7], [8], adaptive control [9], [10], robust control [11], [12] and predictive control [13]. According to the other maintenance, or performance requirements, observers [14] and filters [15] are designed. However, the above papers are all aimed at the single closed-loop nonlinear networked control system. However, the cascade structure is rarely mentioned, so the cascade structure is added to the nonlinear system in this paper.

Cascade control System (CCS) [16] is a kind of control structure with unique advantages. When the primary loop has a strong disturbance, the secondary loop can quickly suppress the disturbance. With the primary loop, the system can achieve the desired output. Networked cascade control systems [17] combines most of the advantages of CCSs and NCSs, which can eliminate the interference in the system in time and quickly, and at the same time, improve the efficiency of the system. However, the single closed-loop structure of the networked control system is easier to study, and the addition of the cascade structure further complicates the model. That makes theoretical research very difficult. However, the addition of the cascade structure makes the model more complicated, making theoretical research very difficult.

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Regarding the nonlinear term in the system, scholars have studied a lot. Among them, the Lurie system is a widely existing nonlinear system. In Lurie system, the nonlinear system is divided into two parts, one part is a linear time-invariant system, and the other part is a nonlinear system that satisfies the sector constraint, and its sector boundary can be selected freely. In actual industrial production, the controlled index changes nonlinearly due to the actual influence of various factors. As long as the appropriate fan-shaped boundary is selected, the Lurie system can be used for simulation. Since 1944, Lurie has proposed the system. Its research has been in progress. For example, in [18], the state of the chaotic Lurie system is estimated, and a new communication channel is established and verified by Chua's circuit. In [19], the absolute stability of Lurie nonlinear systems with both sector and slope constraints is studied. The paper [20] combines Lurie system with networked control system, and gives the stability condition. Furthermore, the paper [21] adds robust control, and its stability is verified by simulation. However, most scholars focus on the stability of the Lurie system structure itself. Few people combine it with the networked cascade control system.

On the other hand, over-saturated signals are more likely to cause packet loss and disorder, which can lead to instability of the control system. In order to reduce the transmission frequency of the signal in the network, the event-triggered control method [22]–[24] is proposed. When the signal is filtered by the event-triggered controller, the signal that is meaningful to the system control will be sent, which reduces the transmission pressure of the network. However, under the conditions of some trigger controls, when the time interval for detecting the system state is minimal, it is easy to happen the unreasonable situation of triggering an infinite number of times in a limited time, that is, the Zeno phenomenon [25], [26]. To this end, many scholars have proposed a discrete state event-triggered condition for defining sampling time [27], [28], thereby avoiding this phenomenon. In addition, some scholars have proposed different event-triggered scheme to deal with some special situations [29]–[32]. In order to allow the control system to operate stably, and avoid failures caused by network pressure, the event-triggered scheme is also added. The selected event-triggered conditions in this paper can avoid the Zeno phenomenon.

In the process of industrial production, there will be control links for specific indicators. Let this index stabilize at the expected value, and the control system should have the advantages of anti-disturbance, optimized resource utilization, and easy maintenance as much as possible. For unavoidable nonlinear factors, idealized considerations sometimes cannot be applied to actual production. This paper is based on a class of nonlinear NCCSs, add an event-triggered scheme and H_∞ control, completing the collaborative design of the primary state feedback controller and secondary one, and obtain event-triggered parameters. The main contributions of this paper are summarized as follows:

1) To the best of our knowledge, few scholars combine nonlinear term and event-triggered scheme with networked cascade control systems simultaneously. This paper is the first time to combine Lurie nonlinear cascade control system and event-triggered scheme with networked control.

2) A new model which takes nonlinear terms and event-triggered control is constructed. This model can describe the actual industrial production process more closely and practically. And the system has all the advantages of event-triggered control and cascade structure.

3) In the presence or absence of disturbances, the co-design methods of the primary and secondary controller parameters and event-triggered parameters are obtained simultaneously. It can be applied to industrial process control systems with the same structure.

4) The event-triggered scheme and H_∞ control of the system is considered. The obtained results can be further applied to the industrial production of this kind of nonlinear NCCSs model.

The rest of this paper mainly describes the following contents: In section II, the model of nonlinear NCCS is established. In section III, the sufficient conditions of the system without disturbance are obtained, and the co-design method of controller parameters and event-triggered parameters is given. In section IV, based on the previous section, we complete the H_∞ control with disturbance. In section V, a simulation example of marine level boiler with nonlinear condition is selected to verify the effectiveness of the proposed method. Section VI is the conclusion.

II. MODELING OF NONLINEAR NCCS

A control system configuration diagram is shown in Figure 1. The event-triggered scheme is introduced into nonlinear NCCSs. In the networked cascade system, the response speed of the inner loop is fast and the communication channel is short, so the network delay can be ignored. Therefore, we construct a model in which the network is in the outer loop. The secondary plant in the outer loop is regarded as nonlinear. An event generator is added between the primary sensor and the primary controller to decide whether to transmit the new sampling signal to the primary controller.

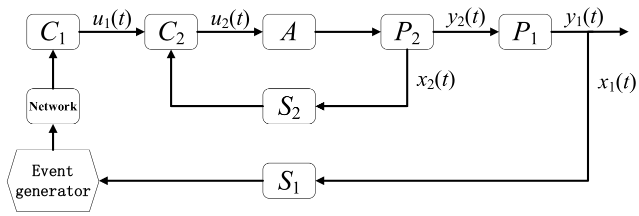

FIGURE 1. Configuration diagram of event-triggered controller of nonlinear network cascade control system.

P_1 and P_2 are two plants. C_1 and C_2 are the primary controller and secondary controller. S_1 is the primary sensor and S_2 is the secondary one. A is actuator. It can be seen from the figure that the output of P_2 is the input of P_1, the output of C_1 is the set value of C_2, and the states of P_1 and P_2 are respectively transmitted to C_1 and C_2 through S_1 and S_2.

First, consider the primary controlled object, as follows

$$P1:\begin{cases}\dot{x}_1(t)=A_1x_1(t)+B_1y_2(t)\\ y_1(t)=C_1x_1(t)+C_3\Delta(t).\end{cases}\tag{1}$$

$x_1(t)$ is the state vector of the primary plant, $y_1(t)$ is its output, $\Delta(t)$ is a finite disturbance that satisfies the $L_2[0,\infty)$. A_1, B_1, C_1 and C_3 are known matrices.

Considering the following secondary plant, which is nonlinearity and can be described as

$$P2:\begin{cases}\dot{x}_2(t)=f(x_1(t),x_2(t))+B_4\Delta(t)\\ y_2(t)=C_2x_2(t)+C_4\Delta(t).\end{cases}\tag{2}$$

$x_2(t)$ is the state vector of the secondary plant, and $y_2(t)$ is its output, $f(x_1(t),x_2(t))$ satisfies Lurie system and can be expressed as a linear system part and a nonlinear part, so the secondary plant can be described as

$$P2:\begin{cases}\dot{x}_2(t)=A_2x_2(t)+B_2u_2(t)+B_3w(t)+B_4\Delta(t)\\ y_2(t)=C_2x_2(t)+C_4\Delta(t).\end{cases}\tag{3}$$

In this formula, $u_2(t)$ is the output of the secondary controller, A_2, B_2, B_3, B_4, C_2 and C_4 are constant matrices with appropriate dimensions, and the nonlinear term $w(t)$ can be expressed as $-\phi(t,y_1(t))$. $\phi(t,y_1(t))$ is piecewise continuous on t and satisfies local Lipschitz condition on $y_1(t)$, and $\phi(t,0)=0$, then the following conditions are satisfied for $t\geq 0$ and $y_1(t)$

$$[\phi(t,y_1(t))-N_1y_1(t)]^T\,[\phi(t,y_1(t))-N_2y_1(t)]\leq 0\tag{4}$$

where $N_2-N_1>0$, that is, the nonlinear function satisfies the sector area $[N_1,N_2]$, and $N=N_2-N_1$, a special constraint condition of the sector of the nonlinear function can be obtained as

$$\phi^T(t,y_1(t))\,[Ny_1(t)-\phi(t,y_1(t))]\leq 0.\tag{5}$$

Considering that in practical application, the network generally exists in the primary loop. It is assumed that there is no time delay in the secondary loop, and the time delay only exists in the primary loop. The delay is time-varying and may be larger than one sampling period.

In this paper, primary controller $u_1(t)$ and secondary controller $u_2(t)$ are state feedback controllers. Considering the network transmission delay of the primary loop, it can be expressed as

$$\begin{cases}u_1(t)=K_1x_1(t_kh)\\ u_2(t)=u_1(t)+K_2x_2(t)\\ t\in[t_kh+\tau_k,\,t_{k+1}h+\tau_{k+1}]\end{cases}\tag{6}$$

where τ_k is the time-varying delay and bounded in the network communication.

Define a function $\tau(t)$ which is

$$t-\tau(t)\in[t_kh,\,t_{k+1}h)$$

where $t\in[t_kh+\tau_k,\,t_{k+1}h+\tau_{k+1})$, then according to the above formula, we can choose $\tau(t)$ as

$$0\leq\tau_k\leq\tau(t)\leq\tau_M\tag{7}$$

where τ_M denotes the upper delay bounds. It means for any $t\in[t_kh+\tau_k,\,t_{k+1}h+\tau_{k+1})$, there exists the corresponding $\tau(t)\in[0,\tau_M]$ to make $t-\tau(t)\in[t_kh,\,t_{k+1}h)$ hold.

Combine the event-triggered scheme with cascade networked cascade control system, which will select the signal according to the following conditions and send it to the controller.

$$[x_1(i_kh)-x_1(t_kh)]^T\,\Omega\,[x_1(i_kh)-x_1(t_kh)]\leq\sigma x_1^T(i_kh)\Omega x_1^T(i_kh)\tag{8}$$

where $\Omega>0,\sigma\in[0,1)$, $x_1(i_kh)$ is the current sampling signal at the moment of i_kh, where $i_kh=t_kh+jh$, $j=1,2,\cdots$, and $x_1(t_kh)$ is the last signal sent at the time of triggering t_kh.

Remark 1: Only when the transmitted signal $x_1(i_kh)$ satisfies inequality (8), the data will be sent to the controller. Therefore, this can reduce the times of signal transmission in the network. And the trigger condition (8) can effectively avoid unlimited triggers in a short time, which means the Zeno phenomenon can be avoided.

Remark 2: It is assumed that the system is completely observable, and there is no data packet loss and mis-sequence.

Define $e_k(t)$ as the difference between the current sampled signal and the last successfully sent signal, that is, $e_k(t)=x_1(i_kh)-x_1(t_kh)$. Therefore, the event-triggered scheme in this paper can be obtained

$$e_k^T(t)\Omega e_k(t)\leq\sigma x_1^T(t-\tau(t))\Omega x_1(t-\tau(t))\tag{9}$$

where $\Omega>0,\sigma\in[0,1)$.

When the event-triggered scheme controller is applied to the primary loop of the cascade control system connected through the network, the closed-loop system corresponding to system (1) and (2) can be expressed

$$\begin{cases}\dot{x}_1(t)=A_1x_1(t)+B_1C_2x_2(t)+B_1C_4\Delta(t)\\ \dot{x}_2(t)=(A_2+B_2K_2)x_2(t)+B_2K_1x_1(t-\tau(t))\\ \qquad\quad +B_2K_1e_k(t)+B_3w(t)+B_4\Delta(t)\\ y_1(t)=C_1x_1(t)+C_3\Delta(t)\\ y_2(t)=C_2x_2(t)+C_4\Delta(t)\\ u_1(t)=K_1x_1(t-\tau(t))+K_1e_k(t)\\ u_2(t)=u_1(t)+K_2x_2(t).\end{cases}\tag{10}$$

In order to design the corresponding controller, the following lemma is needed.

Lemma 1 ([33]): For constant matrix X_3 and symmetric matrix X_1, X_2 with appropriate dimensions, then the inequality $\begin{bmatrix}X_1 & X_3^T\\ X_3 & X_2\end{bmatrix}<0$, if one of the following two conditions is true

$$1)X_2<0,\quad X_1-X_3X_2^{-1}X_3^T<0,$$
$$2)X_1<0,\quad X_2-X_3^TX_1^{-1}X_3<0.$$

III. DESIGN OF NONLINEAR NCCS EVENT-TRIGGED CONTROLLER WITHOUT DISTURBANCE

This paper analyzes the stability of the networked cascade control system and designs the corresponding primary controller and secondary one based on event-triggered control. It is assumed that there is no disturbance, and we will give the sufficient condition of stability for nonlinear NCCS (10) in this section.

Theorem 1: For the known parameters σ and τ_M, and the corresponding parameter K_1, K_2 of the primary and secondary controller, if there exists matrices Y, W with proper dimension and symmetric positive definite matrices Z, P, Q, R, Ω, such that inequality (11) holds

$$\begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & \varphi_{14} & \varphi_{15} & \varphi_{16} & \varphi_{17} \\ * & \varphi_{22} & \varphi_{23} & 0 & 0 & \varphi_{26} & \varphi_{27} \\ * & * & \varphi_{33} & \varphi_{34} & 0 & 0 & 0 \\ * & * & * & \varphi_{44} & 0 & 0 & 0 \\ * & * & * & * & \varphi_{55} & 0 & 0 \\ * & * & * & * & * & \varphi_{66} & 0 \\ * & * & * & * & * & * & \varphi_{77} \end{bmatrix} < 0 \tag{11}$$

where

$$\begin{aligned}
\varphi_{11} &= PA_2 + A_2^T P + PB_2K_2 + K_2^T B_2^T P, \\
\varphi_{12} &= C_2^T B_1^T Z + Y, \quad \varphi_{22} = ZA_1 + A_1^T Z + Q, \\
\varphi_{13} &= PB_2K_1 - Y, \quad \varphi_{23} = W^T, \\
\varphi_{33} &= -Q - W^T - W + \sigma\Omega^T + \sigma\Omega, \quad \varphi_{14} = -\tau_M Y, \\
\varphi_{34} &= -\tau_M W, \quad \varphi_{44} = -\tau_M R, \varphi_{15} = PB_2K_1, \\
\varphi_{55} &= -\Omega - \Omega^T, \\
\varphi_{16} &= PB_3, \quad \varphi_{26} = -C_1^T N^T, \ \varphi_{66} = -2I, \\
\varphi_{17} &= \tau(t)C_2^T B_1^T R, \quad \varphi_{27} = \tau_M A_1^T R, \ \varphi_{77} = -\tau_M R.
\end{aligned}$$

Proof: The Lyapunov function $V(t)$ is defined as follows, where P, Z, R, Q are symmetric and positive definite matrices.

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) \tag{12}$$

where

$$V_1(t) = x_1^T(t)Zx_1(t), V_2(t) = \int_{t-\tau(t)}^{t} x_1^T(s)Qx_1(s)ds,$$
$$V_3(t) = \int_{-\tau(t)}^{0}\int_{t+s}^{t} \dot{x}_1^T(v)R\dot{x}_1(v)dvds, \quad V_4(t) = x_2^T(t)Px_2(t).$$

Taking the derivative of $V(t)$, we can obtain:

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t) \tag{13}$$

where

$$\begin{aligned}
&\dot{V}_1(t) \\
&= 2x_1^T(t)Z(A_1x_1(t) + B_1C_2x_2(t)) \\
&= \frac{1}{\tau(t)}\int_{t-\tau(t)}^{t} [2x_1^T(t)ZA_1x_1(t) + 2x_2^T(t)C_2^T B_1^T Zx_1(t)]dv, \\
&\dot{V}_2(t) \\
&= x_1^T(t)Qx_1(t) - x_1^T(t-\tau(t))Qx_1(t-\tau(t)) \\
&= \frac{1}{\tau(t)}\int_{t-\tau(t)}^{t} [x_1^T(t)Qx_1(t) - x_1^T(t-\tau(t))Qx_1(t-\tau(t))]dv, \\
&\dot{V}_3(t) \\
&= \tau(t)\dot{x}_1^T(t)R\dot{x}_1(t) - \int_{t-\tau(t)}^{t} \dot{x}_1^T(v)R\dot{x}_1(v)dv \\
&= \tau(t)\dot{x}_1^T(t)R[A_1x_1(t) + B_1C_2x_2(t)] - \int_{t-\tau(t)}^{t} \dot{x}_1^T(v)R\dot{x}_1(v)dv \\
&= \frac{1}{\tau(t)}\int_{t-\tau(t)}^{t} [\tau(t)x_1^T(t)A_1^T RA_1x_1(t) \\
&\quad + 2\tau(t)x_2^T(t)C_2^T B_1^T RA_1x_1(t) \\
&\quad + \tau(t)x_2^T(t)C_2^T B_1^T RB_1C_2x_2(t) - \tau(t)\dot{x}_1^T(v)R\dot{x}_1(v)]dv, \\
&\dot{V}_4(t) \\
&= 2x_2^T(t)P[(A_2 + B_2K_2)x_2(t) + B_2K_1x_1(t-\tau(t)) \\
&\quad + B_2K_1e_k(t) + B_3w(t)] \\
&= 2x_2^T(t)(PA_2 + PB_2K_2)x_2(t) + 2x_2^T(t)PB_2K_1x_1(t) \\
&\quad - 2x_2^T(t)PB_2K_1\int_{t-\tau(t)}^{t} \dot{x}_1(v)dv \\
&\quad + 2x_2^T(t)PB_2K_1e_k(t) + 2x_2^T(t)PB_3w(t) \\
&= 2x_2^T(t)(PA_2 + PB_2K_2)x_2(t) + 2x_2^T(t)PB_2K_1x_1(t) \\
&\quad + 2x_2^T(t)(Y - PB_2K_1)\int_{t-\tau(t)}^{t} \dot{x}_1(v)dv \\
&\quad + 2x_1^T(t-\tau(t))W\int_{t-\tau(t)}^{t} \dot{x}_1(v)dv - [2x_2^T(t)Y \\
&\quad \times \int_{t-\tau(t)}^{t} \dot{x}_1(v)dv \\
&\quad + 2x_1^T(t-\tau(t))W\int_{t-\tau(t)}^{t} \dot{x}_1(v)dv] \\
&\quad + 2x_2^T(t)PB_2K_1e_k(t) + 2x_2^T(t)PB_3w(t) \\
&= 2x_2^T(t)(PA_2 + PB_2K_2)x_2(t) + 2x_2^T(t)Yx_1(t) \\
&\quad + 2x_2^T(t)(PB_2K_1 - Y)x_1(t-\tau(t)) \\
&\quad + 2x_1^T(t)W^T x_1(t-\tau(t)) \\
&\quad - 2x_1^T(t-\tau(t))Wx_1(t-\tau(t)) - [2x_2^T(t)Y\int_{t-\tau(t)}^{t} \dot{x}_1(v)dv \\
&\quad + 2x_1^T(t-\tau(t))W\int_{t-\tau(t)}^{t} \dot{x}_1(v)dv] \\
&\quad + 2x_2^T(t)PB_2K_1e_k(t) + 2x_2^T(t)PB_3w(t) \\
&= \frac{1}{\tau(t)}\int_{t-\tau(t)}^{t} [2x_2^T(t)(PA_2 + PB_2K_2)x_2(t) + 2x_2^T(t)Yx_1(t) \\
&\quad + 2x_2^T(t)(PB_2K_1 - Y)x_1^T(t-\tau(t)) \\
&\quad + 2x_1^T(t)W^T x_1(t-\tau(t)) - 2x_1^T(t-\tau(t))Wx_1(t-\tau(t)) \\
&\quad - 2\tau(t)x_2^T(t)Y\dot{x}_1(v) - 2\tau(t)x_1^T(t-\tau(t))W\dot{x}_1(v) \\
&\quad + 2x_2^T(t)PB_2K_1e_k(t) + 2x_2^T(t)PB_3w(t)]dv.
\end{aligned}$$

From the formulas (6), we can define

$$l_1(t) = \sigma x_1^T(t-\tau(t))\Omega x_1(t-\tau(t)) - e_k^T(t)\Omega e_k(t) \geq 0. \tag{14}$$

From the formulas (5), we can define

$$l_2(t) = -x_1^T(t)C_1^T N^T w(t) - w^T(t)Iw(t) \geq 0. \quad (15)$$

Using (13), (14), (15), we can obtain

$$\dot{V}(t) \leq \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t) + 2l_1(t) + 2l_2(t). \quad (16)$$

Define a matrix

$$\zeta^T(t,v) = [x_2^T(t) \;\; x_1^T(t) \;\; x_1^T(t-\tau(t)) \;\; \dot{x}_1^T(v) \;\; e_k^T(t) \;\; w^T(t)].$$

Separate this matrix in formula (16), and define Ψ as a symmetric matrix, then we can obtain

$$\dot{V}(t) = \frac{1}{\tau(t)}\int_{t-\tau}^{t} [\xi^T(t,v)\Psi\xi(t,v)]dv,$$

$$\Psi = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & -\tau(t)Y & PB_2K_1 & PB_3 \\ * & \varphi_{22} & W^T & 0 & 0 & -C_1^T N^T \\ * & * & \varphi_{33} & -\tau(t)W & 0 & 0 \\ * & * & * & -\tau(t)R & 0 & 0 \\ * & * & * & * & -\Omega-\Omega^T & 0 \\ * & * & * & * & * & -2I \end{bmatrix} \quad (17)$$

where

$$\varphi_{11} = \tau(t)C_2^T B_1^T RB_1C_2 + PA_2 + A_2^T P + PB_2K_2 + K_2^T B_2^T P,$$
$$\varphi_{12} = C_2^T B_1^T Z + \tau(t)C_2^T B_1^T RA_1 + Y,$$
$$\varphi_{22} = ZA_1 + A_1^T Z + Q + \tau(t)A_1^T RA_1, \quad \varphi_{13} = PB_2K_1 - Y,$$
$$\varphi_{33} = -Q - W^T + W + \sigma\Omega^T + \sigma\Omega.$$

According to Lemma 1, $\Psi < 0$ in Equation (17) is equivalent to the following inequality (18).

$$\begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & \varphi_{14} & \varphi_{15} & \varphi_{16} & \varphi_{17} \\ * & \varphi_{22} & \varphi_{23} & 0 & 0 & \varphi_{26} & \varphi_{27} \\ * & * & \varphi_{33} & \varphi_{34} & 0 & 0 & 0 \\ * & * & * & \varphi_{44} & 0 & 0 & 0 \\ * & * & * & * & \varphi_{55} & 0 & 0 \\ * & * & * & * & * & \varphi_{66} & 0 \\ * & * & * & * & * & * & \varphi_{77} \end{bmatrix} < 0 \quad (18)$$

where

$$\varphi_{11} = PA_2 + A_2^T P + PB_2K_2 + K_2^T B_2^T P,$$
$$\varphi_{12} = C_2^T B_1^T Z + Y, \quad \varphi_{22} = ZA_1 + A_1^T Z + Q,$$
$$\varphi_{13} = PB_2K_1 - Y, \varphi_{23} = W^T,$$
$$\varphi_{33} = -Q - W^T - W + \sigma\Omega^T + \sigma\Omega, \quad \varphi_{14} = -\tau_M Y,$$
$$\varphi_{34} = -\tau_M W, \varphi_{44} = -\tau_M R, \varphi_{15} = PB_2K_1,$$
$$\varphi_{55} = -\Omega - \Omega^T, \quad \varphi_{16} = PB_3, \; \varphi_{26} = -C_1^T N^T, \; \varphi_{66} = -2I,$$
$$\varphi_{17} = \tau(t)C_2^T B_1^T R, \quad \varphi_{27} = \tau_M A_1^T R, \; \varphi_{77} = -\tau_M R.$$

Thus, if the inequality (18) holds, then the system (10) is asymptotically stable in a given sector interval, and the theorem is proved.

According to Theorem 1, we give the design method of primary controller and secondary one of the system (10).

Theorem 2: For the given parameters σ and $\tau_M > 0$, if there are matrices of appropriate dimensions, W, Y and symmetric positive definite matrices P, Z, R, Q, Ω, such that the following inequality (19) holds, then the system (10) is asymptotically stable, and the expected gain matrix of the primary controller can be obtained:

$$K_1 = \tilde{X}_1\tilde{Z}^{-1},$$

and the gain matrix of the secondary controller is

$$K_2 = \tilde{X}_2\tilde{P}^{-1}.$$

$$\begin{bmatrix} \tilde{\varphi}_{11} & \tilde{\varphi}_{12} & \tilde{\varphi}_{13} & \tilde{\varphi}_{14} & \tilde{\varphi}_{15} & \tilde{\varphi}_{16} & \tilde{\varphi}_{17} \\ * & \tilde{\varphi}_{22} & \tilde{\varphi}_{23} & 0 & 0 & \tilde{\varphi}_{26} & \tilde{\varphi}_{27} \\ * & * & \tilde{\varphi}_{33} & \tilde{\varphi}_{34} & 0 & 0 & 0 \\ * & * & * & \tilde{\varphi}_{44} & 0 & 0 & 0 \\ * & * & * & * & \tilde{\varphi}_{55} & 0 & 0 \\ * & * & * & * & * & \tilde{\varphi}_{66} & 0 \\ * & * & * & * & * & * & \tilde{\varphi}_{77} \end{bmatrix} < 0 \quad (19)$$

where

$$\tilde{\varphi}_{11} = A_2\tilde{P} + \tilde{P}A_2^T + B_2\tilde{X}_2 + \tilde{X}_2^T B_2^T, \quad \tilde{\varphi}_{12} = \tilde{P}C_2^T B_1^T + \tilde{Y}_1,$$
$$\tilde{\varphi}_{22} = A_1\tilde{Z} + \tilde{Z}A_1^T + \tilde{Q}, \tilde{\varphi}_{13} = B_2\tilde{X}_1 - \tilde{Y}_1, \tilde{\varphi}_{23} = \tilde{W}_1,$$
$$\tilde{\varphi}_{33} = -\tilde{Q} - \tilde{W}_1 - \tilde{W}_1^T + \sigma\tilde{\Omega} + \sigma\tilde{\Omega}^T, \quad \tilde{\varphi}_{14} = -\tau_M\tilde{Y}_2,$$
$$\tilde{\varphi}_{34} = -\tau_M\tilde{W}_2, \quad \tilde{\varphi}_{44} = -\tau_M\tilde{R}, \; \tilde{\varphi}_{15} = B_2\tilde{X}_1,$$
$$\tilde{\varphi}_{55} = -\tilde{\Omega} - \tilde{\Omega}^T,$$
$$\tilde{\varphi}_{16} = B_3, \quad \tilde{\varphi}_{26} = -\tilde{Z}C_1^T N^T, \; \tilde{\varphi}_{66} = -2I,$$
$$\tilde{\varphi}_{17} = \tau_M\tilde{P}C_2^T B_1^T, \quad \tilde{\varphi}_{27} = \tau_M\tilde{Z}A_1^T, \; \tilde{\varphi}_{77} = -\tau_M\tilde{R}.$$

Proof: Define a matrix $\Pi = diag\{P^{-1}, Z^{-1}, Z^{-1}, R^{-1}, Z^{-1}, I, R^{-1}\}$. According to Theorem 1, pre- and post-multiplying (11) by Π, so we can obtain

$$\begin{bmatrix} \tilde{\varphi}_{11} & \tilde{\varphi}_{12} & \tilde{\varphi}_{13} & \tilde{\varphi}_{14} & \tilde{\varphi}_{15} & \tilde{\varphi}_{16} & \tilde{\varphi}_{17} \\ * & \tilde{\varphi}_{22} & \tilde{\varphi}_{23} & 0 & 0 & \tilde{\varphi}_{26} & \tilde{\varphi}_{27} \\ * & * & \tilde{\varphi}_{33} & \tilde{\varphi}_{34} & 0 & 0 & 0 \\ * & * & * & \tilde{\varphi}_{44} & 0 & 0 & 0 \\ * & * & * & * & \tilde{\varphi}_{55} & 0 & 0 \\ * & * & * & * & * & \tilde{\varphi}_{66} & 0 \\ * & * & * & * & * & * & \tilde{\varphi}_{77} \end{bmatrix} < 0, \quad (20)$$

where

$$\tilde{\varphi}_{11} = A_2P^{-1} + A_2^T P^{-1} + B_2K_2P^{-1} + P^{-1}K_2B_2^T,$$
$$\tilde{\varphi}_{12} = P^{-1}C_2^T B_1^T + P^{-1}YZ^{-1},$$
$$\tilde{\varphi}_{22} = A_1Z^{-1} + Z^{-1}A_1^T + Z^{-1}QZ^{-1},$$
$$\tilde{\varphi}_{13} = B_2K_1Z^{-1} - P^{-1}Y_1Z^{-1}, \quad \tilde{\varphi}_{23} = Z^{-1}W_1Z^{-1},$$
$$\tilde{\varphi}_{33} = -Z^{-1}QZ^{-1} - Z^{-1}W_1Z^{-1} - Z^{-T}W_1^T Z^{-T} + \sigma Z^{-1}\Omega Z^{-1} + \sigma Z^{-T}\Omega Z^{-T},$$
$$\tilde{\varphi}_{14} = -\tau_M P^{-1}YR^{-1}, \quad \tilde{\varphi}_{34} = -\tau_M Z^{-1}WR^{-1},$$
$$\tilde{\varphi}_{44} = -\tau_M R^{-1},$$
$$\tilde{\varphi}_{15} = B_2K_1Z^{-1}, \quad \tilde{\varphi}_{55} = -Z^{-1}\Omega Z^{-1} - Z^{-1}\Omega^T Z^{-1},$$
$$\tilde{\varphi}_{16} = B_3,$$

$$\tilde{\varphi}_{26} = -Z^{-1}C_1^T N^T, \tilde{\varphi}_{66} = -2I, \quad \tilde{\varphi}_{17} = \tau_M P^{-1}C_2^T B_1^T,$$
$$\tilde{\varphi}_{27} = \tau_M Z^{-1}A_1^T, \quad \tilde{\varphi}_{77} = -\tau_M R^{-1}.$$

Define $\tilde{P} = P^{-1}, \tilde{R} = R^{-1}, \tilde{Z} = Z^{-1}$, $\tilde{Q} = Z^{-1}QZ^{-1}, \tilde{X}_1 = K_1Z^{-1}, \tilde{X}_2 = K_2P^{-1}, \tilde{\Omega} = Z^{-1}\Omega Z^{-1}, \tilde{Y}_1 = P^{-1}YZ^{-1}, \tilde{Y}_2 = P^{-1}YR^{-1}, \tilde{W}_1 = Z^{-1}WZ^{-1}, \tilde{W}_2 = Z^{-1}W^TR^{-1}$.

If there are matrices P, Z, R, Q, Ω, W, Y with appropriate dimensions, so that the inequality (19) holds, then the system (10) is asymptotically stable, and the expected gain matrices of the primary controller and secondary controller are $K_1 = \tilde{X}_1\tilde{Z}^{-1}$ and $K_2 = \tilde{X}_2\tilde{P}^{-1}$, respectively. This completes the proof.

Remark 3: Scholars have completed the work of only considering event-triggered control or only considering nonlinear systems, but these two problems are considered in networked cascade control system in this paper. The above part considers the stability of the system without disturbance. But zero disturbance is a special case of disturbance. Therefore, when the disturbance is not constant zero, the specific influence of the disturbance on the system should be considered.

IV. DESIGN OF NONLINEAR NCCS EVENT-TRIGGED CONTROLLER WITH DISTURBANCE

In the previous section, we studied the sufficient condition of stability for nonlinear NCCS (10) without disturbance. We presented sufficient conditions for system stability and the controller design method through Theorem 1 and Theorem 2. This section will study the stability problem and H_∞ control of NCCS (10) with disturbances.

Under the zero-initial condition, if the constant $\gamma > 0$, the output of the primary controller $y_1(t)$ and the disturbance $\Delta(t)$ satisfy the H_∞ norm bounded constraint $\|y_1(t)\|_2 \leq \gamma||\Delta(t)||_2$, and $\Delta(t)$ boundary satisfies the L_2 norm bounded, that is, $\Delta(t) \in L_2[0, \infty)$. It ensures that the system (10) is stable under the H_∞ performance index γ.

Using calculation methods such as Theorem 1 and Theorem 2, we can obtain sufficient conditions and controller design methods for system stability in the presence of disturbances. The corresponding theorem proposed is as follows.

Theorem 3: For the known parameters σ, τ_M and $\gamma > 0$, and the corresponding parameter K_1, K_2, of the primary and secondary controller, if there exists matrices Y, W with proper dimension and symmetric positive definite matrices Z, P, Q, R, Ω, such that inequality (21) holds

$$\begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & \varphi_{14} & \varphi_{15} & \varphi_{16} & \varphi_{17} & \varphi_{18} & 0 \\ * & \varphi_{22} & \varphi_{23} & 0 & 0 & \varphi_{26} & \varphi_{27} & \varphi_{28} & \varphi_{29} \\ * & * & \varphi_{33} & \varphi_{34} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \varphi_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \varphi_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \varphi_{66} & \varphi_{67} & 0 & 0 \\ * & * & * & * & * & * & \varphi_{77} & \varphi_{78} & \varphi_{79} \\ * & * & * & * & * & * & * & \varphi_{88} & 0 \\ * & * & * & * & * & * & * & * & \varphi_{99} \end{bmatrix} < 0, \quad (21)$$

where

$$\varphi_{11} = PA_2 + A_2^TP + PB_2K_2 + K_2^TB_2^TP,$$
$$\varphi_{12} = C_2^TB_1^TZ + Y, \quad \varphi_{22} = ZA_1 + A_1^TZ + Q,$$
$$\varphi_{13} = PB_2K_1 - Y,$$
$$\varphi_{23} = W^T, \quad \varphi_{33} = -Q - W^T - W + \sigma\Omega^T + \sigma\Omega,$$
$$\varphi_{14} = -\tau_MY, \quad \varphi_{34} = -\tau_MW, \ \varphi_{44} = -\tau_MR,$$
$$\varphi_{15} = PB_2K_1,$$
$$\varphi_{55} = -\Omega - \Omega^T \quad, \varphi_{16} = PB_3, \ \varphi_{26} = -C_1^TN^T,$$
$$\varphi_{66} = -2I,$$
$$\varphi_{17} = PB_4, \quad \varphi_{27} = ZB_1C_4, \ \varphi_{76} = -NC_3, \ \varphi_{77} = -\gamma^2I,$$
$$\varphi_{18} = \tau_MC_2^TB_1^TR, \quad \varphi_{28} = \tau_MA_1^TR, \ \varphi_{78} = \tau_MC_4^TB_1^TR,$$
$$\varphi_{88} = -\tau_MR, \quad \varphi_{29} = C_1^T, \ \varphi_{79} = C_3^T, \ \varphi_{99} = -I.$$

Proof: Due to the addition of disturbance, the equality (14) can be rewritten as

$$l_2(t) = -x_1^T(t)C_1^TN^Tw(t) - w^T(t)NC_3\Delta(t) - w^T(t)Iw(t) \geq 0. \quad (22)$$

Select the same Lyapunov function as in Theorem 1 and derive it, where P, Z, R, Q are symmetric matrices and are positive definite matrices.

$$\dot{V}(t) \leq \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t) + 2l_1(t) + 2l_2(t). \quad (23)$$

For the system (10) with disturbance, define a function

$$J = \int_0^\infty \left[y_1^T(t)y_1(t) - \gamma^2\Delta^T(t)\Delta(t)\right]dt$$

Assuming $y_1^T(t)y_1(t) - \gamma^2\Delta^T(t)\Delta(t) + \dot{V}(t) = \xi_k^T\Upsilon\xi_k$, then for all nonzero matrices ξ_k, if $\gamma > 0$, then we can obtain $J \leq \xi_k^T\Upsilon\xi_k$. Define $\tilde{\Psi}$ as a symmetric matrix, according to formula (22), it can be rewritten in the following form

$$\begin{aligned} J &= \int_0^\infty \left[y_1^T(t)y_1(t) - \gamma^2\Delta^T(t)\Delta(t) + \dot{V}(t)\right]dt \\ &\quad - \int_0^\infty \dot{V}(t)dt \\ &= \int_0^\infty \xi^T(t,v)\tilde{\Psi}\xi(t,v)dt - V(\infty) + V(0). \end{aligned} \quad (24)$$

The inequality $\tilde{\Psi} < 0$, and can be described as

$$\tilde{\Psi} = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & \varphi_{14} & \varphi_{15} & \varphi_{16} & \varphi_{17} \\ * & \varphi_{22} & \varphi_{23} & 0 & 0 & \varphi_{26} & \varphi_{27} \\ * & * & \varphi_{33} & \varphi_{34} & 0 & 0 & 0 \\ * & * & * & \varphi_{44} & 0 & 0 & 0 \\ * & * & * & * & \varphi_{55} & 0 & 0 \\ * & * & * & * & * & \varphi_{66} & \varphi_{67} \\ * & * & * & * & * & * & \varphi_{77} \end{bmatrix} < 0, \quad (25)$$

where

$$\varphi_{11} = \tau(t)C_2^TB_1^TRB_1C_2 + PA_2 + A_2^TP + PB_2K_2 + K_2^TB_2^TP$$

$$\varphi_{12} = C_2^T B_1^T Z + \tau(t) C_2^T B_1^T R A_1 + Y,$$
$$\varphi_{22} = Z A_1 + A_1^T Z + Q + \tau(t) A_1^T R A_1 + C_1^T C_1,$$
$$\varphi_{13} = P B_2 K_1 - Y,$$
$$\varphi_{23} = W^T, \quad \varphi_{33} = -Q - W^T + W + \sigma\Omega^T + \sigma\Omega,$$
$$\varphi_{14} = -\tau_M Y, \quad \varphi_{34} = -\tau_M W, \ \varphi_{44} = -\tau_M R,$$
$$\varphi_{15} = P B_2 K_1,$$
$$\varphi_{55} = -\Omega - \Omega^T, \varphi_{16} = P B_3, \varphi_{26} = -C_1^T N^T, \varphi_{66} = -2I,$$
$$\varphi_{17} = \tau(t) C_2^T B_1^T R B_1 C_4 + P B_4,$$
$$\varphi_{27} = Z B_1 C_4 + \tau(t) A_1^T B_1^T R C_4 + C_1^T C_3, \quad \varphi_{67} = -N C_3,$$
$$\varphi_{77} = \tau(t) C_4^T B_1^T R B_1 C_4 + C_3^T C_3 - \gamma^2 I.$$

According to Lemma 1, the inequality $\tilde{\Psi} < 0$ can be transformed into the following inequality

$$\begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & \varphi_{14} & \varphi_{15} & \varphi_{16} & \varphi_{17} & \varphi_{18} & 0 \\ * & \varphi_{22} & \varphi_{23} & 0 & 0 & \varphi_{26} & \varphi_{27} & \varphi_{28} & \varphi_{29} \\ * & * & \varphi_{33} & \varphi_{34} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \varphi_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \varphi_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \varphi_{66} & \varphi_{67} & 0 & 0 \\ * & * & * & * & * & * & \varphi_{77} & \varphi_{78} & \varphi_{79} \\ * & * & * & * & * & * & * & \varphi_{88} & 0 \\ * & * & * & * & * & * & * & * & \varphi_{99} \end{bmatrix} < 0, \quad (26)$$

where

$$\varphi_{11} = P A_2 + A_2^T P + P B_2 K_2 + K_2^T B_2^T P,$$
$$\varphi_{12} = C_2^T B_1^T Z + Y, \quad \varphi_{22} = Z A_1 + A_1^T Z + Q,$$
$$\varphi_{13} = P B_2 K_1 - Y,$$
$$\varphi_{23} = W^T, \quad \varphi_{33} = -Q - W^T - W + \sigma\Omega^T + \sigma\Omega,$$
$$\varphi_{14} = -\tau_M Y, \ \varphi_{34} = -\tau_M W, \ \varphi_{44} = -\tau_M R, \varphi_{15} = P B_2 K_1,$$
$$\varphi_{55} = -\Omega - \Omega^T, \quad \varphi_{16} = P B_3, \ \varphi_{26} = -C_1^T N^T,$$
$$\varphi_{66} = -2I,$$
$$\varphi_{17} = P B_4, \quad \varphi_{27} = Z B_1 C_4, \ \varphi_{67} = -N C_3, \ \varphi_{77} = -\gamma^2 I,$$
$$\varphi_{18} = \tau_M C_2^T B_1^T R, \quad \varphi_{28} = \tau_M A_1^T R, \ \varphi_{78} = \tau_M C_4^T B_1^T R,$$
$$\varphi_{88} = -\tau_M R, \quad \varphi_{29} = C_1^T, \ \varphi_{79} = C_3^T, \ \varphi_{99} = -I.$$

Since $\tilde{\Psi} < 0$ and $V(0) = 0$ under zero initial condition, and $\lim_{t\to\infty} V(t) \geq 0$, $J < 0$ is verified by LMI. This means that the system (9) has an H_∞ performance index γ. As a result, if the inequality (25) is true, then the system (9) is asymptotically stable in the known sector interval.

According to Theorem 3, we give the design method of the primary controller and secondary one of the system (9) with disturbance.

Theorem 4: For the given parameters σ, $\tau_M > 0$ and $\gamma > 0$, if there are matrices of appropriate dimensions, W, Y and symmetric positive definite matrices P, Z, R, Q, Ω, such that the following inequality (26) holds, then the system (10) is asymptotically stable, and the expected gain matrix of the primary controller can be obtained

$$K_1 = \tilde{X}_1 \tilde{Z}^{-1},$$

and the gain matrix of the secondary controller is

$$K_2 = \tilde{X}_2 \tilde{P}^{-1}.$$

$$\begin{bmatrix} \tilde{\varphi}_{11} & \tilde{\varphi}_{12} & \tilde{\varphi}_{13} & \tilde{\varphi}_{14} & \tilde{\varphi}_{15} & \tilde{\varphi}_{16} & \tilde{\varphi}_{17} & \tilde{\varphi}_{18} & 0 \\ * & \tilde{\varphi}_{22} & \tilde{\varphi}_{23} & 0 & 0 & \tilde{\varphi}_{26} & \tilde{\varphi}_{27} & \tilde{\varphi}_{28} & \tilde{\varphi}_{29} \\ * & * & \tilde{\varphi}_{33} & \tilde{\varphi}_{34} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \tilde{\varphi}_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \tilde{\varphi}_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \tilde{\varphi}_{66} & \tilde{\varphi}_{67} & 0 & 0 \\ * & * & * & * & * & * & \tilde{\varphi}_{77} & \tilde{\varphi}_{78} & \tilde{\varphi}_{79} \\ * & * & * & * & * & * & * & \tilde{\varphi}_{88} & 0 \\ * & * & * & * & * & * & * & * & \tilde{\varphi}_{99} \end{bmatrix} < 0, \quad (27)$$

where

$$\tilde{\varphi}_{11} = A_2 \tilde{P} + \tilde{P} A_2^T + B_2 \tilde{X}_2 + \tilde{X}_2^T B_2^T,$$
$$\tilde{\varphi}_{12} = \tilde{P} C_2^T B_1^T + \tilde{Y}_1, \quad \tilde{\varphi}_{22} = A_1 \tilde{Z} + \tilde{Z} A_1^T + \tilde{Q},$$
$$\tilde{\varphi}_{13} = B_2 \tilde{X}_1 - \tilde{Y}_1,$$
$$\tilde{\varphi}_{23} = \tilde{W}_1, \tilde{\varphi}_{33} = -\tilde{Q} - \tilde{W}_1 - \tilde{W}_1^T + \sigma\tilde{\Omega} + \sigma\tilde{\Omega}^T,$$
$$\tilde{\varphi}_{14} = -\tau_M \tilde{Y}_2, \quad \tilde{\varphi}_{34} = -\tau_M \tilde{W}_2, \ \tilde{\varphi}_{44} = -\tau_M \tilde{R},$$
$$\tilde{\varphi}_{15} = B_2 \tilde{X}_1, \quad \tilde{\varphi}_{55} = -\tilde{\Omega} - \tilde{\Omega}^T, \ \tilde{\varphi}_{16} = B_3,$$
$$\tilde{\varphi}_{26} = -\tilde{Z} C_1^T N^T, \tilde{\varphi}_{66} = -2I,$$
$$\tilde{\varphi}_{17} = B_4, \quad \tilde{\varphi}_{27} = B_1 C_4, \ \tilde{\varphi}_{67} = -N C_3, \tilde{\varphi}_{77} = -\gamma^2 I,$$
$$\tilde{\varphi}_{18} = \tau_M \tilde{P} C_2^T B_1^T, \tilde{\varphi}_{28} = \tau_M \tilde{Z} A_1^T, \tilde{\varphi}_{78} = \tau_M C_4^T B_1^T,$$
$$\tilde{\varphi}_{88} = -\tau_M \tilde{R}, \quad \tilde{\varphi}_{29} = C_1^T, \ \tilde{\varphi}_{79} = C_3^T, \ \tilde{\varphi}_{99} = -I.$$

Proof: Define a matrix $\tilde{\Pi} = diag\{P^{-1}, Z^{-1}, Z^{-1}, R^{-1}, Z^{-1}, I, I, R^{-1}, I\}$. According to Theorem 1, pre- and post-multiplying (25) by $\tilde{\Pi}$, and define: $\tilde{P} = P^{-1}$, $\tilde{R} = R^{-1}$, $\tilde{Z} = Z^{-1}$, $\tilde{Q} = Z^{-1} Q Z^{-1}$, $\tilde{X}_1 = K_1 Z^{-1}$, $\tilde{X}_2 = K_2 P^{-1}$, $\tilde{\Omega} = Z^{-1} \Omega Z^{-1}$, $\tilde{Y}_1 = P^{-1} Y Z^{-1}$, $\tilde{Y}_2 = P^{-1} Y R^{-1}$, $\tilde{W}_1 = Z^{-1} W Z^{-1}$, $\tilde{W}_2 = Z^{-1} W^T R^{-1}$. Then, if there are matrices P, Z, R, Q, Ω, W, Y with appropriate dimensions, so that the inequality (26) holds, then the system (10) is asymptotically stable, and the expected gain matrices of the primary controller and secondary one are $K_1 = \tilde{X}_1 \tilde{Z}^{-1}$ and $K_2 = \tilde{X}_2 \tilde{P}^{-1}$, respectively. This completes the proof.

Remark 4: The consideration of nonlinear terms will make the system more universal. The addition of event-triggered scheme will significantly reduce the burden of the network. The cascade structure of the system can quickly restrain the disturbance in the loop. Finally, adding H_∞ control to make the system more suitable for practical production. Next, the effectiveness of the above design method will be verified by a simulation example of a marine boiler liquid level cascade control system.

V. SIMULATION EXAMPLE

Nonlinear systems are ubiquitous in industrial production with cascade structure, which is of practical significance. And nowadays, many cascade systems are connected through the network. In this paper, we choose a marine boiler liquid level cascade control system model, use the linear matrix inequality method to design the primary and secondary controllers, and verify the feasibility of the design method.

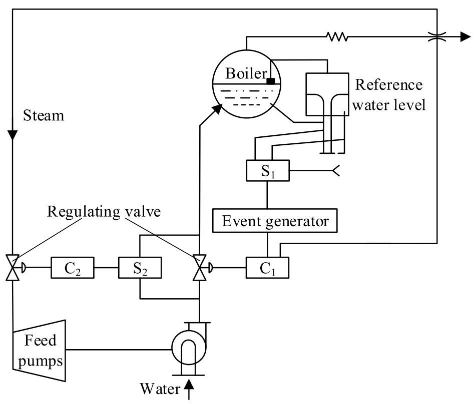

FIGURE 2. Marine boiler liquid level cascade control system.

As shown in fig. 2, C_1 and C_2 are primary and secondary controllers, and S1 and S_2 are corresponding sensors. The primary loop adjusts the steam valve's water supply pressure difference control loop according to the pressure difference before and after the water supply valve. According to the water level deviation, the secondary loop adjusts the water level regulation loop of the water supply valve. The given value of the water level is input. In order to facilitate the calculation, the given value is set to 0. The steam regulating valve is used as the primary plant P_1, and the water pump unit is the secondary plant P_2.

In this paper, the nonlinear function is selected as: $w(t) = y_1(t) + \sin(y_1(t))$. Assume that $\tau_M = 0.25s$, $\sigma = 0.2$, The initial conditions of the system are

$$x_1(0) = \begin{bmatrix} 1 & -1 \end{bmatrix}^T, \quad x_2(0) = \begin{bmatrix} 1 & -1 \end{bmatrix}^T.$$

The known matrices parameters in the model can be described as follows

$$A_1 = \begin{bmatrix} -0.3667 & -0.3 \\ 1 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1 & 2 \\ 1 & 0 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_3 = \begin{bmatrix} -0.5 \\ -0.2 \end{bmatrix},$$
$$B_4 = \begin{bmatrix} 0.02 \\ 0.05 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 0 & 0.11 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0 & 0.1 \end{bmatrix}, \quad C_3 = -0.3, C_4 = 0.2.$$

A. FOR NONLINEAR NCCS WITHOUT DISTURBANCE

When there is no disturbance, the matrices B_4, C_3 and C_4 are all zero matrices. According to Theorem 2, the feasible solution is obtained by using MATLAB/LMI toolbox

$$\tilde{P} = \begin{bmatrix} 145.8903 & -4.1333 \\ -4.1333 & -0.1545 \end{bmatrix}, \quad \tilde{Z} = \begin{bmatrix} 0.8926 & -0.6837 \\ -0.6837 & 2.2769 \end{bmatrix},$$
$$\tilde{\Omega} = \begin{bmatrix} 0.2508 & -0.2579 \\ -0.2579 & 0.7091 \end{bmatrix},$$
$$\tilde{X}_1 = \begin{bmatrix} 0.4183 & -0.2769 \end{bmatrix},$$
$$\tilde{X}_2 = \begin{bmatrix} -384.8342 & 12.0015 \end{bmatrix}.$$

The gain matrix of the primary controller and secondary one can be obtained according to the $K_1 = \tilde{X}_1 Z$ and $K_2 = \tilde{X}_2 P$

$$K_1 = \begin{bmatrix} 0.4183 & -0.2769 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -2.7527 & -4.0456 \end{bmatrix}.$$

In addition, the event-triggered parameter can be obtained as follows

$$\Omega = \begin{bmatrix} 0.3384 & 0.0729 \\ 0.0729 & 0.1500 \end{bmatrix}.$$

The sampling period is $h = 0.2s$. According to Theorem 2, the trigger matrix Ω is obtained. Out of 200 sampled signals, only 54 sampled signals are sent to the primary controller. In addition, the average signal sending interval after adding event-triggered can be calculated to be 0.7370s. It can be seen that the addition of event-triggered controller can filter the signals and save the network resources. Figure 3 show the state response of the system's primary and secondary loop, and Figure 4 shows the signal transmission interval.

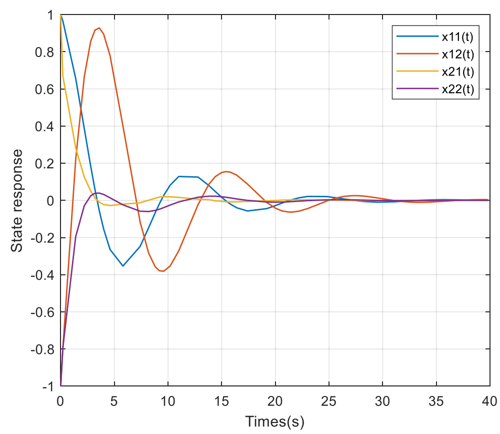

FIGURE 3. The state response of system without disturbance.

It can be seen that the system reaches a stable state at 40s. It indicates that the primary loop reaches the steady-state at 40s, and the secondary loop reaches the steady-state at 25s. Therefore, it can be proved that the design method proposed in this paper is feasible.

The above-described controller design methods in such cascade control systems can be obtained the feasible solutions directly, thereby avoiding the traditional methods of experimentation. For example, literature [34], in order to find the optimal gain of the controller, it is tested repeatedly in a range.

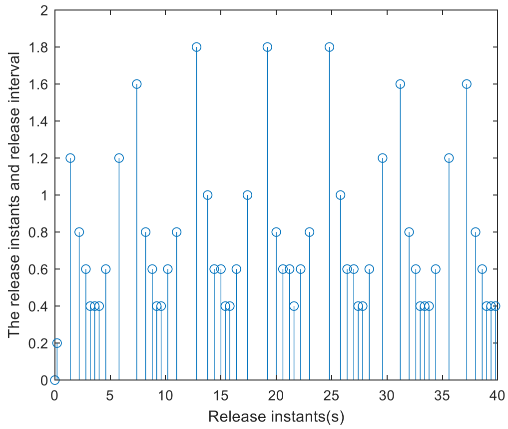

FIGURE 4. **The release instants and interval without disturbance.**

If using the design method in this paper, the result can be obtained directly, and the controller gain can be accurate to 4 decimal places.

B. FOR NONLINEAR NCCS WITH DISTURBANCE

The disturbance of the system mainly comes from the external disturbance caused by the sudden change of the controlled value and the change of the working environment during the operation of the system.

Irregular changes in the controlled value of the system itself can be included in the nonlinear term. So we assumed an external disturbance to impact the system to verify the stability and the constraints of H_∞ control on system disturbance is proved.

$$\Delta(t) = \begin{cases} \sin(t), & 3 < t \leq 10 \\ 0, & \text{otherwise.} \end{cases}$$

Setting $\tau_M = 0.25s, \sigma = 0.2, \gamma = 3$. According to Theorem 4, the following matrix can be obtained

$$\tilde{P} = \begin{bmatrix} 19.5422 & -8.9235 \\ -8.9235 & 9.0744 \end{bmatrix},$$
$$\tilde{Z} = \begin{bmatrix} 1.2459 & -0.9594 \\ -0.9594 & 3.6266 \end{bmatrix},$$
$$\tilde{\Omega} = \begin{bmatrix} 0.2504 & -0.2574 \\ -0.2574 & 0.7292 \end{bmatrix},$$
$$\tilde{X}_1 = \begin{bmatrix} -0.6709 & -0.5821 \end{bmatrix},$$
$$\tilde{X}_2 = \begin{bmatrix} -22.9306 & -13.3143 \end{bmatrix}.$$

The gain matrix of the primary controller and secondary one can be obtained according to the $K_1 = \tilde{X}_1 Z$ and $K_2 = \tilde{X}_2 P$ as follows

$$K_1 = \begin{bmatrix} -0.8315 & -0.3805 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -3.3457 & -4.7573 \end{bmatrix}.$$

In addition, the event-triggered parameter can be obtained as follows

$$\Omega = \begin{bmatrix} 0.1679 & 0.0265 \\ 0.0265 & 0.0577 \end{bmatrix}.$$

The sampling period is $h = 0.2s$. According to Theorem 2, the trigger matrix Ω is obtained. Out of 200 sampled signals, only 40 sampled signals are sent to the primary controller. In addition, the average signal sending interval after adding event-triggered scheme can be calculated to be 0.9950s. Figure 5 show the state response of the system, and Figure 6 shows the signal transmission interval.

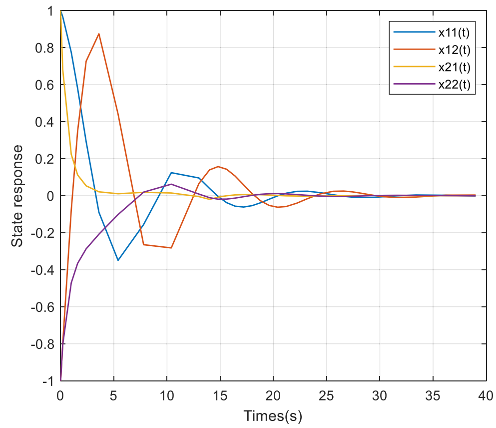

FIGURE 5. **The state response of system with disturbance.**

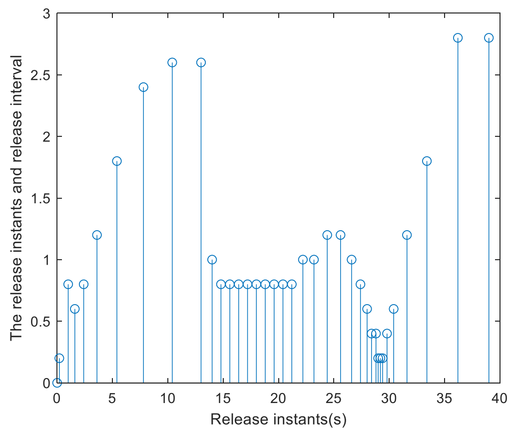

FIGURE 6. **The release instants and interval with disturbance.**

The nonlinear system model with disturbance is more suitable for practical industrial production. If it is a simple nonlinear system, the system will be fragile when applied to practice. For example, the [35] does not consider the influence of disturbance, so it has certain limitations.

Remark 5: We can see that the system can obtain sufficient conditions for stability after considering nonlinearity. Moreover, the simultaneous addition of event-triggered mechanism and H_∞ control can make the system resist disturbance to a certain extent while saving network resources.

VI. CONCLUSION

Considering the needs of practical industrial production, based on a class of nonlinear systems and event-triggered scheme, the primary and secondary controllers of NCCSs have designed in this paper. The method is applied to a liquid level of marine boiler control system, which is connected through the network. This is the first time that the event-triggered scheme is added to the Lurie nonlinear networked cascade control system, and the event-triggered scheme can avoid Zeno phenomenon based on reducing the burden of network bandwidth. A new model of the networked cascade control system is established, and the collaborative design method of primary controller and secondary one parameters and event-triggered parameters is completed through Lyapunov theorem and linear matrix inequality. Then add H_∞ control to effectively suppress the extra disturbance in the nonlinear system. Finally, an example of a marine boiler liquid level cascade control system is used to verify the stability of the system and the optimization of the utilization rate of network resources. Explains the feasibility of this method.

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