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Research on Gear Flank Twist Compensation of Continuous Generating Grinding Gear Based on Flexible Electronic Gearbox

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ABSTRACT Gear flank twist occurs when threaded wheel grinds crowning lead modified helical gears, which reduces gear machining accuracy. To overcome this problem, a flank twist compensation method based on flexible electronic gearbox is proposed in this paper, the method can reduce the degree of twist and improve gear machining accuracy. Firstly, flank equations of gear and threaded wheel are established, coordinates and normal vectors of gear flank are used to represent twist. Secondly, coordinate transformation matrix between workpiece and tool coordinate system is established based on multi-body system theory and homogeneous coordinate transformation, and relationship between coordinates and normal vectors of gear flank and master-slave axis (B-axis, C-axis, X-axis, Y-axis, Z-axis) movement amount is obtained by inverse kinematics and electronic gearbox, then master-slave axis movement compensation amount between crowning lead modified flank and twisted flank is obtained. Finally, twist is compensated by compensating for master-slave axis movement amount, and flank contact analysis based on KISSsoft is carried out to observe normal force and contact stress of flanks. This paper uses inverse kinematics and electronic gearbox to realize flank twist compensation, which makes up for long period and high cost of traditional flank twist compensation by changing shape of threaded wheel. Simulation results show that this method can effectively compensate for flank twist and improve accuracy and efficiency when threaded wheel grinds crowning lead modified helical gears.

INDEX TERMS Gear grinding, generating, flank twist, multi-axis movement, flexible electronic gearbox.

I. INTRODUCTION

Gears are widely used in automobiles, ships, construction machinery, aerospace, industrial robots and other fields as vital parts. Continuous generating gear grinding (threaded wheel gear grinding) is the most commonly used technology for gear finishing, which not only increase productivity of gears, but also improve quality and accuracy of gear machining. In order to improve the working performance of involute gears, such as load distribution [1], meshing impact, operating noise, lubrication [2], it is necessary to modify gear flank [3]. However, flank twist [4] will occur when threaded wheel grinds crowning lead modified helical gears, which will reduce flank accuracy, cause problems such as increasing side backlash, increasing vibration and shock, increasing working noise, and decreasing transmission accuracy. As a result, it is necessary to adjust and compensate for flank twist.

Gear modification methods can be divided into profile modification [5], lead modification [6] and flank modification commonly. Profile modification can be divided into tip and root relief, profile angle modification and profile crowning, lead modification can be divided into end relief, lead angle modification and lead crowning, and flank modification is the combination of profile modification and lead modification. Litvin *et al.* [7] verified that modified gears are superior to conventional helical gears in terms of gear drives

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through computerized design, generation methods, meshing simulation, stress analysis. İmrek *et al.* [8] made pressure distribution of gear flank approximately uniform by modifying width of spur gear, and found that wear depth of modified gear along meshing area was more uniform than unmodified gear. Combining proposed ideas in [9] and [10], flank wear had a bad effect on the dynamic behavior of gear transmission system in the off-resonance speed range, with a reduced effect at the resonance peak, uneven wear lead to stress concentration, and gears with uniform wear worked with lower noise and vibration and longer life. Han *et al.* [11] proposed a method to obtain profile crowning and lead crowning flank by setting the movement of A-axis and B-axis as fourth-order polynomial functions of axial feed of work gear in internal power gear honing machine tool, studied the influence of polynomial coefficients on the flank through sensitivity matrix, adjusted polynomial coefficients to minimize flank error. Tran *et al.* [12] generated double-crowning involute helical gear by setting diagonal feed movement as a second-order function of axial movement of hob, modified the profile of hob cutter into a double-lead form by changing the pressure angle in lead direction, verified that gear pairs produced by dual-lead hob cutter is superior to those produced by conventional modified hob cutter in terms of contact performance and transmission performance by three computer simulation examples. Hsu *et al.* [13] proposed a method that setting a crossed angle between honing cutter and gear axis as a linear function of honing cutter's traverse feed motion to achieve lead crowning flank in honing process. In summary, gear modification can significantly optimize stress distribution of flank and improve transmission performance of gear. However, when threaded wheel grinds crowning lead modified helical gears, since grinding amount on the same contact trace is equal, grinding amount of gear at different radii of same cross section along lead direction is not equal, which often results in flank twist.

Researches on flank twist compensation can be mainly divided into changing of threaded wheel shape, optimizing of modification curve, changing of movement amount of machine tool axis. Graf *et al.* [14] introduced a counter-twist or added a specific twist to counteract the flank twist by changing pressure angle or pitch of threaded wheel. Stadtfeld *et al.* [15] eliminated or controlled flank twist amount by dressing threaded wheel with a circular arc hollow-crowning generating front and shifting workpiece across threaded wheel along a feed shift vector. Faulstich *et al.* [16] created twist required for workpiece by changing amount and course of tool crowning and diagonal ratio, and superimposed it on natural twist to reduce twist. Wu *et al.* [17] proposed a method to compensate for twist of lead crowning flank by applying a linear swivel angle function of honing wheel and a variable pressure angle honing wheel. Li *et al.* [18] proposed a design method for lead modified curve, presented a calculating method of flank twist and analyzed the influence of crowning amount and helix angle on twist amount, then modification curve is divided into three sections, and proportion factor and crowning amount of three sections are adjusted to reduce flank twist. Tran *et al.* [19] proposed a closed-loop topology modification method to obtain double-crowning and anti-twist flank of cylindrical gears of CNC internal gear honing machines, additional motions are added to three axes including radial feed axis and swivel axis of honing wheel and rotating axis of gear in the form of polynomials, and polynomial coefficients of additional motions for desired gear flank are obtained through sensitivity matrix and Levenberg-Marquardt algorithm.

In addition, there exists another precompensation method for flank twist. According to twist generating mechanism, gear anti-twist flank is reversely designed to offset twist. For example, engineers in KISSsoft [20] demonstrated this method in the 3rd International Conference on Gear Production 2019, mentioned an accurate formula of twist amount and calculated the twist amount automatically in KISSsoft, so if twist can be acceptable or if additional profile modification or lead modification are needed to reduce twist can be decided.

Methods of changing the shape of threaded wheel and optimizing the modification curve to achieve flank twist compensation exist deficiencies including long dressing cycle, high cost, low utilization rate of threaded wheel and limited twist compensation effect. Because changing movement of machine tool axis can make up for above deficiencies and improve convenience of flank twist compensation, and previous studies exist problems of incomplete compensation and not considering linkage relationship of machine tool axis movement. Therefore, a method for flank twist compensation by compensating for master-slave axis movement amount based on flexible electronic gearbox is proposed in this paper.

Firstly, coordinates and normal vectors of gear flank are used to express flank twist, then relationship between coordinates and normal vectors of gear flank and master-slave axis movement amount is established based on electronic gearbox, finally on the basis of realizing lead modification, flank twist is compensated by compensating for master-slave axis movement amount. This method can reduce the cost of dressing threaded wheels, improve accuracy and efficiency in gear grinding.

II. GENERATING MECHANISM AND MATHEMATICAL MODEL OF FLANK TWIST

A. GENERATING MECHANISM OF FLANK TWIST

As shown in Fig. 1(a), flank twist refers to the phenomenon that transverse profile of gear is twisted along lead direction. When threaded wheel grinds gear, numerous contact traces are formed during meshing process and grinding amount on the same contact trace is equal. If flank to be ground is involute spur gear flank or involute helical gear flank without crowning lead modification, grinding amount of entire flank is equal, and flank twist does not occur. If flank to be ground is involute helical gear flank with crowning lead modification, grinding amount of flank at different radii of the same cross

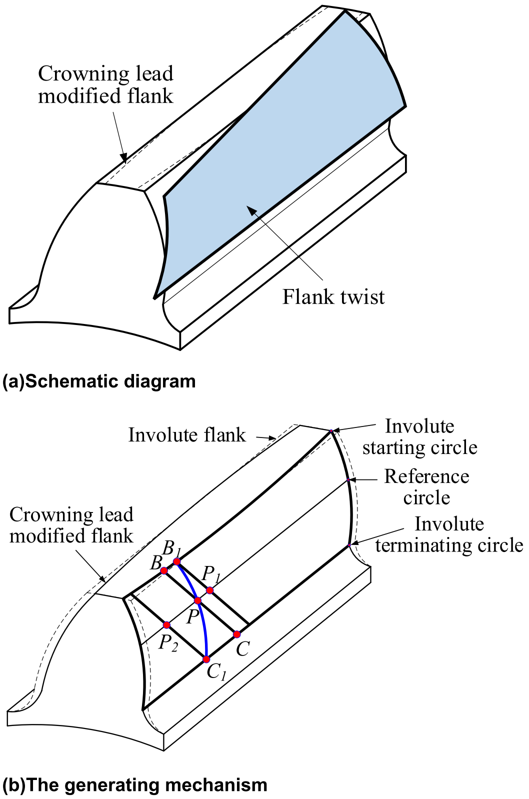


FIGURE 1. Flank twist.

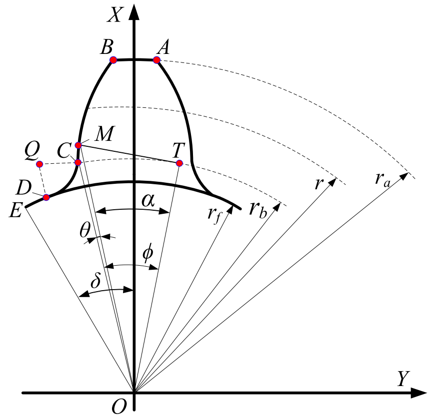


FIGURE 2. Transverse profile of gear.

section along lead direction is not equal, and flank twist occurs.

As shown in Fig. 1(b), taking the crowning lead modified flank as example, assuming modification curve is parabola, because grinding amount on the same contact trace is equal, grinding amount of point B_1 is equal to point P_1, and grinding amount of point C_1 is equal to point P_2, so grinding amount of the cross section where point P (B_1PC_1) is located is not equal. In the same way, grinding amount of the same cross section along lead direction is not equal, and flank is twisted. On both ends of gear, modification amount of lead direction is largest, and tangent slope of modification curve is largest, and torsion is largest, so the maximum torsion at both ends of gear can be defined as flank twist amount.

B. ESTABLISHMENT OF GEAR FLANK MODEL

Fig. 2 shows the transverse profile of gear, including top arc AB, involute BC, root transition arc CD and root arc DE. Establish coordinate system XOY, taking the gear center as coordinate origin O, horizontal right direction as positive direction of Y-axis and vertical upward direction as positive direction of X-axis. Point C is the starting point of involute, point B is the terminating point of involute, δ is the angle between OE and positive direction of X-axis, r_b is base circle radius, and ϕ is the sum of spread angle θ and pressure angle α.

Taking the left flank as example, parameter equation of involute can be obtained as follows:

$$\begin{cases} x_0 = r_b[cos\,(\delta+\phi)+\phi sin\,(\delta+\phi)] \\ y_0 = r_b[sin\,(\delta+\phi)-\phi cos\,(\delta+\phi)] \end{cases} \tag{1}$$

Involute flank is formed by spiraling involute BC around gear axis, and equation of it can be obtained as follows:

$$\begin{cases} x = r_b[cos\,(\delta+\phi+\varphi)+\phi sin\,(\delta+\phi+\varphi)] \\ y = r_b[sin\,(\delta+\phi+\varphi)-\phi cos\,(\delta+\phi+\varphi)] \\ z = p\varphi \end{cases} \tag{2}$$

where φ is the rotation angle of involute BC rotating around gear axis, and $p = r_b/tan\beta_b$ is spiral parameter. Assuming modified curve equation is $G = \rho(p\varphi - b/2)^2$, ρ is parabolic coefficient, and b is face width, because modified curve whose normal distance to involute is G and the curve whose rotation angle to base circle center is $\Delta\varphi = G/r_b$ are the same curve, so crowning lead modified flank equation can be obtained as follows:

$$\begin{cases} x = r_b[cos\,(\delta+\phi+\varphi-\Delta\varphi) \\ \qquad +\phi sin\,(\delta+\phi+\varphi-\Delta\varphi)] \\ y = r_b[sin\,(\delta+\phi+\varphi-\Delta\varphi) \\ \qquad -\phi cos\,(\delta+\phi+\varphi-\Delta\varphi)] \\ z = p\varphi \end{cases} \tag{3}$$

By obtaining partial derivatives of variables ϕ and φ respectively, normal vector of involute flank and crowning lead modified flank can be obtained respectively.

C. ESTABLISHMENT OF THREADED WHEEL FLANK MODEL

Fig. 3 shows the spatial meshing coordinate system of gear and threaded wheel, coordinate system $S(O-X, Y, Z)$ and $S_p(O_p - X_p, Y_p, Z_p)$ are fixed in space, coordinate system $S_1(O_1 - X_1, Y_1, Z_1)$ and $S_2(O_2 - X_2, Y_2, Z_2)$ are fixedly connected with threaded wheel and gear respectively, Z_1-axis and Z_2-axis coincides with the rotation axis of threaded wheel and gear respectively, $OO_p = a$ is center distance, Σ is

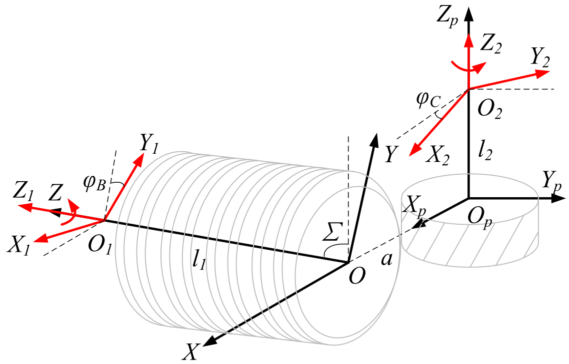


FIGURE 3. Meshing coordinate system.

crossed angle, $OO_1 = l_1$ is threaded wheel shifting distance, $O_pO_2 = l_2$ is moving distance of threaded wheel along gear axis, which can be represented by the moving distance of gear along gear axis.

According to the gear flank meshing equation $v_{12} = n$ [21], equation of threaded wheel flank meshed with gear flank can be obtained as follows:

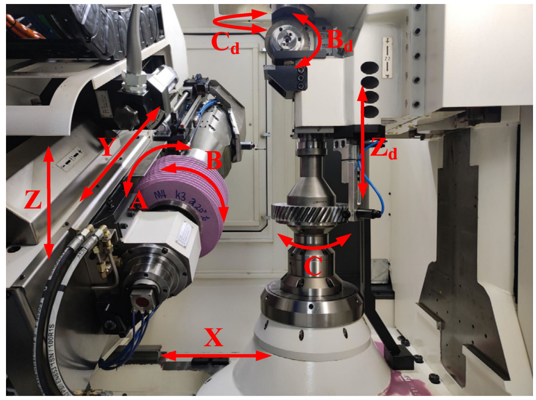
$$\begin{cases} x_t = \left[x_0 \cos\varphi'_C - y_0 sin\varphi'_C - a\right] cos\varphi_B \\ \quad + \left\{\left[x_0 \sin\varphi'_C + y_0 cos\varphi'_C\right] cos\sum + z_C sin\sum\right\} sin\varphi_B \\ y_t = -\left[x_0 \cos\varphi'_C - y_0 sin\varphi'_C - a\right] sin\varphi_B \\ \quad + \left\{\left[x_0 \sin\varphi'_C + y_0 cos\varphi'_C\right] cos\sum + z_C sin\sum\right\} cos\varphi_B \\ z_t = -\left[x_0 \sin\varphi'_C + y_0 cos\varphi'_C\right] sin\sum + z_C cos\sum - l_1 \end{cases} \tag{4}$$

Among them, v_{12} is the relative velocity of a certain point in spatial meshing coordinate system, and n is normal vector of the point. x_0 and y_0 represent coordinates of transverse profile of gear, when gear rotates the angle of φ_C, rotation angle of threaded wheel is $\varphi_B = \varphi_C/i_{21} + l_1/p_1$, and $\varphi'_C = \varphi + \varphi_C$, z_C represents the Z-axis coordinate of gear flank, $i_{21} = z_1/z_2$ represents the ratio of head number of threaded wheel z_1 and teeth number of gear z_2, in the same way, normal vector of threaded wheel flank can be obtained as $N = (N_X, N_Y, N_Z)'$.

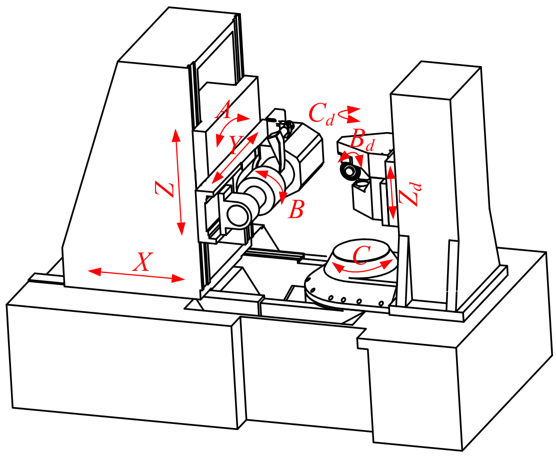
III. COMPENSATION PRINCIPLE OF FLANK TWIST

A. ESTABLISHMENT OF KINEMATICS MODEL OF MACHINE TOOL

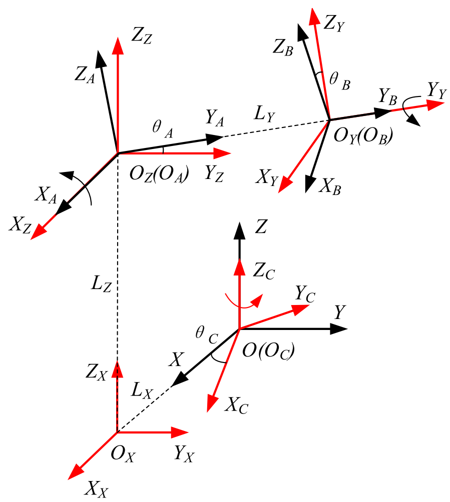
The YW7232CNC gear grinding machine of Chongqing Machine Tool Group is adopted as the gear processing machine tool in this paper, main movement axes of it are radial feed axis X-axis of threaded wheel, tangential feed axis Y-axis of threaded wheel, axial feed axis Z-axis of tool post, swing axis A-axis of tool post, threaded wheel spindle B-axis, and workpiece spindle C-axis, test rig is shown in Fig. 4(a), schematic diagram of the most commonly used axes are shown in Fig. 4(b). According to multi-body system theory, coordinate transformation relationship between adjacent axis can be established as shown in Fig. 4(c), and coordinate transformation matrices can be obtained.



(a) Test rig



(b) Schematic diagram



(c) Coordinate transformation relationship

FIGURE 4. CNC threaded wheel gear grinding machine X: Radial feed axis of threaded wheel Y: Tangential feed axis of threaded wheel Z: Axial feed axis of tool post A: Swing axis of tool post B: Threaded wheel spindle C: Work spindle B_d: Dressing wheel rotating spindle C_d: Dressing wheel swing axis Z_d: External stent moving axis.

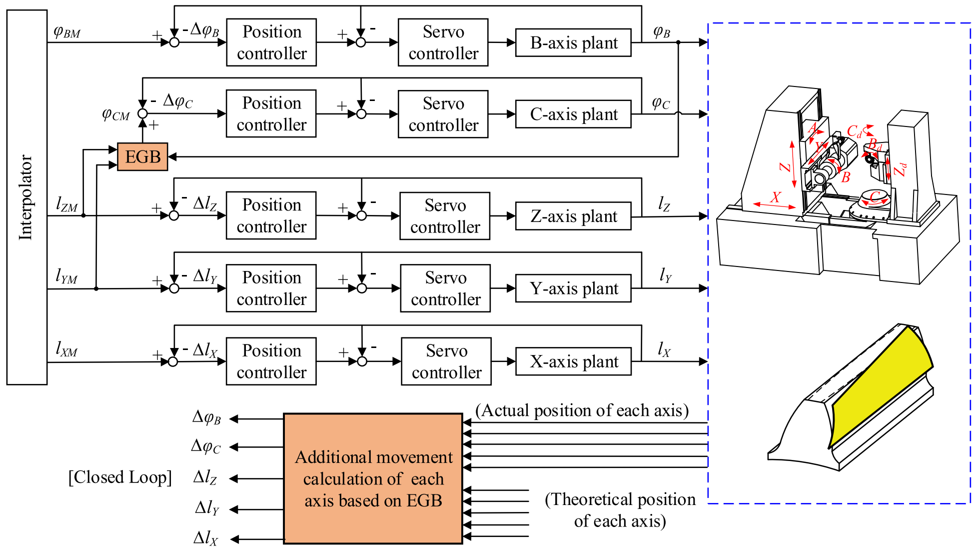


FIGURE 5. Flank twist compensation principle.

According to homogeneous coordinate transformation, coordinate transformation matrix from tool to workpiece coordinate system can be obtained as shown in Equation 5. $\boldsymbol{M}_{CO}$ is the transformation matrix from machine bed to C-axis, $\boldsymbol{M}_{XO}$ is the transformation matrix from machine bed to X-axis, $\boldsymbol{M}_{ZX}$ is the transformation matrix from X-axis to Z-axis, $\boldsymbol{M}_{AZ}$ is the transformation matrix from Z-axis to A-axis, $\boldsymbol{M}_{YA}$ is the transformation matrix from A-axis to Y-axis, $\boldsymbol{M}_{BY}$ is the transformation matrix from Y-axis to B-axis. Among them, φ_A, φ_B and φ_c are rotation angle of A-axis, B-axis and C-axis respectively, l_x, l_y and l_z are displacement of X-axis, Y-axis and Z-axis respectively.

$$\boldsymbol{M}_{CB} = \boldsymbol{M}_{CO}\boldsymbol{M}_{XO}^{-1}\boldsymbol{M}_{ZX}^{-1}\boldsymbol{M}_{AZ}^{-1}\boldsymbol{M}_{YA}^{-1}\boldsymbol{M}_{BY}^{-1}$$

$$\boldsymbol{M}_{CO} = \begin{bmatrix} \cos(\varphi_c) & \sin(\varphi_c) & 0 & 0 \\ -\sin(\varphi_c) & \cos(\varphi_c) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\boldsymbol{M}_{XO} = \begin{bmatrix} 1 & 0 & 0 & -l_x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\boldsymbol{M}_{ZX} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -l_z \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\boldsymbol{M}_{AZ} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\varphi_A) & \sin(\varphi_A) & 0 \\ 0 & -\sin(\varphi_A) & \cos(\varphi_A) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\boldsymbol{M}_{YA} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -l_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\boldsymbol{M}_{BY} = \begin{bmatrix} \cos(\varphi_B) & 0 & -\sin(\varphi_B) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(\varphi_B) & 0 & \cos(\varphi_B) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{5}$$

B. FLANK TWIST COMPENSATION BASED ON FLEXIBLE ELECTRONIC GEARBOX

Electronic gearbox can replace the internal transmission chain of machine tool and maintain multi-axis movement to realize linkage relationship of fixed speed ratio or varying speed ratio. Structure of electronic gearbox can be divided into master-slave or parallel. When threaded wheel grinds gear, spindle participates in linkage, its speed is large, its position and speed are strictly required, so master-slave and parallel structure are combined to use in this paper. Flank twist compensation principle based on electronic gearbox is shown in Fig. 5.

Linkage relationship of B-axis, C-axis, Y-axis, Z-axis of flexible electronic gearbox [22] of threaded wheel gear grinding machine tool can be obtained as follows:

$$n_C = k_B\frac{z_1}{z_2}n_B + k_Z\frac{sin\beta}{\pi m_n z_2}v_Z + k_Y\frac{cos\lambda}{\pi m_n z_2}v_Y \tag{6}$$

Among them, n_B and n_C are rotation speed of B-axis and C-axis respectively; v_Y and v_Z are movement speed of Y-axis and Z-axis respectively, unit is mm/min; z_1 and z_2 are head number of threaded wheel and teeth number of gear respectively; β and λ are helix angle of gear and tool installation angle respectively, unit is degree (°), helix angle is positive when gear is right-hand and negative when gear is left-hand; m_n is normal module of gear, unit is mm; k_B, k_Z and k_Y are constant coefficients, when threaded wheel is right-hand, $k_B = 1$, when threaded wheel is left-hand, $k_B = -1$; when $v_Z < 0$ and $\beta > 0$, $k_Z = 1$; when $v_Z < 0$ and $\beta < 0$, $k_Z = -1$; when $v_Z > 0$ and $\beta > 0$, $k_Z = -1$; when $v_Z > 0$ and $\beta < 0$, $k_Z = 1$; when $v_Y > 0$, $k_Y = 1$; when $v_Y < 0$, $k_Y = -1$.

Define coordinates and normal vectors of gear flank as $\boldsymbol{P}_C = \begin{bmatrix} n & r \\ 0 & 1 \end{bmatrix}$ ($n = (n_x, n_y, n_z)'$, $r = (x, y, z)'$) in workpiece coordinate system and as $\boldsymbol{P}_B = \begin{bmatrix} N & R \\ 0 & 1 \end{bmatrix}$ ($N = (-N_X, -N_Y, -N_Z)'$, $R = (x_t, y_t, z_t)'$) in tool coordinate system, $\boldsymbol{P}_C = \boldsymbol{M}_{CB}\boldsymbol{P}_B$ can be obtained, where $\boldsymbol{P}_B$ and coordinates and normal vectors of threaded wheel flank satisfy the relationship that coordinates are same and directions of normal vectors are opposite at each point. Assuming rotation angle of A-axis is a constant, using electronic gearbox model to ensure linkage relationship between B-axis, C-axis, Y-axis and Z-axis, replacing the speed of axes by the displacement of axes, according to inverse kinematics [23], relationship between coordinates and normal vectors of gear flank and master-slave axis movement amount can be obtained by solving the equation $\boldsymbol{P}_C = \boldsymbol{M}_{CB}\boldsymbol{P}_B$.

According to above relationships, master-slave axis movement amount corresponding to crowning lead modified flank and twisted flank can be obtained respectively, and additional movement amount of master-slave axis can be obtained by subtracting them as follows:

$$\begin{cases} \Delta\varphi_B = \varphi_{BM} - \varphi_{BT} \\ \Delta\varphi_C = \varphi_{CM} - \varphi_{CT} \\ \Delta l_X = l_{XM} - l_{XT} \\ \Delta l_Y = l_{YM} - l_{YT} \\ \Delta l_Z = l_{ZM} - l_{ZT} \end{cases} \tag{7}$$

Among them, $\varphi_{BM}, \varphi_{CM}, l_{XM}, l_{YM}, l_{ZM}$ represent the movement amount of master-slave axis corresponding to crowning lead modified flank, φ_{BT}, φ_{CT}, l_{XT}, l_{YT}, l_{ZT} represent the movement amount of master-slave axis corresponding to twisted flank. When compensating for the flank twist, φ_{BT}, φ_{CT}, l_{XT}, l_{YT}, l_{ZT} combined with their respective additional movement amount are their actual movement amount, as shown in Equation. 8. Among them, $k_i (i = 1, 2, 3, 4, 5)$ are compensation coefficients, and they can become different values.

$$\begin{cases} \varphi_{BA} = \varphi_{BT} + k_1\Delta\varphi_B \\ \varphi_{CA} = \varphi_{CT} + k_2\Delta\varphi_C \\ l_{XA} = l_{XT} + k_3\Delta l_X \\ l_{YA} = l_{YT} + k_4\Delta l_Y \\ l_{ZA} = l_{ZT} + k_5\Delta l_Z \end{cases} \tag{8}$$

IV. NUMERICAL SIMULATION RESEARCH

In this part, numerical examples based on MATLAB simulation are used to verify the correctness of above theories and models, basic parameters of gear and threaded wheel in the simulation are shown in Table 1. Appropriate crowning amount calculated by KISSsoft software for the gear at this parameter is 1.58 μm, so parabolic coefficient ρ is -3.95×10^{-6}. When flank is gridded, sampling area of gear flank is the area where indent of tip and root are taken as 5% of working tooth height respectively, and indent of both ends

TABLE 1. Basic parameters.

Gear		Threaded wheel	
Normal pressure angle α	20°	Normal pressure angle α	20°
Number of teeth z_2	48	Number of head z_1	3
Normal module m_n	4 mm	Normal module m_n	4 mm
Face width b	40 mm	Threaded wheel length l	160 mm
Helical direction	right-hand	Helical direction	right-hand
Helix angle β	30°	Helix angle β	87.5°

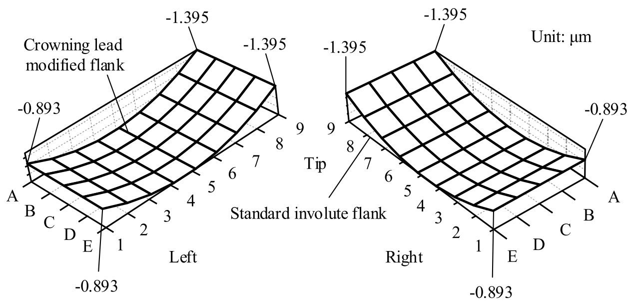


FIGURE 6. Crowning lead modified flank.

TABLE 2. Normal deviations of crowning lead modified flank (unit: μm).

	A	B	C	D	E
1	-0.893	-0.893	-0.893	-0.893	-0.893
2	-0.461	-0.461	-0.461	-0.461	-0.461
3	-0.171	-0.171	-0.171	-0.171	-0.171
4	-0.022	-0.022	-0.022	-0.022	-0.022
5	-0.014	-0.014	-0.014	-0.014	-0.014
6	-0.147	-0.147	-0.147	-0.147	-0.147
7	-0.422	-0.422	-0.422	-0.422	-0.422
8	-0.838	-0.838	-0.838	-0.838	-0.838
9	-1.395	-1.395	-1.395	-1.395	-1.395

are taken as 5% of working face width respectively, and flank is gridded to obtain 5 × 9 points. Crowning lead modified flank simulation is shown in Fig. 6, taking the left flank for example, normal deviations of crowning lead modified flank compared with standard involute flank can be obtained as shown in Table 2.

Twisted flank is constructed according to the twist amount formula $C = 8C_\beta L_\alpha \tan(\beta_b)/b$ [18], C represents twist amount, C_β represents crowning amount, L_α represents involute length, β_b represents helix angle of base circle, b represents face width. Fig. 7 shows simulation result of twisted flank, since flank twist generally reaches maximum amount at both ends of gear, four vertices of upper, lower, left, and right sides of flank are used as research object to observe twist amount. Taking the left flank for example, normal deviations of twisted flank compared with standard involute flank can be obtained as shown in Table 3.

Final compensation result simulation is shown in Fig. 8. When compensating for movement amount of master-slave

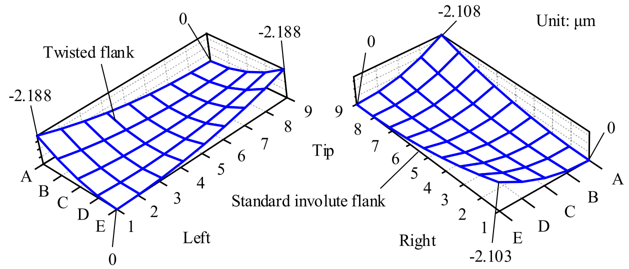


FIGURE 7. Twisted flank.

TABLE 3. Normal deviations of twisted flank (unit: μm).

	A	B	C	D	E
1	-2.188	-1.230	-0.547	-0.137	0
2	-1.675	-0.855	-0.308	-0.034	-0.034
3	-1.230	-0.547	-0.137	0	-0.137
4	-0.854	-0.308	-0.034	-0.034	-0.307
5	-0.546	-0.137	0	-0.137	-0.547
6	-0.307	-0.034	-0.034	-0.308	-0.854
7	-0.137	0	-0.137	-0.547	-1.230
8	-0.034	-0.034	-0.308	-0.855	-1.675
9	0	-0.137	-0.547	-1.230	-2.188

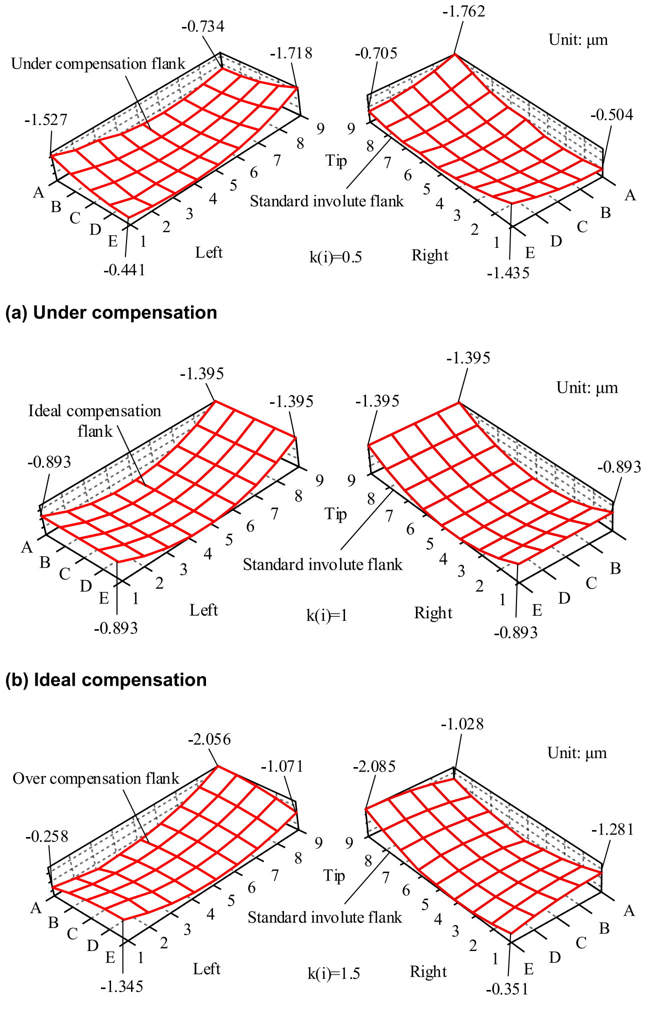
TABLE 4. Basic parameters of single gear pair.

Parameters	Driving gear	Driven gear
Number of teeth	48	30
Helix angle	30°	30°
Helical direction	right-hand	left-hand
Normal module	4 mm	4 mm
Normal pressure angle	20°	20°
Face width	40 mm	40 mm

axis, for compensation coefficients, $k_i (i = 1, 2, 3, 4, 5)$ are 0.5, 1 and 1.5 in Figs. 8(a), 8(b) 8(c) respectively, it means that for compensation movement amount of master-slave axis, Figs. 8(a), 8(b) 8(c) represent under compensation, ideal compensation and over compensation respectively.

V. FLANK CONTACT ANALYSIS

A single gear pair transmission system is constructed with basic parameters shown in Table 4. In gear analysis software KISSsoft, above-mentioned standard involute flank, crowning lead modified flank, twisted flank and flank after ideal twist compensation are established respectively, taking the left flank as working flank, and parameters are set as shown in Table 5, results of normal force and contact stress distribution on flanks are obtained as shown in Figs. 9∼12.



(a) Under compensation

(b) Ideal compensation

(c) Over compensation

FIGURE 8. Compensation result.

TABLE 5. Basic parameters of flank contact analysis.

Input Parameters	Values
Power (P)	45 kW
Rotation speed (n)	1500 r/min
Friction coefficient (μ)	0.05
Alternating bending coefficient (Y_M)	1.0
Service life (H)	200000 h
Usage factor (K_A)	1.25
Inter-tooth load distribution coefficient (K_{H_α})	1.0
Flank load distribution coefficient (K_{H_β})	1.04
Dynamic load factor (K_V)	1.03

VI. RESULTS AND DISCUSSIONS

For the main process of flank twist compensation in this paper, the first step is to analyze flank twist generating mechanism, and express flank twist by coordinates and normal vectors of gear flank. The second step is to establish equations of involute gear flank, crowning lead modified flank and threaded wheel flank, and verify the correctness of above equations based on MATLAB simulation. The third step is to establish coordinate transformation matrix between tool and workpiece coordinate system, and obtain relationship between coordinates and normal vectors of gear flank and master-slave axis movement amount by inverse kinematics and electronic gearbox, and obtain master-slave axis movement compensation amount between crowning lead modified flank and twisted flank. Finally realize twist compensation by compensating for master-slave axis movement amount, and carry out flank contact analysis by KISSsoft software.

Simulation results as shown in Figs. 6∼8 show that twist has been compensated. When $k_i(i = 1, 2, 3, 4, 5)$ are 0.5, twist amount is reduced, but not completely compensated. When $k_i(i = 1, 2, 3, 4, 5)$ are 1, crowning lead modified flank and flank after ideal twist compensation almost coincide, and error between them is very small. When $k_i(i = 1, 2, 3, 4, 5)$ are 1.5, twist directions of flanks in Figs. 7 and 8(c) are opposite. And ideal compensation will be adopted when compensate for twist actually, and it can be concluded that this twist compensation method is effective.

According to flank contact analysis results in Figs. 9∼12, for standard involute flank, after crowning lead modification, maximum normal force decreases from 145.635 N/mm to 117.450 N/mm, and maximum contact stress decreases from 622.564 N/mm^2 to 559.533 N/mm^2. When twist occurs, maximum normal force increases to 182.696 N/mm, and

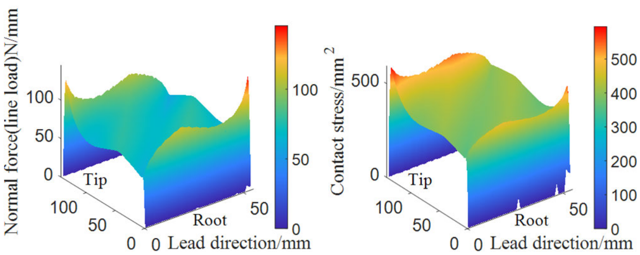


FIGURE 9. Normal force (left) and contact stress (right) of standard involute flank.

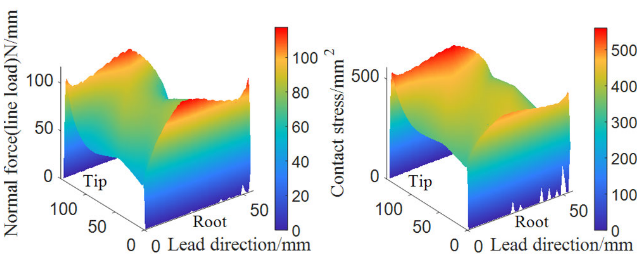


FIGURE 10. Normal force (left) and contact stress (right) of crowning lead modified flank.

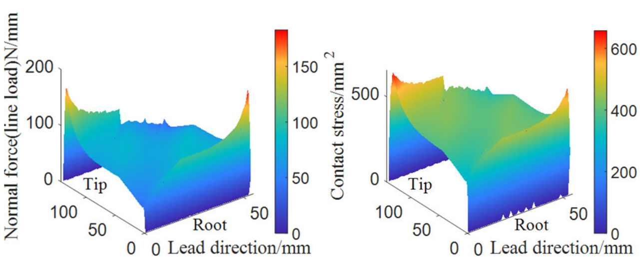


FIGURE 11. Normal force (left) and contact stress (right) of twisted flank.

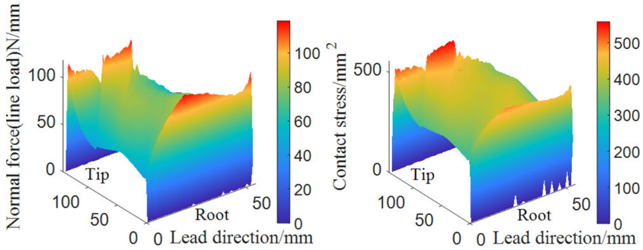


FIGURE 12. Normal force (left) and contact stress (right) of flank after ideal twist compensation.

maximum contact stress increases to 675.087 N/mm^2. After ideal twist compensation, maximum normal force decreases to 124.672 N/mm, and maximum contact stress decreases to 558.210 N/mm^2. Compare normal force and contact stress of crowning lead modified flank and flank after ideal twist compensation, meshing performance of them is basically same, and expected effect of twist compensation is achieved.

VII. CONCLUSION

A method for flank twist compensation in continuous generating grinding gear based on flexible electronic gearbox is proposed. Analysis of flank twist generating mechanism is carried out, and the flank twist is expressed by coordinates and normal vectors of gear flank, mathematical models of involute gear flank, crowning lead modified flank and threaded wheel flank are established. Coordinate transformation matrix between tool and workpiece coordinate system is established according to multi-body system theory and homogeneous coordinate transformation, relationship between coordinates and normal vectors of gear flank and master-slave axis movement amount is obtained by inverse kinematics and electronic gearbox, flank twist is compensated by compensating for master-slave axis movement amount, and the validity and correctness of this twist compensation method is showed by MATLAB simulation. Linkage relationship of B-axis, C-axis, Y-axis, Z-axis is guaranteed by electronic gearbox when compensating for master-slave axis movement amount, and reliability and accuracy of flank twist compensation is improved. Flank contact analysis shows that after ideal twist compensation, normal force and contact stress of twisted flank are reduced, and expected effect of the twist compensation is achieved. For the future research, practical experiments will be conducted to verify the correctness of above theories and models.

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