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Autonomous Vehicle Evaluation: A Comprehensive Survey on Modeling and Simulation Approaches

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ABSTRACT In recent years, autonomous vehicles (AVs), which observe the driving environment and lead a few or all of the driving tasks, have garnered tremendous success. The field of AVs has been rapidly developing and has found many applications. As a safety requirement established by policymakers, these vehicles must be evaluated before their deployment. The evaluation process for AVs is challenging because crashes are rare events, and AVs can escape passing predefined test scenarios. Therefore, capturing crashes and creating real test scenarios should be considered in order to develop an evaluation approach that represents real-world scenarios. One evaluation approach is based on the naturalistic field operational test (N-FOT), in which prototype AVs are driven on roads by volunteers or test engineers. Unfortunately, this approach is time-consuming and costly because thousands of miles need to be driven to experience a police-reported collision and nearly millions of miles for a fatal crash. Another approach is the accelerated evaluation method. The core idea of the accelerated evaluation approach is to modify the statistics of naturalistic driving so that safety-critical events are emphasized. This paper presents a brief survey of the advances that have occurred in the area of the evaluation of partially or fully autonomous vehicles, starting with naturalistic field operational tests (N-FOTs). The review covers the test matrix evaluation, worst-case scenario evaluation (WCSE), Monte Carlo simulations, and accelerated evaluation (AE). We also present all the simulation-based and agent-based modeling approaches that do not follow any evaluation protocol listed above. This study provides a scientific analysis of each evaluation techniques, focusing on their advantages/disadvantages, inherent restrictions, practicability, and optimality. The results reveal that the accelerated evaluation approach outperforms naturalistic field operational tests (N-FOTs), test matrix evaluation, worst-case scenario evaluation (WCSE), and Monte Carlo simulation methods in some of the car-following and lane-change studies when using specific models. Moreover, the agent-based model and augmented and virtual reality approaches show promising results in AV evaluation. Furthermore, integrating machine and deep learning into the available AV evaluation methods can improve their performance and generate encouraging outcomes.

INDEX TERMS Accelerated evaluation, agent-based modeling, autonomous vehicles, evaluation, modeling, Monte Carlo simulations, N-FOTs, safety, simulation, simulation-based model, test matrix, testing, worst-case scenario, augmented reality, virtual reality.

I. INTRODUCTION

Decades-long mobile robot navigation and more recent artificial intelligence (AI) and wireless communication advances have created technological possibilities to make the semi-autonomous road vehicles of today possible and have brought the fully autonomous intelligent transportation systems (ITS) of tomorrow within reach. This body of research in AI also offers excellent potential to substantially increase the efficiency and safety of future transportation. Autonomous vehicles (AVs) can help to save fuel, decrease traffic crashes, reduce traffic congestion, and provide better transportation services to older people and people with disabilities [1]. There are many legal challenges in developing AVs,

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TABLE 1. AV readiness index results [7], [294].

Rank						
2020	2019	2018	Country	2020 Scores	2019 Scores	2018 Scores
1	2	2	Singapore	25.45	24.32	26.08
2	1	1	The Netherlands	25.22	25.05	27.73
3	3	n/a	Norway	24.25	23.75	n/a
4	4	3	United States	23.99	22.58	24.75
5	6	n/a	Finland	23.58	22.28	n/a
6	5	4	Sweden	23.17	22.48	24.73
7	13	10	South Korea	22.71	19.79	20.71
8	9	8	United Arab Emirates	22.23	20.69	20.89
9	7	5	United Kingdom	21.36	21.58	23.99
10	n/a	n/a	Denmark	21.21	n/a	n/a
11	10	11	Japan	20.88	20.53	20.28
12	12	7	Canada	20.68	19.80	22.61
13	n/a	n/a	Taiwan	19.97	n/a	n/a
14	8	6	Germany	19.88	21.15	22.74
15	15	14	Australia	19.70	19.01	19.40
16	14	n/a	Israel	19.40	19.60	n/a
17	11	9	New Zealand	19.19	19.87	20.75
18	16	12	Austria	19.16	18.85	20.00
19	17	13	France	18.59	18.46	19.44
20	20	16	China	16.42	14.41	13.94
21	n/a	n/a	Belgium	16.23	n/a	n/a
22	18	15	Spain	16.15	15.50	14.58
23	19	n/a	Czech Republic	13.99	14.46	n/a
24	n/a	n/a	Italy	12.70	n/a	n/a
25	21	n/a	Hungary	11.66	11.99	n/a
26	22	18	Russia	11.45	8.55	7.09
27	n/a	n/a	Chile	11.28	n/a	n/a
28	23	19	Mexico	7.42	7.73	6.51
29	24	20	India	6.95	6.87	6.14
30	25	17	Brazil	5.49	6.41	7.17

and efforts are being made to remove these challenges. Since 2012, at least 41 states and D.C. have considered legislation related to AVs [2]. Twenty-nine states have proceeded with laws authorizing the testing of AVs on public roads [2]. In Arizona, Delaware, Hawaii, Idaho, Illinois, Maine, Massachusetts, Minnesota, Ohio, Washington, and Wisconsin, executive orders have been issued regarding AVs [2]–[4], as shown in Figure 1.

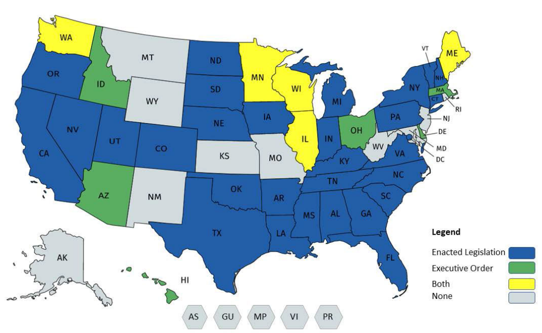

FIGURE 1. States with AV legislation [2].

In Europe, the United Kingdom allowed the testing of AVs on public streets beginning in January 2015 [5]. On February 6, 2019, a formal statement issued by the Department for Transport (DfT) said that the UK is "on track to meet its commitment to have fully self-driving vehicles on UK roads by 2021" [6]. Table 1 shows the AV readiness index for 30 countries [7], [294]. This index shows the level of preparedness for AVs [7]. It is a compound index that combines 28 individual measures from various sources into a single score [7].

As shown in Table 22, up to 30 countries have evaluated AV readiness, based on 28 measures collected into four pillars: policy and legislation, technology and innovation, infrastructure, and consumer acceptance [7], [294]. These references rely on public data, such as media reports, press releases, and other materials. All measures are given equal weight in computing the index, except for the mobile connection speed and broadband measures in the infrastructure pillar [294]. These two measures have half the weight of the other measures. The collected data were normalized before being combined using the min-max approach. The normalization makes all the measures within a range between 0 (worst) and 1 (best) [294]. Figure 2 shows the pillars and the associated measures.

As the pace of AV innovation increases, cities have become the proving grounds of choice. Tech giants, automakers, and startups alike are focused on cities because that is where future customers live and work [8]–[13].

Many major car companies have begun research and development programs for AVs. Table 2 presents the reported AV production schemes [14], [15]. Three features are considered, namely traffic jam assist (TJA), autonomous parking assist (APA), and automated highway driving (AHD). On October 14, 2015, Tesla operated the autopilot function

TABLE 2. Announced autonomous vehicle technologies until 2016 [14], [15].

OEM	Automation Level	Launch Year	First Model	TJA	APA	AHD	Region of Introduction
BMW	Semi-Highly	2014–2017	X5 - 5-/7-series	Yes	Yes	Yes	Europe, North America
Mercedes Benz	Semi-Highly	2014–2017	E-Class S-Class	Yes	Yes	Yes	Europe, North America& Australia
Audi	Semi-Highly	2016	A8	Yes	No	Yes	Europe& North America
Volkswagen	Semi-	2017	Passat	Yes	No	Yes	Europe& North America
General Motors (Cadillac)	Semi-Highly	2018 >2020	SRX, Cadillac CT6 ATS & XTS	No	Yes	Yes	North America
Ford	Semi-Highly	2017 2020	Fusion Explorer	Yes	Yes	Yes	Europe & North America
Lexus	Highly	2020	LS	Yes	Yes	Yes	Japan, Europe & North America
Nissan	Highly	2020	Leaf	Yes	Yes	Yes	Europe & North America
Volvo	Semi	2015	XC90	Yes	Yes	Yes	Europe & North America
Google	Fully-	2018	Model/OEM agnostic	Yes	Yes	Yes	North America

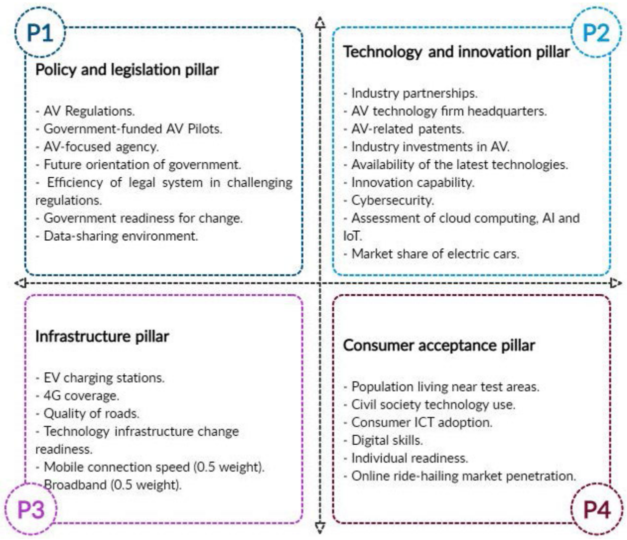

FIGURE 2. AVs readiness index pillars and the associated measures [294].

on the model S through an over-the-air software update [16], enabling features such as adaptive cruise control, lane-keeping, auto lane change, autopark, and automatic emergency steering [17].

After 2016, a large number of companies entered the AV industry, such as Amazon, Apple, Microsoft, Nvidia, Tesla, Toyota, Uber, Volvo, Huawei, and many more [18], [19]. Recent developments in the area of advanced driver assistance systems (ADAS) have shown vast improvements in the accessibility of autonomous driving. Many companies have raised their levels of autonomy over the last few years. Several projects are targeting SAE level 4 or higher. A list of the definition of SAE levels of AVs is explained in [20] and is shown in Figure 3. Advancements in autonomous driving require high-level algorithms that are efficient enough to solve complicated scenarios, especially urban scenarios, such as intersections with multiple pedestrians, pedestrians with unknown intent, traffic lights, cars, and bicycles, which are a real challenge to predict. These high-level algorithms include pattern recognition (classification) [232], [233], [297]-[299], clustering [236], [237], [238], decision matrix algorithms [239], [240], [296], pedestrian intention prediction [234], [235] and many other algorithms.

Driving in urban environments has been both a potential and a hot area of research due to the high density of vehicles and many obstacles that must be avoided. There have been several in-depth efforts to study this problem, such as the DARPA Urban Challenge [21], the V-Charge Project [22], and at least three US military applications—urban operations (UO), manned-unmanned teaming (MUM-T), and AGR [23]. The challenge of driving in an urban environment is complex because it considers increasing the speed of AVs and environmental complexity [24].

By increasing the level of automation, the evaluation process becomes challenging because the AV system will become more complex. An AV may have 100 million lines of code, while Boeing 787 has only 6.5 million [28] (Figure 4). It is a real challenge for a company and also for evaluation authorities, such as the National Highway Traffic Safety Administration (NHTSA) [30], to check every line of code. Many problems may be uncovered after product release, which could cost the company a lot of money [31], [32]. Therefore, it is necessary to evaluate the AV system during the design process. In this paper, we focus on the evaluation of AVs in level 3 to level 5.

The term "autonomous vehicle" is used in this article instead of "automated vehicle." We have chosen to utilize "autonomous" because it is a common term, and the general public is familiar with it. However, the term "automated" implies control or operation by a machine and refers to connected vehicles, while "autonomous" implies more intelligence than the term "automated" and suggests that the vehicle is acting independently [295]. The following sections describe several AV evaluation methods.

II. NATURALISTIC FIELD OPERATIONAL TESTS

Naturalistic field operational tests (N-FOTs) [33] have been used to evaluate AVs. In this test, several ADAS-equipped vehicles are deployed on the road and are driven under natural conditions [34]. During the driving time, the data were collected for evaluation purposes. An advantage of naturalistic field operational tests (N-FOTs) observation is that it allows

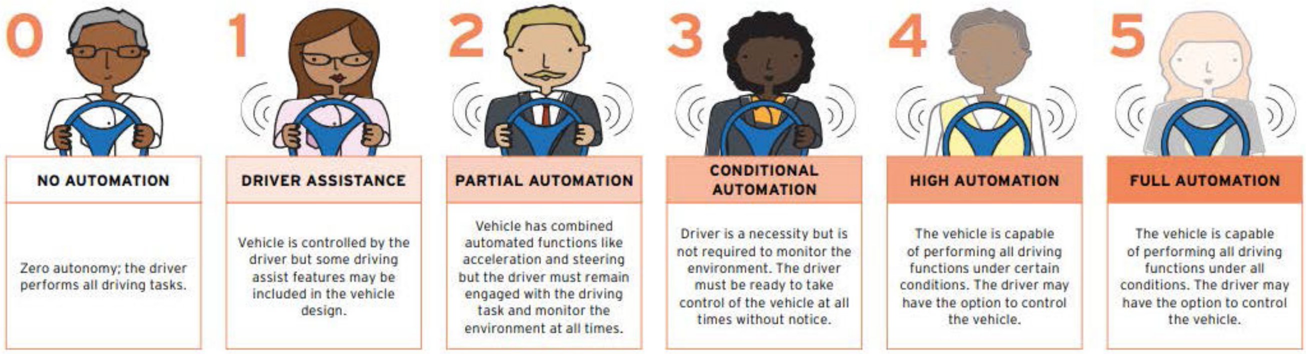

FIGURE 3. Summary of levels of driving automation for on-road vehicles [20], [26], [27].

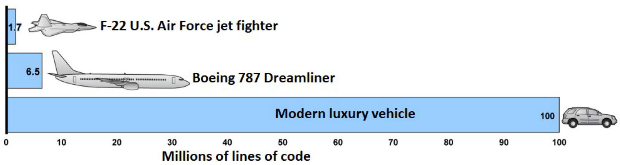

FIGURE 4. Aircraft and automobile software code compared [29].

the investigators to directly observe CAVs and AVs in a natural setting. A naturalistic driving study of 100 vehicles was conducted by Virginia Polytechnic Institute and Virginia Tech to investigate the major contributing factors to crashes. The collected data were used to inspect many factors, such as driver performance, surrounding environment, driving conditions, and other components that are related to critical incidents, near collisions, and collisions [35]–[39]. A list of large-scale N-FOT projects carried out in the United States is shown in Table 3.

Some companies, such as Waymo (formerly the Google self-driving car project), have designed several SAE level 4 AVs [41] and have evaluated the entire autonomous system on actual roads since 2012. Up to January 2020, the Waymo AVs logged nearly 20 million miles of self-driven operation on public roads in 25 cities and tens of billions of miles through computer simulations, with thousands of scenarios and different individual test tracks [42], [304]. In the N-FOT test, the drivers were trained and knew where to drive. Thus, the evaluation process involved non-intrusive driving conditions. The N-FOT test approach has many restrictions, such as the time required to conduct this test. In addition, the probability of expected critical events under natural conditions is very low. For example, in the U.S., the vehicles should travel a total of 0.53 million miles for a police-reported collision and 99 million miles for a fatal collision [43]. Therefore, N-FOT projects require many vehicles, a lot of time, and large budgets. In [44], it is noted that an N-FOT "cannot be conducted with less than $10,000,000." An efficient test approach for AV evaluation is required.

III. TEST MATRIX EVALUATION

A test matrix evaluation is defined as a series of test scenarios defined at the start of the process. Then, the autonomous vehicles go through each test and are assessed objectively or subjectively [40]. Figure 5 shows an example of the test matrix evaluation process. In Figure 5, the design cycle starts with the use of specific scenarios or cases. The functional and technical specifications are then constructed from these cases. The final design is then verified at the component and function level. In the evaluation cycle, a function description is established using the functional and technical specifications. Three types of tests are then applied to the constructed function. These tests are potential safety tests, human factors, and technical tests. At the end of the process, a validation test and a safety impact analysis were conducted.

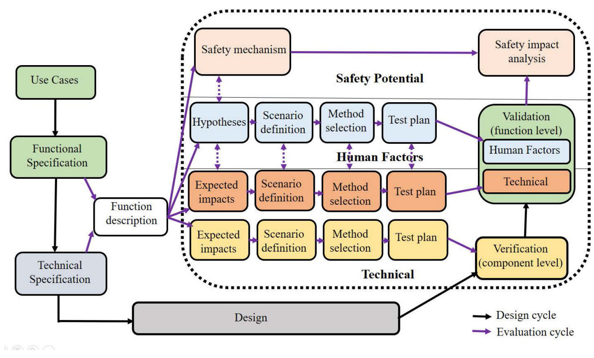

FIGURE 5. Test matrix evaluation diagram [45].

Test matrix evaluation scenarios can be applied in field tests, hardware-in-the-loop (HIL) tests, driving simulator tests, and computer simulations [40]. All certification authorities use field tests [40]. Driving simulator tests and computer simulations have also been used to decrease the cost and time. The test matrix evaluations were mainly based on the crash databases. The pre-crash scenarios were investigated extensively in many studies [241]–[245]. Figure 6 shows the General Motors 44-crashes typology. The United States Department of Transportation designed the pre-crash

TABLE 3. Major N-FOT projects in the U.S. [40], [304].

Name	Conductor	Period	Mileage [mile]	Vehicle	Sensor	Drivers	Research Topic
100-Car Naturalistic Driving Study	VT	2001–2009	2,000,000	100 sedans	Camera	109 primary drivers, 100 secondary drivers	Rear end collision
ACAS	UM	2004–2005	137,000	11 sedans	Camera, Radar	96 drivers	Forward collision warning
RDCW	UM	2005–2006	83,000	11 sedans	Camera, Radar	11 drivers	Lane departure warning
SeMiFOT	UM	2008–2009	106,528	10 sedans, 4 trucks	Camera, Radars	39 drivers	Forward collision warning, lane departure warning, blind spot information systems, electronic stability control, and impairment warning
IVBSS	UM	2010–2011	N/A	sedans: 213,309; trucks: 601,944	16 sedans, 10 heavy trucks	108 drivers for sedans, 19 professional truck drivers	Integrated warning
SPMD	UM	2012–2014	Over 34 millions	2,800 vehicles of various types	Camera, DSRC	2,700 volunteer drivers and several professional bus and truck drivers	Connected vehicle
Waymo-(Google driverless car)	Google	2009-present	Over 20 millions	At least 50 sedans and SUVs	Lidar, Camera, Radar	Google technicians and volunteers	Fully self-driven vehicle

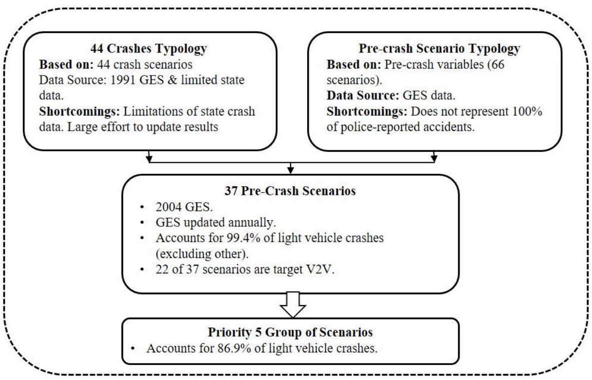

FIGURE 6. Pre-crash scenarios defined by NHTSA [244].

typology based on the NASS crash databases GES (General Estimates System) [246] and CDS (Crashworthiness Data System) [247]. The Volpe National Transportation Systems Center integrated these two typologies to create 37 pre-crash scenarios and capture the vehicle movements and dynamics in real-world and pre-crash critical scenarios. The top five scenario groups, namely, car-following, lane change, left turn, crossing, and opposite directions, were generated by Volpe using the GES, NMVCCS (National Motor Vehicle Crash Causation Survey) [248], and EDR (Event Data Recorder) [249] databases. Table 4 presents the major crash databases in the USA and Europe.

To learn more about crash analysis, reference [250] presents extensive reviews and covers all related works. The test matrix forms scenarios from the data acquired from the NFOTS and is acquired by technical document analysis [251], [252], [253]. Several programs and research projects have begun to develop evaluation policies using the test matrix technique, such as the collision scenarios designed in the crash avoidance metrics partnership (CAMP) [252], the critical scenarios created through the classification tree method for ADAS [253], and the scenarios constructed based on ontologies [254]. Table 5 lists all the test matrix projects. The significant advantages of the test matrix technique are that the determined test policy is repeatable, well-grounded, and quick to complete [46]. Nevertheless, several challenges are encountered, such as predetermined test scenarios. Thus, the AV control system can attain excellent results in these test scenarios, but the results under real-life scenarios may not be satisfactory. In an analogy, "Having a standard test is akin to holding an SAT exam for students with all problems pre-announced. Students do well in the test, but the score may tell very little about how much they learn" [45].

In addition, the test matrix scenarios are usually chosen from collision databases in which most of the collisions are caused by human-controlled vehicles (HVs). Therefore, the test scenarios and evaluation processes applied to AVs may not accurately capture the safety-critical events of AVs [40]. Moreover, according to the CAMP, ADAS, and ontologies projects, the results indicated that the test matrix is more

TABLE 4. Crash databases in the USA and Europe – databases with larger 5000 crashes [40].

Database	Country	Crashes	Data Years	Owner
National Automotive Sampling System - Crashworthiness Data System (NASS-CDS [247])	USA	3300-5000 per year	1988-present	NHTSA
National Automotive Sampling System - General Estimated System (NASS-GES [246])	USA	50,000 per year	1988-present	NHTSA.
Fatal Accidents Recording System (FARS [256])	USA	33,000 per year	1975-present	NHTSA.
National Motor Vehicle Crash Causation Survey (NMVCCS [257])	USA	5,470	2005-2007	NHTSA.
Community Road Accident Database (CARE [258])	EU & CH, IS, NO	1,000,000+	1991-present	European Commission.
German In- Depth Accident Study (GIDAS [259])	DE	22000	1999-present	BASt and several manufacturers.
ADAC Accident Investigation Study (ADAC [260])	DE	11456	2015-present	ADAC.
Co-operative Crash Injury Study (CCIS [261])	UK	15000	1983-2010	Department for Transport.
Vehicule Occupant Infrastructure Etudes de la Sécurité des Usagers de la Route (VOIESUR [262])	FR	9000	2011	CEESAR, CETE NC, IFSTTAR, LAB.

TABLE 5. Projects studying the test matrix technique [40].

Project	Institute	Year	Test scenario	Evaluation approach	Basis
CAMP	Ford/GM/NHTSA	1999	26 tests to evaluate FCW system performance.	Test track.	GM "44 Crashes."
HASTE	HASTE consortium	2005	Three levels of road complexities.	Simulator test track.	Real-world scenarios.
AIDE	AIDE consortium	2005	Three levels of road complexities.	Simulator.	HMI conflict scenarios in previous research.
TRACE	LMS, LAB, INRETS, VW, TNO, ALLIANZ	2006	Scenarios from crash database.	Simulator test track, computer simulation.	National statistics combined with in-depth crash data.
Pre-Crash Scenario Typology for Crash Avoidance Research	Volpe	2007	37 pre-crash scenarios for light vehicles.	Crash data analysis.	GES (General Estimates System).
APROSYS	APROSYS consortium	2007	Scenarios for cars, heavy trucks, motorcyclists, pedestrians, and pedal cyclists.	Simulator Test track.	SAVE-U COMPOSE PReVENT INVENT project GIDAS data.
CICAS	U of Minnesota, PATH, VTech	2008	Scenarios for intersection conflict.	Test track.	
ASSESS	ASSESS consortium	2010	Stopped lead vehicle, slow lead vehicle with constant speed, braking lead vehicle with constant deceleration.	Test track	GIDAS.
interactIVe	interactIVe consortium	2011	Multiple scenarios for active intervention systems.	Hardware-in-the-loop testing, simulator test track.	Previous studies based on real traffic crashes.
Accident data study in support of development of Autonomous Emergency Braking (AEB) test procedures	Loughborough University	2012	EURO NCAP AEB test procedure.	Test track	STATS 19 (2008) OTS (2000-2009).
Depiction of priority light-vehicle pre-crash scenarios for safety applications based on vehicle-to-vehicle communications	Volpe	2013	37 pre-crash scenarios for light vehicle impact.	Crash data analysis.	GES NMVCCS EDR.
Development of Performance Evaluation Procedures for Active Safety Systems	UMTRI	2013	11 scenarios to assess DBS and CIB.	Test track.	Real traffic accidents.

appropriate for autonomous driving system evaluation with the availability of low-cost and high-controllability scenarios. However, the generation of test scenarios using the traditional test matrix approach is usually based on a few influence factors. These factors are generally integrated simultaneously to generate the ultimate scenario [252]. The influencing factors are usually taken from the NFOTS database, WCS database, test standards, and many more.

The influencing factors then be divided into surrounding environment parameters, AV parameters, and road users parameters [131], [132], [40], [255]. Thus, adding additional factors will show a geometric growth in the number of scenarios and, as a result, increase the test cost.

An accident analysis report by Tesla revealed that faults in a critical system might be generated by mixtures of particular values of some factors. Moreover, test scenarios that integrate these mixtures of values can help in the evaluation process by revealing new problems [255]. Therefore, using the traditional test matrix approach, which is based on exhaustive testing of all the influence factors, is redundant and ineffective.

Furthermore, specific test scenarios should be generated using certain influencing factors (elements) to evaluate AV systems. For example, the generated test scenarios to evaluate the lane departure warning (LDW) system consider the traffic environment parameters, subject vehicle driver's behavior, and traffic participants' state [263]. The traffic environment parameters are subdivided into lighting environment (weather, time, and rapid changes in light), lane line parameters (lane line clarity, lane line integrity, lane line number, lane line color, etc.), and road parameters (curvature, lane number, lane marks, slope, and roadside facilities). Every parameter, such as weather conditions, has many values such as sunny, cloudy, rainy, and foggy. To design test scenarios according to ISO 17361, the total number of test scenarios is only eight [8], which are not sufficient to determine the system failure. According to the test matrix method using the exhaustive testing approach, the total number of test scenarios is 497,664,000 [264]. Assuming an average running time of 30 s for each scenario, a total time of 473 years is required to complete all scenarios [263]. That is undoubtedly an inefficient and unacceptable testing approach.

The authors in [263] proposed a combinatorial testing scenario generation method based on complexity (CTBC) to improve the effectiveness of the traditional test matrix technique. The proposed method considers decreasing the number of test scenarios and improving the overall complexity of the scenarios. The results revealed that scenarios with high complexity were effective in finding system failures. Moreover, the CTBC method reduces the number of test scenarios and generates more complex scenarios than the traditional test matrix methods. On the other hand, each AV system has unique influence elements, and the coupling relationship between these elements from different systems has not yet been investigated. Therefore, the defects of the system under the coupling relationship condition have not yet been explored by the CTBC method. Thus, many AV systems, subsystems, and advanced features have not yet been tested and evaluated by the CTBC method and traditional test matrix methods.

IV. WORST-CASE SCENARIO EVALUATION

The worst-case scenario evaluation (WCSE) technique is suggested to recognize highly challenging scenarios for any car, with or without active control systems [40]. In [47] Ma, [48] Ungoren and Peng, attempted WCSE on rollover (overturning of a vehicle) and jackknifing of articulated cars using a dynamic game theory. The term jackknifing refers to the folding of an articulated vehicle so that it resembles the acute angle of a folding pocket knife. This approach suggests that control inputs and disturbance inputs take part in a two-player game condition. In [49], Ungoren proposed another approach as a one-player game by considering the car and its control system as a joint dynamic structure. Then, to solve the WCSE problem numerically, an iterative dynamic programming technique was applied [40]. This technique was used in [50] to assess the integrated chassis control (ICC) system. Therefore, a mathematical model of the vehicle is established, and the WCSE is defined as a horizon optimization problem to resolve for a trajectory (e.g., a sequence of steering inputs) that minimizes or maximizes the cost function (e.g., rollover index) [50]. A solution for the two different systems is conducted. For a linear system (SISO linear time-invariant and MIMO systems), the worst-bounded inputs are acquired from the convolution of the impulse responses [51]. For a nonlinear system (nonlinear dynamical system for a control problem), the solution of the Hamilton-Jacobi-Bellman equations is obtained by the calculus of variations to resolve the optimal trajectory task [52].

Even though the WCSE approach can recognize the weakness of a car and a car control system, it does not consider the occurrence probability of such worst-case scenarios [53], [54]. Therefore, WCSE results do not provide sufficient data on critical real-world situations. Furthermore, there are some limitations when using complicated control algorithms or when the control algorithms are not in numerical form. As a result, the WCSE techniques may either face difficulties finding the worst scenarios or be time-consuming.

V. MONTE CARLO SIMULATIONS

Monte Carlo simulation or the Monte Carlo method is a mathematical procedure utilized to predict the likely results of an unpredictable event. The Monte Carlo simulation creates a model of potential outcomes by leveraging a probability distribution (uniform or normal distribution etc.) for any variable with uncertainty [300]. In this approach of AV evaluation, the N-FOTs data are used to construct stochastic models, and Monte Carlo simulation is applied to assess partially or fully autonomous vehicles. Table 6 presents a list of all the papers related to this method, with the objectives, techniques & models, and scenarios.

In addition to Table 6, Table 7 presents all the references with the associated AV tasks.

In [55], collision avoidance systems were evaluated by establishing an "errorable" driver model to mimic human distraction based on road-departure crash-warning (RDCW) FOT and intelligent cruise control (ICC) FOT naturalistic driving databases. In [56], heavy trucks' collision warning and collision mitigation braking technologies were assessed

TABLE 6. Monte Carlo simulations studies: objectives, techniques and models, and scenarios.

Reference	Objectives	Techniques and Models	Scenarios
Yang et al. 2010, [55]	Evaluate the performance of several collision warning/collision avoidance (CW/CA) algorithms using an errable driver model.	Errorable driver model: perceptual limitation, time delay, and distraction. Stochastic driver model, and a forward multi-step prediction.	Car-following scenario.
Woodrooffe et al. 2013, [56]	Evaluate the safety performance of Commercial Vehicle Forward Collision Avoidance and Mitigation Systems (F-CAM).	Collision warning/collision mitigation braking technologies. Naturalistic driving data conflicts to build test scenarios.	Rear-end crashes using five crash threat scenarios: lead vehicle moving slower at a constant speed, lead vehicle decelerating, lane change (cut-in/out), lead vehicle is not moving when the ego vehicle's radar detects the lead vehicle.
Althoff et al. 2011, [265]	Compare the use of Markov chain abstraction and Monte Carlo simulation for the safety evaluation of fully AVs.	Markov chain abstraction & Monte Carlo simulation.	Car-following scenarios.
Koren et al. 2019, [266]	Extend the adaptive stress evaluation technique to test the AVs and compare the Markov decision process (MDP) and Monte Carlo tree search (MCTS).	Markov decision process (MDP) and Monte Carlo tree search (MCTS).	An AV with noisy sensors approaching a pedestrian crosswalk.
Calzolari et al. 2017, [268]	Investigate the behavior of eight tracking controllers under extreme situations, uncertain parameters, and sensor noises.	Monte Carlo simulations & Rapid Exploring Random Trees (RRTs).	Single and double lane change scenarios.
McAree et al. 2017, [269]	Compare the univariate Gaussian probability density functions to Monte Carlo simulations in estimating the future discrete state transitions.	Univariate Gaussian Probability Density & Monte Carlo simulations. Uncertain mental model.	A vehicle crossing the path of an AV in a roundabout. An autonomous UAS approaching a busy airfield with the intention of landing.
Wang et al. 2019, [270]	Utilize model predictive control and its optimization function to find a smoother trajectory. Use Monte Carlo simulations as a safety assessment.	Model predictive control and Monte Carlo simulations.	Lane-change on the straight road and turning at the intersection.
Broadhurst et al. 2005, [277]	Analyze the safety of complex road scenes using a reasoning framework and Monte Carlo integration.	Reasoning framework, Monte Carlo integration, and Runga-Kutta-Fehlberg algorithm.	A road scene with a parked car and a moving bicycle.
Eidehall et al. 2008, [278]	Present a threat evaluation algorithm for general road scenes.	A dynamic driver model, Monte Carlo sampling, and an extended Kalman filter.	Vehicle approaching an obstacle at a high velocity. Two vehicles are moving, with one approaching the other from behind with a higher velocity.
Okamoto et al. 2017, [279]	Present an algorithm to predict the driver intention of other vehicles using a random-forests classifier. Compute possible future trajectories with a sequential Monte Carlo method.	A sequential Monte Carlo method.	Lane change scenario.
Chen et al. 2020, [284]	Propose a reinforcement learning-based deep-MCTS algorithm for vision-based autonomous driving control.	Deep-MCTS.	Race road with four sharp curves with and without obstacles.
Norden et al. 2019, [285]	Propose a risk-based framework to evaluate AV systems and subsystems as a black box.	Multilevel splitting method and adaptive importance-sampling methods.	A highway scenario consisting of six agents (five are part of the environment).
O'Kelly et al. 2021, [286]	Implement a simulation framework to evaluate a fully AV system using adaptive importance-sampling methods.	Adaptive importance-sampling methods.	A highway scenario.
Rajakumar et al. 2020, [287]	Proposes an enhanced driver model (EDM) that predicts the driver action in an urban environment. Using the EDM model, a Monte Carlo simulation is used to identify the statistical distribution of fuel consumption and travel time.	EDM.	A Mixed Urban Route.
Jönsson et al. 2020, [293]	implement and validate an autonomous agent based on a Monte Carlo tree search	MCTS.	A race track at Halmstad University Sports Center.

by building 1.5 million forward-collision test scenarios from naturalistic driving data conflicts. The main advantage of this approach is that naturalistic driving data were used to create all scenarios/models. Therefore, these scenarios/models represent real-world scenarios. As a result, Monte Carlo simulation models may decrease the assessment cost compared to field tests. Moreover, this method evaluates data collected from human driving databases without any actual crashes [55], [56]. Therefore, using Monte Carlo simulations directly may result in an inefficient simulation model because of the dominance of non-safety-critical portions of naturalistic driving data.

TABLE 7. Summary of the main Monte Carlo studies and the associated tasks.

Reference	Collision Avoidance	Safety Assessment	Trajectory&Prediction	Driver Behavior	Control	End-to-End Testing	Fuel Consumption
Yang et al. 2010, [55]	✓	-	-	-	-	-	-
Woodrooffe et al. 2013, [56]	✓	-	-	-	-	-	-
Althoff et al. 2011, [265]	-	✓	-	-	-	-	-
Koren et al. 2019, [266]	-	✓	-	-	-	-	-
Calzolari et al. 2017, [268]	-	-	✓	-	-	-	-
McAree et al. 2017, [269]	-	-	✓	-	-	-	-
Wang et al. 2019, [270]	-	-	✓	-	-	-	-
Broadhurst et al. 2005, [277]	-	-	✓	-	-	-	-
Eidehall et al. 2008, [278]	-	✓	-	✓	-	-	-
Okamoto et al. 2017, [279]	-	-	-	✓	-	-	-
Chen et al. 2020, [284]	-	-	-	-	✓	-	-
Norden et al. 2019, [285]	-	✓	-	-	-	✓	-
O'Kelly et al. 2021, [286]	-	-	-	-	-	✓	-
Rajakumar et al. 2020, [287]	-	✓	-	✓	-	-	✓
Jönsson et al. 2020, [293]	-	✓	-	-	-	-	-

In [55], model coefficients are usually acquired by fitting a collection of driving data. Then, the errorable model can be improved in real-time to achieve a higher level of false positives and false negatives for better modification to the driver. However, according to [55] and [56], tuning the models under open-loop usually yields inefficient results with human or hardware-in-the-loop. Therefore, when the hardware or human is in the loop, this method may not speed up the evaluation procedure.

The security problem in the autonomous driving domain, especially trajectory planning, has been heavily investigated in literature reviews. It is essential to estimate the surrounding road users and predict the probabilistic occupancy of each road user to identify any future risk [271]. Achieving accurate estimations helps the AV navigate to the final destination with the lowest crash probability. According to some literature reviews, it is commonly proposed that all road users maintain their initial motion state [272], [273]. Therefore, the actual motion of the road user might be different from the estimated motion because of the uncertainty of the road user detection and future intent. In literature reviews, there are many estimation techniques based on kinematic or dynamic models [274] – [276]. Kinematic models have some limitations in neglecting the forces that influence road user movement [274]. At the same time, dynamic models consider various forces that affect driving, such as tire forces and air friction. The use of dynamic models is extremely complex. Moreover, it requires having a different model for each vehicle [270]. Therefore, Monte Carlo simulation [55], [56], [265], [266], [268]–[270], [277]–[279], Gaussian distribution [280], [281], and Markov chain abstraction [265], [282], [283] are usually used to tackle the above issue.

In [270], the motion prediction of road users was studied. The authors focus on trajectory planning in two typical lane-change scenarios (lane change on a straight road and turning at intersections). The Monte Carlo simulation is used as a safety assessment to estimate the probabilistic path planning of road users and then produce a map from probability statistics to actual scenarios. Furthermore, Monte Carlo simulations have a limitation because of probabilistic errors from random sampling [265]. According to literature reviews, the use of more samples is required to achieve accurate outcomes by Monte Carlo simulations. However, using more samples will result in more errors, and the results will not be accurate [265], [270]. The outcomes in [270] revealed that the Monte Carlo simulation is inefficient in real-time computation.

The authors in [265] compared the Markov chain abstraction and Monte Carlo simulation for the safety evaluation of fully autonomous vehicles. The two methods have common differences in terms of their error sources. The significant errors in the Markov chain approach are only systematic errors from the discretization of the state and input space [265]. Moreover, the Markov chain has no probabilistic errors because random sampling is not implemented [265]. In the Monte Carlo method, the main errors are probabilistic errors from sampling the initial states and input sequences [265].

Furthermore, the Monte Carlo simulation has no systematic errors because each simulation uses the system's main dynamical equations [265]. The results show that the Markov chain resulting probability distributions outperformed the Monte Carlo simulation approach in terms of accuracy and simulation speed [265]. On the other hand, the Monte Carlo simulation produces superior outcomes to the Markov chain method when computing crash probabilities [265]. Thus, the outstanding performance of Monte Carlo simulations in crash probabilities is due to the absence of systematic errors in the Monte Carlo simulation.

Koren *et al.* extended the adaptive stress evaluation technique that was used to evaluate the aircraft collision avoidance system to test the AVs [266]. Adaptive stress testing (AST) is an approach used to find critical scenarios using a Markov decision process (MDP) [266]. The original AST approach utilizes a Monte Carlo tree search (MCTS) with double progressive widening (DPW) to explore any failure in the system [267]. The test scenario in [266] includes an

TABLE 8. Numerical results from DRL and MCTS methods. Reward with and without noise to show the difficulty MCTS has with reducing sensor noise. DRL produce a more likely path than MCTS with a less number in calls to STEP [266].

Scenario	MCTS Calls to STEP	MCTS-Reward	MCTS-Reward w/o noise	DRL Calls to STEP	DRL-Reward
1	4.91×10^8	-131	-71	8×10^5	-62
2	1.85×10^6	-38	-15	8×10^5	-1.7
3	1.61×10^9	-161	-104	1×10^6	-52

AV with noisy sensors approaching a pedestrian crosswalk. A deep reinforcement learning solver was used to improve the efficiency of AST instead of using the Monte Carlo tree search (MCTS). The authors claimed that the deep reinforcement learning approach is more efficient and can discover more critical events than the Monte Carlo tree search [266]. The results reveal that both methods can recognize the failure trajectories in an AV-pedestrian conflict. The two solvers produce many events in which the AV hits the pedestrian. A major problem with the MCTS is that the MCTS has non-zero noise that increases over time. As a result, the MCTS does not reduce this noise to zero, leading to a considerable probability error with time. Thus, the AV will not detect and predict the pedestrian and result in a critical collision in the generated scenarios. Table 8 shows the numerical results of the two solvers of adaptive stress testing.

In Table 8, the number of calls to STEP for MCTS is the required number of calls to find a critical accident in the AV-pedestrian conflict. In other words, it refers to the algorithm's computational capability to find a critical conflict. Moreover, this number of calls represents the required number of calls to trust the results. The approach presented in [266] consists of three scenarios, as shown in Figure 7. In scenario 1, the event generated by the MCTS and DRL sends one pedestrian into the scene towards the AV to establish a conflict. The DRL approach produces a straightforward path for a pedestrian toward the vehicle, which is better than the MCTS method. In short, according to the results presented in [266], the DRL solver for AST outperformed the MCTS solver, especially in higher-dimensional scenarios.

Jönsson and Stenbäck implemented and validated an autonomous agent based on a Monte Carlo tree search [293]. Three action generators and two reward functions are compared. The outcomes revealed that the MCTS performs well and converges to a driving agent under static conditions. Moreover, the results showed that the MCTS succeeds only at low speeds in real-time driving [293].

Reference [268] investigated the behavior of eight tracking controllers under extreme situations, uncertain parameters, and sensor noise. Different tests from the single and double lane change scenarios were generated to evaluate the tracking controllers. Monte Carlo simulations and rapid exploring random trees (RRTs) were utilized to assess the controllers' average and worst-case performance [268]. The authors state that most controller properties (e.g., stability, noise rejection, robustness to model variations) are not strongly compromised during the turning phase [268]. Moreover, the authors concluded that the outcomes obtained by Monte Carlo

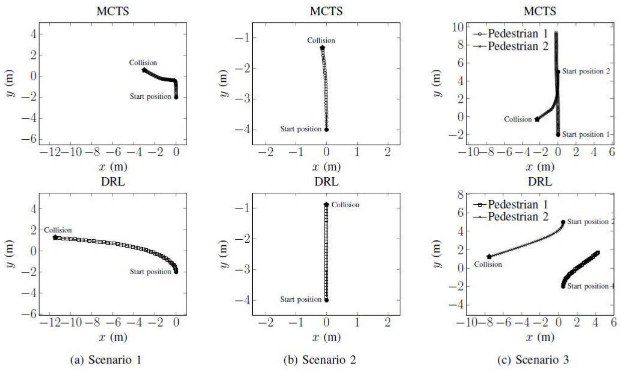

FIGURE 7. The generated scenarios for MCTS and DRL. The collision point is the point of conflict between the AV and the pedestrian. In scenario 3, pedestrian 1 trajectory is far from the collision point [266].

simulations and rapid exploring random trees (RRTs) may not be perfect but can help choose suitable controllers.

In [269], the univariate Gaussian probability density Functions were used to estimate future discrete state transitions such as the beginning of a turn by other agents. Then, the outcomes were compared to Monte Carlo simulations. The results showed a remarkable correlation between the proposed prediction distributions and Monte Carlo simulations, especially over long prediction horizons [269]. Although the outcomes revealed an excellent correlation between the two methods, more investigations and validations are required for this model.

Reference [278] presents a risk evaluation algorithm for public road scenes:

1) The driver behavior was modeled as a probabilistic prior.
2) Monte Carlo sampling was used to approximate the distribution of future scenarios.
3) Different safety measures were computed based on the distribution of future scenarios.
4) A variety of techniques were implemented to increase the performance of this algorithm.

The results showed that the algorithm was tested on simulated data and sensor data and was able to differentiate between safe and non-safe road scenes. However, the dataset used in this study was not sufficient to achieve optimal risk assessment because the data did not have a lot of variation. Moreover, this algorithm removes samples with conflicts with other objects and replaces them with non-conflict samples. Thus, a real-time assessment is required to determine the efficiency of this algorithm.

Reference [279] captured the driver behavior of other vehicles using a random-forests classifier. Then, the likely future trajectories are computed using a sequential Monte Carlo simulation followed by a possible risk assessment. This method was tested by conducting numerical simulations. The simulation results revealed that the algorithm was able to recognize the driver's behavior. However, a limitation of this method is that no real-time deployment or evaluation is conducted.

Reference [277] presented a reasoning framework for the future movement of multiple road users. The probability distribution of every vehicle's future motion was generated using Monte Carlo planning. The synthetic data that are based on a real-world scenario are used to test this approach. The suggested approach shows excellent outcomes but requires more improvements and validation to handle more V2V and V2P complex scenarios.

Reference [284] developed a reinforcement learning-based Monte Carlo tree search (deep-MCTS) control technique for an AV vision-based system. Two deep neural networks (DNNs) were utilized to predict action probabilities and then fed to deep-MCTS to reconstruct multiple future trajectories. The deep-MCTS method outperforms existing methods and shows 50.0%, 66.30%, and 59.06% improvement in training efficiency, steering control stability, and driving trajectory stability, respectively [284]. The deep-MCTS was evaluated using the USS and Torcs simulators.

In [285], the authors presented a simulation testing platform to evaluate the entire AV system as a black box. The multilevel splitting method and adaptive importance-sampling methods were used to address the shortcomings of naïve Monte Carlo simulations for estimating rare event probabilities. The approach in [285] outperforms the naïve Monte Carlo method for events with a probability lower than 10^{-3}. Moreover, the variance of the failure probability is decreased by up to 10x [285]. In contrast, the naïve Monte Carlo method performs well compared to the above method in predicting non-rare events [285].

In [286], O'Kelly *et al.* implemented an end-to-end AV testing framework using adaptive importance-sampling methods to speed up the rare-event probability validation. As a result, the system validation is accelerated by 2-20 times compared to naïve Monte Carlo methods and 10-300p times (where P is the number of processors) over a real-world evaluation [286].

Reference [287] proposed an enhanced driver model (EDM) that predicts the driver action in an urban environment. The effects of signal phasing and timing (SPaT) were considered by presenting the concept of line-of-sight (LOS). Signal phasing and timing (SPaT) data provide information on the signal states by motion [288]. Detailed studies on SPaT and LOS are presented in [288]–[292]. The EDM model was then validated against data collected from vehicles equipped with different drivers. Using the EDM model, a Monte Carlo simulation was used to identify the statistical distribution of fuel consumption and travel time under other conditions such as traffic conditions, SPaT, and driver behavior. This study evaluates the influence of uncertainties related to real-world driving on fuel consumption in connected vehicles [287].

In short, based on the literature reviews presented in this section, Monte Carlo simulations performed well in evaluating AV in some scenarios and failed to produce outstanding outcomes in other scenarios compared to other methods. Thus, more improvements are required for this method using new techniques such as deep learning and reinforcement learning.

VI. ACCELERATED EVALUATION

In [40], Zhao proposed an accelerated evaluation test. The main objective of this test is to establish a method that can speed up the AV evaluation course of action. Moreover, this method can precisely demonstrate AVs' real-life safety benefits. The main idea of the accelerated evaluation approach is to reduce the evaluation process time and eliminate the safe parts of daily driving by skewing the statistics of the surrounding vehicles. This process consists of the following steps.

- Collect a massive amount of real-world driving data.
- Take out events that have possible conflicts between AVs and surrounding human-driven vehicles.
- Model the surrounding human-driven vehicle behaviors as the main distraction to AVs. Then, a modeling of the randomness as random variables vector x with probabilistic distribution $f(x)$ is conducted.
- Skew the disturbance statistics to minimize the safe part of daily driving by replacing $f(x)$ with the accelerated distribution $f^*(x)$.
- Run Monte Carlo tests with the accelerated probability density function $f^*(x)$. The results will provide more intense interactions/collisions between AVs and human-driven vehicles.
- "Skew back" the outcomes of the accelerated tests to understand the performance of AVs under real-life driving scenarios using the statistical analysis.

Figure 8 shows the accelerated evaluation procedure.

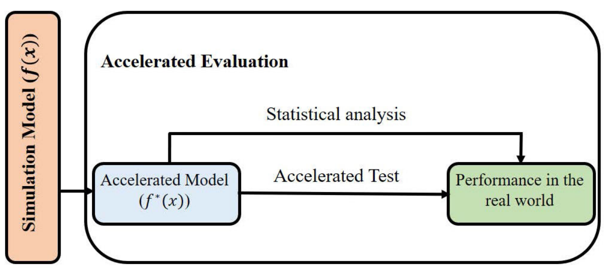

FIGURE 8. Concept of the accelerated evaluation technique [40].

The proposed method can be used in computer simulations, human-in-the-loop tests with driving simulators, hardware-in-the-loop tests, or vehicle tests. Four procedures were established to shape the foundation of the accelerated

TABLE 9. Accelerated rates of crash, conflict and injury events – car-following scenarios [59].

Method	Crash	Injury	Conflict
N_{nature}	4.30×10^8	4.20×10^8	1.07×10^6
N_{acc}	3.84×10^3	3.1×10^3	3.26×10^3
$r_{acc} = N_{nature}/N_{acc}$	1.12×10^5	1.35×10^5	3.28×10^2

TABLE 10. Accelerated rates of crash, conflict and injury events – lane change scenarios [57].

Method	Crash	Injury	Conflict
D_{nature}[mile]	4.53×10^4	4.71×10^7	4.70×10^7
D_{acc} [mile]	16.4	4.02×10^3	2.53×10^3
$r_{acc} = D_{nature}/D_{acc}$	2.77×10^3	1.17×10^4	1.86×10^4

evaluation method. The first technique depends on the likelihood analysis of naturalistic driving. A probabilistic model approach based on time-series driving data was used to build the test scenarios. The assessment policy is sped up by decreasing the relatively safe events that are highly likely to occur. The second technique gives a mathematical base for the "skewing back" mechanism depending on the importance sampling theory, such that the statistical equivalence between the accelerated tests and the naturalistic driving tests can be rigorously demonstrated. The third technique is adaptive accelerated evaluation. This technique shows a policy to recursively discover the best way to skew the probabilistic density functions of human-driven vehicles to maximally decrease the evaluation duration. Finally, an accelerated evaluation method for analyzing the dynamic interactions between AVs and human-driven vehicles was established based on stochastic optimization procedures.

In [59], three indicators-crash, injury, and conflict rates were calculated to test the accuracy and performance of the accelerated evaluation. The crash and conflict cases are binary events. The injury event was modeled as a probabilistic function [59]. Two types of simulations were performed: accelerated evaluation and naturalistic driving simulations (non-accelerated, based on Monte Carlo simulation). Table 9 shows the accelerated rates of crash, injury, and conflict events in car-following scenarios [59]. N_nature, N_acc, and r_acc in Table 9 represent the number of naturalistic driving simulations, the number of accelerated tests, and the accelerated rate, respectively. In crash and injury events, the accelerated approach expedites the evaluation by five orders of magnitude [59]. In a conflict event, the acceleration rate is 300 times lower [59]. Table 10 shows the accelerated rates of crash, injury, and conflict events in lane change scenarios [57]. D_nature, D_acc, and r_acc in Table 10 represent the driving distance needed in the naturalistic test, the driving distance required in the accelerated test, and the accelerated rate, respectively.

The simulation outcomes in car-following and lane change scenarios in [59] and [57] revealed that the accelerated tests could decrease the assessment time of the collision, injury, or conflict events by 300 to 100,000 times. Otherwise stated, driving for 1,000 miles can show the AVs challenging scenarios that would take 300 thousand to 100 million miles to encounter in the real world [57], [59]. As a result, the development and validation time for AVs will be reduced.

Table 11 presents a list of all the papers related to this method, with the objectives, techniques &models, and scenarios.

Even though accelerated evaluation methods can produce excellent results and reduce the duration of the evaluation, they do not consider the following tasks:

- The AVs to AVs and human-driven vehicles to human-driven vehicles interactions are not studied and are used only as a benchmark.
- The AV sensors and controls have been suggested to work accurately. Thus, the measurements, perception errors, and control are assumed to be accurate.
- AVs are assumed to look like human-driven vehicles. Therefore, human drivers' reactions to AVs are the same as to the other human-driven vehicles.
- The secondary impacts of crashes are not considered in these methods.
- The human-driven vehicles model is not accurate enough to mimic real-world scenarios.
- Many real-world scenarios have not been investigated and developed in this technique, such as sensing/detection fail scenarios (e.g., fog, snow, low light), perception failure scenarios (e.g., hand gesture, eye contact, blinking lights), vehicles/pedestrians/pedal-cyclists conflict scenarios (e.g., running, red light, cut-in, jaywalk), making-decisions scenarios (e.g., low confidence, multiple threats), and so on.

Therefore, more improvements for this method are required to have a well-rounded and efficient technique to evaluate AVs.

VII. SIMULATION-BASED MODEL APPROACH

The goal of autonomous driving is mainly to decrease the number of deadly accidents in a highly uncertain environment as well as to provide a high quality of comfort and efficiency and create unprecedented intelligent transportation for individuals within cities. In the interest of getting the AV to navigate safely and dependably in uncertain environments, many challenges need to be considered.

TABLE 11. Accelerated evaluation studies: objectives, techniques and models, and scenarios.

Ref #	Objectives	Techniques and Models	Scenarios
[40]	Accelerated evaluation framework for highly AVs.	Likelihood analysis, importance sampling techniques, adaptive accelerated evaluation, multi-step stochastic sampling.	Lane change scenarios and car-following scenarios.
[57], [58]	Accelerated evaluation of AV safety.	Importance sampling techniques.	Lane changes scenarios. The main crash type is the frontal collision due to unsafe cut-ins.
[59], [60]	Accelerated evaluation of AVs safety.	Extracted naturalistic driving data are used to build the statistics of the motion of the primary other vehicles (POV).	Car-following scenarios.
[61]	Accelerated evaluation of AVs safety.	Piecewise mixture models.	Lane - change scenarios.
[62]	Accelerated evaluation of AVs safety.	Piecewise mixture distribution models.	Lane - change scenarios.
[63]	An Accelerated testing approach for AVs.	Joint statistical models, accelerated distributions for Gaussian mixture models using importance sampling techniques.	Lane - change scenarios.
[64]	Accelerated evaluation and validation of complicated control systems within AVs.	Kernel methods.	Lane - change scenarios.
[65]	Accelerated evaluation of AVs: rare-event simulation.	Learning-based approach.	Unknown rare-event sets.
[66]	Accelerated evaluation of AVs: rare-event.	Statistical learning models.	Rare-event sets.
[67]	Evaluation uncertainty in data-driven self-driving testing.	classical bootstrap method with likelihood ratio-based scheme.	Lane change test scenario.
[68]	On-track testing for AV.	Kriging-based statistical approach.	Lane change scenario.
[69]	An accurate, affordable, and safe way to evaluate a design of an AV.	Co-Kriging-based statistical model.	Lane change test scenario.
[302]	A rare-event simulation for neural network and random forest predictors is presented.	Importance sampling. Sequential mixed integer programming. Random forest. Neural network.	Rare-event sets.
[303]	Present a deep probabilistic accelerated evaluation (Deep-PrAE) to estimate rare-event probabilities in safety-critical applications.	Deep-learning-based importance sampling. Lazy-learner importance sampling. Cross entropy method (CE), adaptive multilevel splitting (AMS), and naive Monte Carlo (NMC).	Rare-events: car-following scenarios.

Modeling AVs is one of these challenges and is regarded as an essential step toward accurately validating AVs in highly uncertain environments. The interaction between AVs and surrounding vehicles or vulnerable road users should then be investigated and validated in various real-world scenarios. A well-established validation approach is required to fill many gaps in the AV evaluation process during the design, pre-deployment, and deployment stages. However, based on previous AV validation techniques, real-world data are limited, and many safety-critical scenarios are difficult to capture in real life. Therefore, a simulation-based model approach was introduced to tackle these challenges.

In 1934, Greenshield *et al.* introduced the first traffic model [307]. Since 1934, three major model classes have been presented: microscopic, macroscopic, and mesoscopic models depending on the level of details needed for network analysis [305]. Microscopic models study the behavior and interaction of individual vehicles based on car-following, lane-changing, and gap-acceptance algorithms [305], [306]. Microscopic models are used to model sophisticated urban street networks, intersections, vulnerable road users (VRU) movements, traffic lights, multi-model systems, and many more. The macroscopic models represent traffic as a continuous sequence and are based on the relationships of flow, speed, and density of traffic stream [306]. Macroscopic models focus on modeling large-scale traffic networks such as freeways, corridors, surface-street grid networks, and rural highways on a section-by-section basis instead of following individual vehicles [306]. Moreover, the macro simulation-based model approach requires a traffic assignment policy, which can be implemented by utilizing activity-based models [70]–[73] or modified traditional four-step models [74]. Finally, mesoscopic models integrate the properties of micro and macroscopic simulation models and allow less fidelity than the micro models for individual vehicles [306]. An example of mesoscopic simulation studies can be found in [310]–[312].

The focus of this section is mainly on the connected and AVs' micro simulation-based model studies that consider longitudinal and lateral dynamic. The micro-simulation-based model produces valuable data for the future development of AVs' based on the level of details in the model. Moreover, the interactions of AVs with human-driven vehicles and vulnerable road users (VRUs) are presented and discussed. In addition, an agent-based simulation modeling of AVs is presented.

A. MICRO-SIMULATION-BASED MODELS

In microscopic simulation models, the behaviors of AV's can be modeled by adopting the available human drivers' models or by inventing new intelligent models that consider V2X capabilities [110], [313]. Moreover, the car-following model is required in collaboration with driver models or any new innovative model for V2X communications to represent how the simulated vehicles interact with the vehicle ahead [110]. Therefore, the car-following model is an essential part of modeling the behavior of human-driven vehicles (HVs), connected autonomous vehicles (CAVs), and AVs in micro-simulation modeling [110], [313]. A car-following model can be designed based on these assumptions by maintaining a safe distance between the lead and host vehicles and controlling the vehicles' speed and accelerations [135].

This section presents all the available car-following models for CAVs and AVs in micro-simulation modeling in the lateral and longitudinal directions. Then, the AV simulation platforms are taken into the spotlight. Finally, the agent-based modeling studies are presented.

1) CAR-FOLLOWING MODELS FOR HVs, CAVs, AND AVs

The car-following, lane change, and distance headway are the three significant behaviors of any vehicle in micro-simulation modeling. Figure 9 illustrates these major behaviors. In addition, the interaction between vehicles on the road is determined by many factors such as lateral vehicle maneuvers, driver behavior, and surrounding vehicle behavior [317]. This section discusses the longitudinal and lateral dynamics of human-driven vehicles (HVs), CAVs, and AVs. The following vehicle will follow the lead vehicle with a proper distance headway, speed, and intended acceleration or deceleration in the longitudinal scenario. In the lateral scenario, a lane-change maneuver is performed, as shown in Figure 9. This chapter covers all the significant contributions in the microscopic analysis of traffic flow and safety evaluation, and how the diverse traffic flow modeling has been presented and developed from homogeneous microscopic modeling to mimic the real-world environment.

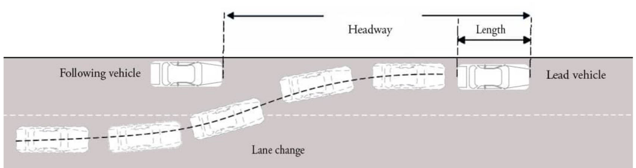

FIGURE 9. Major behaviors of any vehicle in micro-simulation modeling.

In micro-simulation modeling, all vehicle parameters such as the maximum, minimum, intended acceleration or deceleration, and desired speed values are defined using many statistical distributions and functions [315]. In 1945, researchers introduced vehicle trajectories for car-following modeling instead of using speeds and distances between two vehicles [314]. They proposed the safe driving distance between the lead and following vehicles that the following vehicle driver must maintain. The car-following models can then be applied after determining the safe gap between the lead and the following vehicle in every scenario. The popular car-following models are GHR models, safe distance models, intelligent driver model (IDM), ACC & CACC models, optimal velocity models, psychophysical models, fuzzy logic models, and cellular automata (CA) models. Car-following models are classified into types or categories depending on the utilized logic [316]. For example, in [318], the authors presented five different classes while other researchers, such as [319], suggested three more types to include the 21st-century models. Figure 10 illustrates the available car-following models.

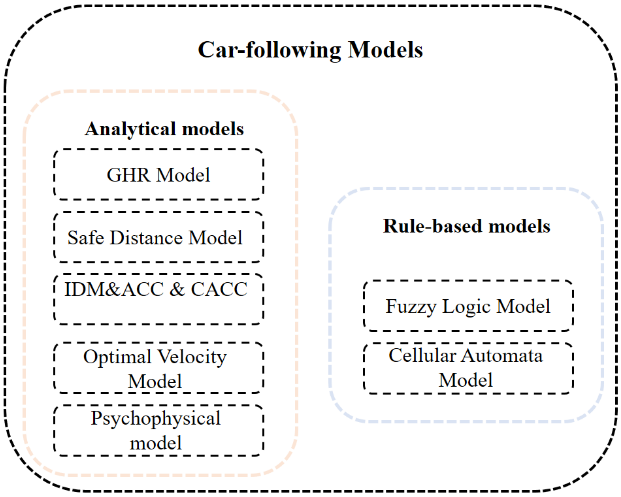

FIGURE 10. Car-following models.

The following subsections will describe each type of car-following model.

a: GHR MODEL

In 1958, Chandler introduced the first version of the GHR model to determine the relative velocity between two vehicles, known as a stimulus [324]. In 1961, General Motors (GM) presented a car-following model using the acceleration/deceleration values as a stimulus [328], [329]. The GM model used the speed of the leader and follower vehicles to estimate the acceleration/deceleration values. Then, the estimated acceleration/deceleration values were used to calculate the driver's reaction time. The GM model is a simple linear car-following model with a constant sensitivity parameter, and the acceleration of the following vehicle can be approximated [324], [330]. In this model, the gap between the leader and follower vehicles affects the stimulus, making the GM linear model impractical [315]. Moreover, this model does not consider the driver's acceleration and deceleration reactions in lane change maneuvers [325]. Furthermore, the diversity of vehicles on the road is neglected in the reaction time calculation [326], [327]. Therefore, the GM and GHR models fail to address the diverse conditions of drivers and vehicles' [321]. Other restrictions in these models include the non-availability of acceleration/deceleration limits [320]. Recently, many researchers have suggested a wide range of solutions to expand the original models and overcome some limitations of the GM model. For example, in [322], the following driver is permitted to accelerate if the relative speed of the lead vehicle has increased. Moreover, the authors also introduced various acceleration and deceleration parameters

TABLE 12. GHR model studies.

Reference	Objective	Vehicle Types	Year	Parameters	Main Results
Chandler et al. 1958, [324]	Investigate car-following scenarios on a highway (without passing).	Cars	1958	Relative velocity, and acceleration.	The acceleration of a car that is attempting to follow a leader is proportional to the difference in velocity of the two cars.
Siuhi et al. 2016, [332]	Include vehicle mix in stimulus-response car-following models.	Cars, trucks	2016	Acceleration/deceleration, driver response time lag, movement state, and vehicle type for different pairs of leader-followers (car-truck, truck-car, and car-car).	The speed, relative speed, space headway parameters are estimated.
Choudhury et al. 2016, [333]	Present a latent leader acceleration model.	LMV, HMV,2W.	2016	Acceleration/deceleration, relative speed, headway.	Predicting acceleration behavior in presence of weak lane discipline where the subject driver may have multiple vehicles in its front and there may not be a distinct leader vehicle.

*Table legend: LMV(Private car, Microbus, Human Hauler, CNG Auto-rickshaw),HMV(SUV, Bus, Truck),2W(Motorcycle, Bicycle, Cycle Rickshaw).

TABLE 13. Safety-distance model studies.

Reference	Objective	Vehicle Types	Year	Parameters	Simulator	Main Results
Banihan, 2007, [334]	Incorporate lateral discomfort in car-following modeling.	Cars	2007	Acceleration, deceleration, vehicle widths/lengths, desired speeds, lateral separation, frictional clearance, and veering distance.	C++	Estimating speed, flow, density diagrams, space-time trajectories.
Banihan, 2009, [335]	Develop a simulation model with various types of vehicular interactions, including lane-change, and perform certain validation tests.	Cars	2009	Lateral separation, frictional clearance, veering distance, maximum escape speed.	C++	Estimating speed, flow, density diagrams, space-time trajectories.
Xu, 2015, [336]	Propose a full velocity difference model for traffic flow, including driver's reaction time delay.	Cars	2015	Driver's reaction time delay, headway, velocity, velocity difference, and traffic flow.	Matlab	Estimating the classification of traffic jams and space-time diagrams.
Ravishankar et al. 2011, [337]	Extend the widely used Gipps's car-following model to include vehicle-type dependent parameters.	3W, trucks, buses.	2011	Driver reaction time, acceleration, desired speed of follower vehicle, size of leader vehicle, vehicle type.	C	Estimating the follower behavior and suggest the need of incorporating vehicle-type combination specific parameters into traffic simulation models.
Lenorzer et al. 2015, [338]	Developed a new mixed-flow model for the micro simulator of Aimsun.	3W, trucks, buses.	2015	Driver reaction time, acceleration, desired speed of follower vehicle, size of leader vehicle, vehicle type.	AIMSUN	Providing a robust behavioral model to simulate Asian traffic conditions. Estimating the capacity and jam density.

to improve the original models in making an efficient decision. The authors in [323] also proposed an extension to speed up the driver reaction under deceleration scenarios as compared to the acceleration cases. Furthermore, the authors in [331] suggested a critical headway value for estimating the state of driving behavior. Finally, another extension to the GM model is presented in [329] to consider the nonlinear behavior in terms of relative speed and distance between the lead and following vehicles. Despite all the extensions, the GM model has its behavioral limitations, such as the drivers' reactions to random changes in the stimulus, and the actions of the leader vehicle keep impacting the relative speed and the driver of the following vehicles even when the gap between these vehicles is high [315]. Table 12 presents an outline of the GHR model studies.

b: SAFETY-DISTANCE OR COLLISION AVOIDANCE MODEL

The safety-distance model maintains a safe distance between the lead and follower vehicles. This model is based on the fundamental motion equation [318]. Pipes explains the term of a safety distance in 1953 [112] as ''a good rule for following another vehicle at a safe distance is to allow yourself at least the length of a car (about 15 ft) between you and the vehicle ahead for every 10 miles of hour speed at which you are traveling''. In 1981, Gipps presented the first acceleration model that documents car-following and non-car-following maneuvers [339]. However, a limitation of the Gipps model is that it requires keeping the safe distance headway and not exceeding the desired speed. As a result, many researchers have performed extensive extensions, modifications, and calibration studies on the Gipps model [340]-[342]. Table 13 lists these studies. Currently, the Gipps model is widely used in micro-simulations modeling because of its basic calibration assumptions about human driving behavior [315].

c: PSYCHOPHYSICAL MODELS

The psychophysical or action point model [318] utilizes the space headway and relative velocity for the following vehicle as a threshold. Effort are made by the drivers when the threshold values are reached. The threshold values of the spacing or relative velocity should be reached to obtain a reaction from the drivers. The psychophysical model also helps to record if the drivers are paying attention to the small spacing and the associated effect when there is a large spacing on the following behavior [330]. In [343], the psychophysical model was implemented on a simulation platform using a framework called "MISSION." Moreover, the interaction between the lead and follower vehicles was investigated by defining four thresholds and regimes, as shown in Figure 11.

Thresholds and regimes	Explanations
Free-flowing	The following vehicle is not affected by the lead vehicle.
Approaching/closing	The following vehicle is consciously affected by the slower-leading vehicle.
Following/closing	The following vehicle unconsciously affected by being in the following process.
Emergency	The headway between vehicle pair drops below the desired safety distance.

FIGURE 11. **Wiedemann model - thresholds and regimes.**

The drivers' behavior in the psychophysical model is suggested to be naturally distributed and can be represented as normal distributions [343]. This means that each driver has unique driving skills for perception, reaction, and prediction of the surrounding environment. The authors in [343] also proposed that vehicles have different abilities to perform simple techniques such as maximum velocity and maximum acceleration/deceleration values. The Wiedemann model in [343] presented various ranges of other random parameters to be used to calculate the threshold values and driving functions. Examples of these parameters are, namely, the desired distance (AX), the desired minimum following distance (ABX), the maximum following distance (SDX), the perception threshold (SDX), and the decreasing and increasing speed differences (CLDV, OPDV) [315]. More improvements are required in this model because the calibration of the parameters is challenging [315], [321]. Examples of simulators that use psychophysical models include VISSIM and PARAMICS. The VISSIM platform is widely used to model heterogeneous traffic conditions. However, using VISSIM to simulate 2D traffic conditions is not recommended because it produces inefficient outcomes [321].

d: OPTIMAL VELOCITY MODEL (OVM)

The optimal velocity model (OVM) was proposed to model traffic flow instabilities that cause congestion in public roads. The OVM model was developed to describe the dynamical behavior of traffic flow using the motion equation of each vehicle. The model is based on the relative distance to the lead vehicle. Moreover, the acceleration of the following vehicle is controlled in a way that the final velocity is modified according to the trajectory of the lead vehicle [321]. Several modifications and extensions have been proposed to overcome some of the limitations of the OVM approach for modeling mixed traffic conditions. A significant improvement in the OVM approach is the two-velocity difference model (TVDM) [345]. The TVDM approach was developed to integrate an intelligent transport system (ITS) with the OVM approach. The integration of the two models provides a comprehensive car-following model that incorporates multiple leading vehicles [321]. Table 14 shows a list of all the OVM modification and extension studies.

e: INTELLIGENT DRIVER MODEL (IDM), ADAPTIVE CRUISE CONTROL (ACC), AND COOPERATIVE ADAPTIVE CRUISE CONTROL (CACC) MODELS

Recently, with the advancement of the intelligent driving assistance system (IDAS), vehicles have become more intelligent and are expected to perform many driving tasks. The cruise control (CC) system is an early step in the intelligent driving assistance system of connected AVs. More improvements are added to the cruise control (CC) system to have more advanced methods, such as adaptive cruise control (ACC) and cooperative adaptive cruise control (CACC) systems. Intelligent driving assistance systems such as CC, ACC, and CACC are essential for supporting acceleration control for longitudinal motions based on the gap distance and speed difference between the lead and host vehicles. Therefore, many researchers have utilized the simulation-based model approach to evaluate and study the impacts of connected and AVs. Furthermore, the simulation-based model analysis provides the flexibility to build safety-critical scenarios and validate the AVs during their development to avoid mistakes before public road deployment. Many of the micro-simulation papers in this section established their own ACC, CACC, AV, or CAV car-following models. Furthermore, each article implemented a unique method and produced distinct performance indicators. Figure 12 presents the four main intelligent vehicle types and their definitions.

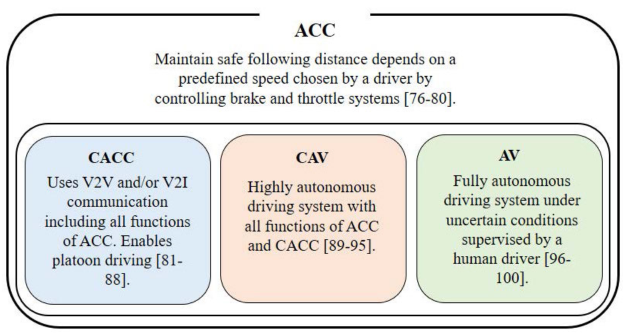

FIGURE 12. **AVs definition and their categorizations [101].**

TABLE 14. Optimal velocity model (OVM) studies.

Reference	Objective	Vehicle Types	Year	Parameters	Simulator	Main Results
Sheng et al. 2010, [344]	Presents a non-lane-based car following model by incorporating the effects of the lane width in traffic.	Cars	2010	Lane width, lateral separation, critical headway.	Numerical simulation	Estimating the evolution of traffic congestion. Lane width effects on the car-following model can lower critical headway. The lateral separation effects greatly enhance the realism of car-following models.
Zheng et al. 2019, [346]	Evaluate the vehicle type-dependent car-following heterogeneity from micro- and macro-aspects by using Next Generation Simulation (NGSIM) trajectory data.	Cars	2019	Time headway (TH), time to collision (TTC), and safety margin (SM)).	Numerical simulation	During the close car-following process with the same speed, the driver generally keeps a higher gap distance with the leader when following or driving a larger vehicle.
Jin et al. 2012, [347]	Develop a new staggered car-following model taking into consideration lateral separation effects.	Cars	2012	Time-to-collision, visual angle variables, lateral separation distance.	Numerical simulation	Incorporating lateral separation effects into the car-following model leads to the suppression of traffic jams and greatly enhances the realism of the model.
Li et al. 2015, [348]	Propose a new car-following model that considers the effects of two-sided lateral gaps on a road without lane discipline.	Cars.	2015	Lateral gaps, Two-sided lateral separation.	Numerical simulation	The two-sided lateral gaps car-following model has a larger stable region compared to a one-sided lateral gap car-following model.
He et al. 2016, [349]	Present a new model that considers the effects of lateral separation and overtaking expectation based on OVM.	Cars	2016	Lateral headway, escape corridor.	Numerical simulation	Considering the lateral separation and overtaking expectation in the model can better simulate car-following behavior, especially in some complex driving conditions.

The ACC system extends the existing CC system to incorporate a headway sensor that observes the distance between the host vehicle and the vehicle in front of it. The essential function of the CC system is to maintain a constant vehicle speed that is adjusted by the driver. In contrast, the principal purpose of the ACC system is to control vehicle's acceleration based on a distance gap and a speed difference between lead and host vehicles. Moreover, the ACC system can accelerate or decelerate based on the speed changes of the lead vehicle. Figure 13 shows the ACC scenario.

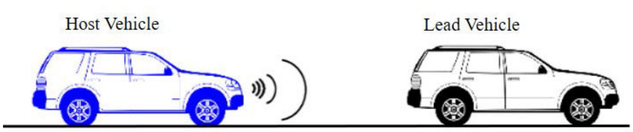

FIGURE 13. Adaptive cruise control [103].

Furthermore, communication capabilities are added to the ACC system. The modified ACC system with V2V and V2I communications is called the CACC system. The CACC system shares the acceleration, deceleration, braking capability, and vehicle positions using V2V and V2I communications [75], [102]. The communications capabilities of the CACC provide shorter headway time compared to the ACC. Figure 14 shows the setup for CACC scenario.

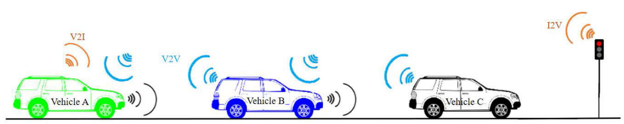

FIGURE 14. Cooperative adaptive cruise control [102].

The ultimate goal of the intelligent driving assistance system (IDAS) is to fully control vehicles. Connected AVs have all the AV functions along with V2V and V2X communications. Figure 15 shows the CAV and AV scenarios. One significant difference between CACC and CAV is automated lateral movement. The standard car-following motions established for human-driven vehicles are old-fashioned compared to CC, ACC, CACC, and CAV. Thus, the literature studies' related terms and approaches are slightly different and are not profoundly classified.

Table 15 presents the simulation-based modeling studies of ACC and CACC and their validation analysis. This table provides the objective of each review, the base model, modeling and validation scenarios, vehicle types, assessment basis, and primary outcomes.

TABLE 15. Simulation-based AVs studies: ACC and CACC studies [101].

Ref #	Objectives	Base Model(s)	Scenarios	Vehicle Types	Evaluation Criteria	Main Results
[77]	Investigate the ACC vehicles' lane-changing effects compared to manual vehicles.	Pipes [112], ACC [113], and comprehensive modal emissions model (CMEM).	Position of ACC vehicles and Market-penetration rate of ACC (5%, 10%, 15% and 30%)	Manual vehicle, ACC	Fuel consumption and emission (CO, HC, CO2, NOx)	The smooth response of the ACC vehicles has a beneficial effect on the environment. These benefits vary with the levels of the disturbance, the position of the ACC vehicle in the string of manually driven vehicles, and the ACC vehicle penetration.
[78]	Present the new ACC car-following model with its impact analysis.	IDM with constant-acceleration heuristic (CAH).	Market-penetration rate of ACC (10%, 20%, 30%, 40%, and 50%)	Manual vehicle and ACC	Throughput	1% more ACC vehicles will lead to an increase in the road capacities of about 0.3%.
[79]	Present the ACC-based traffic-assistance system intended to improve traffic flow and road capacity.	IDM	Market-penetration rate of ACC (0%, 5%, 15% and 25%)	Manual vehicle and ACC	Throughput	A small proportion (5%) of ACC vehicles can improve the traffic flow. An increasing proportion of ACC vehicles reduces traffic congestion.
[80]	Establish the ACC and CACC car-following models and estimate their impact.	An error-based control law for ACC and CACC. The lane change is under human control.	A 100% market-penetration rate of each vehicle type	Manual vehicle, ACC, and CACC	Throughput	Throughput of the manual, ACC, and CACC vehicles was, respectively, 2,050, 2,200, and 4,550 vehicles/h.
[82]	Approximate the effect on highway capacity of varying market penetrations of vehicles with ACC and the CACC.	NGSIM model [111]. ACCs: Proprietary to Nissan. CACCs: Car-following behavior[83].	The ACC and CACC vehicles 10% increase proportion	Manual vehicle, ACC, and CACC and Here-I-Am (HIA) vehicle	Highway throughput	The use of ACC was unlikely to change lane capacity significantly. The CACC can increase capacity greatly after its market penetration reaches moderate to high percentages. The capacity benefits of CACC can be accelerated at somewhat lower market penetrations if the rest of the vehicles are equipped with VADs.
[86]	Investigate the impact of the CACC vehicle string operation on the capacity of multi-lane highway with merging bottlenecks.	ACC and CACC car-following models [83].	Market-penetration rate of CACC (0%, 20%, 40%, 60%, 80% and 100%)	Manual vehicle and CACC	Bottleneck capacity	The freeway capacity increases quadratically as the CACC increases, with a maximum of 3080 vehicles/hour/lane at 100% CACC penetration. A rapid increase in bottleneck capacity from 80% to 100% CACC penetration, especially with high on-ramp inputs.
[87]	Extend the CACC to incorporate new algorithms describing the interactions between CACC and manual vehicles.	The CACC model reported in [114]. The anticipatory lane change (ALC) for lane changing.	Market-penetration rate of CACC (0%, 20%, 40%, 60%, 80% and 100%)	Manual vehicle and CACC	Throughput	Freeway capacity is 90% higher at a 100% CACC penetration compared to 0%. The capacity increase is insignificant under low to medium CACC market penetrations (e.g., 20–60%) in the absence of additional management strategies.
[105]	Inspect the CACC vehicles' impacts on traffic flow characteristics of.	IDM.	Arrival rate scenarios: 7,000v/h (moderate), 8,000v/h (saturated), 9,000v/h (oversaturated), 10,000v/h (oversaturated).	Manual vehicle and CACC	Throughput	At a low-to-moderate penetration rate of CACC, the CACC impact is not statistically significant. A very large improvement is noticed at a high penetration rate of CACC, especially in high traffic conditions.
[106]	Present an ACC-based traffic assistance system.	IDM.	Market-penetration rate of ACC (0%, 5%, 15% and 25%)	Manual vehicle and ACC	Throughput	ACC vehicles improve the traffic stability and the road capacity. A penetration rate of 25% ACC eliminates traffic congestion during simulation.
[110]	Examine the impact of the CACC on traffic-flow characteristic.	MIXIC model.	Market-penetration rate of CACC (0%, 20%, 40%, 60%, 80% and 100%)	Manual vehicle and CACC	Throughput	CACC can improve traffic-flow characteristics. A low market-penetration rate of CACC (< 40%) would not have an impact on the throughput.
[118]	Present an up-to-date algorithm for CACC systems for collaborative driving.	Effective CACC (ECACC) algorithm	Market-penetration rate of CACC (0%, 20%, 40%, 60%, 80% and 100%)	Manual vehicle and CACC	Throughput	The congestion reduction is higher when the market-penetration rate of the CACC-equipped vehicles increases. At a low penetration rate, the effect of the CACC on traffic dynamics is not significant.
[119]	Establish models of both ACC and CACC control systems based on real-world experimental data.	IDM.	Ten consecutive CACC and five consecutive ACC vehicles. Mixed case: the first two followers are ACC-equipped and the next seven are CACC-equipped.	Four ACC and CACC experimental vehicles	Speed, distance gap, time gap	The IDM controller in the experimental test vehicles does not perceptibly follow the speed changes of the preceding vehicle. Strings of consecutive ACC vehicles are unstable, amplifying the speed variations of preceding vehicles. Strings of consecutive CACC vehicles overcome these limitations, providing smooth and stable car-following responses.

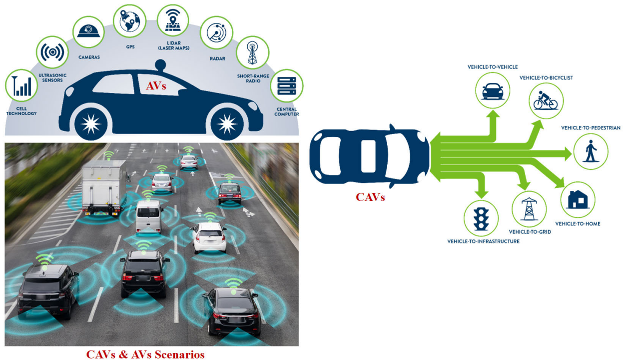

FIGURE 15. CAVs&AVs [308], [350].

According to literature reviews, the IDM and MIXIC models are widely used as benchmark car-following models. Some modifications to the IDM [106], [78], [79], [95] and MIXIC [94], [116], [133] models are added to fully understand the longitudinal motion of AV. The car-following model is an essential model that is widely used to simulate AVs. Research efforts have been conducted to establish AV car-following models by improving the traditional car-following models (IDM and MIXIC). The intelligent driver model (IDM) [134] is an uncomplicated safety model that produces practical outcomes [135]. The main goal of developing an intelligent driver model (IDM) is to tackle the modeling of mixed traffic conditions. The initial development of the IDM was performed by Treiber *et al.* [134] for a single-lane scenario. Furthermore, the acceleration is described as a function of the gap, velocity, and space difference between the lead and following vehicles [321]. The maximum acceleration and minimum headway are considered to maintain the minimum gap and acquire the required velocity. Additional extensions to the model are required to cover multilane traffic modeling and consider potential risk elements [135]. The IDM and linear approaches do not support the modeling and validation of 2D traffic scenarios [321]. The modified IDM can be used as an ACC model or as a human-driven vehicle model. Furthermore, the improved version of the IDM can be utilized to simulate connected autonomous vehicles (CAVs) [95].

To fully understand and estimate the impact of AVs using a simulation-based model, autonomous modeling should be able to examine the performance of the AV under highly uncertain conditions. Moreover, the model should evaluate AV safety, fuel consumption and emissions, noise emissions, and traffic performance. The MICroscopic model for simulation of intelligent cruise control (MIXIC) is then suggested and developed as a stochastic simulation model to overcome these challenges. The MIXIC is implemented widely for cooperative AV simulations because it uses V2V communication and can optimize traffic capacity under real-world conditions. The MIXIC allows interaction between the lead and host vehicles to share the actual speed, acceleration, maximum potential braking, and warnings. In [94], the CAV was established based on the MIXIC model. This technique utilizes a smart-micro automotive radar (UMRR-00 Type 30) (90 m ± 2.5% detection range and ± 35 horizontal FOV) as an input for the MIXIC model. The sensor update rate was 50 ms and could track up to 64 objects. The AV speed should be low enough to apply a complete stop when the lead vehicle is detected and has reached a full stop. Using the maximum deceleration of the AV (host) and lead vehicle, the AV's maximum safe speed and acceleration and the minimum safe distance can be calculated. In [89], the authors presented a hardware-in-the-loop (HIL) testing system for CAV applications. The results showed the effectiveness of the CACC in absorbing certain disturbances and oscillations of speeds. Moreover, the speed oscillation decreased as the vehicle position in the string increased. In addition, a perfect communication/radar contributed to string stability.

f: CELLULAR AUTOMATA MODEL

In 1992, Negal and Schrecknberg presented the cellular automata (CA) model [351]. The road segment in this model is divided into cells with the same size of almost 7.5 meters long [351]. Each cell can fit a single vehicle or be unoccupied. The longitudinal dynamics of the vehicles are integrated into the CA model by including the acceleration, braking time, and randomization of vehicle types [321]. The CA model is then extended to include two-lane traffic conditions [352]. As a result, large-scale dynamic traffic modeling is easily achieved using this model. A limitation to this model is the loss of information due to the discretization of cells [321]. Due to the discretization of cells that have the same size, vehicles are required to update their parameters, such as velocity and acceleration/deceleration in multiple cells. Another limitation is the difficulty in representing all vehicle types within the cells. Cell size is an essential factor in the CA model. The representation of all vehicle types cannot be achieved if a large cell is utilized. Moreover, the computational workload might be increased when using a small cell. Furthermore, this model cannot validate the changeability in the headway distance between the lead and follower vehicles regarding vehicle velocity due to the same cell size assumption. Therefore, a wide range of modifications and extensions to the CA model were performed by incorporating many parameters such as vehicle type, vehicle size, mechanical properties, lateral arrangement, lateral gaps between vehicles, flow, velocity, occupancy of a vehicle in a cell, cell size, and acceleration&vehicle type [353]–[362]. As a result, the improved CA model utilizes a cell size of 0.5 m in length, a safe gap at the front and back of vehicles, the relationship area of occupancy, interaction rate, and structure of vehicles [361], [362]. However, according to the outcomes presented in [353]–[362], further validation and investigation of the model in various traffic conditions and lateral and longitudinal interactions is required to expand its application.

g: FUZZY LOGIC MODEL

In 1992, Kikuchi *et al.* incorporated the relative headway distribution, velocity, and acceleration into the fuzzy model [363]. The development of the fuzzy logic model continued with time to include the car-following model [364]–[367]. The generated outcomes show several issues due to the inadequate establishment of drivers' perception. Moreover, the mixed traffic flow in the car-following behavior in this model was not included [321]. An advantage of the fuzzy logic model in car-following maneuvers is the ability to determine the lane shifting behavior of vehicles [368]–[370].

2) OTHER MICRO-SIMULATION STUDIES

Lane-changing models are essential elements for modeling HVs, CAVs, and AVs in traffic micro-simulation tools. In 1978, Sparmanns proposed a lane-change model to classify lane-change behavior as slower-to-faster and faster-to-slower lanes based on driver needs [321]. In 1986, Gipps presented a well-known lane-change model for urban driving that considers the effects of elements such as traffic signals, obstructions, and heavy vehicles in traffic flow [371]. The main focus of the Gipps model is to investigate the critical interaction between vehicle to vehicle, vehicle to obstructions, and other real-world driving behaviors [371]. More studies on lane-change modeling in micro-simulation can be found in [372]-[373].

The intersection is one of the challenging environments for AVs because of the unpredicted interactions among pedestrians, bicycles, and vehicles and the intersection users' highly complicated design and behavior. As a result, many studies have covered a wide range of research related to intersection scenarios. For example, in [123], a turning vehicle was modeled, and its surrogate safety indicators were investigated at mixed-flow intersections. The authors in [124] validated automated intersection traffic management applications using a vehicle-in-the-loop (VIL) verification environment. In [127], Zulkefli *et al.* evaluated CAV and AV applications and fuel consumption and emission using a hardware-in-the-loop testbed. The outcomes showed fast data transfer every 200 ms, and the optimized engine operating points and the desired vehicle speed are tracked precisely. Shao *et al.* [126] assessed CAVs and AVs using a hardware-in-the-loop testbed and a living lab, focusing on fuel consumption and emissions. The outcomes revealed that the error between the virtual vehicle and the actual testing vehicle was 1%. Thus, the results support the use of the HIL testbed to evaluate CAVs in real-world scenarios. Furthermore, in [128], Li *et al.* presented an advanced intersection control system to support CAVs and AVs trajectories and validate their safety and performance at intersections. Table 16 shows a list of studies related to the modeling and validation of AVs in different scenarios.

With the advancements toward fully autonomous vehicles, human drivers will not control and understand the surrounding environment. Therefore, AVs should have a social understanding of the interaction between their control systems and road users to ensure a safe driving environment [137]. The meaning of interaction in driving involves many tasks such as identifications, behavior analysis, future action prediction, and so on, and taking the right actions to avoid any severe collisions. Behavioral psychology studies have investigated the social aspects of driving and have shown the factors that can significantly impact road users' decisions [138]–[140]. These factors include pedestrian demographics [140], road conditions [139], social factors [139], and traffic characteristics [142]. Thus, a deep understanding of pedestrian crossing behavior, the extent of these factors, and how they are connected is required.

In the case of autonomous driving, intent prediction algorithms have been established to estimate the next moves of pedestrians [143] and drivers [144]. A wide range of technologies has been developed to assist AVs in communicating with road users, such as V2V [145] and V2P [146] communications. Moreover, visual intent interfaces such as LED lights [147] or projectors [148] are used. The problem with all of these studies is that they consider the technologies a rigid active thing rather than a social interaction [149]. Pedestrian behavior studies are classified into two categories, classical studies and AV conflicts studies. The traditional methods focus on studying pedestrian behavior when interacting with human drivers. A wide range of data-collection methods is used in classical pedestrian behavior studies, such as observation, police reports, video recording, photography, simulation, scripted observation, questionnaires, literature surveys, and interviews. The focus of this section is the simulation-based method. A study conducted by Caird and Hancock [151], which involved 48 men and women, showed that the road users misjudged the vehicle arrival time as the size of the vehicle increased. In [152], Sun *et al.* studied the relationship between pedestrian waiting time before crossing and gap acceptance. The outcomes showed that a long pedestrian wait time results in a low acceptance gap. Another study investigated the impact of vehicle size on pedestrian behavior and showed that pedestrians are more careful when interacting with a larger vehicle [153]. Wiedemann [154] showed that pedestrian flow and pedestrian speed have a linear relationship with no interaction between pedestrians. Rasouli and Tsotsos [150] classified the factors that impact pedestrian behavior into two groups: pedestrian and environmental factors. Figure 16 shows a list of these factors and how they are connected under classical studies.

In contrast, Figure 17 presents a list of factors that impact pedestrian behavior when facing AVs and how they are connected. Various methods are used to collect data that are used in pedestrian behavior studies involving AVs, namely, observation, video recording, photography, simulation, scripted observation, questionnaires, literature surveys, interviews, and Wizard of Oz (a research experiment in which subjects interact with a computer framework that subjects believe to be autonomous but is actually run or partially run by a hidden human being).

The simulation data collection methods used in pedestrian intention studies involving AVs are briefly discussed in this section. Beggiato *et al.* [155] investigated the indirect forms

TABLE 16. Additional simulation-based AVs studies [101].

Ref #	Objectives	Base Model(s)	Scenarios	Vehicle Types	Evaluation Criteria	Main Results
[76]	Suggest a pedestrian collision avoidance system based on V2X and another one based on V2P information.	Pedestrian collision avoidance model.	A pedestrian is crossing in front of two vehicles. The host vehicle has three different speeds and is in a different lane from the vehicle in the front.	CAVs & AVs	Pedestrian safety. V2X and V2P technologies	Vehicle emergency braking method is tested. A vehicle that is using V2V and V2P technologies will engage in a severe conflict with a pedestrian when the delay or packet loss rate increases to a certain value. The communication delay and packet loss rate should be reduced to avoid deadly conflicts with pedestrians.
[104]	Examine the safety impacts of leader-follower AVs	Bicycle mode. It comes with a linear tire model. ACC.	Connected autonomous semi-trucks under different weather conditions.	ACC& platooning	Fuel consumption, safety margin	Connected-autonomous semi-trucks that are operating in a leader-follower configuration under different weather conditions.
[121]	Inspect roundabout safety level in circumstances where different numbers of AVs are mixed with manual vehicles.	Surrogate safety assessment model (SSAM), Wiedemann 74 & VISSIM.	Market-penetration rate of AVs (0%, 10%, 25%, and 50%).	Manual vehicle, heavy commercial v-HGV &AVs	Average travel speed	An increase of travel speed and decrease of average stop delay with an increase of the percentage of AVs. Increases in estimated crash numbers at roundabouts in terms of rear-end conflict when the AVs percentage is increased.
[122]	Create a decision-making CAV control algorithm in VISSIM for safety assessments.	Safety impact: SSAM. CAV: External driver model API written in C++. Manual vehicles: Wiedemann 99.	Market-penetration rate of CAVs (0%, 25%, 50%, 75%, and 100%). Daily based estimation, Monday to Friday.	Manual vehicle and CAV	Conflicts based on the threshold values of TTC (1.5 s) and PET (5 s).	CAVs bring about compelling benefit to road safety as traffic conflicts significantly reduce even at relatively low market-penetration rates (12–47%, 50–80%, 82–92% and 90–94% for 25%, 50%, 75%, and 100% CAV penetration rates, respectively).
[123]	Model the turning vehicles at mixed-flow intersections and investigate their surrogate safety measures.	Car-following, yielding &SSM models.	Turning vehicles at mixed-flow intersections.	Regular vehicle: autonomous vehicles in the future.	Evaluate turning vehicles at intersections	Great capability of reproducing four features of real turning trajectories. The travel time of vehicles using simulation does not show significant difference from that in the field. The PET for vehicles and VRU does not show significant difference from the real data.
[128]	Propose an advanced intersection control system to coordinate vehicle trajectories and verify safety.	Temporal-spatial trajectory model. Two signal-free control algorithms.	Typical 4-way intersection.	CAVs&AVs	Evaluate safety and performance efficiency at intersections	The total traffic delays at intersections are reduced. The intersection capacity and performance efficiency are increased.
[129]	Model AVs and validate their safety and performance in a highly uncertain environment.	Pedestrian protection system and bicycle model (roll dynamics).	The effects of rain and pedestrian interaction on AVs.	AV	Throughput	With an increase in rainfall, the detection and classification of lanes and obstacles in the environment takes a longer time to complete. The time-to-collision decreases with increasing rainfall.
[130]	Study different unsafe scenarios for leader-follower to understand the critical driving conditions of AV platoons.	ACC, DSRC.	American Center for Mobility (ACM): No vehicle cut-in (Radar only & Radar and DSRC). Vehicle cut-in (Radar only &Radar and DSRC).	CAVs: truck platoons	Evaluate unsafe scenarios	Having DSRC communications between two trucks in a convoy is an efficient way of decreasing the amount of “blind time,” where the following vehicle is operating without awareness of where the lead vehicle is.
[131]	Model the SSM of pedestrian-AV conflicts.	pedestrian protection system (PPS) model. Bicycle model.	Pedestrian, child walking and running, and bicycle crossing in front of the AVs.	AVs	AV-pedestrian evaluation&SSM	Pedestrian Classification Time to Collision (PCT), Total Braking Time to Collision, and Total Minimum Time to Collision can be used as safety margins for AV-pedestrian conflicts.
[132]	Investigate the SSM of AV-VRU conflict-weather conditions and sensor failure.	Pedestrian protection system (PPS) model. Bicycle model.	Two test scenarios: pedestrian walking and running in front of the AVs.	AVs	AV-VRU conflicts evaluation &SSM	Pedestrian classification time to collision (PCT) and post encroachment (PET) can be utilized as safety indicators for AV-VRU conflicts.
[160]	Present a new technique for developing a pedestrian CIB simulation model.	Pedestrian protection system (PPS)	AV (speed varies) in a straight road with a pedestrian crossing (diff weather&light).	AVs	Throughput	The stop at distance is very sensitive to the error of warning starting TTC, braking starting TTC, and average deceleration.
[161]	Present a V2V-PAEB system that permits the vehicles to share the data of pedestrians observed by the PAEB system in the V2V network.	Pedestrian automatic emergency braking system (PAEB), bicycle model.	Intersection with 5 vehicles and a pedestrian. Vehicle 5 didn't view the pedestrian while crossing because the view is blocked by vehicle 2.	CAVs and AVs	Pedestrian safety at intersection evaluation using PAEB and V2V	Two cases are studied: Vehicle 5 uses PAEB only, and vehicle 5 uses PAEB and V2V from vehicle 1 and 2. When vehicle 5 uses a V2V-PAEB system, the vehicle can detect pedestrians much earlier than in case 1. A V2V-PAEB system can react quickly to avoid a collision.

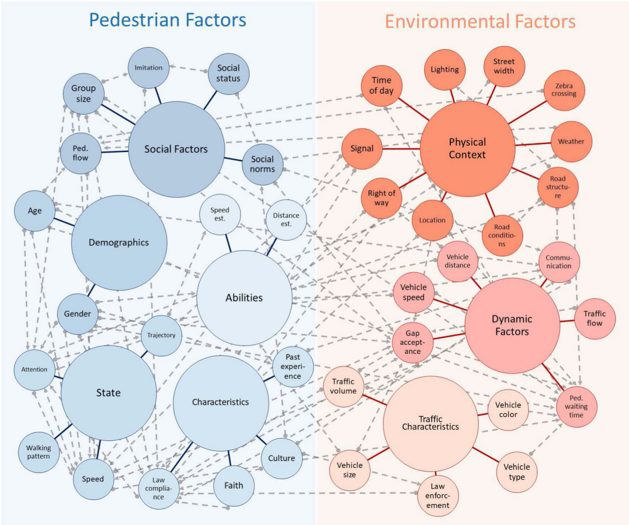

FIGURE 16. Factors that impact pedestrian behavior at the time of crossing. Major factors are represented as large circles. The small circles that are connected with solid lines are sub-factors. The connection between various factors is represented by dashed lines [150].

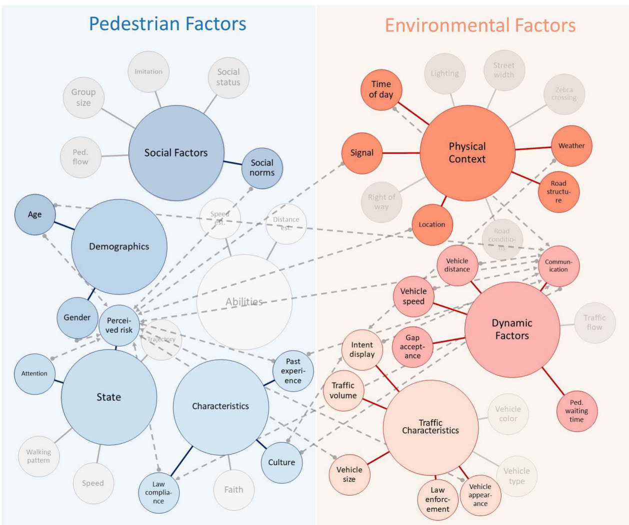

FIGURE 17. Factors that impact pedestrian behavior when facing AVs. Major factors are represented as large circles. The small circles that are connected with solid lines are sub-factors. The connection between various factors is represented by dashed lines. The dim gray drawing in the background represents the classical studies factors [150].

of communication between vehicle and pedestrian, such as vehicle speed and distance. The authors claimed that many factors impacted the interpretation of the signal, such as vehicle speed, road users' age, and time of day.

Jayaraman *et al.* [156] investigated how the availability of traffic signals at crosswalks slightly influences pedestrian crossing behavior while the AV's driving decisions significantly impact such behavior. In [157], Chang *et al.* suggested a method for intent display by placing moving eyes at the front part of the vehicles. Based on data collected from 15 participants, the authors concluded that more participants chose to cross with the availability of rolling eyes, and increased the number of participants by 20% if the eyes are staring toward them. Another study by Pillai [158] suggested that pedestrians' crossing decisions depend on the erratic behavior of the vehicle (speed and distance) and that under specific weather conditions with low visibility, the use of intent display will be helpful. Finally, Pillai concluded that culture is an essential factor that should be considered when designing any intent displays. According to literature reviews [150], pedestrian behavior under autonomous driving conditions needs more focus to include signal, location, road structure, gap acceptance, and social norms factors. Moreover, some elements from classical studies, such as group size, pedestrian speed, and street width, should be evaluated under autonomous driving circumstances. These factors are essential for understanding pedestrian intention to cross the road. A deep understanding and consideration of these factors will result in safe autonomous driving. In short, V2V, V2P, and V2I communications can provide a safe environment for autonomous driving and road users. However, although using these technologies is advantageous, several questions have been raised regarding the sharing of pedestrians' data via these technologies [159].

3) AUTONOMOUS VEHICLE SIMULATION PLATFORMS

Simulation and modeling platforms are well-developed tools for the design and validation of autonomous or non-autonomous vehicle developments. V-model is one of the most popular simulation methods used to cover the testing and design of the entire AV development process [164]. In the development process of autonomous vehicles, virtual simulation methods are applied at different stages, and various testing setups are achieved, such as model-in-the-loop (MIL), software-in-the-loop (SIL), and hardware-in-the-loop (HIL). ISO 26262 is based on the V-model and does not match the agile development process. As a result, a wide range of simulation platforms exist. In [163], Rosique *et al.* classified the simulation platforms into four different approaches that can be considered when selecting a simulator for autonomous vehicles, namely, vehicle test simulation, games, and physics engines for simulation, robotics simulators, and perception simulators.

Autonomous vehicle development is based on the v-model development. Model v has several phases of development and testing, such as the model-in-the-loop (MIL) [165], software-in-the-loop (SIL) [166], hardware-in-the-loop (HIL) [167], and the vehicle-hardware-in-the-loop (VeHIL) approach. The vehicle test simulation approach is based on v model criteria. Several factors must be considered when choosing an autonomous vehicle simulator, such as the availability and compatibility of models, subsystems that can be tested, availability of real-time simulation communications protocols, and compliance with ISO 26262 [163]. Table 17 shows some of the simulation platforms used to validate and test autonomous vehicles based on the vehicle test simulation approach.

TABLE 17. Summary of the main features of simulation platforms for AV [163].

Simulator	License	Open Models	ISO 26262 Compliant	MIL	SIL	HIL
PaTAVTT [170]	GPL	Y	n/a	X	X	Y
Simulink & Matlab [169]	Commercial	n/a	n/a	Y	Y	Y
dSpace GmbH [168]	Commercial	X	Y	Y	Y	Y
LabVIEW [171]	Commercial	X	Y	Y	Y	Y
CarSim [172]	Commercial	X	Y	Y	Y	Y
CAT Vehicle [173]	GPL/Open Source	Y	n/a	Y	Y	Y

*Table legend: Y—Yes, n/a—Unknown or could not be determined, X —No.

TABLE 18. Summary of the main features of robotic simulator platforms for AVs [163].

Simulator	License	Simulation engine	Graphical engine	External Agent
Gazebo	GPL/Open Source	ODE, Bullet, Simbody Art	Ogre3D	Yes
V-Rep	GPL/Open Source, Commercial	ODE, Bullet, Vortex	OpenGL	Yes
Webots	Commercial	ODE	-	Yes
MRDS	Commercial	PhysX	DirectX	No
USARSim	GPL	Unreal Engine	Karma	Yes
BlenSor	GPL/Open Source	-	OpenGL	No
MORSE	GPL/Open Source	Blender, Bullet	OpenGL	Yes

TABLE 19. Summary of the main sensors simulated by robotic simulator platforms for AVs [163].

Simulator	GPS	IMU	LIDAR	Ultrasonic	Radar	Infrared	Stereo Camera	ToF Camera
Gazebo	Y	Y	Y	Y	Y	Y	Y	Y
V-Rep	Y	Y	Y	Y	Y	Y	Y	n/a
Webots	Y	n/a	Y	Y	X	Y	Y	X
MRDS	Y	n/a	Y	Y	n/a	Y	n/a	n/a
USARSim	Y	Y	Y	Y	Y	Y	Y	n/a
BlenSor	Y	Y	Y	Y	n/a	n/a	Y	Y
MORSE	Y	Y	Y	X	X	Y	Y	n/a

*Table legend: Y—Yes, n/a—Unknown or could not be determined, X —No.

Another ordinary simulator is the use of an available game engine. A game engine is defined as the software part of a computer game that has a rendering engine, a physics engine, collision detection and response, sound, scripting, animation, artificial intelligence, networking, streaming, memory control, threading, localization support, and scene graph. In addition, it might incorporate video support for cinematic and virtual reality (VR) simulation [163]. Game engines provide some features that are advantageous for autonomous vehicles and robotics researchers, such as physical fidelity, distributed architecture, cutting-edge graphics, and scriptable environments [163]. The main game engines that are used widely in the development of autonomous vehicle systems or subsystems are Unity 3D [174], Unreal Engine [175], Blender [176], and Cry Engine [177]. The physics engine is an essential component when simulating an autonomous vehicle perception system. This engine provides less fidelity and works according to the detection of collisions. Examples of high-performance physics engines that are used in AV simulation include Open Dynamics Engine (ODE) [178], bullet physics [179], NVidia PhysX [180], and PreScan [231].

Robotics simulation platforms are also used in autonomous vehicle simulation. Models of all sensors and actuators should be provided for the effective use of this type of simulation[181]–[183]. Moreover, a realistic environment for testing and validating all types of algorithms and subsystems should be provided as well. Many features should be considered when choosing a robotics simulator, such as 3D rendering, license, external agent support, sensor noise, parallelism/distribution, level of maturity, fault-tolerance, realistic scenario simulation, and HIL simulation techniques [163]. Examples of current robotics simulators that incorporate data simulation sensors are Gazebo [184], V-REP [185], Webots [186], and Microsoft Robotics Developer Studio (MRDS). In the robotics domain, USARSim [187], BlenSor [188], and MORSE [189] are the three leading simulators that are used extensively for research. For example, MORSE was used by Ford Motor Company to test the 2021 Ford Mustang Mach-E [190]. Table 18 presents a broader list of some robotics simulators that integrate simulation data. Table 19 provides a comparison between sensors that are simulated using robotics simulators.

Simulation platforms should mimic real-world environments to model and validate any perception algorithm. Therefore, the available simulation platforms tend to have these features: fast prototyping, physics engines for realistic motions, realistic 3d rendering, and dynamics with scripting.

TABLE 20. Summary of the features of specific perception simulation platforms for AVs [163].

Simulator	License	Simulation engine	Graphical engine	Scripting Language	External Agent	Notes
CARLA [191]	GPL/Open Source	Unreal Engine	GPU	Python	Yes	Driving
AirSim [200]	GPL/Open Source	Unreal Engine	n/a	C++, Python, C, Java	Yes	Driving/HIL, SIL
DeepDrive [192]	GPL/Open Source	Unreal Engine	n/a	C++, Python	Yes	Driving
Udacity * [193]	GPL/Open Source	Unity	n/a	C++, Python	n/a	Driving
Constellation [194]	Restricted	PhysX/CUDA	GPU	C/C++, Python	Yes	Cloud, HIL, VR
Carcraf/Waymo [195]	Restricted	n/a	n/a	n/a	Yes	Driving
SIMLidar [196]	GPL/Open Source	n/a	n/a	C++	n/a	LiDAR
Helios [197]	GPL/Open Source	JMonkey Engine	OpenGL	Java	n/a	LiDAR
GLIDAR [198]	GPL/Open Source	OpenGL	C++	n/a	LiDAR	
RADSim [199]	Commercial	n/a	n/a	MATLAB	n/a	RADAR
SIMSonic [201]	GPL/Open Source	n/a	n/a	R	n/a	Ultrasonic
PreScan [231]	Commercial	Unreal Engine	GPU	Matlab, C++	n/a	Driving/HIL, SIL

*Table legend: n/a—Unknown or could not be determined.

TABLE 21. Summary of the main sensors simulated by specific perception simulators for AVs [163].

Simulator	GPS	IMU	LIDAR	Ultrasonic	Radar	Infrared	Stereo Camera	ToF Camera
CARLA	Y	X	Y	X	X	X	Y	X
AirSim	Y	Y	Y	n/a	n/a	n/a	n/a	n/a
DeepDrive	Y	n/a	Y	X	Y	n/a	n/a	X
Udacity *	Y	Y	Y	n/a	n/a	Y	n/a	n/a
Constellation	Y	Y	Y	Y	Y	Y	Y	n/a
Carcraft/Waymo	Y	Y	Y	Y	Y	Y	Y	n/a
SIMLidar	X	X	Y	X	X	X	X	X
Helios	X	X	Y	X	X	X	X	X
GLIDAR	X	X	Y	X	X	X	X	X
RADSim	X	X	X	X	Y	X	X	X
SIMSonic	X	X	X	Y	X	X	X	X
PreScan	Y	X	Y	Y	Y	Y	Y	X

*Table legend: Y—Yes, n/a—Unknown or could not be determined, X —No.

Tables 20 and 21 show a list of perception algorithms simulation platforms and their features.

B. AGENT-BASED MODELS

Agent-based models integrate activity-based demand generation and dynamic traffic assignments [204]. This approach covers all macroscopic four-step procedures, namely, demand generation, demand distribution, model choice, and traffic assignment [203]. Agent-based models (ABMs) also utilize independent agents with a bottom-up technique to simulate a highly complex system [202]. This type of modeling and simulation is considered a superior simulation approach compared to other methods in terms of flexibility, hierarchy, intuition, and dealing with complex systems. For example, an AV operating on public roads while interacting with human-driven cars, vulnerable road users, and road networks is highly problematic. Within an independent vehicle system, all subsystems are interconnected and work simultaneously. Moreover, a wide range of elements, such as the diverse behavior of agents (people and vehicles), are integrated within agent-based modeling. With high-end computers, an agent-based modeling approach is used to build challenging models with more realistic scenarios.

The autonomous-vehicles agent-based modeling studies are diverse. They include the travel and environmental impacts of autonomous vehicles [70], [206], the parking requirements with the arrival of autonomous vehicles [207], [208], the traffic congestion caused by autonomous vehicles [209], the system performance of the autonomous vehicle [210], [211], [208], [212], [213], and the autonomous vehicles' modal share and travel modes [214], [72], [215]. There are many critical variables that can impact the system performance of the AV, namely, fleet size, demand, strategy, ride-sharing, pricing schemes, configurations of stations, travel mode, vehicle capacity, service area, refuel/recharge time, maximum waiting time, and cruising time [221]. Many researchers consider these variables in the sensitivity analysis and modeling of various simulation scenarios. For example, in the literature review, there are 27 papers related to the fleet-size research area. In fleet-size studies, regular vehicles are replaced by autonomous vehicles (AV) [71], autonomous taxis (ataxi) [216], autonomous mobility on demand (AMOD) [217], autonomous transit on demand (ATOD) [218], shared autonomous vehicles (SAV) [70], or shared autonomous electric vehicles (SAEV) [219].

The fleet size or replacement rate is considered one of the significant outcomes of the agent-based simulation. The replacement rate is used as an indicator to show the efficiency of an autonomous vehicle system. Fagnant *et al.* [220] argued that travel demand, average speeds, and average trip distances impact AV system performance. Moreover, the replacement rate of the autonomous vehicle was investigated with a case study in Austin. Fagnant and Kockelman [70] argued that one autonomous vehicle can replace ten human-driven cars. In [221], the outcomes showed that the replacement rate in [70] is 1:11 with link-level travel time and is 1:9 with constant speed in [220]. In [210], Marczuk *et al.* showed that fleet size relies on many factors, such as service area, average demand, level of service (based on average waiting time, service and reject rate), routing scheme, relocation plan, and design of the facility. Moreover, in [206], the fleet size can be minimized by ride-sharing. Additionally, the average trip distances data in the survey papers are not precise enough. Furthermore, environmental scenarios, such as urban areas or highway, can impact the travel distance. In [221], ride-sharing was considered as an indicator of the routing scheme. In short, many major factors can affect the fleet size or replacement rate, namely, service data, average demand, average speed, average waiting time, service and reject rate, ride-sharing, relocation plan, and design of the facility. The replacement rate in [70], [220], and [216] is the same, considering that one autonomous taxi can replace ten regular vehicles, excluding relocation and travel demand. The outcomes show that the average waiting time is approximately 2.28 minutes, which is considered too large.

In [162], the authors presented an autonomous intersection management algorithm called AIM-ped, which considers vehicles and pedestrians. The total optimal throughput was calculated when incorporated with maximum pressure control. Moreover, the conflict region model conducts a stability analysis of the autonomous intersection management system. The AIM-ped algorithm is implemented by integrating the maximum-pressure control with an existing trajectory optimization algorithm to obtain the optimal vehicle trajectories. The result is that the AIM-ped algorithm can trigger vehicle movements when there is a change in pedestrian demand. The simulation outcomes show that pedestrians and vehicle delays are negatively correlated. In [125], the sequential movements of vehicles at intersections were modeled as a multi-agent Markov decision process (MAMDPS). The outcomes show that the optimal sequential decision from DCL-AIM outperforms all the other control policies. In [205], the authors presented a model to develop the interaction dynamics between drivers and pedestrians in dense traffic areas where pedestrians and/or drivers do not obey traffic laws and regulations. This approach can be used in control systems of AVs and drivers' onboard alert systems [205]. In [92], the performance of many SAV fleets and vehicle sizes serving travelers across France's Rouen Normandie metropolitan area was evaluated. Moreover, the effect of ride-sharing and rebalancing strategies on service was studied. This study emphasized that the performance of SAV is strongly correlated with the fleet size and the shared rides.

Table 22 presents a summary of the features of specific agent-based simulation platforms. Most agent-based simulation research papers use the MATSim simulation platform to model all autonomous vehicle system operations. In the past, MATSim was used to simulate regular vehicles and not autonomous vehicles. With the need to validate AVs, [209] and [222] establish the AV toolkit for MATSim. In the agent-based simulation, the autonomous vehicles are likewise simulated, and the difference between all the simulated vehicles is the data source. For example, for an autonomous taxi, taxi data are used, and travel surveys are taken into account for other car-sharing services. Private AV simulations in literature surveys are limited. In addition to agent-based modeling, augmented and virtual reality methods have great potential as essential methods for AV evaluation. Recent studies on these methods have been presented in [374]–[377]. In short, the agent-based simulation approach for AVs is in its infancy stage. More focus on agent-based simulation of private autonomous vehicles is required for this approach to compete with other AV validation methods.

VIII. DISCUSSION AND FUTURE RESEARCH DIRECTIONS

Two main tests are currently used to evaluate CAVs and AVs, namely naturalistic field operational tests (N-FOTs) and virtual tests. The virtual tests include test matrix evaluation, worst-case scenario evaluation (WCSE), Monte Carlo simulations, accelerated evaluation (AE), simulation-based and agent-based modeling approaches. In some cases, both N-FOTs and virtual tests are combined simultaneously to evaluate CAVs and AVs. In N-FOTs, vehicles are equipped with the required sensors and are driven in naturalistic conditions, which is not the case in virtual tests. The N-FOTs allow the investigators to observe CAVs and AVs in a natural setting. The data collected from N-FOTs are utilized to investigate many elements, such as driver performance, surrounding environment, driving conditions, and other components related to critical incidents, near collisions, and collisions. However, the N-FOTs have many restrictions, such as the time required to conduct the test, the need for trained drivers, and the low probability of critical events. Moreover, the test requires many vehicles, a lot of time, and large budgets. Therefore, virtual tests are used as an efficient alternative

TABLE 22. Summary of the features of specific agent-based simulation platforms for AVs [221].

Simulator	Country	Scripting Language	License
MATSim [223]	German	Java	Open-source
Matlab	USA	C	Commercial
AnyLogic	Russia	UML	Commercial
SimMobility[224]	Singapore	C++	Open-source
Commuter[225]	US/Autodesk	Java	Commercial
Aimsun [226]	Spain/German	Python	Commercial
Mobility Testbed [227]	USA	Java	Open-source
Multi-agent middleware JADE [228]	Italy	Java	Open-source
mobiTopp [229]	n/a	n/a	n/a
Gurobi [230]	USA/Gurobi	Python	Open-source
GAMA	n/a	n/a	n/a
NetLogo	USA	LOGO	Open-source

*Table legend: n/a—Unknown or could not be determined.

approach to model and validate CAVs and AVs. Many questions are raised regarding virtual tests and how these tests can be reliable and replace the naturalistic field operational tests. For example, urban scenes are essential in virtual tests, which inevitably involve pedestrians, vehicles, cyclists, motorcyclists, etc. Simulating traffic congestion, lane-change scenarios, car-following scenarios, pedestrian-vehicle conflict, vehicle-vehicle conflict, pedestrian behavior, driver behavior, human-driven vehicle behavior, weather conditions, and so many scenarios in a large-scale traffic scenario is a complicated multi-layer task. Usually, the resulting movements of each object in the simulation rarely follow the physical laws. Moreover, accessing vehicle trajectories and including them in virtual tests or applications in real-time is challenging. Furthermore, road networks generation and representation is also a fundamental task in traffic simulation and modeling. Several simulation tools support road networks, but the outcomes do not resemble real-world traffic at the street level. Therefore, in virtual testing methods, model verification in terms of the similarity between the simulated traffic model and real-world scenarios is always a concern. In this review, we presented the advantages and disadvantages of each evaluation method. The review papers presented in this survey show a clear gap in the research area of CAV and AV evaluation. Each method has its own strengths and weaknesses. For example, many techniques focus on car-following and lane-change modeling and evaluation and neglect the remaining traffic conditions and the modeling of severe weather conditions. Furthermore, the V2V, V2P, and V2I technologies are still under investigation and require more validation.

Moreover, the CAV and AV modeling and evaluation is a task that requires integrating many models simultaneously with a wide range of parameters and variables. Choosing suitable models will produce satisfying outcomes. Based on our findings, different models related to behavior, car-following, lane-change, vehicle dynamics, etc. are being used in every research paper. Therefore, establishing a comparison study is a very challenging task. For example, some car-following models outperform other models. Using the superior car-following model in a research paper with a specific evaluation method will produce promising outcomes for this method. Moreover, several architecture models have been developed, from completely modular to fully end-to-end, each with its limitations. The optimal algorithms for localization, mapping, and perception still lack accuracy and efficiency. In short, for safe autonomous driving, a high-fidelity driving simulator, which includes realistic traffic streams and complicated traffic conditions, is necessary. Such a simulator can construct critical training environments in an efficient and reproducible manner. New evaluation methods need to be developed for more scenarios to provide a thorough validation of AVs. The community has not fully understood the full failure modes of AVs to design a complete list of test scenarios, but the possible elements to incorporate are as follows:

1) Challenges in sensing/detection under severe weather conditions such as heavy snow, rain, fog, etc.
2) Aggression of surrounding vehicles/vulnerable road users such as running a red light, cut-in, jaywalk, etc.
3) Challenges in making decisions such as under low confidence, multiple threats at a time, and so on.
4) Challenges due to road types and vehicles types

Moreover, these simulators and evaluation methods should provide clear answers to the following questions:

1) What are scalable driving policies to control many AVs in mixed traffic comprised of human-driven vehicles (HVs), CAVs, AVs, vulnerable road users, etc.?
2) How do we estimate human driver behaviors, pedestrian behaviors, surrounding vehicles (HVs, CAVs, and AVs)?
3) How to ensure that the behaviors of drivers and pedestrians are accurate and capture the real-world behaviors?
4) How should the driving behavior of HVs, CAVs, AVs be modeled in the environment?
5) How are the interactions between human-driven vehicles (HVs) and AVs characterized?
6) How are the interactions between CAVs and AVs characterized?
7) How are the interactions between pedestrians and AVs characterized?

8) How are the interactions between other vulnerable road users (VRU) and AVs characterized?
9) How should pedestrian behavior be modeled in the environment?
10) How should the severe weather conditions be modeled in the environment?

Many methods showed promising outcomes but did not provide answers to all of these questions.

IX. CONCLUSION

It is critical to evaluate AVs thoroughly before their release and deployment to the general public. However, because most trips are not safety-critical in naturalistic driving, testing AVs on public roads is time-consuming, inefficient, and expensive. In this study, we surveyed all evaluation methods. These methods include naturalistic field operational tests, test matrix evaluation, worst-case scenario evaluation, Monte Carlo simulations, accelerated evaluation, and simulation-based model approach. This survey showed that there is a clear gap in the field of AV evaluation. Many factors affect our judgment on what is the best approach to evaluate AVs. These factors include:

1) The AVs to AVs and HVs to HVs interactions have not been studied and used only as a benchmark.
2) The AV sensors and controls have been suggested to work accurately in many papers, and the measurements are presumed to be accurate.
3) The drivers' reactions to AVs are assumed to be the same as to HVs.
4) The vehicle models are not accurate to mimic the real-world scenarios.
5) Many real-world conditions have not yet been investigated.
6) Different models related to behavior, car-following, lane-change, vehicle dynamics, etc. are being used in every research paper.

The accelerated evaluation approach outperforms naturalistic field operational tests (N-FOTs), test matrix evaluation, worst-case scenario evaluation (WCSE), and Monte Carlo simulations methods in some of the car-following, and lane-change studies when using specific models in terms of the assessment time of the collision, injury, or conflict event. In addition, some studies show that integrating machine and deep learning techniques with test matrix evaluation, Monte Carlo simulations, and accelerated evaluation can reveal significant improvements. In the simulation-based model approach, the agent-based modeling approach was investigated and shown to be advantageous for AV modeling and validation. However, more work is needed to implement an agent-based modeling approach to cover a wide range of self-driving vehicle research. Another promising approach for AV evaluation is the augmented and virtual reality methods. The development of AVs depends on advancements in scientific disciplines and new technologies. Therefore, AV research development has a high impact on AV driving technology by overcoming the weaknesses of the available evaluation methods and by inventing new evaluation methods.

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