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Assessment of Solid Waste Management Strategies Using an Efficient Complex Fuzzy Hypersoft Set Algorithm Based on Entropy and Similarity Measures

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ABSTRACT Solid waste management has gained a reputation among environmentalists as it poses a significant threat to the environment when done incorrectly and leading to effects longing for more than a century. Current solid waste management (SWM) concerns are inextricably linked to maintaining mandated organic waste treatment and reusing objectives following European directive regulations. Characterizing and spreading uncertainty, as well as verifying forecasts, are all challenges in decision-making. This study presents a multi-attribute decision-making approach based on entropy and similarity measures to evaluate SWM strategies. This research examined the novelty of the complex fuzzy HyperSoft set (CFHSS), which may respond to instabilities, ambiguity, and vagueness of facts in knowledge by simultaneously putting into consideration the amplitude and phase characteristics (P-terms) of complex numbers (C-numbers). The presented structure is the most suitable option for exploring SWM concerns as it allows for a more comprehensive array of membership values, and the periodic nature of the content can be expressed in P-terms to widen the content to a unit circle in a dynamic reference frame through the specification of the fuzzy HyperSoft set (FHSS). Secondly, the features in CFHSS may be further sub-divided into attribute values for easier comprehension. The paper also illustrates the apparent connection between CFHSS similarity measures (SM) and entropy (ENT) and explores colloquial meaning. These strategies may be used to determine the best approach from a group of possibilities that have a variety of applications in the field of optimization. The recommended methodology's reliability and effectiveness are examined by evaluating the acquired findings to those of several prior studies. An assessment is done using various parameter values to validate the robustness of the suggested approach.

INDEX TERMS Solid waste management (SWM), fuzzy set (FS), fuzzy hypersoft set (FHSS), complex fuzzy hypersoft set (CFHSS), entropy (ENT), similarity measures (SM).

I. INTRODUCTION

With an exponential increase in the human population, SWM is quite a task as it has become a significant factor in natural resources and environmental conservation. It is essential to deal with the produced solid waste to prevent it from harming the earth's natural ecosystem and preserving the living conditions of the life present on earth. To deal with the sheer amount of solid waste, several methods are being used today like composting, recycling, incineration etc. Each of the methods opted for the job have their set of advantages along with their drawbacks. The analysis has revealed that these methods have to lead to conflicting objectives within the selected attribute set. So, there is a dire need for a plan for assessing these SWM methods for the optimal selection of the method that is best suited for the job while keeping the environment safe.

The optimal selection of the SWM process requires proper administration as the range of methods from which the ranking process is to be applied differ significantly in cost, time,

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the technology used, reliability, and the range of the population in effects. Most of the parameters selected for this job have uncertain values, and improper handling of this uncertainty may lead to a suboptimal result. An example of the above phenomenon is when the decision-maker is considering the importance of the parameters as an approximation, the values are given in the form of a range, or the data may not be in scientific terms but is instead expressed in linguistic terms like good, better, and best, this leaves room for uncertainty and if not dealt with adequately may result in the improper selection process. To address this issue, tools from FS theory are opted for as they extensively can deal with uncertainty in a precise manner [1], [2].

The tool most suitable for this type of job is decision making, where more than one decision-maker decide with respect to the present alternatives which are being characterised. In this way, the final decision is a collective contribution of all the decision-making entities. The literature review revealed the application of numerous MCDM methods in the field of SWM. Yesilnacar *et al.* [3] used 3 MCDM methods for the evaluation of 10 disposal alternatives that were further subdivided and assessed under 18 different criteria. Mixed alternative linear programming methods were applied to evaluate various landfill sites for the optimal selection process in SWM [4]. Another approach in MCDM was opted by Mir *et al.* [5] to rank the possible method best suited for SWM. Generowicz *et al.* [6] used MCDM strategies that were used in planning procedures of SWM systems in European nations. In addition to the above-listed procedures, MCDM was applied in SWM in the following studies too [7]–[9].

The idea of probability, the idea of FS [10], [11], the concept of intuitionistic FS [12], the scientific theory of vague sets [14], and the theory of interval mathematics [13] are all essential extant theories. The idea of rough sets [16] can be conceived as numerical strategies for mitigating risk. However, as [17] points out, each of these concepts has its own host of problems. Perhaps the insufficiency of the parametrisation strategies is responsible for these complications. Molodtsov [17] proposed the soft set (SS) theory as a new mathematical tool for dealing with uncertainty or unpredictability that is exempt from the struggles mentioned above. SS refers to (binary, elementary, primitive) nearby systems [18] and is a good demonstration of setting subordinate FS, as Thielle [19] was described. Maji *et al.* [20] came up with the notion of the fuzzy soft set (FSS) by blending SS and FS. Maji *et al.* [21] initiated the request of FSS theory in visual recognition concerns.

By incorporating SS and FS, Maji *et al.* [20] conceptualised FSS. Yang *et al.* [22] proposed the concept of an FSS by combining the FS and SS and implementing it to the MCDM. Dey and Pal [23] broadened the concept of FSS. Zhang and Shu [66] developed the concept of FSS and developed the concept of potential FSS, which they used to an MCDM.

In FS and their mixture frameworks mentioned in [25], the fuzzy event's probability indicators have contributed a lot. For fuzzy ENT, De Luca and Termini [26] suggested a certain configuration of hypotheses. On the other hand, SM, an essential tool for determining the amount of SM between two items, has gained more attention than ENT. Pappis and his colleagues published a series of papers [27], [28] that took a closer look at the SM. The ENT and SM for various sets, such as interval-valued FS [29], FSS [34], and intuitionistic FSS [31], have been extensively used in overcoming problems related to decision making, cognition, and sensor fusion.

Al-Qudah and Hassan [32] devised the CFS (Complex fuzzy set) as a generalisation of FS in the complex setting. This methodology facilitates in the resolution of aspects of complex two-dimensional depiction qualities. They will modify it into the complex fuzzy soft set (CFSS) to shore up the strengths of SS and employ them in the CFS models to allow it more valuable and to provide new powerful effects. By capturing the A-terms and P-terms of the C-numbers simultaneously, their recommended model is expected to deal with these situations, ambiguity, and unclearness of 2D fuzzy data. Al-Qudah and Hassan [33] proposed the concept of CFSS in 2018, which provides a combination of both the CFS and the soft set.

In a range of practical implementations, the attributes should be sub-partitioned into attribute values for easier comprehension. This requirement was met by Smarandache [35], who created the Hypersoft set (HSS) as an expansion of the SS. He broadened this perspective by restoring SS into a multi-attribute mechanism and extrapolating it to the HSS.

In a neutrosophic atmosphere, Saeed *et al.* [30], [36], [65] presented some fundamental concepts such as Hypersoft (HS) and employed similarity measure strategies for a clinical condition. HS subset, HS complement, not HSS, absolute HSS, union, intersection, and several matrix operations were explained by Saeed *et al.* [65]. Saeed *et al.* [36], [62], [65]–[68] several uses of SS, neutrosophic set, neutrosophic HSS in object recognition, biosensors, judgement, and specified mapping in a HSS paradigm. Abbas *et al.* [61] investigated hypersoft points in a spectrum of fuzzy-like contexts. Rahman *et al.* [62] defined complex HSS in 2020 and built hybrids of the HS set with a complex fuzzy set, complex intuitionistic fuzzy set, and complex neutrosophic set, respectively. Rahman *et al.* [63] conceived convexity cum concavity on HSS in 2020 and produced visualisation tools with illustrations.

The following are the major priorities of our investigation. First, we introduced the meanings of ENT and SM of CFHSS are described, as well as the underlying propositions. Mathematical models are also available to check the ability and superiority of the strategy. Secondly, extensive comparisons between available methodology and established ideas are shown. Finally, the mathematical structures are displayed in order to establish the reliability and relevance of the measurements given. The decision making committee will evaluate the data in the form of CFHSS by considering the degree of the influence and the total time of the influence as a complex number; along with the deep evaluation of the information by taking sub parametric values of assigned attributes as hypersoft structure; where all the data can be taken in numeric value between 0 (degree of zero percent match) and 1 (degree of hundred percent match).

A. MOTIVATION

The main purpose of this study is to predict feasible circumstances for SWM strategies, as well as their effective identifying treatment method because it is difficult to ascertain the certain kind of SWM strategies using prior, current theory and procedures [10], [17], [20], [32], and [33] because these tools are curtailed to achieve configurations. The strategies described in [10], [17], [20], [32] and [33] are inadequate to examine the data in a deep sense for better comprehension and correct decisions. In [10], [17], [20], these theories fails to manage complex (two-dimensional) information/data (the degree of the influence and total time of influence) when the parameters have their sub-parameters types of values. In [32], [33], they can manage the 2D information but fails to deal when the parameters have their sub-parameters types of values. To accomplish these objectives, we evolved these frameworks into a complex process constituted by a fusion of fuzzy sets and HSS. In two important respects, this approach is more versatile. First, it enables a variety of membership function values to the unit circle in a complex plane by revising the CFHSS to include an extra term, the P-term, to account for the data's periodic nature. Secondly, for a better understanding, the qualities in CFHS may be further sub-divided into attributive values. The proposed strategy evaluates different SWM strategies based on economic, cultural, scientific, and environmental variables. When paired with scientific modeling, these theories are as effective and essential for an objective as a possible scenario.

B. PAPER PRESENTATION

Section II focuses on some basic definitions and terminologies used in the paper. In Section III, we present the proverbial meaning of ENT for CFHSS, supported by an example. In Section IV, the SM between CFHSS and the connection between the ENT and SM are examined. Section V concludes the paper.

II. PRELIMINARIES

In this part of the paper, several core ideas are explained including FS, SS, ENT, SM, FHSS, and CFHSS.

Definition 1 [10]: The FS, $R = \{(b, \zeta(b))| \ b \in H\}$ such that

$$\zeta : H \to [0, 1],$$

where H signifies objects gathering and $\zeta(b)$ signifies the membership grade of $b \in H$.

Definition 2 [17]: SS is the pair (ζ, A) over H, where ζ is a function that looks like this:

$$\zeta : A \to P(H),$$

for $\epsilon \in A$, $\zeta(\epsilon)$ can be conceived as ϵ approximate components of the SS (ζ, A).

Definition 3 [34]: A real-valued map χ from $FS(\zeta, A)$ to $[0, \infty)$ for FSS is termed an ENT, if χ matches the following requirements,

1) $\chi(\zeta, A) = 0$ if (ζ, A) is a SS,
2) $\chi(\zeta, A) = 1$ if $\zeta(e) = 0.5$, for $e \in A$, where $[0.5]$ is the FS having membership function $[0.5](b) = 0.5$, for every $b \in H$,
3) Suppose (ζ, A) be crisp set than that of (ψ, B) which is, for $e \in A$ and $b \in H$, $\zeta(e)(b) \leq \psi(e)(b)$ if $\psi(e)(b) \leq 0.5$ and $\zeta(e)(b) \geq \psi(e)(b)$ if $\psi(e)(b) \geq 0.5$. Then $\chi(\zeta, A) \leq \chi(\psi, B)$,
4) $\chi(\zeta, A) = \chi(\zeta^c, A)$, where (ζ^c, A) is the complement of FSS (ζ, A), which can be written as $\zeta^c(e) = (\zeta(e))^c$, for every $e \in A$.

Definition 4 [34]: If a map V from $FS(H, E) \times FS(H, E)$ to $[0, 1]$ meets the standard criteria, it is designated an SM for FSS.

1) $V(X_H, \Phi_H) = 0$, for any $H \in E$, and $V((\zeta, A), (\zeta, A)) = 1$ for any $(\zeta, A) \in FS(H, E)$,
2) $V((\zeta, A), (\psi, B)) = V((\psi, B), (\zeta, A))$, for any $(\zeta, A), (\psi, B) \in FS(H, E)$,
3) For any $(\zeta, A), (\psi, B), (H, O) \in FS(H, E)$ if $(\zeta, A) \subseteq (\psi, B) \subseteq (H, O)$, then $V((H, O), (\zeta, A)) = min(V((H, O), (\psi, B)), V((\psi, B), (\zeta, A)))$.

Definition 5 [36]: Consider that H and $\zeta(H)$ are the collection and all imprecise subsets of H respectively, let $l_1, l_2, l_3, \cdots, l_n$ be distinctive features with attributes matching to the sets $Q_1, Q_2, Q_3, \ldots, Q_n$, respectively. where $Q_i \cap Q_j = \Phi$ for $i \neq j$ and i, j belongs to $\{1, 2, 3, \ldots, n\}$. Then the FHSS is the pair (Σ_L, L) over H defined by a map $\Sigma_L : L \to \zeta(H)$, where $L = Q_1 \times Q_2 \times Q_3 \times \ldots \times Q_n$.

Definition 6 [62]: Let $Q_1, Q_2, Q_3, \ldots\ldots, Q_n$ be disjoint sets having attribute values of n distinct attributes $l_1, l_2, l_3, \ldots\ldots, l_n$ respectively for $n \geq 1$, $G = Q_1 \times Q_2 \times Q_3 \times \cdots\cdots \times Q_n$ and $\vartheta(\underline{y})$ be a CF-set over H for all $\underline{o} = (n_1, n_2, n_3, \ldots\ldots, n_n) \in G$. Then, CFHSS φ_G over H is underlying as:

$$\varphi_G = \{(\underline{o}, \vartheta(\underline{o})) : \underline{o} \in G, \vartheta(\underline{o}) \in B(H)\}$$

where

$$\vartheta : G \to B(H), \quad \vartheta(\underline{o}) = \emptyset \ if \ \underline{o} \notin G.$$

is a CF-approximate relation of φ_G and its value $\vartheta(\underline{o})$ is called $\underline{o}$-element of CFHSS $\forall \ \underline{o} \in G$.

Example 1: Assume an individual desired to acquire money from the account for a specific significant period of time. Let $H = \{b_1 = \text{Lloyds}, b_2 = \text{NatWest}, b_3 = \text{HSBC}\}$ be a collection of three London banks. It is often assumed that a year is divided into four periods, each with a distinct risk premium. Let a_1 = Degree of repaying, a_2 = lending rate, a_3 = Contents, separate features with attributes that are component of the sets Q_1, Q_2, Q_3. Let $Q_1 = \{\eta_1 = \text{Flexible}, \eta_2 = \text{Difficult}\}$, $Q_2 = \{\eta_3 = \text{High}, \eta_4 = Low\}$, $Q_3 = \{\eta_5 = \text{Quick}\}$. Now, we develop CFHSS as per observing above information.

$$\psi(\eta_1,\eta_3,\eta_5) = \{b_1/(0.9e^{i2\pi(2/4)}), b_2/(0.8e^{i2\pi(1/4)}), b_3/(0.4e^{i2\pi(3/4)})\},$$
$$\psi(\eta_1,\eta_4,\eta_5) = \{b_1/(0.8e^{i2\pi(2/4)}), b_2/(0.5e^{i2\pi(1/4)}), b_3/(0.1e^{i2\pi(3/4)})\},$$
$$\psi(\eta_2,\eta_3,\eta_5) = \{b_1/(0.1e^{i2\pi(2/4)}), b_2/(0.8e^{i2\pi(2/4)}), b_3/(0.04e^{i2\pi(1/4)})\},$$
$$\psi(\eta_2,\eta_4,\eta_5) = \{b_1/(0.2e^{i2\pi(2/4)}), b_2/(0.7e^{i2\pi(1/4)}), b_3/(0.1e^{i2\pi(3/4)})\},$$

In this scenario, the A-terms describe the degrees of sense of belonging to the exchange rate structure, while the P-terms describe the percentages of connectedness to the season period with respects to the attribute values. In the CFHSS value $b_1/(0.8e^{i2\pi(2=4)}, b_2/0.2e^{i2\pi(4=4)}, b_3/0.3e^{i2\pi(3=4)}$. The first number $(0.8e^{i2\pi(2=4)}$ reveals that the bank interest level is increasing in spring season, the A-term and P-term are 0.8, $(2 = 4)$ respectively identifies the spring season with the following phase w.r.t (η_1,η_3,η_5). Whereas this subsequent degree of membership $0.2e^{i2\pi(4=4)}$ illustrates that the incentive rate is low in the winter since the P-term 0.2 is nearly zero and the P-term $(4 = 4)$ pertains to the final season of year (the winter season) in reference to the attributes value (η_1,η_3,η_5). Now, we shall go through the CFHSS's central concept and functioning.

III. ENTROPY (ENT) ON CFHS-SETS

ENT is among the most crucial components of FS since it handles the essential matter related to FS governance. What is the extent of vagueness in an FS? ENT is a tool for obtaining the uncertainty and ambiguity of FS. This part developed the notion of ENT in the framework of CFHSS. To exemplify the reliability and usefulness of the newly constructed ENT-based CFHSS, certain corresponding theorems and implementation are discussed.

Definition 7: A map E: $CFHSS(H) \to [0,1]$ is said to be ENT on *CFHSS*, if E passes all of the prerequisites

1) $E(\psi,\mathcal{F}) = 0 \Leftrightarrow \upsilon_{F(e)}(b) = 1$ and $\omega_{F(e)}(e)(b) = 2\pi$, $\forall\, e \in \mathcal{F}, b \in H$,.
2) $E(\psi,\mathcal{F}) = 1 \Leftrightarrow \upsilon_{\psi(e)}(b) = 0.5$ and $\omega^j_\psi(e)(b) = \pi$, $\forall\, e \in \mathcal{F},\ b \in H$.
3) $E(\psi,\mathcal{F}) = E(\psi,\mathcal{F})^c$.
4) if $(\psi,\mathcal{F}) \subseteq (\varphi,\mathcal{F})$, i.e, $\upsilon_{\psi(e)}(b) \le \upsilon_{\varphi(e)}(b)$ and $\omega_{\psi(e)}(b) \le \omega_{\varphi(e)}(b)$, $e \in E$, $b \in H$, then $E(\psi,\mathcal{F}) \ge E(\varphi,\mathcal{F})$.

Theorem 1: Let $H = \{b_1, b_2, \ldots, b_g\}$ be the set of elements and $\mathcal{F}$ be the set of parameters. Then $(\psi,\mathcal{F}) = \{\mathcal{F}(e) = r_{\psi(e)}(b).e^{i\omega_{\psi(e)}}(b)|l = 1,2,3,\ldots,m\}$, where $e \in \mathcal{F}$, is a class of *CFHSS*. Consider $E(\psi,\mathcal{F})$ given in such a way:

$$E(\psi,\mathcal{F}) = \frac{1}{2m}\Sigma_{l=1}^m[E_l^r(\psi,\mathcal{F}) + E_l^\omega\frac{(\psi,\mathcal{F})}{2\pi}], \tag{1}$$

here,

$$E_l^r(\psi,\mathcal{F}) = \frac{1}{n}\Sigma_{p=1}^n[1 - |\upsilon_{\psi(e_l)}(b_g) - \upsilon_{\psi^c(e_l)}(b_g)|], \tag{2}$$

and

$$E_l^\omega(\psi,\mathcal{F}) = \frac{1}{n}\Sigma_{p=1}^n[1 - |\omega_{\psi(e_l)}(b_g) - \omega_{\psi^c(e_l)}(b_g)|] \tag{3}$$

then $E(\psi,\mathcal{F})$ is an ENT of *CFHSS*.

Proof: We show that the $E(\psi,\mathcal{F})$ meets all of the requirements in Definition 7.

1) $E(\psi,\mathcal{F}) = 0, \Leftrightarrow \frac{1}{2m}\Sigma_{l=1}^m[E_l^r(\psi,\mathcal{F}) + E_l^\omega\frac{(\psi,\mathcal{F})}{2\pi}] = 0$, $\Leftrightarrow E_l^r(\psi,\mathcal{F}) = 0$ and $E_l^\omega(\psi,\mathcal{F}) = 0 \Leftrightarrow \forall\ e_l \in \mathcal{F}, b_g \in H$, $\Sigma_{p=1}^n[1 - |\upsilon_{\psi(e_l)}(b_g) - \upsilon_{\psi^c(e_l)}(b_g)|] = 0$, and $\Sigma_{p=1}^n[1 - |\omega_{\psi(e_l)}(b_g) - \omega_{\psi^c(e_l)}(b_g)|] = 0$, $\Leftrightarrow \forall\ e_l \in \mathcal{F}, b_g \in H$, $|\upsilon_{\psi(e_l)}(b_g) - \upsilon_{\psi^c(e_l)}(b_g) = 1$, $|\omega_{\psi(e_l)}(b_g) - \omega_{\psi^c(e_l)}(b_g)| = 2\pi$, $\Leftrightarrow \forall\ e_l \in \mathcal{F}, b_g \in H$, $\upsilon_{\psi(e_l)}(b_g) = 1$, $\omega_{\psi(e_l)}(b_g) = 2\pi$,
2) For $(\psi,\mathcal{F}) \in CFSS(H)$, we have $E(\psi,\mathcal{F}) = 1$, $\Sigma_{l=1}^m[E_l^r(\psi,\mathcal{F}) + E_l^\omega\frac{(\psi,\mathcal{F})}{2\pi}] = 2m$, $\Leftrightarrow E_l^r(\psi,\mathcal{F}) = 1$, and $E_l^\omega(\psi,\mathcal{F})] = 2\pi$, $\Leftrightarrow\ \forall\, e_l \in \mathcal{F}, b_g \in H$, $\frac{1}{n}\Sigma_{p=1}^n[1 - |\upsilon_{\psi(e_l)}(b_g) - \upsilon_{\psi^c(e_l)}(b_g)|] = 1$, and $\frac{1}{n}\Sigma_{p=1}^n[1 - |\omega_{\psi(e_l)}(b_g) - \omega_{\psi^c(e_l)}(b_g)|] = 2\pi$, $\Leftrightarrow\ \forall\ e_l \in \mathcal{F}, b_g \in H$, $\Sigma_{p=1}^n[1 - |\upsilon_{\psi(e_l)}(b_g) - \upsilon_{\psi^c(e_l)}(b_g)|] = n$, and $\Sigma_{p=1}^n[2\pi - |\omega_{\psi(e_l)}(b_g) - \omega_{\psi^c(e_l)}(b_g)|] = 2\pi(n)$, $\Leftrightarrow\ \forall\ e_l \in \mathcal{F}, b_g \in H, [1 - |\upsilon_{\psi(e_l)}(b_g) - \upsilon_{\psi^c(e_l)}(b_g)|] = 1$, and $[2\pi - |\omega_{\psi(e_l)}(b_g) - \omega_{\psi^c(e_l)}(b_g)|] = 2\pi$, $\Leftrightarrow$ $\forall\ e_l \in \mathcal{F}, b_g \in H$, $|\upsilon_{\psi^c(e_l)}(b_g) - \upsilon_{\psi(e_l)}(b_g)| = 0$, and $|\omega_{\psi^c(e_l)}(b_g) - \omega_{\psi(e_l)}(b_g)| = 0$, $\Leftrightarrow \forall\ e_l \in \mathcal{F}, b_g \in H$, $\upsilon_{\psi(e_l)}(b_g) = \frac{1}{2}$ and $\omega_{\psi(e_l)}(b_g) = \pi$,
3) For $E(\psi,\mathcal{F}) \in CFSS(H)$, we have, $E_l^r(\psi,\mathcal{F}) = \frac{1}{n}\Sigma_{p=1}^n[1 - |\upsilon_{\psi(e_l)}(b_g) - \upsilon_{\psi^c(e_l)}(b_g)|]$, $\frac{1}{n}\Sigma_{p=1}^n[1 - |\upsilon_{\psi^c(e_l)}(b_g) - \upsilon_{\psi(e_l)}(b_g)|]$, $= E_l^r(\psi,\mathcal{F})^c$, Similarly, we show that $E_l^r(\psi,\mathcal{F}) = E_l^r(\psi,\mathcal{F})^c$ it is clear that $E(\psi,\mathcal{F}) = E(\psi,\mathcal{F})^c$.
4) Assume $(\psi,\mathcal{F})$ and $(\varphi,\mathcal{F}) \in CFSS(H)$. If $(\psi,\mathcal{F}) \subseteq (\varphi,\mathcal{F})$, $\Rightarrow \forall\ e_l \in \mathcal{F}, b \in H$, $\upsilon_{\psi(e_l)}(b_g) \le \upsilon_{\varphi(e_l)}(b_g)$ and $\omega_{\psi(e_l)}(b_g) \le \omega_{\varphi(e_l)}(b_g) \Rightarrow \forall\ e_l \in \mathcal{F}, b \in H$, $|\upsilon_{\psi(e_l)}(b_g) - \upsilon_{\psi^c(e_l)}(b_g)| \le |\upsilon_{\varphi(e_l)}(b_g) - \upsilon_{\varphi^c(e_l)}(b_g)|$, and $|\omega_{\psi(e_l)}(b_g) - \omega_{\psi^c(e_l)}(b_g)| \le |\omega_{\varphi(e_l)}(b_g) - \omega_{\varphi^c(e_l)}(b_g)|$, $\Rightarrow\ \forall e_l \in \mathcal{F}, b \in H$, $1 - |\upsilon_{\psi(e_l)}(b_g) - \upsilon_{\psi^c(e_l)}(b_g)| \ge 1 - |\upsilon_{\varphi(e_l)}(b_g) - \upsilon_{\varphi^c(e_l)}(b_g)|$, and $2\pi - |\omega_{\psi(e_l)}(b_g) - \omega_{\psi^c(e_l)}(b_g)| \ge 2\pi - |\omega_{\varphi(e_l)}(b_g) - \omega_{\varphi^c(e_l)}(b_g)|$, $\Rightarrow \frac{1}{n}\Sigma_{p=1}^n([1 - |r_{\psi(e_l)}(b_g) - \upsilon_{\psi^c(e_l)}(b_g)|]), \ge \frac{1}{n}\Sigma_{p=1}^n([1 - |\upsilon_{\varphi(e_l)}(b_g) - \upsilon_{\varphi^c(e_l)}(b_g)|])$, and $\frac{1}{n}\Sigma_{p=1}^n([2\pi - |\omega_{\psi(e_l)}(b_g) - \omega_{\psi^c(e_l)}(b_g)|]) \ge \frac{1}{n}\Sigma_{p=1}^n([2\pi - |\omega_{\varphi(e_l)}(b_g) - \omega_{\varphi^c(e_l)}(b_g)|]), \Rightarrow E_l^r(\psi,\mathcal{F}) \ge E_l^r(\varphi,\mathcal{F})$, and $\Rightarrow E_l^\omega(\psi,\mathcal{F}) \ge E_l^\omega(\varphi,\mathcal{F})$, $\Rightarrow E_l^r(\psi,\mathcal{F}) + E_l^\omega(\psi,\mathcal{F}) \ge E_l^r(\varphi,\mathcal{F}) + E_l^\omega(\varphi,\mathcal{F})$, $\Rightarrow \frac{1}{2m}\Sigma_{l=1}^m[E_l^r(\psi,\mathcal{F}) + E_l^\omega\frac{(\psi,\mathcal{F})}{2\pi}] \ge \frac{1}{2m}\Sigma_{l=1}^m[E_l^r(\varphi,\mathcal{F}) + E_l^\omega\frac{(\varphi,\mathcal{F})}{2\pi}], \Rightarrow E(\psi,\mathcal{F}) \ge E(\varphi,\mathcal{F})$.

A. USING THE DESCRIBED STRATEGY, RANK SWM APPROACH

In this article, we use the recommended ENT-based CFHSS judgment strategy to rank several SWM strategies. To establish the reliability and effectiveness of the working establish, we compared the suggested methodology with some of the existent investigations.

1) THE ENT-BASED CFHSS OFFERED WITH IMPLEMENTATION

With an ever-increasing human species, industrialization has intensified, and waste disposal creation has soared. As a reason, SWM has increasingly become a significant issue in urbanized environments, especially those in developing countries. SWM is classified as the subject that deals with the development, preservation, collection, transit or transference, cleaning, and waste dumping waste materials in a way that incorporates population health, sustainability, aesthetics, construction, and other environmental exposures. Insufficient substantial way to dispose of is one of the critical environmental problems emerging nations are now struggling with. Since the scientific revolution, it has been a serious global issue. The following are some of the known SWM procedures (marked as alternatives).

B. THE EXPLORATION OF SWM STRATEGY AND ITS ASPECTS

The environmental impact of analytic SWM investigation and computational mathematics is significant. There are three numerous varieties of SWM strategies that are reviewed.

- Composting
- Recovery and Recycling
- Incineration

1) COMPOSTING

This procedure is a biological function in which degrading organic waste is processed into manure by microorganisms, particularly mushrooms and microbes. It provides a soil-like quality with good carbon and nitrogen concentrations. Decomposition produces large, eco sustainable manure, which is a fantastic platform for growing crops and may also be use it for economic uses. For more detail see Fig. 1, 2.

2) RECOVERY AND RECYCLING

This strategy is a strategy of repurposing precious but abandoned objects. Plastic bottles, jars, glasses, and canisters are typically recycled immediately since they are likely to be restricted resources in many circumstances. For more detail see Fig. 3, 4.

3) INCINERATION

This method involves the increased combustion of sewage sludge until they are turned to dust. When smoldering trashes,

FIGURE 1. Composting. ***Source***: https://www.greenmatters.com/food/2018/12/07/ZboPlt/what-is-composting

FIGURE 2. Composting. ***Source***: https://www.ecofarmingdaily.com/build-soil/soil-inputs/compost/real-world-composting-making-life-death-cycle-work-operation/

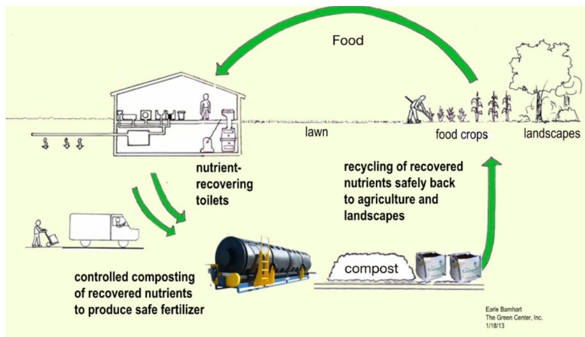

FIGURE 3. Recovery and Recycling. ***Source***: https://resource.temarry.com/blog/examples-of-resource-recovery-and-recycling

effluents are built in such a method that they do not really emit an amounts of energy. For more detail see Fig. 5, 6.

4) ALGORITHM

Consider $H \neq \Phi$ is the arrangement of choices under study, supplied by $H = \{x_1, x_2, \ldots, x_m\}$. Suppose $\mathcal{F} = A_1 \times A_2 \times \cdots \times A_n$, where $n \geq 1$ and A_i be sub attributes of a_i, $i = 1, 2, 3, \ldots, n$. The following are the development phases for the suggested CFHSS-based ENT, or see Fig. 11.

1) Each of the CFHSS should be specified.
2) Using the strategy, determine the ENT for each CFHSS $E(\psi, \mathcal{F}) = \frac{1}{2m}\Sigma_{l=1}^{m}[E_l^r(\psi, \mathcal{F}) + E_l^{\omega}\frac{(\psi,\mathcal{F})}{2\pi}]$, where $E_l^r(\psi, \mathcal{F}) = \frac{1}{n}\Sigma_{p=1}^{n}[1 - |\upsilon_{\psi(e_l)}(b_g) - \upsilon_{\psi^c(e_l)}(b_g)|]$, and $E_l^{\omega}(\psi, \mathcal{F}) = \frac{1}{n}\Sigma_{p=1}^{n}[1 - |\omega_{\psi(e_l)}(b_g) - \omega_{\psi^c(e_l)}(b_g)|]$.

FIGURE 4. **Recovery and Recycling.** ***Source*****: https://www.ecodepur.eu/company/circular-economy/circular-economy-reduction-reuse-recovery-recycling**

FIGURE 5. **Incineration.** ***Source*****: https://earth911.com/business-policy/how-incineration-works/**

FIGURE 6. **Incineration.** ***Source*****: https://ipen.org/news/australian-waste-export-ban-signals-green-light-dangerous-waste-incineration-industry/**

3) Find a CFHSS with the minimal possible of ENT and choose it for the optimum possible scenario.
4) If it obtained more than one maximum, choose any of them.

Example 2: The intricacy of SWM is one of the most key things in environmental research. Solid waste is defined as the unavoidable by-product of domestic, corporate, and organizational actions. SWM is amongst the most pertinent problems today, given the increasing population and rapid industrialization. Using the suggested strategy, the study showed the assessment criteria for sustainable SWM. Assume Covanta holding corporation CEO who have three waste management strategies (Composting, Recovery and Recycling, Incineration) and three decision makers $X = \{x, y, z\}$, let a_1 = Particle size distribution, a_2 = Moisture and organic matter content, a_3 = Geometry and classification of the waste be separate attributes with matching attribute values that are constituents of the collections Q_1, Q_2, Q_3. He wants to choose best optimal alternatives for SWM. Let $Q_1 = \{\eta_1 =$ Gravel $(> 2mm)$, $\eta_2 =$ Sand $(2 - .05mm)\}$, $Q_2 = \{\eta_3 = 10 - 45$ Percent $\}$, $Q_3 = \{\eta_4 =$ Liquid waste, $\eta_5 =$ Organic waste$\}$. With the help of a decision makers, CEO can encode these information in the form of CFHSS $(\psi, \mathcal{F})$, $(\varphi, \mathcal{F})$ and $(\chi, \mathcal{F})$ respectively, where $0 \leq \theta \leq 2\pi$.

1) This can be done with the support of public.

$$
\begin{aligned}
\text{Composting} = (\psi, \mathcal{F}) = \Big\{&\psi(\eta_1, \eta_3, \eta_4) \\
= \Big\{&\frac{(0.3e^{i0.4\theta})}{x}, \frac{(0.8e^{i0.2\theta})}{y}, \frac{(0.8e^{i0.2\theta})}{z}\Big\}, \\
&\Big\{\psi(\eta_1, \eta_3, \eta_5) = \Big\{\frac{(0.3e^{i0.3\theta})}{x}, \frac{(0.8e^{i0.2\theta})}{y}, \\
&\frac{(0.2e^{i0.9\theta})}{z}\Big\}, \Big\{\psi(\eta_2, \eta_3, \eta_4) = \Big\{\frac{(0.3e^{i0.9\theta})}{x}, \\
&\frac{(0.2e^{i0.4\theta})}{y}, \frac{(0.9e^{i0.3\theta})}{z}\Big\}\Big\{\psi(\eta_2, \eta_3, \eta_5) \\
= \Big\{&\frac{(0.3e^{i0.5\theta})}{x}, \frac{(0.3e^{i0.4\theta})}{y}, \frac{(0.4e^{i0.7\theta})}{z}\Big\}\Big\},
\end{aligned}
$$

$$
\begin{aligned}
\text{Recovery and Recycling} &= (\varphi, \mathcal{F}) \\
= \Big\{&\psi(\eta_1, \eta_3, \eta_4) = \Big\{\frac{(0.2e^{i0.7\theta})}{x}, \frac{(0.7e^{i0.5\theta})}{y}, \\
&\frac{(0.4e^{i0.3\theta})}{z}\Big\}, \Big\{\psi(\eta_1, \eta_3, \eta_5) = \Big\{\frac{(0.6e^{i0.9\theta})}{x}, \\
&\frac{(0.2e^{i0.8\theta})}{y}, \frac{(0.3e^{i0.6\theta})}{z}\Big\}, \Big\{\psi(\eta_2, \eta_3, \eta_4) \\
= \Big\{&\frac{(0.3e^{i0.9\theta})}{x}, \frac{(0.8e^{i0.5\theta})}{y}, \frac{(0.6e^{i0.9\theta})}{z}\Big\} \\
&\Big\{\psi(\eta_2, \eta_3, \eta_5) = \Big\{\frac{(0.6e^{i0.5\theta})}{x}, \\
&\quad \frac{(0.3e^{i0.4\theta})}{y}, \frac{(0.4e^{i0.7\theta})}{z}\Big\}\Big\},
\end{aligned}
$$

$$
\begin{aligned}
\text{Incineration} = (\chi, \mathcal{F}) = \Big\{&\psi(\eta_1, \eta_3, \eta_4) = \Big\{\frac{(0.4e^{i0.8\theta})}{x}, \\
&\frac{(0.2e^{i0.9\theta})}{y}, \frac{(0.3e^{i0.2\theta})}{z}\Big\}, \Big\{\psi(\eta_1, \eta_3, \eta_5)
\end{aligned}
$$

$$= \left\{ \frac{(0.7e^{i0.2\theta})}{x}, \frac{(0.5e^{i0.2\theta})}{y}, \frac{(0.2e^{i0.6\theta})}{z} \right\},$$
$$\left\{ \psi(\eta_2, \eta_3, \eta_4) = \left\{ \frac{(0.5e^{i0.9\theta})}{x}, \frac{(0.8e^{i0.6\theta})}{y}, \frac{(0.2e^{i0.9\theta})}{z} \right\} \right\} \left\{ \psi(\eta_2, \eta_3, \eta_5) = \left\{ \frac{(0.2e^{i0.5\theta})}{x}, \frac{(0.5e^{i0.4\theta})}{y}, \frac{(0.2e^{i0.5\theta})}{z} \right\} \right\}.$$

2) Calculate the Entropies of $(\psi, \mathcal{F})$, $(\varphi, \mathcal{F})$ and $(\chi, \mathcal{F})$ using the formula mention in algorithm, see Table 1.

TABLE 1. Entropies.

$E_1^{\tau}(\psi, \mathcal{F})$	0.46
$E_2^{\tau}(\psi, \mathcal{F})$	0.46
$E_3^{\tau}(\psi, \mathcal{F})$	0.4
$E_4^{\tau}(\psi, \mathcal{F})$	0.66
$E_1^{\omega}(\psi, \mathcal{F})$	0.53
$E_2^{\omega}(\psi, \mathcal{F})$	0.933
$E_3^{\omega}(\psi, \mathcal{F})$	1.066
$E_4^{\omega}(\psi, \mathcal{F})$	1.066
$E_1^{\tau}(\varphi, \mathcal{F})$	0.6
$E_2^{\tau}(\varphi, \mathcal{F})$	0.6
$E_3^{\tau}(\varphi, \mathcal{F})$	0.6
$E_4^{\tau}(\varphi, \mathcal{F})$	0.73
$E_1^{\omega}(\varphi, \mathcal{F})$	1
$E_2^{\omega}(\varphi, \mathcal{F})$	1.533
$E_3^{\omega}(\varphi, \mathcal{F})$	1.533
$E_4^{\omega}(\varphi, \mathcal{F})$	1.066
$E_1^{\tau}(\chi, \mathcal{F})$	0.6
$E_2^{\tau}(\chi, \mathcal{F})$	0.6
$E_3^{\tau}(\chi, \mathcal{F})$	0.6
$E_4^{\tau}(\chi, \mathcal{F})$	0.6
$E_1^{\omega}(\chi, \mathcal{F})$	1.26
$E_2^{\omega}(\chi, \mathcal{F})$	0.66
$E_3^{\omega}(\chi, \mathcal{F})$	1.6
$E_4^{\omega}(\chi, \mathcal{F})$	2.51

Hence the Entropies of the *CFSSs* $(\psi, \mathcal{F})$, $(\varphi, \mathcal{F})$ and $(\chi, \mathcal{F})$ are underlying as $E(\psi, \mathcal{F}) = 0.31$, $E(\varphi, \mathcal{F}) = 0.4148$, $E(\chi, \mathcal{F}) = 0.42$ respectively.

3) Optimal solution is to choose $((\psi, \mathcal{F})$ as it hs minimum value of ENT.
4) Composting is best SWM strategy.

All alternatives are ranked by ENT based CFHSS depicted in the following clustered cone 7.

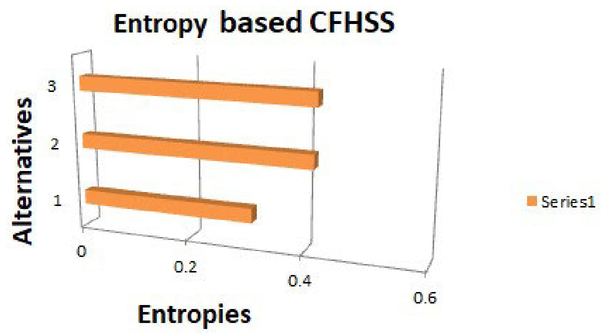

FIGURE 7. Ranking of alternative by ENT Based CFHSS.

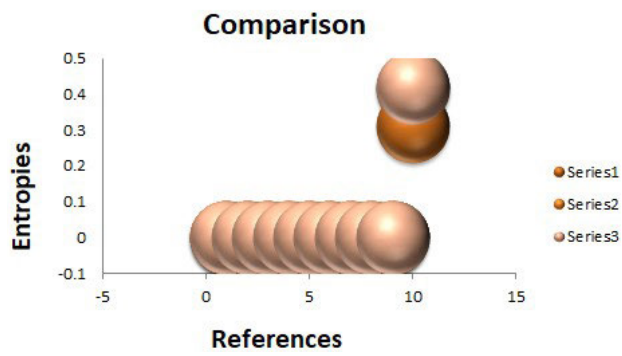

FIGURE 8. The envisioned ENT-based CFHSS is examined to established entropies.

C. COMPARATIVE STUDIES

A few evaluations of the initial methodologies with deficiencies are explored to determine the suggested methodology's reliability and supremacy. In addition, we will compare our proposed ENT-based CFHSS to nine different entropies already in use, Szmidt and Kacprzyk [50] offered a non-probabilistic-type ENT gauge for intuitionistic FS, Zhang *et al.* [29] delivered an ontological description of ENT, interaction, and the similarity measure for IVFSS set, Majumdar and Samanta [51] studied about ENT based on neutrosophic set, and Ye *et al.* [52] presented ENT measurements for interval valued neutrosophic set, and Aydodu *et al.* [53] focused on the ENT and similarity measure of interval-valued neutrosophic sets and the argument presented by Lvqing *et al.* [56] based on two classes of ENT measures for complex FS, and the suggestion proffered by Kumar *et al.* [57] determined on complex intuitionistic FSS with different algorithms and entropies, and also the indication presented by Athira *et al.* [55] based on ENT and distance measures of Pythagorean FSS and their implementations, and the suggestion, and the paradigm offered by Selvachandran *et al.* [58] for sophisticated, imprecise soft sets depending on an imprecise ENT measure when the features would be further split into attribute values and concerns that comprise two-dimensional content, all preceding restraints are abolished. The predicted ENT-based CFHSS will satisfy this necessity. For more detailed see Table 2, Fig. 8.

IV. SIMILARITY MEASURE AMONG CFHS-SETS

SM quantifies how similar distinct patterns, images, or combinations are. These sorts of indicators are often used in use of FSS. The following is a derivation of an SM for *CFHSS*.

TABLE 2. The envisioned ENT-based CFHSS is examined to existent entropies.

SN	References	Entropies	Ranking
1	[50]	Not valid	$\times$
2	[29]	Not valid	$\times$
3	[51]	Not valid	$\times$
4	[52]	Not valid	$\times$
5	[53]	Not valid	$\times$
6	[55]	Not valid	$\times$
7	[56]	Not valid	$\times$
8	[57]	Not valid	$\times$
9	[58]	Not valid	$\times$
10	Proposed Method in this paper	$E(\psi,\mathcal{F}) = 0.31$, $E(\varphi,\mathcal{F}) = 0.4148$, $E(\chi,\mathcal{F}) = 0.42$	$E(\psi,\mathcal{F}) \geq E(\varphi,\mathcal{F}) \geq E(\chi,\mathcal{F})$

Definition 8: A function S: $CFHSS(H) \times CFHSS(H) \to [0,1]$ is said to be SM between two $CFHSS$ $(\psi,\mathcal{F})$ and $(\varphi,\mathcal{F})$, if S satisfies the following axiomatic requirements

1) $S((\psi,\mathcal{F}),(\varphi,\mathcal{F})) = S((\varphi,\mathcal{F}),(\psi,\mathcal{F}))$,
2) $S((\psi,\mathcal{F}),(\varphi,\mathcal{F})) = 1 \Leftrightarrow (\psi,\mathcal{F}) = (\varphi,\mathcal{F})$,
3) $S((\psi,\mathcal{F}),(\varphi,\mathcal{F})) = 0 \Leftrightarrow \forall\ e \in \mathcal{F}, x \in H$, the following restrictions are fulfilled $\upsilon_{\psi(e)} = 1, \upsilon_{\varphi(e)} = 0$ or $\upsilon_{\psi(e)} = 0, \upsilon_{\varphi(e)} = 1$ and $\omega_{\psi(e)} = 2\pi, \omega_{\varphi(e)} = 0$ or $\omega_{\psi(e)} = 0, \omega_{\varphi(e)} = 2\pi$,
4) $\forall\ (\psi,\mathcal{F}),(\varphi,\mathcal{F})$ and $(\chi,\mathcal{F}) \in CFHSS$, if $(\psi,\mathcal{F}) \subseteq (\varphi,\mathcal{F}) \subseteq (\chi,\mathcal{F})$, then $S((\psi,\mathcal{F}),(\chi,\mathcal{F})) \leq S((\psi,\mathcal{F}),(\varphi,\mathcal{F}))$ and $S((\psi,\mathcal{F}),(\chi,\mathcal{F})) \leq S((\varphi,\mathcal{F}),(\chi,\mathcal{F}))$. The following is the formula for computing the SM among two $CFHSS$.

Theorem 2: Let $H = \{b_1, b_2, \ldots, b_g\}$ be the gathering of alternatives and $\mathcal{F}$ is the set of parameters.

$(\psi,\mathcal{F}) = \{\mathcal{F}(e) = \upsilon_{\psi(e)}(b).e^{i\omega_{\psi(e)}}(b)|\}$, and $(\varphi,\mathcal{F}) = \{\mathcal{F}(e) = \upsilon_{\varphi(e)}(b).e^{i\omega_{\varphi(e)}}(b)|\}$, are two families of $CFHSS$.

Define $S((\psi,\mathcal{F}),(\varphi,\mathcal{F}))$ as follows,

$$S((\psi,\mathcal{F}),(\varphi,\mathcal{F})) = \frac{1}{2m}\Sigma_{l=1}^{m}[S_l^r((\psi,\mathcal{F}),(\varphi,\mathcal{F})) + \frac{S_l^\omega((\psi,\mathcal{F}),(\varphi,\mathcal{F}))}{2\pi}], \tag{4}$$

where,

$$S_{l=1}^r((\psi,\mathcal{F}),(\varphi,\mathcal{F})) = 1 - \frac{1}{n}\Sigma_{l=1}^{n} max\{(|\upsilon_{\psi(e)}(b_g) - r_{\varphi(e)}(b_g)|)\}, \tag{5}$$

and

$$S_{l=1}^\omega((\psi,\mathcal{F}),(\varphi,\mathcal{F})) = 2\pi - \frac{1}{n}\Sigma_{l=1}^{n} max\{(|\omega_{\psi(e)}(b_g) - \omega_{\varphi(e)}(b_g)|)\}. \tag{6}$$

then $S((\psi,\mathcal{F}),(\varphi,\mathcal{F}))$ is a SM between two $CFHSS$ $(\psi,\mathcal{F})$ and $(\varphi,\mathcal{F})$.

Proof: It is necessary to show that $S((\psi,\mathcal{F}),(\varphi,\mathcal{F}))$ fulfill the properties listed in definition 8.

1) For $S_{l=1}^r((\psi,\mathcal{F}),(\varphi,\mathcal{F})) = 1 - \frac{1}{n}\Sigma_{p=1}^n max\{(|\upsilon_{\psi(e_l)}(b_g) - \upsilon_{\varphi(e_l)}(b_g)|)\}, = 1 - \frac{1}{n}\Sigma_{p=1}^n max\{(|\upsilon_{\varphi(e_l)}(b_g) - \upsilon_{\psi(e_l)}(b_g)|)\} = S_{l=1}^r((\varphi,\mathcal{F}),(\psi,\mathcal{F}))$, and $S_{l=1}^\omega((\psi,\mathcal{F}),(\varphi,\mathcal{F})) = 2\pi - \frac{1}{n}\Sigma_{p=1}^n max\{(|\omega_{\psi(e_l)}(b_g) - \omega_{\varphi(e_l)}(b_g)|)\}, = 2\pi - \frac{1}{n}\Sigma_{p=1}^n max\{(|\omega_{\varphi(e_l)}(b_g) - \omega_{\psi(e_l)}(b_g)|)\} = S_{l=1}^\omega((\varphi,\mathcal{F}),(\psi,\mathcal{F}))$, So we have $S((\psi,\mathcal{F}),(\varphi,\mathcal{F})) = \frac{1}{2m}\Sigma_{l=1}^m[S_l^r((\psi,\mathcal{F}),(\varphi,\mathcal{F})) + \frac{S_l^\omega((\psi,\mathcal{F}),(\varphi,\mathcal{F}))}{2\pi}], = \frac{1}{2m}\Sigma_{l=1}^m[S_l^r((\varphi,\mathcal{F}),(\psi,\mathcal{F})) + \frac{S_l^\omega((\varphi,\mathcal{F}),(\psi,\mathcal{F}))}{2\pi}] = S((\varphi,\mathcal{F}),(\psi,\mathcal{F}))$.
2) $S((\psi,\mathcal{F}),(\varphi,\mathcal{F})) = 1 \Leftrightarrow \frac{1}{2m}\Sigma_{l=1}^m[S_l^r((\psi,\mathcal{F}),(\varphi,\mathcal{F})) + \frac{S_l^\omega((\psi,\mathcal{F}),(\varphi,\mathcal{F}))}{2\pi}] = 1, \Leftrightarrow S_l^r((\psi,\mathcal{F}),(\varphi,\mathcal{F})) = 1, \Leftrightarrow S_l^\omega((\psi,\mathcal{F}),(\varphi,\mathcal{F})) = 2\pi, \Leftrightarrow S_{l=1}^r((\psi,\mathcal{F}),(\varphi,\mathcal{F})) = 1 - \frac{1}{n}\Sigma_{p=1}^n max\{(|\upsilon_{\psi(e_l)}(b_g) - \upsilon_{\varphi(e_l)}(b_g)|)\}, 2\pi - \frac{1}{n}\Sigma_{p=1}^n max\{(|\omega_{\psi(e_l)}(b_g) - \omega_{\varphi(e_l)}(b_g)|)\} = 2\pi, \forall\, e_l \in \mathcal{F}, b \in H, \Leftrightarrow \frac{1}{n}\Sigma_{p=1}^n max\{(|\upsilon_{\psi(e_l)}(b_g) - \upsilon_{\varphi(e_l)}(b_g)|) = 0$, and $\Leftrightarrow \frac{1}{n}\Sigma_{p=1}^n max\{(|\omega_{\psi(e_l)}(b_g) - \omega_{\varphi(e_l)}(b_g)|) = 0, \forall\ e_l \in \mathcal{F}, b \in H, \Leftrightarrow \Sigma_{p=1}^n max\{(|\upsilon_{\psi(e_l)}(b_g) - \upsilon_{\varphi(e_l)}(b_g)|) = 0, \Leftrightarrow \Sigma_{p=1}^n max\{(|\omega_{\psi(e_l)}(b_g) - \omega_{\varphi(e_l)}(b_g)|) = 0, \forall\, e_l \in \mathcal{F}, b \in H, \Leftrightarrow \upsilon_{\psi(e_l)}(b_g) = \upsilon_{\varphi(e_l)}(b_g), \Leftrightarrow \omega_{\psi(e_l)}(b_g) = \omega_{\varphi(e_l)}(b_g), \forall\ e_l \in \mathcal{F}, b \in H, \Leftrightarrow (\psi,\mathcal{F}) = (\varphi,\mathcal{F})$.
3) $S((\psi,\mathcal{F}),(\varphi,\mathcal{F})) = 0, \frac{1}{2m}\Sigma_{l=1}^m[S_l^r((\psi,\mathcal{F}),(\varphi,\mathcal{F})) + \frac{S_l^\omega((\psi,\mathcal{F}),(\varphi,\mathcal{F}))}{2\pi}] = 0, \Leftrightarrow S_l^r((\psi,\mathcal{F}),(\varphi,\mathcal{F})) = 0$, and $\Leftrightarrow S_l^\omega((\psi,\mathcal{F}),(\varphi,\mathcal{F})) = 0, \Leftrightarrow 1 - \frac{1}{n}\Sigma_{p=1}^n max\{(|\upsilon_{\psi(e_l)}(b_g) - \upsilon_{\varphi(e_l)}(b_g)|)\} = 0$, and $\Leftrightarrow 2\pi - \frac{1}{n}\Sigma_{p=1}^n max\{(|\omega_{\psi(e_l)}(b_g) - \omega_{\varphi(e_l)}(b_g)|)\} = 0, \forall\ e_l \in \mathcal{F}, b \in H, \Leftrightarrow \frac{1}{n}\Sigma_{p=1}^n max\{(|\upsilon_{\psi(e_l)}(b_g) - \upsilon_{\varphi(e_l)}(b_g)|)\} = 1, \Leftrightarrow \frac{1}{n}\Sigma_{p=1}^n max\{(|\omega_{\psi(e_l)}(b_g) - \omega_{\varphi(e_l)}(b_g)|)\} = 2\pi, \forall\, e_l \in \mathcal{F}, b \in H, \Leftrightarrow max\{(|\upsilon_{\psi(e_l)}(b_g) - \upsilon_{\varphi(e_l)}(b_g)|)\} = 1$, and $\Leftrightarrow max\{(|\omega_{\psi(e_l)}(b_g) - \omega_{\varphi(e_l)}(b_g)|)\} = 2\pi, \forall\ e_l \in \mathcal{F}, b \in H, \Leftrightarrow \upsilon_{\psi(e_l)} = 0, \upsilon_{\varphi(e_l)} = 1, \upsilon_{\psi(e_l)} = 1, \upsilon_{\varphi(e_l)} = 0$ and $\omega_{\psi(e_l)} = 0, \omega_{\varphi(e_l)} = 2\pi$ or $\omega_{\psi(e_l)} = 2\pi, \omega_{\varphi(e_l)} = 0$.
4) $(\psi,\mathcal{F}) \subseteq (\varphi,\mathcal{F}) \subseteq (\chi,\mathcal{F}), \Rightarrow \upsilon_{\psi(e_l)}(b_g) \leq \upsilon_{\varphi(e_l)}(b_g) \leq \upsilon_{\chi(e_l)}(b_g)$ and $\omega_{\psi(e_l)}(b_g) \leq \omega_{\varphi(e_l)}(b_g) \leq \omega_{\chi(e_l)}(b_g), \forall\ e_l \in \mathcal{F}, b \in H, \Rightarrow |\upsilon_{\psi(e_l)}(b_g) - \upsilon_{\chi(e_l)}(b_g)| \leq |\upsilon_{\psi(e_l)}(b_g) - r_{\varphi(e_l)}(b_g)|$, and $\Rightarrow |\omega_{\psi(e_l)}(b_g) - \omega_{\chi(e_l)}(b_g)| \leq |\omega_{\psi(e_l)}(b_g) - \omega_{\varphi(e_l)}(b_g)|, \forall\ e_l \in \mathcal{F}, b \in H, \Leftrightarrow 1 - \frac{1}{n}\Sigma_{p=1}^n max\{(|\upsilon_{\psi(e_l)}(b_g) - \upsilon_{\chi(e_l)}(b_g)|)\} \leq 1 - \frac{1}{n}\Sigma_{p=1}^n max\{(|\upsilon_{\psi(e_l)}(b_g) - \upsilon_{\varphi(e_l)}$

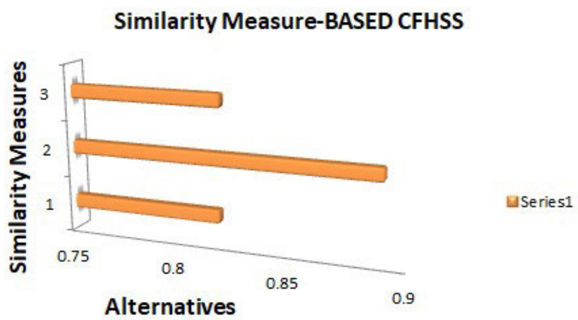

FIGURE 9. Ranking of alternative by SM Based CFHSS.

$(b_g)|)\}$, $\Leftrightarrow$ $2\pi - \frac{1}{n}\Sigma_{p=1}^{n} max\{(|\omega_{\psi(e_l)}(b_g) - \omega_{\chi(e_l)}(b_g)|)\} \leq 2\pi - \frac{1}{n}\Sigma_{p=1}^{n} max\{(|\omega_{\psi(e_l)}(b_g) - \omega_{\varphi(e_l)}(b_g)|)\}$, $\Rightarrow$ $S_{l=1}^{r}((\psi,\mathcal{F}),(\chi,\mathcal{F})) \leq S_{l=1}^{r}((\psi,\mathcal{F}),(\varphi,\mathcal{F}))$, and $\Rightarrow S_{l=1}^{\omega}((\psi,\mathcal{F}),(\chi,\mathcal{F})) \leq S_{l=1}^{\omega}((\psi,\mathcal{F}),(\varphi,\mathcal{F}))$, $\Rightarrow$ $S_{l=1}^{r}((\psi,\mathcal{F}),(\chi,\mathcal{F})) + S_{l=1}^{\omega}((\psi,\mathcal{F}),(\chi,\mathcal{F})) \leq S_{l=1}^{r}((\psi,\mathcal{F}),(\varphi,\mathcal{F})) + S_{l=1}^{\omega}((\psi,\mathcal{F}),(\varphi,\mathcal{F}))$, $\Rightarrow$ $\frac{1}{2m}\Sigma_{l=1}^{m}[S_l^{r}((\psi,\mathcal{F}),(\chi,\mathcal{F})) + \frac{S_l^{\omega}((\psi,\mathcal{F}),(\chi,\mathcal{F}))}{2\pi}] \leq \frac{1}{2m}\Sigma_{l=1}^{m}[S_l^{r}((\psi,\mathcal{F}),(\varphi,\mathcal{F})) + \frac{S_l^{\omega}((\psi,\mathcal{F}),(\varphi,\mathcal{F}))}{2\pi}]$, $S((\psi,\mathcal{F}),(\chi,\mathcal{F})) \leq S((\psi,\mathcal{F}),(\varphi,\mathcal{F}))$.

A. DEPLOYMENT OF PROPOSED SM-BASED CFHSS

In this part, we use the paradigm of CFHSS to establish a new mechanism and tactic called SM-based CFHSS, wherein we broaden SM utilizing CFHSS in a fuzzy situation. Additionally, a practical choice issue is established to show the validity and necessity of the recently developed ENT-based CFHSS.

1) ALGORITHM

Assume $H \neq \Phi$ be the universe indicated by $H = \{b_1, b_2, \ldots, b_m\}$. Let $\mathcal{F} = A_1 \times A_2 \times \cdots \times A_n$, where $n \geq 1$ and Ai is the collection of all attribute of the feature a_i, $i = 1, 2, 3, \ldots, n$. The following are the design methods for the proposed CFHSS-based similarity or see Fig. 12:

1) Input each of the CFHSS.
2) By using formula, determine the similarity measure for each CFHSS, $S((\psi,\mathcal{F}),(\varphi,\mathcal{F})) = \frac{1}{2m}\Sigma_{l=1}^{m}[S_l^{r}((\psi,\mathcal{F}),(\varphi,\mathcal{F})) + \frac{S_l^{\omega}((\psi,\mathcal{F}),(\varphi,\mathcal{F}))}{2\pi}]$, where $S_{l=1}^{r}((\psi,\mathcal{F}),(\varphi,\mathcal{F})) = 1 - \frac{1}{n}\Sigma_{l=1}^{n} max\{(|\upsilon_{\psi(e)}(b_g) - r_{\varphi(e)}(b_g)|)\}$, and $S_{l=1}^{\omega}((\psi,\mathcal{F}),(\varphi,\mathcal{F})) = 2\pi - \frac{1}{n}\Sigma_{l=1}^{n} max\{(|\omega_{\psi(e)}(b_g) - \omega_{\varphi(e)}(b_g)|)\}$.
3) Choose the CFHSS that bears the most similarities.
4) If it scored more than one optimum, simply pick one.

Example 3: From Example 2,

1) Our objective is to determine the optimum SWM tactics based on established of factors. The following tables have included CFHSS paradigm.

Recovery and Recycling $= (\varphi, \mathcal{F})$

$$= \left\{\varphi(\eta_1,\eta_3,\eta_4) = \left\{\frac{(0.2e^{i0.7\theta})}{x}, \frac{(0.7e^{i0.5\theta})}{y}, \frac{(0.4e^{i0.3\theta})}{z}\right\}, \varphi(\eta_1,\eta_3,\eta_5) = \left\{\frac{(0.6e^{i0.9\theta})}{x}, \frac{(0.2e^{i0.8\theta})}{y}, \frac{(0.3e^{i0.6\theta})}{z}\right\}, \varphi(\eta_2,\eta_3,\eta_4) = \left\{\frac{(0.3e^{i0.7\theta})}{x}, \frac{(0.9e^{i0.2\theta})}{y}, \frac{(0.3e^{i0.6\theta})}{z}\right\}, \varphi(\eta_2,\eta_3,\eta_5) = \left\{\frac{(0.6e^{i0.9\theta})}{x}, \frac{(0.6e^{i0.8\theta})}{y}, \frac{(0.4e^{i0.7\theta})}{z}\right\},\right\},$$

$$\text{Composting} = (\chi,\mathcal{F}) = \left\{\chi(\eta_1,\eta_3,\eta_4) = \left\{\frac{(0.2e^{i0.7\theta})}{x}, \frac{(0.3e^{i0.9\theta})}{y}, \frac{(0.2e^{i0.5\theta})}{z}\right\}, \chi(\eta_1,\eta_3,\eta_5) = \left\{\frac{(0.7e^{i0.9\theta})}{x}, \frac{(0.1e^{i0.7\theta})}{y}, \frac{(0.1e^{i0.6\theta})}{z}\right\}, \chi(\eta_2,\eta_3,\eta_4) = \left\{\frac{(0.8e^{i0.9\theta})}{x}, \frac{(0.8e^{i0.8\theta})}{y}, \frac{(0.2e^{i0.6\theta})}{z}\right\}, \chi(\eta_2,\eta_3,\eta_5) = \left\{\frac{(0.6e^{i0.9\theta})}{x}, \frac{(0.2e^{i0.8\theta})}{y}, \frac{(0.3e^{i0.6\theta})}{z}\right\},\right\},$$

$$\text{Incineration} = (\mu,\mathcal{F}) = \left\{\mu(\eta_1,\eta_3,\eta_4) = \left\{\frac{(0.2e^{i0.7\theta})}{x},\right.\right.$$

TABLE 3. Similarity measures.

$S_{l=1}^{r}((\psi,\mathcal{F}),(\varphi,\mathcal{F}))$	0.8
$S_{l=2}^{r}((\psi,\mathcal{F}),(\varphi,\mathcal{F}))$	0.66
$S_{l=3}^{r}((\psi,\mathcal{F}),(\varphi,\mathcal{F}))$	0.63
$S_{l=4}^{r}((\psi,\mathcal{F}),(\varphi,\mathcal{F}))$	0.73
$S_{l=1}^{\omega}((\psi,\mathcal{F}),(\varphi,\mathcal{F}))$	6.01
$S_{l=2}^{\omega}((\psi,\mathcal{F}),(\varphi,\mathcal{F}))$	5.78
$S_{l=3}^{\omega}((\psi,\mathcal{F}),(\varphi,\mathcal{F}))$	6.08
$S_{l=4}^{\omega}((\psi,\mathcal{F}),(\varphi,\mathcal{F}))$	5.74
$S_{l=1}^{r}((\psi,\mathcal{F}),(\chi,\mathcal{F}))$	0.8
$S_{l=2}^{r}((\psi,\mathcal{F}),(\chi,\mathcal{F}))$	0.86
$S_{l=3}^{r}((\psi,\mathcal{F}),(\chi,\mathcal{F}))$	0.76
$S_{l=4}^{r}((\psi,\mathcal{F}),(\chi,\mathcal{F}))$	0.83
$S_{l=1}^{\omega}((\psi,\mathcal{F}),(\chi,\mathcal{F}))$	6.08
$S_{l=2}^{\omega}((\psi,\mathcal{F}),(\chi,\mathcal{F}))$	6.24
$S_{l=3}^{\omega}((\psi,\mathcal{F}),(\chi,\mathcal{F}))$	6.01
$S_{l=4}^{\omega}((\psi,\mathcal{F}),(\chi,\mathcal{F}))$	6.24
$S_{l=1}^{r}((\psi,\mathcal{F}),(\mu,\mathcal{F}))$	0.73
$S_{l=2}^{r}((\psi,\mathcal{F}),(\mu,\mathcal{F}))$	0.7
$S_{l=3}^{r}((\psi,\mathcal{F}),(\mu,\mathcal{F}))$	0.83
$S_{l=4}^{r}((\psi,\mathcal{F}),(\mu,\mathcal{F}))$	0.56
$S_{l=1}^{\omega}((\psi,\mathcal{F}),(\mu,\mathcal{F}))$	5.91
$S_{l=2}^{\omega}((\psi,\mathcal{F}),(\mu,\mathcal{F}))$	5.78
$S_{l=3}^{\omega}((\psi,\mathcal{F}),(\mu,\mathcal{F}))$	6.013
$S_{l=4}^{\omega}((\psi,\mathcal{F}),(\mu,\mathcal{F}))$	5.94

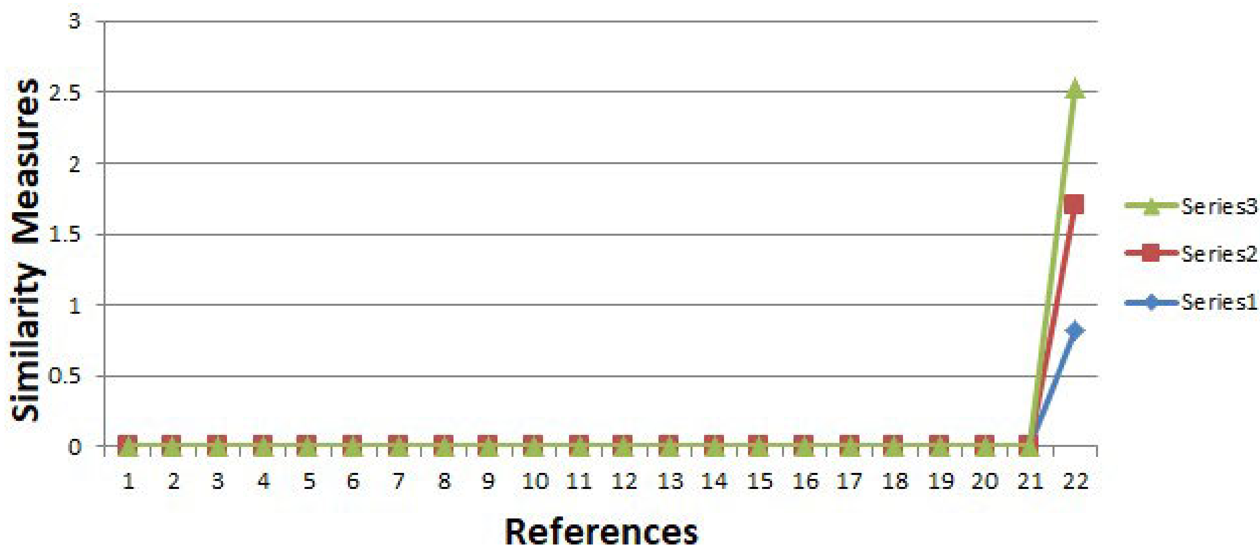

FIGURE 10. Comparison of the proposed SM based CFHSS with existing Similarity measure.

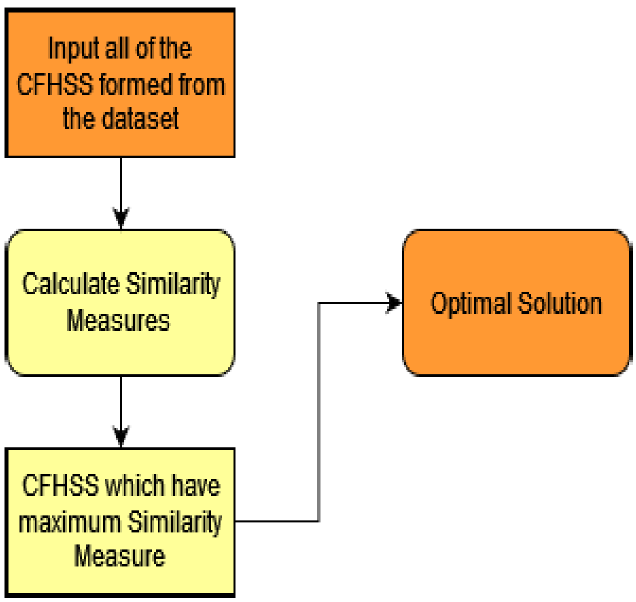

FIGURE 11. Construction steps for the proposed CFHSS-based ENT.

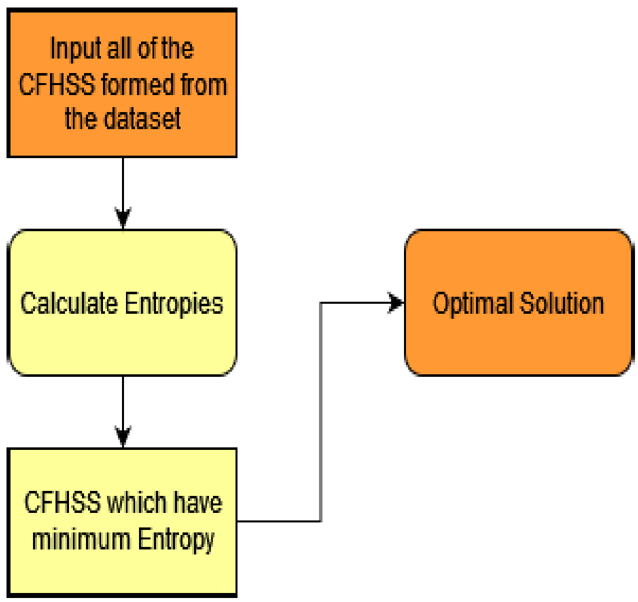

FIGURE 12. Construction steps for the proposed CFHSS-based SM.

$$\frac{(0.2e^{i0.6\theta})}{y}, \frac{(0.7e^{i0.6\theta})}{z}\Bigg\}, \mu(\eta_1, \eta_3, \eta_5) = \Bigg\{\frac{(0.6e^{i0.9\theta})}{x},$$
$$\frac{(0.2e^{i0.8\theta})}{y}, \frac{(0.2e^{i0.6\theta})}{z}\Bigg\}, \mu(\eta_2, \eta_3, \eta_4) = \Bigg\{\frac{(0.6e^{i0.9\theta})}{x},$$
$$\frac{(0.2e^{i0.8\theta})}{y}, \frac{(0.3e^{i0.6\theta})}{z}\Bigg\}, \mu(\eta_2, \eta_3, \eta_5) = \Bigg\{\frac{(0.3e^{i0.4\theta})}{x},$$
$$\frac{(0.2e^{i0.8\theta})}{y}, \frac{(0.3e^{i0.6\theta})}{z}\Bigg\}, \Bigg\},$$

and ideal SWM strategy in the form of CFHSS is

$$(\psi, \mathcal{F}) = \Bigg\{\psi(\eta_1, \eta_3, \eta_4) = \Bigg\{\frac{(0.3e^{i0.4\theta})}{x}, \frac{(0.8e^{i0.2\theta})}{y},$$
$$\frac{(0.8e^{i0.2\theta})}{z}\Bigg\}, \psi(\eta_1, \eta_3, \eta_5) = \Bigg\{\frac{(0.3e^{i0.3\theta})}{x},$$
$$\frac{(0.8e^{i0.2\theta})}{y}, \frac{(0.2e^{i0.9\theta})}{z}\Bigg\}, \psi(\eta_2, \eta_3, \eta_4) = \Bigg\{\frac{(0.4e^{i0.8\theta})}{x},$$
$$\frac{(0.2e^{i0.4\theta})}{y}, \frac{(0.6e^{i0.9\theta})}{z}\Bigg\}, \psi(\eta_2, \eta_3, \eta_5) = \Bigg\{\frac{(0.3e^{i0.3\theta})}{x},$$
$$\frac{(0.8e^{i0.2\theta})}{y}, \frac{(0.7e^{i0.3\theta})}{z}\Bigg\}, \Bigg\},$$

2) Calculate the SM of $(\psi, \mathcal{F})$, $(\varphi, \mathcal{F})$ and $(\chi, \mathcal{F})$ using the formula mention in algorithm in Step (2), see Table 3.

TABLE 4. Comparison of the proposed similarity measure based CFHSS with existing SM.

SN	References	SM	Ranking
1	[38]	Not valid	$\times$
2	[39]	Not valid	$\times$
3	[40]	Not valid	$\times$
4	[59]	Not valid	$\times$
5	[59]	Not valid	$\times$
6	[59]	Not valid	$\times$
7	[41]	Not valid	$\times$
8	[42]	Not valid	$\times$
9	[43]	Not valid	$\times$
10	[44]	Not valid	$\times$
11	[44]	Not valid	$\times$
12	[44]	Not valid	$\times$
13	[45]	Not valid	$\times$
14	[52]	Not valid	$\times$
15	[60]	Not valid	$\times$
16	[46]	Not valid	$\times$
17	[47]	Not valid	$\times$
18	[47]	Not valid	$\times$
19	[47]	Not valid	$\times$
20	[48]	Not valid	$\times$
21	[49]	Not valid	$\times$
22	Proposed Method in this paper	$S_1 = 0.82$, $S_2 = 0.89$, $S_3 = 0.82$.	$S_2 \geq S_1 \geq S_3$

TABLE 5. A comparison of the anticipated CFHSS relying on a similarity measure to present SM.

SN	References	2D information	Sub-parameter	SM	Ranking
1	[38]	$\times$	$\times$	Not valid	$\times$
2	[39]	$\times$	$\times$	Not valid	$\times$
3	[40]	$\times$	$\times$	Not valid	$\times$
4	[59]	$\times$	$\times$	Not valid	$\times$
5	[41]	$\times$	$\times$	Not valid	$\times$
6	[42]	$\times$	$\times$	Not valid	$\times$
7	[43]	$\times$	$\times$	Not valid	$\times$
8	[44]	$\times$	$\times$	Not valid	$\times$
9	[45]	$\times$	$\times$	Not valid	$\times$
10	[52]	$\times$	$\times$	Not valid	$\times$
11	[60]	$\times$	$\times$	Not valid	$\times$
12	[46]	$\times$	$\times$	Not valid	$\times$
13	[47]	$\times$	$\times$	Not valid	$\times$
14	[48]	$\times$	$\times$	Not valid	$\times$
15	[49]	$\times$	$\times$	Not valid	$\times$
16	[57]	$\checkmark$	$\times$	Not valid	$\times$
17	[56]	$\checkmark$	$\times$	Not valid	$\times$
18	[33]	$\checkmark$	$\times$	Not valid	$\times$
19	[32]	$\checkmark$	$\times$	Not valid	$\times$
20	[35]	$\times$	$\checkmark$	Not valid	$\times$
21	Proposed Method	$\checkmark$	$\checkmark$	$S_1 = 0.35$, $S_2 = 0.40$, $S_3 = 0.35$	$S_2 \geq S_1 \geq S_3$

Hence the degree of similarity between $(\psi, \mathcal{F})$ and $(\varphi, \mathcal{F})$, $(\chi, \mathcal{F})$, $(\mu, \mathcal{F})$ respectively is given by $S_1 = S((\psi, \mathcal{F}), (\varphi, \mathcal{F})) = 0.82$, $S_2 = S((\psi, \mathcal{F}), (\chi, \mathcal{F})) = 0.89$, $S_3 = S((\psi, \mathcal{F}), (\mu, \mathcal{F})) = 0.82$.

3) Thus, $(\chi, \mathcal{F})$ have highest similarity measure so composting is most optimal SWM strategy.

The association of the suggested measures with current measures supplied by Li *et al.* is illustrated in Table 4, Fig. 9, and 10, Chen [39], Chen *et al.* [40], Hung *et al.* [59], Hong *et al.* [41], Dengfeng [42], Li *et al.* [43], Liang *et al.* [44], Mitchell [45], Ye [52], Wei [60], Zhang [46], Peng *et al.* [47], Boran *et al.* [48] and Begam *et al.* [49].

B. THE CFHSS's FEATURES AND A COMPARATIVE ANALYSIS

A few assessments of the launched procedures with shortcomings are described in the following portions to test the validity and predominance of the recommended tactics. Additionally, the intended SM is compared to other current measures and is found to have two disadvantages, which are explained with an example, comprising the ideas provided in the previous sections. However, all existing flaws fail to answer challenges with 2D data/information, i.e., two different forms of data/information relative to the relevant parameters. With the assistance of scenario 3, the equivalence of

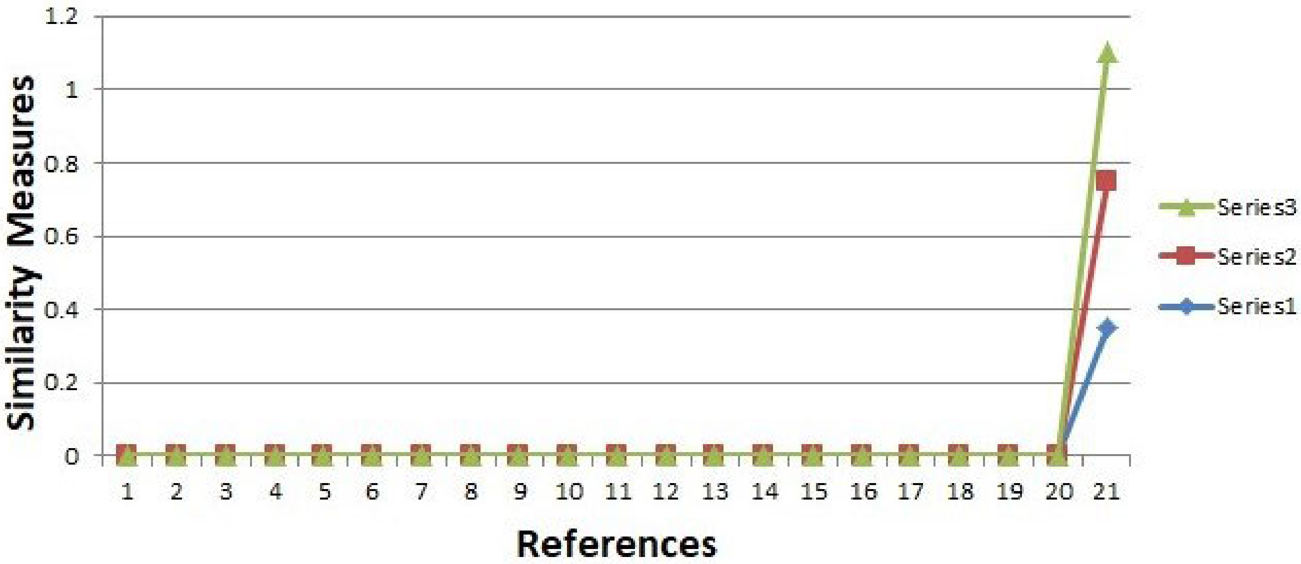

FIGURE 13. The anticipated SM-based CFHSS is linked to present SM.

suggested approaches is illustrated; see the results in Table 5 and Fig. 13.

Example 4: Consider 3 if we only have one-dimensional data

$$(\varphi,\mathcal{F})=\Big\{\varphi(\eta_1,\eta_3,\eta_4)=\Big\{\frac{0.2e^{i2\theta(0.0)}}{x},\frac{0.7e^{i2\theta(0.0)}}{y},\frac{0.4e^{i2\theta(0.0)}}{z}\Big\},$$
$$\varphi(\eta_1,\eta_3,\eta_5)=\Big\{\frac{0.6e^{i2\theta(0.0)}}{x},\frac{0.2e^{i2\theta(0.0)}}{y},\frac{0.3e^{i2\theta(0.0)}}{z}\Big\},$$
$$\varphi(\eta_2,\eta_3,\eta_4)=\Big\{\frac{0.3e^{i2\theta(0.0)}}{x},\frac{0.9e^{i2\theta(0.0)}}{y},\frac{0.3e^{i2\theta(0.0)}}{z}\Big\},$$
$$\varphi(\eta_2,\eta_3,\eta_5)=\Big\{\frac{0.6e^{i2\theta(0.0)}}{x},\frac{0.6e^{i2\theta(0.0)}}{y},\frac{0.4e^{i2\theta(0.0)}}{z}\Big\}\Big\},$$
$$(\chi,\mathcal{F})=\Big\{\chi(\eta_1,\eta_3,\eta_4)=\Big\{\frac{0.2e^{i2\theta(0.0)}}{x},\frac{0.3e^{i2\theta(0.0)}}{y},\frac{0.2e^{i2\theta(0.0)}}{z}\Big\},$$
$$\chi(\eta_1,\eta_3,\eta_5)=\Big\{\frac{0.7e^{i2\theta(0.0)}}{x},\frac{0.1e^{i2\theta(0.0)}}{y},\frac{0.1e^{i2\theta(0.0)}}{z}\Big\},$$
$$\chi(\eta_2,\eta_3,\eta_4)=\Big\{\frac{0.8e^{i2\theta(0.0)}}{x},\frac{0.8e^{i2\theta(0.0)}}{y},\frac{0.2e^{i2\theta(0.0)}}{z}\Big\},$$
$$\chi(\eta_2,\eta_3,\eta_5)=\Big\{\frac{0.6e^{i2\theta(0.0)}}{x},\frac{0.2e^{i2\theta(0.0)}}{y},\frac{0.3e^{i2\theta(0.0)}}{z}\Big\}\Big\},$$
$$(\mu,\mathcal{F})=\Big\{\mu(\eta_1,\eta_3,\eta_4)=\Big\{\frac{0.2e^{i2\theta(0.0)}}{x},\frac{0.2e^{i2\theta(0.0)}}{y},\frac{0.7e^{i2\theta(0.0)}}{z}\Big\},$$
$$\mu(\eta_1,\eta_3,\eta_5)=\Big\{\frac{0.6e^{i2\theta(0.0)}}{x},\frac{0.2e^{i2\theta(0.0)}}{y},\frac{0.2e^{i2\theta(0.0)}}{z}\Big\},$$
$$\mu(\eta_2,\eta_3,\eta_4)=\Big\{\frac{0.6e^{i2\theta(0.0)}}{x},\frac{0.2e^{i2\theta(0.0)}}{y},\frac{0.3e^{i2\theta(0.0)}}{z}\Big\},$$
$$\mu(\eta_2,\eta_3,\eta_5)=\Big\{\frac{0.3e^{i2\theta(0.0)}}{x},\frac{0.2e^{i2\theta(0.0)}}{y},\frac{0.3e^{i2\theta(0.0)}}{z}\Big\}\Big\},$$

and ideal CFHSS are

$$(\psi,\mathcal{F})=\Big\{\psi(\eta_1,\eta_3,\eta_4)=\Big\{\frac{0.3e^{i2\theta(0.0)}}{x},\frac{0.8e^{i2\theta(0.0)}}{y},\frac{0.8e^{i2\theta(0.0)}}{z}\Big\},$$
$$\psi(\eta_1,\eta_3,\eta_5)=\Big\{\frac{0.3e^{i2\theta(0.0)}}{x},\frac{0.8e^{i2\theta(0.0)}}{y},\frac{0.2e^{i2\theta(0.0)}}{z}\Big\},$$
$$\psi(\eta_2,\eta_3,\eta_4)=\Big\{\frac{0.4e^{i2\theta(0.0)}}{x},\frac{0.2e^{i2\theta(0.0)}}{y},\frac{0.6e^{i2\theta(0.0)}}{z}\Big\},$$
$$\psi(\eta_2,\eta_3,\eta_5)=\Big\{\frac{0.3e^{i2\theta(0.0)}}{x},\frac{0.8e^{i2\theta(0.0)}}{y},\frac{0.7e^{i2\theta(0.0)}}{z}\Big\}\Big\},$$
$$S_1=S((\psi,\mathcal{F}),(\varphi,\mathcal{F}))=0.35,$$
$$S_2=S((\psi,\mathcal{F}),(\chi,\mathcal{F}))=0.40,$$
$$S_3=S((\psi,\mathcal{F}),(\mu,\mathcal{F}))=0.35.$$

C. SENSITIVITY ANALYSIS

1) The intended CFHSS was transformed to a fuzzy hypersoft set by omitting the imaginary elements [35].
2) By removing the imaginary elements and $Q_1=Q_2=Q_3\ldots=Q_n$, the suggested CFHSS was then trimmed to a fuzzy soft set [20].
3) When $Q_1=Q_2=Q_3\ldots=Q_n$, then the proposed CFHSS reduced to CFSS [33].

The recommended CFHSS-based strategies are more prominent and comprehensive than previous strategies, as shown in [22], [33]. We are now exploring establishing a more leading conceptual base for comparative measures, with intentions to broaden this to other types of SM in the future. We are encouraged by [33] and organize to enlarge our exploration to other assertions of CFHSS, such as Intuitionistic CFHSS, Neutrosophic CFHSS, Plithogenic CFHSS, Plithogenic Intuitionistic CFHSS, Plithogenic Intuitionistic CFHSS, and Plithogenic Neutrosophic CFHSS, as well as diagnostic and therapeutic imaging, analytical thinking, decision support paradigms, socioeconomic, and fiscal concerns.

V. CONCLUSION

Utilizing effective and appropriate SWM strategies is necessary to regulate many forms of pollution, prevent infectious illnesses, conserve natural resources, and recycle toxic substances. As a result, several researchers and academics have begun to work on SWM. The existence of ambiguity in nearly every real-world system has prompted scholars to use fuzzy set theory and its variants to handle the problem of SWM. A novel scientific tool is produced in this study to reveal factual information in inherent complexity. Blending a FS and HSS described in a complex structure provides the CFHSS set. This paradigm is dynamic in double ways. First, it expands the membership by converting it into a unit circle with phase and amplitude aspects. Second, for a more profound comprehension, the features in CFHSS may be further sub-divided into attribute values. In this scenario, the provided framework is able to analyze the relative importance of each method with the help of weights of numerous factors that influence SWM strategies based on their characteristics. This investigation should provide a theoretical background for managing vagueness and periodicity in construction, healthcare, nanotechnology, transportation, and other sectors. This new understanding of the P-terms offers many possibilities for application in scientific theory and other social disciplines, where P-terms could depict temperature, tension, proximity, or any other criterion that influences and colludes with the respective A-terms in the round of choices. Additionally, the metaphoric interpretations of ENT and SM of CFHSS were discussed and the associated concerns. Mathematical models are also presented to check the ability and supremacy of the configuration strategies. The consequences of the suggested measures and comparability to current systems are also described in detail. Eventually, mathematical models that illustrate the suitability of the strategic planning process are supplied.

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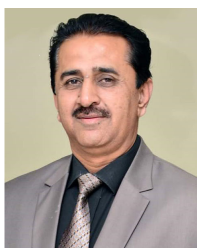

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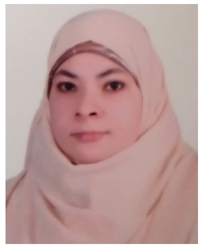

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