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Intuitionistic Fuzzy PRI-AND and PRI-OR Aggregation Operators Based on the Priority Degrees and Their Applications in Multi-Attribute Decision Making

RUI HUA[1], BOQUAN LI[1], AND YONGYI LI[2]
[1]School of Mathematics and Statistics, Anhui Normal University, Wuhu, Anhui 241003, China
[2]School of Transportation Engineering, Nanjing Tech University, Nanjing 210009, China

Corresponding authors: Yongyi Li (liyongyi@njtech.edu.cn) and Boquan Li (010902014@163.com)

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ABSTRACT Multi-attribute decision making (MADM) problems that exist a prioritization relationship between the attributes get more and more scholars' attention. Considering that the priority relationship of attributes, the concept of priority degree was applied to assign a non-negative real number to each priority order and such a non-negative real number is called the priority degree. In this paper, we introduce two new kinds of intuitionistic fuzzy prioritized aggregation operators based on the priority degrees: intuitionistic fuzzy prioritized "and" aggregation operators (PRI-AND) based on the priority degrees and intuitionistic fuzzy prioritized "or" aggregation operators (PRI-OR) based on the priority degrees and also establish some important properties of these aggregation operators in their different particular cases. Next, we develop a new method for addressing the multi-attribute decision making problems in which the attributes are in different priority levels based on intuitionistic fuzzy prioritized "or" aggregation operators based on the priority degrees. The new proposed methods can provide more choices for decision makers and many decision makers choose the appropriate priority degrees according to their own preference. Finally, an illustrative example is provided to prove the rationality of the proposed approach.

INDEX TERMS Intuitionistic fuzzy PRI-AND aggregation operator, intuitionistic fuzzy PRI-OR aggregation operator, priority degree, multi-attribute decision making, intuitionistic fuzzy set.

I. INTRODUCTION

Multi-attribute decision making (MADM) is the behavior of using decision-making information to evaluate, rank and select the best alternatives through a certain decision-making method. The actual decision making process mainly includes the giving of decision-making information and the selection of decision-making methods. Due to the complexity of society and the limitations of experts' knowledge, enriching the form of decision information in the decision-making process and improving the multi-attribute decision making method has become a hot spot in the decision-making field. Zadeh [1], [2] introduced the theory of fuzzy sets to express the uncertainty of actual problems, and fuzzy sets provide a very effective method for dealing with the ambiguity and uncertainty in the decision-making process. Then Atanassov et.al developed the concept of intuitionistic fuzzy sets (IFS) used membership function and non-membership function to express ambiguity in the decision-making process [3]–[7]. Some scholars handled multi-criteria fuzzy decision making problems based on vague set theory and they provided some functions to measure the degree of suitability of each alternative with respect to a set of criteria presented by vague values in [8], [9]. Later, decision makers began to give information about their preference in the decision-making problem for alternatives in the form of intuitionistic fuzzy numbers [10]–[12]. Then many authors applied intuitionistic fuzzy preference relations [13]–[17] to express the experts'

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preference and developed methods for solving multi-attribute decision making problems. Thus intuitionistic fuzzy information attracts more and more scholars' attention.

After obtaining the decision information, we need to use some aggregation operators to integrate the intuitionistic fuzzy information. Xu [18] developed some aggregation operators, such as intuitionistic fuzzy weighted averaging operator, intuitionistic fuzzy ordered weighted averaging operator, and intuitionistic fuzzy hybrid aggregation operator. In [19], Xu and Yager proposed some geometric aggregation operators such as intuitionistic fuzzy weighted geometric operator, intuitionistic fuzzy ordered weighted geometric operator and intuitionistic fuzzy hybrid geometric operator. Then Zhao [20] developed some new generalized aggregation operators, such as generalized intuitionistic fuzzy weighted averaging operator, generalized intuitionistic fuzzy ordered weighted averaging operator, generalized intuitionistic fuzzy hybrid averaging operator. Since then, these operators have been widely used in decision-making fields where attributes are at the same level. But in the actual multi-attribute problem, attributes are often at the different priority levels.

Suppose a student needs to be evaluated based on academic performance and moral cultivation, this student has good academic performance, but his moral cultivation is not necessarily good. This student has good moral cultivation, but his academic performance is not necessarily good. Therefore we cannot evaluate this student objectively. In this situation, we usually think that moral cultivation is more important to a person. In a word, there exists a priority relationship among these attributes and lack of satisfaction by the higher priority attribute cannot be readily compensated for by satisfaction by lower priority attribute.

In recent years, Yager [21] showed how this prioritization of criteria can be modeled by using importance weights in which the weights associated with the lower priority criteria are related to the satisfaction of the higher priority criteria. Then Yu [22] and Chen [26] developed some methods in prioritized multi-criteria decision making based on preference relations. Yager [23], [24] and Chen [25] established prioritized information fusion methods and generalized model for prioritized multi-criteria decision making systems. Some scholars proposed some basic prioritized aggregation operators and studied the above decision-making problems that attributes are at the different priority level and proposed some prioritized aggregation operators [27]–[31], for instance, prioritized scoring aggregation operator, prioritized averaging aggregation operator, prioritized "and" aggregation operator, prioritized "or" aggregation operator in which there exists a prioritization relationship among the attributes. Later, Li [32] think that the importance weights of the attributes are represented by Atanassov's intuitionistic fuzzy values (IFVs) and proposed intuitionistic fuzzy prioritized "and" aggregation operator and intuitionistic fuzzy prioritized "or" aggregation operator for aggregating intuitionistic fuzzy information.

In [33], Li and Xu consider that the multi-criteria decision making (MCDM) problems that exists a prioritization relationship between the criteria. Then the decision maker assign a non-negative real number to each priority order, and call such a non-negative real number the priority degree. In addition, they used the concept of priority degree to propose some new prioritized aggregation operators, such as prioritized averaging operator with the priority degrees, prioritized scoring operator with the priority degrees, and prioritized ordered weighted averaging operator with the priority degrees.

This study aims to developing some new prioritized aggregation operators based on the priority degrees [33] for aggregating intuitionistic fuzzy information in decision making problems in which the attributes are at different priority levels. Therefore, we apply the concept of priority degrees to propose two kinds of intuitionistic fuzzy prioritized aggregation operators: intuitionistic fuzzy PRI-AND aggregation operators based on the priority degrees and intuitionistic fuzzy PRI-OR aggregation operators based on the priority degrees. The new prioritized aggregation operators are different from intuitionistic fuzzy PRI-AND aggregation operators and intuitionistic fuzzy PRI-OR aggregation operators [32], they can solve MADM problems and enrich the form of previous prioritized aggregation operators.

The overall structure of this paper takes the form of seven sections. In Section 2, we review some basic concepts about intuitionistic fuzzy sets and some prioritized aggregation operators, introduce intuitionistic fuzzy prioritized "and" aggregation operator and intuitionistic fuzzy prioritized "or" aggregation operator in [32] and several cases based on t-norm and t-conorm [35]. In Section 3, based on the priority degrees, we proposed two kinds of intuitionistic fuzzy prioritized aggregation operators: intuitionistic fuzzy PRI-AND aggregation operators based on the priority degrees and intuitionistic fuzzy PRI-OR aggregation operators based on the priority degrees. We also establish their particular cases of these operators and discuss some propositions about them. In Section 4, we develop new methods for addressing the MADM problems. Section 5 give an example of choosing the best engineer project to prove the rationality of the proposed approach. Section 6 concludes the paper.

II. PRELIMINARIES

A. BASIC KNOWLEDGE

The concept of intuitionistic fuzzy set was introduced by Atanassov and was further published in an extended form in Atanassov [3].

Definition 1 [1], [3]: Let X be a finite set, an intuitionistic fuzzy set (IFS) A in X having the following form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}, \tag{1}$$

where the functions $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ are the degree of membership μ_A and the degree of non-membership ν_A respectively and for every $x \in X$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1. \tag{2}$$

For each given IFS A, we take $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x), x \in X, \pi_A(x)$ as a hesitancy degree of x to A. It is clear that $\pi_A(x) \in [0, 1]$. If $\pi_A(x) = 0$, i.e. $\mu_A(x) = 1 - \nu_A(x)$, then IFS A reduces to a fuzzy set.

Definition 2 [4], [19]: An IFV α has the form: $\alpha = \langle \mu_\alpha, \nu_\alpha \rangle$, where $\mu_\alpha \in [0, 1]$, $\nu_\alpha \in [0, 1]$ and $\mu_\alpha + \nu_\alpha \leq 1$.

Next, L^* stands for the family of all IFVs. The ordinary partial ordered relation "$\prec$" is defined as: for any two IFVs $\alpha = \langle \mu_\alpha, \nu_\alpha \rangle$ and $\beta = \langle \mu_\beta, \nu_\beta \rangle$, then

$$\alpha \prec \beta \Leftrightarrow \mu_\alpha \leq \mu_\beta \wedge \nu_\alpha \geq \nu_\beta. \tag{3}$$

The smallest element in L^* is $\langle 0, 1 \rangle$, usually denoted by **0**, and the largest element in L^* is $\langle 1, 0 \rangle$, usually denoted by **1**.

Definition 3 [8], [9]: Let $\alpha = \langle \mu_\alpha, \nu_\alpha \rangle$ be IFV, $s(\alpha) = \mu_\alpha - \nu_\alpha$ can be expressed as its score function, and $h(\alpha) = \mu_\alpha + \nu_\alpha$ can be expressed as its accuracy function.

Definition 4 [19]: Let α_1 and α_2 be two IFVs:

(1) If $s(\alpha_1) < s(\alpha_2)$, then α_2 is lager than α_1, denoted by $\alpha_2 > \alpha_1$;

(2) If $s(\alpha_1) = s(\alpha_2)$, then

(a) If $h(\alpha_1) = h(\alpha_2)$, then there is no difference between α_1 and α_2, denoted by $\alpha_1 \sim \alpha_2$;

(b) If $h(\alpha_1) > h(\alpha_2)$, then α_1 is lager than α_2, denoted by $\alpha_1 > \alpha_2$.

Let $\alpha = \langle u_\alpha, \nu_\alpha \rangle$, $\alpha_1 = \langle u_{\alpha_1}, \nu_{\alpha_1} \rangle$ and $\alpha_2 = \langle u_{\alpha_2}, \nu_{\alpha_2} \rangle$ be three IFVs, $k > 0$, [18], [19] defined the following operation laws as:

(1) $\alpha_1 \oplus \alpha_2 = \langle u_{\alpha_1} + u_{\alpha_2} - u_{\alpha_1}u_{\alpha_2}, \nu_{\alpha_1}\nu_{\alpha_2} \rangle$;
(2) $\alpha_1 \otimes \alpha_2 = \langle u_{\alpha_1}u_{\alpha_2}, \nu_{\alpha_1} + \nu_{\alpha_2} - \nu_{\alpha_1}\nu_{\alpha_2} \rangle$;
(3) $k\alpha = \langle 1 - (1 - u_\alpha)^k, \nu_\alpha^k \rangle$;
(4) $\alpha^k = \langle u_\alpha^k, 1 - (1 - \nu_\alpha)^k \rangle$.

Definition 5 [32]: Let $\alpha = \langle u_\alpha, \nu_\alpha \rangle$ and $\lambda = \langle \omega, \rho \rangle$ be two IFVs, define the following operation laws on IFVs as:

$$\lambda\alpha = \langle 1 - (1 - u_\alpha)^\omega, \nu_\alpha^{1-\rho} \rangle, \tag{4}$$

$$\alpha^\lambda = \langle u_\alpha^{1-\rho}, 1 - (1 - \nu_\alpha)^\omega \rangle. \tag{5}$$

Definition 6 [32]: (1) Let $\alpha = \langle u_\alpha, \nu_\alpha \rangle$ and $\lambda = \langle \omega, \rho \rangle$ be two IFVs, then $\lambda\alpha$ and α^λ are IFVs.

(2) Let $\alpha = \langle u_\alpha, \nu_\alpha \rangle$, $\lambda_1 = \langle \omega_1, \rho_1 \rangle$ and $\lambda_2 = \langle \omega_2, \rho_2 \rangle$ be three IFVs, if $\lambda_1 \prec \lambda_2$, then $\lambda_1\alpha \prec \lambda_2\alpha$, $\alpha^{\lambda_1} \succ \alpha^{\lambda_2}$.

In [35], if function T: $[0, 1]^2 \to [0, 1]$ is commutative and associative, which is increasing in both arguments and satisfy $T(x, 1) = x$ for all x in [0,1], then T is t-norm. If function S: $[0, 1]^2 \to [0, 1]$ is commutative and associative, which is increasing in both arguments and satisfy $S(x, 0) = x$ for all x in [0,1], then S is t-conorm. A t-norm T and a t-conorm S that satisfy the De Morgan law simultaneously are called dual.

Definition 7 [34], [35]: A t-norm function $T(x, y)$ is called Archimedean t-norm if it is continuous and $T(x, x) < x$ for all $x \in (0, 1)$. An Archimedean t-norm is called strictly Archimedean t-norm if it is strictly increasing in each variable for $x, y \in (0, 1)$.

It is well known [35] that a strict Archimedean t-norm T is expressed via its additive generator g as $T(x, y) = g^{-1}(g(x) + g(y))$, and its dual t-conorm $S(x, y) = h^{-1}(h(x) + h(y))$ with $h(t) = g(1 - t)$. The additive generator g is a strictly decreasing function $g : [0, 1] \to [0, +\infty)$ such that $g(1) = 0$. The following are four specific forms of the function g, and some well-known t-norms and t-conorms can be obtained correspondingly.

(1) Let $g(t) = -\ln t$, then $h(t) = -\ln(1 - t)$, $g^{-1}(t) = e^{-t}$, $h^{-1}(t) = 1 - e^{-t}$, and Algebraic t-norm and t-conorm are obtained as follows:

$$T(x, y) = xy, S(x, y) = x + y - xy. \tag{6}$$

(2) Let $g(t) = \ln(\frac{2-t}{t})$, then $h(t) = \ln(\frac{1+t}{1-t})$, $g^{-1}(t) = \frac{2}{e^t+1}$, $h^{-1}(t) = 1 - \frac{2}{e^t+1}$, and Einstein t-norm and t-conorm are obtained as follows:

$$T(x, y) = \frac{xy}{1 + (1 - x)(1 - y)}, \quad S(x, y) = \frac{x + y}{1 + xy}. \tag{7}$$

(3) Let $g(t) = \ln(\frac{\gamma+(1-\gamma)t}{t})$, $\gamma > 0$, then $h(t) = \ln(\frac{\gamma+(1-\gamma)(1-t)}{1-t})$, $g^{-1}(t) = \frac{\gamma}{e^t+\gamma-1}$, $h^{-1}(t) = 1 - \frac{\gamma}{e^t+\gamma-1}$, and Hamacher t-norm and t-conorm are obtained as follows:

$$T(x, y) = \frac{xy}{\gamma + (1 - \gamma)(x + y - xy)}, \tag{8}$$

$$S(x, y) = \frac{x + y - (2 - \gamma)xy}{1 - (1 - \gamma)xy}. \tag{9}$$

Especially, if $\gamma = 1$, then Hamacher t-norm and t-conorm reduce to the Algebraic t-norm and t-conorm respectively; if $\gamma = 2$, then Hamacher t-norm and t-conorm reduce to the Einstein t-norm and t-conorm respectively.

(4) Let $g(t) = \ln(\frac{\gamma-1}{\gamma^t-1})$, $\gamma > 1$, then $h(t) = \ln(\frac{\gamma-1}{\gamma^{1-t}-1})$, $g^{-1}(t) = \frac{\log(\frac{\gamma-1+e^t}{e^t})}{\log\gamma}$, $h^{-1}(t) = 1 - g^{-1}(t) = \frac{\log(\frac{\gamma e^t}{\gamma-1+e^t})}{\log\gamma}$ and Frank t-norm and t-conorm are obtained as follows:

$$T(x, y) = log_\gamma(1 + \frac{(\gamma^x - 1)(\gamma^y - 1)}{\gamma - 1}), \tag{10}$$

$$S(x, y) = 1 - log_\gamma(1 + \frac{(\gamma^{1-x} - 1)(\gamma^{1-y} - 1)}{\gamma - 1}). \tag{11}$$

Especially, if $\gamma \to 1$, then Frank t-norm and t-conorm reduce to the Algebraic t-norm and t-conorm respectively.

Definition 8 [34]: An intuitionistic fuzzy t-norm (IF t-norm) $\mathcal{T}$ in L^* is called representable if there exists a t-norm T and a t-conorm S such that for all $\alpha = \langle \mu_\alpha, \nu_\alpha \rangle, \beta = \langle \mu_\beta, \nu_\beta \rangle \in L^*$, $\mathcal{T} = \langle T(\mu_\alpha, \mu_\beta), S(\nu_\alpha, \nu_\beta) \rangle$, T and S are called the representant of $\mathcal{T}$.

An intuitionistic fuzzy t-conorm (IF t-conorm) $\mathcal{S}$ in L^* is called representable if there exists a t-norm T' and a t-conorm S' such that for all $\alpha = \langle \mu_\alpha, \nu_\alpha \rangle, \beta = \langle \mu_\beta, \nu_\beta \rangle \in L^*$, $\mathcal{S} = \langle S'(\mu_\alpha, \mu_\alpha), T'(\nu_\alpha, \nu_\beta) \rangle$, S' and T' are called the representant of $\mathcal{S}$.

Xia *et al.* [35] defined some operations of IFVs based on Archimedean t-norm and Archimedean t-conorm [2], as follows:

(1) $\alpha_1 \oplus \alpha_2 = (S(\mu_{\alpha_1}, \mu_{\alpha_2}), T(\nu_{\alpha_1}, \nu_{\alpha_2}))$
$= (h^{-1}(h(\mu_{\alpha_1}) + h(\mu_{\alpha_2})), g^{-1}(g(\nu_{\alpha_1}) + g(\nu_{\alpha_2})))$,
(2) $\alpha_1 \otimes \alpha_2 = (T(\mu_{\alpha_1}, \mu_{\alpha_2}), S(\nu_{\alpha_1}, \nu_{\alpha_2}))$
$= (g^{-1}(g(\mu_{\alpha_1}) + g(\mu_{\alpha_2})), h^{-1}(h(\nu_{\alpha_1}) + h(\nu_{\alpha_2})))$,
(3) $\lambda\alpha = (h^{-1}(\lambda h(\mu_\alpha)), g^{-1}(\lambda g(\nu_\alpha))), \lambda > 0$,
(4) $\alpha^\lambda = (g^{-1}(\lambda g(\mu_\alpha)), h^{-1}(\lambda h(\nu_\alpha))), \lambda > 0$.

Based on the above operations, we apply Arichimedean t-conorm and t-norm to intuitionistic fuzzy aggregation operators and develop the extended Archimedean t-conorm and t-norm based intuitionistic fuzzy weighted averaging (E-ATS-IFWA) operator and the extended Archimedean t-conorm and t-norm based intuitionistic fuzzy weighted geometric (E-ATS-IFWG) operator. Let $\alpha_i = \langle\mu_{\alpha_i}, \nu_{\alpha_i}\rangle$ be n IFVs, define the following operators as:

(1) The extended Archimedean t-conorm and t-norm based intuitionistic fuzzy weighted averaging (E-ATS-IFWA) operator:

$$\begin{aligned}&\text{E-ATS-IFWA}(\alpha_1, \alpha_2, \ldots, \alpha_n)\\ &= \bigoplus_{i=1}^{n}(\lambda_i\alpha_i)\\ &= \lambda_1\alpha_1 \oplus \lambda_2\alpha_2 \oplus \ldots \oplus \lambda_n\alpha_n\\ &= \langle h^{-1}(\sum_{i=1}^{n}\mu_{\lambda_i}h(\mu_{\alpha_i})), g^{-1}(\sum_{i=1}^{n}(1-\nu_{\lambda_i})g(\nu_{\alpha_i}))\rangle, \end{aligned} \tag{12}$$

where $\lambda_i = \langle\mu_{\lambda_i}, \nu_{\lambda_i}\rangle$ is the intuitionistic fuzzy weight of $\alpha_i (i = 1, 2, \ldots, n)$.

(2) The extended Archimedean t-conorm and t-norm based intuitionistic fuzzy weighted geometric (E-ATS-IFWG) operator:

$$\begin{aligned}&\text{E-ATS-IFWG}(\alpha_1, \alpha_2, \ldots, \alpha_n)\\ &= \bigotimes_{i=1}^{n}(\lambda_i\alpha_i)\\ &= \alpha_1^{\lambda_1} \otimes \alpha_2^{\lambda_2} \otimes \ldots \otimes \alpha_n^{\lambda_n}\\ &= \langle g^{-1}(\sum_{i=1}^{n}(1-\nu_{\lambda_i})g(\mu_{\alpha_i})), h^{-1}(\sum_{i=1}^{n}\mu_{\lambda_i}h(\nu_{\alpha_i}))\rangle, \end{aligned} \tag{13}$$

where $\lambda_i = \langle\mu_{\lambda_i}, \nu_{\lambda_i}\rangle$ is the intuitionistic fuzzy weight of $\alpha_i (i = 1, 2, \ldots, n)$.

Case 1: If $g(t) = -ln(t)$, then we have:
(1) $\lambda\alpha = \langle 1 - (1-\mu_\alpha)^{\mu_\lambda}, (\nu_\alpha)^{1-\nu_\lambda}\rangle$;
(2) $\alpha^\lambda = \langle(\mu_\alpha)^{1-\nu_\lambda}, 1 - (1-\nu_\alpha)^{\mu_\lambda}\rangle$.

Thus under this circumstance, the corresponding E-ATS-IFWA operators and E-ATS-IFWG operators are as follows:

$$\begin{aligned}&\text{E-ATS-IFWA}(\alpha_1, \alpha_2, \ldots, \alpha_n)\\ &= \bigoplus_{i=1}^{n}(\lambda_i\alpha_i)\\ &= \langle 1 - \prod_{i=1}^{n}(1-\mu_{\alpha_i})^{\mu_{\lambda_i}}, \prod_{i=1}^{n}(\nu_{\alpha_i})^{1-\nu_{\lambda_i}}\rangle, \end{aligned} \tag{14}$$

$$\begin{aligned}&\text{E-ATS-IFWG}(\alpha_1, \alpha_2, \ldots, \alpha_n)\\ &= \bigotimes_{i=1}^{n}(\lambda_i\alpha_i)\\ &= \langle\prod_{i=1}^{n}(\mu_{\alpha_i})^{1-\nu_{\lambda_i}}, 1 - \prod_{i=1}^{n}(1-\nu_{\alpha_i})^{\mu_{\lambda_i}}\rangle. \end{aligned} \tag{15}$$

Case 2: If $g(t) = -ln(\frac{2-t}{t})$, then we have:
(1) $\lambda\alpha = \langle\frac{(1+\mu_\alpha)^{\mu_\lambda}-(1-\mu_\alpha)^{\mu_\lambda}}{(1+\mu_\alpha)^{\mu_\lambda}+(1-\mu_\alpha)^{\mu_\lambda}}, \frac{2(\nu_\alpha)^{1-\nu_\lambda}}{(2-\nu_\alpha)^{1-\nu_\lambda}+(\nu_\alpha)^{1-\nu_\lambda}}\rangle$;
(2) $\alpha^\lambda = \langle\frac{2(\mu_\alpha)^{1-\nu_\lambda}}{(2-\mu_\alpha)^{1-\nu_\lambda}+(\mu_\alpha)^{1-\nu_\lambda}}, \frac{(1+\nu_\alpha)^{\mu_\lambda}-(1-\nu_\alpha)^{\mu_\lambda}}{(1+\nu_\alpha)^{\mu_\lambda}+(1-\nu_\alpha)^{\mu_\lambda}}\rangle$.

Thus under this circumstance, the corresponding E-ATS-IFWA operators and E-ATS-IFWG operators are as follows:

$$\begin{aligned}&\text{E-ATS-IFWA}(\alpha_1, \alpha_2, \ldots, \alpha_n)\\ &= \bigoplus_{i=1}^{n}(\lambda_i\alpha_i)\\ &= \langle\frac{\prod_{i=1}^{n}(1+\mu_{\alpha_i})^{\mu_{\lambda_i}} - \prod_{i=1}^{n}(1-\mu_{\alpha_i})^{\mu_{\lambda_i}}}{\prod_{i=1}^{n}(1+\mu_{\alpha_i})^{\mu_{\lambda_i}} + \prod_{i=1}^{n}(1-\mu_{\alpha_i})^{\mu_{\lambda_i}}},\\ &\quad \frac{2\prod_{i=1}^{n}(\nu_{\alpha_i})^{1-\nu_{\lambda_i}}}{\prod_{i=1}^{n}(2-\nu_{\alpha_i})^{1-\nu_{\lambda_i}} + \prod_{i=1}^{n}(\nu_{\alpha_i})^{1-\nu_{\lambda_i}}}\rangle, \end{aligned} \tag{16}$$

$$\begin{aligned}&\text{E-ATS-IFWG}(\alpha_1, \alpha_2, \ldots, \alpha_n)\\ &= \bigotimes_{i=1}^{n}(\lambda_i\alpha_i)\\ &= \langle\frac{2\prod_{i=1}^{n}(\mu_{\alpha_i})^{1-\nu_{\lambda_i}}}{\prod_{i=1}^{n}(2-\mu_{\alpha_i})^{1-\nu_{\lambda_i}} + \prod_{i=1}^{n}(\mu_{\alpha_i})^{1-\nu_{\lambda_i}}},\\ &\quad \frac{\prod_{i=1}^{n}(1+\nu_{\alpha_i})^{\mu_{\lambda_i}} - \prod_{i=1}^{n}(1-\nu_{\alpha_i})^{\mu_{\lambda_i}}}{\prod_{i=1}^{n}(1+\nu_{\alpha_i})^{\mu_{\lambda_i}} + \prod_{i=1}^{n}(1-\nu_{\alpha_i})^{\mu_{\lambda_i}}}\rangle. \end{aligned} \tag{17}$$

Case 3: If $g(t) = ln(\frac{\gamma+(1-\gamma)t}{t}), \gamma > 0$, then we have:
(1) $\lambda\alpha = \langle\frac{(1+(\gamma-1)\mu_\alpha)^{\mu_\lambda}-(1-\mu_\alpha)^{\mu_\lambda}}{(1+(\gamma-1)\mu_\alpha)^{\mu_\lambda}+(\gamma-1)(1-\mu_\alpha)^{\mu_\lambda}}, \frac{\gamma(\nu_\alpha)^{1-\nu_\lambda}}{(1+(\gamma-1)(1-\nu_\alpha))^{1-\nu_\lambda}+(\gamma-1)(\nu_\alpha)^{1-\nu_\lambda}}\rangle$;
(2) $\alpha^\lambda = \langle\frac{\gamma(\mu_\alpha)^{1-\nu_\lambda}}{(1+(\gamma-1)(1-\mu_\alpha))^{1-\nu_\lambda}+(\gamma-1)(\mu_\alpha)^{1-\nu_\lambda}}, \frac{(1+(\gamma-1)\nu_\alpha)^{\mu_\lambda}-(1-\nu_\alpha)^{\mu_\lambda}}{(1+(\gamma-1)\nu_\alpha)^{\mu_\lambda}+(\gamma-1)(1-\nu_\alpha)^{\mu_\lambda}}\rangle$.

Thus under this circumstance, the corresponding E-ATS-IFWA operators and E-ATS-IFWG operators are as follows:

$$\begin{aligned}&\text{E-ATS-IFWA}(\alpha_1, \alpha_2, \ldots, \alpha_n)\\ &= \bigoplus_{i=1}^{n}(\lambda_i\alpha_i)\end{aligned}$$

$$=\langle\frac{\prod_{i=1}^{n}(1+(\gamma-1)\mu_{\alpha_i})^{\mu_{\lambda_i}}-\prod_{i=1}^{n}(1-\mu_{\alpha_i})^{\mu_{\lambda_i}}}{\prod_{i=1}^{n}(1+(\gamma-1)\mu_{\alpha_i})^{\mu_{\lambda_i}}+(\gamma-1)\prod_{i=1}^{n}(1-\mu_{\alpha_i})^{\mu_{\lambda_i}}},$$
$$\frac{\gamma\prod_{i=1}^{n}(\nu_{\alpha_i})^{1-\nu_{\lambda_i}}}{\prod_{i=1}^{n}(1+(\gamma-1)(1-\nu_{\alpha_i}))^{1-\nu_{\lambda_i}}+(\gamma-1)\prod_{i=1}^{n}(\nu_{\alpha_i})^{1-\nu_{\lambda_i}}}\rangle \tag{18}$$

$$\text{E-ATS-IFWG}(\alpha_1,\alpha_2,\ldots,\alpha_n)$$
$$=\bigotimes_{i=1}^{n}(\lambda_i\alpha_i)$$
$$=\langle\frac{\gamma\prod_{i=1}^{n}(\mu_{\alpha_i})^{1-\nu_{\lambda_i}}}{\prod_{i=1}^{n}(1+(\gamma-1)(1-\mu_{\alpha_i}))^{1-\nu_{\lambda_i}}+(\gamma-1)\prod_{i=1}^{n}(\mu_{\alpha_i})^{1-\nu_{\lambda_i}}},$$
$$\frac{\prod_{i=1}^{n}(1+(\gamma-1)\nu_{\alpha_i})^{\mu_{\lambda_i}}-\prod_{i=1}^{n}(1-\nu_{\alpha_i})^{\mu_{\lambda_i}}}{\prod_{i=1}^{n}(1+(\gamma-1)\nu_{\alpha_i})^{\mu_{\lambda_i}}+(\gamma-1)\prod_{i=1}^{n}(1-\nu_{\alpha_i})^{\mu_{\lambda_i}}}\rangle. \tag{19}$$

Especially, if $\gamma=1$, then Eq.(18) and Eq.(19) reduce to Eq.(14) and Eq.(15) respectively; if $\gamma=2$, then Eq.(18) and Eq.(19) reduce to Eq.(16) and Eq.(17) respectively.

Case 4: If $g(t)=ln(\frac{\gamma-1}{\gamma^t-1})$, $\gamma>1$, then we have:

(1) $\lambda\alpha=\langle 1-\log_\gamma(1+\frac{(\gamma^{1-\mu_\alpha}-1)^{\mu_\lambda}}{(\gamma-1)^{\mu_\gamma-1}}),\log_\gamma(1+\frac{(\gamma^{\nu_\alpha}-1)^{1-\nu_\lambda}}{(\gamma-1)^{\nu_\lambda-1}})\rangle$;

(2) $\alpha^\lambda=\langle\log_\gamma(1+\frac{(\gamma^{\mu_\alpha}-1)^{1-\nu_\lambda}}{(\gamma-1)^{\nu_\lambda-1}}),1-\log_\gamma(1+\frac{(\gamma^{1-\nu_\alpha}-1)^{\mu_\lambda}}{(\gamma-1)^{\mu_\lambda-1}})\rangle$.

Thus under this circumstance, the corresponding E-ATS-IFWA operators and E-ATS-IFWG operators are as follows:

$$\text{E-ATS-IFWA}(\alpha_1,\alpha_2,\ldots,\alpha_n)$$
$$=\bigoplus_{i=1}^{n}(\lambda_i\alpha_i)$$
$$=\langle 1-\log_\gamma(1+\frac{\prod_{i=1}^{n}(\gamma^{1-\mu_{\alpha_i}}-1)^{\mu_{\lambda_i}}}{\gamma-1}),$$
$$\log_\gamma(1+\frac{\prod_{i=1}^{n}(\gamma^{\nu_{\alpha_i}}-1)^{1-\nu_{\lambda_i}}}{\gamma-1})\rangle, \tag{20}$$

$$\text{E-ATS-IFWG}(\alpha_1,\alpha_2,\ldots,\alpha_n)$$
$$=\bigotimes_{i=1}^{n}(\lambda_i\alpha_i)$$
$$=\langle\log_\gamma(1+\frac{\prod_{i=1}^{n}(\gamma^{\mu_{\alpha_i}}-1)^{1-\nu_{\lambda_i}}}{\gamma-1}),$$
$$1-\log_\gamma(1+\frac{\prod_{i=1}^{n}(\gamma^{1-\nu_{\alpha_i}}-1)^{\mu_{\lambda_i}}}{\gamma-1})\rangle. \tag{21}$$

Especially, if $\gamma\to 1$, then Eq.(20) and Eq.(21) reduce to Eq.(14) and Eq.(15) respectively.

The above intuitionistic fuzzy aggregation operators have often been applied to decision making under the assumption that the attributes are at the same priority level. However, in practical MADM problem, the attributes are in different priority levels. Consider the situation, we will develope some extended forms of intuitionistic fuzzy prioritized "and" aggregation operator (IFPRI-AND) and intuitionistic fuzzy prioritized "or" aggregation operator (IFPRI-OR). Then assume that we have a collection of attributes $C=\{C_1,C_2,\ldots,C_n\}$, a set of alternatives $X=\{x_1,x_2,\ldots,x_m\}$. Assume that the collection of attributes C are partitioned into q distinct categories, $H_1,H_2,\ldots,H_q$ such that $H_i=\{C_{i1},C_{i2},\ldots,C_{in_i}\}$. Here $C_{ij}\in C(i=1,2,\ldots,q;j=1,2,\ldots,n_i)$ are the attributes in the category H_i. We also assume a prioritization relationship among these categories:

$$H_1\succ H_2\succ\cdots\succ H_q.$$

The attributes in the category H_i have higher priority than those in the category H_k if $i<k$. Then the universal set of attributes is $C=\bigcup_{i=1}^{q}H_i$ and the total number of attributes is $n=\sum_{i=1}^{q}n_i$. We have an attribute value $C_{ij}(x)=\langle\mu_{ij}(x),\nu_{ij}(x)\rangle\in L^*$ based on any alternative $x\in X$ for each attribute C_{ij}.

B. INTUITIONISTIC FUZZY PRIORITIZED "AND" OPERATOR

Let $s_0(x)=\langle 1,0\rangle$ and

$$s_i(x)=\mathcal{T}_M\{C_{ij}(x)\mid j=1,2,\ldots,n_i\}, \tag{22}$$

for $i=1,2,\ldots,q$.

We take $w_{ij}(x)$ as the priority weight associated with C_{ij}, then

$$w_{ij}(x)=t_i(x)=\bigotimes_{k=1}^{i}s_{k-1}(x)$$
$$=s_0(x)\otimes s_1(x)\otimes\ldots\otimes s_{i-1}(x). \tag{23}$$

Then we calculate the priority weights $w_{ij}(x)$ for $i=1,2,\ldots,q;j=1,2,\ldots,n_i$. By the definition of the intuitionistic fuzzy t-norm (IF t-norm) $\mathcal{T}_M$, we get

$$s_i(x)=\mathcal{T}_M\{C_{ij}(x)\mid j=1,2,\ldots,n_i\}$$
$$=\langle\min(\mu_{i1}(x),\ldots,\mu_{in_i}(x)),\max(\nu_{i1}(x),\ldots,\nu_{in_i}(x))\rangle.$$

Assume that $\min(\mu_{i1}(x),\ldots,\mu_{in_i}(x))=\mu_{s_i}(x)$, $\max(\mu_{i1}(x),\ldots,\mu_{in_i}(x))=\nu_{s_i}(x)$, then we have $s_i(x)=\langle\mu_{s_i}(x),\nu_{s_i}(x)\rangle$,

$$t_i(x)=\bigotimes_{k=1}^{i}s_{k-1}(x)=\langle 1,0\rangle\otimes s_1(x)\otimes\ldots\otimes s_{i-1}(x)$$
$$=s_1(x)\otimes\ldots\otimes s_{i-1}(x)$$

$$= \langle \mu_{s_1}(x), \nu_{s_1}(x)\rangle \otimes \langle \mu_{s_2}(x), \nu_{s_2}(x)\rangle \otimes \ldots \otimes \langle \mu_{s_{i-1}}(x), \nu_{s_{i-1}}(x)\rangle = \langle \prod_{k=1}^{i-1} \mu_{s_k}(x), 1 - \prod_{k=1}^{i-1}(1 - \nu_{s_k}(x))\rangle.$$

Assume that $\prod_{k=1}^{i-1} \mu_{s_k}(x) = u_{t_i}(x)$ and $1 - \prod_{k=1}^{i-1}(1 - \nu_{s_k}(x) = \nu_{t_i}(x)$, then we have $t_i(x) = \langle \mu_{t_i}(x), \nu_{t_i}(x)\rangle$.

Thus we get the weights $w_{ij}(x) = t_i(x) = \langle \prod_{k=1}^{i-1} \mu_{s_k}(x), 1 - \prod_{k=1}^{i-1}(1 - \nu_{s_k}(x))\rangle$.

For each $C_{ij}(x)$ and the weights $w_{ij}(x)$, by Eq.(4), we can calculate

$$C_{ij}(x)^{w_{ij}(x)} = \langle \mu_{ij}(x), \nu_{ij}(x)\rangle^{\langle \mu_{t_i}(x), \nu_{t_i}(x)\rangle} = \langle \mu_{ij}(x)^{1-\nu_{t_i}(x)}, 1 - (1 - \nu_{ij}(x))^{1-\mu_{t_i}(x)}\rangle.$$

Definition 9 [32]: Let $\{(C_{11}, \ldots, C_{1n_1}), \ldots, (C_{q1}, \ldots, C_{qn_q})\}$ be the collection of attributes with different priority level and $\{(C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x), \ldots, C_{qn_q}(x))\}$ be the collection of preference values, which take the form of IFVs for the alternative $x \in X$ with respect to the attributes $\{(C_{11}, \ldots, C_{1n_1}), \ldots, (C_{q1}, \ldots, C_{qn_q})\}$. $\mathcal{T}$ is an IF t-norm. Define the operator IFPRI-AND: $(L^*)^n \rightarrow L^*$ as follow:

$$C(x) = \text{IFPRI-AND}((C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x), \ldots, C_{qn_q}(x))) = \mathcal{T}(\mathcal{T}(C_{11}^{w_{11}(x)}(x), \ldots, C_{1n_1}^{w_{1n_1}(x)}(x)), \ldots, \mathcal{T}(C_{q1}^{w_{q1}(x)}(x), \ldots, C_{qn_q}^{w_{qn_q}(x)}(x))), \tag{24}$$

where the weight $w_{ij}(x)$ can be obtained by Eq.(22). IFPRI-AND is called an intuitionistic fuzzy prioritized "and" aggregation operator.

Since $w_{ij}(x) = t_i(x)$, then we also have

$$C(x) = \mathcal{T}(\mathcal{T}(C_{11}^{t_1(x)}(x), \ldots, C_{1n_1}^{t_1(x)}(x)), \ldots, \mathcal{T}(C_{q1}^{t_q(x)}(x), \ldots, C_{qn_q}^{t_q(x)}(x))).$$

If we select "$\otimes$" operation for IFVs as the IF t-norm $\mathcal{T}$, according to (13), then we have

$$C(x) = \mathcal{T}(\mathcal{T}(C_{11}^{t_1(x)}(x), \ldots, C_{1n_1}^{t_1(x)}(x)), \ldots, \mathcal{T}(C_{q1}^{t_q(x)}(x), \ldots, C_{qn_q}^{t_q(x)}(x))) = (C_{11}^{t_1(x)}(x) \otimes \ldots \otimes C_{1n_1}^{t_1(x)}(x)) \otimes \ldots \otimes (C_{q1}^{t_q(x)}(x) \otimes \ldots \otimes C_{qn_q}^{t_q(x)}(x)).$$

Thus if we select "$\otimes$" as the IF t-norm $\mathcal{T}$, then we get the special intuitionistic fuzzy prioritized "and" aggregation operator as follows:

$$\begin{aligned} C(x) &= \text{IFPRI-AND}((C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x), \ldots, C_{qn_q}(x))) \\ &= \mathcal{T}(\mathcal{T}(C_{11}^{w_{11}(x)}(x), \ldots, C_{1n_1}^{w_{1n_1}(x)}(x)), \ldots, \mathcal{T}(C_{q1}^{w_{q1}(x)}(x), \ldots, C_{qn_q}^{w_{qn_q}(x)}(x))) \\ &= (C_{11}^{t_1(x)}(x) \otimes \ldots \otimes C_{1n_1}^{t_1(x)}(x)) \otimes \ldots \otimes (C_{q1}^{t_q(x)}(x) \otimes \ldots \otimes C_{qn_q}^{t_q(x)}(x)) \\ &= \langle g^{-1}(\sum_{i=1}^{q}\sum_{j=1}^{n_i}(1 - \nu_{t_i}(x))g(\mu_{ij}(x))), h^{-1}(\sum_{i=1}^{q}\sum_{j=1}^{n_i}\mu_{t_i}(x)h(\nu_{ij}(x)))\rangle. \end{aligned} \tag{25}$$

Case 1: If $g(t) = -\ln(t)$, then we have

$$C(x) = \text{IFPRI-AND}((C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x), \ldots, C_{qn_q}(x))) = \langle \prod_{i=1}^{q}(\prod_{j=1}^{n_i} u_{ij}(x)^{1-\nu_{t_i}(x)}), 1 - \prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1-\nu_{ij}(x))^{u_{t_i}(x)})\rangle. \tag{26}$$

Case 2: If $g(t) = \ln(\frac{2-t}{t})$, then we have

$$C(x) = \text{IFPRI-AND}((C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x), \ldots, C_{qn_q}(x))) = \langle \frac{2\prod_{i=1}^{q}(\prod_{j=1}^{n_i} \mu_{ij}(x)^{1-\nu_{t_i}(x)})}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(2 - \mu_{ij}(x))^{1-\nu_{t_i}(x)}) + \prod_{i=1}^{q}(\prod_{j=1}^{n_i} \mu_{ij}(x)^{1-\nu_{t_i}(x)})}, \frac{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1 + \nu_{ij}(x))^{\mu_{t_i}(x)}) - \prod_{i=1}^{q}(\prod_{j=i}^{n_i}(1 - \nu_{ij}(x))^{\mu_{t_i}(x)})}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1 + \nu_{ij}(x))^{\mu_{t_i}(x)}) + \prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1 - \nu_{ij}(x))^{\mu_{t_i}(x)})}\rangle. \tag{27}$$

Case 3: If $g(t) = \ln(\frac{\gamma+(1-\gamma)t}{t})$, $\gamma > 0$, then we have

$$C(x) = \text{IFPRI-AND}((C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x), \ldots, C_{qn_q}(x))) = \langle \frac{\gamma\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(\mu_{ij}(x))^{1-\nu_{t_i}(x)})}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1+(\gamma-1)(1-\mu_{ij}(x)))^{1-\nu_{t_i}(x)}) + (\gamma-1)\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(\mu_{ij}(x))^{1-\nu_{t_i}(x)})}, \frac{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1+(\gamma-1)\nu_{ij}(x))^{\mu_{t_i}(x)}) - \prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1-\nu_{ij}(x))^{\mu_{t_i}(x)})}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1+(\gamma-1)\nu_{ij}(x))^{\mu_{t_i}(x)}) + (\gamma-1)\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1-\nu_{ij}(x))^{\mu_{t_i}(x)})}\rangle. \tag{28}$$

Especially, if $\gamma = 1$, then Eq.(28) reduces to Eq.(26); if $\gamma = 2$, then Eq.(28) reduces to Eq.(27).

Case 4: If $g(t) = \ln(\frac{\gamma-1}{\gamma^t-1})$, $\gamma > 1$, then we have

$$C(x) = \text{IFPRI-AND}((C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x),$$

$$\ldots, C_{qn_q}(x)))$$
$$= \langle \log_\lambda(1 + \frac{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(\gamma^{\mu_{ij}(x)} - 1)^{1-\nu_{t_i}(x)})}{\gamma - 1}),$$
$$1 - \log_\gamma(1 + \frac{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(\gamma^{1-\nu_{ij}(x)} - 1)^{\mu_{t_i}(x)})}{\gamma - 1})\rangle. \quad (29)$$

Especially, if $\gamma \to 1$, then Eq.(29) reduces to Eq.(26).

C. INTUITIONISTIC FUZZY PRIORITIZED "OR" OPERATOR

Let $s_0(x) = \langle 1, 0\rangle$ and

$$s'_i(x) = \mathcal{S}_M\{C_{ij}(x) \mid j = 1, 2, \ldots, n_i\}, \quad (30)$$

for $i = 1, 2, \ldots, q$.

Define $w'_{ij}(x)$ as the priority weight associated with C_{ij}, then

$$w'_{ij}(x) = t'_i(x) = \bigotimes_{k=1}^{i} s'_{k-1}(x)$$
$$= s'_0(x) \otimes s'_1(x) \otimes \ldots \otimes s'_{i-1}(x). \quad (31)$$

Now we first calculate the weights $w'_{ij}(x)$ for $i = 1, 2, \ldots, q; j = 1, 2, \ldots, n_i$. By the definition of the IF t-norm $\mathcal{S}_M$, we get $s'_i(x) = \mathcal{S}_M\{C_{ij}(x) \mid j = 1, 2, \ldots, n_i\} = \langle\max(\mu_{i1}(x), \ldots, \mu_{in_i}(x)), \min(\nu_{i1}(x), \ldots, \nu_{in_i}(x))\rangle$.

Assume that $\max(\mu_{i1}(x), \ldots, \mu_{in_i}(x)) = \mu_{s'_i}(x)$, $\min(\mu_{i1}(x), \ldots, \mu_{in_i}(x)) = \nu_{s'_i}(x)$, then $s'_i(x) = \langle\mu_{s'_i}(x), \nu_{s'_i}(x)\rangle$,

$$t'_i(x) = \bigotimes_{k=1}^{i} s'_{k-1}(x) = \langle 1, 0\rangle \otimes s'_1(x) \otimes \ldots \otimes s'_{i-1}(x)$$
$$= s'_1(x) \otimes \ldots \otimes s'_{i-1}(x)$$
$$= \langle\mu_{s'_1}(x), \nu_{s'_1}(x)\rangle \otimes \langle\mu_{s'_2}(x), \nu_{s'_2}(x)\rangle \otimes \ldots$$
$$\otimes \langle\mu_{s'_{i-1}}(x), \nu_{s'_{i-1}}(x)\rangle$$
$$= \langle\prod_{k=1}^{i-1} \mu_{s'_k}(x), 1 - \prod_{k=1}^{i-1}(1 - \nu_{s'_k}(x))\rangle.$$

Assume that $\prod_{k=1}^{i-1} \mu_{s'_k}(x) = u_{t'_i}(x)$ and $1 - \prod_{k=1}^{i-1}(1 - \nu_{s'_k}(x)) = \nu_{t'_i}(x)$, then we have $t'_i(x) = \langle\mu_{t'_i}(x), \nu_{t'_i}(x)\rangle$.

Thus we get the weights

$$w'_{ij}(x) = t'_i(x) = \langle\prod_{k=1}^{i-1} \mu_{s'_k}(x), 1 - \prod_{k=1}^{i-1}(1 - \nu_{s'_k}(x))\rangle.$$

For each $C_{ij}(x)$ and the weights $w'_{ij}(x)$, by Eq.(30), we can calculate

$$w'_{ij}(x)C_{ij}(x) = \langle\mu_{t'_i}(x), \nu_{t'_i}(x)\rangle\langle\mu_{ij}(x), \nu_{ij}(x)\rangle$$
$$= \langle 1 - (1 - \mu_{ij}(x))^{\mu_{t'_i}(x)}, \nu_{ij}(x)^{1-\nu_{t'_i}(x)}\rangle.$$

Definition 10 [32]: $\{(C_{11}, \ldots, C_{1n_1}), \ldots, (C_{q1}, \ldots, C_{qn_q})\}$ is the collection of attributes with different priority levels, $\{(C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x), \ldots, C_{qn_q}(x))\}$ is the collection of preference values which take the form of IFVs for the alternative $x \in X$ with respect to the attributes $\{(C_{11}, \ldots, C_{1n_1}), \ldots, (C_{q1}, \ldots, C_{qn_q})\}$. $\mathcal{S}$ is an IF t-conorm. Define the operator IFPRI-OR: $(L^*)^n \to L^*$ as follow:

$$C(x) = \text{IFPRI-OR}((C_{11}(x), \ldots, C_{1n_1}(x)),$$
$$\ldots, (C_{q1}(x),$$
$$\ldots, C_{qn_q}(x)))$$
$$= \mathcal{S}(\mathcal{S}(w'_{11}(x)C_{11}(x), \ldots, w'_{1n_1}(x)C_{1n_1}(x)),$$
$$\mathcal{S}(w'_{q1}(x)C_{q1}(x), \ldots, w'_{qn_q}(x)C_{qn_q}(x))), \quad (32)$$

where the weight $w'_{ij}(x)$ can be obtained by Eq.(31). IFPRI-OR is called an intuitionistic fuzzy prioritized "or" aggregation operator.

Since $w'_{ij}(x) = t'_i(x)$, then we also have

$$C'(x) = \mathcal{S}(\mathcal{S}(t'_1(x)C_{11}(x), \ldots, t'_1(x)C_{1n_1}(x)), \ldots,$$
$$\mathcal{S}(t'_q(x)C_{q1}(x), \ldots, t'_q(x)C_{qn_q}(x))).$$

If we select "$\oplus$" operation for IFVs as the IF t-conorm $\mathcal{S}$, according to (12), then we have

$$t'_i(x)C_{i1}(x) \oplus \ldots \oplus t'_i(x)C_{in_i}(x)$$
$$= \mathcal{S}(\mathcal{S}(w'_{11}(x)C_{11}(x), \ldots, w'_{1n_1}(x)C_{1n_1}(x)), \ldots,$$
$$\mathcal{S}(w'_{q1}(x)C_{q1}(x), \ldots, w'_{qn_q}(x)C_{qn_q}(x)))$$
$$= (t'_1(x)C_{11}(x) \oplus \ldots \oplus t'_1(x)C_{1n_1}(x)) \oplus \ldots \oplus$$
$$(t'_q(x)C_{q1}(x) \oplus \ldots \oplus t'_q(x)C_{qn_q}(x))$$
$$= \langle 1 - \prod_{j=1}^{n_i}(1 - \mu_{ij}(x))^{\mu_{t'_i}(x)}, \prod_{j=1}^{n_i} \nu_{ij}(x)^{1-\nu_{t'_i}(x)}\rangle.$$

Thus if we select "$\oplus$" as the IF t-conorm $\mathcal{S}$, then we get the special intuitionistic fuzzy prioritized "and" aggregation operator as follows:

$$C'(x) = \text{IFPRI-OR}((C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x),$$
$$\ldots, C_{qn_q}(x)))$$
$$= (t'_1(x)C_{11}(x) \oplus \ldots \oplus t'_1(x)C_{1n_1}(x)) \oplus \ldots$$
$$\oplus(t'_q(x)C_{q1}(x) \oplus \ldots \oplus t'_q(x)C_{qn_q}(x))$$
$$= \langle h^{-1}(\sum_{i=1}^{q}\sum_{j=1}^{n_i} \mu_{t'_i}(x)h(\mu_{ij}(x))),$$
$$g^{-1}(\sum_{i=1}^{q}\sum_{j=1}^{n_i}(1 - \nu_{t'_i}(x))g(\nu_{ij}(x)))\rangle. \quad (33)$$

Case 1: If $g(t) = -\ln(t)$, then we have

$$C'(x) = \text{IFPRI-OR}((C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x),$$
$$\ldots, C_{qn_q}(x)))$$
$$= \langle 1 - \prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1 - u_{ij}(x))^{u_{t'_i}(x)}),$$
$$\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(\nu_{ij}(x))^{1-\nu_{t'_i}(x)})\rangle. \quad (34)$$

Case 2: If $g(t) = \ln(\frac{2-t}{t})$, then we have

$$
\begin{aligned}
&C'(x)\\
&= \text{IFPRI-OR}((C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x), \\
&\quad \ldots, C_{qn_q}(x)))\\
&= \langle \frac{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1+\mu_{ij}(x))^{\mu_{t_i'}(x)}) - \prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1-\mu_{ij}(x))^{\mu_{t_i'}(x)})}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1+\mu_{ij}(x))^{\mu_{t_i'}(x)}) + \prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1-\mu_{ij}(x))^{\mu_{t_i'}(x)})},\\
&\quad \frac{2\prod_{i=1}^{q}(\prod_{j=1}^{n_i}\nu_{ij}(x)^{1-\nu_{t_i'}(x)})}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(2-\nu_{ij}(x))^{1-\nu_{t_i'}(x)}) + \prod_{i=1}^{q}(\prod_{j=1}^{n_i}\nu_{ij}(x)^{1-\nu_{t_i'}(x)})}\rangle. \quad (35)
\end{aligned}
$$

Case 3: If $g(t) = \ln(\frac{\gamma+(1-\gamma)t}{t})$, $\gamma > 0$, then we have

$$
\begin{aligned}
&C'(x)\\
&= \text{IFPRI-OR}((C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x), \\
&\quad \ldots, C_{qn_q}(x)))\\
&= \langle \frac{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1+(\gamma-1)\mu_{ij}(x))^{\mu_{t_i'}(x)}) - \prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1-\mu_{ij}(x))^{\mu_{t_i'}(x)})}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1+(\gamma-1)\mu_{ij}(x))^{\mu_{t_i'}(x)}) + (\gamma-1)\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1-\mu_{ij}(x))^{\mu_{t_i'}})},\\
&\quad \frac{\gamma\prod_{i=1}^{q}(\prod_{j=1}^{n_i}\nu_{ij}(x)^{1-\nu_{t_i'}(x)})}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1+(\gamma-1)(1-\nu_{ij}(x)))^{1-\nu_{t_i'}(x)}) + (\gamma-1)\prod_{i=1}^{q}(\prod_{j=1}^{n_i}\nu_{ij}(x)^{1-\nu_{t_i'}(x)})}\rangle. \quad (36)
\end{aligned}
$$

Especially, if $\gamma = 1$, then Eq.(36) reduces to Eq.(34); if $\gamma = 2$, then Eq.(36) reduces to Eq.(35).

Case 4: If $g(t) = \ln(\frac{\gamma-1}{\gamma^t-1})$, $\gamma > 1$, then we have Eq.(37).

$$
\begin{aligned}
C'(x) &= \text{IFPRI-OR}((C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x), \ldots, \\
&\quad C_{qn_q}(x)))\\
&= \langle 1 - \log_\gamma(1 + \frac{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(\gamma^{1-\mu_{ij}(x)}-1)^{\mu_{t_i'}(x)})}{\gamma-1}),\\
&\quad \log_\gamma(1 + \frac{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(\gamma^{\nu_{ij}(x)}-1)^{1-\nu_{t_i'}(x)})}{\gamma-1})\rangle \quad (37)
\end{aligned}
$$

Especially, if $\gamma \to 1$, then Eq.(37) reduces to Eq.(34).

III. INTUITIONISTIC FUZZY PRI-AND AND PRI-OR AGGREGATION OPERATORS BASED ON THE PRIORITY DEGREES

In the previous section, we reviewed some basic knowledge about intuitionistic fuzzy prioritized aggregation operators. Li and He [32] introduced the intuitionistic fuzzy PRI-AND and PRI-OR aggregation operators. After that, Li [33] proposed concept of the priority degrees and gave three kinds of prioritized aggregation operators based on the priority degrees. In this section, we shall apply the priority degrees to intuitionistic fuzzy PRI-AND and PRI-OR aggregation operators and introduce two kinds of intuitionistic fuzzy prioritied aggregation using the new concept of priority degrees, extend some particular cases for intuitionistic fuzzy PRI-AND and PRI-OR aggregation operators based on the priority degrees under different forms of t-norm and t-conorm.

Now we consider the multi-attribute decision making problems which attributes are represented by intuitionistic fuzzy information. Suppose that we have a collection of attributes $C = \{C_1, C_2, \ldots, C_n\}$ and a set of alternatives $X = \{x_1, x_2, \ldots, x_m\}$. Then we divide a collection of attributes C into q distinct categories $H_1, H_2, \ldots, H_q$ such that $H_i = \{C_{i1}, C_{i2}, \ldots, C_{in_i}\}$ and $C_{ij} \in C$, $(i = 1, 2, \ldots, q; j = 1, 2, \ldots, n_i)$ are the attributes in the category H_i. The attributes in the category H_i may have "higher" priorities than the attributes in the category H_{i+1}. In addition, the attributes in the category H_i sometimes may have "particularly high" priorities than the attributes in H_{i+1}. Based on the above consideration, Li [33] introduce the concept of the priority degrees to describe the degree of this priority relationship.

Definition 11 [33]: The attributes in $C = \{C_1, C_2, \ldots, C_n\}$ are partitioned into q distinct categories $H_1, H_2, \ldots, H_q$, and there are $q - 1$ priority orders "$\succ$" in the prioritization relationship $H_1 \succ H_2 \succ \cdots \succ H_q$. We assign the ith priority order "$\succ$" with a real nonnegative number d_i, that is, $d_i > 0$, and d_i is called the degree of the ith priority order "$\succ$", $i = 1, 2, \ldots, q - 1$. We get $q - 1$ priority degrees $d_1, d_2, \ldots, d_{q-1}$. We then get a prioritization relationship among these categories:

$$H_1 \succ_{d_1} H_2 \succ_{d_2} \cdots \succ_{d_{q-1}} H_q, \quad (38)$$

where $H_i \succ_{d_i} H_{i+1}$ indicates that the attributes in the category H_i have a d_i-higher priority than those in H_{i+1}. When each priority level has only one attribute, we can get $n_i = 1$ for each $i = 1, 2, \ldots, q$. In this case, $q = n$, $H_i = \{C_i\}$ for each $i = 1, 2, \ldots, n$, and prioritization relationship $\{C_1\} \succ_{d_1} \{C_2\} \succ_{d_2} \succ \cdots \succ_{d_{n-1}} \{C_n\}$ can be expressed as

$$C_1 \succ_{d_1} C_2 \succ_{d_2} \succ \cdots \succ_{d_{n-1}} C_n. \quad (39)$$

Based on the above analysis, we have an IFV $C_{ij}(x) = \langle \mu_{ij}(x), \nu_{ij}(x) \rangle \in L^*$ for each attribute C_{ij}, where $\mu_{ij}(x)$ represents the degree of satisfaction of the alternative x with the attribute C_{ij} and $\nu_{ij}(x)$ represents the degree of dissatisfaction of the alternative x with the attribute C_{ij}. In the following, we will develop the intuitionistic fuzzy PRI-AND aggregation operator and PRI-OR aggregation operator based on the priority degrees.

A. INTUITIONISTIC FUZZY PRIORITIZED "AND" AGGREGATION OPERATORS BASED ON THE PRIORITY DEGREES

In this section, we will extend the priority degrees to intuitionistic fuzzy prioritized "and" aggregation operator and intuitionistic fuzzy prioritized "or" aggregation operator motivated by Li [32]. First, we develop intuitionistic fuzzy prioritized "and" aggregation operator based on priority degrees (IFPRI-AND$_d$).

Let $s_0(x) = \langle 1, 0\rangle$ and

$$s_i(x) = \mathcal{T}_M\{C_{ij}(x) \mid j = 1, 2, \ldots, n_i\}, \tag{40}$$

for $i = 1, 2, \ldots, q$.

Define $w_{ij}(x)$ as the prioritized weight associated with $C_{ij}(x)$, then

$$w_{ij}(x) = t_i(x) = \bigotimes_{k=1}^{i} s_{k-1}^{d_{k-1}}(x)$$
$$= s_0^{d_0}(x) \otimes s_1^{d_1}(x) \otimes \ldots \otimes s_{i-1}^{d_{i-1}}(x). \tag{41}$$

Now we calculate the weights $w_{ij}(x)$ for $i = 1, \ldots, q; j = 1, \ldots, n_i$, define the IF t-norm as $\mathcal{T}$, we have

$$s_i(x) = \mathcal{T}_M\{C_{ij}(x) \mid j = 1, 2, \ldots n_i\}$$
$$= \langle \min(u_{i1}(x), \ldots, u_{in_i}(x)), \max(v_{i1}(x), \ldots, v_{in_i}(x))\rangle.$$

Assume that $\min(u_{i1}(x), \ldots, u_{in_i}(x)) = u_{s_i}(x)$, $\max(v_{i1}(x), \ldots, v_{in_i}(x)) = v_{s_i}(x)$, then we have $s_i(x) = \langle \mu_{s_i}(x), \nu_{s_i}(x)\rangle$. By the IF t-norm, we can compute the weights as follows:

$$t_i(x) = \bigotimes_{k=1}^{i} s_{k-1}^{d_{k-1}}(x)$$
$$= \langle 1, 0\rangle^{d_0} \otimes s_1^{d_1}(x) \otimes \ldots \otimes s_{i-1}^{d_{i-1}}(x)$$
$$= s_1^{d_1}(x) \otimes \ldots \otimes s_{i-1}^{d_{i-1}}(x)$$
$$= \langle \mu_{s_1}^{d_1}(x), 1 - (1 - \nu_{s_1}(x))^{d_1}\rangle \otimes \langle \mu_{s_2}^{d_2}(x), 1 - (1 - \nu_{s_2}(x))^{d_2}\rangle \otimes \ldots \otimes \langle \mu_{s_{i-1}}^{d_{i-1}}(x), 1-(1 - \nu_{s_{i-1}}(x))^{d_{i-1}}\rangle$$
$$= \langle \prod_{k=1}^{i-1} \mu_{s_k}^{d_k}(x), 1 - \prod_{k=1}^{i-1}(1 - \nu_{s_k}(x))^{d_k}\rangle.$$

Assume that $\prod_{k=1}^{i-1} \mu_{s_k}^{d_k}(x) = \mu_{t_i}(x)$, $1 - \prod_{k=1}^{i-1}(1 - \nu_{s_k}(x))^{d_k} = \nu_{t_i}(x)$, then we get $s_i(x) = \langle \mu_{s_i}(x), \nu_{s_i}(x)\rangle$. Thus we get

$$w_{ij}(x) = t_i(x) = \langle \mu_{s_i}(x), \nu_{s_i}(x)\rangle$$
$$= \langle \prod_{k=1}^{i-1} \mu_{s_k}(x), 1 - \prod_{k=1}^{i-1}(1 - \nu_{s_k}(x))\rangle\rangle.$$

The weights of the attributes in the same priority level are the same. Let $\mathcal{T}$ be an IF t-norm, then define the IFPRI-AND operator based on the priority degrees as follow:

Definition 12: Assume that $\{(C_{11}, \ldots, C_{1n_1}), \ldots, (C_{q1}, \ldots, C_{qn_q})\}$ is the collection of attributes with different priority levels. $\{(C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x), \ldots, C_{qn_q}(x))\}$ is the colletion of preference values, which take the form of IFVs, for the alternative $x \in X$ with respect to the attributes $\{(C_{11}, \ldots, C_{1n_1}), \ldots, (C_{q1}, \ldots, C_{qn_q})\}$. Define the operator IFPRI-AND$_d$: $(L^*)^n \to L^*$ as follow:

$$C(x) = \text{IFPRI-AND}_d((C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x), \ldots, C_{qn_q}(x)))$$
$$= \mathcal{T}(\mathcal{T}(C_{11}^{w_{11}(x)}(x), \ldots, C_{1n_1}^{w_{1n_1}(x)}(x)), \ldots, \mathcal{T}(C_{q1}^{w_{q1}(x)}(x), \ldots, C_{qn_q}^{w_{qn_q}(x)}(x))), \tag{42}$$

where the weight $w_{ij}(x)$ can be obtained by Eq.(41). IFPRI-AND$_d$ is called an intuitionistic fuzzy priority "and" operator based on the priority degrees.

Since $w_{ij} = t_i(x)$, then we also have

$$C(x) = \mathcal{T}(\mathcal{T}(C_{11}^{t_1(x)}(x), \ldots, C_{1n_1}^{t_1(x)}(x)), \ldots, \mathcal{T}(C_{q1}^{t_q(x)}(x), \ldots, C_{qn_q}^{t_q(x)}(x))). \tag{43}$$

If we select "$\otimes$" as IF t-norm, according to Eq.(13), we can get

$$C(x) = \text{IFPRI-AND}_d((C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x), \ldots, C_{qn_q}(x)))$$
$$= \mathcal{T}(\mathcal{T}(C_{11}^{w_{11}(x)}(x), \ldots, C_{1n_1}^{w_{1n_1}(x)}(x)), \ldots, \mathcal{T}(C_{q1}^{w_{q1}(x)}(x), \ldots, C_{qn_q}^{w_{qn_q}(x)}(x)))$$
$$= (C_{11}^{t_1(x)}(x) \otimes \ldots \otimes C_{1n_1}^{t_1(x)}) \otimes \ldots \otimes (C_{q1}^{t_q(x)}(x) \otimes \ldots \otimes C_{qn_q}^{t_q(x)}(x))$$
$$= \langle g^{-1}(\sum_{i=1}^{q}\sum_{j=1}^{n_i}(1 - \nu_{t_i}(x))g(\mu_{ij}(x))), h^{-1}(\sum_{i=1}^{q}\sum_{j=1}^{n_i} \mu_{t_i}(x)h(\nu_{ij}(x)))\rangle. \tag{44}$$

Case 1: If $g(t) = -\ln(t)$, then we have

$$C(x) = \text{IFPRI-AND}_d((C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x), \ldots, C_{qn_q}(x)))$$
$$= \langle \prod_{i=1}^{q}(\prod_{j=1}^{n_i} u_{ij}(x)^{1-\nu_{t_i}(x)}), 1 - \prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1 - \nu_{ij}(x))^{u_{t_i}(x)})\rangle$$
$$= \langle \prod_{i=1}^{q}(\prod_{j=1}^{n_i} \mu_{ij}(x)^{\prod_{k=1}^{i-1}(1-\nu_{s_k}(x))^{d_k}}), 1 - \prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1 - \nu_{ij}(x))^{\prod_{k=1}^{i-1}\mu_{s_k}(x)^{d_k}}\rangle. \tag{45}$$

Case 2: If $g(t) = \ln(\frac{2-t}{t})$, then we have

$$C(x) = \text{IFPRI-AND}_d((C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x), \ldots, C_{qn_q}(x)))$$
$$= \langle \frac{2\prod_{i=1}^{q}(\prod_{j=1}^{n_i} \mu_{ij}(x)^{1-\nu_{t_i}(x)})}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(2 - \mu_{ij}(x))^{1-\nu_{t_i}(x)}) + \prod_{i=1}^{q}(\prod_{j=1}^{n_i} \mu_{ij}(x)^{1-\nu_{t_i}(x)})},$$

$$1-\frac{2\prod_{i=1}^{q}(\prod_{j=i}^{n_i}(1-\nu_{ij}(x))^{\mu_{t_i}(x)})}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1+\nu_{ij}(x))^{\mu_{t_i}(x)})+\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1-\nu_{ij}(x))^{\mu_{t_i}(x)})}\rangle$$

$$=\langle\frac{2\prod_{i=1}^{q}(\prod_{j=1}^{n_i}\mu_{ij}(x)^{\prod_{k=1}^{i-1}(1-\nu_{s_k}(x))^{d_k}})}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(2-\mu_{ij}(x))^{\prod_{k=1}^{i-1}(1-\nu_{s_k}(x))^{d_k}})+\prod_{i=1}^{q}(\prod_{j=1}^{n_i}\mu_{ij}(x)^{\prod_{k=1}^{i-1}(1-\nu_{s_k}(x))^{d_k}})},$$

$$1-\frac{2\prod_{i=1}^{q}(\prod_{j=i}^{n_i}(1-\nu_{ij}(x))^{\prod_{k=1}^{i-1}\mu_{s_k}(x)^{d_k}})}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1+\nu_{ij}(x))^{\prod_{k=1}^{i-1}\mu_{s_k}(x)^{d_k}})+\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1-\nu_{ij}(x))^{\prod_{k=1}^{i-1}\mu_{s_k}(x)^{d_k}})}\rangle. \quad (46)$$

Case 3: If $g(t)=\ln(\frac{\gamma+(1-\gamma)t}{t}), \gamma>0$, then we have Eq.(47), as shown at the bottom of the next page.

Especially, if $\gamma=1$, then Eq.(47) reduces to Eq.(45); if $\gamma=2$, then Eq.(47) reduces to Eq.(46).

Case 4: If $g(t)=\ln(\frac{\gamma-1}{\gamma^t-1}), \gamma>1$, then we have

$$C(x)=\text{IFPRI-AND}_d((C_{11}(x),\ldots,C_{1n_1}(x)),\ldots,(C_{q1}(x),\ldots,C_{qn_q}(x)))$$
$$=\langle\log_\gamma(1+\frac{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}\gamma^{\mu_{ij}(x)}-1)^{1-\nu_{t_i}(x)}}{\gamma-1}),$$
$$1-\log_\gamma(1+\frac{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(\gamma^{1-\nu_{ij}(x)}-1)^{\mu_{t_i}(x)})}{\gamma-1})\rangle$$
$$=\langle\log_\gamma(1+\frac{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}\gamma^{\mu_{ij}(x)}-1)^{\prod_{k=1}^{i-1}(1-\nu_{s_k}(x))^{d_k}}}{\gamma-1}),$$
$$1-\log_\gamma(1+\frac{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(\gamma^{1-\nu_{ij}(x)}-1)^{\prod_{k=1}^{i-1}\mu_{s_k}^{d_k}(x)})}{\gamma-1})\rangle. \quad (48)$$

Especially, if $\gamma\to1$, then Eq.(48) reduces to Eq.(45).

In the above intuitionistic fuzzy prioritized "and" aggregation operator based on the priority degrees, we can stand a_{ij} for $C_{ij}(x)$, $i=1,2,\ldots,q; j=1,2,\ldots,n_i$. Then $w_{ij}=t_i=\bigotimes_{k=1}^{i-1}s_k^{d_k}=s_0^{d_0}\otimes s_1^{d_1}\otimes\ldots\otimes s_{i-1}^{d_{i-1}}$. If all the priority degrees $d_1=d_2=\ldots=d_{q-1}=1$, we have $t_1=\langle1,0\rangle$, $t_2=s_1, t_3=s_1\otimes s_2, t_q=s_1\otimes s_2\otimes\ldots\otimes s_{q-1}$. Then IFPRI-AND$_d$ aggregation operators reduce to IFPRI-AND aggregation operators. Therefore we shall get some properties of the IFPRI-AND$_d$ aggregation operators according to changes of the priority degrees d_k as follows:

Proposition 13:

$$\lim_{(d_1,d_2,\ldots,d_{n-1})\to(1,1,\ldots,1)} IFPRI-AND_d((a_{11},\ldots,a_{1n_1}),\ldots,(a_{q1},\ldots,a_{qn_q}))$$
$$=\mathcal{T}(\mathcal{T}(a_{11}^{t_1},\ldots,a_{1n_1}^{t_1}),\ldots,\mathcal{T}(a_{q1}^{t_q},\ldots,a_{qn_q}^{t_q})).$$

Proposition 14:

$$\lim_{(d_1,d_2,\ldots,d_{n-1})\to(0,0,\ldots,0)} IFPRI-AND_d((a_{11},\ldots,a_{1n_1}),\ldots,(a_{q1},\ldots,a_{qn_q}))$$
$$=\mathcal{T}(\mathcal{T}(a_{11},\ldots,a_{1n_1}),\ldots,\mathcal{T}(a_{q1},\ldots,a_{qn_q})).$$

Proof: If $(d_1,d_2,\ldots,d_{n-1})\to(0,0,\ldots,0)$, $t_1=\langle1,0\rangle$, then we have:

For each $i=2,3,\ldots,n$, $w_{ij}=t_i={s_1}^{d_1}\otimes\ldots\otimes{s_{i-1}}^{d_{i-1}}\to{s_1}^0\otimes{s_2}^0\otimes\ldots\otimes{s_{i-1}}^0=\langle1,0\rangle$.

Thus IFPRI-AND$_d((a_{11},\ldots,a_{1n_1}),\ldots,(a_{q1},\ldots,a_{qn_q})\to\mathcal{T}(\mathcal{T}(a_{11},\ldots,a_{1n_1}),\ldots,\mathcal{T}(a_{q1},\ldots,a_{qn_q}))$.

Especially, if we select "$\otimes$" as the IF t-norm $\mathcal{T}$, we have

$$\text{IFPRI-AND}_d((a_{11},\ldots,a_{1n_1}),\ldots,(a_{q1},\ldots,a_{qn_q}))$$
$$=\mathcal{T}(\mathcal{T}(a_{11}^{t_1},\ldots,a_{1n_1}^{t_1}),\ldots,\mathcal{T}(a_{q1}^{t_q},\ldots,a_{qn_q}^{t_q}))$$
$$\to\mathcal{T}(\mathcal{T}(a_{11},\ldots,a_{1n_1}),\ldots,\mathcal{T}(a_{q1},\ldots,a_{qn_q}))$$
$$=(a_{11}^{t_1}\otimes\ldots\otimes a_{1n_1}^{t_1})\otimes\ldots\otimes(a_{q1}^{t_q}\otimes\ldots\otimes a_{qn_q}^{t_q})$$
$$\to(\langle\mu_{11}(x),1-(1-\nu_{11}(x))\rangle^{\langle1,0\rangle}\otimes\ldots\otimes\langle\mu_{1n_1}(x),1-(1-\nu_{1n_1}(x))\rangle^{\langle1,0\rangle})\otimes\ldots\otimes(\langle\mu_{q1}(x),1-(1-\nu_{q1}(x))\rangle^{\langle1,0\rangle}\otimes\ldots\otimes\langle\mu_{qn_q}(x),1-(1-\nu_{qn_q}(x))\rangle^{\langle1,0\rangle})$$
$$=(\langle\mu_{11}(x),1-(1-\nu_{11}(x))\rangle\otimes\ldots\otimes\langle\mu_{1n_1}(x),1-(1-\nu_{1n_1}(x))\rangle)\otimes\ldots\otimes(\langle\mu_{q1}(x),1-(1-\nu_{q1}(x))\rangle\otimes\ldots\otimes\langle\mu_{qn_q}(x),1-(1-\nu_{qn_q}(x))\rangle)$$
$$=\langle\prod_{i=1}^{q}(\prod_{j=1}^{n_1}u_{ij}(x)),1-\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1-\nu_{ij}(x))\rangle.$$

Proposition 15:

$$\lim_{d_1\to+\infty} IFPRI-AND_d((a_{11},\ldots,a_{1n_1}),\ldots,(a_{q1},\ldots,a_{qn_q}))=\mathcal{T}(\mathcal{T}(a_{11},\ldots,a_{1n_1})).$$

Proof: If $d_1\to+\infty$, $t_1=\langle1,0\rangle$, then we can get $w_{ij}=t_i={s_1}^{d_1}\otimes\ldots\otimes{s_{i-1}}^{d_{i-1}}\to\langle0,1\rangle$ for each $i=2,3,\ldots,q$, and $w_{1j}=t_1=\langle1,0\rangle$ for $i=1$.

Thus IFPRI-AND$_d((a_{11},\ldots,a_{1n_1}),\ldots,(a_{q1},\ldots,a_{qn_q}))\to\mathcal{T}(\mathcal{T}(a_{11},\ldots,a_{1n_1}))$.

Especially, if we select "$\otimes$" as the IF t-norm $\mathcal{T}$, we have

$$\text{IFPRI-AND}_d((a_{11},\ldots,a_{1n_1}),\ldots,(a_{q1},\ldots,a_{qn_q}))$$
$$=\mathcal{T}(\mathcal{T}(a_{11}^{t_1},\ldots,a_{1n_1}^{t_1}),\ldots,\mathcal{T}(a_{q1}^{t_q},\ldots,a_{qn_q}^{t_q}))$$
$$\to\mathcal{T}(\mathcal{T}(a_{11},\ldots,a_{1n_1})$$
$$=a_{11}^{t_1}\otimes\ldots\otimes a_{1n_1}^{t_1}\to\langle\mu_{11}(x),\nu_{11}(x)\rangle^{\langle1,0\rangle}\otimes\ldots\otimes\langle\mu_{1n_1}(x),\nu_{1n_1}(x)\rangle^{\langle1,0\rangle}$$
$$=\langle\mu_{11}(x),\nu_{11}(x)\rangle\otimes\ldots\otimes\langle\mu_{1n_1}(x),\nu_{1n_1}(x)\rangle$$

$$= \langle \prod_{j=1}^{n_1} \mu_{1j}(x), 1 - \prod_{j=1}^{n_1}(1 - \nu_{1j}(x))\rangle.$$

Proposition 16:

$$\lim_{(d_1,d_2,\ldots,d_{k+1})\to(\underbrace{0,\ldots,0}_{k},+\infty)} IFPRI-AND_d((a_{11},\ldots, a_{1n_1}),\ldots,(a_{q1},\ldots,a_{qn_q})) = \mathcal{T}(\mathcal{T}(a_{11},\ldots,a_{1n_1}),\ldots,\mathcal{T}(a_{k+1,1},\ldots,a_{k+1,n_{k+1}})).$$

Proof: If $(d_1,d_2,\ldots,d_{k+1}) \to (\underbrace{0,\ldots,0}_{k},+\infty)$, we have $t_i = {s_1}^{d_1} \otimes {s_2}^{d_2} \otimes \ldots \otimes {s_{i-1}}^{d_{i-1}} \to {s_0}^0 \otimes {s_1}^0 \otimes {s_2}^0 \otimes \ldots \otimes {s_{i-1}}^0 = \langle 1,0\rangle$ for each $i = 2,3,\ldots,k+1$, and $t_i = {s_1}^{d_1} \otimes {s_2}^{d_2} \otimes \ldots \otimes {s_{i-1}}^{d_{i-1}} \to {s_1}^0 \otimes \ldots \otimes {s_k}^0 \otimes {s_{k+1}}^{+\infty} \otimes \ldots \otimes {s_{i-1}}^{+\infty} = \langle 0,1\rangle$ for $i = k+2,k+3,\ldots,q$.

Thus IFPRI-AND$_d((a_{11},\ldots,a_{1n_1}),\ldots,(a_{q1},\ldots,a_{qn_q})) \to \mathcal{T}(\mathcal{T}(a_{11},\ldots,a_{1n_1}),\ldots,\mathcal{T}(a_{k+1,1},\ldots,a_{k+1,n_{k+1}}))$.

Especially, if we select "$\otimes$" as the IF t-norm $\mathcal{T}$, we have

$$\begin{aligned}
&\text{IFPRI-AND}_d((a_{11},\ldots,a_{1n_1}),\ldots,(a_{q1},\ldots,a_{qn_q}))\\
&\to \mathcal{T}(\mathcal{T}(a_{11},\ldots,a_{1n_1}),\ldots,\mathcal{T}(a_{k+1,1},\ldots,a_{k+1,n_{k+1}}))\\
&= (a_{11}^{t_1} \otimes \ldots \otimes a_{1n_1}^{t_1}) \otimes \ldots \otimes (a_{k+1,1}^{t_{k+1}} \otimes \ldots \otimes a_{k+1,n_{k+1}}^{t_{k+1}})\\
&\to (\langle \mu_{11}(x),\nu_{11}(x)\rangle^{\langle 1,0\rangle} \otimes \ldots \otimes \langle \mu_{1n_1}(x),\nu_{1n_1}(x)\rangle^{\langle 1,0\rangle})\\
&\quad \otimes \ldots \otimes (\langle \mu_{k+1,1}(x),\nu_{k+1,1}(x)\rangle^{\langle 1,0\rangle}\\
&\quad \otimes \ldots \otimes \langle \mu_{k+1,n_{k+1}}(x),\\
&\quad \nu_{k+1,n_{k+1}}(x)\rangle^{\langle 1,0\rangle})\\
&= (\langle \mu_{11}(x),\nu_{11}(x)\rangle \otimes \ldots \otimes \langle \mu_{1n_1}(x),\nu_{1n_1}(x)\rangle)\\
&\quad \otimes \ldots \otimes (\langle \mu_{k+1,1}(x),\nu_{k+1,1}(x)\rangle \otimes \ldots \otimes \langle \mu_{k+1,n_{k+1}}(x),\\
&\quad \nu_{k+1,n_{k+1}}(x)\rangle)\\
&= \langle \prod_{i=1}^{k+1}(\prod_{j=1}^{n_i} u_{ij}(x)), 1 - \prod_{i=1}^{k+1}(\prod_{j=1}^{n_1}(1 - \nu_{ij}(x))\rangle.
\end{aligned}$$

Proposition 17:

$$\lim_{(d_1,d_2,\ldots,d_{k+1})\to(\underbrace{1,\ldots,1}_{k},+\infty)} IFPRI-AND_d((a_{11},\ldots,a_{1n_1}), \ldots,(a_{q1},\ldots,a_{qn_q})) = \mathcal{T}(\mathcal{T}(a_{11}^{t_1},\ldots,a_{1n_1}^{t_1}),\ldots,\mathcal{T}(a_{k+1,1}^{t_{k+1}},\ldots,a_{k+1,n_{k+1}}^{t_{k+1}})).$$

Proof: If $(d_1,d_2,\ldots,d_{k+1}) \to (\underbrace{1,\ldots,1}_{k},+\infty)$, we have $t_i = {s_1}^{d_1} \otimes {s_2}^{d_2} \otimes \ldots \otimes {s_{i-1}}^{d_{i-1}} \to {s_1}^1 \otimes {s_2}^1 \otimes \ldots \otimes {s_{i-1}}^1 = s_1 \otimes s_2 \otimes \ldots \otimes s_{i-1}$ for each $i = 1,2,\ldots,k$, and $t_i = {s_1}^{d_1} \otimes {s_2}^{d_2} \otimes \ldots \otimes {s_{i-1}}^{d_{i-1}} \to {s_1}^1 \otimes {s_2}^1 \otimes \ldots \otimes {s_k}^1 \otimes {s_{k+1}}^{+\infty} \otimes \ldots \otimes {s_{i-1}}^{+\infty} = \langle 0,1\rangle$ for $i = k+2,k+3,\ldots,q$.

Thus IFPRI-AND$_d((a_{11},\ldots,a_{1n_1}),\ldots,(a_{q1},\ldots,a_{qn_q})) \to \mathcal{T}(\mathcal{T}(a_{11}^{t_1},\ldots,a_{1n_1}^{t_1}),\ldots,\mathcal{T}(a_{k+1,1}^{t_{k+1}},\ldots,a_{k+1,n_{k+1}}^{t_{k+1}}))$ = IFPRI-AND$((a_{11},\ldots,a_{1n_1}),\ldots,(a_{k+1,1},\ldots,a_{k+1,n_{k+1}}))$.

Especially, if we select "$\otimes$" as the IF t-norm $\mathcal{T}$, we have

$$\begin{aligned}
&\text{IFPRI-AND}_d((a_{11},\ldots,a_{1n_1}),\ldots,(a_{q1},\ldots,a_{qn_q})\\
&\to (\langle \mu_{11}(x),\nu_{11}(x)\rangle^{t_1} \otimes \ldots \otimes \langle \mu_{1n_1}(x),\nu_{1n_1}(x)\rangle^{t_1}) \otimes\\
&\ldots \otimes (\langle \mu_{k+1,1}(x),\nu_{k+1,1}(x)\rangle^{t_{k+1}} \otimes \ldots \otimes \langle \mu_{k+1,n_{k+1}}(x),\\
&\nu_{k+1,n_{k+1}}(x)\rangle^{t_{k+1}})\\
&= \text{IFPRI-AND}((a_{11},\ldots,a_{1n_1}),\ldots,(a_{k+1,1},\ldots,a_{k+1,n_{k+1}})).
\end{aligned}$$

$$\begin{aligned}
C(x) &= IFPRI-AND_d((C_{11}(x),\ldots,C_{1n_1}(x)),\ldots,(C_{q1}(x),\ldots,C_{qn_q}(x)))\\
&= \langle \frac{\gamma \prod_{i=1}^{q}(\prod_{j=1}^{n_i}(\mu_{ij}(x))^{1-\nu_{t_i}(x)})}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(\gamma + (1-\gamma)\mu_{ij}(x))^{1-\nu_{t_i}(x)}) + (\gamma - 1)(\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(\mu_{ij}(x))^{1-\nu_{t_i}(x)}))},\\
&\quad 1 - \frac{\gamma \prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1-\nu_{ij}(x))^{\mu_{t_i}(x)})}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(\gamma + (1-\gamma)(1-\nu_{ij}(x)))^{\mu_{t_i}(x)}) + (\gamma - 1)(\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1-\nu_{ij}(x))^{\mu_{t_i}(x)}))}\rangle\\
&= \langle \frac{\gamma \prod_{i=1}^{q}(\prod_{j=1}^{n_i}(\mu_{ij}(x))^{\prod_{k=1}^{i-1}(1-\nu_{s_k}(x))^{d_k}})}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(\gamma + (1-\gamma)\mu_{ij}(x))^{\prod_{k=1}^{i-1}(1-\nu_{s_k}(x))^{d_k}}) + (\gamma - 1)(\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(\mu_{ij}(x))^{\prod_{k=1}^{i-1}(1-\nu_{s_k}(x))^{d_k}}))},\\
&\quad 1 - \frac{\gamma \prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1-\nu_{ij}(x))^{\prod_{k=1}^{i-1}\mu_{s_k}(x)^{d_k}})}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(\gamma + (1-\gamma)(1-\nu_{ij}(x)))^{\prod_{k=1}^{i-1}\mu_{s_k}(x)^{d_k}}) + (\gamma - 1)(\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1-\nu_{ij}(x))^{\prod_{k=1}^{i-1}\mu_{s_k}(x)^{d_k}}))}\rangle. \qquad (47)
\end{aligned}$$

B. INTUITIONISTIC FUZZY PRIORITIZED "OR" AGGREGATION OPERATORS BASED ON THE PRIORITY DEGREES

Next we will propose intuitionistic fuzzy prioritized "or" aggregation operators based on the priority degrees (IFPRI-OR$_d$).

Let $s_0(x) = \langle 1, 0\rangle$ and

$$s_i'(x) = \mathcal{S}_M\{C_{ij}(x) \mid j = 1, 2, \ldots, n_i\}, \tag{49}$$

for $i = 1, 2, \ldots, q$.

Define $w_{ij}'(x)$ as the prioritized weight associated with $C_{ij}(x)$, then

$$\begin{aligned} w_{ij}'(x) = t_i'(x) &= \bigotimes_{k=1}^{i} s'^{d_{k-1}}_{k-1}(x) \\ &= s'^{d_0}_0(x) \otimes s'^{d_1}_1(x) \otimes \ldots \otimes s'^{d_{i-1}}_{i-1}(x). \end{aligned} \tag{50}$$

Now we calculate the weights $w_{ij}'(x)$ for $i = 1, \ldots, q; j = 1, \ldots, n_i$ and define the IF t-conorm as $\mathcal{S}$, we have

$$\begin{aligned} s_i'(x) &= \mathcal{S}_M\{C_{ij}(x) \mid j = 1, 2, \ldots n_i\} \\ &= \langle \max(u_{i1}(x), \ldots, u_{in_i}(x)), \min(v_{i1}(x), \ldots, v_{in_i}(x))\rangle. \end{aligned}$$

Assume that $\max(u_{i1}(x), \ldots, u_{in_i}(x)) = u_{s_i'}(x)$, $\min(v_{i1}(x), \ldots, v_{in_i}(x)) = v_{s_i'}(x)$, then we have $s_i'(x) = \langle \mu_{s_i'}(x), v_{s_i'}(x)\rangle$. By the IF t-conorm, we can compute the weights as follow:

$$\begin{aligned} t'_i(x) &= \bigotimes_{k=1}^{i} s'^{d_{k-1}}_{k-1}(x) \\ &= \langle 1, 0\rangle^{d_0} \otimes s'^{d_1}_1(x) \otimes \ldots \otimes s'^{d_{i-1}}_{i-1}(x) \\ &= s'^{d_1}_1(x) \otimes \ldots \otimes s'^{d_{i-1}}_{i-1}(x) \\ &= \langle \mu^{d_1}_{s_1'}(x), 1 - (1 - v_{s_1'}(x))^{d_1}\rangle \otimes \langle \mu^{d_2}_{s_2'}(x), 1 - (1 \\ &\quad - v_{s_2'}(x))^{d_2}\rangle \otimes \ldots \otimes \langle \mu^{d_{i-1}}_{s_{i-1}'}(x), 1-(1-v_{s_{i-1}'}(x))^{d_{i-1}}\rangle \\ &= \langle \prod_{k=1}^{i-1} \mu^{d_k}_{s_k'}(x), 1 - \prod_{k=1}^{i-1}(1 - v_{s_k'}(x))^{d_k}\rangle. \end{aligned}$$

Assume that $\prod_{k=1}^{i-1} \mu^{d_k}_{s_k'}(x) = \mu_{t_i'}(x)$, $1 - \prod_{k=1}^{i-1}(1 - v_{s_k'}(x))^{d_k} = v_{t_i'}(x)$, then we get $s_i'(x) = \langle \mu_{s_i'}(x), v_{s_i'}(x)\rangle$. Thus we have

$$\begin{aligned} w_{ij}'(x) = t_i'(x) &= \langle \mu_{s_i'}(x), v_{s_i'}(x)\rangle \\ &= \langle \prod_{k=1}^{i-1} \mu_{s_k'}(x), 1 - \prod_{k=1}^{i-1}(1 - v_{s_k'}(x))\rangle. \end{aligned}$$

The weights of the attributes in the same priority level are the same. Let $\mathcal{S}$ be an IF t-conorm, then define the IFPRI-OR operator based on the priority degrees as follow:

Definition 18: Assume that $\{(C_{11}, \ldots, C_{1n_1}), \ldots, (C_{q1}, \ldots, C_{qn_q})\}$ is the collection of attributes with different priority levels. $\{(C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x), \ldots, C_{qn_q}(x))\}$ is the colletion of preference values, which take the form of IFVs for the alternative $x \in X$ with respect to the attributes $\{(C_{11}, \ldots, C_{1n_1}), \ldots, (C_{q1}, \ldots, C_{qn_q})\}$. Define the operator IFPRI-OR$_d$: $(L^*)^n \to L^*$ as follow:

$$\begin{aligned} C'(x) &= \text{IFPRI-OR}_d((C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x), \\ &\qquad \ldots, C_{qn_q}(x))) \\ &= \mathcal{S}(\mathcal{S}(w'_{11}(x)C_{11}(x), \ldots, w'_{1n_1}(x)C_{1n_1}(x)), \ldots, \\ &\qquad \mathcal{S}(w'_{q1}(x)C_{q1}(x), \ldots, w'_{qn_q}(x)C_{qn_q}(x))), \end{aligned} \tag{51}$$

where the weight $w_{ij}'(x)$ can be obtained by Eq.(47). IFPRI-OR$_d$ is called an intuitionistic fuzzy prioritized "or" aggregation operator based on the priority degrees.

Since $w_{ij}' = t_i'(x)$, then we also have

$$\begin{aligned} C(x) = \mathcal{S}(&\mathcal{S}(t_1'(x)C_{11}(x), \ldots, t_1'(x)C_{1n_1}(x)), \ldots, \\ &\mathcal{S}(t_q'(x)C_{q1}(x), \ldots, t_q'(x)C_{qn_q}(x))). \end{aligned} \tag{52}$$

If we select "$\oplus$" as IF t-conorm, according to Eq.(12), we can get

$$\begin{aligned} C(x) &= \text{IFPRI-OR}_d((C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x), \\ &\qquad \ldots, C_{qn_q}(x))) \\ &= (t_1'(x)C_{11}(x) \oplus \ldots \oplus t_1'(x)C_{1n_1}(x)) \oplus \ldots \\ &\qquad \oplus(t_q'(x)C_{q1}(x) \oplus \ldots \oplus t_q'(x)C_{qn_q}(x)) \\ &= \langle h^{-1}(\sum_{i=1}^{q}\sum_{j=1}^{n_i} \mu_{t_i'}(x)h(\mu_{ij}(x))), \\ &\qquad g^{-1}(\sum_{i=1}^{q}\sum_{j=1}^{n_i}(1 - v_{t_i'}(x))g(v_{ij}(x)))\rangle. \end{aligned} \tag{53}$$

Case 1: If $g(t) = -\ln(t)$, then we have

$$\begin{aligned} C(x) &= \text{IFPRI-OR}_d((C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x), \\ &\qquad \ldots, C_{qn_q}(x))) \\ &= \langle 1-\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1 - u_{ij}(x))^{u_{t_i'}(x)}), \prod_{i=1}^{q}(\prod_{j=1}^{n_i}(v_{ij}(x))^{1-v_{t_i'}(x)})\rangle \\ &= \langle 1 - \prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1 - u_{ij}(x))^{\prod_{k=1}^{i-1} \mu^{d_k}_{s_k'}(x)}), \\ &\qquad \prod_{i=1}^{q}(\prod_{j=1}^{n_i}(v_{ij}(x))^{\prod_{k=1}^{i-1}(1-v_{s_k'}(x))^{d_k}})\rangle. \end{aligned} \tag{54}$$

Case 2: If $g(t) = \ln(\frac{2-t}{t})$, then we have

$$\begin{aligned} &C(x) \\ &=\text{IFPRI-OR}_d((C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x), \\ &\quad \ldots, C_{qn_q}(x))) \\ &=\langle 1 - \frac{2\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1-\mu_{ij}(x))^{\mu_{t_i'}(x)})}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1 + \mu_{ij}(x))^{\mu_{t_i'}(x)})+\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1 - \mu_{ij}(x))^{\mu_{t_i'}(x)})}, \end{aligned}$$

$$\frac{2\prod_{i=1}^{q}(\prod_{j=1}^{n_i}\nu_{ij}(x)^{1-\nu_{t_i'}(x)})}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1-\nu_{ij}(x))^{1-\nu_{t_i'}(x)})+\prod_{i=1}^{q}(\prod_{j=1}^{n_i}\nu_{ij}(x)^{1-\nu_{t_i'}(x)})}\rangle$$

$$=\langle 1-\frac{2\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1-\mu_{ij}(x))^{\prod_{k=1}^{i-1}\mu_{s_k'}^{d_k}(x)})}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1+\mu_{ij}(x))^{\prod_{k=1}^{i-1}\mu_{s_k'}^{d_k}(x)})+\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1-\mu_{ij}(x))^{\prod_{k=1}^{i-1}\mu_{s_k'}^{d_k}(x)})},$$

$$\frac{2\prod_{i=1}^{q}(\prod_{j=1}^{n_i}\nu_{ij}(x)^{\prod_{k=1}^{i-1}(1-\nu_{s_k'}(x))^{d_k}})}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1-\nu_{ij}(x))^{\prod_{k=1}^{i-1}(1-\nu_{s_k'}(x))^{d_k}})+\prod_{i=1}^{q}(\prod_{j=1}^{n_i}\nu_{ij}(x)^{\prod_{k=1}^{i-1}(1-\nu_{s_k'}(x))^{d_k}})}\rangle. \tag{55}$$

Case 3: If $g(t)=\ln(\frac{\gamma+(1-\gamma)t}{t}), \gamma>0$, then we have Eq.(56), as shown at the bottom of the next page.

Especially, if $\gamma=1$, then Eq.(56) reduces to Eq.(54); if $\gamma=2$, then Eq.(56) reduces to Eq.(55).

Case 4: If $g(t)=\ln(\frac{\gamma-1}{\gamma^t-1}), \gamma>1$, then we have

$$C(x)=\text{IFPRI-OR}_d((C_{11}(x),\ldots,C_{1n_1}(x)),\ldots,(C_{q1}(x),\ldots,C_{qn_q}(x)))$$

$$=\langle 1-\log_\gamma(1+\frac{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(\gamma^{1-\mu_{ij}(x)}-1)^{\mu_{t_i'}(x)})}{\gamma-1}),\ \log_\gamma(1+\frac{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(\gamma^{\nu_{ij}(x)}-1)^{1-\nu_{t_i'}(x)})}{\gamma-1})\rangle$$

$$=\langle 1-\log_\gamma(1+\frac{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(\gamma^{1-\mu_{ij}(x)}-1)^{\prod_{k=1}^{i-1}\mu_{s_k'}^{d_k}(x)})}{\gamma-1}),\ \log_\gamma(1+\frac{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(\gamma^{\nu_{ij}(x)}-1)^{\prod_{k=1}^{i-1}(1-\nu_{s_k'}(x))^{d_k}})}{\gamma-1})\rangle. \tag{57}$$

Especially, if $\gamma\to 1$, then Eq.(57) reduces to Eq.(54).

In the above intuitionistic fuzzy prioritized "or" operator on the priority degrees, we can stand a_{ij} for $C_{ij}(x)$, $i=1,2,\ldots,q; j=1,2,\ldots,n_i$. Then $w_{ij}'=t_i'=\bigotimes_{k=1}^{i-1}s_k'^{d_k}=s_0'^{d_0}\otimes s_1'^{d_1}\otimes\cdots\otimes s_{i-1}'^{d_{i-1}}$. If all the priority degrees $d_1'=d_2'=\cdots=d_{q-1}'=1$, we have $t_1'=\langle 1,0\rangle$, $t_2'=s_1$, $t_3'=s_1'\otimes s_2'$, $t_q'=s_1'\otimes s_2'\otimes\cdots\otimes s_{q-1}'$. Then IFPRI-OR$_d$ operators reduce to IFPRI-OR operators. Therefore we shall get some properties of the IFPRI-OR$_d$ aggregation operators according to changes of the priority degree d_k, as follows:

Proposition 19:

$$\lim_{(d_1,d_2,\ldots,d_{n-1})\to(1,1,\ldots,1)} IFPRI-OR_d((a_{11},\ldots,a_{1n_1}),\ldots,(a_{q1},\ldots,a_{qn_q}))$$
$$=\mathcal{S}(\mathcal{S}(t_1'a_{11},\ldots,t_1'a_{1n_1}),\ldots,\mathcal{S}(t_q'a_{q1},\ldots,t_q'a_{qn_q})).$$

Especially, if we select "$\oplus$" as the IF t-conorm $\mathcal{S}$, when $(d_1,d_2,\ldots,d_{n-1})\to(1,1,\ldots,1)$, then IFPRI-OR$_d((a_{11},\ldots,a_{1n_1}),\ldots,(a_{q1},\ldots,a_{qn_q}))\to$ IFPRI-OR$((a_{11},\ldots,a_{1n_1}),\ldots,(a_{q1},\ldots,a_{qn_q}))$.

Proposition 20:

$$\lim_{(d_1,d_2,\ldots,d_{n-1})\to(0,0,\ldots,0)} IFPRI-OR_d((a_{11},\ldots,a_{1n_1}),\ldots,(a_{q1},\ldots,a_{qn_q}))$$
$$=\mathcal{S}(\mathcal{S}(a_{11},\ldots,a_{1n_1}),\ldots,\mathcal{S}(a_{q1},\ldots,a_{qn_q})).$$

Especially, if we select "$\oplus$" as the IF t-conorm $\mathcal{S}$, for $(d_1,d_2,\ldots,d_{n-1})\to(0,0,\ldots,0)$, then IFPRI-OR$_d((a_{11},\ldots,a_{1n_1}),\ldots,(a_{q1},\ldots,a_{qn_q}))\to\langle 1-\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1-u_{ij}(x))), \prod_{i=1}^{q}(\prod_{j=1}^{n_i}\nu_{ij}(x))\rangle$.

Proposition 21:

$$\lim_{d_1\to+\infty} IFPRI-OR_d((a_{11},\ldots,a_{1n_1}),\ldots,(a_{q1},\ldots,a_{qn_q}))$$
$$=\mathcal{S}(\mathcal{S}(a_{11},\ldots,a_{1n_1})).$$

Especially, if we select "$\oplus$" as the IF t-conorm $\mathcal{S}$, when $d_1\to+\infty$, then IFPRI-OR$_d((a_{11},\ldots,a_{1n_1}),\ldots,(a_{q1},\ldots,a_{qn_q}))\to\langle 1-\prod_{j=1}^{n_1}(1-u_{1j}(x)), \prod_{j=1}^{n_1}\nu_{1j}(x)\rangle$.

Proposition 22:

$$\lim_{(d_1,d_2,\ldots,d_{k+1})\to(\underbrace{0,\ldots,0}_{k},+\infty)} IFPRI-OR_d((a_{11},\ldots,a_{1n_1}),\ldots,(a_{q1},\ldots,a_{qn_q}))$$
$$=\mathcal{S}(\mathcal{S}(a_{11},\ldots,a_{1n_1}),\ldots,\mathcal{S}(a_{k+1,1},\ldots,a_{k+1,n_{k+1}})).$$

Especially, if we select "$\oplus$" as the IF t-conorm $\mathcal{S}$, when $(d_1,d_2,\ldots,d_{k+1})\to(\underbrace{0,\ldots,0}_{k},+\infty)$, then IFPRI-OR$_d((a_{11},\ldots,a_{1n_1}),\ldots,(a_{q1},\ldots,a_{qn_q}))\to\langle 1-\prod_{i=1}^{k+1}(\prod_{j=1}^{n_i}(1-\mu_{ij}(x))), \prod_{i=1}^{k+1}(\prod_{j=1}^{n_i}\nu_{ij}(x))\rangle$.

Proposition 23:

$$\lim_{(d_1,d_2,\ldots,d_{k+1})\to(\underbrace{1,\ldots,1}_{k},+\infty)} IFPRI-OR_d((a_{11},\ldots,a_{1n_1}),\ldots,(a_{q1},\ldots,a_{qn_q}))=\mathcal{S}(\mathcal{S}(t_1'a_{11},\ldots,t_1'a_{1n_1}),\cdots,\mathcal{S}(t_{k+1}'a_{k+1,1},\ldots,t_{k+1}'a_{k+1,n_{k+1}})).$$

Especially, if we select "$\oplus$" as the IF t-conorm $\mathcal{S}$, for $(d_1,d_2,\ldots,d_{k+1})\to(\underbrace{1,\ldots,1}_{k},+\infty)$, then IFPRI-OR$_d((a_{11},\ldots,a_{1n_1}),\ldots,(a_{q1},\ldots,a_{qn_q}))\to$ IFPRI-OR$((a_{11},\ldots,a_{1n_1}),\ldots,(a_{k+1,1},\ldots,a_{k+1,n_{k+1}}))$.

IV. APPROACH TO MADM WITH THE IFPRI-OR$_d$ AGGREGATION OPERATOR

In this section, we can apply IFPRI-OR$_d$ operator by using IF t-conorm "$\oplus$" to aggregate evaluation information given by experts. Then according to the score function values, we rank all the alternatives x_k and choose the best alternative. Assume that a set of alternatives $X = \{x_1, x_2, \ldots, x_m\}$ and a collection of attributes $C = \{C_1, C_2, \ldots, C_n\}$.

Procedure 1: We assume that there is a prioritization relationship between the attributes expressed by the strict ordering $C_1 \succ_{d_1} C_2 \succ_{d_2} \cdots \succ_{d_{n-1}} C_n$. $C_i \succ C_j$ indicates that C_i has a higher priority than C_j if $i < j$. Then each category H_i has just one member, i.e, $H_i = \{C_i\}$ for each $i \in \{1, 2, \ldots, n\}$.

For each attribute C_i, we have an IFV $C_i(x_k) = \langle \mu_i(x_k), \nu_i(x_k)\rangle \in L^*$, where $\mu_i(x_k)$ indicating the degree that the alternative x_k satisfies the attribute C_i and $\nu_i(x_k)$ indicating the degree that the alternative x_k does not satisfy the attribute C_i, $i = 1, 2, \ldots, n, k = 1, 2, \ldots, m$.

Step 1: For each $x_k \in X$, $C_0(x_k) = \langle \mu_0(x_k), \nu_0(x_k)\rangle = \langle 1, 0\rangle$. Then

$$
\begin{aligned}
w_i(x_k) &= \bigotimes_{j=1}^{i} C_{j-1}^{d_{j-1}}(x_k) \\
&= C_0^{d_0}(x_k) \otimes C_1^{d_1}(x_k) \otimes \ldots \otimes C_{i-1}^{d_{i-1}}(x_k) \\
&= \langle \prod_{j=1}^{i} \mu_{j-1}^{d_{j-1}}(x_k), 1 - \prod_{j=1}^{i}(1 - \nu_{j-1}(x_k))^{d_{j-1}}\rangle.
\end{aligned}
$$

$w_i(x_k)$ is the prioritized weight associated with $C_i(x_k)$.

Step 2: Apply the intuitionistic fuzzy prioritized "or" aggregation operators based on the priority degrees by using the IF t-conorm "$\oplus$", we can aggregate n IFVs $C_1(x_k), C_2(x_k), \ldots, C_n(x_k)$ as follows:

$$
\begin{aligned}
C(x_k) &= \text{IFPRI-OR}_d(C_1(x_k), C_2(x_k), \ldots, C_n(x_k)) \\
&= w_1(x_k)C_1(x_k) \oplus w_2(x_k)C_2(x_k) \oplus \ldots \oplus w_n(x_k)C_n(x_k) \\
&= \langle 1 - \prod_{i=1}^{n}(1 - \mu_i(x_k))^{\prod_{j=1}^{i} \mu_{j-1}^{d_{j-1}}(x_k)}, \\
&\quad \prod_{i=1}^{n} \nu_i(x_k)^{\prod_{j=1}^{i}(1-\nu_{j-1}(x_k))^{d_{j-1}}}\rangle.
\end{aligned}
$$

Step 3: Calculate the score $s(C(x_k))$ and the accuracy $h(C(x_k))$ of each A-IFV $C(x_k)(k = 1, 2, \ldots, m)$.

Step 4: Rank all the alternative x_k in accordance with $s(C(x_k))$ and $h(C(x_k))(k = 1, 2, \ldots, m)$ and choose the most desirable alternative.

Procedure 2: Assume a set of attributes C are partitioned into q distinct priority levels, $H_1, H_2, \ldots, H_q$ such that $H_1 \succ_{d_1} H_2 \succ_{d_2} \cdots \succ_{d_{q-1}} H_q$. The attributes in the category H_i have a higher priority than those in H_k if $i < k$. C_{ij} are the attributes in the category H_i, $C_{ij} \in C$ $(i = 1, 2, \ldots, q; j = 1, 2, \ldots, n_i)$. The total set of attributes is $C = \bigcup_{i=1}^{q} H_i$, $n = \sum_{i=1}^{q} n_i$ is the total number of attributes.

For any alternative x_k, we have each attribute C_{ij}, an intuitionistic fuzzy values $C_{ij}(x_k) = \langle \mu_{ij}(x_k), \nu_{ij}(x_k)\rangle \in L^*$, where $\mu_{ij}(x_k)$ indicating the degree that the alternative x_k

$$
\begin{aligned}
C(x) &= IFPRI - OR_d((C_{11}(x), \ldots, C_{1n_1}(x)), \ldots, (C_{q1}(x), \ldots, C_{qn_q}(x))) \\
&= \langle 1 - \frac{\gamma \prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1 - \mu_{ij}(x))^{\mu_{t_i'}(x)})}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1 + (\gamma - 1)\mu_{ij}(x))^{\mu_{t_i'}(x)}) + (\gamma - 1)(\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1 - \mu_{ij}(x))^{\mu_{t_i'}(x)}))}, \\
&\quad \frac{\gamma \prod_{i=1}^{q}(\prod_{j=1}^{n_i}(\nu_{ij}(x)^{1-\nu_{t_i'}(x)}))}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1 + (\gamma - 1)\nu_{ij}(x)^{1-\nu_{t_i'}(x)})) + (\gamma - 1)(\prod_{i=1}^{q}(\prod_{j=1}^{n_i} \nu_{ij}(x)^{1-\nu_{t_i'}(x)}))}\rangle \\
&= \langle 1 - \frac{\gamma \prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1 - \mu_{ij}(x))^{\prod_{k=1}^{i-1} \mu_{s_k'}^{d_k}(x)})}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1 + (\gamma - 1)\mu_{ij}(x))^{\prod_{k=1}^{i-1} \mu_{s_k'}^{d_k}(x)}) + (\gamma - 1)(\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1 - \mu_{ij}(x))^{\prod_{k=1}^{i-1} \mu_{s_k'}^{d_k}(x)}))}, \\
&\quad \frac{\gamma \prod_{i=1}^{q}(\prod_{j=1}^{n_i}(\nu_{ij}(x)^{\prod_{k=1}^{i-1}(1-\nu_{s_k'}(x))^{d_k}}))}{\prod_{i=1}^{q}(\prod_{j=1}^{n_i}(1 + (\gamma - 1)\nu_{ij}(x)^{\prod_{k=1}^{i-1}(1-\nu_{s_k'}(x))^{d_k}})) + (\gamma - 1)(\prod_{i=1}^{q}(\prod_{j=1}^{n_i} \nu_{ij}(x)^{\prod_{k=1}^{i-1}(1-\nu_{s_k'}(x))^{d_k}}))}\rangle.
\end{aligned}
\tag{56}
$$

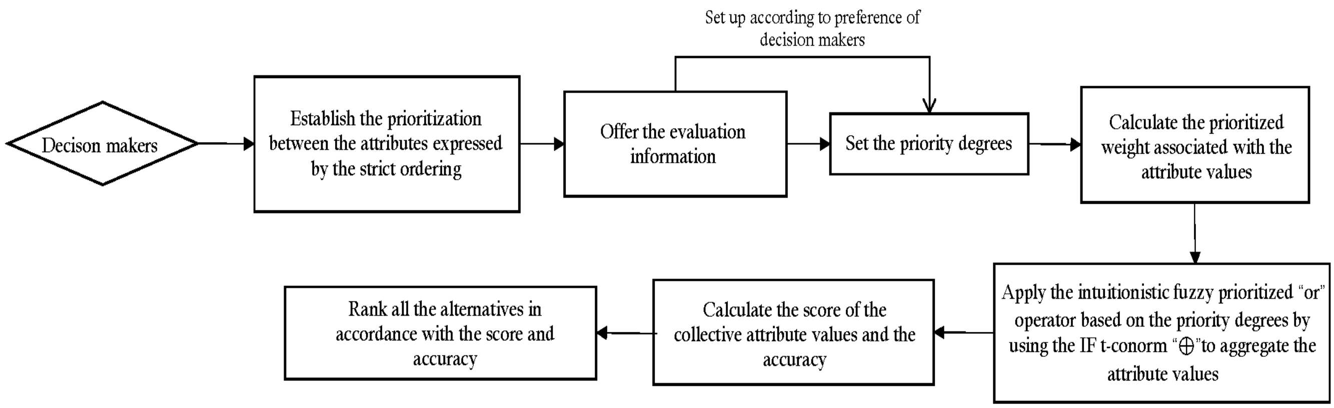

FIGURE 1. Approach to MADM with the IFPRI-OR$_d$ aggregation operator.

satisfies the attribute C_{ij} and $v_{ij}(x_k)$ indicating the degree that the alternative x_k does not satisfy the attribute C_{ij}, $i = 1, 2, \ldots, n, j = 1, 2, \ldots, n_i, k = 1, 2, \ldots, m$.

Step 1: For each $x_k \in X$, $s'_i(x_k) = \mathcal{S}_M\{C_{ij}(x_k)|j = 1, 2, \ldots, n_i\} = \langle \max(u_{i1}(x_k), \ldots, u_{in_i}(x_k)), \min(v_{i1}(x_k), \ldots, v_{in_i}(x_k))\rangle$.

Assume that $s'_i(x_k) = \langle \mu_{s'_i}(x_k), v_{s'_i}(x_k)\rangle$, $t'_i(x_k) = \langle 1, 0\rangle \otimes s_1'^{d_1}(x_k) \otimes \ldots \otimes s_{i-1}'^{d_{i-1}}(x_k) = \langle \prod_{k=1}^{i-1} u_{s'_k}^{d_k}(x_k), 1 - \prod_{k=1}^{i-1}(1 - v_{s'_k}(x_k))^{d_k}\rangle$.

Let $\mu_{t'_i}(x_k) = \prod_{k=1}^{i-1} u_{s'_k}^{d_k}(x_k)$, $v_{t'_i}(x_k) = 1 - \prod_{k=1}^{i-1}(1 - v_{s'_k}(x_k))^{d_k}$, then $t'_i(x_k) = \langle u_{t'_i}(x), v_{t'_i}(x)\rangle$. Thus we have the weights $w'_{ij}(x_k) = t'_i(x_k) = \langle u_{t'_i}(x), v_{t'_i}(x)\rangle$.

Step 2: Apply the intuitionistic fuzzy prioritized "or" aggregation operators based on the priority degrees by using the IF t-conorm "$\oplus$", we can aggregate the n IFVs $C_1(x_k), C_2(x_k), \ldots, C_n(x_k)$ as follows:

$$\begin{aligned} C'(x_k) &= \text{IFPRI-OR}_d((C_{11}(x_k), \ldots, C_{1n_1}(x_k)), \ldots, (C_{q1}(x_k), \\ &\quad \ldots, C_{qn_q}(x_k))) \\ &= (t'_1(x_k)C_{11}(x_k) \oplus \ldots \oplus t'_1(x_k)C_{1n_1}(x_k)) \oplus \ldots \\ &\quad \oplus (t'_q(x_k)C_{q1}(x_k) \oplus \ldots \oplus t'_q(x_k)C_{qn_q}(x_k)) \\ &= \langle h^{-1}(\sum_{i=1}^{q}\sum_{j=1}^{n_i} \mu_{t'_i}(x)h(\mu_{ij}(x))), \\ &\quad g^{-1}(\sum_{i=1}^{q}\sum_{j=1}^{n_i}(1 - v_{t'_i}(x))g(v_{ij}(x)))\rangle. \end{aligned}$$

Then in the real MADM problem, the experts can choose the right additive generator g and h according to their preference.

Step 3: Calculate the score $s(C'(x_k))$ and the accuracy of each IFV $C'(x_k)$, where $k = 1, 2, \ldots, m$.

Step 4: Rank all the alternatives x_k according to $s(C'(x_k))$ and $h(C'(x_k))$ and choose the best alternative.

V. ILLUSTRATIVE EXAMPLE

In order to illustrate the rationality of the proposed method to multi-attribute group decision making problem, we consider a university faculty recruitment group decision-making problem.

Example 1: The department of mathematics in a university wants to appoint outstanding mathematics teachers. Group of decision makers made strict evaluation for five teachers $x_i (i = 1, 2, 3, 4, 5)$ according to the following five attributes: (1) C_1, the past experience, (2) C_2, the teaching skill, (3) C_3, moral quality, (4) C_4, the research capability, (3) C_5, subject knowledge. The evaluation information offered by four decision makers is represented by IFVs, as shown in **Table 1**. The above multi-attribute group decision making problem can be shown as follows:

(1) In the decision makers' opinion, there is the prioritization relationship among these attributes, for example, moral quality is the most important, but the past experience is not more important than other attributes. Define the relationship as follow:

$$C_3 \succ_{d_1} C_4 \succ_{d_2} C_5 \succ_{d_3} C_2 \succ_{d_4} C_1.$$

Assume that $(d_1, d_2, d_3, d_4) = (5, 3, 1, 1)$, the priority relationship between the attributes is expressed by the strict ordering. Thus we apply **Procedure 1** to rank the alternatives.

Step 1: For each $x_k \in X(k = 1, 2, 3, 4, 5)$, Then
$w_1(x_1) = \langle 1, 0\rangle$, $w_2(x_1) = \langle 0.031, 0.922\rangle$,
$w_3(x_1) = \langle 0.002, 0.96\rangle$, $w_4(x_1) = \langle 0.001, 0.976\rangle$,
$w_5(x_1) = \langle 0.01, 0.978\rangle$, $w_1(x_2) = \langle 1, 0\rangle$,
$w_2(x_2) = \langle 0.01, 0.832\rangle$, $w_3(x_2) = \langle 0.001, 0.942\rangle$,
$w_4(x_2) = \langle 0.001, 0.965\rangle$, $w_5(x_2) = \langle 0.001, 0.972\rangle$,
$w_1(x_3) = \langle 1, 0\rangle$, $w_2(x_3) = \langle 0.031, 0.832\rangle$,
$w_3(x_3) = \langle 0.002, 0.942\rangle$, $w_4(x_3) = \langle 0.001, 0.965\rangle$,
$w_5(x_3) = \langle 0.001, 0.976\rangle$, $w_1(x_4) = \langle 1, 0\rangle$,
$w_2(x_4) = \langle 0.031, 0.41\rangle$, $w_3(x_4) = \langle 0.002, 0.798\rangle$,
$w_4(x_4) = \langle 0.001, 0.879\rangle$, $w_5(x_4) = \langle 0.001, 0.903\rangle$,
$w_1(x_5) = \langle 1, 0\rangle$, $w_2(x_5) = \langle 0.078, 0.832\rangle$,
$w_3(x_5) = \langle 0.005, 0.942\rangle$, $w_4(x_5) = \langle 0.003, 0.965\rangle$,
$w_5(x_5) = \langle 0.002, 0.976\rangle$,

TABLE 1. The attribute values $C_i(x_k)(i = 1, 2, 3, 4, 5)$.

	C_1	C_2	C_3	C_4	C_5
x_1	$\langle 0.5, 0.4\rangle$	$\langle 0.4, 0.2\rangle$	$\langle 0.5, 0.4\rangle$	$\langle 0.7, 0.1\rangle$	$\langle 0.6, 0.2\rangle$
x_2	$\langle 0.4, 0.3\rangle$	$\langle 0.5, 0.3\rangle$	$\langle 0.5, 0.4\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.6, 0.4\rangle$
x_3	$\langle 0.5, 0.3\rangle$	$\langle 0.4, 0.3\rangle$	$\langle 0.5, 0.4\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 0.6, 0.2\rangle$
x_4	$\langle 0.5, 0.1\rangle$	$\langle 0.4, 0.3\rangle$	$\langle 0.6, 0.4\rangle$	$\langle 0.6, 0.2\rangle$	$\langle 0.5, 0.1\rangle$
x_5	$\langle 0.6, 0.3\rangle$	$\langle 0.4, 0.3\rangle$	$\langle 0.5, 0.4\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 0.6, 0.3\rangle$

Step 2: Apply the operator IFPRI-OR based on the priority degrees by using the IF t-conorm "$\oplus$", we can aggregate these IFVs $C_1(x_k), C_2(x_k), \ldots, C_5(x_k)(k = 1, 2, 3, 4, 5)$ as follows:

$C(x_1) = \langle 0.508, 0.019\rangle$, $C(x_2) = \langle 0.406, 0.244\rangle$,
$C(x_3) = \langle 0.506, 0.215\rangle$, $C(x_4) = \langle 0.506, 0.043\rangle$,
$C(x_5) = \langle 0.614, 0.245\rangle$.

Step 3: Calculate the score $s(C(x_k))(k = 1, 2, 3, 4, 5)$ of each IFV $C(x_k)(k = 1, 2, 3, 4, 5)$.

$s(C(x_1)) = 0.489$, $s(C(x_2)) = 0.162$,
$s(C(x_3)) = 0.291$, $s(C(x_4)) = 0.463$,
$s(C(x_5)) = 0.369$.

Step 4: Rank all the alternative x_k in accordance with $s(C(x_k))(k = 1, 2, 3, 4, 5)$, $s(C(x_1)) > s(C(x_4)) > s(C(x_5)) > s(C(x_3)) > s(C(x_2))$ i.e. $x_1 > x_4 > x_5 > x_3 > x_2$ and choose the most desirable alternative x_1.

Comparative Analysis: According to the proposed method of Li [32], we can only consider the prioritization relationship among these attributes not priority degrees among attributes. The proposed method in this paper is compared with the previous method of Li [32]. First we define the relationship as follow:

$$C_3 \succ C_4 \succ C_5 \succ C_2 \succ C_1.$$

Step 1: For each $x_k \in X(k = 1, 2, 3, 4, 5)$, we compute the prioritized weights $w_i(x_k)$ associated with $C_i(x_k)$.

$w_1(x_1) = \langle 1, 0\rangle$, $w_2(x_1) = \langle 0.5, 0.4\rangle$,
$w_3(x_1) = \langle 0.35, 0.46\rangle$, $w_4(x_1) = \langle 0.21, 0.57\rangle$,
$w_5(x_1) = \langle 0.08, 0.65\rangle$, $w_1(x_2) = \langle 1, 0\rangle$,
$w_2(x_2) = \langle 0.5, 0.4\rangle$, $w_3(x_2) = \langle 0.35, 0.52\rangle$,
$w_4(x_2) = \langle 0.21, 0.71\rangle$, $w_5(x_2) = \langle 0.11, 0.8\rangle$,
$w_1(x_3) = \langle 1, 0\rangle$, $w_2(x_3) = \langle 0.5, 0.4\rangle$,
$w_3(x_3) = \langle 0.3, 0.58\rangle$, $w_4(x_3) = \langle 0.18, 0.66\rangle$,
$w_5(x_3) = \langle 0.07, 0.76\rangle$, $w_1(x_4) = \langle 1, 0\rangle$,
$w_2(x_4) = \langle 0.6, 0.4\rangle$, $w_3(x_4) = \langle 0.36, 0.52\rangle$,
$w_4(x_4) = \langle 0.18, 0.57\rangle$, $w_5(x_4) = \langle 0.07, 0.7\rangle$,
$w_1(x_5) = \langle 1, 0\rangle$, $w_2(x_5) = \langle 0.5, 0.4\rangle$,
$w_3(x_5) = \langle 0.3, 0.58\rangle$, $w_4(x_5) = \langle 0.18, 0.71\rangle$,
$w_5(x_5) = \langle 0.07, 0.8\rangle$.

Step 2: Apply the operator IFPRI-OR based on the priority degrees by using the IF t-conorm "$\oplus$", we aggregate the n IFVs $C_1(x_k), C_2(x_k), C_3(x_k), C_4(x_k), C_5(x_k)$ $(k = 1, 2, 3, 4, 5)$.

$C(x_1) = \langle 0.77, 0.05\rangle$, $C(x_2) = \langle 0.78, 0.1\rangle$,
$C(x_3) = \langle 0.74, 0.1\rangle$, $C(x_4) = \langle 0.81, 0.06\rangle$,
$C(x_5) = \langle 0.74, 0.11\rangle$,

Step 3: Calculate the score $s(C(x_k))$ and the accuracy $h(C(x_k))$ of each IFV $C(x_k)(k = 1, 2, 3, 4, 5)$.

$s(C(x_1)) = 0.72$, $s(C(x_2)) = 0.77$,
$s(C(x_3)) = 0.73$, $s(C(x_4)) = 0.75$,
$s(C(x_5)) = 0.63$,

Step 4: Rank all the alternative x_k in accordance with $s(C(x_k))$ and $h(C(x_k))(k = 1, 2, 3, 4, 5)$, and choose the most desirable alternative x_2.

$s(C(x_2)) > s(C(x_4)) > s(C(x_3)) > s(C(x_1)) > s(C(x_5))$ i.e $x_2 > x_4 > x_3 > x_1 > x_5$.

From the above example, the ranking results of the proposed method is different from the previous method of Li [32]. In the proposed method, the prioritized weights associated with $C_{ij}(x)$ are changeable if the priority degrees are variable. Then the collective aggregation values are vary with the priority degrees. But in the previous method of Li [32], the collective aggregation results only are related to the attribute values provided by the experts. Thus the priority degrees make the preference of the experts for alternatives more obvious in the decision making process and provide the experts more options. In the rest, we focus on the solution of multi-attribute group decision making problems.

(2)Assume a set of attributes $C = \{C_1, C_2, C_3, C_4, C_5\}$ are partitioned into three distinct priority levels, H_1, H_2, H_3 such that $H_1 \succ_{d_1} H_2 \succ_{d_2} H_3$. The moral quality C_3 is at the first priority level H_1. The research capability C_4 and subject knowledge C_5 are at the second priority level H_2. The teaching skill C_2 and the past experience C_1 are at the third priority level H_3. Then we have $H_1 = \{C_3\}, H_2 = \{C_4, C_5\}, H_3 = \{C_1, C_2\}$ and $H_1 \succ_{d_1} H_2 \succ_{d_2} H_3$, where $(d_1, d_2) = (5, 1)$.

For each $x_k \in X, k = 1, 2, 3, 4, 5$, we can calcuate the priority weights of each alternative.

For the alternative x_1, we have
$s'_0(x_1) = \langle 1, 0\rangle$, $s'_1(x_1) = \langle 0.5, 0.4\rangle$,
$s'_2(x_1) = \langle 0.7, 0.1\rangle$, $s'_3(x_1) = \langle 0.5, 0.2\rangle$.
Thus $t'_1(x_1) = \langle 1, 0\rangle$, $t'_2(x_1) = \langle 0.0313, 0.9222\rangle$,
$t'_3(x_1) = \langle 0.0219, 0.93\rangle$.
For the alternative x_2, we have
$s'_0(x_2) = \langle 1, 0\rangle$, $s'_1(x_2) = \langle 0.5, 0.4\rangle$,
$s'_2(x_2) = \langle 0.7, 0.2\rangle$, $s'_3(x_2) = \langle 0.5, 0.3\rangle$.
Thus $t'_1(x_2) = \langle 1, 0\rangle$, $t'_2(x_2) = \langle 0.0313, 0.9222\rangle$,
$t'_3(x_2) = \langle 0.0219, 0.9378\rangle$.
For the alternative x_3, we have
$s'_0(x_3) = \langle 1, 0\rangle$, $s'_1(x_3) = \langle 0.5, 0.4\rangle$,
$s'_2(x_3) = \langle 0.6, 0.2\rangle$, $s'_3(x_3) = \langle 0.5, 0.3\rangle$.
Thus $t'_1(x_3) = \langle 1, 0\rangle$, $t'_2(x_3) = \langle 0.0313, 0.9222\rangle$,
$t'_3(x_3) = \langle 0.0188, 0.9378\rangle$.
For the alternative x_4, we have $s'_0(x_4) = \langle 1, 0\rangle$,
$s'_1(x_4) = \langle 0.6, 0.4\rangle$, $s'_2(x_4) = \langle 0.6, 0.1\rangle$,
$s'_3(x_4) = \langle 0.5, 0.1\rangle$.
Thus $t'_1(x_4) = \langle 1, 0\rangle$, $t'_2(x_4) = \langle 0.0778, 0.9222\rangle$,
$t'_3(x_4) = \langle 0.0467, 0.93\rangle$.
For the alternative x_5, we have $s'_0(x_5) = \langle 1, 0\rangle$,
$s'_1(x_5) = \langle 0.5, 0.4\rangle$, $s'_2(x_5) = \langle 0.6, 0.3\rangle$,
$s'_3(x_5) = \langle 0.6, 0.3\rangle$.

TABLE 2. The scores of attribute values $C_i(x_k)(i = 1, 2, 3, 4, 5)$ with different additive generators $g(t)$ and the order of alternatives.

g(t)	$s(C'(x_1))$	$s(C'(x_2))$	$s(C'(x_3))$	$s(C'(x_4))$	$s(C'(x_5))$	the order of alternatives
$g(t) = -\ln(t)$	0.1771	0.171	0.1698	0.1745	0.1669	$x_1 > x_4 > x_2 > x_3 > x_5$
$g(t) = \ln(\frac{2-t}{t})$	0.537	0.4754	0.4932	0.6791	0.466	$x_4 > x_1 > x_3 > x_2 > x_5$
$g(t) = \ln(\frac{\gamma+(1-\gamma)t}{t}), \gamma > 0$	0.56	0.5595	0.5536	0.7052	0.5556	$x_4 > x_1 > x_2 > x_5 > x_3$
$g(t) = \ln(\frac{\gamma-1}{\gamma^t-1}), \gamma > 1$	0.8508	0.8286	0.8356	0.884	0.8256	$x_4 > x_1 > x_3 > x_2 > x_5$

TABLE 3. The scores of attribute values $C_i(x_k)(i = 1, 2, 3, 4, 5)$ with different priority degrees and the order of alternatives.

(d_1, d_2)	$s(C'(x_1))$	$s(C'(x_2))$	$s(C'(x_3))$	$s(C'(x_4))$	$s(C'(x_5))$	the order of alternatives
(0,1)	0.9814	0.9791	0.9738	0.9758	0.9776	$x_1 > x_2 > x_5 > x_4 > x_3$
(1,1)	0.8061	0.777	0.6884	0.8466	0.8501	$x_5 > x_4 > x_1 > x_2 > x_3$
(3,1)	0.7314	0.6936	0.6773	0.7836	0.6821	$x_4 > x_1 > x_2 > x_5 > x_3$
(5,1)	0.1771	0.171	0.1698	0.1745	0.1669	$x_1 > x_4 > x_2 > x_3 > x_5$
(9,1)	0.108	0.1081	0.108	0.2123	0.1053	$x_4 > x_2 > x_1 = x_3 > x_5$
$(+\infty, 1)$	0.1	0.1	0.1	0.2	0.1	$x_4 > x_1 = x_2 = x_3 = x_5$

Thus $t'_1(x_5) = \langle 1, 0\rangle$, $t'_2(x_5) = \langle 0.0313, 0.9222\rangle$, $t'_3(x_5) = \langle 0.0188, 0.9455\rangle$.

Then apply the intuitionistic fuzzy prioritized "or" aggregation operators based on the priority degrees by using the IF t-conorm "$\oplus$", we can aggregate five IFVs $C_1(x_k), C_2(x_k), \ldots, C_5(x_k)$ as follows:

$$\begin{aligned} C'(x_k) &= \text{IFPRI-OR}_d(C_3(x_k), (C_4(x_k), C_5(x_k)), \\ &\quad (C_1(x_k), C_2(x_k))) \\ &= t'_1(x_k)C_3(x_k) \oplus t'_2(x_k)(C_4(x_k) \oplus C_5(x_k)) \\ &\quad \oplus t'_3(x_k)(C_1(x_k) \oplus C_4(x_k)) \\ &= \langle h^{-1}(\sum_{i=1}^{q}\sum_{j=1}^{n_i}\mu_{t'_i}(x)h(\mu_{ij}(x))), \\ &\quad g^{-1}(\sum_{i=1}^{q}\sum_{j=1}^{n_i}(1-\nu_{t'_i}(x))g(\nu_{ij}(x)))\rangle. \end{aligned}$$

Case 1: If $g(t) = -\ln(t)$, according to (53),then we have $C'(x_1) = \langle 0.5212, 0.3457\rangle$, $C'(x_2) = \langle 0.5212, 0.3503\rangle$, $C'(x_3) = \langle 0.5196, 0.3498\rangle$, $C'(x_4) = \langle 0.6371, 0.3444\rangle$, $C'(x_5) = \langle 0.5201, 0.3532\rangle$.

Case 2: If $g(t) = \ln(\frac{2-t}{t})$, according to (54),then we have $C'(x_1) = \langle 0.5508, 0.0138\rangle$, $C'(x_2) = \langle 0.5508, 0.0754\rangle$, $C'(x_3) = \langle 0.5448, 0.0516\rangle$, $C'(x_4) = \langle 0.6833, 0.0042\rangle$, $C'(x_5) = \langle 0.5467, 0.0807\rangle$.

Case 3: If $g(t) = \ln(\frac{\gamma+(1-\gamma)t}{t})$, $\gamma > 0$, we assume $\gamma = 4$. According to (55), then we have $C'(x_1) = \langle 0.5601, 0.0001\rangle$, $C'(x_2) = \langle 0.5601, 0.0006\rangle$, $C'(x_3) = \langle 0.5538, 0.0002\rangle$, $C'(x_4) = \langle 0.7053, 0.0001\rangle$, $C'(x_5) = \langle 0.5557, 0.0001\rangle$

Case 4: If $g(t) = \ln(\frac{\gamma-1}{\gamma^t-1})$, $\gamma > 1$, we assume $\gamma = 6$. According to (56), then we have $C'(x_1) = \langle 0.8562, 0.0054\rangle$, $C'(x_2) = \langle 0.8562, 0.0.0276\rangle$, $C'(x_3) = \langle 0.8551, 0.0195\rangle$, $C'(x_4) = \langle 0.8858, 0.0018\rangle$, $C'(x_5) = \langle 0.8561, 0.0305\rangle$.

Then we calculate the scores of attribute values $C_i(x_k)(i = 1, 2, 3, 4, 5)$ with different additive generators $g(t)$ and get the ranking results as **Table 2**. From the above results, we can know that results are possible according to different additive generators g, for example, the best alternative is both x_4 in **Case 2**, **3**, **4** except **Case 1**. In addition, we change the priority degrees to find how the aggregate values change by using intuitionistic fuzzy prioritized "or" aggregation operators based on the priority degrees this moment. Next take **Case 1** as example, then

① Assume $(d_1, d_2) = (0, 1)$, it shows that the category H_1 and the category H_2 do not have a priority relationship and the category H_2 and the category H_3 have a priority relationship. The criteria in the category H_1 and the category H_2 is equally improtant. We can get $C(x_1) = \langle 0.982, 0.0006\rangle$, $C(x_2) = \langle 0.982, 0.0029\rangle$, $C(x_3) = \langle 0.976, 0.0022\rangle$, $C(x_4) = \langle 0.976, 0.0002\rangle$, $C(x_5) = \langle 0.9808, 0.0032\rangle$.

② Assume $(d_1, d_2) = (1, 1)$, then IFPRI-OR$_d$ operator reduces to IFPRI-OR operator. We can get $C(x_1) = \langle 0.8062, 0.0001\rangle$, $C(x_2) = \langle 0.8062, 0.0292\rangle$, $C(x_3) = \langle 0.7866, 0.0982\rangle$, $C(x_4) = \langle 0.8579, 0.0113\rangle$, $C(x_5) = \langle 0.8863, 0.0362\rangle$.

③ Assume $(d_1, d_2) = (3, 1)$, it shows that the priority relationship between the category H_1 and the category H_2 is better than the priority relationship between the category H_2 and the category H_3. We can get $C(x_1) = \langle 0.9281, 0.1967\rangle$, $C(x_2) = \langle 0.9299, 0.2363\rangle$, $C(x_3) = \langle 0.9122, 0.2349\rangle$, $C(x_4) = \langle 0.8927, 0.1091\rangle$, $C(x_5) = \langle 0.9292, 0.2471\rangle$.

④ Assume $(d_1, d_2) = (9, 1)$, it shows that the priority relationship between the category H_1 and the category H_2 is much stronger than the priority relationship between the category H_2 and the category H_3. We can get $C(x_1) = \langle 0.5014, 0.3934\rangle$, $C(x_2) = \langle 0.5014, 0.3933\rangle$, $C(x_3) = \langle 0.5013, 0.3933\rangle$, $C(x_4) = \langle 0.6049, 0.3926\rangle$, $C(x_5) = \langle 0.5013, 0.396\rangle$.

⑤ Assume $(d_1, d_2) = (+\infty, 1)$, it shows that the priority relationship between the category H_1 and the category H_2 is particularly higher priority than the category H_2 and the category H_3. Generally in this case, we only consider that

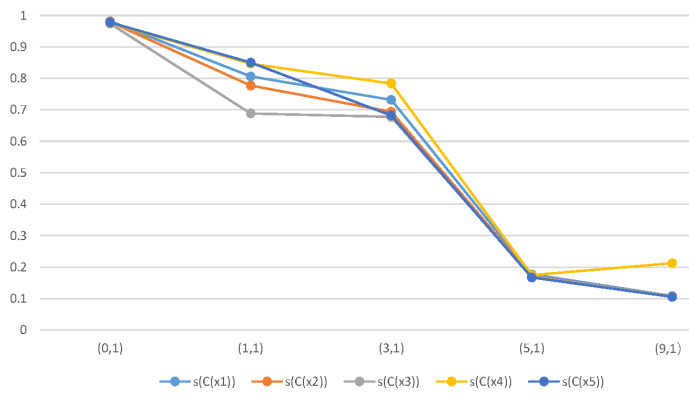

FIGURE 2. Score values of the collective attribute values $C(x_k)$ ($k = 1, 2, 3, 4, 5$) with different priority degrees.

the priority relationship between the category H_1 and the category H_2.

$C(x_1) = \langle 0.5, 0.4\rangle$, $C(x_2) = \langle 0.5, 0.4\rangle$,
$C(x_3) = \langle 0.5, 0.4\rangle$, $C(x_4) = \langle 0.6, 0.4\rangle$,
$C(x_5) = \langle 0.5, 0.4\rangle$.

The scores of attribute values $C_i(x_k)(i = 1, 2, 3, 4, 5)$ by different priority degrees and the order of alternatives is as **Table 3**.

From the above example, we find that the ranking results are different when the priority degree changes, for example, the best alternative is x_4 when the priority degree is $(3, 1)$, $(9, 1)$ and $(+\infty, 1)$, but the best alternative is x_5 when $(d_1, d_2) = (1, 1)$. The attribute values corresponding to each attribute decreases with the increase of the priority degree d_1. Figure 1 can reflect this point more intuitively. The variety of the priority degrees affects the ranking of alternatives and the selection of alternatives. Thus in multi-attribute decision making, decision makers can choose more flexibly and reasonably according to their preference by using our proposed prioritized aggregation operators. But in the practical decision making problems, how to set the priority degrees is difficult for decision makers. This problem needs to be solved properly in the future.

VI. CONCLUSION

In this paper, we first reviewed the basic concept of intuitionistic fuzzy set and introduced the concept of the priority degrees. We further propose two new kinds of prioritized aggregation operators: intuitionistic fuzzy PRI-AND aggregation operators based on priority degrees and intuitionistic fuzzy PRI-OR aggregation operators based on priority degrees. Then we also establish some important properties of these operators and their particular cases. These particular cases can be used to solve the MADM problems with intuitionistic fuzzy information in which the attributes are different priority levels. Finally, we propose an approach to MADM problems with the IFPRI-OR$_d$ operator and observe that the attribute values decrease with the increase of the priority degrees. Thus in the practical application, the decision maker can choose the appropriate priority degrees according to their own preference. Our proposed method in this paper can also make the decision making process more flexible and reasonable.

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RUI HUA is currently pursuing the master's degree with the School of Mathematics and Statistics, Anhui Normal University, Wuhu, China. Her current research interests include decision analysis and aggregation operators.

BOQUAN LI received the Ph.D. degree in fuzzy mathematics from Nanjing Normal University, in 2013. He is currently an Associate Professor with the School of Mathematics and Statistics, Anhui Normal University, Wuhu, China. He has contributed over 20 journal articles to professional journals, such as *Information Sciences* and *Information Fusion*. His current research interests include the theory and applications of fuzzy mathematics, aggregation operators, decision making, and uncertainty.

YONGYI LI received the Ph.D. degree in disaster prevention and reduction engineering and protective engineering from the Institute of Engineering Mechanics, China Earthquake Administration, in 2014. He is currently an Associate Professor with the School of Transportation Engineering, Nanjing Tech University, Nanjing, China. He has contributed over 20 journal articles to professional journals, such as *Journal of Basic Science and Engineering*, *Journal of Disaster Prevention and Mitigation Engineering*, and *Highway*. His current research interests include lifeline earthquake engineering, information fusion, and multi-criteria decision making.