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A Greener Heterogeneous Scheduling Algorithm via Blending Pattern of Particle Swarm Computing Intelligence and Geometric Brownian Motion

SHAOHUI LI[1,2], HONG LIU[1,2], BIN GONG[3], AND JINGLIAN WANG[4]
[1]Department of Information Science and Engineering, Shandong Normal University, Jinan 250014, China
[2]Shandong Key Laboratory of Distributed Computer Software New Technology, Jinan 250014, China
[3]Department of Software, Shandong University, Jinan 250101, China
[4]Department of Information and Electrical Engineering, Ludong University, Yantai 264025, China

Corresponding author: Shaohui Li (lwjl22422@163.com)

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ABSTRACT This paper focuses on the algorithms design of heterogeneous green scheduling for energy conservation and emission reduction in cloud computing. In essence, the real time, dynamic and complexity of heterogeneous scheduling require higher algorithm performance; however, the swarm intelligent algorithms although with some improvements, still exist big imbalances between local exploration and macro development or between route (solution) diversity and faster convergence. In this paper, a greener heterogeneous scheduling algorithm via blending pattern of particle swarm computing intelligence and geometric Brownian motion, is proposed, based on our earlier theoretical breakthroughs on G-Brownian motion and through a series of mathematical derivations or proofs; furthermore, in order for suitable for the hybrid processor architecture of the scheduling management server, the algorithm is designed in parallel with deep fusion of coarse-grained and master-slave models. A large number of experimental results are given. Compared with most newly published scheduling algorithms, there are significant advantages of the proposed algorithm on the dynamic optimization performance for consistent or semiconsistent and large inconsistent scheduling instances, although with lower improvement factors for small inconsistent instances.

INDEX TERMS Heterogeneous green scheduling, swarm intelligence, particle swarm optimization (PSO) algorithm, standard Brownian motion, G-Brownian motion (geometric Brownian motion), blending pattern.

I. INTRODUCTION

At present, the epidemic is still raging around the world, and record breaking extreme weather is also frequent. As a matter of fact, energy conservation and emission reduction is a new pressing demand of cloud heterogeneous computing. Since 2016, the annual power consumption of data centers in China (about 120 billion KWH in 2016) has exceeded the annual power generation of the Three Gorges Hydropower Station (about 100 billion KWH in 2016); and there is a huge waste of energy in China's data centers, whose PUE (Power Usage Effectiveness) is generally greater than 2.2 while that of USA is also about 1.9 in the same period. Then, around the many theoretical or technical hot spots on green scheduling coordination, a large number of studies or discussions have been widely carried out [1].

In essence, the candidate solutions of the scheduling algorithm correspond to the candidate schemes one to one, which means that the real time, dynamic and complexity of heterogeneous scheduling optimization problems require higher optimization performance, such as solution diversity or convergence speed [2], [3].

Concretely, the improvement of swarm intelligence algorithm represented by particle swarm optimization (PSO) algorithm has been systematically carried out from different dimensions, such as parameter selection or optimization [2],

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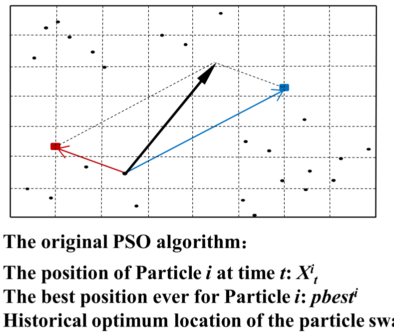

FIGURE 1. The original PSO proposed by Kennedy and Eberhart.

TABLE 1. The related variables and their representative meanings.

Symbol	Description
$\boldsymbol{v}^{i}_{t+1}$	The instantaneous velocity of Particle ***i*** at the given moment ***t*+1**
$\boldsymbol{x}^{i}_{t+1}$	The updated displacement of Particle ***i*** at the given moment ***t*+1**
$pbest^{i}$	The best position ever for Particle ***i*** until the given moment ***t*+1**
$gbest_{t}$	Historical optimum location of the particle swarm
$C_{b,Lip}(R^{k\times d})$	The space of bounded and Lipschitz functions on $R^{k\times d}$
Ω_T	The space of all $\boldsymbol{R}^{d}$-valued continuous paths $\{\boldsymbol{x}^{i}_{t}(\boldsymbol{i}\in \boldsymbol{N},\ \boldsymbol{t}\in[0,\boldsymbol{T}])\}$,starting from origin, equipped with the supremum norm and equal to $C_0([0,T];R^{d})$
$Lip(\Omega_T)$	The set of the bounded and Lipschitz functions on the space Ω_T
$\xi(X)$	The bounded and Lipschitz function on the space Ω_T
φ	The canonical mapping of the bounded and Lipschitz function ($\xi(X)$) on the space Ω_T
$\boldsymbol{B}_{t_k}$	The Borel σ-algebra on the space Ω_T where $t_k\in[0,T]$
H	The Hurst exponent of the time series
$S(d)$	The space of all $d\times d$ symmetric matrices
$G:S(d)\rightarrow R$	Each given monotonic and nonlinear function, referred to as the G-heat function
$\Gamma_{\boldsymbol{H}}$	The Gamma function defined in this paper as the G-heat function for a more generalized co-evolution of the particle swarms
$\mu_{i}(t,x; x_1,\cdots,x_{i-1})$	The viscosity solution of the G-heat function
$\hat{E}_t[\xi(X)]$	A dynamic nonlinear G-expectation space of the bounded and Lipschitz function($\xi(X)$)
$(X^{i}_{t+1},\xi(X),\Gamma_H, \hat{E}_t[\xi(X)]\mid\Gamma_H)$	The equation definition of geometric Brownian stochastic motion with nonlinear G-expectation for a more generalized co-evolution of the particle swarms where $\boldsymbol{x}^{i}_{t+1}\in\boldsymbol{\Omega_T}$ and $\xi(X)\in Lip(\Omega_T)$

and swarm topology restructuring [3]; among them, the fusion of different ideas is the most representative direction currently.

(1) Inspired by thermodynamic molecular motion theory, some studies introduced concepts such as group centroid, acceleration and molecular force into PSO algorithm to transform the particle velocity and displacement [4]–[6].

(2) Other studies refer to human adaptive learning and other mechanisms to realize various information sharing, so as to improve the convergence speed or show stronger ability [7]–[9].

(3) Moreover, the representative achievements are the effective integration of PSO algorithm and Ito process driven by the standard Brownian motion [10].

Substantially, the PSO algorithms aforementioned in **(1)** or **(2)**, are still ***approximately linear*** optimization-dynamic-patterns; the drive definition of the improved PSO algorithms in **(3)**, is reduced to a ***special*** Markov stochastic process with ***constant*** expected drift rate and variance rate, also ***without*** the generality in the dynamic swarm intelligence simulation. All of them mean that the swarm intelligent algorithms although with some improvements, still exist big imbalances between local exploration and macro development or between route (solution) diversity and faster convergence for high dimensional multi-objective scheduling problems.

In this paper, based on our earlier theoretical breakthroughs on G-Brownian (Geometric Brownian) motion and through a series of mathematical derivations or proofs, a greener heterogeneous scheduling algorithm via blending pattern of particle swarm computing intelligence and geometric Brownian motion, i.e., PSO/Rd$^{\mathrm{G}}_{\mathrm{BM}}$, is proposed; furthermore, in order for suitable for the hybrid processor architecture of the scheduling management server, the algorithm is designed in parallel with deep fusion of coarse-grained and master-slave models.

II. RELATED WORK

The combing of related work mainly takes two threads: ① swarm intelligence and PSO algorithms, and ② the standard Brownian motion vs. G-Brownian motion.

A. SWARM INTELLIGENCE AND PSO ALGORITHMS

The PSO algorithm is one of the most popular swarm intelligence algorithms of the computational intelligence theory in recent years. It was first proposed by Kennedy and Eberhart in 1995, and basically adopts the concepts of ''group'' and ''evolution' to search for the optimal solution in complex space through cooperation and competition among particles.

At the same time, as the extension of traditional artificial intelligence, PSO algorithms have been widely applied because of its simple principle, profound background of traditional evolutionary computing and unique high-dimensional objective optimization performance [11]–[17].

With the deepening of application and practice, some PSO researches focus on preserving the diversity of individuals in swarm intelligence algorithms. Inspired by thermodynamic molecular motion theory, the researches [4]–[6] introduced concepts such as group centroid, acceleration and molecular force into PSO algorithm to transform formulas such as particle velocity and displacement. In other words, according to the distance between the particle and the center of mass, the switching between the inductive force and the repulsive force can be realized to control the flight direction of the particle, and the diversity of the population can be maintained to a certain extent.

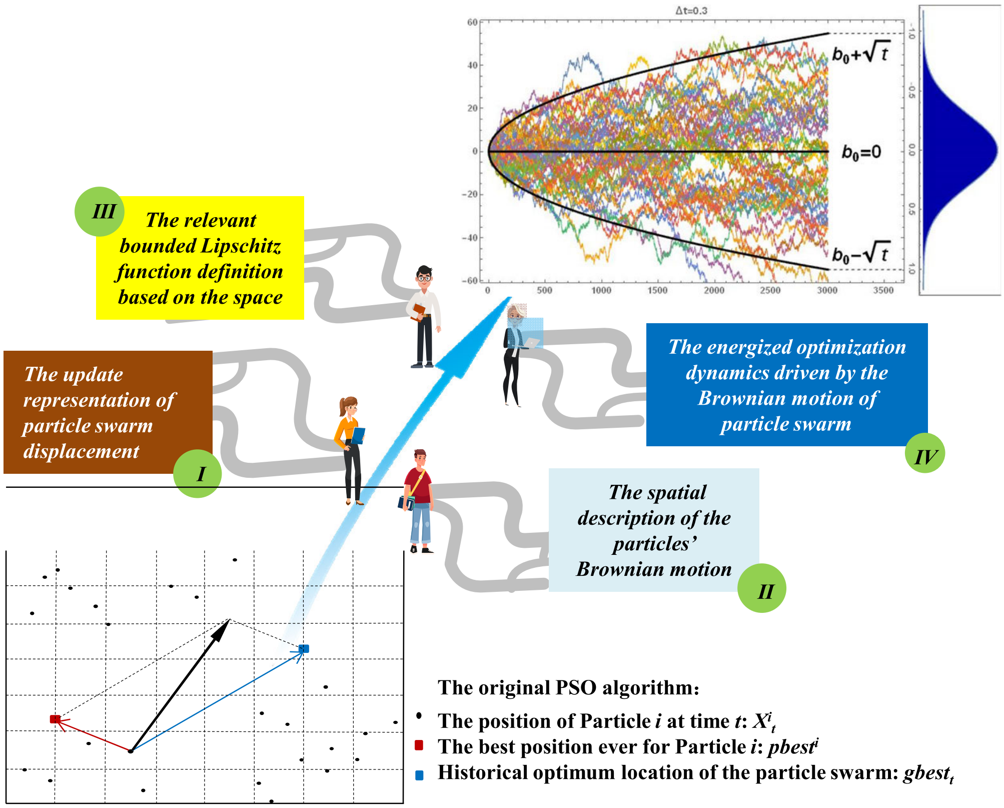

FIGURE 2. The blending pattern of particle swarm computing intelligence and Brownian motion.

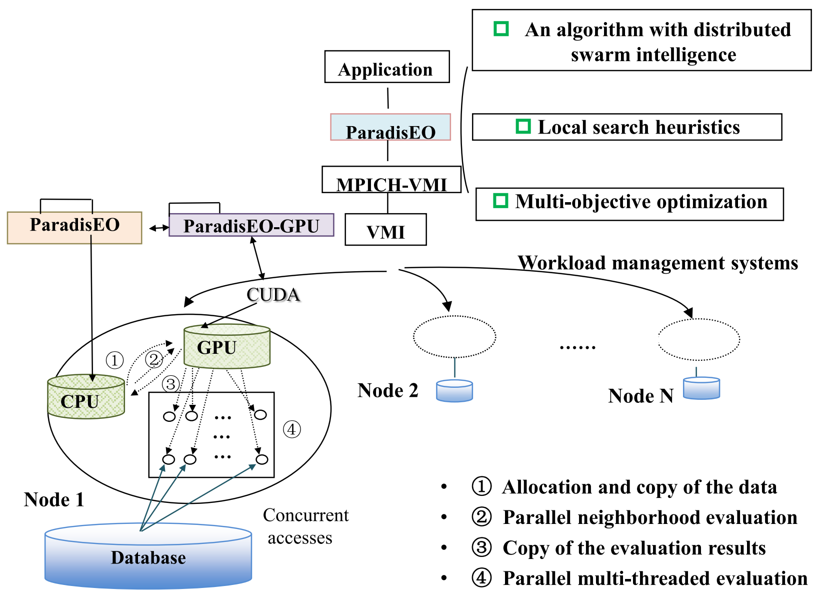

FIGURE 3. The algorithm's parallel design with deep fusion of coarse-grained and master-slave models.

Other studies refer to human adaptive learning and other mechanisms to realize various information sharing in swarm intelligence algorithms, and then to improve the convergence speed or show a stronger ability of later evolution compared with the original swarm intelligence algorithms [7]–[9].

In recent years, the representative achievement of swarm intelligence algorithm improvement is the effective fusion of standard Brownian motion or Ito Process and PSO algorithms [10]. Some experiments show that the interdisciplinarity can improve the convergence speed or maintain the swarm

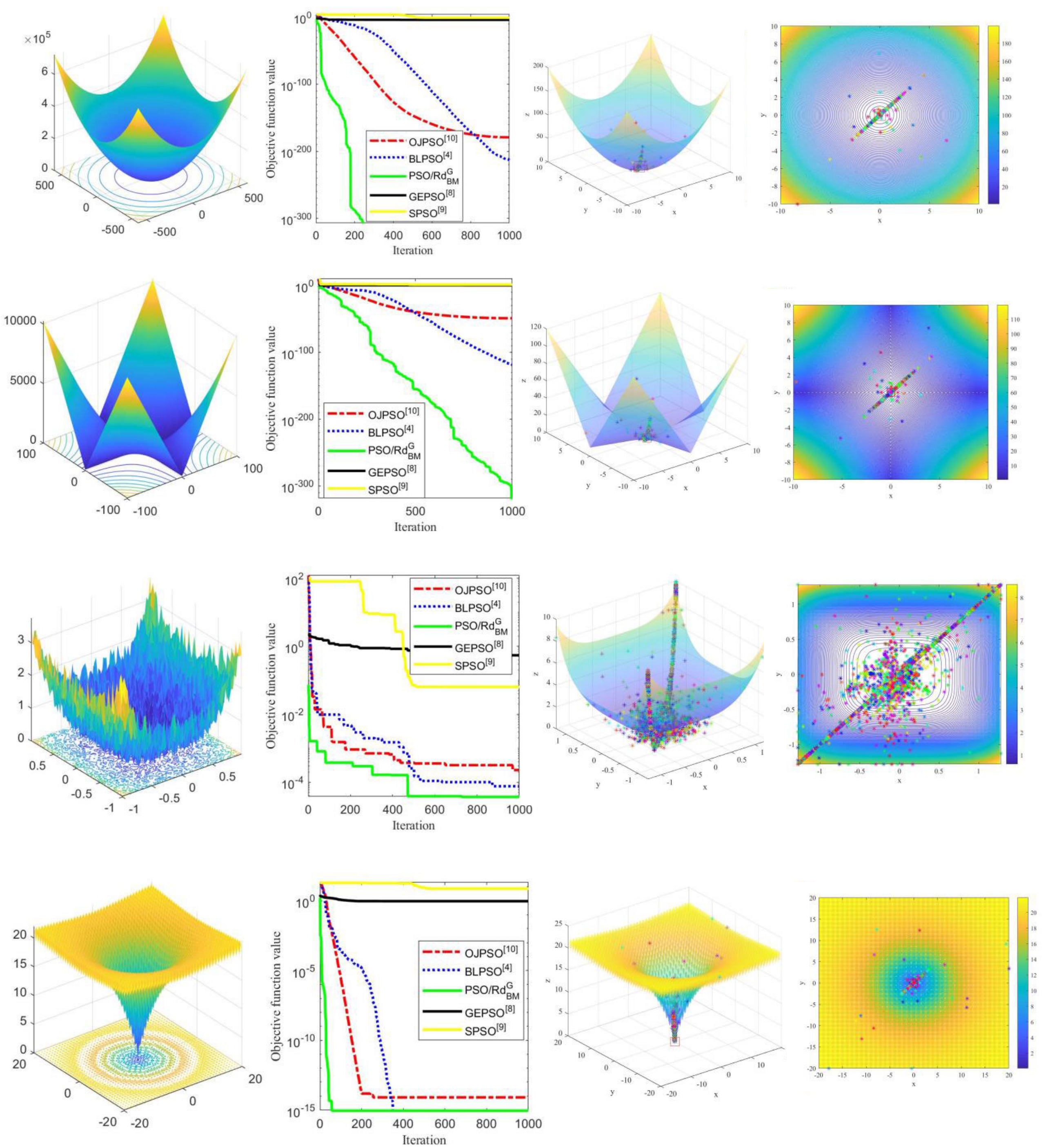

FIGURE 4. The Detail information of static performance comparison results.

diversity effectively, but at the same time, it also shows the shortcomings of the algorithm, such as the lack of stability.

B. STANDARD BROWNIAN MOTION VS. G-BROWNIAN MOTION

Standard Brownian motion was first proposed by British biologist **R.Brown** according to the random movement of pollen on the liquid surface (1827). Later, **Wiener** further studied the standard Brownian motion trajectory, and theoretically gave its spatial measure definition and other accurate descriptions (1918). Then, **Kiyoshi Ito** established the stochastic differential equation with the interference term of the standard Brownian motion, which was widely used in the fields of economy, management and social science; for this process, local stochastic disturbance and macroscopic drift are two obvious characteristics [18]–[22].

On the basis of preserving the core idea of PSO algorithm, this study intends to derive the energetic particle swarm co-evolution drive equation with nonlinear expectation space and G-Brownian motion characteristics.

The team of Academician Peng, Shige from Shandong University, the cooperative unit of this paper, has made world-renowned basic theoretical researches on G-Brownian motion [23] with their unremitting efforts and systematic theoretical advancement over the past 30 years, which are powerful and instrumental in the field of nonlinear stochastic analysis.

They include the uniqueness of solutions of backward stochastic differential equations (BSDEs) (1990), the nonlinear Feynman-Kac formula for the solution correlation between BSDEs and second-order quasilinear PDEs (1991), nonlinear expectation theory with time consistency (2006), G-Brownian motion definition (2007) [24], and G-Brownian motion numerical simulation algorithm (2019) [25].

At the same time, these preliminary works are also the valuable basis of strict theoretical derivation or proof of this topic.

III. AN ENERGIZED HETEROGENEOUS MULTIMODAL OPTIMIZATION ALGORITHM

Generally, in the original PSO proposed by Eberhart and Kennedy, the updated position (x^i_{t+1}) and velocity (v^i_{t+1}) of any particle, as the optimization-dynamic-equations, are defined as Equation **(1)** and Equation **(2)**.

$$v^i_{t+1} = \omega v^i_t + c_1 r_1 (pbest^i - x^i_t) + c_2 r_2 (gbest_t - x^i_t) \quad (1)$$

$$x^i_{t+1} = x^i_t + v^i_{t+1} \quad (2)$$

In Equation **(1)**, the first part of the formula is called the memory item, where ω represents the inertial motion, that is, the influence of the past position on the present; the second part is called self-cognition, where $c_1 r_1$ indicates that the direction of motion of particles comes from their experience; and the third part is called group cognition, where $c_2 r_2$ reflects the cooperation and information sharing between particles (see **Fig. 1**).

A. THE ENERGIZED OPTIMIZATION-DYNAMIC-EQUATIONS DRIVEN BY G-BROWNIAN MOTION

In this paper, based on the core idea of original PSO algorithm (such as memory, self-cognition and social cognition) defined as Equation (1) and Equation (2), the particle swarm evolution equation is expanded to the geometric Brownian motion model with nonlinear G-expectation, as is more generalized than the standard Brownian motion with the invariable expected drift rate or the variance rate.

Then, shown as **Fig. 2**, a series of strict theoretical derivation or proof is key, including the update representation of particle swarm displacement, denoted as $x^i_{t+1} \in \Omega_T$, and the relevant bounded Lipschitz function definition based on the space, denoted as $\xi(X) \in Lip(\Omega_T)$.

In other words, once the update representation of particle swarm displacement, the spatial description of the particles' Brownian motion and the relevant bounded Lipschitz function definition based on the space, are obtained, then the particles are accompanied by G-Brownian motion for the more intelligent optimization swarm.

The related variables and their representative meanings are shown in **Table 1**.

The PSO/Rd$^{\mathbf{G}}_{\mathbf{BM}}$ Algorithm

Step 1: **Initialize** the iteration (ι) and the subgroups, each subgroups of k particles;
Step 2:
Step 3: **Randomly initialize** the velocity and the position of the particle i;
While ($\iota < \iota_{\mathbf{max}}$) and (other termination criteria are not satisfied)
Step 4: **Do in parallel** for each island /*Obtain coarse-grained model, one of parallel and distributed models */
Step 5: $\iota = \iota + 1$;
Step 6: **For** each particle in the subgroups /*Obtain master-slave model, another parallel model */
Step 7: **Velocity update** via Equation **(1)** and make it act on the current Particle i;
Step 8: **Apply** the Equation **(3)** and Equation **(4)** to denote the spatial representation of the continuous path (displacement update) of the Particle i;
Step 9: **Apply** the Equation **(5)**, Equation **(6)** and Equation **(7)** to define the related bounded Lipschitz functions based on the space;
Step 10: **Sort** the particles in the subgroups in a decreasing order of fitness values, and **save** the fittest particle in the external memory;
Step 11: **Perform** local search strategies;
Step 12: **End For**
Step 13: **If** $\iota = \tau$(migration interval) **then**
Step 14: **Create** Ψ_δ for the current subgroups;
Step 15: **Send** Ψ_δ to the neighboring subgroups;
Step 16: **Receive** Ψ_δ from the neighboring subgroups;
Step 17: **Construct** the founding subgroups Ξ;
Step 18: **Select** k particles into Ξ;
Step 19: **Replace** the subgroups Ψ_δ with Ψ^τ_δ;
Step 20: **End If**
Step 21: **End Do in parallel**
Step 22: **End While**
Step 23: **Output** the best particle.

1) THE UPDATE REPRESENTATION OF PARTICLE SWARM DISPLACEMENT

By Equation **(1)**, the PSO algorithm can be regarded as an Ito process driven by Brownian motion and drifting toward "two" attractors, which are the best position ever for Particle i until the given moment $t+1$($pbest^i$) and the

TABLE 2. **Famous unimodal and multimodal test problems.**

Function	Dim	Range	f_{min}	Type
$F_1(x)=\sum_{i=1}^{n} x_i^2$	30	[-100,100]	0	Unimodal
$F_2(x)=\sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$	30	[-10,10]	0	Unimodal
$F_3(x)=\sum_{i=1}^{n} (\sum_{j=1}^{i} x_j)^2$	30	[-100,100]	0	Unimodal
$F_4(x)=\max\{\lvert x_i \rvert, 1 \le i \le n\}$	30	[-100,100]	0	Unimodal
$F_5(x)=\sum_{i=1}^{n-1} [100(x_{i+1}-x_i^2)^2+(x_{i+1}-1)^2]$	30	[-30,30]	0	Unimodal
$F_6(x)=\sum_{i=1}^{n} (\lfloor x_i+0.5 \rfloor)^2$	30	[-100,100]	0	Unimodal
$F_7(x)=\sum_{i=1}^{n} ix_i^4+random[0,1)$	30	[-1.28,1.28]	0	Unimodal
$F_8(x)=\sum_{i=1}^{n} -x_i \sin(\sqrt{\lvert x_i \rvert})$	30	[-500,500]	-12567	Multimodal
$F_9(x)=\sum_{i=1}^{n} [x_i^2-10\cos(2\pi x_i)+10]$	30	[-5.12,5.12]	0	Multimodal
$F_{10}(x)=-20\exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2})-\exp(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i))+20+e$	30	[-32,32]	0	Multimodal

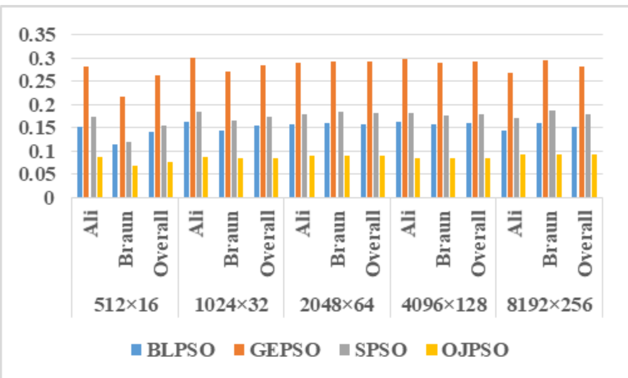

FIGURE 5. **Performance estimation of the solutions of different algorithms summarized for twelve instances.**

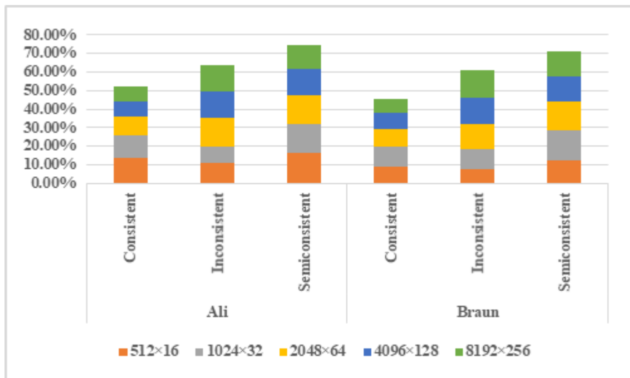

FIGURE 6. **The averaged efficiency improvements of the PSO/Rd$^{G}_{BM}$ algorithm regarding the consistency classification.**

historical optimum location of the particle swarm until the given moment $\boldsymbol{t+1(gbest_t)}$ where $\boldsymbol{c}_1\boldsymbol{r}_1$ and $\boldsymbol{c}_2\boldsymbol{r}_2$ are the drift coefficients.

Then the spatial representation of the continuous path (displacement update) of Particle $\boldsymbol{i}$, which is denoted by the following Equation **(3)**, can be obtained.

$$dX_t^i = v_t^i dt + \hat{\mu} X_t^i dt + \omega(t) X_t^i (dt)^H \tag{3}$$

2) THE SPATIAL DESCRIPTION OF THE PARTICLES' BROWNIAN MOTION

The research team of Academician Peng has made a batch of basic theoretical research results which are internationally renowned and have been shown to be powerful tools in the field of financial mathematics by virtue of nearly 30 years of unremitting efforts and systematic and solid theoretical advancement.

And these preliminary works, represented by the unique proof of solutions of backward stochastic differential equations (BSDES), the nonlinear Feynman-Kac formula for the correlation between BSDES and second-order quasilinear PDES solutions, and the nonlinear expectation theory with time consistency, are also the precious strict theoretical derivation or proof basis of this topic.

Following that, the space of Particle $\boldsymbol{i}$ with Brownian motion, can be denoted as Equation **(4)**.

$$\hat{X}_{t+1}^i = (1+\hat{\mu})X_t^i + \omega(t)X_t^i + v_{t+1}^i \tag{4}$$

TABLE 3. Statistical values of the methods for the 30-variable unimodal/multimodal benchmarks functions (F_1-F_{10}).

Function	Item	BLPSO[4]	GEPSO[8]	SPSO[9]	OJPSO[10]	PSO/Rd$_{BM}{}^G$
F_1	Best	4.22E-01	1.61E-07	1.94E-06	1.06E-19	**0.00E+00**
	Median	9.21E-01	2.68E-05	5.19E-06	1.86E-15	**0.00E+00**
	Mean	1.01E-00	4.61E-04	5.09E-06	4.81E-08	**0.00E+00**
	Worst	1.94E+00	3.08E-03	9.34E-06	2.41E-06	**0.00E+00**
	STD	3.58E-01	8.82E-04	1.82E-06	3.40E-07	**0**
F_2	Best	1.72E-01	2.98E-03	1.47E-08	9.08E-12	**0.00E+00**
	Median	3.70E-01	6.70E-03	1.24E-06	3.00E-11	**0.00E+00**
	Mean	3.73E-01	6.88E-03	6.42E-06	2.97E-11	**0.00E+00**
	Worst	6.54E-01	1.26E-04	7.36E-05	7.37E-11	**0.00E+00**
	STD	1.01E-01	1.94E-03	1.64E-05	1.52E-11	**0**
F_3	Best	9.07E-02	7.18E-02	2.22E-07	5.03E-03	**0.00E+00**
	Median	3.71E-01	2.54E-03	1.07E-03	1.15E-02	**0.00E+00**
	Mean	4.27E-01	2.72E-03	3.68E-02	1.20E-02	**0.00E+00**
	Worst	1.27E-00	7.43E-03	9.70E-01	2.65E-02	**0.00E+00**
	STD	2.59E-01	1.18E-03	1.56E-01	4.30E-03	**0**
F_4	Best	0.13E-117	0.49E-219	1.36E-179	1.68E-224	**6.68E-228**
	Median	1.33E-109	1.79E-208	1.77E-168	1.28E-109	**6.68E-228**
	Mean	2.24E-126	4.49E-128	1.39E-159	3.08E-99	**6.68E-228**
	Worst	7.13E-227	6.49E-208	1.33E-139	0.77E-122	**6.68E-228**
	STD	1.03E-09	1.69E-36	3.36E-09	6.68E-09	**0**
F_5	Best	1.78E-09	2.29E-28	0.28E-17	0.36E-09	**0.00E+00**
	Median	2.24E-17	1.38E-09	2.19E-13	1.36E-17	**0.00E+00**
	Mean	1.22E-17	1.49E-28	1.39E-17	1.62E-19	**0.00E+00**
	Worst	1.22E-09	1.28E-17	0.36E-11	2.19E-09	**0.00E+00**
	STD	1.37E-09	0	2.19E-03	2.36E-17	**0**
F_6	Best	1.62E-19	1.57E-02	6.28E-19	3.16E-09	**0.00E+00**
	Median	2.28E-13	1.39E-09	2.66E-13	1.33E-07	**0.00E+00**
	Mean	1.73E-07	1.49E-17	1.22E-09	1.78E-09	**0.00E+00**
	Worst	0.22E-19	2.28E-19	0.36E-09	2.19E-09	**0.00E+00**
	STD	1.37E-09	0	0	3.36E-17	**0**
F_7	Best	7.22E-22	2.49E-16	3.39E-17	9.62E-13	**1.28E-17**
	Median	2.33E-19	1.09E-18	2.99E-18	6.68E-12	**1.28E-17**
	Mean	1.22E-17	2.26E-17	1.37E-16	3.62E-17	**1.28E-17**
	Worst	4.56E-19	1.37E-08	6.22E-22	1.02E-09	**1.28E-17**
	STD	5.22E-17	3.49E-28	2.39E-17	2.62E-13	**0**
F_8	Best	-3.59E+03	-4.77E+03	-3.34E+03	-1.04E+04	**-1.06E+04**
	Median	-2.11E+04	-6.28E+03	-9.61E+02	-2.73E+03	**-1.00E+04**
	Mean	-4.09E+03	-5.17E+03	-8.07E+02	-2.72E+03	**-1.00E+04**
	Worst	-7.07E+03	-5.87E+03	-6.49E+02	-2.06E+03	**-9.44E+02**
	STD	1.17E+02	2.79E+02	2.53E+02	1.68E+02	**2.49E+02**
F_9	Best	2.57E-19	3.33E-18	2.22E-12	1.28E-09	**2.22E-16**
	Median	4.56E-19	1.37E-08	6.37E-22	1.02E-09	**2.22E-16**
	Mean	1.06E-19	9.39E-08	6.28E-17	1.32E-09	**2.22E-16**
	Worst	5.56E-19	7.37E-08	0.22E-19	0.66E-09	**2.22E-16**
	STD	0.56E-19	1.77E-08	6.28E-07	2.02E-09	**2.22E-16**
F_{10}	Best	7.62E-19	5.57E-02	3.28E-19	2.16E-09	**0.00E+00**
	Median	6.28E-13	4.39E-09	3.66E-13	3.33E-07	**0.00E+00**
	Mean	4.73E-07	4.49E-17	2.22E-09	2.78E-09	**0.00E+00**
	Worst	8.22E-19	8.28E-19	6.36E-09	5.19E-09	**0.00E+00**
	STD	3.37E-19	0	0	2.16E-07	**0**

3) THE RELEVANT BOUNDED LIPSCHITZ FUNCTION DEFINITION BASED ON THE SPACE

Following that, the related bounded Lipschitz functions based on the space are defined as Equation **(5)**, Equation **(6)** and Equation **(7)**,

$$\boldsymbol{Lip}(\Omega_{\boldsymbol{T}})=\{\varphi(\boldsymbol{B}_{t_1},\cdots,\boldsymbol{B}_{t_k}):\boldsymbol{k}\in\boldsymbol{N},\ 0=\boldsymbol{t}_0<\cdots<\boldsymbol{t_k}\leq\boldsymbol{T},\ \varphi\in\boldsymbol{C}_{b,Lip}(\boldsymbol{R}^{k\times d})\} \quad (5)$$

where $t \in [0,T]$ and $C_{b,Lip}(R^{k\times d})$ denotes the space of bounded and Lipschitz functions on $R^{k\times d}$.

$$\xi(\boldsymbol{X})=\varphi(\boldsymbol{B}_{t_1},\cdots,\boldsymbol{B}_{t_k}):\boldsymbol{k}\in\boldsymbol{N},0=\boldsymbol{t}_0<\cdots<\boldsymbol{t_k}\leq\boldsymbol{T},\ \varphi\in\boldsymbol{C}_{b,Lip}(\boldsymbol{R}^{k\times d}), \quad (6)$$

$$\xi(\boldsymbol{X})=\left\{\begin{array}{c}\boldsymbol{B}_{t_0}=\boldsymbol{b}_0\\ \frac{1}{\Gamma(\boldsymbol{H}+0.5)}\\ \boldsymbol{B}_{t_k}-\boldsymbol{B}_{t_0}\\ =\times\{\int_{-\infty}^{0}[|\boldsymbol{t_k}-\Gamma|^{\boldsymbol{H}-0.5}-|\Gamma|^{\boldsymbol{H}-0.5}]\boldsymbol{d}(\boldsymbol{B}(\Gamma))\\ +\int_{-\infty}^{0}|\boldsymbol{t_k}-\Gamma|^{\boldsymbol{H}-0.5}\boldsymbol{d}(\boldsymbol{B}(\Gamma))\}\end{array}\right\} \quad (7)$$

4) THE ENERGIZED OPTIMIZATION DYNAMICS DRIVEN BY THE BROWNIAN MOTION OF PARTICLE SWARM

To summarize briefly, the optimization-dynamic-equation of particle swarm driven by the Brownian stochastic motion with nonlinear expectation, is well defined in Equation **(4)**, Equation **(5)**, Equation **(6)**, and Equation **(7)**, respectively, in order for a more generalized co-evolution of the particle swarms.

B. ALGORITHM DESCRIPTION

In this paper, in order for suitable for the hybrid processor architecture of the scheduling management server, the algorithm is designed in parallel with deep fusion of coarse-grained and master-slave models (see Fig. 3).

Following that, the algorithm can be described as follows

IV. EXPERIMENT RESULTS AND DISCUSSION

In this section, all the experiments have been carried out at national supercomputing center in jinan, china.

A. SIMULATOR AND SIMULATION PARAMETERS

To ensure that the comparison between the algorithms is fair, there are not any special requirements for parameter setting between different methods; in other words, the general parameter values of the PSO algorithm are set normally.

Famous unimodal and multimodal test problems are shown in **Table 2**. They are roughly divided into unimodal functions with changing variables, and multimodal functions with fixed or changing variables; and they are used to verify the static optimization performance of the different methods, where unimodal functions can assist in the global convergence validation and multimodal functions can test the ability of the local search or averting premature convergence.

B. EXPERIMENTAL RESULTS AND ANALYSIS

In this subsection, the multi-objective optimization performance comparison between the different methods including the static and dynamic optimization performance results analysis, is given.

1) STATIC OPTIMIZATION PERFORMANCE RESULTS ANALYSIS

The unimodal or multimodal statistical optimization results of BLPSO [4],GEPSO [8],SPSO [9],OJPSO[10] and PSO/Rd$^{G}_{BM}$, are shown as **Table 3**. It can be found that the quality of the solutions obtained by the PSO/Rd$^{G}_{BM}$ approach is higher than BLPSO [4],GEPSO [8],SPSO [9], and OJPSO [10].

Shown as **Fig. 4**, the 3-D shapes of some selected functions with obvious differences in the response surface, the convergence corves of five methods(BLPSO [4], GEPSO [8], SPSO [9], OJPSO [10] and PSO/RD$^{G}_{BM}$), and 2-D OR 3-D convergence details of the PSO/RD$^{G}_{BM}$ algorithm, are given.

From **Fig. 4**, it can be seen that for the unimodal or multimodal functions, the PSO/RD$^{G}_{BM}$ algorithm can quickly seek out the globally optimal solutions at the early co-evolutionary stage, while the others (BLPSO [4], GEPSO [8],SPSO [9], AND OJPSO [10]) FAIL TO MAKE IT.

2) DYNAMIC OPTIMIZATION PERFORMANCE RESULTS ANALYSIS

Here, to compare the dynamic optimization performance of different algorithms, twelve classic instances of heterogeneous computing and cloud scheduling proposed by [26] are used.

Fig. 5 summarizes the averaged efficiency improvements of the solution of PSO/Rd$^{G}_{BM}$ over that of BLPSO [4], GEPSO [8], SPSO [9], and OJPSO [10], for each dimension and heterogeneity model.

Shown as **Fig. 6**, for semiconsistent instances or for consistent instances, the averaged efficiency preponderance of the PSO/Rd$^{G}_{BM}$ approach over the other four methods (BLPSO [4], GEPSO [8], SPSO [9], and OJPSO [10]) is obvious. Lower improvement factors are obtained for small inconsistent instances, but for large inconsistent instances, the advantage significantly increase.

V. CONCLUSION

As the extension of traditional artificial intelligence, PSO algorithms representing the swarm intelligence algorithms of the computational intelligence theory, have been widely used in heterogeneous multimodal optimization.

Although with some improvements, the PSO algorithms are mostly ***linear*** optimization-dynamic-models; and the PSO algorithm integrated with Ito process driven by the standard Brownian motion, is reduced to a ***special*** Markov stochastic process with ***constant*** expected drift rate and variance rate, also ***without*** the generality in the dynamic swarm intelligence simulation.

In this paper, based on the core idea of original PSO algorithm (such as memory, self-cognition and social cognition) and the strict theories such as nonlinear stochastic analysis, a greener heterogeneous scheduling algorithm via blending pattern of particle swarm computing intelligence and geometric Brownian motion, is proposed, to achieve the balance of ''local exploration and macro development'' or ''route diversity and faster convergence''.

Then, the evaluation indicators can be divided into two categories: static and dynamic optimization performance. A large number of experimental results are given. Compared with most newly published scheduling algorithms (BLPSO [4], GEPSO [8], SPSO [9], and OJPSO [10]), there are significant advantages of the proposed algorithm on the dynamic optimization performance for consistent or semiconsistent and large inconsistent scheduling instances, although with lower improvement factors for small inconsistent instances.

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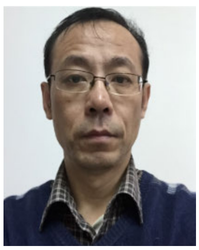
SHAOHUI LI received the B.S. and M.S. degrees in computer science and technology from Shandong Normal University, Jinan, China, in 2002 and 2006, respectively.

Since 2009, he has devoted himself to the researches on the multi-disciplinary direction of green computing, heterogeneous multicore super-systems, cloud scheduling middleware, and distributed artificial intelligence. He currently works as a Teacher with the School of Information Science and Engineering, Shandong Normal University. His work has been supported by the National Natural Science Foundation of China and the National High Technology Research and Development Program of China.

HONG LIU received the Ph.D. degree from the Institute of Computer Science, Chinese Academy of Sciences, China, in 1998.

She is currently a Professor and the Doctoral Supervisor with the School of Information Science and Engineering, Shandong Normal University, Jinan, China. She is also the Director of the Key Laboratory of "Distributed Computer Software New Technology" of Shandong Province, the Information Management Engineering Technology Center of Shandong Province, and the Computer Science and Technology Project with national characteristics. She is mainly engaged in the theory and application research of swarm intelligence algorithm and cooperative computer multi-agent systems. She has successively presided over six projects supported by the National Natural Science Foundation of China and nine provincial and ministerial projects. She has published more than 200 papers in SCI and EI retrieval and other famous academic journals and international conferences.



BIN GONG received the B.S. and M.S. degrees in computer science and technology from Shandong University, Jinan, China, in 1986 and 1989, respectively, and the Ph.D. degree in computer system architecture from Shandong University.

He is currently a Professor, the Doctoral Supervisor, and the Director with the High Performance Computing Center, Shandong University. He is one of the investigators, who have devoted themselves to the researches on supercomputer architecture and high performance computing, China, since 1980s. His work has been supported by the National High Technology Research and Development Program of China (863 Program).

JINGLIAN WANG received the B.S. and M.S. degrees in computer science and technology from Shandong Normal University, Jinan, China, in 2002 and 2006, respectively, and the Ph.D. degree in computer system architecture from Shandong University, Jinan, in 2015.

She is currently an Associate Professor with the College of Information and Electrical Engineering, Ludong University, Yantai, China. She is mainly engaged in the multi-disciplinary direction of green computing, heterogeneous multicore super-systems, cloud scheduling middleware, and distributed artificial intelligence. Her work has been supported by the National Science Foundation for Young Scholars of China and the Talent Introduction Project of Ludong University.