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Delay-Dependent State-Feedback Dissipative Control for Suspension Systems With Constraints Using a Generalized Free-Weighting-Matrix Method

HYUN DUCK CHOI [1] **AND SEOK-KYOON KIM** [2]
[1]Department of ICT Convergence System Engineering, Chonnam University, Gwangju 61186, South Korea
[2]Department of Creative Convergence Engineering, Hanbat National University, Yuseong-gu, Daejeon 34158, South Korea
Corresponding author: Seok-Kyoon Kim (skkim77@hanbat.ac.kr)

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ABSTRACT This paper presents a novel delay-dependent dissipative control synthesis technique with a state-feedback structure for input-delayed suspension systems using a tighter bounding technique. By more accurately estimating the derivative of the LKF, we focus on reducing the conservatism of the state-feedback control synthesis for suspension systems with strict design constraints. New LMI conditions for a desired state-feedback controller are developed by employing a generalized free-weighting-matrix (GFWM) method. By solving the LMIs, the proposed controller for active suspension systems is obtained such that the closed-loop systems have asymptotic stability with guaranteed $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative performances, while also satisfying the design constraint conditions. Numerical simulations effectively confirm the benefits of the proposed control synthesis technique for the design of state-feedback control.

INDEX TERMS Lyapunov–Krasovskii stability, generalized free-weighting-matrix, linear matrix inequality, state-feedback, suspension systems, dissipative control, bonded time-varying delay.

I. INTRODUCTION

Recently, autonomous driving technology has been on the rise because it promises to improve the safety of vehicles and the comfort of drivers and passengers. The demand for electronic stability control systems, such as rollover protection and adaptive cruise control, is growing [1], [2]. In particular, research on suspension systems has received significant attention from academia and industry because it can improve vehicle comfort and steering stability [3]. State-of-the-art suspension systems can be divided into two classes: active and semi-active suspensions [4], [5]. A semi-active suspension can only change the damping coefficient based on the road conditions, whereas an active suspension can raise and lower the chassis through an actuator independently mounted on each wheel [6], [7].

The major concern of active suspension systems is the effective handling of the trade-off between competing performances [8]. This can be regarded as a multi-objective optimization problem, and various control techniques for active suspension systems have been introduced [9]–[11]. Among them, H_∞ control has been recognized as a powerful tool for managing the trade-off so that a compromise between the conflicting performance requirements can be achieved. This is because, based on the Lyapunov stability theory and linear matrix inequality, it can be treated as a single control problem by treating ride comfort as the main target and other performance indicators as constraints [12]–[15].

On the other hand, the notion of dissipativity comes from energy-based control, which generalizes the concepts of energy storage and dissipation. The theory of dissipation inequality in [16], [17] has received considerable attention in many fields, such as circuits, networks, systems, and control engineering [18]–[21]. Dissipative control is advantageous in providing a unified framework that includes conventional controls such as H_∞ control and passive control as special cases, and providing design flexibility through $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ adjustment [22], [23]. This can be an effective solution for

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reducing the conservatism of controller design problems where design constraints exist [24], [25]. Despite this potential, the controller design results utilizing the concept of dissipation in suspension systems are very limited, and in particular, the problem of input-delay state-feedback dissipative control has not been solved yet.

In many practical applications, the time-varying delay is frequently considered a factor causing instability of dynamic systems [26], [27]. Therefore, time delay has been recognized as an essential consideration in stability analysis and control design, and has been extensively studied over the last few decades [28]–[30]. A challenging task in control design and stability analysis for time-delayed systems is extending the allowable upper bound to ensure the stability of closed-loop systems [31], [32]. This can be achieved by reducing the numerically derived stability or conservatism of the controller synthesis conditions [33]. Therefore, some tighter bounding techniques have been proposed in the literature to reduce conservatism considering stability criteria and control design [34]–[37]. Representative bounding techniques for estimating the derivative of the Lyapunov-Krasovskii function include the Jensen inequality [34], Wirtinger-type inequality [35], free-weight matrix approach [36], and reciprocal convex technique [37]. Developing and using effective boundary techniques to contribute to the reduction of conservatism in controller synthesis remains a challenging task.

In active suspension systems, considerable attention has been paid to the issue of actuators input delay [38], [39]. Many studies have provided numerical solutions based on the Lyapunov stability theory and the LMI approach [40]–[44]. The authors in [40] proposed a delay-dependent controller in the context of robustness and disturbance attenuation. Fuzzy sampled-data control for uncertain vehicle suspension systems was addressed in [41]. By employing a delay-dependent Lyapunov function, the conditions of a robust non-fragile controller were derived in [42]. The results of the dynamic output-feedback control for half-vehicle suspensions were obtained in [43]. The problem of sampled-data H_∞ control of nonlinear suspension systems via a fuzzy approach was studied in [44]. However, the aforementioned results were limited to control design conditions that guarantee robust performance in the H_∞ sense. In addition, only a few studies have focused on the conservatism of controller synthesis conditions caused by design constraints, and the conditions presented above are still conservative. In particular, developing an efficient and less conservative analysis method for the design of a controller for a state-feedback structure remains a difficult yet important task.

Based on the above discussions, we focused on developing a less conservative condition for the state-feedback dissipative control strategy for input-delayed suspension systems. To simultaneously satisfy the asymptotic stability, achieve strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative performance, and obtain desirable design constraints of the suspension system, including the state-feedback controller, a novel LMI-based control synthesis method was developed. The proposed approach is beneficial because it reduces the conservatism of the control synthesis problem for suspension systems, which can be achieved by introducing a generalized free weight matrix (GFWM) technique. In addition, the proposed control synthesis technique provides an integrated framework for the controller synthesis of input delay suspension systems with design constraints including conventional H_∞ and passive control as special cases. Numerical simulations are presented to confirm the superiority of the proposed method over other bounding techniques and to demonstrate the importance of the delay-dependent approach when designing the controller of the suspension system.

Notation: Throughout this note, a symmetric and positive definite (semi-definite) matrix is denoted by the notation $P > 0$ ($P \geq 0$). $\|\cdot\|_\infty$ refers to the H_∞ norm for matrices. R^n represents the Euclidean space with dimension n. The superscript T refers to the matrix transposition. We use an asterisk $*$ to represent a symmetric term in the symmetric matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. $\mathrm{sym}\{A\}$ is the shorthand notation for $A + A^T$. $\mathrm{diag}\{\cdot\}$ refers to the block-diagonal matrix. The space of the square-integrable vector function over $[0, \infty)$ is represented by $L_2[0, \infty)$, and for $x = \{x(t)\} \in L_2[0, \infty)$, its norm is represented by $\|x\|_2 = \sqrt{\int_0^\infty |x(t)|^2\, dt}$.

II. SYSTEM MODELING AND PROBLEM DESCRIPTION

Figure 1 illustrates a typical quarter-car suspension system introduced in various studies. The variable descriptions of the systems are presented in Table. 1. Based on Newton's classical laws, the dynamics of the suspension motion are governed by the following equation:

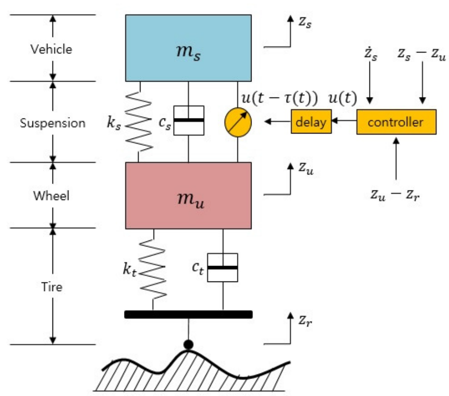

FIGURE 1. Quarter-car suspension system.

$$
\begin{aligned}
&m_s\ddot{z}_c(t) + c_s[\dot{z}_s(t) - \dot{z}_u(t) + k_s[z_s(t) - z_u(t)]] \\
&\quad = u(t - h(t)), \\
&m_u\ddot{z}_u(t) - c_s[\dot{z}_s(t) - \dot{z}_u(t) - k_s[z_s(t) - z_u(t)]] \\
&\qquad + c_t[\dot{z}(t) - \dot{z}_r(t) + k_t[z_u(t) - z_r(t)]] \\
&\quad = -u(t - h(t))
\end{aligned} \tag{1}
$$

where $h(t)$ is the bounded time-varying delay with the following conditions:

$$0 \leq h(t) \leq d \quad (2)$$
$$\dot{h}(t) \leq \mu \quad (3)$$

where d, and μ are prescribed scalar; Before proceeding,

TABLE 1. Description of variables.

Description	Variable	Units
Sprung mass	m_s	kg
Unsprung mass	m_u	kg
Displacements (sprung mass)	z_s	m
Displacements (unsprung mass)	z_u	m
Road displacement	z_r	m
Stiffness of suspension	k_s	N/m
Stiffness of tire	k_u	N/m
Damping of suspension	c_s	Ns/m
Damping of tire	c_u	Ns/m
Active input	u	N

we must explore the key performance metrics of the suspension system and the control objective. The representative performance metrics for the control design of the active suspension are ride comfort, road holding, and suspension stroke. It is well known that body acceleration is a quantitative measure of vehicle ride comfort. Thus, we set $\ddot{z}_s(t)$ as the main control variable. Other performance metrics for the quantitative evaluation of suspension systems include suspension and tire deflection, which are generally considered constraint variables. Owing to physical constraints, the suspension stroke only allows movement within the maximum allowable limits. This can be represented by $|z_s(t) - z_u(t)| \leq z_{max}$. Moreover, to guarantee solid contact between the wheels and road, the dynamic and static tire loads must satisfy the condition $k_t(z_u(t) - z_r(t)) < (m_s + m_u)g$. The design of an active suspension can be regarded as a multi-objective optimization problem that minimizes vertical acceleration within the constraints defined above. Based on the above performance criteria, the body acceleration is selected as the first control output $z_1(t)$, and the suspension deflection and dynamic tire load are defined as the second control outputs $z_2(t)$. Furthermore, by choosing the state vector as $x(t) = [z_u(t) - z_r(t)\; z_u(t) - z_r(t)\; \dot{z}_s(t)\; \dot{z}_u(t)]^T$ and $w(t) = \dot{z}_r(t)$, the dynamics of the suspension motion can be expressed by the following state equation:

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + B_u u(t - h(t)) + B_w w(t),\\
z_1(t) &= C_1 x(t) + D_{1u} u(t - h(t)),\\
z_2(t) &= C_2 x(t),\\
y(t) &= Cx(t),
\end{aligned}$$

where

$$\begin{aligned}
A &= \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -\frac{k_s}{m_s} & 0 & -\frac{c_s}{m_s} & \frac{c_s}{m_s} \\ \frac{k_s}{m_u} & -\frac{k_t}{m_u} & \frac{c_s}{m_u} & -\frac{c_s + c_t}{m_u} \end{bmatrix},\\
B_u &= \begin{bmatrix} 0 & 0 & \frac{1}{m_s} & -\frac{1}{m_u} \end{bmatrix}^T,\\
B_w &= \begin{bmatrix} 0 & -1 & 0 & \frac{c_t}{m_u} \end{bmatrix}^T,\\
C &= \begin{bmatrix} 1 & 1 & 1 & 0 \end{bmatrix}\\
C_1 &= \begin{bmatrix} -\frac{k_s}{m_s} & 0 & -\frac{c_s}{m_s} & \frac{c_s}{m_s} \end{bmatrix}, \quad D_1 = \frac{1}{m_s}\\
C_2 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{k_t}{(m_s + m_u)g} & 0 & 0 \end{bmatrix}
\end{aligned} \quad (4)$$

It is assumed that an online measurement of all states is available. Thus, the controller with the state-feedback form is given as follows:

$$u(t) = Kx(t). \quad (5)$$

By applying a controller of the form (5) to the suspension model in (4), we can represent a closed-loop system as follows:

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + B_u Kx(t - h(t)) + B_w w(t),\\
z_1(t) &= C_1 x(t) + D_{1u} Kx(t - h(t)),\\
z_2(t) &= C_2 x(t),\\
y(t) &= Cx(t),
\end{aligned} \quad (6)$$

In this study, we introduced the notion of $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ dissipativity, which was used as a performance criterion.

Definition 1: For the given matrices $\mathcal{Q}$, $\mathcal{S}$, and $\mathcal{R}$, the closed-loop system (6) is said to be strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative if there exists a scalar α, such that the following inequality holds under the zero initial condition:

$$E(t) \geq \alpha \int_0^t w^T(s) w(s) ds \quad (7)$$

where $E(t) = z^T(t)\mathcal{Q}z(t) + 2z^T(t)\mathcal{S}w(t) + w^T(t)\mathcal{R}w(t)$ is called energy supply function, and α is the dissipativity performance index. Without loss of generality, $\mathcal{Q}$ and $\mathcal{R}$ are symmetric, and $\mathcal{Q} \leq 0$ and $-\mathcal{Q} = \mathcal{Q}_-^T \mathcal{Q}_-$ for any $\mathcal{Q}_- \geq 0$.

Remark 1: In terms of the energy dissipation in a suspension system, the energy associated with the $(w(t), z_1(t))$ pair stored in the system must be strictly less than the energy supplied. This can be guaranteed with dissipative performance by providing an integrated framework that includes conventional H_∞ and passivity control as special cases.

Remark 2: The notion of $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipativity introduced in Definition 1 includes some well-known performance indices as special cases by changing the weighting matrices:

- *Case 1: By choosing $\mathcal{Q} = -I$, $\mathcal{S} = 0$, and $\mathcal{R} = (\alpha^2 + \alpha)I$, the $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative performance can be changed to H_∞ performance.*

- *Case 2: By choosing $\mathcal{Q} = 0, \mathcal{S} = I$, and $\mathcal{R} = 2\alpha I$, the $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative performance can be changed to passivity performance.*
- *Case 3: By choosing $\mathcal{Q} = -\epsilon I, \mathcal{S} = (1-\epsilon)I$, and $\mathcal{R} = [(\alpha^2 - \alpha)\epsilon + 2\alpha]I$, the $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative performance can be changed to mixed passivity/H_∞ performance. ϵ is a design parameter that allows flexible tuning between passivity and H_∞ performance.*

This study aimed to find the gain matrix K of the controller in (5), which guarantees that

1) an asymptotic stability of the closed-loop systems (6) with the bounded time-varying delay $h(t)$ satisfying (2) and (3);
2) $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipativity related to the transfer function from the disturbance $w(t)$ to the control output $z_1(t)$ under zero initial conditions, for all nonzero $w \in L_2[0, \infty)$;
3) the constraints for the control output and input are as follows:

$$|z_2(t)| \leq z_{2,max}, \quad |u(t)| \leq u_{max}, \; t > 0 \tag{8}$$

III. GFWM-BASED LMI CONDITIONS FOR THE STATE-FEEDBACK H_∞ CONTROLLER

A. PRELIMINARY

Before proceeding, it is necessary to address the following lemmas:

Lemma 1 (Wirtinger-Type Inequality): For the well-defined vector $\rho : [a, b] \mapsto \mathcal{R}^n$, scalar $a < b$, and symmetric $n \times n$ matrix $R > 0$; thus, the following inequality holds:

$$\int_a^b \rho^T(s)R\rho(s)ds \geq \frac{1}{b-a}\begin{bmatrix}\eta_1\\ \eta_2\end{bmatrix}^T \begin{bmatrix}R & 0\\ * & 3R\end{bmatrix}\begin{bmatrix}\eta_1\\ \eta_2\end{bmatrix} \tag{9}$$

where $\eta_1 = \int_a^b \rho(s)ds$ and $\eta_2 = \eta_1 - \frac{2}{b-a}\int_b^a \int_a^s \rho(u)duds = -\eta_1 + \frac{2}{b-a}\int_b^a \int_s^b \rho(u)duds$.

Lemma 2 (Free-Matrix-Based Inequality): For the well-defined vector $\omega : [a, b] \mapsto \mathcal{R}^n$, symmetric $n \times n$ matrix R and $3n \times 3n$ matrices S_1, S_3, any $3n \times 3n$ matrices S_2 and $3n \times n$ matrices L_1, L_2 such that

$$\begin{bmatrix}S_1 & S_2 & L_1\\ * & S_3 & L_2\\ * & * & R\end{bmatrix} > 0 \tag{10}$$

the following inequality holds:

$$\int_a^b \dot{\omega}^T(s)R\dot{\omega}(s)ds \geq -(b-a)\upsilon_0^T\left(\frac{3S_1+S_3}{3}\right)\upsilon_0 \\ -\mathrm{Sym}\left\{\upsilon_0^T L_1 \upsilon_1 + \upsilon_0^T L_2 \upsilon_2\right\} \tag{11}$$

where $\upsilon_0 = [\omega^T(b)\ \omega^T(a)\ \int_a^b \frac{\omega^T(s)}{b-a}ds]^T$, $\upsilon_1 = \omega(b) - \omega(a)$, and $\upsilon_2 = \omega(b) + \omega(a) - 2\int_a^b \frac{\omega^T(s)}{b-a}ds$.

Lemma 3 (Reciprocally Convex Approach): Given constants $0 \leq \alpha \leq 1$, any vectors β_1 and β_2, symmetric matrix R, and any matrix Z such that $\begin{bmatrix}R & Z\\ * & R\end{bmatrix} \geq 0$, the following inequality holds:

$$\frac{1}{\alpha}\beta_1^T R\beta_1 + \frac{1}{1-\alpha}\beta_2^T R\beta_2 \geq \begin{bmatrix}\beta_1\\ \beta_2\end{bmatrix}^T \begin{bmatrix}R & Z\\ * & R\end{bmatrix}\begin{bmatrix}\beta_1\\ \beta_2\end{bmatrix} \tag{12}$$

Lemma 4: For any symmetric matrices Ξ_0, Ξ_1, and Ξ_2, and a scalar function $\kappa(t) \in [0, \gamma]$ with constant γ, the following inequality holds:

$$\left.\begin{array}{r}\gamma^2\Xi_0 + \gamma\Xi_1 + \Xi_2 \leq 0\\ \gamma\Xi_1 + \Xi_2 \leq 0\\ \Xi_2 \leq 0\end{array}\right\} \Rightarrow \kappa^2(t)\Xi_0 + \kappa(t)\Xi_1 + \Xi_2 \leq 0$$
$$\left.\begin{array}{r}\Xi_0 \geq 0\\ \gamma^2\Xi_0 + \gamma\Xi_1 + \Xi_2 \leq 0\\ \Xi_2 \leq 0\end{array}\right\} \Rightarrow \kappa^2(t)\Xi_0 + \kappa(t)\Xi_1 + \Xi_2 \leq 0$$
$$\left.\begin{array}{r}\Xi_0 \leq 0\\ \gamma\Xi_1 + \Xi_2 \leq 0\\ \Xi_2 \leq 0\end{array}\right\} \Rightarrow \kappa^2(t)\Xi_0 + \kappa(t)\Xi_1 + \Xi_2 \leq 0 \tag{13}$$

Lemma 5 (GFWM-Based Inequality): For the well-defined vector $\rho : [a, b] \mapsto \mathcal{R}^n$, symmetric $n \times n$ matrix $R > 0$, and any matrices M, N. Thus, the following inequality holds:

$$\int_a^b \rho^T(s)R\rho(s)ds \geq -\mathrm{Sym}\left\{\eta_0^T M\eta_1 + \eta_0^T N\eta_2\right\} \\ -(b-a)\eta_0^T\left(\frac{3MR^{-1}M^T + NR^{-1}N^T}{3}\right)\eta_0 \tag{14}$$

where η_0 is any vector and η_1, η_2 are prescribed in Lemma 1.

Remark 3: As the GFWM technique does not require $\rho = \dot{\omega}$, it can handle single integral terms in a more general form than the FWM. In addition, the GFWM is more advantageous for the generation of less conservative LMI conditions than the FWM because it includes more zero-value terms that reduce the gap between the integral term and the estimate.

Remark 4: The Wirtinger-based inequality is a special case of the GFWM-based inequality [45] and can be obtained by fixing slack matrices. Thus, the GFWM approach is less conservative because slack matrices provide additional freedom.

B. DESIGN OF A STATE-FEEDBACK CONTROLLER USING THE GFWM APPROACH

In this section, the LMI conditions for the design of a robust state-feedback controller and a $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative performance analysis are presented by employing the GFWM approach. Before proceeding, we first introduce the following notation to simplify the expressions in this section:

$$\bar{h}(t) = d - h(t)$$
$$\chi_1(t) = \int_{t-h(t)}^{t} \frac{x(s)}{h(t)}ds$$
$$\chi_2(t) = \int_{t-d}^{t-h(t)} \frac{x(s)}{\bar{h}(t)}ds$$

$$\begin{aligned}\zeta(t) &= [x^T(t)\,x^T(t-h(t))\,x^T(t-d)\\ &\qquad \chi_1^T(t)\quad \chi_2^T(t)\quad w^T(t)]^T\\ e_0 &= [0\,0\,0\,0\,0\,0]\\ e_i &= [0_{n\times(i-1)n}\, I_{n\times n}\, 0_{n\times(6-i)n}],\quad i=1,2,\cdots,6\\ e_s &= [A\; B_uK\; 0\,0\,0\; B_w]\\ e_y &= [C_1\; D_{1u}K\; 0\,0\,0\,0]\\ e_g &= [e_1^T\, e_2^T\, e_3^T\, e_4^T\, e_5^T\, e_0^T]^T \end{aligned}\tag{15}$$

Theorem 1: Given constants d, μ, ρ and θ, if there exist symmetric $n\times n$ matrices $P>0$, $X>0$, $Y>0$, $Z>0$ and any $6n\times n$ matrices M_i, N_i, $i=1,2$, the following conditions hold:

$$\begin{bmatrix} \Phi(h(t))|_{h(t)=d} - d\Theta_5 & e_y^T\mathcal{Q}_- & \sqrt{d}e_s^TP & de_g^TM_1 & de_g^TN_1\\ * & -I & 0_{n\times n} & 0_{n\times n} & 0_{n\times n}\\ * & * & \theta^2Z-2\theta P & 0_{n\times n} & 0_{n\times n}\\ * & * & * & -dZ & 0_{n\times n}\\ * & * & * & * & -3dZ \end{bmatrix}\le 0,\tag{16}$$

$$\begin{bmatrix} \Phi(h(t))|_{h(t)=0} - d\Theta_6 & e_y^T\mathcal{Q}_- & \sqrt{d}e_s^TP & de_g^TM_2 & de_g^TN_2\\ * & -I & 0_{n\times n} & 0_{n\times n} & 0_{n\times n}\\ * & * & \theta^2Z-2\theta P & 0_{n\times n} & 0_{n\times n}\\ * & * & * & -dZ & 0_{n\times n}\\ * & * & * & * & -3dZ \end{bmatrix}\le 0,\tag{17}$$

$$\begin{bmatrix} -I & \sqrt{\rho}(C_2)_i\\ * & -(z_{2,max})_i^2P\end{bmatrix}<0,\quad i=1,2\tag{18}$$

$$\begin{bmatrix} -I & \sqrt{\rho}K\\ * & -u_{max}^2P\end{bmatrix}<0\tag{19}$$

where

$$\begin{aligned}\Phi(h(t)) &= \Theta_0 + h(t)\Theta_5 + \bar{h}(t)\Theta_6,\\ \Theta_0 &= \Theta_1+\Theta_2+\Theta_4,\\ \Theta_1 &= \mathrm{Sym}\left\{e_1^TPe_s\right\} + e_1^T(X+Y)e_1\\ &\quad -e_3^TYe_3 - (1-\mu)e_2^TXe_2,\\ \Theta_2 &= -e_y^T\mathcal{Q}e_y - \mathrm{Sym}\left\{e_6^T\mathcal{S}e_y\right\} - e_6^T(\mathcal{R}-\alpha I)e_6,\\ \Theta_3 &= de_s^TZe_s,\\ \Theta_4 &= \mathrm{Sym}\left\{e_g^TM_1(e_1-e_2)\right.\\ &\quad \left.+\, e_g^TN_1(e_1+e_2-2e_4)\right\}\\ &\quad +\mathrm{Sym}\left\{e_g^TM_2(e_2-e_3)\right.\\ &\quad \left.+\, e_g^TN_2(e_2+e_3-2e_5)\right\},\\ \Theta_5 &= e_g^T\left(M_1Z^{-1}M_1^T + \frac{1}{3}N_1Z^{-1}N_1^T\right)e_g,\\ \Theta_6 &= e_g^T\left(M_2Z^{-1}M_2^T + \frac{1}{3}N_2Z^{-1}N_2^T\right)e_g,\end{aligned}\tag{20}$$

Then, 1) the closed-loop system in (6) is asymptotically stable for the bounded time-varying delay $h(t)$ satisfying (2) and (3); 2) a $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity performance related to the transfer function $\|T_{z_1w}\|$ is guaranteed; and 3) the hard constraints in (8) are satisfied.

Proof: We select the following Lyapunov-Krasovskii function candidate:

$$\begin{aligned}V(t) &= x^T(t)Px(t) + \int_{t-h(t)}^{t} x^T(s)Xx(s)ds,\\ &\quad + \int_{t-d}^{t} x^T(s)Yx(s)ds,\\ &\quad + \int_{-d}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Z\dot{x}(s)dsd\theta,\end{aligned}\tag{21}$$

By differentiating $V(s)$, we can get

$$\begin{aligned}\dot{V}(t) &\leqslant 2x^T(t)P\dot{x}(t) + x^T(t)(X+Y)x(t)\\ &\quad -(1-\mu)x^T(t-h(t))Xx(t-h(t))\\ &\quad -x^T(t-d)Yx(t-d)\\ &\quad +d\dot{x}^T(t)Z\dot{x}(t)\\ &\quad -\int_{t-d}^{t}\dot{x}^T(s)Z\dot{x}(s)ds\end{aligned}\tag{22}$$

Adding $-z_1^T(t)\mathcal{Q}z_1(t) - 2z_1^T(t)\mathcal{S}w(t) - w^T(t)(\mathcal{R}-\alpha I)w(t)$ on both sides of (22) results in

$$\begin{aligned}&\dot{V}(t) - z_1^T(t)\mathcal{Q}z_1(t) - 2z_1^T(t)\mathcal{S}w(t) - w^T(t)(\mathcal{R}-\alpha I)w(t)\\ &\qquad \le \zeta^T(t)(\Theta_1+\Theta_2+\Theta_3)\zeta(t) - \dot{V}_z(t)\end{aligned}\tag{23}$$

where $\zeta(t)$ and Θ_i, $i=1,2,3$ are defined in (20), and

$$\dot{V}_z(t) = \int_{t-h(t)}^{t}\dot{x}^T(s)Z\dot{x}(s)ds + \int_{t-d}^{t-h(t)}\dot{x}^T(s)Z\dot{x}(s)ds\tag{24}$$

Conversely, let us define η_0 in (14) as

$$\begin{aligned}\eta_0 = \varsigma_0 &= \Big[x^T(t)\,x^T(t-h(t))\,x^T(t-d)\\ &\qquad \chi_1^T(t)\,\chi_2^T(t)\Big]^T\\ &= e_g\zeta(t).\end{aligned}\tag{25}$$

For some matrices $M_i, N_i, \in \mathcal{R}^{6n\times n}$, $i=1,2$, by employing the inequality (14) in Lemma 5 to accurately estimate (24), we obtain

$$\begin{aligned}\dot{V}_z(t) &\ge -\mathrm{Sym}\left\{\varsigma_0^TN_1\varsigma_1 + \varsigma_0^TM_1\varsigma_2\right\}\\ &\quad -h(t)\varsigma_0^T\left(\frac{3N_1Z^{-1}N_1^T + M_1Z^{-1}M_1^T}{3}\right)\varsigma_0\end{aligned}$$

$$-\mathrm{Sym}\left\{\varsigma_0^T N_2\varsigma_3+\varsigma_0^T M_2\varsigma_4\right\}$$
$$-\bar{h}(t)\varsigma_0^T\left(\frac{3N_2Z^{-1}N_2^T+M_2Z^{-1}M_2^T}{3}\right)\varsigma_0 \quad (26)$$

where

$$\varsigma_1(t)=x(t)-x(t-h(t)),$$
$$\varsigma_2(t)=x(t)+x(t-h(t))-2\chi_1(t),$$
$$\varsigma_3(t)=x(t-h(t))-x(t-d),$$
$$\varsigma_4(t)=x(t-h(t))+x(t-d)-2\chi_2(t)$$

This expression is rewritten as

$$-\dot{V}_s(t)\le\zeta^T(t)(\Theta_4+h(t)\Theta_5+\bar{h}(t)\Theta_6)\zeta(t) \quad (27)$$

where $\Theta_i, i = 4, 5, 6$ are defined in (20). Consequently, by adding (23) and (27), we obtain

$$\dot{V}(t)-z_1^T(t)\mathcal{Q}z_1(t)-2z_1^T(t)\mathcal{S}w(t)$$
$$-w^T(t)(\mathcal{R}-\alpha I)w(t)$$
$$\le\zeta^T(t)\left(\sum_{i=1}^{4}\Theta_i+h(t)\Theta_5+\bar{h}(t)\Theta_6\right)\zeta(t) \quad (28)$$

Based on the Schur complement and convex combination method, satisfying the LMI conditions (16) and (17) along with (6) guarantees

$$\left(\sum_{i=1}^{4}\Theta_i+h(t)\Theta_5+\bar{h}(t)\Theta_6\right)|_{h(t)=d}\le 0,$$
$$\left(\sum_{i=1}^{4}\Theta_i+h(t)\Theta_5+\bar{h}(t)\Theta_6\right)|_{h(t)=0}\le 0, \quad (29)$$

Thus, from (28) we can have

$$\dot{V}(t)-z_1^T(t)\mathcal{Q}z_1(t)-2z_1^T(t)\mathcal{S}w(t)$$
$$-w^T(t)(\mathcal{R}-\alpha I)w(t)\le 0 \quad (30)$$

for any nonzero $w \in L_2[0,\infty)$. We know that $V(0) = 0$ and $V(\infty) \ge 0$ under zero initial conditions. Integrating both sides of (30), we have $\alpha\int_0^t w^T(s)w(s)ds \le E(t)$ for any nonzero $w \in L_2[0,\infty)$. Thus, the strict $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipative performance is satisfied from Definition 1. In addition, if $w(t) = 0$, $\dot{V}(t) \le z_1^T(t)\mathcal{Q}z_1(t) \le 0$, which shows the asymptotic stability of the closed-loop system (6). The hard constraints in (8) are then derived. From (30) and Young's inequality, we have

$$x^T(t)Px(t)$$
$$<\int_0^t z_1^T(s)(\mathcal{Q}+\mathcal{S})z_1(s)ds$$
$$+\int_0^t w^T(s)(\mathcal{S}+\mathcal{R}-\alpha I)w(s)ds+V(0)$$
$$\le\Lambda_{\max}(\mathcal{Q}+\mathcal{S})a_{\max}+\Lambda_{\max}(\mathcal{S}+\mathcal{R}-\alpha I)w_{\max}, \quad (31)$$

where $a_{\max}\triangleq\max_{t\ge0}\int_0^t z_1^T(s)z_1(s)ds$; $\Lambda_{max}(\cdot)$ denotes the maximal eigenvalue. Let $\rho\triangleq\Lambda_{\max}(\mathcal{Q}+\mathcal{S})a_{\max}+\Lambda_{\max}(\mathcal{S}+\mathcal{R}-\alpha I)w_{\max}+V(0)$. Similar to [38]; therefore, the following conditions hold:

$$\max_{t>0}|(z_2(t))_i|^2$$
$$=\max_{t>0}\left\|x^T(t)P^{\frac{1}{2}}P^{-\frac{1}{2}}(C_2)_i^T(C_2)_iP^{-\frac{1}{2}}P^{\frac{1}{2}}x(t)\right\|_2$$
$$<\rho\cdot\left(P^{-\frac{1}{2}}(C_2)_i^T(C_2)_iP^{-\frac{1}{2}}\right),\qquad i=1,2,$$
$$\max_{t>0}|u(t)|^2$$
$$=\max_{t>0}\left\|x^T(t)K^TKx(t)\right\|_2$$
$$<\rho\cdot\left(P^{-\frac{1}{2}}K^TKP^{-\frac{1}{2}}\right) \quad (32)$$

Thus, the hard constraints in (8) can be guaranteed, if

$$\rho\cdot P^{-\frac{1}{2}}(C_2)_i^T(C_2)_iP^{-\frac{1}{2}}<(z_{2,max})_i^2I,\quad i=1,2$$
$$\rho\cdot P^{-\frac{1}{2}}K^TKP^{-\frac{1}{2}}<u_{max}^2I. \quad (33)$$

This can be obtained using (18) and (19). The proof is completed. □

Remark 5: The term $-PZ^{-1}P$ that occurs when solving Θ_3 is not an LMI condition. This nonlinear matrix inequality can be changed to an LMI condition using the inequality $-PZ^{-1}P \le \theta^2 Z - 2\theta P$. Here, θ becomes a tunable parameter; that affords flexibility when solving the LMI.

Now, let us explain how to obtain the desired controller matrix K in (5) through the appropriate transformation of (16) and (17).

Theorem 2: Given constants d, μ, ρ, and θ, if symmetric $n\times n$ matrices $\mathcal{P}>0$, $\mathcal{X}>0$, $\mathcal{Y}>0$, $\mathcal{Z}>0$, and any $6n\times n$ matrices $\mathcal{M}_i, \mathcal{N}_i$, $i=1,2$ such that the following conditions hold:

$$\begin{bmatrix}\Psi(h(t))|_{h(t)=d}-d\Omega_5 & \Upsilon_2^Te_y^T\mathcal{Q}_- & \sqrt{d}\Upsilon_2^Te_s^T & de_g^T\mathcal{M}_1 & de_g^T\mathcal{N}_1\\ * & -I & 0_{n\times n} & 0_{n\times n} & 0_{n\times n}\\ * & * & \theta^2\mathcal{Z}-2\theta\mathcal{P} & 0_{n\times n} & 0_{n\times n}\\ * & * & * & -d\mathcal{Z} & 0_{n\times n}\\ * & * & * & * & -3d\mathcal{Z}\end{bmatrix}\le 0, \quad (34)$$

$$\begin{bmatrix}\Psi(h(t))|_{h(t)=0}-d\Omega_6 & \Upsilon_2^Te_y^T\mathcal{Q}_- & \sqrt{d}\Upsilon_2^Te_s^T & de_g^T\mathcal{M}_2 & de_g^T\mathcal{N}_2\\ * & -I & 0_{n\times n} & 0_{n\times n} & 0_{n\times n}\\ * & * & \theta^2\mathcal{Z}-2\theta\mathcal{P} & 0_{n\times n} & 0_{n\times n}\\ * & * & * & -d\mathcal{Z} & 0_{n\times n}\\ * & * & * & * & -3d\mathcal{Z}\end{bmatrix}\le 0, \quad (35)$$

$$\begin{bmatrix}-I & \sqrt{\rho}\,\{C_{2i}\}_j\mathcal{P}\\ * & -\{z_{2,max}\}_j^2\mathcal{P}\end{bmatrix}<0,\quad i,j=1,2 \quad (36)$$

$$\begin{bmatrix} -I & \sqrt{\rho}\mathcal{K} \\ * & -u_{max}^2\mathcal{P} \end{bmatrix} < 0 \tag{37}$$

where

$$\begin{aligned}
\Upsilon_1 &= \mathrm{diag}\left\{P^{-1}, P^{-1}, P^{-1}\right\} \\
\Upsilon_2 &= \mathrm{diag}\left\{\Upsilon_1, I, I, I\right\} \\
\Psi(h(t)) &= \Upsilon_2^T \Phi(h(t))\Upsilon_2, \\
\Omega_k &= \Upsilon_2^T \Theta_k \Upsilon_2, \quad k = 1, \cdots, 6 \\
\mathcal{P} &= P^{-1}, \mathcal{K} = KP^{-1}, \\
\mathcal{X} &= P^{-1}XP^{-1}, \mathcal{Y} = P^{-1}YP^{-1}, \mathcal{Z} = P^{-1}ZP^{-1}, \\
\mathcal{M}_i &= \Upsilon_2^T M_i P^{-1}, \ \mathcal{N}_i = \Upsilon_2^T N_i P^{-1}, \quad i = 1, 2
\end{aligned} \tag{38}$$

Then, 1) the closed-loop system in (6) is asymptotically stable for the bounded time-varying delay $h(t)$ satisfying (2) and (3); 2) the $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipativity performance related to the transfer function $\|T_{z_1w}\|$ is guaranteed; and 3) the hard constraints in (8) are satisfied.

Proof: The problem of the $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative performance analysis with the given controller matrices can be changed to the problem of obtaining the controller matrices with some modifications. To obtain the controller matrices, transforming the LMI condition (16)-(19) through the pre- and post-multiplication of diag$\{\Upsilon_2, I, \Upsilon_1\}$, diag$\{\Upsilon_2, I, \Upsilon_1\}$, diag$\{I, P^{-1}\}$, diag$\{I, P^{-1}\}$, respectively, the state-feedback controller synthesis condition (34)-(37) can then be obtained using the change of matrix variable technique.

When the LMI conditions (34)-(37) in Theorem 2 are feasible, the control gain in (5) is calculated as

$$K = \mathcal{K}\mathcal{P}^{-1} \tag{39}$$

This completes the proof. □

Remark 6: State feedback requires the assumption that all the state variables are accessible. However, online measurement of all state information is difficult and impractical in terms of cost and complexity. In this case, it is more practical to achieve an appropriate feedback loop by estimating the state information from the measured output. Therefore, our future work will include a suitable filter design and its practical implementation.

IV. NUMERICAL SIMULATIONS

This section provides design examples to demonstrate the validity of the controller design technique described in the previous section. The H_∞ performance is not only a useful measure of the ride comfort of the suspension system under external disturbances, but it also shows how conservative the problem of state-feedback control design of suspension systems with hard constraints is. Table 2 presents the parameters of the quarter-car suspension system used to design the following controllers. We assume that $z_{2,max}$ and u_{max} are 0.035m and 2000N, respectively. By selecting $\mathcal{Q} = -I, \mathcal{S} = 0$, and $\mathcal{R} = (\alpha^2 + \alpha)I$, the $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative controller can be changed to an H_∞ controller. Here, the dissipativity performance index α can be regarded as the disturbance attenuation level γ. Under $d = 0.005, \mu = 0.1, \rho = 1$, and $\theta = 1$, the desired gain matrix for the H_∞ controller with the structure (5) can be derived from Theorem 1 and is given as follows:

$$K = 10^4 \times \left[-4.8610 \ 2.3070 \ -1.0768 \ 0.0975\right]$$

First, we compare the minimum disturbance attenuation level with other bounding techniques to confirm the reduced conservatism of the proposed GFWM-based controller design condition. The desired state-feedback robust controller for a delayed suspension system can be designed based on Theorem 1. Table 3 shows the minimum disturbance attenuation level γ^* of the state-feedback controllers for input delayed suspension systems based on different boundary techniques. As shown in the table, the design of a state-feedback controller that meets all the different requirements of a suspension system is a very conservative problem. Only controllers based on Theorem 1 overcome the conservatism of the state-feedback controller design and provide a feasible solution. We also provide a result for the maximum upper bound d of the delay $h(t)$ under different γ for controllers using different bounding techniques. It can be seen that only the proposed GFWM-based control design method provides a feasible solution for the state-feedback controller of active suspension systems under certain design conditions. Next, some evaluations are performed on different road surfaces to demonstrate the performance requirements of suspension systems, such as ride comfort, road retention, physical constraints of the suspension space, and constraints on control.

TABLE 2. Parameters for the quarter-car model.

Parameter	k_s	k_t	c_s	c_u
Unit	N/m	N/m	Ns/m	Ns/m
Value	42720	101115	1095	14.6

TABLE 3. Minimum disturbance attenuation level γ^* corresponding to different bounding techniques.

d	5(ms)	10(ms)	15(ms)	20(ms)
Jensen	x	x	x	x
Wirtinger	x	x	x	x
Reciprocal	x	x	x	x
GMWM	7.3851	8.2202	10.0075	14.3058

TABLE 4. Maximum upper bound d of the delay under different γ.

γ	10	15	20	25
Jensen	x	x	x	x
Wirtinger	x	x	x	x
Reciprocal	x	x	x	x
GMWM	14(ms)	20(ms)	22(ms)	22(ms)

A. BUMP RESPONSE

Let us consider the time responses for isolated bump-road profiles. The corresponding disturbance can be expressed as:

$$z_r(t) = \begin{cases} \frac{H}{2}\left(1 - \cos\left(\frac{2\pi V}{L}t\right)\right), & \text{if } 0 \le t \le \frac{L}{V}, \\ 0, & \text{if } t > \frac{L}{V}, \end{cases} \tag{40}$$

Here, we set the height and length of the bump as $H = 50$ mm and $L = 6$ m respectively, and the vehicle speed as $V = 35$ km/h. Figure 2 shows the comparison between the open- and closed-loop systems (the proposed delay-dependent controller applied) in terms of the suspension performance response. It can be confirmed that the closed-loop system where the controller derived from Theorem 1 is applied exhibits a better performance with regard to all the key performance metrics of suspension systems. We define Controller I as a controller designed without considering the input delay and Controller II as the controller proposed in Theorem 1, which is designed considering the input delay. When Controller I was compared with Controller II, we confirmed the suitability of the proposed delay-dependent control synthesis method. Figure 4 illustrates the response of the state value for suspension performances related to ride comfort, steering, and physical constraints under the input delay condition. As shown in the figure, only the delay-dependent control design method can maintain a stable performance even in the input delay situation. Thus, the proposed delay-dependent control design technique should be considered when designing a practical suspension controller.

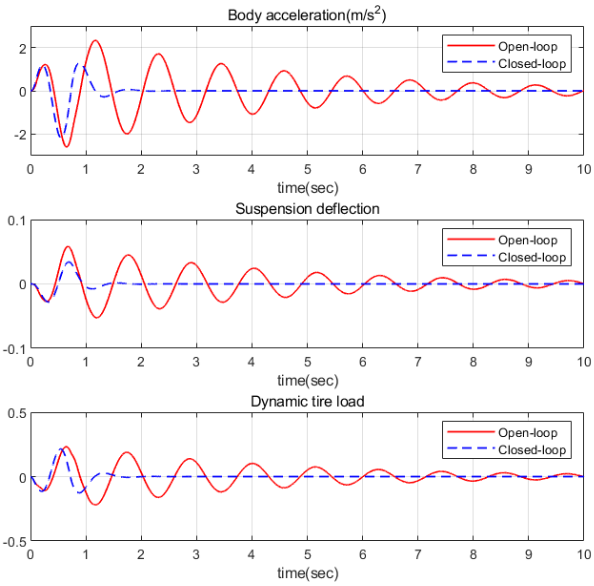

FIGURE 2. Bump responses of the open- and closed-loop systems.

In addition, It can be seen from Figure 3 that the controller is synthesized within a range that satisfies the design constraints of the suspension system.

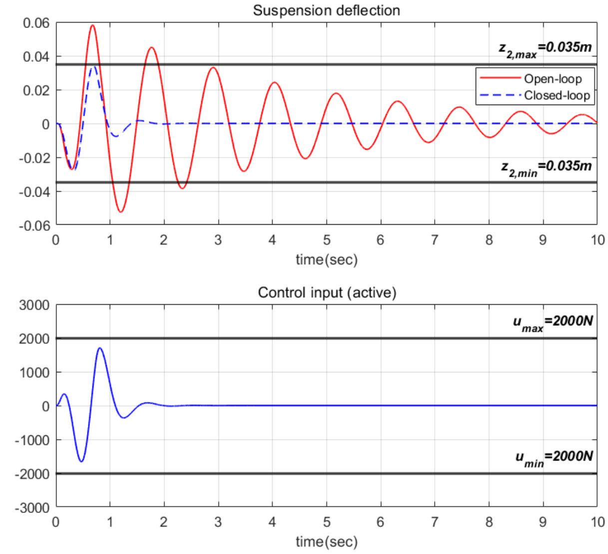

FIGURE 3. Constraints for the control output $z_2(t)$ and input $u(t)$.

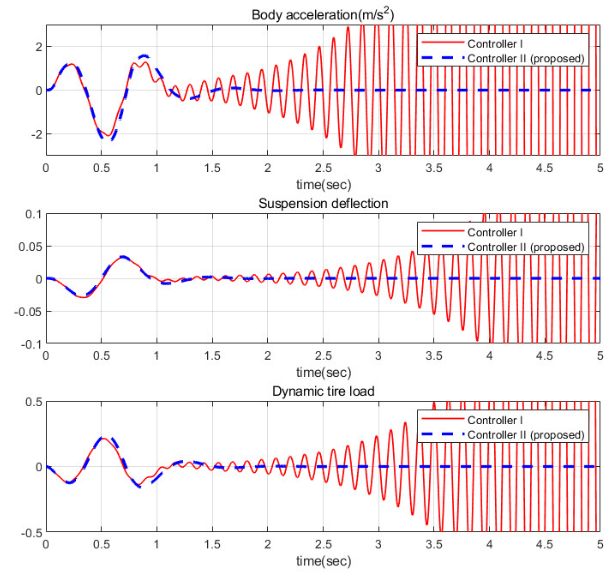

FIGURE 4. Bump responses of controllers I and II under an input delay.

B. RANDOM RESPONSE

Furthermore, to evaluate the performance requirements of the proposed controller in suspension systems, we evaluated the response of the state under random vibrations. According to [14], an irregular disturbance on the road can be described by random noise with a power spectral density of the ground displacement, as follows:

$$\dot{z}_r(t) = 2\pi n_0 \sqrt{G_q(n_0)V} w(t), \tag{41}$$

where $w(t)$ is the zero-mean unit-variance white noise process; $n_0 = 0.1(1/\text{m})$ denotes the reference spatial frequency, and $G_q(n_0)$ denotes the coefficient of road roughness. The driving speed was set to $V = 35$(km/h). In this study, four levels of road roughness were selected according to

ISO 2631 specifications; these are presented in Table 5. The root mean square (RMS) values of the state with respect to body ride comfort, suspension stroke constraint, and road holding property under different levels of road roughness are listed in Tables 6, 7, and 8, respectively. We calculated the RMS value of $x(t)$ as $\sqrt{\frac{1}{T}\int_0^T |x(t)|^2\, dt}$, where $T = 100$(s). As shown in tables 6, 7, and 8, compared with that of open-loop systems, the ride comfort of the proposed system is significantly better while meeting stringent constraints under various load conditions.

TABLE 5. Classification standards of road roughness.

Grade	$G_q(n_0)$	Driving speed
B (Good)	$16 \times 10^{-6}(\mathrm{m}^3)$	$V = 35$(km/h)
C (Average)	$64 \times 10^{-6}(\mathrm{m}^3)$	$V = 35$(km/h)
D (Poor)	$256 \times 10^{-6}(\mathrm{m}^3)$	$V = 35$(km/h)
E (Very Poor)	$1024 \times 10^{-6}(\mathrm{m}^3)$	$V = 35$(km/h)

TABLE 6. RMS values of the acceleration of the car body under different $G_q(n_0)$.

Roughness $G_q(n_0)$	open-loop	closed-loop
B grade	0.0084	0.0046
C grade	0.0163	0.0093
D grade	0.0282	0.0185
E grade	0.0594	0.0376

TABLE 7. RMS values of the suspension deflection under different $G_q(n_0)$.

Roughness $G_q(n_0)$	open-loop	closed-loop
B grade	1.8530×10^{-4}	6.6051×10^{-5}
C grade	3.6091×10^{-4}	1.3562×10^{-4}
D grade	6.2146×10^{-4}	2.6575×10^{-4}
E grade	0.0013	5.4468×10^{-4}

TABLE 8. RMS values of the relative dynamic tire load under different $G_q(n_0)$.

Roughness $G_q(n_0)$	open-loop	closed-loop
B grade	8.9878×10^{-5}	5.2584×10^{-5}
C grade	1.7671×10^{-4}	1.0546×10^{-4}
D grade	3.1163×10^{-4}	2.0962×10^{-4}
E grade	6.4883×10^{-4}	4.2369×10^{-4}

V. CONCLUSION

This paper introduced new conditions for $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative controller synthesis of input delayed suspensions with a state-feedback structure under design constraints. By introducing a novel bounding technique called the GFWM, new, less conservative delay-dependent conditions for state-feedback controllers were established. By solving the proposed sets of LMIs, the desired controller with state-feedback structure could be obtained such that the closed-loop systems had asymptotic stability with guaranteed performance in $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative senses. Finally, the effectiveness of the developed robust controller design was verified using numerical simulations. This study aims to provide the latest theoretical findings prior to practical application. In future work, we will focus on identifying practical ways to implement the proposed methods, including state estimators, into vehicle platforms to close the gap between theoretical techniques and practical implementations.

REFERENCES

[1] J. Cao, H. Liu, P. Li, and D. J. Brown, "State of the art in vehicle active suspension adaptive control systems based on intelligent methodologies," *IEEE Trans. Intell. Transp. Syst.*, vol. 9, no. 3, pp. 392–405, Sep. 2008.

[2] J. Lin and R.-J. Lian, "Intelligent control of active suspension systems," *IEEE Trans. Ind. Electron.*, vol. 58, no. 2, pp. 618–628, Feb. 2011.

[3] Y.-J. Liu, Q. Zeng, S. Tong, C. L. P. Chen, and L. Liu, "Adaptive neural network control for active suspension systems with time-varying vertical displacement and speed constraints," *IEEE Trans. Ind. Electron.*, vol. 66, no. 12, pp. 9458–9466, Dec. 2019.

[4] H. Du, N. Zhang, and J. Lam, "Parameter-dependent input-delayed control of uncertain vehicle suspensions," *J. Sound Vibrat.*, vol. 317, nos. 3–5, pp. 537–556, Nov. 2008.

[5] X. Ma, P. K. Wong, and J. Zhao, "Practical multi-objective control for automotive semi-active suspension system with nonlinear hydraulic adjustable damper," *Mech. Syst. Signal Process.*, vol. 117, pp. 667–688, Feb. 2019.

[6] J. Zhao, "Chassis integrated control for active suspension, active front steering and direct yaw moment systems using hierarchical strategy," *Veh. Syst. Dyn.*, vol. 55, no. 1, pp. 72–103, 2017.

[7] H. Pang, Y. Wang, X. Zhang, and Z. Xu, "Robust state-feedback control design for active suspension system with time-varying input delay and wheelbase preview information," *J. Franklin Inst.*, vol. 356, no. 4, pp. 1899–1923, Mar. 2019.

[8] H. D. Choi, C. J. Lee, and M. T. Lim, "Fuzzy preview control for half-vehicle electro-hydraulic suspension system," *Int. J. Control, Autom. Syst.*, vol. 16, no. 5, pp. 2489–2500, Oct. 2018.

[9] P. C. Chen and A. C. Huang, "Adaptive sliding control of active suspension systems with uncertain hydraulic actuator dynamics," *Vehicle Syst. Dyn.*, vol. 44, no. 5, pp. 268–357, May 2006.

[10] H. Du and N. Zhang, "Fuzzy control for nonlinear uncertain electrohydraulic active suspensions with input constraint," *IEEE Trans. Fuzzy Syst.*, vol. 17, no. 2, pp. 343–356, Apr. 2009.

[11] S. Liu, R. Hao, D. Zhao, and Z. Tian, "Adaptive dynamic surface control for active suspension with electro-hydraulic actuator parameter uncertainty and external disturbance," *IEEE Access*, vol. 8, pp. 156645–156653, 2020.

[12] S. K. Nguang and P. Shi, "H_∞ fuzzy output feedback control design for nonlinear systems: An LMI approach," *IEEE Trans. Fuzzy Syst.*, vol. 11, no. 3, pp. 331–340, Jun. 2003.

[13] H. Chen and K. Guo, "Constrained H_∞ control of active suspensions: An LMI approach," *IEEE Trans. Control Syst. Technol.*, vol. 13, no. 3, pp. 412–421, May 2005.

[14] H. Li, X. Jing, and H. Karimi, "Output-feedback-based H_∞ control for vehicle suspension systems with control delay," *IEEE Trans. Ind. Electron.*, vol. 61, no. 1, pp. 436–446, Jan. 2014.

[15] V. Vembarasan, P. Balasubramaniam, and E. M. Joo, "H_∞ state-feedback control of time-delay systems using reciprocally convex approach," *J. Process Control*, vol. 24, no. 6, pp. 892–904, Jun. 2014.

[16] J. C. Willems, "Dissipative dynamical systems: I. General theory," *Arch. Ration. Mech. Anal.*, vol. 45, pp. 321–351, 1972.

[17] J. C. Willems, "Dissipative dynamical systems—Part II: Linear systems with quadratic supply rates," *Arch. Rational Mech. Anal.*, vol. 45, no. 5, pp. 352–393, 1972.

[18] Z. Li, J. Wang, and H. Shao, "Delay-dependent dissipative control for linear time-delay systems," *J. Franklin Inst.*, vol. 339, nos. 6–7, pp. 529–542, Sep. 2002.
[19] C. K. Ahn, P. Shi, and M. V. Basin, "Two-dimensional dissipative control and filtering for Roesser model," *IEEE Trans. Autom. Control*, vol. 60, no. 7, pp. 1745–1759, Jul. 2015.
[20] H.-B. Zeng, J. H. Park, C.-F. Zhang, and W. Wang, "Stability and dissipativity analysis of static neural networks with interval time-varying delay," *J. Franklin Inst.*, vol. 352, no. 3, pp. 1284–1295, Jan. 2015.
[21] H.-C. Lin, W. Wang, and H.-Q. Xiao, "Dissipativity analysis for neural networks with time-varying delays based on augmented second-order delay-product-type functionals," *IEEE Access*, vol. 8, pp. 171154–171161, 2020.
[22] C. K. Ahn, P. Shi, and M. V. Basin, "Deadbeat dissipative FIR filtering," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 63, no. 8, pp. 1210–1221, Aug. 2016.
[23] H. D. Choi, C. K. Ahn, P. Shi, L. Wu, and M. T. Lim, "Dynamic output-feedback dissipative control for T–S fuzzy systems with time-varying input delay and output constraints," *IEEE Trans. Fuzzy Syst.*, vol. 25, no. 3, pp. 511–526, Jun. 2017.
[24] H. D. Choi, C. K. Ahn, H. R. Karimi, and M. T. Lim, "Filtering of discrete-time switched neural networks ensuring exponential dissipative and $l_2 - l_\infty$ performances," *IEEE Trans. Cybern.*, vol. 47, no. 10, pp. 3195–3207, Oct. 2017.
[25] R. Sakthivel, S. Mohanapriya, C. K. Ahn, and P. Selvaraj, "State estimation and dissipative-based control design for vehicle lateral dynamics with probabilistic faults," *IEEE Trans. Ind. Electron.*, vol. 65, no. 9, pp. 7193–7201, Sep. 2018.
[26] L. Zhang, M. Chen, Q.-X. Wu, and B. Wu, "Fault tolerant control for uncertain networked control systems with induced delays and actuator saturation," *IEEE Access*, vol. 4, pp. 6574–6584, 2016.
[27] I. Birs, C. Muresan, I. Nascu, and C. Ionescu, "A survey of recent advances in fractional order control for time delay systems," *IEEE Access*, vol. 7, pp. 30951–30965, 2019.
[28] Y. S. Moon, P. Park, W. H. Kwon, and Y. S. Lee, "Delay-dependent robust stabilization of uncertain state-delayed systems," *Int. J. Control*, vol. 74, no. 14, pp. 1447–1455, 2001.
[29] H. Karimi, B. Moshiri, and C. Lucas, "Robust fuzzy linear control of a class of stochastic nonlinear time-delay systems," *Nonlinear Dyn. Syst. Theory*, vol. 4, no. 3, pp. 317–332, Jan. 2004.
[30] W. Zheng, H. Wang, H. Wang, S. Wen, and Z.-M. Zhang, "Fuzzy dynamic output feedback control for T-S fuzzy discrete-time systems with multiple time-varying delays and unmatched disturbances," *IEEE Access*, vol. 6, pp. 31037–31049, 2018.
[31] E. Fridman and U. Shaked, "Delay-dependent stability and H_∞ control: Constant and time-varying delays," *Int. J. Control*, vol. 76, no. 1, pp. 48–60, Jan. 2003.
[32] Z. Feng, Z. Wu, B. Zhang, K. Liu, M. Park, L. V. Hien, X. Meng, and C. Fiter, "IEEE access special section: Analysis and synthesis of time-delay system," *IEEE Access*, vol. 8, pp. 205913–205918, 2020, doi: 10.1109/ACCESS.2020.3035460.
[33] P. Park and J. W. Ko, "Stability and robust stability for systems with a time-varying delay," *Automatica*, vol. 43, no. 10, pp. 1855–1858, Oct. 2007.
[34] H. Shao, "Delay-dependent stability for recurrent neural networks with time-varying delays," *IEEE Trans. Neural Netw.*, vol. 19, no. 9, pp. 1647–1651, Sep. 2008.
[35] A. Seuret and F. Gouaisbaut, "Wirtinger-based integral inequality: Application to time-delay systems," *Automatica*, vol. 49, no. 9, pp. 2860–2866, Sep. 2013.
[36] H.-B. Zeng, Y. He, M. Wu, and J. She, "Free-matrix-based integral inequality for stability analysis of systems with time-varying delay," *IEEE Trans. Autom. Control*, vol. 60, no. 10, pp. 2768–2772, Oct. 2015.
[37] P. G. Park, J. W. Ko, and C. Jeong, "Reciprocally convex approach to stability of systems with time-varying delays," *Automatica*, vol. 47, no. 1, pp. 235–238, 2011.
[38] H. Gao, W. Sun, and P. Shi, "Robust sampled-data H_∞ control for vehicle active suspension systems," *IEEE Trans. Control Syst. Technol.*, vol. 18, no. 1, pp. 238–245, Jan. 2010.
[39] J. Na, Y. Huang, X. Wu, S. Su, and G. Li, "Adaptive finite-time fuzzy control of nonlinear active suspension systems with input delay," *IEEE Trans. Cybern.*, vol. 50, no. 6, pp. 2639–2650, Jan. 2019.
[40] H. Du and N. Zhang, "H_∞ control of active vehicle suspensions with actuator time delay," *J. Sound Vibrat.*, vol. 301, nos. 1–2, pp. 236–252, 2007.
[41] H. Li, X. Jing, H.-K. Lam, and P. Shi, "Fuzzy sampled-data control for uncertain vehicle suspension systems," *IEEE Trans. Cybern.*, vol. 44, no. 7, pp. 1111–1126, Jul. 2014.
[42] Y. Kong, D. Zhao, B. Yang, C. Han, and K. Han, "Robust non-fragile H_∞/L_2-L_∞ control of uncertain linear system with time-delay and application to vehicle active suspension," *Int. J. Robust Nonlinear Control*, vol. 25, no. 13, pp. 2122–2141, Sep. 2015.
[43] H. D. Choi, C. K. Ahn, M. T. Lim, and M. K. Song, "Dynamic output-feedback H_∞ control for active half-vehicle suspension systems with time-varying input delay," *Int. J. Control, Autom. Syst.*, vol. 14, no. 1, pp. 59–68, Feb. 2016.
[44] M.-J. Hu, J. H. Park, and J. Cheng, "Robust fuzzy delayed sampled-data control for nonlinear active suspension systems with varying vehicle load and frequency-domain constraint," *Nonlinear Dyn.*, vol. 105, no. 3, pp. 2265–2281, Aug. 2021.
[45] C.-K. Zhang, Y. He, L. Jiang, W.-J. Lin, and M. Wu, "Delay-dependent stability analysis of neural networks with time-varying delay: A generalized free-weighting-matrix approach," *Appl. Math. Comput.*, vol. 294, pp. 102–120, Feb. 2017.

HYUN DUCK CHOI received the B.S. and integrated M.S. and Ph.D. degrees in electrical engineering from Korea University, Seoul, South Korea, in 2011 and 2017, respectively. He was a Senior Research Engineer with Hyundai Mobis, Yongin, South Korea, and Samsung Electronics, Suwon, South Korea. He is currently an Assistant Professor with the School of Electronics and Computer Engineering, Chonnam University, Gwangju, South Korea. His current research interests include robust control, estimation, fuzzy systems, neural networks, nonlinear systems, and their application to vehicle suspension systems.

SEOK-KYOON KIM received the B.S. degree in electronic and IT media engineering from the Seoul National University of Science and Technology, Seoul, South Korea, in 2004, and the Ph.D. degree in electrical engineering from Korea University, Seoul, in 2014. He worked a Senior Research Engineer with LG Electronics, from 2015 to 2016. He joined the Department of Creative Convergence Engineering, Hanbat National University, Daejeon, South Korea, in 2017. His research interests include the tracking problem of power electronic applications, such as power converters and motor drive, the power system stabilization problem, and the development of control theory, such as passivity-based control and nonlinear adaptive control.