IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received September 30, 2021, accepted October 22, 2021, date of publication October 26, 2021, date of current version November 3, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3123282

Exponential Quasi-Incremental-(Q, S, R)-Dissipativity and Practically Incremental Stability for Switched Nonlinear Systems

HONGBO PANG AND SHUO LIU

College of Science, Liaoning University of Technology, Jinzhou 121001, China

Corresponding author: Hongbo Pang (dongdaphb@163.com)

This work was supported in part by the Education Scientific Research Funding Project Foundation of Liaoning Province under Grant JQL202015408, in part by the Doctoral Scientific Research Foundation of Liaoning Province under Grant 2019-BS-122, in part by the Natural Scientific Research Foundation Guiding Plan of Liaoning Province under Grant 2019-ZD-0698, and in part by the National Natural Science Foundation of China under Grant 61903167.

ABSTRACT This paper addresses the problems of exponentially quasi-incremental-(Q, S, R)-dissipativity and practically incremental stability analysis for a switched nonlinear system. First, exponential quasi-incremental-(Q, S, R)-dissipativity for switched nonlinear systems is first defined. Each subsystem may be not exponentially quasi-incremental-(Q, S, R)- dissipative. Based on this dissipativity property, practically incremental stability is obtained for switched nonlinear systems. Second, a state-dependent switching law is designed to establish exponential quasi-incremental-(Q, S, R)- dissipativity criterion. Finally, the effectiveness of the obtained results is verified by a numerical example.

INDEX TERMS Switched nonlinear systems, exponentially quasi-incremental-(Q, S, R)-dissipativity, practically incremental stability.

I. INTRODUCTION

In recent years, the dissipativity introduced by Willems [1] has received more attention from nonlinear control areas due to its practical applications. In general, a dissipative system does not produce energy by itself. Since the storage functions of dissipative systems often can be selected as Lyapunov function candidates, dissipativity is closely related to stability [2]–[5]. Especially, (Q, S, R)-dissipativity was investigated in [4]. In [6] and [7], dissipativity was extended to exponential dissipativity. Corresponding to the dissipativity, exponential stability was obtained. However, the conventional dissipativity theory can only apply to the stability of nonlinear system with respect to one particular equilibrium. For a more broad physical system with an equilibrium point or not, incremental passivity and some preliminary properties in state space form were studied in [8]. It was originally defined from an input-output of view in [9]. The concept of incremental passivity was extended to incremental dissipativity [10]. In particular, the supply rate in (Q, S, R) form implies the gain and phase-related conditions [10], [11]. The incremental dissipativity was verified to preserve under feedback interconnection in [11]. Moreover, incremental dissipativity was often applied to output tracking [12], output regulation [8], network synchronization [10]. Nevertheless, it was difficult to achieve dissipativity due to the large disturbance in practical systems. Hence, the quasi-dissipativity, or almost dissipativity was proposed in [13]–[15]. Compared with dissipative systems, quasi-dissipative systems may contain sources of energy. Hence, the trajectories of quasi-dissipative systems were bounded by a simple output feedback. Similar concept, semi-passivity was studied to derive set stability [16], [17]. Many physical or biological systems are semi-passive.

On the other hand, switched systems have been widely studied due to their application in recent years [18]–[23]. Stability has been focused on the study of switched systems [20], [21], [24]–[31]. However, switched systems do not inherit the property of subsystems. Even if each subsystem is stable, the switched system may be unstable. Hence, [20], [22] verify stability, when each subsystem was stable. In [19], [28], and [29], at least a subsystem was assumed to be stable, stability was obtained via average dwell time method. Nevertheless, many practical systems are stable. Hence, [20], [30], [31] solved stability problem,

The associate editor coordinating the review of this manuscript and approving it for publication was Jianquan Lu.

even if all subsystems were unstable. Incremental stability of switched systems is one of the hot issues [32]–[36]. Incremental stability means that the distance between any two solutions of switched systems with bounded input variables has stable behavior and converges to zero under the same switching signal. The methods to verify stability for switched systems were also useful for incremental stability of switched nonlinear systems, such as the common Lyapunov function [32], [34], [36], multiple Lyapunov functions [32], [34], [37], average dwell time method [33], [35].

Dissipativity is a useful tool to obtain stability as nonswitched systems [37]–[39]. Incremental dissipativity was expected to be useful for switched nonlinear systems [40]–[43]. In [40], incremental passivity theory and the incremental passivity-based output tracking problem for switched nonlinear systems were established using multiple storage functions and multiple incremental supply rates. But the adjacent storage functions were required to be connected at each switching time, which was a strong requirement. Hence, [41]–[44] extended incremental passivity to incremental dissipativity, which allowed the adjacent storage functions to increase. Moreover, the corresponding incremental stability was obtained for switched nonlinear systems. Nevertheless, it is hard to achieve incremental dissipativity due to large uncertainties or switching. As the quasi-dissipativity for switched systems investigated in [45] and [46], this paper will study quasi-incremental dissipativity. Unlike incremental dissipative systems which do not produce energy by itself, quasi-incrementally dissipative systems often contain energy sources. Thus, it is impossible to obtain incremental stability. Therefore, it is necessary to address how to achieve stability for a quasi-incremental dissipative system. This motivates the present study.

In this paper, we will investigate exponential quasi-incremental-(Q, S, R)-dissipativity and practically incremental stability. The contributions are in three aspects. First, an exponential quasi-incremental-(Q, S, R)-dissipativity concept for switched nonlinear systems is first proposed. Weakening the incremental dissipativity concept in [41]–[44] leads to a broader perspective application. Second, compared with [19], where the common Lyapunov function method was adopted, while in this paper, practically incremental stability is obtained using multiple Lyapunov functions, which provides us a new method to verify incremental stability. Third, an exponential quasi-incremental-(Q, S, R)- dissipativity criterion is established. It is a generalization of the nonlinear versions of the Kalman-Yakubovich-Popove lemma and Hamilton–Jacobi Inequality. The designed state-dependent switching law is more general than [19], [41]–[44]. Compared with [19], this switching law allows the storage functions to increase at each switching point. Compared with [41]–[44], which requires the selection of active region is dependent on a storage function, the selection of active region here is dependent on a continuous function associated with a storage function. This provides more freedom for the design of the switching law.

Notations: U: a set of any measurable, locally essentially bounded function of time; $\|x\| = \left(x^T x\right)^{\frac{1}{2}} = \left(\sum_{i=1}^{n} |x_i|^2\right)^{\frac{1}{2}}$: the norm of a vector $x = (x_1, x_2 \cdots x_n)^T$; I_m: m-order identity matrix;

II. PRELIMINARIES AND PROBLEM FORMULATION

Consider a switched nonlinear system

$$\begin{aligned} \dot{x} &= f_\sigma (x, u_\sigma), \\ y &= h_\sigma (x, u_\sigma) \end{aligned} \tag{1}$$

with the state $x \in R^n$ and a switching signal $\sigma : [0, \infty) \to I = \{1, 2, \ldots, M\}$, which has a finite number of switches in any finite time interval. $u_i \in R^p$ and $h_i(x, u_i) \in R^m$ are the input vector and the output vector of the i-th subsystem, respectively. f_i, h_i are continuous functions for all $i \in I$. The switching sequence generated by the switching signal can be characterized as follows:

$$\sum = \{(i_0, t_0), (i_1, t_1), \ldots, (i_k, t_k) \ldots | i_k \in I, k \in N\}, \tag{2}$$

where t_0 denotes the initial time and N is the set of nonnegative integers. When $\sigma(t) = i_k$, the i_k-th subsystem is active on $[t_k, t_{k+1})$. In addition, the state of system (1) is assumed to be continuous at the switching instants.

We first review the definition of class $\mathcal{GK}$ function.

Definition 1 [47]: A function $\alpha : R^+ \to R^+$ is said to be a class $\mathcal{GK}$ function if it is increasing and right continuous at the origin with $\alpha(0) = 0$.

Many real systems can be modeled as switched systems. We take a switched RLC circuit for example [41].

Example 1: Consider a switched RLC circuit with M input power sources, M resistances R_i and M capacitors C_i that could be switched between each other. The models are given by

$$\begin{aligned} \dot{x}_1 &= \frac{1}{L_\sigma} x_2, \\ \dot{x}_2 &= -\frac{1}{C_\sigma} x_1 - \frac{R_\sigma}{L_\sigma} x_2 + u_\sigma + v_d, \\ y &= \frac{1}{L_\sigma} x_2, \end{aligned} \tag{3}$$

where the two state variables are the charge in the capacitor and the flux in the inductance $x = \left[q_{c_i}, \phi_{L_i}\right]^T$, the input u_i and v_d denote the voltage and the bounded disturbance with $\|v_d\| \leq d$ and d is a positive constant, $I = \{1, 2, \ldots, M\}$.

The energy function of ith submode is given as $S_i = \frac{1}{2C_i}\left(x_1 - \hat{x}_1\right)^2 + \frac{1}{2L_i}\left(x_2 - \hat{x}_2\right)^2$.

When $\sigma(t) = i$, i.e. the i th submode is active, differentiating S_i yields

$$\dot{S}_i \leq (1 - R_i)\left(y - \hat{y}\right)^2 + \left(u_i - \hat{u}_i\right)\left(y - \hat{y}\right) + d^2.$$

Thus, the active submode is quasi-incrementally (Q, S, R)-dissipative.

Now, we define exponential quasi-incremental-dissipativity for system (1) as follows:

Definition 2: System (1) is said to be quasi-incremental-dissipative if for a given switching signal $\sigma(t)$, there exists a nonnegative function $V(\sigma(t), x, \hat{x}) : I \times R^{2n} \to R^{+}$, called storage function, and class $\mathcal{GK}$ function α and locally integerable functions $w_i(\Delta u_i, \Delta y)$, called incremental supply rates, where $\Delta u_i = u_i - \hat{u}_i$, $\Delta y = y - \hat{y}$, constants $\lambda > 0$, $c \geq 0$ such that for any two inputs u_σ, $\hat{u}_\sigma$ and any two solutions of system (1)$x, \hat{x}$ corresponding to these inputs, the respective outputs $y = h_\sigma(x, u_\sigma)$ and $\hat{y} = h_\sigma(\hat{x}, \hat{u}_\sigma)$ satisfy the inequality

$$e^{\lambda t} V(\sigma(t), x(t), \hat{x}(t)) - e^{\lambda t_0} V(\sigma(t_0), x(t_0), \hat{x}(t_0)) \leq \int_{t_0}^{t} e^{\lambda \tau}(w_{\sigma(\tau)}(\Delta u_{\sigma(\tau)}(\tau), \Delta y(\tau)) + c_{\sigma(\tau)})\, d\tau + e^{\lambda t_0}\alpha(\|x_0 - \hat{x}_0\|), \quad (4)$$

where x_0 and $\hat{x}_0$ are the initial states. If the incremental supply rates are given by

$$w_i(\Delta u_i, \Delta y) = \Delta y^T Q_i \Delta y + 2\Delta y^T S_i \Delta u_i + \Delta u_i^T R_i \Delta u_i, \quad (5)$$

then, system (1) is said to be exponentially quasi-incrementally (Q, S, R)-dissipative, where $Q = (Q_1, Q_2, \cdots Q_M)$, $Q_i = Q_i^T \in R^{m \times m}$, $S = (S_1, S_2, \cdots S_M)$, $R = (R_1, R_2, \cdots R_M)$, $S_i \in R^{m \times p}$, $R_i = R_i^T \in R^{p \times p}$ are constant matrices.

Remark 1: (4) means that the overall system is quasi-incremental-dissipative. Each subsystem of a quasi-incremental-dissipative switched system may contain a source of energy with c_i being seen as the interior supply rate. This dissipativity property balances the total energy throughout the overall process, while each active subsystem is not required to be quasi-incremental dissipative. In many practical systems, some active subsystems are not incrementally dissipative due to fault or external disturbance, such as power system or network control system. According to Definition 2, the energy may increase at some switching time. The item $e^{\lambda t_0}\alpha(\|x_0 - \hat{x}_0\|)$ is used for bounding the total change of "energy" at the switching times. If $c_i = 0$ then Definition 2 can degenerate into the exponential quasi-dissipativity defintion in [44]. If $c_i = 0, \lambda = 0$ then Definition 2 can degenerate into the incremental quasi-dissipativity definition in [41].

Remark 2: For system with an equilibrium (0, 0), Definition 2 is reduced to the quasi-dissipativity definition in [46] by setting $\hat{x} = 0$ and $\hat{u} = 0$.

Next, based on Definition 2, exponentially incremental finite power gain and exponentially incremental quasi-passivity are defined:

Definition 3: System (1) is said to have exponentially vector quasi-incremental finite power gain $(\gamma_1, \gamma_2, \ldots \gamma_M)$, if system (1) is exponentially quasi-incrementally dissipative with $Q_i = -I_m$, $R_i = \gamma_i^2 I_p$, $S_i = 0$ for some constants $\gamma_i > 0$ and $\forall i \in I$.

Definition 4: System (1) with $m = p$ is said to be exponentially quasi-incrementally passive if system (1) is exponentially quasi-dissipative with $Q_i = 0, R_i = 0, S_i = \frac{1}{2} I_m$ for $\forall i \in I$.

Remark 3: The quasi-incremental (Q, S, R)-dissipativity is more general than the quasi-incremental passivity and vector quasi-incremental L_2-gain, since its incremental supply rate carries the information on both the incremental phase (given by the bilinear term $2\Delta y^T S_i \Delta u_i$) and the incremental gain (given by the quadratic terms $\Delta y^T Q_i \Delta y_i$ and $\Delta u_i^T R_i \Delta u_i$).

This paper will study quasi-incremental-(Q, S, R)-dissipativity and practically incremental stability for switched nonlinear systems.

III. PRACTICAL INCREMENTAL STABILITY ANALYSIS

This section will show practically incremental stability based on quasi-incremental (Q, S, R)- dissipativity.

First, a concept of practically incremental stability is proposed as follows:

Definition 5: System (1) is practically incrementally stable, if for any given constant $\delta \geq 0$, a switching signal $\sigma(t)$ and any $u_i \in U$, $i \in I$, the closed-loop system possesses the following properties

(a) (Uniform boundedness) there exists $\varepsilon > 0$ such that for all $t_0 \geq 0$, $\|x(t, x_0, u_\sigma) - \hat{x}(t, \hat{x}_0, u_\sigma)\| < \varepsilon$, when $\|x_0 - \hat{x}_0\| < \delta$.

(b) (Uniform ultimate boundedness) for every initial condition $x(t_0), \hat{x}(t_0)$, there exist constants R > 0 and $T = T(x_0, \hat{x}_0, R) \geq 0$ such that $\|x(t) - \hat{x}(t)\| \leq R$ holds for $t \geq t_0 + T$.

Remark 4: If all vector fields share a common equilibrium, system (1) is practical stability. In particular, system (1) with equilibrium 0 is practical stable.

Next, we will show that an quasi-incrementally (Q, S, R)-dissipative switched system is practically incrementally stable.

Theorem 1: Suppose that there exist class K_∞ functions α_1, α_2 and nonnegative continuous functions $V_i(x, \hat{x})$, $i \in I$ satisfying $\alpha_1(\|x - \hat{x}\|) \leq V_i(x, \hat{x}) \leq \alpha_2(\|x - \hat{x}\|)$. If system (1) is quasi-incrementally (Q, S, R)-dissipative with $Q_i \leq 0, i \in I$ and $V(\sigma(t), x, \hat{x}) = V_{\sigma(t)}(x, \hat{x})$ then system (1) is practically incrementally stabilized.

Proof: For $t \geq t_0$, $\forall t \in [t_k, t_{k+1})$, $k \in N$, substituting $\hat{u}_i = u_i \in U$ into the inequality (4) gives

$$e^{\lambda t} V(\sigma(t), x(t), \hat{x}(t)) - e^{\lambda t_0} V(\sigma(t_0), x(t_0), \hat{x}(t_0)) \leq \int_{t_0}^{t} e^{\lambda \tau} c_{\sigma(\tau)}\, d\tau + e^{\lambda t_0}\alpha(\|x_0 - \hat{x}_0\|), \quad (6)$$

For $t > t_0$, there exists positive integer k such that $t \in [t_k, t_{k+1})$. By (5), we get:

$$\begin{aligned} e^{\lambda t} V(\sigma(t), x(t), \hat{x}(t)) &- e^{\lambda t_0} V(\sigma(t_0), x(t_0), \hat{x}(t_0)) \\ &= e^{\lambda t} V_{i_k}(x(t), \hat{x}(t)) - e^{\lambda t_0} V_{i_0}(x(t_0), \hat{x}(t_0)) \\ &\leq \int_{t_0}^{t} e^{\lambda \tau} c_{\sigma(\tau)}\, d\tau + e^{\lambda t_0}\alpha(\|x_0 - \hat{x}_0\|), \end{aligned} \quad (7)$$

Combining $\alpha_1\left(\left\|x-\hat{x}\right\|\right) \leq V_i\left(x,\hat{x}\right) \leq \alpha_2\left(\left\|x-\hat{x}\right\|\right)$ with (7) gives:

$$\begin{aligned}
&\alpha_1\left(\left\|x(t)-\hat{x}(t)\right\|\right)\\
&\leq V_{i_k}\left(x(t),\hat{x}(t)\right)\\
&\leq e^{-\lambda(t-t_0)}V_{i_0}\left(x(t_0),\hat{x}(t_0)\right)+e^{-\lambda(t-t_0)}\alpha\left(\left\|x_0-\hat{x}_0\right\|\right)\\
&\quad+\frac{c}{\lambda}\left(1-e^{-\lambda(t-t_0)}\right)\\
&\leq e^{-\lambda(t-t_0)}\left(V_{i_0}\left(x(t_0),\hat{x}(t_0)\right)-\frac{c}{\lambda}+\alpha\left(\left\|x_0-\hat{x}_0\right\|\right)\right)+\frac{c}{\lambda}\\
&\leq e^{-\lambda(t-t_0)}\overline{\alpha}\left(\left\|x_0-\hat{x}_0\right\|\right)+\frac{c}{\lambda},
\end{aligned}$$

where $c=\max_{i\in I}\{c_i\}$, $\overline{\alpha}=\alpha+a_2$.

i) For $\forall\delta>0$, there exists $\varepsilon=\alpha_1^{-1}\left(\overline{\alpha}(\delta)+\frac{c}{\lambda}\right)$ such that $\left\|x(t,x_0,u_\sigma)-\hat{x}(t,\hat{x}_0,u_\sigma)\right\|\leq\varepsilon$, when $\left\|x_0-\hat{x}_0\right\|\leq\delta$.

ii) When $t\to\infty$, $\left\|x(t)-\hat{x}(t)\right\|\leq\alpha_1^{-1}\left(\frac{c}{\lambda}\right)$.

By Definition 5, system (1) is practically incrementally stabilized.

IV. SUFFICIENT CONDITIONS OF QUASI–INCREMENTAL-(Q, S, R)-DISSIPATIVITY

In this section, a state-dependent switching law will be designed to achieve exponential quasi-incremental-(Q, S, R)-dissipativity.

Theorem 2: Suppose that there exist nonnegative smooth functions $V_i\left(x,\hat{x}\right)$, functions $\beta_{ij}\left(x,\hat{x}\right)\leq 0$, smooth functions $\mu_{ij}\left(x-\hat{x}\right)$ with $\mu_{ij}(0)=0$ and $\mu_{ii}\left(x-\hat{x}\right)=0$, matrices $Q_i=Q_i^T\in R^{m\times m}, S_i\in R^{m\times p}, R_i=R_i^T\in R^{p\times p}$ and constants $c_i\geq 0$, $\lambda_i>0$ for all $i,j\in I$ such that for any two inputs u_i and $\hat{u}_i$, any two solutions of system (1) x and $\hat{x}$ corresponding to these two inputs and the respective outputs y and $\hat{y}$

$$\frac{\partial V_i}{\partial x}f_i(x,u_i)+\frac{\partial V_i}{\partial\hat{x}}f_i\left(\hat{x},\hat{u}_i\right)+\sum_{j=1}^{M}\beta_{ij}\left(x,\hat{x}\right)\left(V_i-V_j+\mu_{ij}\right)$$

$$\leq\Delta y^TQ_i\Delta y+2\Delta y^TS_i\Delta u_i+\Delta u_i^TR_i\Delta u_i+c_i-\lambda_iV_i,\quad(8)$$

$$\frac{\partial\mu_{ij}}{\partial x}f_i(x,u_i)+\frac{\partial\mu_{ij}}{\partial\hat{x}}f_i\left(\hat{x},\hat{u}_i\right)+\mu_{\mathrm{ij}}\lambda\leq 0,\ \lambda=\min_{i\in I}\{\lambda_i\},\quad(9)$$

$$\mu_{ij}\left(x-\hat{x}\right)+\mu_{jk}\left(x-\hat{x}\right)\leq\min\left\{0,\mu_{ik}\left(x-\hat{x}\right)\right\},\ \forall i,j,k\quad(10)$$

hold, where $\Delta u_i=u_i-\hat{u}_i$, $\Delta y=y-\hat{y}$. Design the state-dependent switching law as follows:

$$\begin{aligned}
\sigma(t)&=i\text{ if }\sigma\left(t^-\right)=i\text{ and }\left(x(t),\hat{x}(t)\right)\in\Omega_i,\\
\sigma(t)&=\min\left\{j\left|\left(x(t),\hat{x}(t)\right)\in\Omega_{ij}\right.\right\}\text{ if }\sigma\left(t^-\right)\\
&=i\text{ and }\left(x(t),\hat{x}(t)\right)\in\tilde{\Omega}_{ij},
\end{aligned}\quad(11)$$

where $\Omega_i=\left\{\left(x,\hat{x}\right)\middle|\,V_i\left(x,\hat{x}\right)-V_j\left(x,\hat{x}\right)+\mu_{ij}\left(x-\hat{x}\right)\leq 0,\ j\in I\right\}$ and

$$\tilde{\Omega}_{ij}=\left\{\left(x,\hat{x}\right)\middle|\,V_i\left(x,\hat{x}\right)-V_j\left(x,\hat{x}\right)+\mu_{ij}\left(x-\hat{x}\right)=0,i\neq j\right\}.\quad(12)$$

Then, system (1) is quasi-incrementally (Q,S,R)-dissipative under the switching law (11).

Proof: Similar to [47], we can show that $\{\Omega_i|\,i\in I\}$ makes a partition of R^{2n}.

When $V_i\left(x,\hat{x}\right)-V_j\left(x,\hat{x}\right)+\mu_{ij}\left(x-\hat{x}\right)\leq 0$, namely, $\left(x,\hat{x}\right)\in\Omega_i$, differentiating $V_i\left(x,\hat{x}\right)$ together with (8) gives

$$\begin{aligned}
\dot{V}_i&=\frac{\partial V_i}{\partial x}f_i(x,u_i)+\frac{\partial V_i}{\partial\hat{x}}f_i\left(\hat{x},\hat{u}_i\right)\\
&\leq-\lambda V_i+\Delta y^TQ_i\Delta y+2\Delta y^TS_i\Delta u_i+\Delta u_i^TR_i\Delta u_i+c_i,
\end{aligned}\quad(13)$$

where $\lambda=\min_{i\in I}\{\lambda_i\}$. Multiplying both sides of (13) by $e^{\lambda t}$, respectively, yields:

$$\begin{aligned}
&\frac{d}{dt}\left(e^{\lambda t}V_i\right)\\
&\leq e^{\lambda t}\left(\Delta y^TQ_i\Delta y+2\Delta y^TS_i\Delta u_i+\Delta u_i^TR_i\Delta u_i+c_i\right).
\end{aligned}\quad(14)$$

Integrating (14) over $[s,t]$ for $\forall t>s\geq t_0$ gives:

$$\begin{aligned}
&e^{\lambda t}V_i(x(t),\hat{x}(t))-e^{\lambda s}V_i(x(s),\hat{x}(s))\\
&\leq\int_s^t e^{\lambda\tau}\left(\Delta y^TQ_i\Delta y+2\Delta y^TS_i\Delta u_i+\Delta u_i^TR_i\Delta u_i+c_i\right)d\tau
\end{aligned}\quad(15)$$

By the switching law (11), we can obtain the switching sequence (2) with the property

$$\begin{aligned}
&V_{i_{k+1}}\left(x(t_{k+1}),\hat{x}(t_{k+1})\right)-V_{i_k}\left(x(t_{k+1}),\hat{x}(t_{k+1})\right)\\
&\qquad=\mu_{i_ki_{k+1}}\left(x(t_{k+1})-\hat{x}(t_{k+1})\right).
\end{aligned}\quad(16)$$

(9) tells us that $e^{\lambda t}\mu_{i_kj}\left(x(t)-\hat{x}(t)\right)$ are decreasing on $[t_k,t_{k+1})$.

Let $V\left(\sigma(t),x,\hat{x}\right)=V_{\sigma(t)}\left(x,\hat{x}\right)$. For $t_0\leq t<\infty$, there exists positive integer k such that $t\in[t_k,t_{k+1})$. From (15) and (16), we have

$$\begin{aligned}
&e^{\lambda t}V\left(\sigma(t),x(t),\hat{x}(t)\right)-e^{\lambda t_0}V\left(\sigma(t_0),x(t_0),\hat{x}(t_0)\right)\\
&=e^{\lambda t}V_{i_k}\left(x(t),\hat{x}(t)\right)-e^{\lambda t_k}V_{i_k}\left(x(t_k),\hat{x}(t_k)\right)\\
&\quad+\sum_{p=0}^{k-1}\left(e^{\lambda t_{p+1}}V_{i_p}\left(x\left(t_{p+1}\right),\hat{x}\left(t_{p+1}\right)\right)\right.\\
&\qquad\left.-e^{\lambda t_p}V_{i_p}\left(x\left(t_p\right),\hat{x}\left(t_p\right)\right)\right)\\
&\quad+\sum_{p=1}^{k}e^{\lambda t_p}\left(V_{i_p}\left(\left(x\left(t_p\right),\hat{x}\left(t_p\right)\right)\right)-V_{i_{p-1}}\left(\left(x\left(t_p\right),\hat{x}\left(t_p\right)\right)\right)\right)
\end{aligned}\quad(17)$$

$$\begin{aligned}
&\leq\int_{t_0}^t e^{\lambda\tau}\left(\Delta y^TQ_\sigma\Delta y+2\Delta y^TS_\sigma\Delta u_\sigma+\Delta u_\sigma^TR_\sigma\Delta u_\sigma+c_\sigma\right)d\tau\\
&\quad+\sum_{p=1}^{k}e^{\lambda t_p}\mu_{i_{p-1}i_p}\left(x\left(t_p\right)-\hat{x}\left(t_p\right)\right)
\end{aligned}$$

$$\leq \begin{cases} \int_{t_0}^{t} e^{\lambda\tau}\left(\Delta y^T Q_\sigma \Delta y + 2\Delta y^T S_\sigma \Delta u_\sigma \right. \\ \qquad \left. + \Delta u_\sigma^T R_\sigma \Delta u_\sigma + c_\sigma\right) d\tau \text{ if k is odd} \\ \int_{t_0}^{t} e^{\lambda\tau}\left(\Delta y^T Q_\sigma \Delta y + 2\Delta y^T S_\sigma \Delta u_\sigma \right. \\ \qquad \left. + \Delta u_\sigma^T R_\sigma \Delta u_\sigma + c_\sigma\right) d\tau \\ + e^{\lambda t_0}\mu_{i_0 i_1}\left(x_0 - \hat{x}_0\right) \text{ if k is even} \end{cases} \tag{18}$$

where $\alpha(s) = \max_{\|x-\hat{x}\|\leq s}\left\{\left|\mu_{ij}(x-\hat{x})\right| |i,j \in I\right\}$ is a class $\mathcal{GK}$ function. Then, system (1) is exponentially quasi-incrementally (Q, S, R)-dissipative under switching law (11).

Consider a switched system of the form

$$\begin{aligned} \dot{x} &= f_\sigma(x) + g_\sigma(x) u_\sigma, \\ y &= h_\sigma(x) + J_\sigma(x) u_\sigma. \end{aligned} \tag{19}$$

Theorem 3: Suppose that there exist nonnegative smooth functions $V_i(x, \hat{x})$, continuous functions $\beta_{ij}(x, \hat{x}) \leq 0$, $l_i(x, \hat{x}) : R^{2n} \to R^q$, $W_i(x, \hat{x}) : R^{2n} \to R^{q\times m}$, constants $c_i \geq 0$, $\lambda_i > 0$ and smooth functions $\mu_{ij}(x-\hat{x})$ with $\mu_{ij}(0) = 0$ and $\mu_{ii}(x-\hat{x}) = 0$ for all $i, j \in I$ and some integer q such that inequality (10) and

$$\begin{aligned} &\frac{\partial V_i}{\partial x} f_i(x) + \frac{\partial V_i}{\partial \hat{x}} f_i(\hat{x}) - \left(h_i(x) - h_i(\hat{x})\right)^T Q_i \left(h_i(x) - h_i(\hat{x})\right) \\ &\quad + \lambda_i V_i + \sum_{j=1}^{M} \beta_{ij}\left(V_i(x, \hat{x}) - V_j(x, \hat{x}) + \mu_{ij}(x-\hat{x})\right) \\ &\quad \leq -l_i^T l_i + c_i, \end{aligned} \tag{20}$$

$$\begin{aligned} &\frac{1}{2} g_i^T(x) \frac{\partial V_i}{\partial x} - \hat{S}_i^T(x, \hat{x})\left(h_i(x) - h_i(\hat{x})\right) \\ &\quad = W_i^T(x, \hat{x}) l_i(x, \hat{x}), \end{aligned} \tag{21}$$

$$\begin{aligned} &\frac{\partial V_i}{\partial x} g_i(x) + \frac{\partial V_i}{\partial \hat{x}} g_i(\hat{x}) = 0, \hat{R}_i(x, \hat{x}) \\ &\quad = W_i^T(x, \hat{x}) W_i(x, \hat{x}), \end{aligned} \tag{22}$$

$$\begin{aligned} &\frac{\partial \mu_{ij}}{\partial x} f_i(x) + \frac{\partial \mu_{ij}}{\partial \hat{x}} f_i(\hat{x}) + \mu_{ij}\lambda \leq 0, \ \frac{\partial \mu_{ij}}{\partial x} g_i(x) \\ &\quad = \frac{\partial \mu_{ij}}{\partial \hat{x}} g_i(\hat{x}) = 0 \end{aligned} \tag{23}$$

hold, where

$$\begin{aligned} \hat{R}_i(x, \hat{x}) &= R_i + \left(J_i(x) - J_i(\hat{x})\right)^T S_i + S_i^T\left(J_i(x) - J_i(\hat{x})\right) \\ &\quad + \left(J_i(x) - J_i(\hat{x})\right)^T Q_i \left(J_i(x) - J_i(\hat{x})\right) \\ \hat{S}_i(x, \hat{x}) &= Q_i\left(J_i(x) - J_i(\hat{x})\right) + S_i. \end{aligned}$$

Then, system (19) is exponentially quasi-incrementally (Q, S, R)- dissipative under the switching law (11).

Proof: When $V_i(x, \hat{x}) - V_j(x, \hat{x}) + \mu_{ij}(x-\hat{x}) \leq 0$, differentiating $V_i(x, \hat{x})$ together with (20), (21) and (22) gives

$$\begin{aligned} &\dot{V}_i - \left(\Delta y^T Q_i \Delta y + 2\Delta y^T S_i \Delta u_i + \Delta u_i^T R_i \Delta u_i\right) \\ &= \frac{\partial V_i}{\partial x} f_i(x) + \frac{\partial V_i}{\partial \hat{x}} f_i(\hat{x}) + \frac{\partial V_i}{\partial x} g_i(x) \Delta u_i \\ &\quad - \left(\Delta y^T Q_i \Delta y + 2\Delta y^T S_i \Delta u_i + \Delta u_i^T R_i \Delta u_i\right) \\ &= -\Delta u_i^T \hat{R}_i(x, \hat{x}) \Delta u_i + \frac{1}{2} B_i(x, \hat{x}) \Delta u_i + \frac{1}{2}\Delta u_i^T B_i^T(x, \hat{x}) \\ &\quad + C_i(x, \hat{x}) \\ &\leq -\left(l_i(x, \hat{x}) + W_i(x, \hat{x}) \Delta u_i\right)^T \\ &\quad \times \left(l_i(x, \hat{x}) + W_i(x, \hat{x}) \Delta u_i\right) + c_i - \lambda V_i \\ &\leq c_i - \lambda V_i, \end{aligned}$$

where $\Delta u_i = u_i - \hat{u}_i$, $\Delta y = y - \hat{y}$, $B_i(x, \hat{x}) = \frac{\partial V_i}{\partial x} g_i(x) + 2\left(h_i(x) - h_i(\hat{x})\right)^T \hat{S}_i$ and

$$\begin{aligned} C_i(x, \hat{x}) &= \frac{\partial V_i}{\partial x} f_i(x) + \frac{\partial V_i}{\partial \hat{x}} f_i(\hat{x}) \\ &\quad - \left(h_i(x) - h_i(\hat{x})\right)^T Q_i \left(h_i(x) - h_i(\hat{x})\right). \end{aligned}$$

The rest proof is similar to that of Theorem 2.

If $\hat{R}_i(x, \hat{x}) > 0$ the Hamilton-Jacobi inequalities to judge quasi-incremental (Q, S, R)- dissipativity property of system (19) can be obtained as follows.

Theorem 4: Suppose that there exist nonnegative smooth functions $V_i(x, \hat{x})$, continuous functions $\beta_{ij}(x, \hat{x}) \leq 0$ and smooth functions $\mu_{ij}(x-\hat{x})$ with $\mu_{ij}(0) = 0$ and $\mu_{ii}(x-\hat{x}) = 0$ for $i, j \in I$ such that (10), (23) and

$$\begin{aligned} &\frac{\partial V_i}{\partial x} f_i(x) + \frac{\partial V_i}{\partial \hat{x}} f_i(\hat{x}) - \left(h_i(x) - h_i(\hat{x})\right)^T Q_i \left(h_i(x) - h_i(\hat{x})\right) \\ &\quad + \left(\frac{1}{2}\frac{\partial V_i}{\partial x} g_i - \left(h_i(x) - h_i(\hat{x})\right)^T \hat{S}_i\right) \hat{R}_i^{-1} \\ &\quad \times \left(\frac{1}{2}\frac{\partial V_i}{\partial x} g_i - \left(h_i(x) - h_i(\hat{x})\right)^T \hat{S}_i\right)^T + \lambda_i V_i \\ &\quad + \sum_{j=1}^{M} \beta_{ij}(x, \hat{x})\left(V_i(x, \hat{x}) - V_j(x, \hat{x}) + \mu_{ij}(x-\hat{x})\right) \leq c_i, \end{aligned} \tag{24}$$

$$\frac{\partial V_i}{\partial x} g_i(x) + \frac{\partial V_i}{\partial \hat{x}} g_i(\hat{x}) = 0, \quad \hat{R}_i(x, \hat{x}) > 0 \tag{25}$$

hold, where

$$\begin{aligned} \hat{R}_i(x, \hat{x}) &= R_i + \left(J_i(x) - J_i(\hat{x})\right)^T S_i + S_i^T\left(J_i(x) - J_i(\hat{x})\right) \\ &\quad + \left(J_i(x) - J_i(\hat{x})\right)^T Q_i \left(J_i(x) - J_i(\hat{x})\right) \end{aligned}$$

and $\hat{S}_i(x, \hat{x}) = Q_i\left(J_i(x) - J_i(\hat{x})\right) + S_i$. Then, system (19) is incrementally (Q, S, R)- dissipative under the switching law (11).

Proof: Similar to the proof of Theorem 3, when $V_i(x, \hat{x}) - V_j(x, \hat{x}) + \mu_{ij}(x-\hat{x}) \leq 0$, differentiating $V_i(x, \hat{x})$ together with (24) and (25) gives

$$\begin{aligned} &\dot{V}_i - \left(\Delta y^T Q_i \Delta y + 2\Delta y^T S_i \Delta u_i + \Delta u_i^T R_i \Delta u_i\right) \\ &= -\Delta u_i^T \hat{R}_i(x, \hat{x}) \Delta u_i + \frac{1}{2} B_i(x, \hat{x}) \Delta u_i + \frac{1}{2}\Delta u_i^T B_i^T(x, \hat{x}) \\ &\quad + C_i(x, \hat{x}) = -\left(\Delta u_i^T \hat{R}_i^{\frac{1}{2}} - \frac{1}{2} B_i(x, \hat{x}) \hat{R}_i^{-\frac{1}{2}}\right) \\ &\quad \times \left(\Delta u_i^T \hat{R}_i^{\frac{1}{2}} - \frac{1}{2} B_i(x, \hat{x}) \hat{R}_i^{-\frac{1}{2}}\right)^T \end{aligned}$$

$$+\frac{1}{4}B_i\left(x,\hat{x}\right)\hat{R}_i^{-1}\left(x,\hat{x}\right)B_i^T\left(x,\hat{x}\right)+C_i\left(x,\hat{x}\right)$$
$$\leq c_i-\lambda V_i.$$

The rest of proof is similar to that of Theorem 3.

Remark 5: (8), or (20) or (24) tells us that exponentially quasi-incremental (Q, S, R)-dissipativity is unnecessary for each subsystem. If $c_i = 0$, $\lambda = 0$ in (8) then the conditions in Theorem 3 can degenerate into incremental (Q, S, R)-dissipativity conditions [41].

Remark 6: The conditions (8), (20), (24) are commonly adopted for switched nonlinear systems [39]–[47]. However, checking these conditions may be difficult in practice, so that some literatures proposed an associated sufficient condition corresponding to a feasibility problem over Linear Matrix Inequalities. In general, it is very hard to get the exact analytical solution of PDIs (Partial Differential Inequalities). Hence, several methods for finding approximate solutions on a compact set or numeral solutions were proposed in [48]–[53]. In some special cases, this problem can be recasted as a convex optimization problem or state-dependent matrix inequality problem over bilinear matrix inequality. These problems can be solved by the developed approaches in [48] and [49]. Moreover, Sum of Squares method is an effective analytic tool for constructing storage functions of systems described by polynomial vector fields [50]–[53].

V. EXAMPLE

Example 2: This section will give an example to demonstrate the effectiveness of the results. Consider system (1) with two subsystems described by

$$\begin{aligned}
&f_1\left(x,u_1\right)\\
&=\begin{pmatrix}-x_1\left(x_1^2+4.3\right)+0.5x_2-0.6x_2^2+3.2+0.45u_1\\ 0.3x_1+0.6x_2+7.8x_1^2+0.3u_1\end{pmatrix},\\
&y=x_1+2x_2+0.5u_1,\\
&f_2\left(x,u_2\right)\\
&=\begin{pmatrix}0.78x_1+0.781111x_2-2.984+0.549u_2\\ 1.6x_1-20.39798x_2+38.4+0.92u_2\end{pmatrix},\\
&y=x_1+0.99495x_2-0.2u_1. \qquad (26)
\end{aligned}$$

The storage functions are selected as

$$V_1\left(x,\hat{x}\right)=\frac{1}{2}\left(x-\hat{x}\right)^TP_1\left(x-\hat{x}\right)\text{ and}$$
$$V_2\left(x,\hat{x}\right)=\frac{1}{2}\left(x-\hat{x}\right)^TP_2\left(x-\hat{x}\right), \qquad (27)$$

where $P_1=\begin{bmatrix}1&0\\0&3\end{bmatrix}$ and $P_2=\begin{bmatrix}2&-0.01\\-0.01&1\end{bmatrix}$.

The derivative of S_i is given as follows:

$$\begin{aligned}
\dot{V}_1&\leq-\beta_{12}\left(V_1-V_2\right)-2V_1+1.2+0.6\left(y-\hat{y}\right)^2\\
&\quad+0.5\left(u_1-\hat{u}_1\right)\left(y-\hat{y}\right)-0.3\left(u_1-\hat{u}_1\right)^2,\\
\dot{V}_2&\leq-\beta_{21}\left(V_2-V_1\right)-V_2+1\\
&\quad+1.2\left(u_2-\hat{u}_2\right)\left(y-\hat{y}\right)+0.3\left(u_2-\hat{u}_2\right)^2. \qquad (28)
\end{aligned}$$

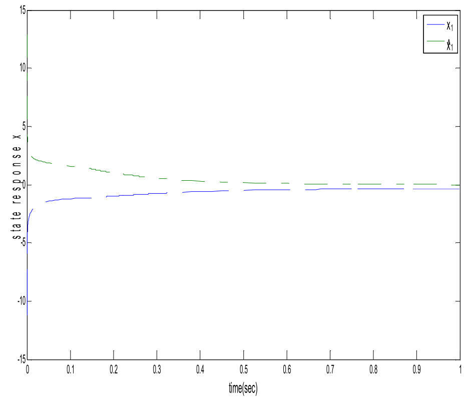

FIGURE 1. State response $x_1, \hat{x}_1$ of the switched system.

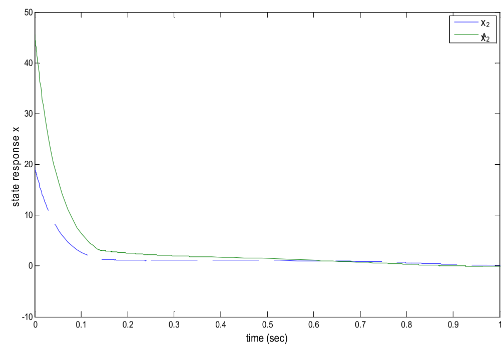

FIGURE 2. State response $x_2, \hat{x}_2$ of the switched system.

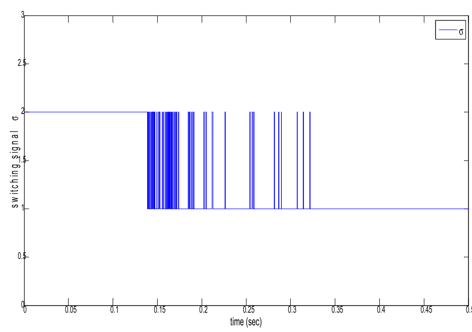

FIGURE 3. Switching signal.

where $\beta_{12} = -3.5$, $\beta_{21} = -10$. The switching law is designed as

$$\sigma\left(t\right)=1,\text{ when }V_1-V_2\leq0,\ \ \sigma\left(t\right)=2,\text{ when }V_2-V_1\leq0. \qquad (29)$$

By Theorem 2, system (26) is quasi- incrementally (Q, S, R)-dissipative.

According to Theorem 1, the resulting closed-loop system is practically incrementally stable.

Let $u_1 = \hat{u}_1 = -10$, $u_2 = \hat{u}_2 = 2.59$ and the initial states $\left(x_1(0), x_2(0)\right) = (-11.2, 19.4)$, $\left(\hat{x}_1(0), \hat{x}_2(0)\right) = (-12.9, 45.5)$. The simulation of system (26) was performed in MATLAB using the "ode45s" solver. The simulation results are presented in Figs. 1 - 3. Figs.1, 2 show the state response of the switched system is bounded under the switching signal described by Fig. 3. Figs.1, 2 imply that two system trajectories $x(t)$ and $\hat{x}(t)$ converge to a ball, which indicates that the closed-loop system is practically incrementally stable. Thus, the simulation results well illustrate the theory presented.

VI. CONCLUSION

This paper has investigated quasi-incremental (Q, S, R)- dissipativity and practicallly incremental stability for switched nonlinear systems. A state-dependent switching law has been designed to establish exponential quasi-incremental-(Q, S, R)- dissipativity criterion. The designed state-dependent switching law is more general than the well-known min-switching or max-switching. This gives more design freedom of stabilizing switched systems. However, the state-dependent switching law may cause frequent switching. How to design a state-dependent switching law with a desirable dwell time is a significant issue. This issue will be our future topic of research.

REFERENCES

[1] J. C. Willems, "Dissipative dynamical systems—Part I: General theory," *Arch. Rational Mech. Anal.*, vol. 45, no. 5, pp. 321–351, 1972.

[2] F. Zhu, M. Xia, and P. J. Antsaklis, "On passivity analysis and passivation of event-triggered feedback systems using passivity indices," *IEEE Trans. Autom. Control*, vol. 62, no. 3, pp. 1397–1402, Mar. 2017.

[3] D. Hill and P. Moylan, "The stability of nonlinear dissipative systems," *IEEE Trans. Autom. Control*, vol. AC-21, no. 5, pp. 708–711, Oct. 1976.

[4] D. J. Hill and P. J. Moylan, "Dissipative dynamical systems: Basic input-output and state properties," *J. Franklin Inst.*, vol. 309, no. 5, pp. 327–357, May 1980.

[5] Y. Yan, L. Su, V. Gupta, and P. Antsaklis, "Analysis of two-dimensional feedback systems over networks using dissipativity," *IEEE Trans. Autom. Control*, vol. 65, no. 8, pp. 3241–3255, Aug. 2020.

[6] A. L. Fradkov and D. J. Hill, "Exponential feedback passivity and stabilizability of nonlinear systems," *Automatica*, vol. 34, no. 6, pp. 697–703, Jun. 1998.

[7] V. S. Chellaboina and W. M. Haddad, "Exponentially dissipative nonlinear dynamical systems: A nonlinear extension of strict positive realness," *Math. Problems Eng.*, vol. 2003, no. 1, pp. 25–45, 2003.

[8] A. Pavlov and L. Marconi, "Incremental passivity and output regulation," *Syst. Control Lett.*, vol. 57, no. 5, pp. 400–409, May 2008.

[9] C. A. Desoer and M. Vidyasagar, *Feedback Systems: Input-Output Properties*. New York, NY, USA: Academic, 1975.

[10] H. Santoso, D. Hioe, J. Bao, and P. L. Lee, "Operability analysis of nonlinear processes based on incremental dissipativity," *J. Process Control*, vol. 22, no. 1, pp. 156–166, Jan. 2012.

[11] A. Hamadeh, G.-B. Stan, R. Sepulchre, and J. Gonçalves, "Global state synchronization in networks of cyclic feedback systems," *IEEE Trans. Autom. Control*, vol. 57, no. 2, pp. 478–483, Feb. 2012.

[12] C. Wu, A. van der Schaft, and J. Chen, "Robust trajectory tracking for incrementally passive nonlinear systems," *Automatica*, vol. 107, pp. 595–599, Sep. 2019.

[13] I. G. Polushin and J. H. Marquez, "Boundedness properties of nonlinear quasi-dissipative systems," *IEEE Trans. Autom. Control*, vol. 49, no. 12, pp. 2257–2261, Jan. 2004.

[14] P. M. Dower, "A variational inequality for measurement feedback almost-dissipative control," *Syst. Control Lett.*, vol. 50, pp. 21–38, Sep. 2003.

[15] P. M. Dower and M. R. James, "Dissipativity and nonlinear systems with finite power gain," *Int. J. Robust Nonlinear Control*, vol. 8, pp. 699–724, Dec. 1998.

[16] A. Pogromsky and H. Nijmeijer, "Cooperative oscillatory behavior of mutually coupled dynamical systems," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 48, no. 2, pp. 152–162, Mar. 2001.

[17] E. Steur, I. Tyukin, and H. Nijmeijer, "Semi-passivity and synchronization of diffusively coupled neuronal oscillators," *Phys. D, Nonlinear Phenomena*, vol. 238, no. 21, pp. 2119–2128, 2009.

[18] S. Li, C. K. Ahn, and Z. Xiang, "Sampled-data adaptive output feedback fuzzy stabilization for switched nonlinear systems with asynchronous switching," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 1, pp. 200–205, Jan. 2019.

[19] S. Li, C. K. Ahn, and Z. Xiang, "Command-filter-based adaptive fuzzy finite-time control for switched nonlinear systems using state-dependent switching method," *IEEE Trans. Fuzzy Syst.*, vol. 29, no. 4, pp. 833–845, Apr. 2021.

[20] S. Li, C. K. Ahn, J. Guo, and Z. R. Xiang, "Global output feedback sampled-data stabilization of a class of switched nonlinear systems in the p-normal form," *IEEE Trans. Syst., Man Cybern., Syst.*, vol. 51, no. 2, pp. 1075–1084, Feb. 2021.

[21] S. Li, J. Guo, and Z. Xiang, "Global stabilization of a class of switched nonlinear systems under sampled-data control," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 49, no. 9, pp. 1912–1919, Sep. 2019.

[22] S. Li, C. K. Ahn, J. Guo, and Z. Xiang, "Neural-network approximation-based adaptive periodic event-triggered output-feedback control of switched nonlinear systems," *IEEE Trans. Cybern.*, vol. 51, no. 8, pp. 4011–4020, Aug. 2021.

[23] S. Li, C. K. Ahn, M. Chadli, and Z. Xiang, "Sampled-data adaptive fuzzy control of switched large-scale nonlinear delay systems," *IEEE Trans. Fuzzy Syst.*, early access, Jan. 15, 2021, doi: 10.1109/TFUZZ.2021.3052094.

[24] A. Lu and G. Yang, "Stabilization of switched systems with all modes unstable via periodical switching laws," *Automatica*, vol. 112, Dec. 2020, Art. no. 109150.

[25] J. Fu, R. Ma, and T. Chai, "Global finite-time stabilization of a class of switched nonlinear systems with the powers of positive odd rational numbers," *Automatica*, vol. 54, pp. 360–373, Apr. 2015.

[26] J. Fu, R. Ma, and T. Chai, "Adaptive finite-time stabilization of a class of uncertain nonlinear systems via logic-based switchings," *IEEE Trans. Autom. Control*, vol. 62, no. 11, pp. 5998–6003, Nov. 2017.

[27] L. Zhang and P. Shi, "Stability, l_2-gain and asynchronous H_∞ control of discrete-time switched systems with average dwell time," *IEEE Trans. Autom. Control*, vol. 54, no. 9, pp. 2192–2199, Sep. 2009, doi: 10.1109/TAC.2009.2026841.

[28] Y. Mao, H. Zhang, and S. Xu, "The exponential stability and asynchronous stabilization of a class of switched nonlinear system via the T–S fuzzy model," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 4, pp. 817–828, Aug. 2014.

[29] G. Zhai, B. Hu, K. Yasuda, and A. N. Michel, "Stability analysis of switched systems with stable and unstable subsystems: An average dwell time approach," *Int. J. Syst. Sci.*, vol. 32, no. 8, pp. 1055–1061, Nov. 2001.

[30] Y. Mao, H. Zhang, and Z. Zhang, "Finite-time stabilization of discrete-time switched nonlinear systems without stable subsystems via optimal switching signal design," *IEEE Trans. Fuzzy Syst.*, vol. 25, no. 1, pp. 172–180, Mar. 2017.

[31] L. Long, Z. Wang, and J. Zhao, "Switched adaptive control of switched nonlinearly parameterized systems with unstable subsystems," *Automatica*, vol. 54, no. 4, pp. 217–228, Apr. 2015.

[32] A. Girard, G. Pola, and P. Tabuada, "Approximately bisimilar symbolic models for incrementally stable switched systems," *IEEE Trans. Autom. Control*, vol. 55, no. 1, pp. 116–126, Dec. 2010.

[33] H. Yang and B. Jiang, "Incremental stability of switched non-linear systems," *IET Control Theory Appl.*, vol. 10, no. 2, pp. 220–225, Jan. 2016.

[34] A. Girard, G. Gössler, and S. Mouelhi, "Safety controller synthesis for incrementally stable switched systems using multiscale symbolic models," *IEEE Trans. Autom. Control*, vol. 61, no. 6, pp. 1537–1549, Jun. 2016.

[35] D. Fiore, S. J. Hogan, and M. di Bernardo, "Contraction analysis of switched systems via regularization," *Automatica*, vol. 73, pp. 279–288, Nov. 2016.

[36] Y. Li, S. Phillips, and R. G. Sanfelice, "Basic properties and characterizations of incremental stability prioritizing flow time for a class of hybrid systems," *Syst. Control Lett.*, vol. 90, pp. 7–15, Apr. 2016.

[37] J. Zhao and D. J. Hill, "Dissipativity theory for switched systems," *IEEE Trans. Autom. Control*, vol. 53, no. 4, pp. 941–953, May 2008.

[38] D. Yang and J. Zhao, "Dissipativity for switched LPV systems and its application: A parameter and dwell time-dependent multiple storage functions method," *IEEE Trans. Syst., Man Cybern., Syst.*, vol. 50, no. 2, pp. 502–513, Oct. 2017.

[39] H. Wang and J. Zhao, "Finite-time passivity of switched non-linear systems," *IET Control Theory Appl.*, vol. 12, no. 3, pp. 338–345, Nov. 2018.
[40] X. Dong and J. Zhao, "Incremental passivity and output tracking of switched nonlinear systems," *Int. J. Control*, vol. 85, no. 10, pp. 1477–1485, May 2012.
[41] H. Pang and J. Zhao, "Incremental (Q,S,R)-dissipativity and incremental stability for switched nonlinear systems," *J. Franklin Inst.*, vol. 353, no. 17, pp. 4542–4564, Nov.,2016.
[42] H. Pang, "Vector incremental L_2-gain and incremental stability for switched nonlinear systems," *J. Franklin Inst.*, vol. 356, no. 6, pp. 3508–3530, Apr. 2019.
[43] H. Pang and C. Li, "Incremental passivity and stabilization for switched nonlinear systems," *Trans. Inst. Meas. Control*, vol. 42, no. 11, pp. 2088–2102, Apr. 2020.
[44] Y. Ren, W. Wang, and Y. Wang, "Exponentially incremental (Q,S,R)-dissipativity and incremental stability for switched time-varying nonlinear systems," *Int. J. Syst. Sci.*, vol. 48, no. 15, pp. 3201–3214, Aug. 2017.
[45] H. Pang and S. Liu, "Robust exponential quasi-passivity and global stabilization for uncertain switched nonlinear systems," *Int. J. Robust Nonlinear Control*, vol. 30, no. 18, pp. 8117–8138, Dec. 2020.
[46] S. Liu and H. Pang, "Exponential quasi-(Q,S,R)-dissipativity and practical stability for switched nonlinear systems," *Asian J. Control*, vol. 22, no. 5, pp. 2029–2040, Jan. 2020.
[47] J. Zhao and D. J. Hill, "On stability, L_2-gain and $H\infty$ control for switched systems," *Automatica*, vol. 44, no. 5, pp. 1220–1232, May 2008.
[48] N. Sakamoto and A. J. van der Schaft, "Analytical approximation methods for the stabilizing solution of the Hamilton–Jacobi equation," *IEEE Trans. Autom. Control*, vol. 53, no. 10, pp. 2335–2350, Nov. 2008.
[49] J. G. Van Antwerp and R. D. Braatz, "A tutorial on linear and bilinear matrix inequalities," *J. Process Control*, vol. 10, no. 4, pp. 363–385, 2000.
[50] E. Mojica-Nava, N. Quijano, A. Gauthier, and N. Rakoto-Ravalontsalama, "Stability analysis of switched polynomial systems using dissipation inequalities," in *Proc. 47th IEEE Conf. Decis. Control*, Dec. 2008, pp. 32–36.
[51] M. J. McCourt and P. J. Antsaklis, "SOS methods for demonstrating dissipativity for switched systems," in *Proc. Amer. Control Conf.*, 2014, pp. 3936–3941, doi: 10.1109/ACC.2014.6858757.
[52] S. Coogan and M. Arcak, "A dissipativity approach to safety verification for interconnected systems," *IEEE Trans. Autom. Control*, vol. 60, no. 6, pp. 1722–1727, Jun. 2015.
[53] M. Soodavi, J. Ghaisari, and F. Sheikholeslam, "L_2-gain analysis for switched systems with a state dependent switching signal," *IET Control Theory Appl.*, vol. 9, no. 11, pp. 1723–1739, Mar. 2011.

HONGBO PANG received the Ph.D. degree in control theory and control engineering from the College of Information Science and Engineering, Northeastern University, Shenyang, China, in 2016. He is currently working with the College of Science, Liaoning University of Technology, Jinzhou, China. His research interests include switched nonlinear systems and adaptive control.

SHUO LIU received the master's degree in mathematics from the College of Science, Northeastern University, Shenyang, China, in 2006. She is currently working with the College of Science, Liaoning University of Technology, Jinzhou, China. Her current research interests include switched systems, nonlinear systems, robust control, and adaptive control.