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Circuit-Based Mathematical Model of an Arc Heater for Control System Development

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ABSTRACT A novel circuit-based mathematical model of an electric arc heater is presented so that an arc heater system can be modeled, and a control algorithm can be developed and simulated. Due to inherent arc nonlinearities and complexities, as well as low amounts of arc heater data, the new model was developed by establishing a holistic approach to implementing the arc as a circuit element, where common circuit analysis and control techniques can be easily applied. The response of the arc heater system was examined at various voltage and current operating points that represent different regions of operation within the arc's characteristic curve. The simulated data of the arc heater model were compared to the arc characteristics of the experimental data. The experimental data set used for comparison was collected at the Hypersonic Materials Environmental Test System (HyMETS) arc-jet wind tunnel by the NASA Langley Research Center in Hampton, Virginia. Data analysis and simulations were executed utilizing MATLAB and Simulink to compare the newly developed model with the experimental data. The simulations demonstrate a strong correlation between these datasets, indicating the model's ability to accurately replicate the physical system, while also allowing initial control system development to begin with simplistic proportional-integral-derivative (PID) control of the arc heater.

INDEX TERMS Arc heater, arc heater control, arc heater model, arc heater simulation, arc in a circuit.

I. INTRODUCTION

The occurrence of the electric arc is analyzed from many areas within industry, academics, and research. In each, the arc is either considered a nuisance or hazard in which rapid extinguishment is examined and preferred, or arc sustainment that has been proven useful for an application is desired. Circuit breakers are one such common area of research where the arc is viewed from the standpoint of a quick extinguishment for overcurrent protection. For example, the time taken to extinguish an arc is of particular importance within arc flash analysis where the goal is to reduce the incident energy in which an employee may be exposed [1], [2]. The useful areas of the arc include technologies such as welders, gas lighting, arc furnaces, and arc heaters. For this paper, arc heater applications are the primary focus; specifically, the response of the system while examining the sustainment and stability of an arc. The control of the arc is important not only for the application through modeling and simulation, but the reduction of commissioning time and risk to the real-world hardware and facility.

Initially it may be helpful to consider a broad categorization of the arc models from the perspective of the generic overview mentioned above. At a high level, arc models can be placed into two broad categories: physics/plasma based, and circuit based. The physics/plasma-based models are mostly utilized for circuit breaker applications around current-zero occurrence [5]. The fundamental equations governing these models are based on the Tseng *et al.* [3], Cassie [4], Mayr [3], Amato *et al.* [15], Cassie-Mayr combinations/hybrids [3], [6]–[8], [26], or some other black-box type examination of the arc conductance based on experimental data. Alternatively, within circuit-based models the arc may be examined and analyzed as a circuit element. That is, a device within the circuit that can then be examined using well established circuit analysis techniques such as Kirchoff's Voltage and Current Laws for modeling and simulation. The holistic approach of implementing the arc as a circuit element is more amenable to electrical and control engineers when the underlying physics, power losses, and time constants are mostly irrelevant from an arc sustainment perspective.

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TABLE 1. Arc heaters from around the world.

Organization	Location	Type (Power)
NASA	Langley, VA	Huels (20 MW)
NASA	Langley, VA	Segmented (75 MW)
NASA	Mt. View, CA	Huels/Segmented (75 MW)
U. of Texas	Arlington, TX	Huels (1.6 MW)
Dept. of Defense	Tullahoma, TN	Segmented (70 MW)
CIRA	Scirocco, Italy	Segmented (70 MW)
ISAS/JAXA	Sagamihara City, Japan	Segmented (1 MW)
DLR	Cologne, Germany	Huels (6 MW)

MW = Megawatt

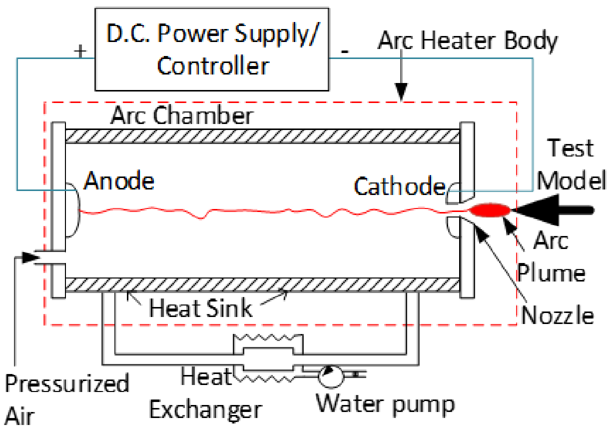

FIGURE 1. Diagram of an Arc heater system [9].

Arc heaters, such as those located at NASA, DoD, and arc heater facilities around the world (Table 1), convert electrical energy to thermal energy to replicate flight test conditions where operating parameters are often changed to obtain the desired environmental conditions. Such heaters can operate from a few kilowatts to 75 megawatt and are increasing in size as technology advances. Arc heaters are not commonly encountered systems. Electric Arc Furnaces are similar, but have extremely different applications and control algorithms, and tend to be much more stable in operation.

The overall system is shown in Fig. 1, consisting of an arc heater and the various subsystems such as the DC Power Supply/Controller, pressurized air, and heat exchanger. The focus of this research is on modeling the arc heater to control the DC Power Supply, for the purposes of maintaining stability, while also maintaining the operating conditions of the system. In order to develop the control system, a mathematical model of the entire electric arc heater system was developed for analysis. In practice, this allows the control system to be developed in a lab environment which significantly reduces commissioning times and asset risks.

II. LITERATURE REVIEW

Literature for this research was collected mostly from the IEEE and the AIAA, along with textbooks providing expertise in electric arcs, control theory, and also modeling and simulation. Research of existing arc models indicates that most models published are not intended for transient analysis in circuit simulation nor control system development [3], [10]. Instead, the models were usually used to research the interaction between a switching arc and the electrical circuit during the interruption process of circuit breakers [1], [3]. The Cassie and Mayr models include differential equations that depend on a set of parameters which should be obtained from the experimental data [12]. The Cassie model is applicable at high currents, while the Mayr model is applicable at low currents near current zero [12]. Some authors have combined Cassie and Mayr models, or hybrid models as they're often referred, to represent the arc characteristics in a wide current region [13], [26]. The Cassie-Mayr type models produce good qualitative descriptions of the arc but may not provide very good quantitative results [6], [7].

Major gaps exist between the available models and the applications for which they were intended. At present, black box models are commonly used to describe the interaction of the arc with the electric network during an interruption process [12]. However, these arc models have some voltage inaccuracies in the high current regions [13], and even when suitable parameters are selected, there are still variances between experimental data and model outputs [14]. Problems with models focused primarily on circuit breaker operations are that they never completely predict the behavior of the arc. The physics do not seem to be scalable from very short contact gaps to the lengths of several feet inherent to arc heaters. Further, the temperatures and pressures of circuit breaker applications do not directly correlate to arc heater operations. To remedy these nonconformities, researchers often add fit parameters that modify the models and equations, in such a way, that the modified model is extremely disassociated from the fundamental equations. The current-voltage characteristics of the electric arc depend on many parameters such as electric arc dimensions, material of current carrying parts, chemical composition, and pressure [27]. In some instances, over 10 such parameters are applied to describe the specific arc phenomenon [15]–[18]. Fitting the model's simulated results to experimental data is accomplished by selecting arc parameters such as the time constant and the cooling power, which are normally a function of the arc current and voltage [7]. All arcs exhibit a highly complex nonlinear nature affected by such parameters [11]. Increasing the number of parameters can lead to difficulties in obtaining their value [12], while also injecting complexity and uncertainty into the modeling effort.

III. BACKGROUND

The electric arc is a self-sustained discharge with low resistance and high currents [19]. The electric arc itself can be considered as a single element of the entire electric power system, having a resistive nature (i.e., as a pure resistance) [11]. Due to its nonlinear characteristics and very complex arc phenomena, the modeling of an arc is additionally a complex

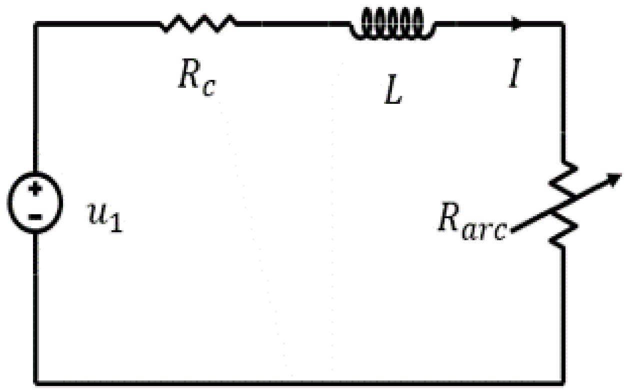

FIGURE 2. Simplified series circuit.

task [8], [10], [11]. The simplified series circuit of an arc heater electrical system is shown in Fig. 2.

In Fig. 2, u_1 represents the supplied voltage from the rectifier, R_c the circuit resistance, L the smoothing inductance, I the current through the circuit, and R_{arc} represents the resistance of the arc heater. However, this circuit cannot be easily modeled for control system development utilizing the existing tools and techniques applicable to Linear Time-Invariant (LTI) systems. A new approach is to apply circuit theory with the arc as one of the system elements, and then linearize the arc characteristic by decomposing the arc into local elements, thus representing the global model in its entirety. The variable nonlinear resistor (R_{arc}) can be replaced by multiple linearized resistors. The resistance function is a negative differential resistance dependent on the location and associated slopes relative to the arc's characteristic curve.

IV. CONTROL OF ARC HEATERS

Control of arc heaters is of particular importance when examining the sustainment, stability, and performance of the system. In order to more adequately match the desired environments, a thyristor-based power supply is preferred for system flexibility. These systems are oftentimes prototypes, experimental, and very costly. For these reasons, the ability to model and simulate the arc heater and associated power supply is invaluable for complete system knowledge and hardware risk mitigation.

Arc models do not usually consider the arc within the entire electrical system but are often developed for specific applications of plasma physics [6]. Therefore, a better approach for control design with modeling and simulation, is to examine the arc within the circuit in which it is applied. Thus, the focus is shifted from discharge and ignition characteristics, to control performance and stability. Such a holistic approach may allow the designer to engineer the optimal components and control algorithms for the system in which the arc exists, as part of the overall circuit.

Various types of power supplies are utilized for arc sustainment. The way in which the power supply is designed to operate will determine, to a great extent, how the arc can be controlled. One important feature being a determination of

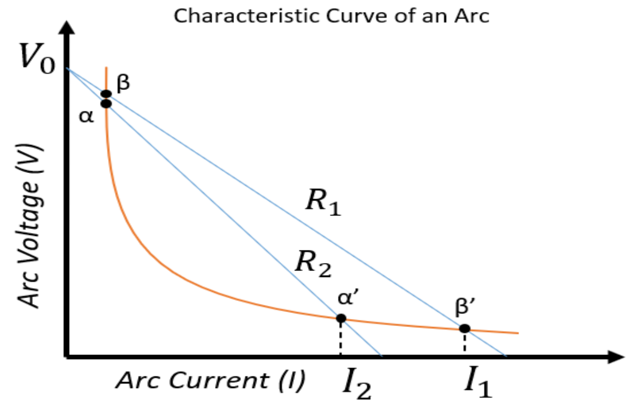

FIGURE 3. A change in operating points based on circuit resistance.

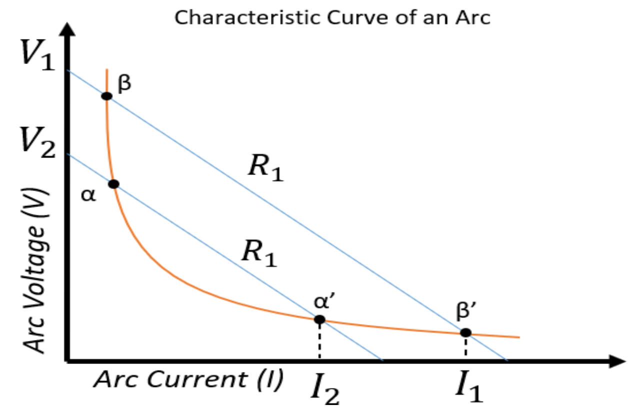

FIGURE 4. A change in operating points based on applied voltage.

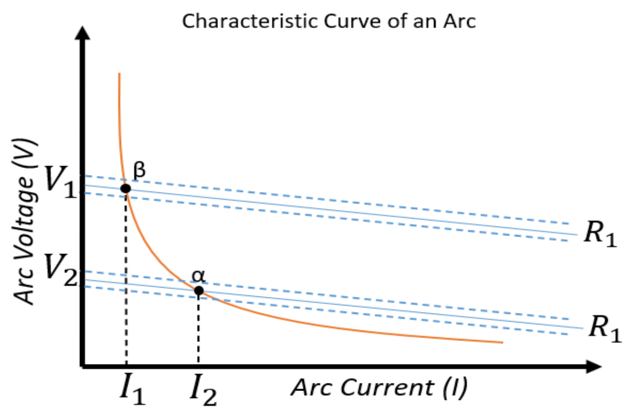

FIGURE 5. Control of the Arc around unstable points.

where on the arc characteristic curve the system is to operate. This selection also determines the stability of the system. Within [16], [17] it is shown that for a given resistance there coincides an unsteady operation point and a steady operation point, both after the arc's ignition. The first two examples, Fig. 3 and Fig. 4, are passively stable at the steady operation point. Whereas, in Fig. 5, the system must be actively stabilized by a controller to operate at the unsteady operating points.

As shown in Fig. 3, for an applied voltage and various resistances, there exists unsteady operation points (points α and β) and their respective steady operating points (α' and β'). By maintaining the supplied voltage (V_0) and varying

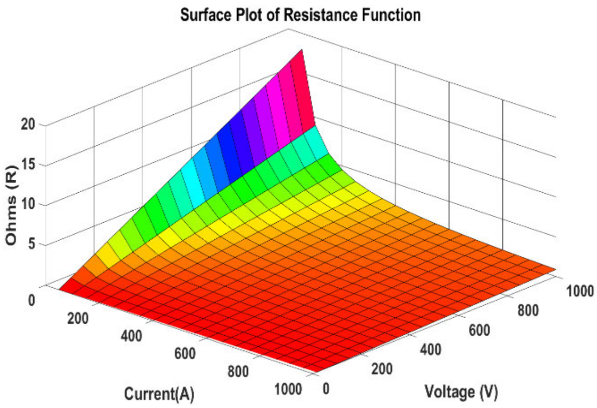

FIGURE 6. Plot of the resistance function *R (I, V)*.

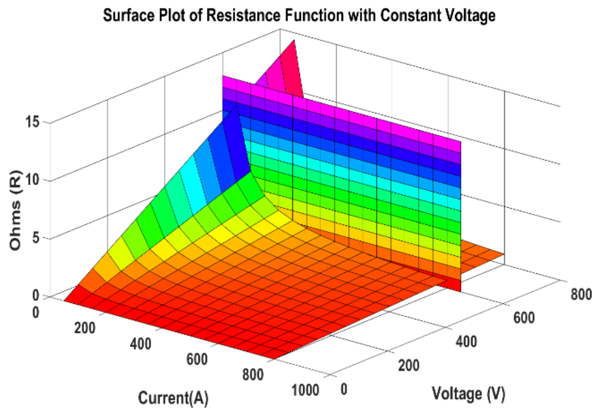

FIGURE 7. Resistance function *R(I,V)* with linear voltage plane.

the circuit resistance (R_1 to R_2), the current in the system and through the arc can be changed (I_1 to I_2) by adjusting the characteristic slope of the power supply impedance. Another method of operation is to hold the circuit resistance constant (R_1) and vary the voltage (V_1 to V_2) as in Fig. 4. This method is more amenable to thyristor voltage control and provides system flexibility when operating an arc with various currents.

In Fig. 5 there is shown yet another mode of operation. The system is operated around the unsteady points, and the controller must continually bring the process to the desired setpoint and maintain it. A low impedance power supply is utilized with a very shallow slope, where V_Δ (V_1 to V_2) determines I_Δ (I_1 to I_2) inversely and follows the shape of the arc characteristic curve.

V. DETERMINING THE CHARACTERISTIC CURVE OF AN ARC

The arc heater is a nonlinear time invariant system in which the electric arc is a self-sustained discharge with a low voltage drop, able to support large currents [19], [20]. One important arc parameter describing its complex nature is the arc resistance [11]. The variable resistance of the arc was modeled from the derived equation presented in [11], [21]. The equation is presented as follows:

$$R_{arc} = \frac{2\sqrt{2}V_{arc}}{\pi I_{arc}} \tag{1}$$

where (V_{arc}) is a function of the arc voltage gradient multiplied by the arc length, and (I_{arc}) is the arc RMS current. This function is plotted in Fig. 6 over a typical range of values for voltage and current in an arc heater.

The plot definitely appears to validate that the volt-ampere characteristics of the arc discharge has a negative slope [19], and the resistance of the arc, for a given length, decreases as current increases [22]. This implies the relationship between voltage and current is not proportional. The arc is considered non-ohmic and does not obey Ohm's Law of proportionality and is thus inversely nonlinear. This research pertains only to arc characteristics post arc ignition and omits the discharge processes and starting transients.

The first step in analyzing a nonlinear system is usually to linearize it, if possible, around some nominal operating point and analyze the resulting linear model [23]. The initial phase in the linearization process was to take the partial derivatives of the Resistance function R (I, V), and determine which variable has the most effect (or highest rate of change) on the arc resistance as shown below in equation (2, 3).

$$\frac{\partial R(I,V)}{\partial I} = \frac{-2\sqrt{2}V_{arc}}{\pi I^2} \tag{2}$$

$$\frac{\partial R(I,V)}{\partial V} = \frac{2\sqrt{2}}{\pi I} \tag{3}$$

Substituting a typical operating point of [340, 621] for (I_{arc}, V_{arc}) into the partial derivatives from Equations (2, 3) yields:

$$\frac{\partial R(I,V)}{\partial I}|_{[340,621]} = \frac{-2\sqrt{2}\,V_{arc}}{\pi I_{arc}^2} = -0.0048$$

$$\frac{\partial R(I,V)}{\partial V}|_{[340,621]} = \frac{2\sqrt{2}}{\pi I_{arc}} = 0.0026$$

In the calculation, it is observed that the current has almost twice greater effect on the resistance rate of change than that of the voltage, examined at this operating point. This result agrees with Fig. 6; therefore, it was determined that the voltage variable would be held constant for the purpose of linearization in this region. With the voltage held constant, the partial derivative with respect to voltage goes to zero, and the tangent line slope of the resistance is now only a function of current. An experimental operating point was chosen at V equals 600 volts, that creates a surface plane through the original 3-D surface plot. The intersection of these two surfaces creates a curve of the function $R(I)$ as shown in Fig. 7, in which now the 2-D curve of the arc is visible.

The shape of the arc's characteristic curve was studied extensively, and it was observed that the documented variations could be based on the Nottingham equation (4) [3], [10],

$$V_{arc} = A + \frac{B}{I^\alpha} \tag{4}$$

where A and B are constants, and α is a fit parameter that is based on experimental data. The constant α cannot be

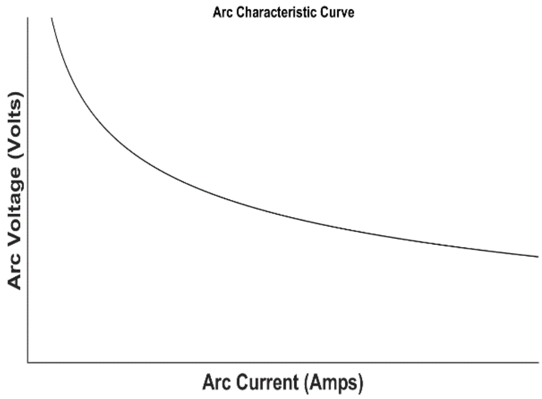

FIGURE 8. Generic Arc characteristic curve.

determined with any great degree of accuracy because of the variable nature of the phenomenon [19]. Since there are many constants with each specific arc system (column length, column diameter, pressure, enthalpy, etc. [15]–[18], [27]), the constants can be combined to produce the well-known and widely applied Warrington equation for the general formula for arc voltage:

$$V_{arc} = AI^{-\alpha} \tag{5}$$

where the constant A is based on the geometries and other parameters estimated from system measurements [21], [24]. The plot of such generic arc characteristic curve is shown in Fig. 8, which agrees with the results and plot from Fig. 7.

VI. MODELING

Modeling has been found to be indispensable for rapid development and analysis of new and existing systems, as well as for monitoring, failure prediction, fault detection, and the design of process controllers [20]. Modeling the system reduces design time and can be continually iterated to increase accuracy and applicability of the model [28]. Due to the inherent complexities and nonlinearities of real systems, the standard approach is to utilize the linearized model (as extensively discussed in the previous section) to provide a valid system approximation. It is imperative to obtain a thorough understanding of the system to make sound decisions pertaining to the applicability of the model, the bounds in which the approximation is valid, and the regions in which the model may or may not render useful data for analysis. Specifically, for control system analysis the engineer or designer must be able to interpret the results of the model compared to other data and data types. In addition, the engineer or designer must determine how much the modeling process can deviate from the actual process for performance or any other analysis.

The state of the system is a set of variables whose values, together with the input signals and the equations describing the dynamics, will provide the future state and output of the system [25]. The state-space representation of the schematic of the arc system is described in equations (6, 7) as:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{Ax}(t) + \boldsymbol{Bu}(t) \tag{6}$$

$$y(t) = \boldsymbol{Cx}(t) + \boldsymbol{Du}(t) \tag{7}$$

and,

$$\Delta I(0) = \Delta I_0 \tag{8}$$

$$\Delta V(0) = \Delta V_0 \tag{9}$$

where ΔI_0 and ΔV_0 are the linearization points.

By Kirchoff's Voltage Law:

$$u_1 = R_c I + L\frac{dI}{dt} + V_{arc}$$

$$u_1 = R_c I + L\frac{dI}{dt} + \left(\frac{dV}{dI}|_{\Delta I_0} I + \Delta V_0\right)$$

$$u_1 = I\left(R_c + \frac{dV}{dI}|_{\Delta I_0}\right) + L\frac{dI}{dt} + \Delta V_0$$

Let:

$$x_1 = I$$

Then,

$$\dot{x}_1 = \frac{dI}{dt} = \frac{u_1}{L} - \frac{R_c x_1}{L} - \left(\frac{\frac{dV}{dI}|_{\Delta I_0} x_1 + \Delta V_0}{L}\right) \tag{10}$$

Substituting (10) into the state-space equations of (6, 7):

$$[\dot{\boldsymbol{x}}_1] = -\left[\frac{\frac{dV}{dI}|_{\Delta I_0} + R_c}{L}\right][x_1] + \left[\frac{1}{L}\right][u_1] + \frac{\Delta V_0}{L} \tag{11}$$

$$y = [1][x_1] + \frac{\Delta I_0}{L} \tag{12}$$

where,

$$\boldsymbol{Ax}(t) = -\left[\frac{\frac{dV}{dI}|_{\Delta I_0} + R_c}{L}\right]; \quad \boldsymbol{Bu}(t) = \frac{1}{L},$$

$$\boldsymbol{Cx}(t) = 1; \quad \boldsymbol{Du}(t) = 0.$$

And,

$$\frac{\Delta I_0}{L}; \quad \frac{\Delta V_0}{L}$$

are the linearization offsets.

This describes an unstable system with a single pole in the right-hand plane. The open loop transfer function of the arc heater is then:

$$G_p(s) = \frac{1}{sL + \left(\frac{dV}{dI}|_{\Delta I_0} + R_c\right)} \tag{13}$$

The pole is in the right-half plane at $\left[\left(dV/dI|_{\Delta I_0} + R_c\right)/L\right]$ for the arc system when $\left[dV/dI|_{\Delta I_0} + R_c\right] < 0$, as shown by the unstable operating points of Fig. 5. This reinforces the fact that the arc is unstable within the non-thermal operating region of the arc, where the differential resistance is negative within a low impedance circuit. Within the system, the circuit parameter of the inductor (L) was determined to be 30 mH, and the circuit resistance (R_c) was calculated to be 10 mΩ. The arc voltage ranged from approximately 600 VDC to 650 VDC, with current ranging from 200A to 400A. The circuit parameters of the experimental data were the same as those implemented within the simulations.

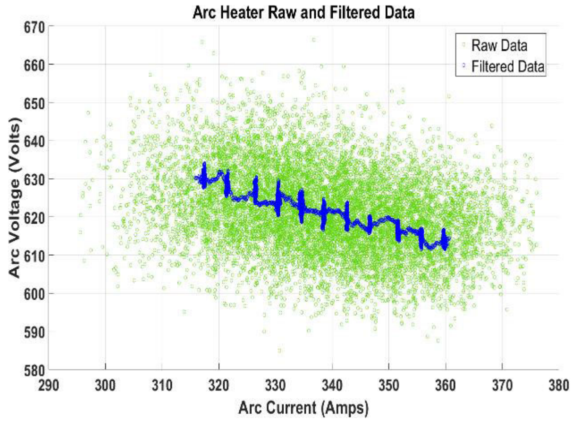

FIGURE 9. Raw and filtered Data for multiple current changes.

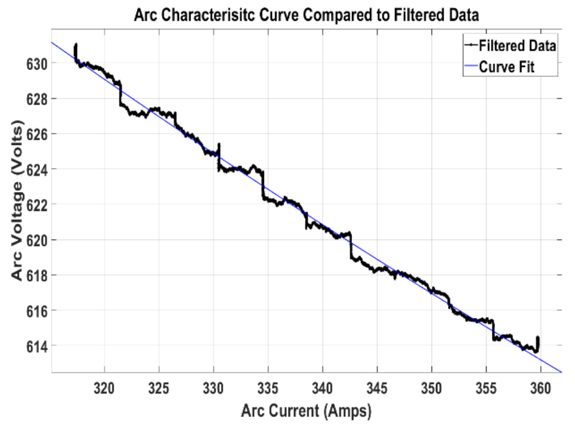

FIGURE 10. Curve fit power function to filtered data.

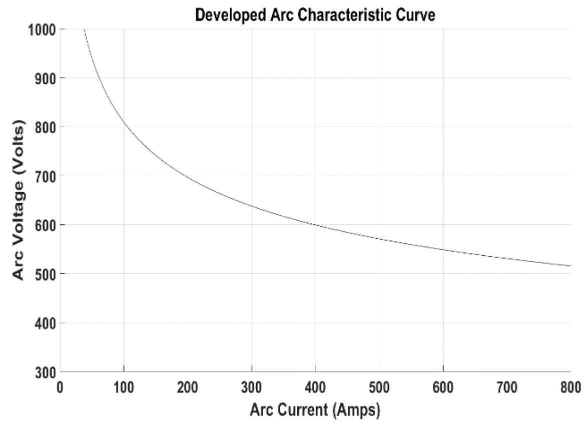

FIGURE 11. Developed Arc characteristic results.

VII. DATA

The utilized data was provided by the Hypersonic Materials Environmental Test System (HyMETS) arc-jet wind tunnel at the NASA Langley Research Center in Hampton, Virginia. The first set of data was utilized to establish system characteristics for the specific heater, heater geometry, and test condition(s) that ultimately determine the arc's characteristic curve (5) as shown in Fig. 8. The focus was the arc's current and voltage elements as they change through several setpoint changes, with electrical currents ranging from 316A to 358A as shown below in Fig. 9. All data were processed using MATLAB's 'smoothdata' function, with a 50-sample window, moving average filter. The solid line indicates filtered data.

The data shown in Fig. 9 exhibits the inverse relationship between current and voltage. As the supplied voltage increases the current decreases, and vice versa. This is not the entire characteristic curve of the arc, but a segment in which this heater was setup to run. Hardware constraints usually dictate which segments are feasible to operate relative to the arc characteristic curve. Subsequently, other operational segments are integrated to interpolate and extrapolate the arc characteristic curve in its entirety. Each operational segment could encapsulate a few hundred amps. Extreme low currents are undesired due to the increased instability caused by the high slope on the arc characteristic curve, as well as proving less useful from the thermal perspective. Oppositely, high currents are constrained by the limitations of the arc heater, power supply, and other subsystems. Therefore, only a specific section of the arc characteristic curve is operationally valid.

Next the filtered data was utilized to develop the alpha parameter as explained in (5) by curve fitting to the experimental data shown in Fig. 10 [29]. This approach will allow the model to predict the arc behavior about a specific linearized operating point while also allowing the engineer or designer to begin control system development and preliminary system tuning. As with many systems, the more supplementary data available for this step, the more accurate the curve fit will approximate the alpha parameter.

Now that the A and the α parameters are known in (5), from curve fitting the experimental data, the arc characteristic curve for the arc heater can be reconstructed as shown in Fig. 11. The developed arc characteristic curve may not be operationally obtainable in its entirety based on the physical system as previously mentioned. However, this curve well defines how the arc will behave relative to the voltage and current elements for a designated arc heater and arc heater configuration.

By taking the derivative of the arc characteristic curve at a specific current point, the necessary $dV/dI|_{\Delta I_0}$ parameter is calculated as required in (11, 13), which is utilized in the model.

VIII. RESULTS

Simulations of the newly developed mathematical model, based on the physics previously described, were executed to analyze the fidelity of the model as compared to the experimental data. Using Simulink software, the model's simulated results were compared to the experimental data, as well as running additional simulations above and below the initial current data set. The system contained a discretized PID controller with a sample rate of 1.4 ms, with inputs that matched the step inputs of the data. Implementing a simple feedback loop, and lookup tables to calculate the slope and linearization offset, the block diagram is shown in Fig. 12.

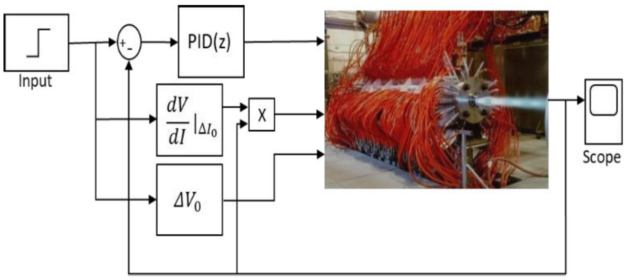

FIGURE 12. Simulink model control and feedback loop.

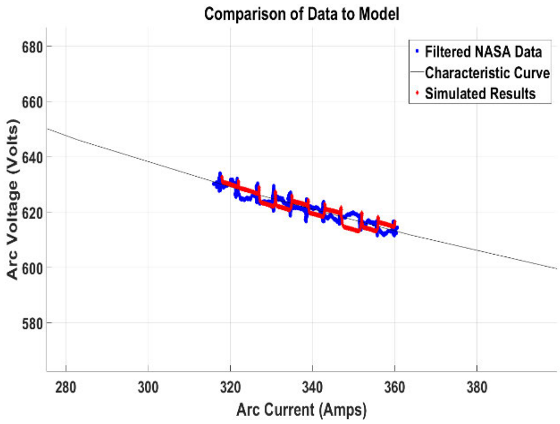

FIGURE 13. Comparison of experimental data to simulation.

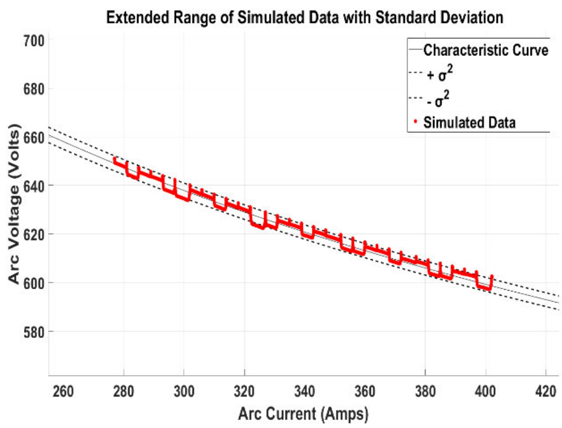

FIGURE 14. Comparison of simulated data to characteristic curve, with standard deviation.

The discrete PID controller was tuned in Simulink using the transfer function as derived in (13). The first simulation compared a data set that ranged from 316A to 358A to the mathematical model for validation. The step inputs to the controller replicated the step inputs of the data set to simulate the actual system as closely as possible. The current output produced favorable results where the mathematical model tracks closely to the data as shown in Fig. 13.

The next simulations compared data that were higher and lower in current magnitude than the initial dataset, from 275A to 316A in a lower region, and 358A to 400A in a higher region. It is shown in Fig. 14 that there exists a high correlation between the simulated data to the developed characteristic curve in all regions.

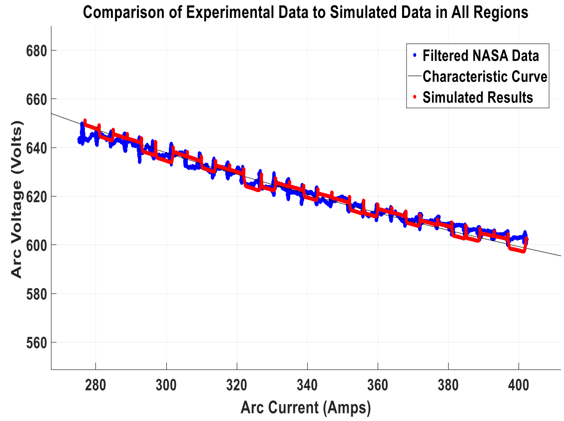

FIGURE 15. Comparison of experimental data to simulated data in all regions.

To quantitatively analyze the accuracy and precision of the model, the variance and standard deviation of the simulated data was calculated, relative to the developed characteristic curve. The calculations produced a variance of 9.003V and a standard deviation of 3.001V, or +/-0.5% of the smoothed data. These values were plotted with dashed lines in Fig. 14 for visual comparison.

The final simulation compared the simulated data to the experimental data for the entire range of data, along with the characteristic curve. It is shown in Fig. 15 that once again there exists a high correlation between the simulated data to the experimental data in this region.

It is discernible that the experimental data and simulated data begin to diverge slightly from the developed arc characteristic curve at the data extrema. It is up to the designer or engineer to determine how much divergence or error is tolerable for the specific application. Further, the additional data sets could be utilized to develop the arc characteristic curve more accurately by refitting the curve to a broader spectrum of data. In practice, these simulation results are vital for establishing the control system algorithm and performance criteria that ultimately provide the designer or engineer a baseline for control system development. By showing that the model accurately replicates the system, a certain level of confidence can be obtained pertaining to the system's response.

IX. CONCLUSION

A holistic approach to modeling an electric arc as an integral part of the circuit has been presented. A new model for the V-I characteristics of the arc was developed and compared to the experimental data. The model seems to fit very well to the inherently non-linear V-I characteristics obtained from the experimental data and was supported by multiple simulations. If one can design and simulate controllability at these various regions, one can expect stability over the entire operating region bounded by the constraints of the physical system. As expected, there will be performance tradeoffs along the various slopes where more complex control algorithms could be utilized within the linearized regions of the new model. The system will be more stable as the slope decreases, and pole placement will largely depend on the amount of inductance in the circuit. Since linearization is an approximation in the area of an operating point, it can only predict the ''local'' behavior of the nonlinear system in the vicinity of that point [23]. Therefore, the question of linearity and the range of applicability must be considered for each system [25]. The number of linear sections versus controllability and performance could also be investigated. It is shown, however, that the mathematical model, implementing the arc within the electric circuit, produces favorable results for modeling the electric arc for control system development. Future work could include a comparison of known arc models such as Cassie, Mayr, or Cassie-Mayr hybrids, with the linearized model presented in this work, for a possible inclusion in the circuit of an arc heater system over an operating range.

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REFERENCES

[1] L. Yuan, L. Sun, and H. Wu, ''Simulation of fault arc using conventional arc models,'' *Energy Power Eng.*, vol. 5, no. 4, pp. 833–837, 2013.
[2] T. Gammon, W.-J. Lee, Z. Zhang, and B. C. Johnson, ''A review of commonly used DC arc models,'' *IEEE Trans. Ind. Appl.*, vol. 51, no. 2, pp. 1398–1407, Mar. 2015, doi: 10.1109/tia.2014.2347456.
[3] K.-J. Tseng, Y. Wang, and D. M. Vilathgamuwa, ''An experimentally verified hybrid Cassie–Mayr electric arc model for power electronics simulations,'' *IEEE Trans. Power Electron.*, vol. 12, no. 3, pp. 429–436, May 1997, doi: 10.1109/63.575670.
[4] A. M. Cassie, ''Arc rupture and circuit severity: A new theory,'' in *Proc. Conf. Internationale des Grands Reseaux Electriques Haute Tension*, Paris, France, Jun. 1939.
[5] L. van der Sluis, W. R. Rutgers, and C. G. A. Koreman, ''A physical arc model for the simulation of current zero behavior of high-voltage circuit breakers,'' in *Proc. IEEE Power Eng. Soc. Transmiss. Distrib. Conf.*, Sep. 1991, pp. 485–491.
[6] J. Andrea, P. Besdel, O. Zirn, and M. Bournat, ''The electric arc as a circuit component,'' in *Proc. 41st Annu. Conf. IEEE Ind. Electron. Soc. (IECON)*, Nov. 2015, pp. 3027–3034.
[7] *Optimization of Arc Models Parameter Using Genetic Algorithm*. Accessed: Jul. 22, 2020. [Online]. Available: https://www.researchgate.net/publication/224116172_Optimization_of_arc_models_parameter_using_genetic_algorithm
[8] J. Dai, R. Hao, X. You, H. Sun, X. Huang, and Y. Li, ''Modeling of plasma arc for the high power arc heater in MATLAB,'' in *Proc. 5th IEEE Conf. Ind. Electron. Appl.*, Jun. 2010, pp. 463–468.
[9] R. Hao, T. Q. Zheng, X. You, and W. Guo, ''Development and experiment research of high-power arc heater power supply utilizing thyristor converter,'' in *Proc. 2nd IEEE Conf. Ind. Electron. Appl.*, May 2007, pp. 1559–1563.
[10] K. J. Tseng, Y. Wang, and D. M. Vilathgamuwa, ''Development of a dynamic model of electric arc for power electronics simulations,'' in *Proc. IEEE Ind. Appl. Conf. 31st IAS Annu. Meeting (IAS)*, Oct. 1996, pp. 2173–2180.
[11] V. V. Terzija and H. J. Koglin, ''On the modeling of long arc in still air and arc resistance calculation,'' *IEEE Trans. Power Del.*, vol. 19, no. 3, pp. 1012–1017, Jul. 2004.
[12] S. Maximov, V. Venegas, and J. L. Guardado, ''A method of obtaining of electric arc model parameters for SF_6 power circuit breakers,'' in *Proc. IEEE EUROCON*, May 2009, pp. 458–463.
[13] J. L. Guardado, S. G. Maximov, E. Melgoza, J. L. Naredo, and P. Moreno, ''An improved arc model before current zero based on the combined Mayr and Cassie arc models,'' *IEEE Trans. Power Del.*, vol. 20, no. 1, pp. 138–142, Jan. 2005.
[14] S. Golestani and H. Samet, ''Generalised Cassie–Mayr electric arc furnace models,'' *IET Gener., Transmiss. Distrib.*, vol. 10, no. 13, pp. 3364–3373, Oct. 2016.
[15] F. Amato, M. Mattei, and A. Pironti, ''Robust control of a power supply system for an arc heater,'' in *Proc. IEEE Int. Conf. Control Appl., IEEE Int. Conf. Control Appl. Held Together With IEEE Int. Symp. Intell. Control, IEEE Int. Symp. Comput.-Aided Contro*, Sep./Nov. 1996, pp. 882–887.
[16] W. Guo, F. Lin, X. You, and T. Zheng, ''Nonlinear control of power supply for an arc heater,'' in *Proc. 1st IEEE Conf. Ind. Electron. Appl.*, May 2006, pp. 1–4.
[17] W. Guo, F. Lin, X. You, and T. Q. Zheng, ''Analysis of nonlinear control for high power arc heater utilizing thyristor converter,'' in *Proc. 37th IEEE Power Electron. Spec. Conf.*, Jun. 2006, pp. 1–5.
[18] G. Ambrosino, G. Celentano, M. Mattei, and L. Verde, ''A model based control system for the scirocco plasma wind tunnel,'' in *Proc. 8th AIAA Int. Space Planes Hypersonic Syst. Technol. Conf.*, Apr. 1998, p. 1639.
[19] J. D. Cobine, *Gaseous Conductors: Theory and Engineering Applications*. New York, NY, USA: Dover, 1958.
[20] A. Adebowle and S. Ferik, ''Modeling and identification of nonlinear systems,'' *IEEE Trans. Syst., Man, Cybern.*, vol. 47, no. 7, p. 1149, Jul. 2017.
[21] V. V. Terzija and H.-J. Koglin, ''New approach to arc resistance calculation,'' in *Proc. IEEE Power Eng. Soc. Winter Meeting*, Jan./Feb. 2001, pp. 781–787.
[22] H. Ayrton, *The Electric Arc*. London, U.K.: Printing and Publishing Co., 1902.
[23] H. K. Khalil, *Nonlinear Control*. London, U.K.: Pearson, 2015.
[24] A. R. Van and C. Warrington, ''Reactance relays negligibly affected by arc impedance,'' *Elect. World*, vol. 98, pp. 502–505, Sep. 1931.
[25] R. C. Dorf and R. H. Bishop, *Modern Control Systems*. Essex, U.K.: Pearson, 2011.
[26] P. H. Schavemaker and L. van der Slui, ''An improved Mayr-type arc model based on current-zero measurements [circuit breakers],'' *IEEE Trans. Power Del.*, vol. 15, no. 2, pp. 580–584, Apr. 2000.
[27] P. S. Kireev and S. V. Sarry, ''Mathematical and physical modeling of arc transient resistance for relay protection operating estimation,'' in *Proc. 2nd Int. Conf. Ind. Eng., Appl. Manuf. (ICIEAM)*, 2016, pp. 1–4, doi: 10.1109/icieam.2016.7911472.
[28] B. P. Singh, ''Mathematical modeling of electronic devices and circuits,'' *AIP Conf.*, vol. 1324, p. 65, Dec. 2010, doi: 10.1063/1.3526268.
[29] T. Papallo, ''Arc flash calculations using a physics-based circuit model,'' *IEEE Trans. Ind. Appl.*, vol. 48, no. 4, pp. 1230–1236, Jul. 2012, doi: 10.1109/tia.2012.2199272.

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