IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received September 21, 2021, accepted October 7, 2021, date of publication October 13, 2021, date of current version October 26, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3119663

Urban Traffic Light Control Considering Capacity Difference Between Public Bus and Private Vehicles

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This work was supported in part by the Science and Technology Development Fund (FDCT), Macau, under Grant 0017/2019/A1 and Grant 0015/2020/AMJ; and in part by the National Natural Science Foundation of China under Grant 62173356.

ABSTRACT Due to the rapid growth of transportation demand for economic development, the importance of effective urban traffic signal control can never be underestimated, and the severe space saturation greatly limit the operations of a transportation system. In this paper, public buses and other vehicles are treated as different traffic flows. We address the urban traffic signal control problem in a scheduling framework by considering that the capacity of a public bus is much greater than that of other vehicles. The dynamics of an urban traffic network controlled by traffic lights are described by a novel model, which is formed by inserting mixed logical constraints into a cell transmission flow dynamic model. It includes the nonlinear relationship among the current link volume from the upstream, the current remaining link capacity from the downstream, and the state of the traffic lights. With the objective of minimizing the total delay time in a traffic network for people in the private vehicles and buses, we obtain a mixed integer linear programming formulation, which is built from the traffic signal control problem. We further analyze the influence of the number of passengers on buses and the time interval. The efficiency of the proposed method is verified by making comparisons between the models with and without considering the public transport regular. When the average number of passengers on a bus increases from 10 to 60, the degree of optimization ranges from 17% to 51% for the two-stage model and from 4% to 26% for the four-stage model. Finally, we consider the case without bus lanes and show the importance of dedicated bus lanes.

INDEX TERMS Public transportation system, urban traffic signal scheduling, mixed integer linear programming.

I. INTRODUCTION

A. BACKGROUND

With the increasing demand for transportation due to the fast economic development, the execution of effective urban traffic signal control becomes more and more important. However, the increasingly severe space saturation greatly restricts the performance of a transportation system. With the rapid increase in population and vehicles, traffic congestion in urban areas is getting worse and worse. The reduction of traffic congestion can improve traffic safety and efficiency and reduce environmental pollution as well. Therefore, how to effectively arrange traffic lights becomes more and more important to big cities. Reasonable traffic signal scheduling is one of the ways to reduce delay time for urban traffic network systems [1]–[5].

A set of road links forms an urban traffic network and the road links are connected with each other by intersections. A number of approaches and crossing areas form an intersection [6], [7]. There is at least one lane in an approach. At each approach, the queue is unique and independent. An intersection can be crossed safely by two compatible streams simultaneously and cannot be crossed by antagonistic streams. By traditional traffic signal control, one repetition of the basic series of stages at an intersection is called a signal cycle. At each stage, a predefined compatible traffic streams cross the intersection simultaneously in the instruction of simultaneous traffic light signals. The cycle time is the duration

The associate editor coordinating the review of this manuscript and approving it for publication was Seyedali Mirjalili.

of a cycle [8], [9]. For safety, in order to avoid interference between antagonistic streams, among the consecutive stages, it is necessary to insert a constant lost (or inter-green) time with a few seconds. For each traffic light, the split is the ratio of the green-light time and the red-light time within one cycle, and the offset is the delay between the starting time of green periods of two neighboring traffic lights along the same traffic route [10], [11].

In China, by the National Middle and Long-term Science and Technology Development Program, it proposes preferential policies for developing public transit. More and more cities endeavor to explore effective approaches to develop public transit, including some small-medium-scale cities. There is a sustained growth of traffic volume in big cities, while public transit in small-medium-scale cities develops at a quite slow rate [12], [13].

B. LITERATURE REVIEW

For the traffic signal control, four basic strategies are considered, e.g., the combination of fixed time strategies or traffic responsive strategies, and isolated strategies or coordinated strategies. For isolated fixed-time strategies, they are applicable to under-saturated traffic conditions only. Stage-based strategies belong to this class, such as the work proposed in [8], [9]. This kind of strategies determines the optimal splits and cycle time to minimize the total delay or maximize the intersection capacity utilization. Phase-based strategies belong to this class too, such as the work in [10]. This kind of strategies determines not only the optimal split and cycle time but also the optimal staging, which may be an important feature of complex intersections. For isolated traffic-responsive strategies, the work in [6] and [11] are the representative ones. These strategies use real-time measurements. Some more or less sophisticated vehicle-actuation logic is executed by using inductive loop detectors. In practice, the widely used strategies are the fixed-time coordinated ones. Typical work in this direction includes the one in [7], which considers a two-way arterial with several signals (intersections) and specifies the corresponding offsets to maximize the number of vehicles that can travel within a given speed range without stopping at any signal (green light). The study in [14] utilizes platoon dispersion (i.e., a dynamic first-order time-delay system) to model flow progression along a link and uses heuristic optimization algorithms such as hill-climbing to determine the splits, offsets, and cycle time, and optimize the corresponding performance index such as the total number of vehicle stops.

In practice, the traffic situations are highly dynamic, and the fixed-time strategies become ineffective. Hence, there has been a widespread concern on coordinated traffic-responsive strategies. For coordinated traffic-responsive strategies, the study in [15] is a representative one. The authors in [15] update the splits, offsets, and cycle time based on actual traffic measurements, which can be considered as a traffic-responsive version. Model-based optimization methods are used in coordinated traffic-responsive strategies in [16]–[19]. They do not consider explicitly the splits, offsets, or cycles. Starting from the current time and the currently applied stage, they calculate the optimal values of the next few switching time in real-time according to the realistic traffic models. Generally, over a future time horizon, the models have 2-5 seconds as the sampling time. Control-theoretical-based approaches are also used in coordinated traffic-responsive strategies. For example, the work in [20], [21] uses the store-and-forward modeling for traffic networks with a linear quadratic regulator optimal control solution.

Based on Daganzo's cell transmission models [22], [23], the study in [24] addresses the urban traffic signal control in a scheduling framework with the objective of minimizing the total delay time in the traffic network. This model describes each outgoing flow rate in the way of a nonlinear mixed logical switching function, which is the novelty point. This function describes the density of the source link, the density and capacity of the destination link, and the response to the past situation of the traffic light from the driver's potential psychology. The adoption of this dynamic flow model makes it applicable to situations including the under-saturated and over-saturated situations. In this model, the traditional concepts of cycles, offsets, and splits are not adopted. In this way, the urban traffic light control problem becomes a class of model-based optimization problems, where each traffic light is assigned with a green-light period in a real-time way by the network controller. In the recent work for this problem [25] develops a harmony search with the ensemble of a local search operators for solving the urban traffic light control problem to minimize the total network-wise delay time. The study in [26] describes a heterogeneous traffic system with signalized and non-signalized intersections. The work in [27] addresses a bi-objective urban traffic light scheduling problem to minimize both the total network-wise delay time for the vehicles and total delay time of all pedestrians. The work in [28] presents a bi-objective urban traffic light scheduling problem to minimize the total network-wise delay time for the vehicles and optimize the control for drivers' unhappy. The study in [29] addresses the traffic signal scheduling problem with a heterogeneous traffic network of signalized and non-signalized intersections in order to minimize the total network-wise delay time of all vehicles within a given finite-time window by using meta-heuristics.

The aforementioned studies do not consider the factors of the public transport regular. The authors in [30] evaluate the performance of bus routes within a public transportation system, concluding that the operation in the off-peak period is better than that in the peak period, and the average running time has the greatest impact on performance. For the buses' running time, they are affected by traffic signal control, and reasonable traffic signal control can effectively reduce running time. The facility-design-based measures provide priority to buses mainly through appropriate arrangements of the available infrastructure. The most common means in this category is the use of dedicated bus lanes [31], [32]. As for priority strategies of public transport, it includes fixed-time strategies such as the work in [33], [34] and real-time strategies such as the work in [35], [36]. The real-time strategies include rule-based strategies such as the work in [35] and optimization-based strategies such as the work in [36]. The main difficulty faced by the rule-based strategies comes from the requirements for responding to multiple requests, which can be solved by optimization-based strategies. Meanwhile, the computational cost may be significantly increased. The work in [36] develops a real-time, traffic-responsive signal control strategy in order to minimize the total person delay at traffic signals in the under-saturated traffic conditions.

The work in [35] considers the bus headway for the bus priority at traffic signals. The study in [36] considers different vehicle arrivals, including deterministic and stochastic, for reducing the total person delay at one intersection. They all consider public transport from a micro perspective. In this work, for a large traffic network, we analyze this problem from a macro perspective.

Recently, the work in [37] presents a bi-objective algorithm to minimize both passenger delay and the deviation from bus schedule, where an example with two adjacent signalized intersections, a bus stop, and five bus routes in Nanjing is tested. The authors in [38] maximize bus service reliability, including minimizing the bus schedule discrepancies and the total squared headway deviations by setting optimal signal timings. A case problem with five intersections and 20 buses in London is presented for numerical experiments. The study in [39] describes a signal timing model for intersections with a center transit lane and bus stops considering private vehicles, buses, crossing pedestrians, and passengers, where an intersection in Shanghai is used as the example to verify the model. The work in [40] sets up a dynamic headway control to minimize the weight addition of deviation of bus headways and ratio of signal cycle length scaling by considering a high-frequency route with bus lane. The paper [41] develops a bi-level programming model to minimize average passenger delay at intersections and vehicle delay in lanes simultaneously. An example with one intersection is analyzed. The authors in [42] propose a traffic signal optimization model to minimize total travel delay by considering the interaction of traffic flows between the intersections and upstream curbside bus stops, and one intersection is analyzed to verify the proposed model. The work in [43] proposes a mixed-integer nonlinear problem model to minimize the travel time of Bus Rapid Transit (BRT) considering on-time rate of the bus schedule and analyze the example of Fengxian-Pudong BRT line in Shanghai. The paper [44] presents an adaptive signal controller for managing traffic delays and urban bus service reliability with fully adaptable acyclic timing plans. One example in London with five intersections is analyzed.

C. THE ADDRESSED PROBLEM

In this paper, we cope with the signal control problem based on a novel traffic flow model that is first applied in [24], where the authors compare the proposed scheduling strategy for fixed-time scheduling and the results show that the proposed strategy can significantly reduce delay time.

Since the public transport regular plays an important role in economic development and every one's daily life, we formulate the problem of urban traffic light control considering the bus transportation system in order to reduce the total waiting time for the people in all vehicles including buses over a given finite horizon. Then, in each intersection, we adopt a real-time scheduling strategy to dynamically determine the RED traffic signal or Green traffic signal for each traffic light. Meanwhile, we also consider the situation without dedicated bus lanes. It is important to determine the traffic light signals for intersections according to real-time traffic situations. Hence, the urban traffic light control problem becomes a type of optimization-based problems that are suitable to coordinated traffic-responsive strategies. We convert the traffic light control problem into a mixed integer linear program formulation. The techniques that we use are introduced in [45]. Compared with a mate-heuristic algorithm obtained by using evolutionary computation and reinforcement learning, this method can be realized by several standard optimization tools to get a global optimal solution, and we use the standard optimization tool LINGO to do so.

This study contributes to the performance of urban traffic light control in the following ways: (1) the factors of the public transport regular are taken into consideration and urban traffic signal control models are built based on Daganzo's cell transmission so as to achieve a better result, (2) by converting the traffic signal control problem, we get a mixed integer linear programming model, and (3) we show the importance of dedicated bus lanes for the improvement of the urban transportation.

In this article, the target is to optimize the urban traffic light control in considering the public transport regular. The rest of this paper is organized as follows. Section II develops the model for the traffic light scheduling problem considering the regular public transit system based on a cell transmission model. Meanwhile, the case without dedicated bus lanes is considered. Then, Section III describes how to convert the traffic light scheduling problem into a mixed integer linear programming problem. With the obtained models, the results and analysis are given in Section IV. Finally, Section V concludes this paper.

II. PROBLEM FORMULATION

A set of intersections and links form a traffic network. In Fig. 1, we describe a simple unidirectional traffic network. In this network, there are two antagonistic traffic flows at each intersection. Based on the cell transmission model, a discrete-time model is proposed. In this model, we consider the regular public transit system. The assumptions for the system are given as follows.

- The entrance and exit models for the traffic network are determined.
- The delay for each vehicle in the traffic network is caused by the waiting time only due to the traffic signals, and the vehicles will leave the network.

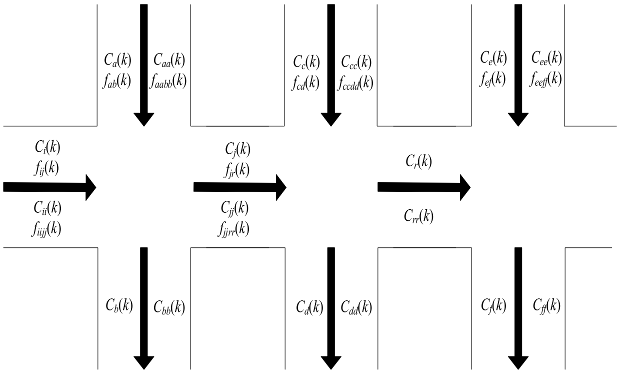

FIGURE 1. **A simple unidirectional traffic network.**

• The link turning ratios at each link in the traffic network are determined.

From Part A to Part D, we consider the situation with dedicated bus lanes. Then we have the following assumption.

• There is a dedicated bus lane for buses only.

In Part E, we consider the situation without dedicated bus lanes.

In order to better illustrate the establishment of the model, we present the notation used in this model as follows.

$C_i(k)$: In the interval k, the number of vehicles except buses in the link i.

$C_{ii}(k)$: In the interval k, the number of buses in the link i.

$\hat{C}_j$: The capacity for vehicles except buses in the link j.

$\hat{C}_{jj}$: The capacity for buses in the link j.

$f_{ij}(k)$: In the interval k, the number of vehicles except buses exiting from links i to j.

$f_{iijj}(k)$: In the interval k, the number of buses exiting from links i to j.

A: The number of average people in vehicles except for those in buses.

B: The number of people in average in buses.

Δ: The sampling interval.

M: The number of sampling intervals in one prediction horizon.

L: The set of all one-way links.

J: The set of all intersections.

ω: A stage in an intersection. At each stage, the predefined compatible traffic streams cross the intersection simultaneously according to the simultaneous traffic light signals.

Ω_a: The set of stages in intersection a, $a \in J$.

$F_a \subseteq L \times L$: In the intersection a, the set of all streams such as $(i, j) \in F_a$ means that there exists a traffic stream except buses from links i to j via the junction a and $(ii, jj) \in F_a$ means that there exists a traffic stream of buses from links i to j via the junction a.

h_a: $\Omega \rightarrow 2^{F_a}$, The association of each stage to relevant compatible streams.

$\theta_\omega(k)$: In the interval k, the traffic situation of stage ω.

$\mathbf{N}$: The set of natural numbers.

L_i: The length of the link i.

v_i^*: The maximum speed allowed on the link i.

$d_j(k)$: In the interval k, the number of entering vehicles except buses of the link j.

$d_{jj}(k)$: In the interval k, the number of entering buses of the link j.

$s_j(k)$: In the interval k, the number of exit vehicles except buses from the link j.

$s_{jj}(k)$: In the interval k, the number of exit buses from the link j.

$\lambda_{ij}(k)$: In the interval k, the turning ratio of vehicles except buses from the link i towards the link j.

$\lambda_{iijj}(k)$: In the interval k, the turning ratio of buses from the link i towards the link j.

l_{ij}^r: The r-th speed profile of vehicles except buses from the link i towards the link j.

l_{iijj}^r: The r-th speed profile of buses from the link i towards the link j.

$l_{ij}(k)$: In the interval k, the speed profile of vehicles except buses from the link i towards the link j.

$l_{iijj}(k)$: In the interval k, the speed profile of buses from the link i towards the link j.

ε: A positive real number that is small enough.

In the proposed traffic flow model, we consider three types of constraints as follows. They are the stage status constraints, the link volume dynamic constraints, and the flow dynamic constraints. Minimizing the delay time for the people in the vehicles in traffic network is the objective. Next, we describe the three types of constraints and the objective function to be optimized.

A. STAGE STATUS CONSTRAINTS

At each sample interval k, there exists only one action stage at intersection a. The constraints are as follows. For vehicles except buses,

$$(\forall \omega \in \Omega_a)\theta_\omega(k) = 0 \Rightarrow (\forall (i, j) \in h_a(\omega))f_{ij}(k) = 0 \quad (1a)$$

For buses,

$$(\forall \omega \in \Omega_a)\theta_\omega(k) = 0 \Rightarrow (\forall (ii, jj) \in h_a(\omega))f_{iijj}(k) = 0 \quad (1b)$$

For the traffic light,

$$\sum\nolimits_{\omega \in \Omega_a} \theta_\omega(k) = 1 \quad (1c)$$

$$(\forall \omega \in \Omega_a)(\forall k \in \mathbf{N})\theta_\omega(k) \in \{0, 1\} \quad (1d)$$

where $\theta_\omega(k) = 0$ and $\theta_\omega(k) = 1$ represent that the RED traffic light and Green traffic light at stage ω in the interval k, respectively.

Constraints (1a) and (1b) indicate that if the RED traffic light is on at stage ω in the interval k, then all the related flow rates must be zero. Constraints (1c) and (1d) state that there is only one GREEN traffic stage at an intersection.

B. LINK VOLUME DYNAMIC CONSTRAINTS

According to conservation principle, each link $j \in L$ for vehicles except buses, there are volume dynamic Constraints (2a) - (2d) as follows.

$$C_j(k + 1) = C_j(k) + (d_j(k) - s_j(k))\Delta \quad (2a)$$

$$(\forall k \in \mathrm{N}) C_j(k) \in \mathrm{N} \tag{2b}$$

$$d_j(k) = \sum\nolimits_{i \in L:(i,j) \in \cup_{a \in J} F_a} f_{ij}(k) \tag{2c}$$

$$s_j(k) = \sum\nolimits_{i \in L:(j,i) \in \cup_{a \in J} F_a} f_{ji}(k) \tag{2d}$$

Each link $j \in L$ for buses, there are volume dynamic Constraints (2e) - (2h) as follows.

$$C_{jj}(k+1) = C_{jj}(k) + (d_{jj}(k) - s_{jj}(k))\Delta \tag{2e}$$

$$(\forall k \in \mathrm{N}) C_{jj}(k) \in \mathrm{N} \tag{2f}$$

$$d_{jj}(k) = \sum\nolimits_{i \in L:(ii,jj) \in \cup_{a \in J} F_a} f_{iijj}(k) \tag{2g}$$

$$s_{jj}(k) = \sum\nolimits_{i \in L:(jj,ii) \in \cup_{a \in J} F_a} f_{jjii}(k) \tag{2h}$$

From Fig. 1, we can get that $d_j(k) = f_{ij}(k)$, $s_j(k) = f_{jr}(k)$, $d_{jj}(k) = f_{iijj}(k)$, and $s_{jj}(k) = f_{jjrr}(k)$.

C. FLOW DYNAMIC CONSTRAINTS

According to the work in [24], for vehicles except buses, at each stage $\omega \in \Omega_a$ and each stream $(i, j) \in h_a(\omega)$, the exit flow $f_{ij}(k)$ is determined by the following factors: (1) the current link volume $C_i(k)$ the upstream, (2) the current remaining link capacity $\hat{C}_j - C_j(k)$ from the downstream, and (3) the past $r+1$ time intervals $\theta_\omega(k-r), \ldots, \theta_\omega(k)$ of the traffic light. Then, we have Constraint (3) as follows.

$$f_{ij}(k) = g_{ij}(C_i(k), \hat{C}_j - C_j(k), \theta_\omega(k-r), \ldots, \theta_\omega(k)) \tag{3}$$

Constraint (3) is a nonlinear function. If the GREEN traffic signal stay for a long time, then the drivers intend to keep a high speed as long as the downstream link can receive the flow with sufficient capacity. Meanwhile, it embodies the delay caused by traffic state transition. To present the relative constraints, we define $g_{ij}(\bullet)$ as follows. Assume that there are $r+1$ speed categories that are monotonically non-increasing. Let them be $l_{ij}^0 \geq \cdots \geq l_{ij}^r > 0$, presenting the speed ranges from high to low. We determine the actual speed category $l_{ij}(k)$ in Constraints (4a) - (4e).

$$l_{ij}(k) = \sum\nolimits_{p=0}^{r} \delta_{ij}^p(k) l_{ij}^p \tag{4a}$$

$$\sum\nolimits_{p=0}^{r} \delta_{ij}^p(k) - \theta_\omega(k) = 0 \tag{4b}$$

$$(\forall q : 0 \leq q \leq r-1)(1 - \theta_\omega(k-q-1)) \prod\nolimits_{p=0}^{q} \theta_\omega(k-p) = 1 \Leftrightarrow \delta_{ij}^{r-q}(k) = 1 \tag{4c}$$

$$\prod\nolimits_{p=0}^{r} \theta_\omega(k-p) = 1 \Leftrightarrow \delta_{ij}^0(k) = 1 \tag{4d}$$

$$(\forall p : 0 \leq p \leq r) \delta_{ij}^p(k) \in \{0, 1\} \tag{4e}$$

Constraint (4b) indicates that if $\theta_\omega(k) = 0$, we have $\sum_{p=0}^{r} \delta_{ij}^p(k) = 0$. Then, according to Constraint (4e), $\delta_{ij}^p(k) = 0$ implies $l_{ij}(k) = 0$, i.e., in the interval k at stage ω, when the traffic light is RED, the related speed must be zero. Similarly, if $\theta_\omega(k) = 1$, we have $\sum_{p=0}^{r} \delta_{ij}^p(k) = 1$, implying that there exists only one p such that $\delta_{ij}^p(k) = 1$, i.e., in the interval k at stage ω, when the traffic light is GREEN, the related speed category $l_{ij}(k)$ can select one corresponding value only. Constraints (4c) and (4d) state that the number of continuously green light intervals from the past to present decides the actual speed category. When the number of continuously green intervals is large, the speed category is high. Especially, when there is a traffic state transition, from Constraint (4c), we have $q = 0$, resulting in $\delta_{ij}^r(k) = 1$. According to Constraint (4a), $l_{ij}(k) = l_{ij}^r$. Since l_{ij}^r is the lowest speed category, it chooses the lowest speed category when there is a traffic state transition.

In order to better illustrate the above constraints, we use an example to explain Constraints (4a) - (4e). In the speed categories, we assume that there are six speed levels (or $r = 5$): $l_{ij}^0, l_{ij}^1, l_{ij}^2, l_{ij}^3, l_{ij}^4, l_{ij}^5$. At an intersection, if the continuously green light interval is 4, $p = r-4+1 = 5-4+1 = 2$. Then, the speed category $\delta_{ij}^2(k) = 1$, and the other speed categories are 0, or $\delta_{ij}^0(k) = 0$, $\delta_{ij}^1(k) = 0$, $\delta_{ij}^3(k) = 0$, $\delta_{ij}^4(k) = 0$ and $\delta_{ij}^5(k) = 0$. Thus, in this intersection, the speed category is selected as $l_{ij}(k) = \sum_{p=0}^{5} \delta_{ij}^p(k) l_{ij}^p = l_{ij}^2$. Similarly, in another intersection, if the continuously green light interval is 1, we have $p = r-1+1 = 5-1+1 = 5$. Then, the speed category $\delta_{ij}^5(k) = 1$, and the other speed category is 0. Then, in this intersection, the speed category is selected as $l_{ij}(k) = l_{ij}^5$. In this case, the chosen speed category is the lowest one. It means that at a traffic state transition, it chooses the lowest speed category, representing a delay caused by a traffic state transition. When the continuously green light interval is large, the selected speed category is high. When the continuously green light number is equal to or greater than 6, according to Constraint (4d), we have $\delta_{ij}^0(k) = 1$, resulting in $l_{ij}(k) = l_{ij}^0$, and the speed category is maintaining the highest one.

Similarly, for buses, the actual speed category $l_{iijj}(k)$ is given by Constraints (4f) - (4j).

$$l_{iijj}(k) = \sum\nolimits_{p=0}^{r} \delta_{iijj}^p(k) l_{iijj}^p \tag{4f}$$

$$\sum\nolimits_{p=0}^{r} \delta_{iijj}^p(k) - \theta_\omega(k) = 0 \tag{4g}$$

$$(\forall q : 0 \leq q \leq r-1)(1 - \theta_\omega(k-q-1)) \prod\nolimits_{p=0}^{q} \theta_\omega(k-p) = 1 \Leftrightarrow \delta_{iijj}^{r-q}(k) = 1 \tag{4h}$$

$$\prod\nolimits_{p=0}^{r} \theta_\omega(k-p) = 1 \Leftrightarrow \delta_{iijj}^0(k) = 1 \tag{4i}$$

$$(\forall p : 0 \leq p \leq r) \delta_{iijj}^p(k) \in \{0, 1\} \tag{4j}$$

After $l_{ij}(k)$ and $l_{iijj}(k)$ are determined, the link flow rates $f_{ij}(k)$ and $f_{iijj}(k)$ are given as follows.

$$f_{ij}(k)\Delta = \left\lfloor \min\{\lambda_{ij}(k) C_j(k), l_{ij}(k)(\hat{C}_j - C_j(k))\} \right\rfloor \tag{5a}$$

$$f_{iijj}(k)\Delta = \left\lfloor \min\{\lambda_{iijj}(k) C_{jj}(k), l_{iijj}(k)(\hat{C}_{jj} - C_{jj}(k))\} \right\rfloor \tag{5b}$$

$$f_{ij}(k)\Delta \in \mathrm{N} \tag{5c}$$

$$f_{iijj}(k)\Delta \in \mathrm{N} \tag{5d}$$

$$\sum\nolimits_{i\in L:(i,j)\in\cup_{a\in J}F_a}\lambda_{ij}(k)=1 \tag{5e}$$

$$\sum\nolimits_{i\in L:(ii,jj)\in\cup_{a\in J}F_a}\lambda_{iijj}(k)=1 \tag{5f}$$

where $\lfloor\bullet\rfloor$ denotes the floor of the input argument, i.e., the largest integer smaller than the input argument, $\lambda_{ij}(k)$ and $\lambda_{iijj}(k)$ are assumed to be known.

From Constraints (5e) and (5f), each vehicle moves from the upstream link to the downstream link. In a one-time interval, $f_{ij}(k)\Delta$ is the number of vehicles except buses moving from links i to j and $f_{iijj}(k)\Delta$ is the number of buses moving from links i to j. According to Constraints (5a)–(5d), $f_{ij}(k)\Delta$ and $f_{iijj}(k)\Delta$ are the largest integers that are not greater than the volume $\lambda_{ij}(k)C_j(k)$ and $\lambda_{iijj}(k)C_{jj}(k)$ from the upstream link i, respectively. Meanwhile, $f_{ij}(k)\Delta$ and $f_{iijj}(k)\Delta$ are the largest integers that are not greater than the remaining capacity $\hat{C}_j - C_j(k)$ and $\hat{C}_{jj} - C_{jj}(k)$ to the downstream link j weighted by the speed categories $l_{ij}(k)$ and $l_{iijj}(k)$, respectively.

For the above constraints, Constraints (1a), (2a) - (2d), (4a) - (4e), (5a), (5c) and (5e) are for vehicles except buses; while Constraints (1b), (2e) - (2h), (4f) - (4j), (5b), (5d) and (5f) are for buses; and Constraints (1c) and (1d) are for vehicles including buses.

D. OBJECTIVE FUNCTION

The total time delay within M time intervals for vehicles except buses in a traffic network can be calculated as.

$$\sum\nolimits_{i\in L}\sum\nolimits_{k=1}^{M}C_i(k)\left(1-\frac{\bar{v}_i(k)}{v_i^*}\right) \tag{6a}$$

$$\bar{v}_i(k)=\frac{s_i(k)}{\frac{C_i(k)}{L_i}} \tag{6b}$$

where $\frac{C_i(k)}{L_i}$ is the average link density.

According to Constraints (2d), (6a), and (6b), the total time delay within M time intervals for vehicles except buses in the traffic network can be calculated as.

$$\sum\nolimits_{i\in L}\sum\nolimits_{k=1}^{M}\left(C_i(k)-\frac{L_i}{v_i^*}\sum\nolimits_{j\in L:(i,j)\in\cup_{a\in J}F_a}f_{ij}(k)\right)\Delta \tag{6c}$$

Similarly, the total time delay within M time intervals for the buses in a traffic network can be calculated as.

$$\sum\nolimits_{i\in L}\sum\nolimits_{k=1}^{M}\left(C_{ii}(k)-\frac{L_i}{v_i^*}\sum\nolimits_{j\in L:(ii,jj)\in\cup_{a\in J}F_a}f_{iijj}(k)\right)\Delta \tag{6d}$$

In this work, we consider the bus transit system. Let the total delay within M time intervals in a traffic network for the people in the vehicles be the objective function. It can be given as

$$\begin{aligned}\min\sum\nolimits_{i\in L}\sum\nolimits_{k=1}^{M}A\left(C_i(k)-\frac{L_i}{v_i^*}\sum\nolimits_{j\in L:(i,j)\in\cup_{a\in J}F_a}f_{ij}(k)\right)\Delta\\ +\sum\nolimits_{i\in L}\sum\nolimits_{k=1}^{M}B\left(C_{ii}(k)-\frac{L_i}{v_i^*}\sum\nolimits_{j\in L:(ii,jj)\in\cup_{a\in J}F_a}f_{iijj}(k)\right)\Delta\end{aligned} \tag{6e}$$

For the above objective function (6e), we can see that the function consists of two parts. The first part is the total delay time within M time intervals in a traffic network for the people in private vehicles, while the second part is the total delay time within M time intervals in the traffic network for the people in buses.

E. THE MODEL FOR THE CASE WITHOUT DEDICATED BUS LANES

We notice that there is no bus lane in some cities. Without dedicated bus lane, the model presented in Part A to Part D in this section should be modified. The buses and private vehicles share the existing lanes, and they are randomly distributed on every road. The buses are converted into private vehicles in the same stage. Then, some parameters are changed. In the above model, we use the delay time for the people as the objective function. Then, in link i, the modified number of buses $C'_{ii}(k)$ can be represented as

$$C'_{ii}(k)=\frac{B}{A}C_{ii}(k) \tag{7a}$$

$$(\forall k\in \mathrm{N})C'_{ii}(k)\in \mathrm{N} \tag{7b}$$

Since private vehicles and public buses share the same traffic lanes, in link i, the total number of vehicles including buses $C'_i(k)$ can be represented as

$$C'_i(k)=C_i(k)+C'_{ii}(k) \tag{7c}$$

In link i, the modified capacity $\hat{C}'_i(k)$ can be given as

$$\hat{C}'_i(k)=\hat{C}_i(k)+C'_{ii}(k) \tag{7d}$$

Then, in the next time interval $K+1$,the modified capacity $\hat{C}'_j(k+1)$ can be represented as Formulas (7e) and (7f).

$$\begin{aligned}\hat{C}'_j(k+1)=\hat{C}'_j(k)+d_j(k)\times\frac{C'_{ii}(k)}{C'_{ii}(k)+C_i(k)}-s_j(k)\\ \times\frac{C'_{jj}(k)}{C'_{jj}(k)+C_j(k)}\end{aligned} \tag{7e}$$

$$(\forall k\in \mathrm{N})C'_j(k+1)\in \mathrm{N} \tag{7f}$$

In the next time interval $K+1$, the modified number of buses $C'_{jj}(k+1)$ can be calculated as

$$\begin{aligned}\mathrm{C}'_{jj}(k+1)=\mathrm{C}'_{jj}(k)+d_j(k)\times\frac{C'_{ii}(k)}{C'_{ii}(k)+C_i(k)}-s_j(k)\\ \times\frac{C'_{jj}(k)}{C'_{jj}(k)+C_j(k)}\end{aligned} \tag{7g}$$

$$(\forall k\in \mathrm{N})C'_{jj}(k+1)\in \mathrm{N} \tag{7h}$$

Similarly, in the next time interval $K+1$,the number of private vehicles $C_j(k+1)$ can be calculated as (7i) and (7j).

$$\begin{aligned}\mathrm{C}_j(k+1)=C_j(k)+d_j(k)\times\frac{C_i(k)}{C'_{ii}(k)+C_i(k)}-s_j(k)\\ \times\frac{C_j(k)}{C'_{jj}(k)+C_j(k)}\end{aligned} \tag{7i}$$

$$(\forall k \in \mathrm{N})C_j(k+1) \in \mathrm{N} \tag{7j}$$

For buses, the modified model ensures that the number of people passing an intersection per unit time is consistent with the proposed model. Then, the modified r-th speed profile of buses from the link i towards the link j is as

$$l_{iijj}^{r'} = l_{iijj}^{r} \times \frac{B}{A} \tag{7k}$$

The modified r-th speed profile of vehicles including buses from the link i towards the link j is as

$$l_{ij}^{r'} = \frac{C'_{ii}(k) \times l_{iijj}^{r'} + C_i(k) \times l_{ij}^{r}}{C'_{ii}(k) + C_i(k)} \tag{7l}$$

Finally, the objective function for the modified model can be given as

$$\min \sum_{i\in L} \sum_{k=1}^{M} A \left(C_i(k) + \frac{B}{A} C_{ii}(k) - \frac{L_i}{v_i^*} \sum_{j\in L:(i,j)\in \cup_{a\in J} F_a} f_{ij}(k) \right) \Delta \tag{8}$$

III. CONVERSION OF MIXED LOGICAL CONSTRAINTS

In last Section, for the situation of considering the dedicated bus lanes, there is a set of constraints including the stage status Constraints (1a) - (1d), link volume dynamic Constraints (2a) - (2h), the actual speed category Constraints (4a) - (4j), and link flow Constraints (5a) - (5f). Among these constraints, some of them are mixed logic constraints or non-linear constraints. In order to efficiently solve the problem, we convert them into mixed integer linear constraints as follows.

We first convert the constraints for vehicles except buses. Constraint (1a) is converted into Constraint (9) as follows.

$$(\forall \omega \in \Omega_a)\theta_\omega(k) = 0 \Rightarrow (\forall (i,j) \in h_a(\omega)) f_{ij}(k) = 0 \tag{1a}$$

$$\Downarrow$$

$$(\forall \omega \in \Omega_a)(\forall (i,j) \in h_a(\omega)) f_{ij}(k) \le M_1 \theta_\omega(k) \tag{9}$$

where $M_1 > \max_{i\in L} \hat{C}_i$, making M_1 big enough.

M_1 is the first value we introduce, which is as big as possible. It ensures the conversion from (1a) to (9) correct.

Proposition 1: For the scheduling formulation of urban network traffic signal, Constraint (1a) is equivalent to Constraint (9).

Proof: When $\theta_\omega(k) = 0$, according to Constraint (9), $f_{ij}(k) \le 0$. With $f_{ij}(k) \ge 0$, we have $f_{ij}(k) = 0$. When $\theta_\omega(k) = 1$, $f_{ij}(k) \le M_1$. According to Constraint (5a), it is trivially true. ■

Constraint (4c) is converted into Constraints (10a) - (10c) as follows.

$$(\forall q : 0 \le q \le r-1)(1-\theta_\omega(k-q-1)) \prod_{p=0}^{q} \theta_\omega(k-p) = 1 \Leftrightarrow \delta_{ij}^{r-q}(k) = 1 \tag{4c}$$

$$\Downarrow$$

$$(\forall q : 0 \le q \le r-1) - (1-\theta_\omega(k-q-1)) + \delta_{ij}^{r-q}(k) \le 0 \tag{10a}$$

$$(\forall q : 0 \le q \le r-1)(\forall p : 0 \le p \le q) - \theta_\omega(k-p) + \delta_{ij}^{r-q}(k) \le 0 \tag{10b}$$

$$(\forall q : 0 \le q \le r-1)(1-\theta_\omega(k-q-1)) + \sum_{p=0}^{q} \theta_\omega(k-p) - \delta_{ij}^{r-q}(k) \le q+1 \tag{10c}$$

Proposition 2: For the scheduling formulation of urban network traffic signal, Constraint (4c) is equivalent to Constraint (10a) - (10c).

Proof: When $(\forall q : 0 \le q \le r-1)(1-\theta_\omega(k-q-1)) \prod_{p=0}^{q} \theta_\omega(k-p) = 1$, we have $(\forall 0 \le q \le r-1)\theta_\omega(k) = \theta_\omega(k-1) = \cdots = \theta_\omega(k-q+1) = \theta_\omega(k-q) = 1$, and $\theta_\omega(k-q-1) = 0$. Then, Constraints (10a) - (10c) can be transformed as

$$(\forall q : 0 \le q \le r-1)\delta_{ij}^{r-q}(k) \le 1 \tag{10d}$$

$$(\forall q : 0 \le q \le r-1)\delta_{ij}^{r-q}(k) \ge 1 \tag{10e}$$

Then, we can get $(\forall q : 0 \le q \le r-1)\delta_{ij}^{r-q}(k) = 1$. When $(\forall q : 0 \le q \le r-1)\delta_{ij}^{r-q}(k) = 1$, Constraints (10a) -(10c) can be transformed as

$$(\forall q : 0 \le q \le r-1)\theta_\omega(k-q-1) \le 0 \tag{10f}$$

$$(\forall q : 0 \le q \le r-1)(\forall p : 0 \le p \le q) - \theta_\omega(k-p) + 1 \le 0 \tag{10g}$$

$$(\forall q : 0 \le q \le r-1)(1-\theta_\omega(k-q-1)) + \sum_{p=0}^{q} \theta_\omega(k-p) \le q+2 \tag{10h}$$

It is obvious that Constraint (10h) is trivially true. From Constraints (10f) and (10g), we have $(\forall q : 0 \le q \le r-1)\theta_\omega(k-q-1) = 0$ and $(\forall q : 0 \le q \le r-1)(\forall p : 0 \le p \le q)\theta_\omega(k-p) = 1$, leading to $(\forall 0 \le q \le r-1)\theta_\omega(k) = \theta_\omega(k-1) = \cdots = \theta_\omega(k-q+1) = \theta_\omega(k-q) = 1$, and $\theta_\omega(k-q-1) = 0$, implying that $(\forall q : 0 \le q \le r-1)(1-\theta_\omega(k-q-1)) \prod_{p=0}^{q} \theta_\omega(k-p) = 1$ holds. ■

Constraint (4d) is converted into Constraints (11a) - (11b) as follows.

$$\prod_{p=0}^{r} \theta_\omega(k-p) = 1 \Leftrightarrow \delta_{ij}^{0}(k) = 1 \tag{4d}$$

$$\Downarrow$$

$$(\forall p : 0 \le p \le r) - \theta_\omega(k-p) + \delta_{ij}^{0}(k) \le 0 \tag{11a}$$

$$\sum_{p=0}^{r} \theta_\omega(k-p) - \delta_{ij}^{0}(k) \le r \tag{11b}$$

Proposition 3: For the scheduling formulation of urban network traffic signal, Constraint (4d) is equivalent to Constraints (11a) - (11b).

Proof: When $\prod_{p=0}^{r} \theta_\omega(k-p) = 1$, we can get $(\forall p : 0 \le p \le r)\theta_\omega(k-p) = 1$. Then, Constraints (11a) and (11b) can be transformed as

$$\delta_{ij}^{0}(k) \le 1 \tag{11c}$$

$$\delta_{ij}^{0}(k) \ge 1 \tag{11d}$$

Then, we have $\delta_{ij}^{0}(k) = 1$. When $\delta_{ij}^{0}(k) = 1$, Constraints (11a) and (11b) can be transformed as

$$(\forall p : 0 \le p \le r)\theta_\omega(k-p) \ge 1 \tag{11e}$$

$$\sum_{p=0}^{r}\theta_\omega(k-p)\leq r+1 \tag{11f}$$

It is clear that Constraint (11f) is trivially true. From Constraint (11e), we can get $(\forall p: 0\leq p\leq r)\theta_\omega(k-p)=1$, implying that $\prod_{p=0}^{r}\theta_\omega(k-p)=1$ holds. ■

Constraints (5a) and (5c) are converted into the mixed logical Constraints (5c) and (12a) - (12c) as

$$f_{ij}(k)\Delta=\left\lfloor \min\{\lambda_{ij}(k)C_j(k),\, l_{ij}(k)(\hat{C}_j-C_j(k))\}\right\rfloor \tag{5a}$$
$$f_{ij}(k)\Delta\in \mathrm{N} \tag{5c}$$
$$\Downarrow$$
$$f_{ij}(k)\Delta\in \mathrm{N} \tag{5c}$$
$$f_{ij}(k)\Delta\leq \lambda_{ij}(k)C_i(k) \tag{12a}$$
$$f_{ij}(k)\Delta\leq l_{ij}(k)(\hat{C}_j-C_j(k)) \tag{12b}$$
$$[f_{ij}(k)\Delta+1\geq \lambda_{ij}(k)C_i(k)+\varepsilon] \cup [f_{ij}(k)\Delta+1\geq l_{ij}(k)(\hat{C}_j-C_j(k))+\varepsilon] \tag{12c}$$

In this conversion and the subsequent conversion, ε is a positive real number that is small enough. Functions (12b) and (12c) are used to make the upper and lower bounds of integer requirements satisfied.

Note that (12c) is a mixed logical constraint. In [24], the authors show that Constraint (12b) needs to be transformed into mixed logical Constraint (13a) as

$$f_{ij}(k)\Delta\leq l_{ij}(k)(\hat{C}_j-C_j(k)) \tag{12b}$$
$$\Downarrow$$
$$(\forall q: 0\leq q\leq r)\delta_{ij}^q(k)=1 \Leftrightarrow f_{ij}(k)\Delta\leq l_{ij}^q(\hat{C}_j-C_j(k)) \tag{13a}$$

However, this is unnecessary since (12b) is already a linear constraint. Furthermore, we can show that (13a) results in a conflict as follows. According to the work in [45], The authors in [24] convert Constraint (13a) into mixed integer linear constraints (13b) - (13c) as

$$(\forall q: 0\leq q\leq r)\delta_{ij}^q(k)=1 \Leftrightarrow f_{ij}(k)\Delta\leq l_{ij}^q(\hat{C}_j-C_j(k)) \tag{13a}$$
$$\Downarrow$$
$$(\forall q: 0\leq q\leq r)f_{ij}(k)\Delta-l_{ij}^q(\hat{C}_j-C_j(k))\leq M_q(1-\delta_{ij}^q(k)) \tag{13b}$$
$$(\forall q: 0\leq q\leq r)f_{ij}(k)\Delta-l_{ij}^q(\hat{C}_j-C_j(k))\geq \varepsilon+(m_q-\varepsilon)\delta_{ij}^q(k) \tag{13c}$$

where $M_q\geq \max_k(f_{ij}(k)\Delta-l_{ij}^q(\hat{C}_j-C_j(k)))$, making M_q big enough, and $m_q\leq \min_k\,(f_{ij}(k)\Delta-l_{ij}^q(\hat{C}_j-C_j(k)))$, making m_q small enough. We have the following result.

Proposition 4: For the scheduling formulation of urban network traffic signal, the replacement of Constraint (12b) with Constraints (13b) - (13c) would result in a conflict.

Proof: When $\theta_\omega(k)=0$, according to Constraints (1a) and (4b), we can get $f_{ij}(k)=0$, $\delta_{ij}^q(k)=0$. Then, Constraints (13b) - (13c) can be transformed as

$$(\forall q: 0\leq q\leq r)-l_{ij}^q(\hat{C}_j-C_j(k))\leq M_q \tag{13d}$$
$$(\forall q: 0\leq q\leq r)-l_{ij}^q(\hat{C}_j-C_j(k))\geq \varepsilon \tag{13e}$$

It can be found that Constraint (13e) is a conflict since the left side is smaller than zero, while the right side is greater than zero. ■

The cause of this error is from Constraint (13a), it is wrong if $(\forall q: 0\leq q\leq r)f_{ij}(k)\Delta\leq l_{ij}^q(\hat{C}_j-C_j(k))\Rightarrow \delta_{ij}^q(k)=1$ holds. This situation is a conflict when $f_{ij}(k)=0$.

With (12c) being a mixed logical constraint, it needs to be converted into a linear constraint. In [24], the authors convert it into mixed integer linear Constraints (14a) - (14e) as

$$[f_{ij}(k)\Delta+1\geq \lambda_{ij}(k)C_i(k)+\varepsilon] \cup [f_{ij}(k)\Delta+1\geq l_{ij}(k)(\hat{C}_j-C_j(k))+\varepsilon] \tag{12c}$$
$$\Downarrow$$
$$m_a(1-\mu_1(k))-f_{ij}(k)\Delta-1+\lambda_{ij}(k)C_i(k)+\varepsilon\leq 0 \tag{14a}$$
$$(\forall q: 0\leq q\leq r)m_b(1-\mu_2(k))-f_{ij}(k)\Delta-1 + l_{ij}^q(\hat{C}_j-C_j(k))+\varepsilon\leq \dot{M}_q(1-\delta_{ij}^q(k)) \tag{14b}$$
$$(\forall q: 0\leq q\leq r)m_b(1-\mu_2(k))-f_{ij}(k)\Delta-1 + l_{ij}^q(\hat{C}_j-C_j(k))\geq \varepsilon+(\dot{m}_q-2\varepsilon)\delta_{ij}^q(k) \tag{14c}$$
$$\mu_1(k)+\mu_2(k)\geq 1 \tag{14d}$$
$$\mu_1(k),\quad \mu_2(k)\in\{0,1\} \tag{14e}$$

where $m_a\leq \min_k\,[f_{ij}(k)+1-\lambda_{ij}(k)C_i(k)-\varepsilon]$, making m_a small enough, $m_b\leq min_k\,(f_{ij}(k)\Delta+1-l_{ij}^q(\hat{C}_j-C_j(k))-\varepsilon)$, making m_b small enough, $\dot{M}_q\geq \max_k\,(m_b(1-\mu_2(k))-f_{ij}(k)\Delta-1+l_{ij}^q(\hat{C}_j-C_j(k))+\varepsilon)$, making $\dot{M}_q$ big enough, and $\dot{m}_q\leq \min_k\,[(m_b(1-\mu_2(k))-f_{ij}(k)\Delta-1+l_{ij}^q(\hat{C}_j-C_j(k))+\varepsilon]$, making $\dot{m}_q$ small enough.

However, this is not correct since it results in a conflict as stated by the following proposition.

Proposition 5: For the scheduling formulation of urban network traffic signal, the replacement of Constraint (12c) with Constraints (14a) - (14e) results in a conflict.

Proof: When $\theta_\omega(k)=0$, according to Constraints (1a) and (4b), we can get $f_{ij}(k)=0$, $\delta_{ij}^q(k)=0$. Then, Constraints (14a) - (14c) can be transformed as

$$m_a(1-\mu_1(k))-1+\lambda_{ij}(k)C_i(k)+\varepsilon\leq 0 \tag{14f}$$
$$(\forall q: 0\leq q\leq r)m_b(1-\mu_2(k))-1+l_{ij}^q(\hat{C}_j-C_j(k))+\varepsilon\leq \hat{M}_q \tag{14g}$$
$$(\forall q: 0\leq q\leq r)m_b(1-\mu_2(k))-1+l_{ij}^q(\hat{C}_j-C_j(k))\geq \varepsilon \tag{14i}$$

When $\mu_1(k)=1$, Constraint (14f) can be transformed as

$$\lambda_{ij}(k)C_i(k)\leq 1-\varepsilon \tag{14i}$$

We know that the value of $\lambda_{ij}(k)C_i(k)$ is a non-zero integer. However, from (14i) we can get that $\lambda_{ij}(k)C_i(k)$ must be zero, leading to a conflict. Then, according to Constraints (14d) and (14e), we should let $\mu_1(k)=0$ and $\mu_2(k)=1$, then Constraint (14i) can be transformed as

$$(\forall q: 0\leq q\leq r)l_{ij}^q(\hat{C}_j-C_j(k))\geq 1+\varepsilon \tag{14j}$$

Then, when the vehicles on the road are saturated, this constraint creates contradictions, which means that the model has no solution. ■

From the above formulation, Constraints (12b) and (12c) are in the constraint set. It is sure that Constraint (5a) is satisfied. For Constraint (12b), there is no prerequisite, such as $(\forall q : 0 \leq q \leq r)\delta_{ij}^{q}(k) = 1$. The reason is that even if $(\forall q : 0 \leq q \leq r)\delta_{ij}^{q}(k) = 0$, Constraint (12b) is still satisfied. Hence, it is no need to convert Constraint (12b) and in fact it is a linear constraint. For Constraint (12c), there are prerequisites, such as $(\forall q : 0 \leq q \leq r)\delta_{ij}^{q}(k) = 1$. The reason is that when $(\forall q : 0 \leq q \leq r)\delta_{ij}^{q}(k) = 0$, Constraint (12c) is not satisfied. We let $\theta_\omega(k) = 1$ be the prerequisite, which is same as $(\forall q : 0 \leq q \leq r)\delta_{ij}^{q}(k) = 1$. Thus, Constraint (12c) can be converted into mixed logical Constraint (15a) as

$$\begin{aligned}&[f_{ij}(k)\Delta + 1 \geq \lambda_{ij}(k)C_i(k) + \varepsilon]\\&\quad \cup [f_{ij}(k)\Delta + 1 \geq l_{ij}(k)(\hat{C}_j - C_j(k)) + \varepsilon] \qquad (12c)\\&\qquad\Downarrow\\&\theta_\omega(k) = 1 \Rightarrow [f_{ij}(k)\Delta + 1 \geq \lambda_{ij}(k)C_i(k) + \varepsilon]\\&\quad \cup [f_{ij}(k)\Delta + 1 \geq l_{ij}(k)(\hat{C}_j - C_j(k)) + \varepsilon] \qquad (15a)\end{aligned}$$

According to the work in [45], Constraint (15a) can be converted into mixed integer linear Constraints (15b) - (15e) as

$$\begin{aligned}&\theta_\omega(k) = 1 \Rightarrow [f_{ij}(k)\Delta + 1 \geq \lambda_{ij}(k)C_i(k) + \varepsilon]\\&\quad \cup [f_{ij}(k)\Delta + 1 \geq l_{ij}(k)(\hat{C}_j - C_j(k)) + \varepsilon] \qquad (15a)\\&\qquad\Downarrow\\&M_2(1-\theta_\omega(k)) - \lambda_{ij}(k)C_i(k) - \varepsilon + f_{ij}(k)\Delta + 1 \geq m_1(1 - \eta_1(k)) \qquad (15b)\\&M_3(1 - \theta_\omega(k)) - l_{ij}(k)(\hat{C}_j - C_j(k)) - \varepsilon + f_{ij}(k)\Delta + 1\\&\quad \geq m_2(1 - \eta_2(k)) \qquad (15c)\\&\eta_1(k) + \eta_2(k) \geq 1 \qquad (15d)\\&\eta_1(k), \eta_2(k) \in \{0, 1\} \qquad (15e)\end{aligned}$$

where $M_2 \geq \max_k(\lambda_{ij}(k)C_i(k) + \varepsilon - f_{ij}(k)\Delta - 1)$, making M_2 big enough; $M_3 \geq \max_k(l_{ij}(k)(\hat{C}_j - C_j(k)) + \varepsilon - f_{ij}(k)\Delta - 1)$, making M_3 big enough; $m_1 \leq \min_k (M_2(1 - \theta_\omega(k)) - \lambda_{ij}(k)C_i(k) - \varepsilon + f_{ij}(k)\Delta + 1)$, making m_1 small enough; and $m_2 \leq \min_k (M_3(1-\theta_\omega(k)) - l_{ij}(k)(\hat{C}_j - C_j(k)) - \varepsilon + f_{ij}(k)\Delta + 1)$, making m_2 small enough.

In order to satisfy the equation $\theta_\omega(k) = 1 \rightarrow f_{ij}(k)\Delta + 1 \geq \lambda_{ij}(k)C_i(k) + \varepsilon$, the left part of (15b) introduces the sufficiently big value M_2. In order to satisfy the equation $\theta_\omega(k) = 1 \rightarrow f_{ij}(k)\Delta + 1 \geq l_{ij}(k)(\hat{C}_j - C_j(k)) + \varepsilon$, the left part of (15c) shows the sufficiently big value M_3. At last, in order to satisfy the "or" relationship, we introduce the sufficiently small values m_1 and m_2, and the binary valves $\eta_1(k)$ and $\eta_2(k)$ that are in Constraints (15d) and (15e), and the right side of Constraints (15b) and (15c). Then, we have the following results.

Proposition 6: For the scheduling formulation of urban network traffic signal, Constraint (15a) is equivalent to Constraints (15b) - (15e).

Proof: When $\theta_\omega(k) = 1$, Constraints (15b) and (15c) can be transformed as

$$-\lambda_{ij}(k)C_i(k) - \varepsilon + f_{ij}(k)\Delta + 1 \geq m_1(1 - \eta_1(k)) \qquad (15f)$$

$$-l_{ij}(k)(\hat{C}_j - C_j(k)) - \varepsilon + f_{ij}(k)\Delta + 1 \geq m_2(1 - \eta_2(k)) \qquad (15g)$$

When $\eta_1(k) = 1$, $f_{ij}(k)\Delta + 1 \geq \lambda_{ij}(k)C_i(k) + \varepsilon$. When $\eta_1(k) = 0$, with the value of m_1, Constraint (15f) is trivially true. When $\eta_2(k) = 1$, $f_{ij}(k)\Delta + 1 \geq l_{ij}(k)(\hat{C}_j - C_j(k)) + \varepsilon$. When $\eta_2(k) = 0$, with the value of m_2, Constraint (15g) is trivially true. According to Constraints (15d) and (15e), there is at least one of $\eta_1(k)$ and $\eta_2(k)$ that can be chosen as 1. That is to say, when $\theta_\omega(k) = 1$, we can get $[f_{ij}(k)\Delta + 1 \geq \lambda_{ij}(k)C_i(k) + \varepsilon] \cup [f_{ij}(k)\Delta + 1 \geq l_{ij}(k)(\hat{C}_j - C_j(k)) + \varepsilon]$. When $\theta_\omega(k) = 0$, $f_{ij}(k) = 0$, Constraints (15b) and (15c) can be transformed as:

$$M_2 - \lambda_{ij}(k)C_i(k) - \varepsilon + 1 \geq m_1(1 - \eta_1(k)) \qquad (15h)$$

$$M_3 - l_{ij}(k)(\hat{C}_j - C_j(k)) - \varepsilon + 1 \geq m_2(1 - \eta_2(k)) \qquad (15i)$$

When $\eta_1(k) = 1$, $M_2 - \lambda_{ij}(k)C_i(k) - \varepsilon + 1 \geq \lambda_{ij}(k)C_i(k) + \varepsilon - 1 - \lambda_{ij}(k)C_i(k) - \varepsilon + 1 = 0$, $m_1(1 - \eta_1(k)) = 0$, Constraint (15h) is trivially true. When $\eta_1(k) = 0$, $m_1(1 - \eta_1(k)) = m_1 \leq M_2 - \lambda_{ij}(k)C_i(k) - \varepsilon + 1$, Constraint (15h) is trivially true. When $\eta_2(k) = 1$, $M_3 - l_{ij}(k)(\hat{C}_j - C_j(k)) - \varepsilon + 1 \geq l_{ij}(k)(\hat{C}_j - C_j(k)) + \varepsilon - 1 - l_{ij}(k)(\hat{C}_j - C_j(k)) - \varepsilon + 1 = 0$, $m_2(1 - \eta_2(k)) = 0$, Constraint (15i) is trivially true. When $\eta_2(k) = 0$, $m_2(1 - \eta_2(k)) = m_2 \leq M_3 - l_{ij}(k)(\hat{C}_j - C_j(k)) - \varepsilon + 1$, Constraint (15i) is trivially true. Hence, no matter what the values of $\eta_1(k)$ and $\eta_2(k)$ are, Constraints (15h) and (15i) are true, resulting in no conflict. ■

Similarly, in the situation with dedicated bus lanes, the transformation for the constraints of buses is as follows. Constraint (1b) can be converted into Constraint (16) as

$$(\forall \omega \in \Omega_a)(\forall (ii, jj) \in h_a(\omega))f_{iijj}(k) \leq M_4\theta_\omega(k) \qquad (16)$$

where $M_4 > \max_{i \in L} \hat{C}_{ii}$, making M_4 big enough.

Constraint (4h) is converted into Constraints (17a) - (17c) as

$$(\forall q : 0 \leq q \leq r - 1) - (1 - \theta_\omega(k - q - 1)) + \delta_{iijj}^{r-q}(k) \leq 0 \qquad (17a)$$

$$(\forall q : 0 \leq q \leq r - 1)(\forall p : 0 \leq p \leq q) - \theta_\omega(k - p) + \delta_{iijj}^{r-q}(k) \leq 0 \qquad (17b)$$

$$\begin{aligned}&(\forall q : 0 \leq q \leq r - 1)(1 - \theta_\omega(k - q - 1))\\&\quad + \sum_{p=0}^{q} \theta_\omega(k - p) - \delta_{iijj}^{r-q}(k) \leq q + 1 \qquad (17c)\end{aligned}$$

Constraint (4i) is converted into Constraints (18a) and (18b) as

$$(\forall p : 0 \leq p \leq r) - \theta_\omega(k - p) + \delta_{iijj}^{0} \leq 0 \qquad (18a)$$

$$\sum_{p=0}^{r} \theta_\omega(k - p) - \delta_{iijj}^{0}(k) \leq r \qquad (18b)$$

Constraints (5b) and (5d) are converted into Constraints (5d) and (19a) - (19c) as follows.

$$f_{iijj}(k)\Delta \in \mathbb{N} \tag{5d}$$
$$f_{iijj}(k)\Delta \leq \lambda_{iijj}(k)C_{ii}(k) \tag{19a}$$
$$f_{iijj}(k)\Delta \leq l_{iijj}(k)(\hat{C}_{jj} - C_{jj}(k)) \tag{19b}$$
$$[f_{iijj}(k)\Delta + 1 \geq \lambda_{iijj}(k)C_{ii}(k) + \varepsilon] \cup [f_{iijj}(k)\Delta + 1 \geq l_{iijj}(k)(\hat{C}_{jj} - C_{jj}(k)) + \varepsilon] \tag{19c}$$

The mixed logical Constraint (19c) can be further converted into mixed logical Constraint (20a) as

$$\theta_\omega(k) = 1 \Rightarrow [f_{iijj}(k)\Delta + 1 \geq \lambda_{iijj}(k)C_{ii}(k) + \varepsilon] \cup [f_{iijj}(k)\Delta + 1 \geq l_{iijj}(k)(\hat{C}_{jj} - C_{jj}(k)) + \varepsilon] \tag{20a}$$

According to the work in [45], mixed logical Constraint (20a) can be converted into mixed integer linear Constraints (20b) - (20e) as

$$M_5(1 - \theta_\omega(k)) - \lambda_{iijj}(k)C_{ii}(k) - \varepsilon + f_{iijj}(k)\Delta + 1 \geq m_3(1 - \eta_3(k)) \tag{20b}$$
$$M_6(1 - \theta_\omega(k)) - l_{iijj}(k)(\hat{C}_{jj} - C_{jj}(k)) - \varepsilon + f_{iijj}(k)\Delta + 1 \geq m_4(1 - \eta_4(k)) \tag{20c}$$
$$\eta_3(k) + \eta_4(k) \geq 1 \tag{20d}$$
$$\eta_3(k), \quad \eta_4(k) \in \{0, 1\} \tag{20e}$$

where $M_5 \geq \max_k (\lambda_{iijj}(k)C_{ii}(k) + \varepsilon - f_{iijj}(k)\Delta - 1)$, making M_5 big enough; $M_6 \geq \max_k (l_{iijj}(k)(\hat{C}_{jj} - C_{jj}(k)) + \varepsilon - f_{iijj}(k)\Delta - 1)$, making M_6 big enough; $m_3 \leq \min_k (M_5(1 - \theta_\omega(k)) - \lambda_{iijj}(k)C_{ii}(k) - \varepsilon + f_{iijj}(k)\Delta + 1)$, making m_3 small enough; and $m_4 \leq \min_k (M_6(1-\theta_\omega(k)) - l_{iijj}(k)(\hat{C}_{jj} - C_{jj}(k)) - \varepsilon + f_{iijj}(k)\Delta + 1)$, making m_4 small enough.

At last, we summary the scheduling problem of urban traffic signals by considering the public transport regular system with bus lanes. The model includes the Objective Function (6e) and the constraints. The constraints include the original ones and the converted ones. The original constraints include (1c), (1d), (2a), (2b), (2c), (2d), (4a), (4b), (4e), (4f), (4g), (4j), (5e) and (5f). The converted constraints include (1a) → (9), (4c) → (10a) - (10c), (4d) →(11a) - (11b), (5a) and (5c) → (5c), (12a) - (12c), and (12c) → (15a) → (15b) - (15e) for vehicles except buses, and (1b) → (16), (4h) → (17a) - (17c), (4i) →(18a) - (18b), (5b) and (5d) → (5d), (19a)- (19c), (19c) → (20a) → (20b) - (20e) for buses. We present them as follows.

$$\min \sum_{i\in L} \sum_{k=1}^{M} A\left(C_i(k) - \frac{L_i}{v_i^*} \sum_{j\in L:(i,j)\in \cup_{a\in J} F_a} f_{ij}(k)\right)\Delta + \sum_{i\in L} \sum_{k=1}^{M} B\left(C_{ii}(k) - \frac{L_i}{v_i^*} \sum_{j\in L:(ii,jj)\in \cup_{a\in J} F_a} f_{iijj}(k)\right)\Delta \tag{6e}$$

subject to

$$\sum_{\omega\in\Omega_a} \theta_\omega(k) = 1 \tag{1c}$$
$$(\forall \omega \in \Omega_a)(\forall k \in \mathbf{N})\theta_\omega(k) \in \{0, 1\} \tag{1d}$$
$$(\forall \omega \in \Omega_a)(\forall (i, j) \in h_a(\omega))f_{ij}(k) \leq M_1\theta_\omega(k) \tag{9}$$
$$C_j(k + 1) = C_j(k) + (d_j(k) - s_j(k))\Delta \tag{2a}$$
$$(\forall k \in \mathbb{N})C_j(k) \in \mathbb{N} \tag{2b}$$
$$d_j(k) = \sum_{i\in L:(i,j)\in \cup_{a\in J} F_a} f_{ij}(k) \tag{2c}$$
$$s_j(k) = \sum_{i\in L:(j,i)\in \cup_{a\in J} F_a} f_{ji}(k) \tag{2d}$$
$$l_{ij}(k) = \sum_{p=0}^{r} \delta_{ij}^{p}(k) l_{ij}^{p} \tag{4a}$$
$$\sum_{p=0}^{r} \delta_{ij}^{p}(k) - \theta_\omega(k) = 0 \tag{4b}$$
$$(\forall q : 0 \leq q \leq r - 1) - (1 - \theta_\omega(k - q - 1)) + \delta_{ij}^{r-q}(k) \leq 0 \tag{10a}$$
$$(\forall q : 0 \leq q \leq r - 1)(\forall p : 0 \leq p \leq q) - \theta_\omega(k - p) + \delta_{ij}^{r-q}(k) \leq 0 \tag{10b}$$
$$(\forall q : 0 \leq q \leq r - 1)(1 - \theta_\omega(k - q - 1)) + \sum_{p=0}^{q} \theta_\omega(k - p) - \delta_{ij}^{r-q}(k) \leq q + 1 \tag{10c}$$
$$(\forall p : 0 \leq p \leq r) - \theta_\omega(k - p) + \delta_{ij}^{0} \leq 0 \tag{11a}$$
$$\sum_{p=0}^{r} \theta_\omega(k - p) - \delta_{ij}^{0}(k) \leq r \tag{11b}$$
$$(\forall p : 0 \leq p \leq r)\delta_{ij}^{p}(k) \in \{0, 1\} \tag{4e}$$
$$f_{ij}(k)\Delta \leq \lambda_{ij}(k)C_i(k) \tag{12a}$$
$$f_{ij}(k)\Delta \leq l_{ij}(k)(\hat{C}_j - C_j(k)) \tag{12b}$$
$$M_2(1 - \theta_\omega(k)) - \lambda_{ij}(k)C_i(k) - \varepsilon + f_{ij}(k)\Delta + 1 \geq m_1(1 - \eta_1(k)) \tag{15b}$$
$$M_3(1 - \theta_\omega(k)) - l_{ij}(k)(\hat{C}_j - C_j(k)) - \varepsilon + f_{ij}(k)\Delta + 1 \geq m_2(1 - \eta_2(k)) \tag{15c}$$
$$\eta_1(k) + \eta_2(k) \geq 1 \tag{15d}$$
$$\eta_1(k), \eta_2(k) \in \{0, 1\} \tag{15e}$$
$$f_{ij}(k)\Delta \in \mathbb{N} \tag{5c}$$
$$\sum_{i\in L:(i,j)\in \cup_{a\in J} F_a} \lambda_{ij}(k) = 1 \tag{5e}$$
$$(\forall \omega \in \Omega_a)(\forall (ii, jj) \in h_a(\omega))f_{iijj}(k) \leq M_4\theta_\omega(k) \tag{16}$$
$$C_{jj}(k + 1) = C_{jj}(k) + (d_{jj}(k) - s_{jj}(k))\Delta \tag{2e}$$
$$(\forall k \in \mathbb{N})C_{jj}(k) \in \mathbb{N} \tag{2f}$$
$$d_{jj}(k) = \sum_{i\in L:(ii,jj)\in \cup_{a\in J} F_a} f_{iijj}(k) \tag{2g}$$
$$s_{jj}(k) = \sum_{i\in L:(jj,ii)\in \cup_{a\in J} F_a} f_{jjii}(k) \tag{2h}$$
$$l_{iijj}(k) = \sum_{p=0}^{r} \delta_{iijj}^{p}(k) l_{iijj}^{p} \tag{4f}$$
$$\sum_{p=0}^{r} \delta_{iijj}^{p}(k) - \theta_\omega(k) = 0 \tag{4g}$$
$$(\forall q : 0 \leq q \leq r - 1) - (1 - \theta_\omega(k - q - 1)) + \delta_{iijj}^{r-q}(k) \leq 0 \tag{17a}$$
$$(\forall q : 0 \leq q \leq r - 1)(\forall p : 0 \leq p \leq q) - \theta_\omega(k - p) + \delta_{iijj}^{r-q}(k) \leq 0 \tag{17b}$$
$$(\forall q : 0 \leq q \leq r - 1)(1 - \theta_\omega(k - q - 1)) + \sum_{p=0}^{q} \theta_\omega(k - p) - \delta_{iijj}^{r-q}(k) \leq q + 1 \tag{17c}$$

$$(\forall p : 0 \le p \le r) - \theta_\omega(k-p) + \delta^0_{iijj} \le 0 \tag{18a}$$
$$\sum\nolimits_{p=0}^{r} \theta_\omega(k-p) - \delta^0_{iijj}(k) \le r \tag{18b}$$
$$(\forall p : 0 \le p \le r)\delta^p_{iijj}(k) \in \{0, 1\} \tag{4j}$$
$$f_{iijj}(k)\Delta \le \lambda_{iijj}(k)C_{ii}(k) \tag{19a}$$
$$f_{iijj}(k)\Delta \le l_{iijj}(k)(\hat{C}_{jj} - C_{jj}(k)) \tag{19b}$$
$$M_5(1-\theta_\omega(k)) - \lambda_{iijj}(k)C_{ii}(k) - \varepsilon + f_{iijj}(k)\Delta + 1 \ge m_3(1-\eta_3(k)) \tag{20b}$$
$$M_6(1-\theta_\omega(k)) - l_{iijj}(k)(\hat{C}_{jj} - C_{jj}(k)) - \varepsilon + f_{iijj}(k)\Delta + 1 \ge m_4(1-\eta_4(k)) \tag{20c}$$
$$\eta_3(k) + \eta_4(k) \ge 1 \tag{20d}$$
$$\eta_3(k), \eta_4(k) \in \{0, 1\} \tag{20e}$$
$$f_{iijj}(k)\Delta \in \mathrm{N} \tag{5d}$$
$$\sum\nolimits_{i \in L:(ii,jj) \in \cup_{a \in J} F_a} \lambda_{iijj}(k) = 1 \tag{5f}$$

Among these constraints, the first two are for vehicles including buses. The next 21 constraints are for vehicles except buses, while the last 21 constraints are for buses.

For the case without bus lane, the modified model is given as

$$\min \sum\nolimits_{i \in L} \sum\nolimits_{k=1}^{M} A\left(C_i(k) + \frac{B}{A}C_{ii}(k) - \frac{L_i}{v_i^*}\sum\nolimits_{j \in L:(i,j) \in \cup_{a \in J} F_a} f_{ij}(k)\right)\Delta \tag{8}$$

The constraints include (1c), (1d), (2c), (2d), (4e), (5c), (5e), (9), (10a) - (10c), (11a), (11b), (15d), (15e) and (7a) - (7l). In addition, the following constraints should be included. Since the total number of vehicles including buses different from the case without bus lane, Constraints (12a), (12b), (15b) and (15c) should be changed as follows, respectively.

$$f_{ij}(k)\Delta \le \lambda_{ij}(k)C'_i(k) \tag{21a}$$
$$f_{ij}(k)\Delta \le l_{ij}(k)(\hat{C}'_j(k) - C'_j(k)) \tag{21b}$$
$$M_2(1-\theta_\omega(k)) - \lambda_{ij}(k)C'_i(k) - \varepsilon + f_{ij}(k)\Delta + 1 \ge m_1(1-\eta_1(k)) \tag{21c}$$
$$M_3(1-\theta_\omega(k)) - l_{ij}(k)(\hat{C}'_j(k) - C'_j(k)) - \varepsilon + f_{ij}(k)\Delta + 1 \ge m_2(1-\eta_2(k)) \tag{21d}$$

IV. NUMERICAL RESULTS AND ANALYSIS

In order to solve the optimization problem formulated in the last section at time k, some information is required. We need to know the past situation of traffic light, the current volume at each link and the incoming flow with the corresponding prediction. In this paper, we assume that the past situation of traffic light and the current volume can be measured by sensors, while the incoming flow with the corresponding prediction are provided by a traffic management system via another model. According to the objective function and the constraints, the solver gets the profile of optimal traffic signal control at time k. After this step, the latest real-time data are provided by the traffic management system and sensors. Then, the solver gets the profile of optimal traffic signal control at time $k+1$. This process is repeated. That is to say, we adopt a receding horizon manner to implement the scheduling. Thus, for the changing traffic conditions, we have the real-time response. In this section, from Part A to Part D, we analyze the model with dedicated bus lanes. In Part E, we analyze the model without dedicated bus lane. A summary is given in Part F.

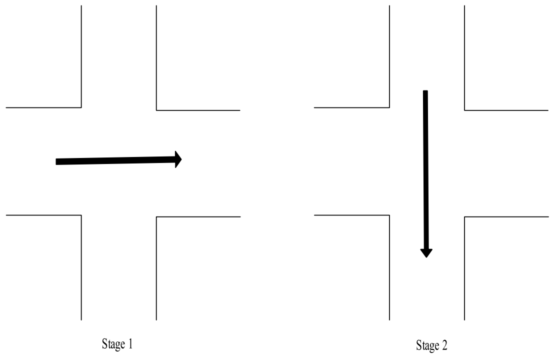

FIGURE 2. The two stages of an intersection.

A. THE CASE WITH TWO STAGES FOR AN INTERSECTION

This situation is the simplest one in the urban traffic network. It is the same as what we shown in Fig. 1. There are two stages in each intersection as shown in Fig. 2.

It can be seen from Fig. 2 that there are two stages and at each stage there is only one stream crossing the intersection. In this part, we consider a traffic network including different number of horizontal and vertical roads. Due to the consideration of the public transport regular, the size of the model is about twice of the original model. There are two performance criteria. The first one is the computational time for different situations and the second one is the time delay reduction for the two models with and without considering the public transport and the maximum improvement degree for the model with public transport regular.

We assume that there are n_h horizontal roads and n_v vertical roads, respectively. For the values of some parameters, we refer to the paper [24]. Then, some other parameters are as follows. The incoming vehicle flow rate $b_i(k)$ is given and $L_i/v_i^* = 1$ is used in this part. For each link j, the maximal volume $\hat{C}_j$ and $\hat{C}_{jj}$ are set as 30 and 15, respectively. The scheduling horizon is $H_p = M\Delta$. The sampling interval is 12 seconds. Two speed categories are chosen. We set the fast speed ratio and the low speed ratio as $l^0_{ij} = 0.8$ and $l^1_{ij} = 0.4$ for the vehicles except the buses. At the same time, $l^0_{iijj} = 0.4$ and $l^1_{iijj} = 0.2$ are set for the buses. The value of A is set to be 4. This is based on the calculation of private vehicles. In the front row, there are a driver and a passenger. Meanwhile, in the rear row, there are two passengers, i.e., four people in total. The value of B is set as 40. During May 2016, in a large city in China, the average number of passengers on

TABLE 1. Computation time for different scale in the case of 2 stages (in seconds).

$n_v = n_h$	$H_p =$ 12(s)	$H_p =$ 24(s)	$H_p =$ 36(s)	$H_p =$ 48(s)	$H_p =$ 60(s)
2	0.05	0.11	0.41	1.03	2.22
4	0.06	0.51	5.98	75.22	-
6	0.07	4.05	91.64	-	-
8	0.09	9.13	-	-	-
10	0.13	11.96	-	-	-

the main road of buses was 39.93. Hence, we choose 40 in the model. The optimization problem is solved by LINGO on an ASUS Laptop Y4200F with an Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz and 1.80GHz CPU. We test cases of $n_v = n_h = 2$, 4, 6, 8, and 10, and $H_p = 12$, 24, 36, 48, and 60 seconds, respectively. The corresponding computation time for the two-stage model is summarized in Table 1. The time values are given in seconds. When the computational time is very long (more than 120s), it is no sense to solve. Then we use a short solid line to represent them.

From Table 1, we can see that when $H_p = 12$s, each case can be solved in a short time. With the increasing of H_p, the computation time also increases rapidly. For a small-scale network, the increase in computation time is relatively slow. To ensure the real-time reaction on the diversification of traffic conditions, we should choose a suitable scheduling horizon according to the size of the network. For a large-scale problem, it can be divided into several small-scale problems. The solver calculates the optimal control signal based on the latest data repeatedly. That is to say, the scheduling is implemented in a receding horizon way.

Next, we compare the two models with and without considering the public transport regular. In some situations, both models achieve the same result. Except for these situations, the model with public transport regular achieves a better result. Therefore, we consider the maximum optimization degree obtained by the model with public transport regular. At this time, the solutions obtained by the two models are different. We use three indicators for evaluation, they are the number of delay people, the total delay time in the road network, and the average delay time per person. We set one-time interval to express it.

Table 2 shows the result of the two models with two stages. It can be seen that, for $n_v = n_h = 2$, the model with public transport regular can reduce the delay of 344 people at most by 12s, leading to 86 people on average at each intersection.

Similarly, for $n_v = n_h = 4$, $n_v = n_h = 6$, $n_v = n_h = 8$, and $n_v = n_h = 10$, the model with public transport regular can reduce the delay of 1340, 2752, 4892, and 7876 people at most by 12s, respectively, leading to 84, 76, 76 and 79 people on average at every intersection, respectively. We can get that if the result of original model and the model with public transport regular is different in one intersection, the model with public transport regular can reduce the delay about 80 people at most in an intersection. In terms of the average delay time per person, for $n_v = n_h = 2$, the model with public

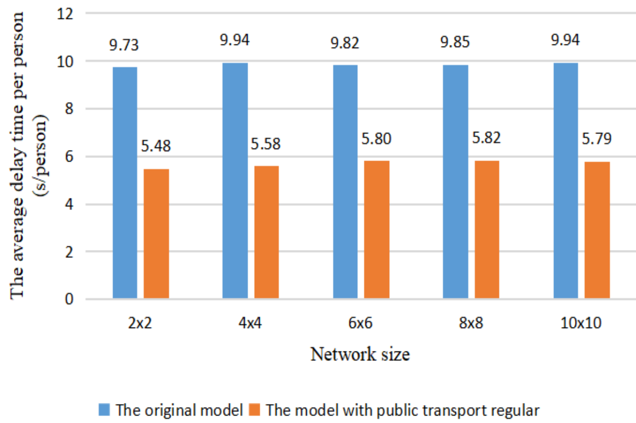

FIGURE 3. The results of the two models in the case of two stages.

transport regular can reduce the average delay time of 4.25s at most in 12s. Similarly, for $n_v = n_h = 4$, $n_v = n_h = 6$, $n_v = n_h = 8$, and $n_v = n_h = 10$, the model with public transport regular can reduce the average delay time of 4.35s, 4.03s, 4.03s, and 4.15s at most in 12s, respectively. If the result of original model and the model with public transport regular is different in the corresponding intersections, the model with public transport regular can reduce the delay time per person about 4.15s at most in 12s.

In order to show the comparisons of the two models more intuitively, Fig. 3 shows the result for the average delay time per person in the two-stage model. It can be seen directly from Fig. 3 that the model with public transport regular performs better than the original model, and the maximum degree can reach about 40%.

B. THE CASE WITH FOUR STAGES FOR AN INTERSECTION

For the four-stage situation, it is the normal one in the urban traffic network. There are four stages in each intersection as shown in Fig. 4.

It can be observed from Fig. 4 that, at each stage, there are two compatible streams crossing an intersection simultaneously. Compared with the two-stage system, with two more stages, the number of constraints for this model is about twice of that for a two-stage system. For each stage, the flow is twice as that of the original one. Overall, the size of the four-stage model is about four times of that for the two-stage model. For the values of the parameters, we set them to be the same as the two-stage model. The corresponding computation time in seconds for the four-stage model are summarized in Table 3. When the computational time is very long (more than 120s), it is no sense for practice. Then we use a short solid line to represent them.

It can be observed from Table 3 that when $H_p = 12$s, each case can be solved in a short time, which is close to the two-stage model. With the increase of H_p, the computation time increases rapidly. The growth rate is significantly greater than that for the two-stage model. For a small-scale network, there is relatively slow increase in computation time. In order to

TABLE 2. The results of the two models for the case of two stages.

$n_v = n_h$	The original model			The model with public transport regular		
	The number of delay people (person)	The total delay time in the road network(s)	The average delay time per person (s/person)	The number of delay people (person)	The total delay time in the road network(s)	The average delay time per person (s/person)
2	788	9456	9.73	444	5328	5.48
4	3060	36720	9.94	1720	20640	5.58
6	6716	80592	9.82	3964	47568	5.80
8	11960	143520	9.85	7068	84816	5.82
10	18868	226416	9.94	10992	131904	5.79

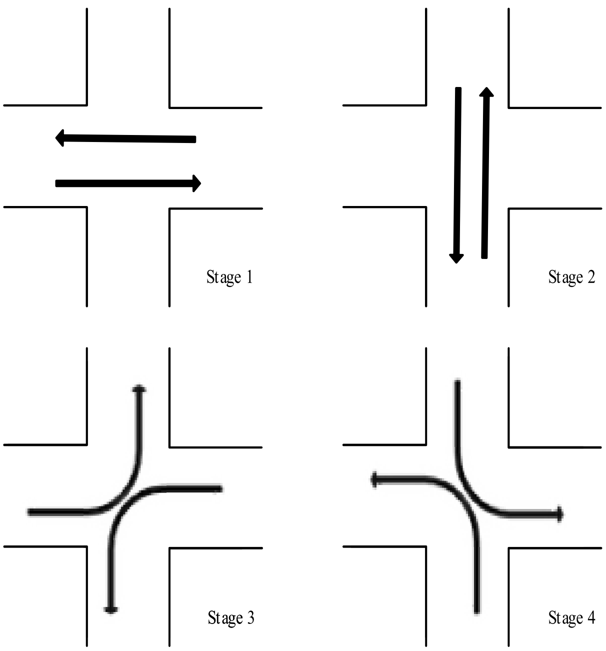

FIGURE 4. The four stages of an intersection.

TABLE 3. Computation time for different scale in the case of 4 stages (in seconds).

$n_v = n_h$	$H_p =$ 12(s)	$H_p =$ 24(s)	$H_p =$ 36(s)	$H_p =$ 48(s)	$H_p =$ 60(s)
2	0.05	0.22	2.44	26.55	73.24
4	0.08	0.66	-	-	-
6	0.11	11.89	-	-	-
8	0.16	23.24	-	-	-
10	0.22	-	-	-	-

ensure the real-time response to the traffic conditions, choosing a suitable scheduling horizon according to the size of the network is necessary. Meanwhile, a large-scale problem can be divided into several small-scale problems. The solver gets the optimal control signal based on the latest data again and again. In other words, the scheduling system performs in the way of receding horizon.

Next, we compare the two models with and without considering the public transport regular. We use the same approach as the two-stage model does with the same indicators for evaluation.

Table 4 shows the results of the two models with four stages. It can be summarized as follows. For $n_v = n_h = 2$, the model with public transport regular can reduce the delay of 828 people at most by 12s, leading to 207 people on average at every intersection. Similarly, for $n_v = n_h = 4$, $n_v = n_h = 6$, $n_v = n_h = 8$, and $n_v = n_h = 10$, the model with public transport regular can reduce the delay of 3364, 7524, 13300, and 20740 people at most by 12s, respectively, leading to 210, 209, 208, and 207 people on average at every intersection, respectively. We can get that if the result of the original model and the model with public transport regular is not the same in one intersection, the model with public transport regular can reduce the delay about 208 people at most in an intersection. For the indicators of the average delay time per person, when $n_v = n_h = 2$, the model with public transport regular can reduce the average delay time of 2.23s at most in 12s. Similarly, when $n_v = n_h = 4$, $n_v = n_h = 6$, $n_v = n_h = 8$, and $n_v = n_h = 10$, the model with public transport regular can reduce the average delay time of 2.33s, 2.37s, 2.36s, and 2.33s at most in 12s, respectively. If the result of original model and the model with public transport regular is different in the corresponding intersection, the model with public transport regular can reduce the delay time per person about 2.3s at most in 12s.

For further comparison, Fig. 5 shows the result for the average delay time per person of the two models with four stages. It can be seen from Fig. 5 that the model with public transport regular is better than the original one, and the maximum degree can be improved by about 20%. Compared to the two-stage model, the optimization degree of the four-stage model is about half of that by the two-stage model. The reason is that the four-stage model passes a quarter of the total stage at each time, and the two-stage model passes one-half of the total stage. The degree of optimization is specific to the part of passing vehicles.

C. THE CASE OF DIFFERENT NUMBER OF PEOPLE ON BUSES

In practice, the number of passengers in buses changes dynamically over time, we further test the difference of the two models. To be more intuitive, we use the average delay time per person as an indicator for evaluation. The average

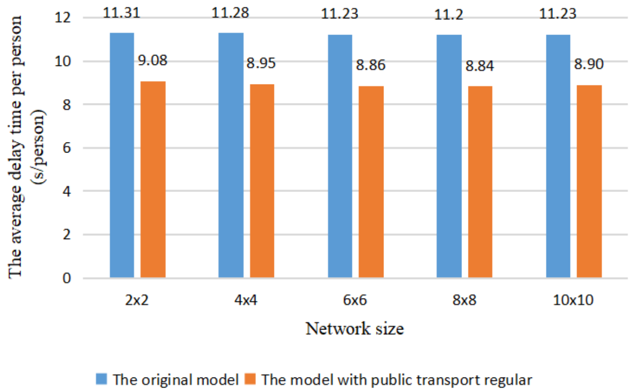

FIGURE 5. The results of the two models in the case of four stages.

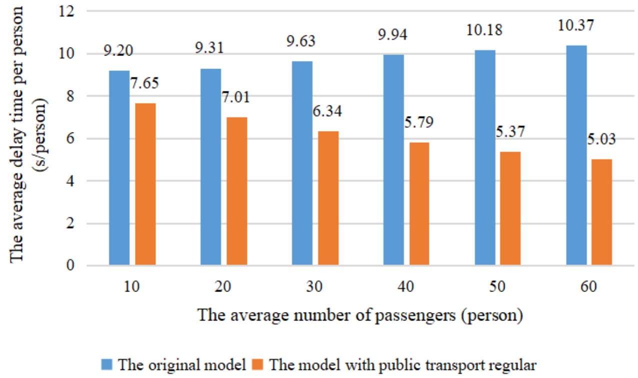

FIGURE 6. The results of the two models in different number of people on buses for the case of two stages in the scale of 10 × 10.

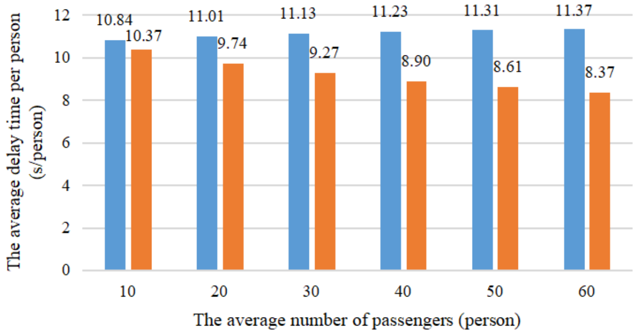

FIGURE 7. The results of the two models in different number of people on buses for the case of four stages in the scale of 10 × 10.

number of people in a bus is set to 10, 20, 30, 40, 50, and 60, meaning that $B = 10, 20, 30, 40, 50$, and 60, respectively. The other indicators as used in above discussion are kept unchanged. Figs. 6 and 7 show the results of two-stage model and four-stage model in the case of $n_v = n_h = 10$, respectively.

It can be seen that with the number of passengers increasing, the average delay time per person becomes less and less in the model with public transport regular, the corresponding value becomes larger and larger in the original model. In the case of $n_v = n_h = 10$, as the average number of passengers in a bus increases from 10 to 60, the difference of these two models is 1.55s, 2.3s, 3.29s, 4.15s 4.81s, and 5.15s, respectively in the two-stage model and 0.46s, 1.27s, 1.86s, 2.33s 2.7s, and 3s, respectively in the four-stage model. With the number of people increasing from 10 to 60, the results of the model with public transport regular can improve those obtained by the original model by 17%, 25%, 34%, 42%, 47%, and 51% respectively in the two-stage model. The corresponding improving values for the four-stage models are about 4%, 12%, 17%, 21%, 24%, and 26%, respectively. The reason is that the objective value is the number of people in all existing vehicles minus the number of people who can pass the intersection. As the number of average people in the bus increases, the number of people in all vehicles increases. Since the original model does not consider public transportation, the number of passing passengers is unchanged, resulting in the average delay time increasing with the increase of the average number of people in the bus. For the model with public transport regular, as the average number of people in the bus increases, the number of passing passengers increases at the same time, leading to the objective value maintaining within a range. When the average number of people in the buses increases, the total number of people increases. As a result, the average delay time is reduced. We can get that with the increase of the number of people in the buses, the model with public transport regular has greater advantages.

D. THE CASE OF DIFFERENT SAMPLE INTERVAL

The choice of scheduling horizon affects the result. Also, the choice of the sample interval and the problem scale affects the choice of scheduling horizon. The choice of the sample interval has greater impact on the result. Here, we analyze the impact of different sampling intervals for these two models. Since the interval time is different, we use the new indicator, the percentage of average delay time per person. It is the ratio of average delay time per person and the sampling interval. The sample intervals are set to 3s, 6s, 9s, 12s, 15s and 18s. The other indicators are the same as that for the above discussion. Figs. 8 and 9 show the results of two-stage model and four-stage model in the case of $n_v = n_h = 10$, respectively.

It can be seen from Figs. 8 and 9 that when $\Delta = 3$s, two models get the same results. The reason is that there are no buses passing the intersection during this time. From $\Delta = 3$s to $\Delta = 15$s, the percentage of average delay time per person drop rapidly for the model with public transport regular, while the percentage of average delay time per person drop slowly for the model without public transport regular. It means that there are buses passing the intersection continuously. From $\Delta = 15$s to $\Delta = 18$s, the percentage of average delay time per person drop slowly for the two models, which means that all buses of the corresponding stages pass through the intersection. With Δ increasing from 3s to 18s, the optimization degree of the model with public transport regular can improve those obtained by the original model about 0%, 21%, 26%, 42%, 54%, and 54%, respectively, in the two-stage model. The corresponding values for the

TABLE 4. The results of the two models in the case of four stages.

	The original model			The model with public transport regular		
$n_v = n_h$	The number of delay people (person)	The total delay time in the road network(s)	The average delay time per person (s/person)	The number of delay people (person)	The total delay time in the road network(s)	The average delay time per person (s/person)
2	4192	50304	11.31	3364	40368	9.08
4	16308	195696	11.28	12944	155328	8.95
6	35632	427584	11.23	28108	337296	8.86
8	63056	756672	11.20	49756	597072	8.84
10	99988	1199856	11.23	79248	950976	8.90

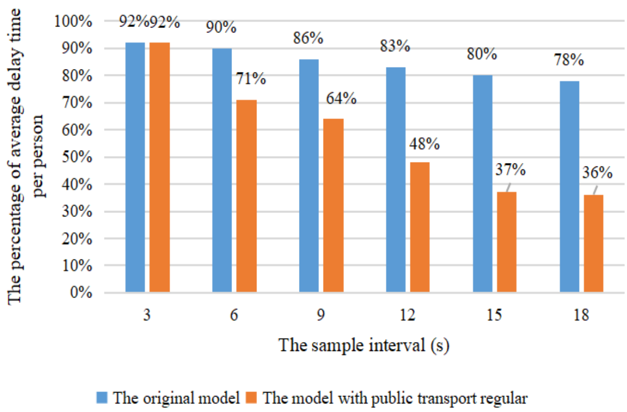

FIGURE 8. The results of the two models in different sampling intervals for the case of two stages in the scale of 10 × 10.

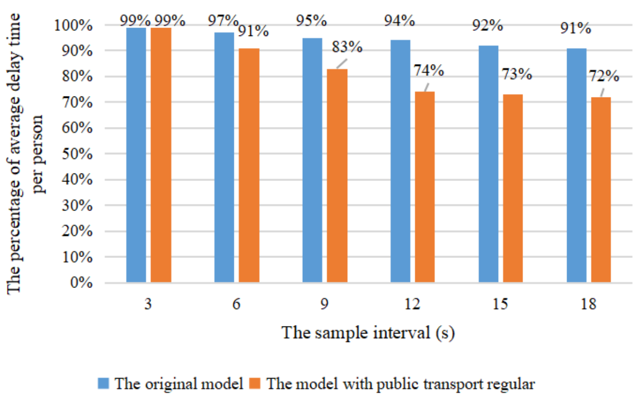

FIGURE 9. The results of the two models in different sampling intervals for the case of four stage in the scale of 10 × 10.

four-stage models are about 0%, 6%, 13%, 21%, 21%, and 21%, respectively. The reason is that the objective value is the number of people in all existing vehicles minus the number people passing the intersection. When there is no bus passing, the results of the two models are same. If there are some buses passing, as the number of buses passing increases, the number of passengers passing the intersection increases. In this situation, the degree of optimization for the model with public transport regular increases. At the end, when all the buses in the current stage are passed, these two models return the same changing degree. In this example, the most reasonable sample interval is to ensure that all buses at the current stage pass.

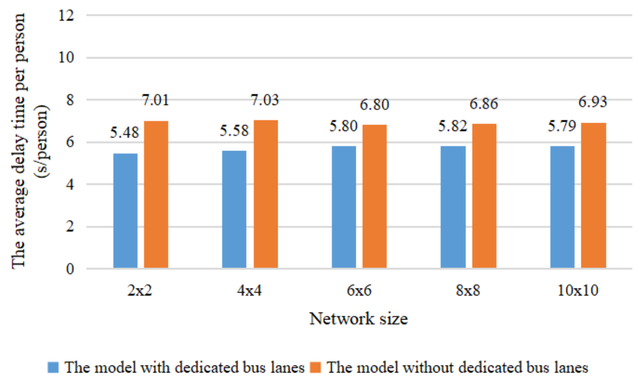

FIGURE 10. The results of the two models with and without dedicated bus lanes for the case of two stages.

E. THE CASE WITHOUT DEDICATED BUS LANES

We then compare the model with and without dedicated bus lanes. To be more intuitive, the average delay time per person is used as the indicator for evaluation. The parameters are the same as previously used. Figs. 10 and 11 illustrate the results of two-stage and four-stage models, respectively.

From the results, it is clear that the model with dedicated bus lanes performs better. When $n_v = n_h = 10$, the difference of these two models is 1.14s and 1.45s, respectively. Furthermore, the four-stage one shows a greater improvement with dedicated bus lanes. We can get that when the number of private vehicles increases, the buses have greater impact. The reason for this is the constraint (7i). When the proportion of private vehicles becomes larger, the speed profile decreases, and the result of the model also become larger. It is of great significance to set dedicated bus lanes, especially for areas with a large number of vehicles.

F. SUMMARY

From the perspective of solvability, as long as the number of vehicles meets the demand for capacity, the model is solvable. Hence, it is suitable for various saturation conditions. From the perspective of the obtained results, as the average number of passengers in a bus from 10 to 60, the degree of optimization is from 17% to 51% for the two-stage model, and 4% to 26% for the four-stage model, respectively. The proposed model is better than the original one. Considering the issue of dedicated bus lanes, it is important to set up

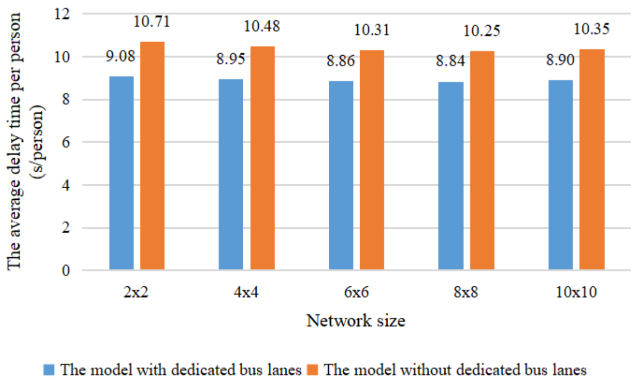

FIGURE 11. The results of the two models with and without dedicated bus lanes for the case of four stages.

dedicated bus lanes especially for areas with a large number of vehicles.

V. CONCLUSION

In this paper, we propose a novel centralized scheduling formulation for minimizing the total delay time for all people in the buses and private vehicles by using urban traffic signal control. The model is built based on a cell transmission model incorporated with novel link flow rate functions of the volume from the upstream, the available capacity from the downstream, and the past situation of traffic signals. We develop a mixed integer linear programming formulation, it is obtained by converting the traffic signal control problem. Meanwhile, we provide the proofs for the conversion. By experiments, we get the computation time for different cases including the different number of stages (two stages and four stages), the different number of intersections (2×2, 4×4, 6×6, 8×8 and 10×10 intersections), and different prediction horizon (12s, 24s, 36s, 48s and 60s). Furthermore, we analyze the impact of the number of people in buses and different sample intervals. In order to perform the efficient real-time scheduling, results indicate that we should choose a reasonable prediction horizon according to the size of the network (the total number of intersections) and the number of stages. Meanwhile, we should choose a reasonable sample interval base on the number of buses. The solver calculates the optimal control signal according to the latest data over and over again, which means that the execution of the scheduling process is in a manner of receding horizon. We compare the model proposed in this study to the original one in [24] and show that the model proposed in this study achieves better results. When the average number of passengers in a bus from 10 to 60, the degree of optimization model is from 17% to 51% in the two-stage model, and 4% to 26% in the four-stage model, respectively. Finally, to analyze the scenarios without dedicated bus lanes, we modify the model to adapt to this situation for comparison. The comparisons and analysis show that it is important to set dedicated bus lanes, especially for the roads with a large number of vehicles.

There are some directions for the future work.

(1) In this paper, we assume that we know the network boundary flow rates and the turning ratios in advance. In fact, these values are stochastic in practice. It is meaningful to study the problem with this situation taken into account in our future work.

(2) This study only considers the effect of the people in vehicles, including the ones in buses. The effect of pedestrians may be considered at the same time. It is meaningful to integrate the effect of pedestrians on the proposed model, which can be seen as a bi-objective optimization problem.

(3) It is assumed that each vehicle in the traffic network is delayed only by traffic signals. However, there may be some emergencies or traffic accidents in the traffic network. It is meaningful to study the traffic signal scheduling under these situations.

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