

# Implementation of PMU-Based Distributed Wide Area Monitoring in Smart Grid

MARWAN AHMED ABDULLAH SUFYAN<sup>1</sup>, MOHD ZUHAIB<sup>ID</sup><sup>1</sup>, (Member, IEEE),  
MOHD ANAS ANEES<sup>1</sup>, (Member, IEEE), ABUL KHAIR<sup>2</sup>,  
AND MOHD RIHAN<sup>1</sup>, (Senior Member, IEEE)

<sup>1</sup>Department of Electrical Engineering, ZHCET, Aligarh Muslim University, Aligarh 202002, India

<sup>2</sup>Department of Electrical Engineering, Saharsa College of Engineering, Saharsa 852201, India

Corresponding authors: Mohd Zuhaib (mohd.zuhaib@zhcet.ac.in) and Abul Khair (abulkhair1986@gmail.com)

This work was supported by the Aligarh Muslim University, India.

**ABSTRACT** Determination of accurate operating states of the power system is one of the most challenging tasks due to integration of large number of solar PVs into the power system. Since these sources of energy are inertialess generations, hence may cause instability issues if highly penetrated. Hence, precise monitoring and control of such systems with higher PV penetration is a critical issue to address both in terms of the total number of PV sources in the system and the percentage of penetration. Phasor Measurement Units (PMUs), with their time synchronizing abilities, have made this task a bit easier. However, traditional centralized data handling architecture at control centers are becoming redundant due to various limitations such as data handling, computational constraints etc. To overcome this, a distributed PMU-PDC architecture approach is utilized in this paper. Since local PDC (Phasor Data Concentrator) in an n-area power system can run separate optimization algorithms, by combining the results of these optimization algorithms of n-area PDCs, we can get a much more accurate global consensus about the current operating state of the system. The presented work is divided into two parts. First, the analysis of power system stability is performed in terms of the total number of Photovoltaic Sources (PV) in the system and the percentage of PV penetration. Then, the accuracy of the method is tested both in terms of the number of PMUs in each area along with their channel capacities. Low-frequency oscillations have been simulated on IEEE-68 standard bus system using MATLAB, and the modes of oscillation are estimated using the Alternating Direction Method of Multipliers (ADMM) algorithm.

**INDEX TERMS** Wide-area measurement system (WAMS), phasor measurement unit (PMU), phasor data concentrator (PDC), alternating direction method of multipliers (ADMM).

## I. INTRODUCTION

Following the power outage of July 2012 in the northern and north-eastern portions of India, PMUs with their ability to provide time-synchronized measurements are gaining popularity and momentum. The government of India has initiated the various projects for the deployment of PMUs across the grid on a larger scale. These synchronized measurements from different locations are sent to the PDC installed at load centres through proper communication for the aim of analysis and usage [1], [2]. The most serious concern about such a system is the generation of gigantic PMU data generated, which are perhaps unattended so far. Therefore, instead of

The associate editor coordinating the review of this manuscript and approving it for publication was Yang Li<sup>ID</sup>.

concentrated data storage and analysis, distributed physical architecture needs to be developed under such conditions. This distributed architecture contains dedicated local PMUs and PDC that determine the operating states of the power system locally [3]. Decisions towards any perturbation in the power system can be accurately done by assessing the locally attained measurements at central PDC sent via proper communication channels. Such a distributed architecture ensures the robustness of the system compared to centralized architecture in terms of computational complexity as well as data handling [1], [4].

Several advanced signal processing methods analyze low-frequency oscillations using centralized approach. These techniques are Fast Fourier Transform (FFT) [5], Matrix Pencil (MP) [6], [7], Prony Analysis (PA) [8], [9], Principal

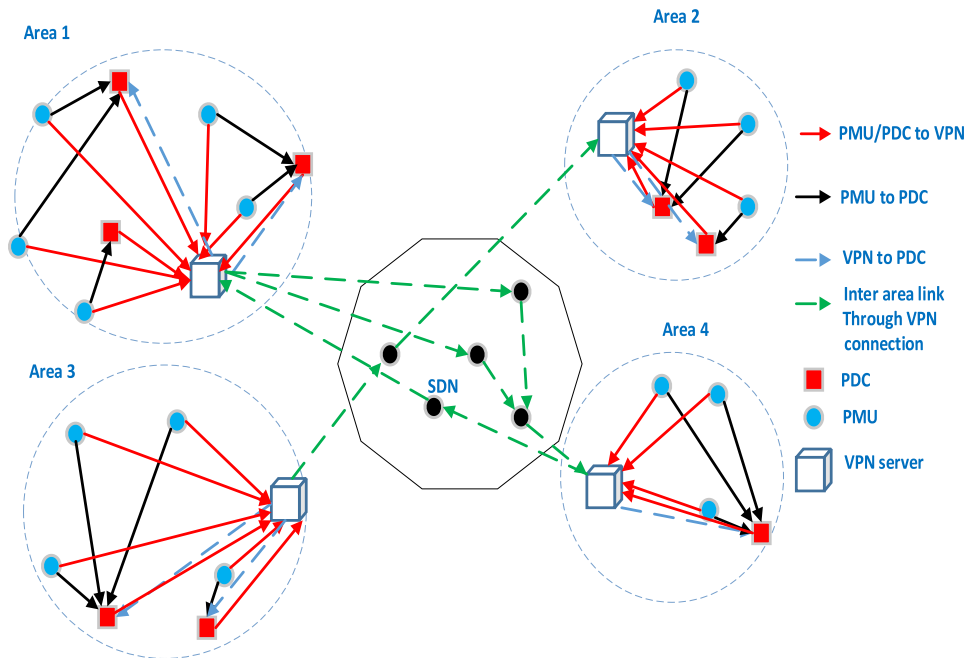


FIGURE 1. Architecture for PMU-PDC communication.

Component Analysis (PCA) [10], Empirical Mode Decomposition (EMD) [11], [12], Multivariate Empirical Mode Decomposition (MEMD) [13], Hilbert Transform (HT) [14], wavelet transform (WT) [15], robust recursive least square (RRLS), regularized robust recursive least square (R3LS) algorithm [16], Eigensystem Realization Algorithm (ERA) [17], Dynamic Mode Decomposition (DMD) Algorithm [18] etc. All these methods suffer computational challenges due to data handling at a centralized stage.

To overcome such challenges, distributed PMU-PDC architecture is used in [19]–[27] to analyze instability caused by low-frequency oscillations due to various faults occurring in the system. Such a system consists of several regions, which might or may not be symmetric, and belong to several companies as shown in Figure 1. Each wide-area contains PMUs that communicate their data in real-time to PDCs positioned at the control station by a Virtual Private Network (VPN). These PDCs can send data into each other and with a main PDC placed at the separate system operators while monitoring such a wide-area using Software Define Network (SDN). The idea is to utilize this distributed grid to work as global assent at the PDCs into every region, iteratively produce a reliable evaluation of the coefficients of the polynomial, and allow the PDCs to interact with each other to reach the global solution using many variants of the ADMM algorithm [28], [29].

In standard distributed algorithms, local PDCs perform their respective optimization steps with equal speed, and the communication delays between the local PDCs and the central PDC are also equal, i.e., they are synchronous. However, in reality, the PDCs may not be perfectly synchronized with

each other due to differences in their processing speeds as well as due to various communication delays such as routing, queuing, and transfer delays in the SDN. This problem can be overcome by the use of recently developed method called Asynchronous ADMM (A-ADMM). In this method, the central PDC receives the updates only from a subset of the  $N$  local PDCs at every iteration  $k$ , referred to as active PDCs. Although both the algorithms ensure data privacy between the  $N$  PDCs, it is not very resilient as the central PDC is directly amenable to failure under extraneous attacks. This problem can be resolved by resorting to a completely distributed version of Architecture distributed ADMM (D-ADMM). In this formulation, each active PDC at each iteration communicates directly with a subset of other active PDCs determined by a communication graph. Therefore, the need for the central PDC no longer exists. However, if  $p$ , the number of PMUs, is large, a better strategy will be to create multiple hierarchical layers of PDCs so that the computational load of the global estimation gets divided. This method is known as hierarchical ADMM (H-ADMM) algorithm [24], [25], [30]–[32].

Since in modern power systems, conventional energy resources are continuously being replaced by renewable energy resources such as solar, wind etc. Continuous efforts are presently being made worldwide to integrate large numbers of solar PV systems into the grid due to the numerous benefits it offers to utility and consumers. The stability of the power system is directly linked with the inertia between generators at the generation side and motors at the load side. Since solar PV system is an inertialess generation replacing it with conventional power sources will cause inertia imbalance, hence can affect the stability of the power system.

However, the present power system cannot handle a large amount of solar penetration without the smart renovation of the grid. It is unanimously accepted that more than 30% Renewable Energy Sources (RES) penetration affects grid stability. Although, large-scale integration of energy storage devices will be helpful to improve the grid stability and can enhance the percentage of penetration for safe operation. Therefore, analysis of power system stability under significant PV penetration is a critical issue to address. In this work, stability analysis of the power system is performed with varying PV penetration using distributed PMU-PDC architecture. The analysis performed in this work is divided into two parts. First, the power system stability analysis is performed under different test conditions viz. percentage of PV penetration and number of PV sources in the system. Then performance on the accuracy of the proposed method is also tested based on the total number of PMUs installed along with their channel capacity.

This paper is divided into the following sections. Section II describes the problem formulation and description of the ADMM algorithm, in section III, the communication architecture for obtaining the measurement from multiple PMU-PDC systems is presented, section IV discusses the test results on the IEEE-68 bus system. The conclusions drawn are presented in section V.

## II. PROBLEM FORMULATION

ADMM algorithm is a strong algorithm that is well appropriate to control distributed optimization. ADMM algorithm can be utilized for many practical convex optimization problems suitable for analyzing stability properties and decomposability [24], [25].

Let a power system be consisted by  $m$  synchronous generators  $m$  and  $m_l$  load buses linked by a particular topology. All generators are modelled via a second-order swing equation. The modelling of every bus is obtained by representing active and reactive power buses by two algebraic equations. The linearized element variable type for the  $m$  machine framework can be expressed as:

$$\begin{aligned} \begin{bmatrix} \Delta \dot{\delta}(t) \\ \Delta \dot{w}(t) \end{bmatrix} &= \begin{bmatrix} 0_{m \times m} & w_s I_m \\ M^{-1} L & M^{-1} D \end{bmatrix} \begin{bmatrix} \Delta \delta(t) \\ \Delta w(t) \end{bmatrix} \\ A &= \begin{bmatrix} 0_{m \times m} & w_s I_m \\ M^{-1} L & M^{-1} D \end{bmatrix} \\ y(t) &= [\Delta \theta_1(t), \dots, \Delta \theta_p(t)]^T \end{aligned} \quad (1)$$

where,  $\Delta \delta = [\Delta \delta_1 \dots \Delta \delta_m]^T$ ,  $\Delta w = [\Delta w_1 \dots \Delta w_m]^T$ ,  $M = \text{diag}(M_1, \dots, M_m)$ ,  $D = \text{diag}(D_1, \dots, D_m)$ ,  $\Delta w_i$  &  $\Delta \delta_i$  are the small-signal angle variation,  $D_i$  is the mechanical damping from generator  $i$ ,  $M_i$  is the inertia,  $I_m$  is the  $(m \times m)$  matrix,  $w_s$  is synchronous speed,  $L$  is the laplacian matrix.  $y(t) \in R^P$ , represent collection of phase angle measurements  $\Delta \theta_{p_i}(t)$ ,  $p = 1, \dots, P$  measured at busses equipped with PMU.  $A$  is the eigenvalues, denoted by  $(-\sigma_l \pm j\Omega_l)$ , ( $j \triangleq \sqrt{-1}$ )  $l = 1, \dots, m$ . The purpose is to

evaluate those second  $n$  eigenvalues of  $A$  from  $y(t)$  in a distributed style utilizing various computational sources. Before this, we need to generalize model assessment algorithm that is well suited for distributed optimization problems [24].

The solution of  $\Delta \theta_p(t)$  in equation 1 can be written as

$$\Delta \theta_p(t) = \sum_{l=1}^m \left( r_{pl} e^{(-\sigma_l + j\Omega_l)t} + \dot{r}_{pl} e^{(-\sigma_l - j\Omega_l)t} \right) \quad (2)$$

where  $(\theta_p(t))$  is a uniform sampling with period  $T$ ,  $(\sigma_l)$  damping,  $(\Omega_l)$  frequency of  $L^{th}$  mode &  $r_{pl}$  is the remains of the  $L^{th}$  mode reflected in the  $P^{th}$  output. A generic expression for the  $Z$  - transform of  $\Delta \theta_p(t) \triangleq \Delta \theta_p(t)|_{kT}$ , ( $k = 0, 1, \dots, n-1$ ) with measured samples, can be rewritten as

$$\Delta \theta_p(z) = \frac{b_0 + b_1 z^{-1} + \dots + b_{2m} z^{-2m}}{1 + a_1 z^{-1} + \dots + a_{2m} z^{-2m}} \quad (3)$$

where (b) and (a) are fixed zero polynomial and characteristic polynomial.

The solution of equation 3 can be achieved in 3 steps [25], [28].

Step one: Finding the values of matrix by using

$$\underbrace{\begin{bmatrix} \Delta \theta_p(2m) \\ \Delta \theta_p(2m+2) \\ \vdots \\ \Delta \theta_p(2m+l) \end{bmatrix}}_{c_p} = \underbrace{\begin{bmatrix} \Delta \theta_p(2m-1) & \dots & \Delta \theta_p(0) \\ \Delta \theta_p(2m) & \dots & \Delta \theta_p(1) \\ \vdots & & \vdots \\ \Delta \theta_p(2m+l-1) & \dots & \Delta \theta_p(l) \end{bmatrix}}_{H_p} \times \underbrace{\begin{bmatrix} -a_1 \\ -a_2 \\ \vdots \\ -a_{2m} \end{bmatrix}}_a \quad (4)$$

where  $l$  is an integer satisfying  $2n + l \leq m - 1$ . For PMU  $p = 1, \dots, P$ , concatenate,  $c_p$  and  $H_p$  and solving by using LS minimization technique.

$$\min_a 0.5 * \left\| \begin{bmatrix} H_1 \\ \vdots \\ H_P \end{bmatrix} a - \begin{bmatrix} C_1 \\ \vdots \\ C_P \end{bmatrix} \right\|^2 \quad (5)$$

where  $\|\cdot\|$  indicates the 2 models of a vector.

Step two: after finding  $a$ , we calculate the roots of discrete-time  $z$  transform from the denominator of (3). Suppose roots be denoted by  $z_l$ ,  $l = 1, \dots, 2m$ , after that the eigenvalues of  $A$  can be find as  $\ln \frac{z_l}{T}$ . The LS of (5) can be reformulated as a global consensus problem over a grid of  $N$  calculation areas spanning the power grid, as shown in Figure 2.

The considered power system consists of 4 areas and each area contains 1 local PDC and 3 PMUs. The local PDCs

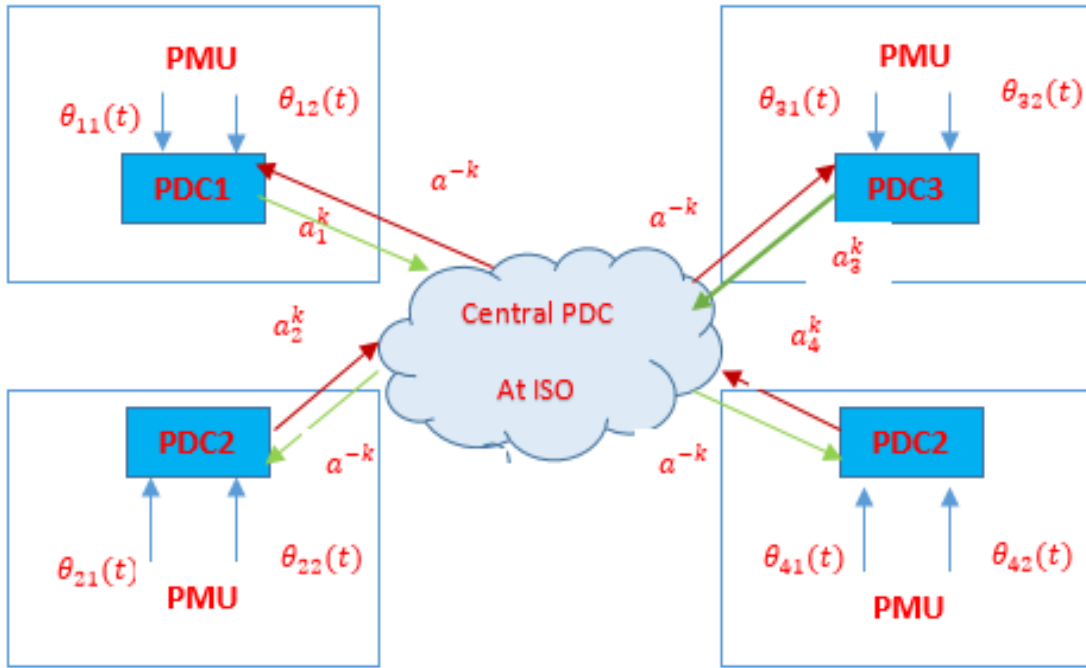


FIGURE 2. Grid architecture for considered 4-area network.

receive PMU measurements of that area and share synchronized time measurements to central PDC at Independent System Operator (ISO). The equation (5) then can be rewritten as:

$$\min_{a_1, \dots, a_N, z} \sum_{i=1}^N 0.5 * \|H^{\wedge}_i a_i - c^{\wedge}_i\|^2 \quad (6)$$

Such that  $a_i - z = 0$ , for  $i = 1, \dots, N$ ,  $H^{\wedge}_i = col(H_p)$ ,  $c^{\wedge}_i = col(c_p)$ ,  $p = 1, \dots, P_i$  ( $P_i$  is the number of sensors in each area),  $a_i$  is a vector of the primal variable and  $z$  is the local variable. From equation 6, we utilize ADMM algorithm, which reduces the next group of recursive update by utilizing an increased lagrangian. We can write

$$w_i^{(k)} = w_i^{(k-1)} + \rho(a_i^{(k-1)} - z^{(k)}) \quad (7)$$

$$a_i^{(k+1)} = \left( (H^{\wedge}_i)^T H^{\wedge}_i + \rho I \right)^{-1} \times \left( (H^{\wedge}_i)^T c^{\wedge}_i - w_i^{(k)} + \rho z^{(k)} \right) \quad (8)$$

$$z^{(k+1)} = \frac{1}{N} \sum_{i=1}^N (a_i^{(k+1)} + \frac{1}{\rho} w_i^{(k)}) \quad (9)$$

where  $w_i^{(k)}$  vector of the dual variables at repetition  $k$ ,  $\rho > 0$  indicates the penalty factor.  $H^{\wedge}_i^{(0)} = H^{\wedge}_i$  as determined in (6).

Step one: at time  $t_{1,i}^{(k)}$ , whatever global PDC ( $i$ ) provide the double first update gain ( $w_i^{(k)}$ ,  $a_i^{(k+1)}$ ) utilize (7) and (8) after attaining the consensus variable  $z^{(k)}$  from the main PDC.

Step two: at time  $t_{2,i}^{(k)}$ , the local PDC  $i$  send updated double -first changing set to ( $w_i^{(k)}$ ,  $a_i^{(k+1)}$ ) main PDC.

Step three: at time  $t_{3,i}^{(k)}$ , the main PDC computes the assent variable  $z^{(k+1)}$  by utilizing (9).

Step four: at time  $t_{4,i}^{(k)}$  the main PDC transmits  $z^{(k+1)}$  to the global PDC in every area. The last step is to form vander monde matrix and solve it for residues up to  $r_{in}$  using

$$\begin{bmatrix} \Delta\theta_i(0) \\ \Delta\theta_i(1) \\ \cdot \\ \cdot \\ \Delta\theta_i(m) \end{bmatrix} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ (\bar{z}_1)^{1/T} & (\bar{z}_1)^{1/T} & \dots & (\bar{z}_{2n})^{m/T} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ (\bar{z}_1)^{m/T} & (\bar{z}_2)^{m/T} & \dots & (\bar{z}_{2n})^{m/T} \end{bmatrix} \times \begin{bmatrix} r_{i1} \\ r_{i1}^* \\ \cdot \\ \cdot \\ r_{in} \\ r_{in}^* \end{bmatrix} \quad (10)$$

### III. ASYNCHRONOUS ADMM (A-ADMM) ALGORITHM

One basic consideration in standard ADMM algorithm is that all the local PDCs are obtaining their particular optimization procedure at a similar speed, and the communication latency between the main PDC and the local PDCs are also similar; thus, we can say that they are synchronous. Although the global PDCs might not get totally synchronized together due to variations in their processing speeds and different connection delays like transfer, queuing, and carry delays in the SDN. The solution to take control of this asynchrony is to put

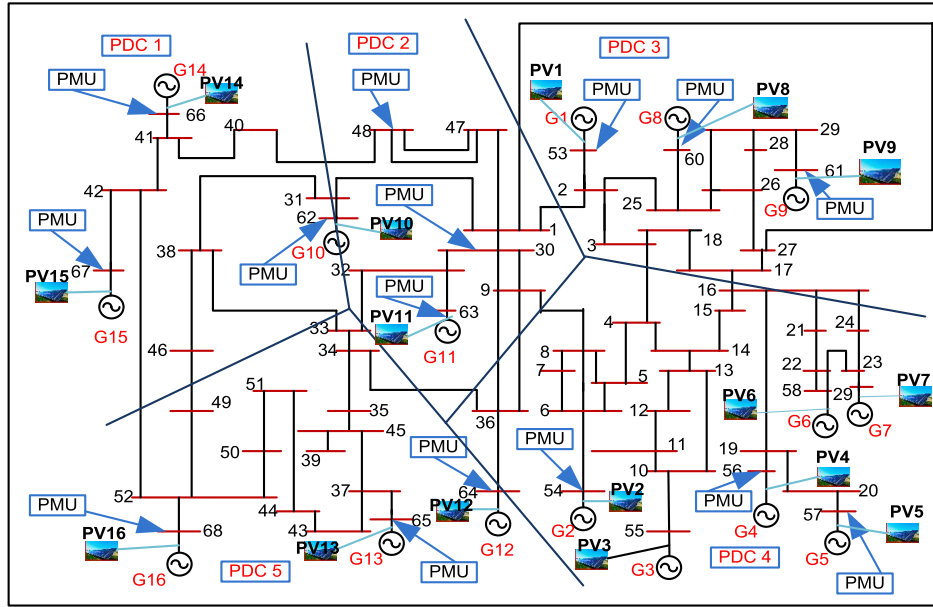


FIGURE 3. IEEE-68 bus system.

the main PDC to stay until it receives all local PDCs. In this state, the latency for every repetition will be dependent on the slow connection link. Under this condition, the algorithm will become slower. The various alternative method for resolving this problem is an asynchronous version of the ADMM algorithm. In this algorithm, the PDC receives the update data only from N global PDCs at each repetition point as active PDCs. For this algorithm, let this set be signified by  $S^k$  And then found  $z^{k+1}$ , utilizing the latest local evaluation from all PDCs. At this time instant  $T^{k+1}$ ,  $z^{k+1}$  is computed. The main PDC then transmit  $(z^{k+1}, T^{k+1})$  to each local PDC. After accepting  $T^{k+1}$  from every local PDC j at that point builds  $H_j^{k+1}$  and  $C_j^{k+1}$ . It can be written as [16], [24], [33], [34].

$$H_j^k \triangleq \left[ \left( H_{j,1}^k \right)^T \dots \left( H_{j,N_j}^k \right)^T \right]^T \quad (11)$$

$$C_j^k \triangleq \left[ \left( C_{j,1}^k \right)^T \dots \left( C_{j,N_j}^k \right)^T \right]^T \quad (12)$$

where,

$$H_{j,1}^k \triangleq \begin{bmatrix} \Delta\theta_{j,i}(2n-1) & \dots & \Delta\theta_{j,i}(0) \\ \vdots & \ddots & \vdots \\ \Delta\theta_{j,i}(m^k-1) & \dots & \Delta\theta_{j,i}(m^k-2n) \end{bmatrix} \quad (13)$$

$$C_j^k \triangleq \left[ \Delta\theta_{j,i}(2n) \ \Delta\theta_{j,i}(2n+1) \dots \Delta\theta_{j,i}(m^k) \right]^T$$

It should be noted that  $\Delta\theta(m^{k+1})$  may not be the latest estimation while building  $H_j^{k+1}, C_j^{k+1}$  matrix. The A-ADMM algorithm adapted for

$$\begin{aligned} &\text{Minimize} \sum_{j=1}^N .5 |H_j a_j - C_j|^2 \\ &\text{Subject to} \ a_j - z = 0, \quad \text{for } j = 1 \dots N. \end{aligned}$$

$$\begin{aligned} H_J &\triangleq [H_{j,1}^T H_{j,2}^T \dots H_{j,N_j}^T]^T, \\ C_j^T &\triangleq [C_{j,1}^T C_{j,2}^T \dots C_{j,N_j}^T]^T \end{aligned} \quad (14)$$

where  $N_j$  is the PMU in Area j [25], [28].

#### Distributed Algorithm Utilizing ADMM Algorithm

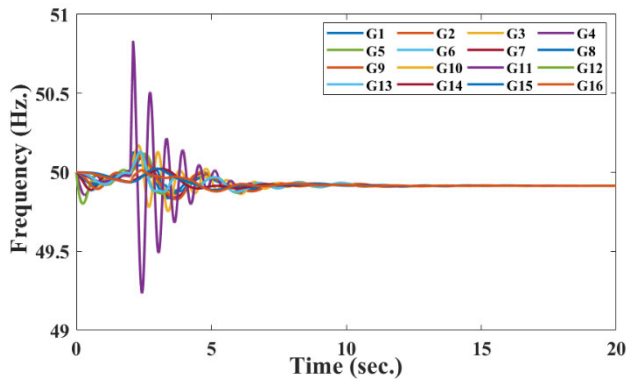
- 1: The main PDC beginning  $T^0$  and sent to every main PDC
- 2: PDCj start  $a_j^0, z^0$  &  $w_j^0$ , for  $j = 1, \dots, N$
- 3: Initialize  $K$ .
- 4: Given  $T^k$ , PDC j construct  $H_j^k$  and  $C_j^k$  from (1) using  $m^k$  decided from  $a_j$  as
 
$$a_j^{k+1} = ((H_j^k)^T H_j^k + \rho I_{2n})^{-1} ((H_j^k)^T c_j^k - w_j^k + \rho z^k)$$
- 5: PDC j sends  $a_j^{k+1}, w_j^k$  &  $k$  to the main PDC and takes the result of  $a_j^{k+1}, w_j^k$  and  $k$  from the PDCs  $j \in S^k$ .
- 6: The main PDC updates  $z$  as
 
$$z^{k+1} = \frac{1}{N} \sum_{j=1}^N (a_j^{k+1} + (\frac{1}{\rho}) w_j^k).$$
- 7: The main PDC sends  $z^{k+1}, k+1$  and  $T^{k+1}$  to local PDCs
- 8: PDC j setup  $w_j$  as
 
$$\begin{aligned} w_j^{k+1} &= w_j^k + \rho(a_j^{k+1} - z^{k+1}), \quad j \in S^k \\ w_j^{k+1} &= w_j^k, \quad j \in S^k \end{aligned}$$

#### IV. RESULT AND DISCUSSION

To verify the results, IEEE-68 standard bus system is considered as shown in Figure 3. The whole system is divided into five areas (PDC), and each area contains 3 PMUs and 1 local PDC. Table 1 shows the optimally placed PMU location for each considered area. For simplicity, sixth-order system generator is considered. A three-phase fault is simulated between

**TABLE 1.** The IEEE 68 bus model with (3 PMUs per area (PDC)).

Area (PDC)	Buses
1	31,38,40,41,42,46,52, <b>62,66,67</b>
2	1,9, <b>30,32,47,48,63</b>
3	2,3,16,17,18,25,26,27,28,29, <b>53,60,61</b>
4	4,5,6,7,8,10,11,12,13,14,15,19,20,21,22,23,24, <b>54,55,56,57</b>
5	58,33,34,35,37,39,43,44,45,49,50,51,52, <b>64,65,68</b>

**FIGURE 4.** Frequency (Hz) with time (sec).

bus 6 and bus 7, began at 0.4 s, cleared at 0.25 s and 0.8 sec at bus 6 and bus 7, respectively. The optimization iteration is started by considering initial 50 samples. The main objective is to assess different inter-area post-fault oscillations of the system. In this simulation, select  $2n = 40$  provides satisfying evaluations for inter-area mode. The value of  $\rho = 10^{-9}$  and  $\varepsilon = 10^{-6}$ . For each case, PDCs are chosen arbitrarily with equal probability of 0.9.

The time per iterations for the algorithm is found to be between 1-2 seconds using MATLAB 2016a tic() toc() command on an Intel(R) Core(TM) i5-7200U CPU @2.50GHz processor with 8.00 GM RAM and 64-bit Windows 10 OS. This is pragmatic since a PMU based communication delay will not be over a few milliseconds if added to the above-mentioned iteration time. The iteration time is comparable to the previous studies. One such study has been reported in [35]. Here Table 1 to Table 5 are of specific importance where the number of iterations and the iteration time in seconds are summarized for the ADMM technique against other techniques. Time per iteration in seconds for our study (between 1 to 2 seconds) is at par with the reported studies.

The frequency plot of all the generation buses is plotted in Figure 4.

#### A. CASE STUDY

Power system stability using eigenvalues during oscillations in power system after PV penetration is monitored by multi-channel PMUs using ADMM technique.

#### 1) SIMULATION METHODOLOGY

The dynamics of the IEEE 68 bus system was analyzed in MATPOWER toolbox in MATLAB. The fault was simulated at various locations, and post-disturbance dynamics was recorded. The recorded dynamics, mixed with PMU error, are portrayed as the PMU data for selective locations. The PMU data is then processed to obtain eigenvalues of the system for stability analysis.

Its hardware counterpart would be the data acquisition directly from the PMUs installed in the network through PDCs. Processing PDC data at a central location to obtain eigenvalues of the system using the ADMM technique.

First, the analysis of power system stability is performed in terms of the total number of PV sources in the system and the percentage of penetration. For analysis with varying PV penetration, a certain percentage of total installed capacity of synchronous generators is replaced by PV system and the effect of size of PV penetration on system stability is determined. For analysis with variation in number of PV sources, the synchronous generators are completely replaced by PV sources of the same capacity and the effect of number of PV sources on the system stability is determined.

Second, the accuracy of the method is tested both in terms of the number of PMUs in each area along with their channel capacities.

The analysis of each case considered is discussed as follows:

#### Case 1: Effect of size of PV penetration on system's vulnerability to disturbances using ADMM technique utilizing PMU data

Table 2 shows the effect of the size of PV on the eigenvalues of the system as monitored by Synchrophasors data. The system's vulnerability is increasing with an increase in the size of the PV. The eigenvalues for the post-disturbance dynamics are closer to the origin for higher PV penetration.

Note that due to the absence of techniques like inertial emulation and energy storage devices, the simulations cannot be run for higher PV penetration as it will make the system unstable. PVs do not provide an inertial response to the system, which leads to such vulnerability unless energy storage devices or virtual inertia techniques are used.

The results for damping ( $\sigma$ ) and frequency ( $\Omega$ ) for each case of varying PV penetration are shown in Figure 5 and Figure 6 respectively.

TABLE 2. Effect the PV on the eigenvalues.

S. No.	Size of PV (% of total generation)	Eigen Values ( $\sigma$ )	Eigen Values ( $j\Omega$ )	Remarks
1.	10	0.85, 0.46, 0.46, 0.62 at the figure of 5(A)	2.41, 3.52, 3.52, 4.95 at the figure of 6(A)	Stable
2.	20	0.45, 0.27, 0.27, 0.32 at the figure of 5(B)	2.00, 3.41, 3.41, 5.20 at the figure of 6(B)	Stable
3.	30	0.01, 0.20, 0.20, 0.22 at the figure of 5(C)	2.30, 3.05, 3.05, 4.68 at the figure of 6(C)	Stable
4.	40	-0.21, 0.10, 0.39, -0.02 at the figure of 5(D)	1.72, 3.12, 3.85, 4.75 at the figure of 6(D)	Unstable

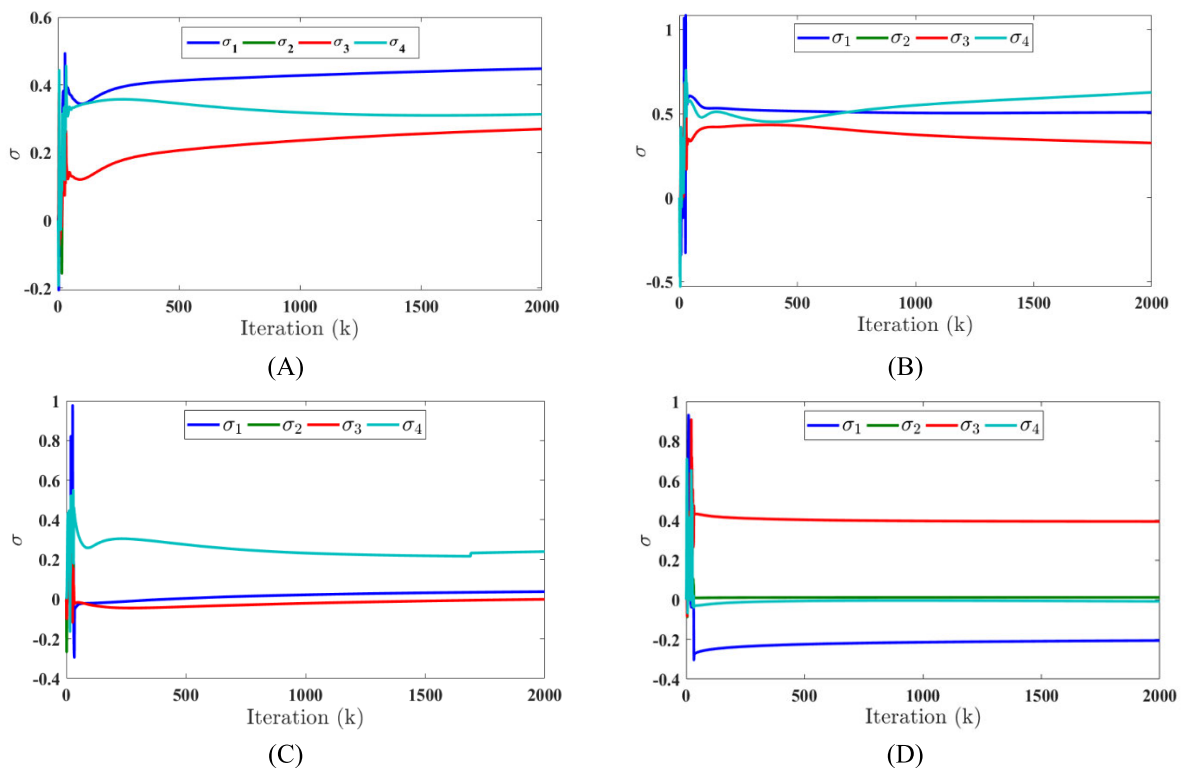


FIGURE 5. Estimation of  $\sigma$  PER iteration K for PV size with (A) 10% of total generation, (B) 20% of total generation, (C) 30% of total generation, (D) 40% of total generation.

**Case 2: Effect of number of PV sources on the system’s vulnerability to disturbances using ADMM**

The stability of the power system reduces as PV sources replace more conventional generators as these PV sources do not compensate the inertia provided by conventional generators. The eigenvalues of the system appear to come closer to the origin as evaluated at the PDC end. When the PV sources replace 6 generators, the system becomes unstable post fault. This indicates that the PV penetration seriously affects the inertial response on the power system; hence, this

vulnerability of distributed generation connected grid should be addressed. In addition, the study suggests the use of PMU based ADMM technique to find eigenvalues for disturbance analysis in power systems. The results for each case with the integration of different numbers of PV sources are tabulated in Table 3, and the results for damping ( $\sigma$ ) and frequency ( $\Omega$ ) for each case of a varying number of PV generation sources are shown in Figure 7 and Figure 8 respectively.

**Case 3: Effect of number of PMUs in each region on the accuracy of ADMM technique**

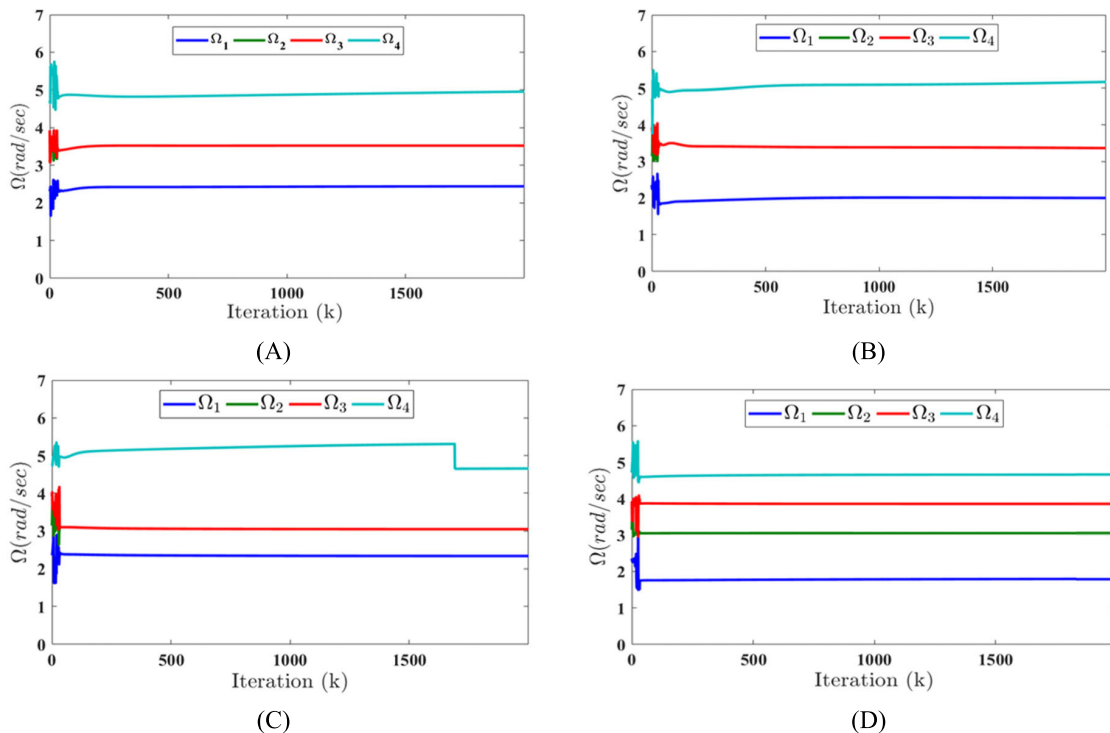


FIGURE 6. Estimation of  $\Omega$  PER iteration  $K$  for PV size with (A) 10% of total generation, (B) 20% of total generation, (C) 30% of total generation, (D) 40% of total generation.

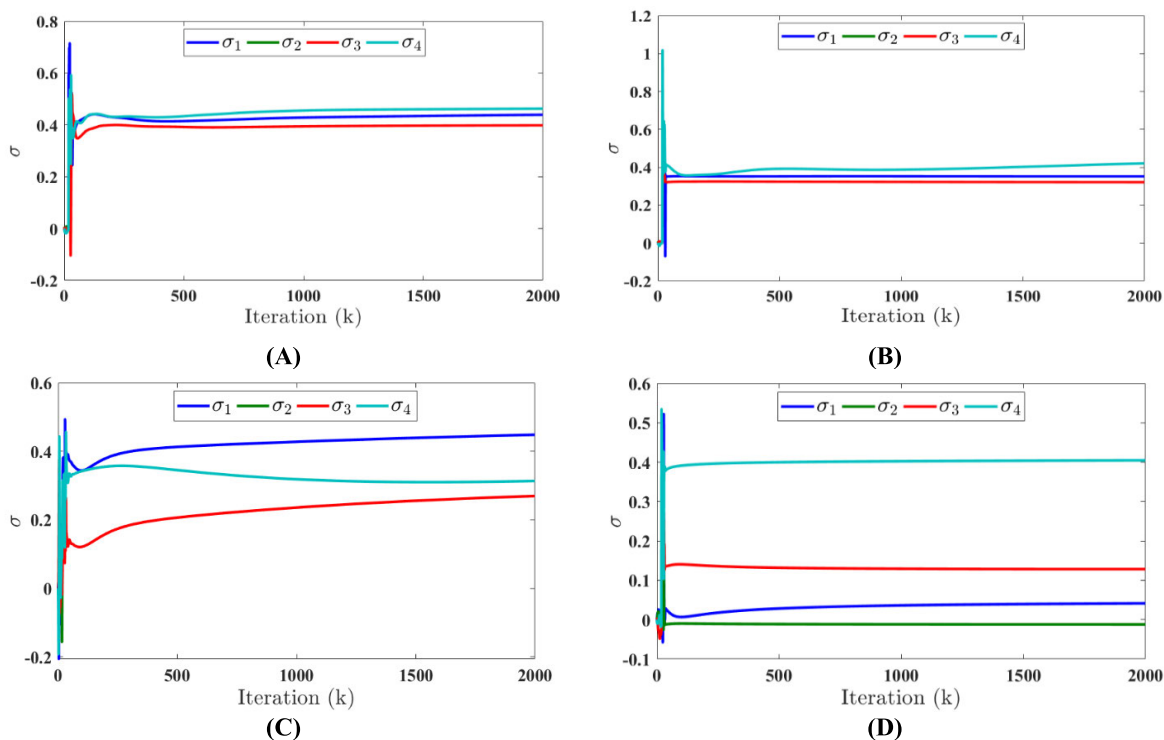


FIGURE 7. Estimation of  $\sigma$  PER iteration  $K$  with (A) 1 PV generation, (B) 2 PV generation, (C) 3 PV generation, (D) 6 PV generation.

As the number of PMUs are increased in each region, the accuracy of the technique to find eigenvalues using synchrophasor data improves. This is evident from Figure 9,

showing the error between the true eigenvalues and those evaluated by ADMM algorithm. However, as the number of PMUs increases in each area, the error margin reduces.



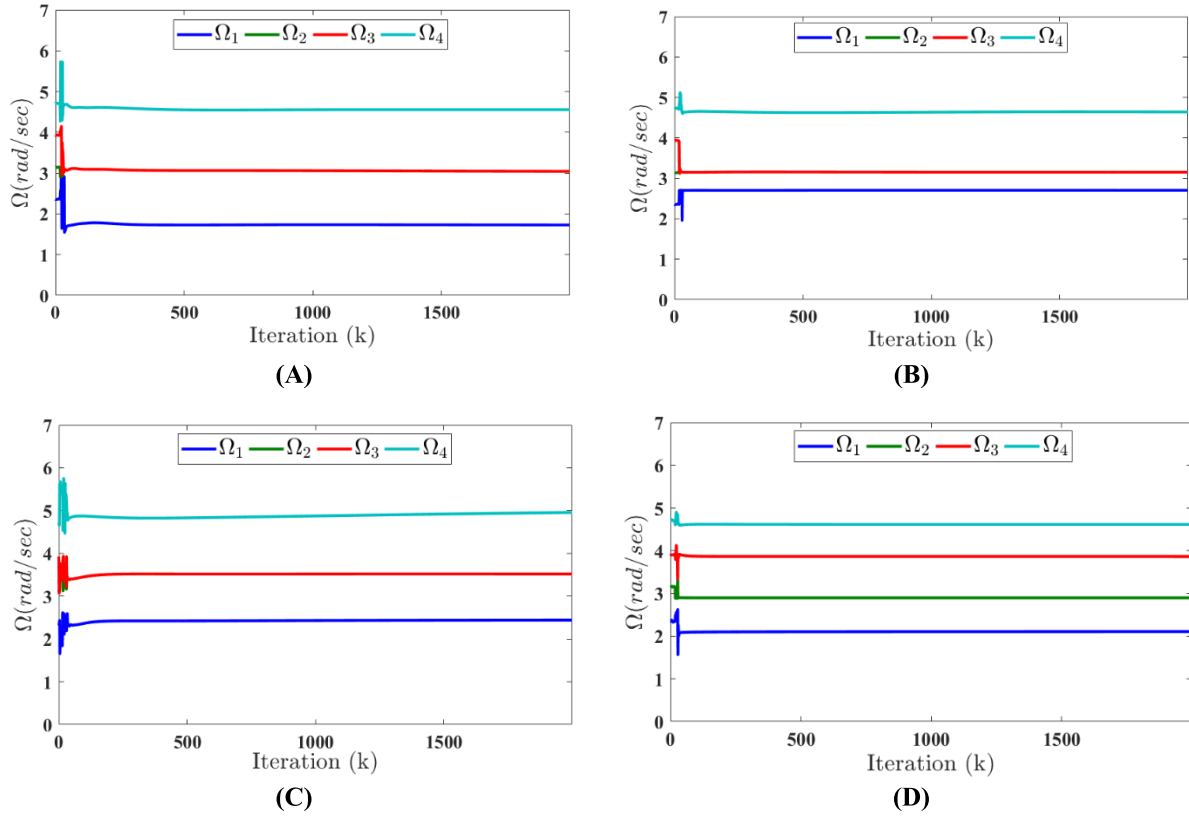


FIGURE 8. Estimation of  $\Omega$  PER iteration K with (A) 1 PV generation, (B) 2 PV generation, (C) 3 PV generation, (D) 6 PV generation.

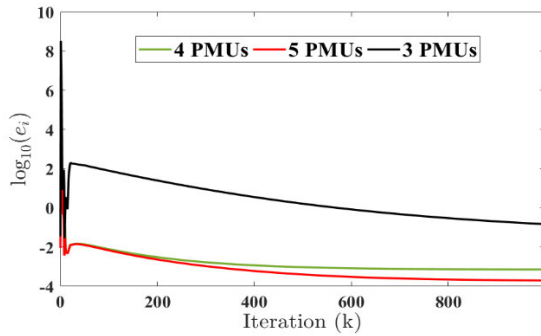
TABLE 3. Effect of number of PV sources on the system.

S. No.	No. of PV generation locations	Eigen Values ( $\sigma$ )	Eigen Values ( $j\Omega$ )	Remarks
1.	1	0.45,0.27,0.27,0.31 at the Figure of 7(A)	2.44,3.52,3.52,4.60 at the figure of 8(A)	Stable
2.	2	0.45,0.32,0.32,0.42 at the Figure of 7(B)	2.70,3.15,3.15,4.64 at the figure of 8(B)	Stable
3.	3	0.46,0.39,0.39,0.46 at the Figure of 7(C)	1.73,3.04,3.04,4.55 at the figure of 8(C)	Stable
4.	6	0.04, -0.01,0.13,0.40 at the Figure of 7(D)	2.10,2.90,3.86,4.61 at the figure of 8(D)	Unstable

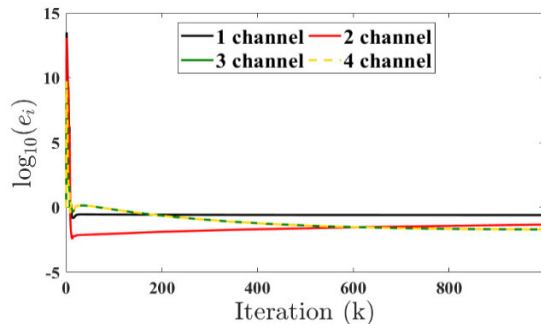
This clearly suggests that a trade-off between the installation cost of PMUs and accuracy can be made conveniently. Therefore, the study suggests that no more than 4 PMUs per area are needed as the error margin is not significant as more PMUs are added. However, if redundancy of the monitoring system is intended more PMUs can be placed.

**Case 4: Effect of number of PMU channels in each region on the accuracy of ADMM technique**

The effect of PMU channels on the accuracy of the technique has a similar pattern as observed in Case IV. More PMU channels reduce the error between the true eigenvalues and those evaluated using the ADMM technique shown in Figure 10. However, as the number of channels increases,



**FIGURE 9.** The error between the true eigenvalues and those evaluated by ADMM with different numbers of PMUs.



**FIGURE 10.** The error between the true eigenvalues and those evaluated by ADMM with different numbers of PMU channels in each region values.

the error margin reduces so much that the 4 PMU channel error curve almost coincides with the 3 PMU channel error curve. Hence, placement of a costly 4-PMU channel will be a needless exercise as 3 channel PMUs will suffice to monitor the system. Further, those locations (buses) which have less than 3 intersecting lines (branches) can have PMUs with lesser channels. For example, a bus having only two interconnections will need only a 2 channel PMU unless redundancy in measurements is intended.

## V. CONCLUSION

Power system stability is one of the most severe concerns due to integrating a large number of inertia-less renewable energy resources, especially solar PV systems. This paper performs stability analysis under such conditions using distributed PMU-PDC architecture. The power system stability is tested under different test conditions that include variations of PV penetration and the number of PV generation sources in the system. The performance of the proposed technique is also tested by varying the number of PMUs in the system along with their channel capacity. The study results obtained on the IEEE-68 bus system demonstrate the effectiveness of the proposed work.

The proposed work demonstrates how the different PDCs installed across the power system can communicate with each other to avoid data handling issues in centralized PDC architecture. The ADMM algorithm has been presented and

the results computed for each mode suggest that the future installation of PMUs and PDCs can be reconfigured into distributed one to enhanced accuracy, reliability and avoid complexity, the most important the data handling.

## REFERENCES

- [1] M. Zuhair and M. Rihan, "PMU installation in power grid for enhanced situational awareness: Challenges, issues and application," in *Proc. 4th IEEE Uttar Pradesh Sect. Int. Conf. Electr., Comput. Electron. (UPCON)*, Oct. 2017, pp. 654–659.
- [2] M. A. A. Sufyan, M. Zuhair, and M. Rihan, "An investigation on the application and challenges for wide area monitoring and control in smart grid," *Bull. Electr. Eng. Informat.*, vol. 10, no. 2, pp. 580–587, Apr. 2021.
- [3] A. Khair, M. Zuhair, and M. Rihan, "Effective utilization of limited channel PMUs for islanding detection in a solar PV integrated distribution system," *J. Inst. Eng. India, B*, vol. 102, no. 1, pp. 75–86, Feb. 2021.
- [4] M. A. A. Sufyan, M. Zuhair, M. Sefid, and M. Rihan, "Analysis of effectiveness of PMU based wide area monitoring system in Indian power grid," in *Proc. 5th IEEE Uttar Pradesh Sect. Int. Conf. Electr., Electron. Comput. Eng. (UPCON)*, Nov. 2018, p. 5.
- [5] T. Hiyama, N. Suzuki, and T. Funakoshi, "On-line identification of power system oscillation modes by using real time FFT," in *Proc. IEEE Power Eng. Soc. Winter Meeting. Conf.*, Jan. 2010, pp. 1521–1526.
- [6] V. Rampurkar, F. Kazi, H. A. Mangalvedekar, P. Pentayya, C. Kumar, and A. Rajkumar, "PMU based identification of low frequency oscillations—A case study," in *Proc. IEEE Innov. Smart Grid Technol.-Asia (ISGT Asia)*, Nov. 2013, pp. 1–5.
- [7] M. L. Crow and A. Singh, "The matrix pencil for power system modal extraction," *IEEE Trans. Power Syst.*, vol. 20, no. 1, pp. 501–502, Feb. 2005.
- [8] J. Khazaei, L. Fan, W. Jiang, and D. Manjure, "Distributed Prony analysis for real-world PMU data," *Electr. Power Syst. Res.*, vol. 133, pp. 113–120, Apr. 2016.
- [9] M. K. Arpanahi, M. Kordi, R. Torkzadeh, H. H. Alhelou, and P. Siano, "An augmented Prony method for power system oscillation analysis using synchrophasor data," *Energies*, vol. 12, no. 7, p. 1267, Apr. 2019.
- [10] M. A. M. Ariff and B. C. Pal, "Coherency identification in interconnected power system—An independent component analysis approach," *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 1747–1755, May 2013.
- [11] A. R. Messina and V. Vittal, "Extraction of dynamic patterns from wide-area measurements using empirical orthogonal functions," *IEEE Trans. Power Syst.*, vol. 22, no. 2, pp. 682–692, May 2007.
- [12] M. Zuhair, M. Rihan, and M. T. Saeed, "A novel method for locating the source of sustained oscillation in power system using synchrophasors data," *Protection Control Mod. Power Syst.*, vol. 5, no. 1, p. 30, Dec. 2020.
- [13] S. You, J. Guo, G. Kou, Y. Liu, and Y. Liu, "Oscillation mode identification based on wide-area ambient measurements using multivariate empirical mode decomposition," *Electr. Power Syst. Res.*, vol. 134, pp. 158–166, May 2016.
- [14] E. Barocio, B. C. Pal, N. F. Thornhill, and A. R. Messina, "A dynamic mode decomposition framework for global power system oscillation analysis," *IEEE Trans. Power Syst.*, vol. 30, no. 6, pp. 2902–2912, Nov. 2015.
- [15] H. Khalilinia and V. M. Venkatasubramanian, "Modal analysis of ambient PMU measurements using orthogonal wavelet bases," *IEEE Trans. Smart Grid*, vol. 6, no. 6, pp. 2954–2963, Nov. 2015.
- [16] N. Zhou, D. J. Trudnowski, J. W. Pierre, and W. A. Mittelstadt, "Electromechanical mode online estimation using regularized robust RLS methods," *IEEE Trans. Power Syst.*, vol. 23, no. 4, pp. 1670–1680, Nov. 2008.
- [17] X. Li, T. Jiang, H. Yuan, H. Cui, F. Li, G. Li, and H. Jia, "An eigensystem realization algorithm based data-driven approach for extracting electromechanical oscillation dynamic patterns from synchrophasor measurements in bulk power grids," *Int. J. Electr. Power Energy Syst.*, vol. 116, Mar. 2020, Art. no. 105549.
- [18] M. Zuhair and M. Rihan, "Identification of low-frequency oscillation modes using PMU based data-driven dynamic mode decomposition algorithm," *IEEE Access*, vol. 9, pp. 49434–49447, 2021.
- [19] S. Kar and G. Hug, "Distributed robust economic dispatch in power systems: A consensus innovations approach," in *Proc. IEEE Power Energy Soc. Gen. Meeting*, Jul. 2012, pp. 1–8.

- [20] Z. Zhang and M.-Y. Chow, "Convergence analysis of the incremental cost consensus algorithm under different communication network topologies in a smart grid," *IEEE Trans. Power Syst.*, vol. 27, no. 4, pp. 1761–1768, Nov. 2012.
- [21] T. Erseghe, "Distributed optimal power flow using ADMM," *IEEE Trans. Power Syst.*, vol. 29, no. 5, pp. 2370–2380, Sep. 2014.
- [22] A. D. Domínguez-García, "Coordination of distributed energy resources for provision of ancillary services: Architectures and algorithms," in *Encyclopedia of Systems and Control*. London, U.K.: Springer, 2014, pp. 1–8.
- [23] E. Dall'Anese, H. Zhu, and G. B. Giannakis, "Distributed optimal power flow for smart microgrids," *IEEE Trans. Smart Grid*, vol. 4, no. 3, pp. 1464–1475, Sep. 2013.
- [24] J. Zhang, S. Nabavi, A. A. Chakraborty, and Y. Xin, "ADMM optimization strategies for wide-area oscillation monitoring in power systems under asynchronous communication delays," *IEEE Trans. Smart Grid*, vol. 7, no. 4, pp. 2123–2133, Jul. 2016.
- [25] S. Nabavi, J. Zhang, and A. Chakraborty, "Distributed optimization algorithms for wide-area oscillation monitoring in power systems using interregional PMU-PDC architectures," *IEEE Trans. Smart Grid*, vol. 6, no. 5, pp. 2529–2538, Sep. 2015.
- [26] I. Khan, Z. Li, Y. Xu, and W. Gu, "Distributed control algorithm for optimal reactive power control in power grids," *Int. J. Electr. Power Energy Syst.*, vol. 83, pp. 505–513, Dec. 2016.
- [27] I. Khan, Y. Xu, S. Kar, and H. Sun, "Compressive sensing-based optimal reactive power control of a multi-area power system," *IEEE Access*, vol. 5, pp. 23576–23588, 2017.
- [28] E. Wei and A. Ozdaglar, "On the  $O(1/k)$  convergence of asynchronous distributed alternating direction method of multipliers," in *Proc. IEEE Global Conf. Signal Inf. Process.*, Dec. 2013, pp. 551–554.
- [29] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Found. Trends Mach. Learn.*, vol. 3, no. 1, pp. 1–122, Jan. 2011.
- [30] S. Nabavi and A. Chakraborty, "A real-time distributed Prony-based algorithm for modal estimation of power system oscillations," in *Proc. Amer. Control Conf. (ACC)*, Portland, OR, USA, Jun. 2014, pp. 729–734.
- [31] S. Nabavi and A. Chakraborty, "Distributed estimation of inter-area oscillation modes in large power systems using alternating direction multiplier method," in *Proc. IEEE PES General Meeting*, National Harbor, MD, USA, Jul. 2014, pp. 1–5.
- [32] J. Zhang, S. Nabavi, A. Chakraborty, and Y. Xin, "Convergence analysis of ADMM-based power system mode estimation under asynchronous wide-area communication delays," in *Proc. IEEE Power Energy Soc. Gen. Meeting*, Denver, CO, USA, Jul. 2015, pp. 1–5.
- [33] R. Zhang and J. T. Kwok, "Asynchronous distributed ADMM for consensus optimization," in *Proc. 31th Int. Conf. Mach. Learn.*, Beijing, China, Jun. 2014, pp. 1701–1709.
- [34] A. R. Messina and V. Vittal, "Nonlinear, non-stationary analysis of inter-area oscillations via Hilbert spectral analysis," *IEEE Trans. Power Syst.*, vol. 21, no. 3, pp. 1234–1241, Aug. 2006.
- [35] M. Liao, D. Shi, Z. Yu, Z. Yi, Z. Wang, and Y. Xiang, "An alternating direction method of multipliers based approach for PMU data recovery," *IEEE Trans. Smart Grid*, vol. 10, no. 4, pp. 4554–4565, Jul. 2019.



**MARWAN AHMED ABDULLAH SUFYAN** was born in Taiz, Yemen, in 1979. He received the B.Tech. degree in electronic engineering (communication division) from IBB University, IBB, Yemen, in 2005, and the M.Tech. degree (Hons.) in control system from JNTUH, Hyderabad, India, in 2015. He is currently a Research Scholar with the Department of Electrical Engineering, AMU, Aligarh, India. His research interests include wide area control systems, PMU, and smart grid.



**MOHD ZUHAIB** (Member, IEEE) was born in Aligarh, Uttar Pradesh, India, in 1990. He received the B.Tech. and M.Tech. degrees in electrical engineering from Aligarh Muslim University, Aligarh, in 2013 and 2015, respectively. He is currently a Research Scholar with the Department of Electrical Engineering, Aligarh Muslim University. His research interests include wide area measurement systems (WAMS), synchrophasor measurements, and smart grid. He was awarded the Senior Research Fellowship from the Ministry of Human Resource Development, Government of India, New Delhi, India, in 2017.



**MOHD ANAS ANEES** (Member, IEEE) was born in Aligarh, India. He received the B.Tech. degree in electrical engineering and the M.Tech. degree in instrumentation and control engineering from Aligarh Muslim University, Aligarh, in 2011 and 2013, respectively. He has been an Assistant Professor with the Department of Electrical Engineering, Aligarh Muslim University, since 2015. During his M.Tech. degree, he worked on phasor measurement units in smart grid and state estimation. His research interests include control systems and its applications in electrical engineering, smart grid, and wide area measurement systems (WAMS).



**ABUL KHAIR** received the B.E. and M.Tech. degrees in electrical engineering and the Ph.D. degree from Aligarh Muslim University (AMU), Aligarh, India, in 2010, 2014, and 2019, respectively. His Ph.D. degree was supervised by Dr. Mohd Rihan at AMU, from 2014 to 2019. He is currently working as an Assistant Professor with the Department of Electrical Engineering, Saharsa College of Engineering, Saharsa. His research interests include wide area measurement systems (WAMS) and smart grid.



**MOHD RIHAN** (Senior Member, IEEE) received the B.Tech. (Hons.) and M.Tech. degrees in electrical engineering and the Ph.D. degree from Aligarh Muslim University (AMU), Aligarh, India, in 2001, 2005, and 2013, respectively. He is currently a Professor and the Member-in-Charge of electricity at AMU. He is also a Visiting Scientist at the National Institute of Solar Energy, an autonomous institute of Ministry of New and Renewable Energy, Government of India. As the Convener of Green University Project, he has been leading the efforts towards integration of solar power with distribution grid at AMU campus for last six years. His efforts led to the installation of 6.5 megawatt solar plants in the campus, which is the largest setup of green and renewable energy in any academic institute in the country. His research interests include wide area measurement systems (WAMS), synchrophasor measurements, smart grid, and control systems and its applications. He is a fellow of IET, U.K., and IE, India.

• • •