

Scalable Distributed State Estimation for a Class of State-Saturated Systems Subject to Quantization Effects

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ABSTRACT This paper investigates the problem of a scalable distributed state estimation for a class of discrete time-variant systems with state-saturation, quantization effects, and two redundant channels over a sensor network. In transmission data from a sensor to its estimator, two phenomena are considered together. First, the data of each sensor is transmitted to its estimator through two redundant communication channels. Second, innovation data is quantized before being used by the estimator. These phenomena are beneficial in alleviating the negative effects on measurements and reducing the energy consumption and bandwidth. In the structure of proposed filter consensus is used on estimations in which consensus is first achieved on the prediction estimation, then the accuracy of computed estimation is improved by two recursive equations. The parameters of the proposed filter are obtained for each sensor node by employing an upper bound for common error covariance, therefore less computational burden is required. Eventually, the comparative simulation results are presented to show that our method has better performance compared with a rival one recently published.

INDEX TERMS Sensor networks, distributed filtering, state-saturated systems, quantization effects.

I. INTRODUCTION

Distributed state estimation problem is a fundamental issue over wireless sensor networks in control engineering and signal processing. There is extensive research related to the study of distributed state estimation because of its extensive engineering applications in many fields, such as environment monitoring, target tracking, intelligent robotics, and battlefield surveillance [1]–[3]. One of the major challenges in distributed filtering is how to incorporate the information obtained from each sensor and its neighbours based on the given communication topology. Accordingly, several approaches have been presented for this challenge considering how to exchange data between sensors or estimators. The most popular method in the distributed state estimation is called consensus which can be classified into three main types: (1) consensus on estimation [4]–[7], in which the sum of weighted differences between the local and neighbouring

estimations is added to the estimator of each node. (2) consensus on measurement [8], wherein consensus is obtained on local measurements and innovation covariance. However, it requires high communication costs and does not assure convergence. Recently, a scalable distributed extended Kalman filter with consensus on measurements is proposed in [9]. (3) consensus on information [10], in which uniform local average of information is used such that the stability of algorithm is provided.

On the other hand, state saturation as a common nonlinear phenomenon appears in many practical systems because of the inherent physical restrictions of components or technological limitations; for instance, a moving vehicle with the state variables (i.e., speed and position) restrictions [11], [12]. The state saturation has a direct effect on the stability and performance of the systems. Consequently, it is essential to take the filtering process into account. So far, the problem of state estimation for state-saturated systems is investigated by a few researchers (see [13]–[19]). A time-varying filter related to the problem of estimation for a class of state-saturated

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systems is considered in [13] by presenting a free matrix and via a certain set of recursive nonlinear matrix inequalities. In [14], an event-based distributed state estimation problem is studied for the state-saturated systems with incomplete measurements and redundant channels. A recursive filter for discrete-time state-saturated systems with both missing measurements and randomly occurring nonlinearities is studied in [16]. In [17], the problem of an event-triggered distributed state estimation over sensor networks is derived for a class of discrete time-varying nonlinear systems, where noises and sensor saturations are supposed to be unknown and bounded. Also, a distributed recursive filtering is derived for the time-varying state-saturated systems under round-robin communication protocol over sensor networks in [19].

Another phenomenon that inevitably occurs in networked systems due to limited bandwidth is quantization. Some approaches are proposed to model this kind of error. For instance in [20], the quantization errors are modelled as sector bound uncertainties without any conservatism. Subsequently, the idea of the proposed approach in [20] is extensively employed in the estimation problems where communication is modelled with quantized transmissions [21]–[24]. A robust extended Kalman filter for a class of discrete time-varying nonlinear complex networks with event-triggered communication and quantization effects is investigated in [24]. In [25], a distributed Kalman filter is proposed for a class of state-saturated systems with fading measurements and quantization effects, and an augmented vector is used to compute the parameters of filter. Therefore, in the mentioned reference the cross-covariance matrices between coupling nodes are obtained recursively and used to obtain the gain matrices of the nodes simultaneously. Hence, a significant amount of communication and a high calculation burden is required. The design of distributed estimator with low computation cost for multi-sensor networks with a large number of nodes is one of the most important challenges in practical applications.

Motivated by the aforementioned discussions, this paper is purposed to derive a novel distributed state estimation for a class of state-saturated systems subject to both quantization effects and two redundant channels over a sensor network. By implementing state-saturation phenomena, two redundant channels and quantization effects in the design of filter bring substantial difficulties to performance analysis, but the problem would be more applicable. The main contributions of proposed approach can be stated as follows:

(1) A novel approach is proposed in which a two-layer structure is employed for the distributed filter such that the consensus is first achieved on the prediction estimation, then this value is refined by using the recursive equations to obtain a more accurate estimation.

(2) In order to transmit data from a sensor to its estimator, two phenomena are tackled simultaneously. First, each sensor sends local information to its estimator over two redundant communication channels. Second, the innovation data is quantized before being used by the estimator.

(3) The parameters of proposed estimator are obtained at each sensor by solving three Riccati-like difference equations using an upper bound for the cross-covariance.

It is worth mentioning that the proposed method is scalable for multi-sensor networked systems with large nodes because of using an upper bound for the cross-covariance.

The remaining sections of this paper are structured as follows: The necessary background materials are recalled and the problem of a class of discrete systems with state-saturation is formulated in section 2. The structure of proposed distributed state estimation is presented in section 3. In section 4, simulation results are provided to illustrate the applicability of the suggested approach. Ultimately, the conclusion is given in section 5.

Notations: The notations employed in this paper are quite standard. For a matrix B , $\|B\|$ and $tr(B)$ show the spectral norm and the trace, respectively. $E\{\xi\}$ symbolizes the expectation of the random variable ξ . I indicates the identity matrix with appropriate dimension.

II. PROBLEM SETUP

In this paper, the sensor network is modelled as an undirected graph $\Omega = (\nu, \vartheta)$ where $\nu = \{1, \dots, m\}$ and $\vartheta \subset \nu \times \nu$ are the nodes and the edges, respectively. Two nodes are connected if and only if they can exchange data between each other. The set of nodes linked to the node i is called the neighborhood of node i and is represented by Λ_i . N_i shows the number of neighbors of node i . It is emphasized that we consider an undirected graph in this study.

Consider the following discrete time-variant system with state-saturation:

$$x_{s+1} = \sigma(A_s x_s + B_s w_s) \quad (1)$$

where $x_s \in R^n$ is the state vector of system that should be estimated, A_s and B_s are known time-variant matrices with appropriate dimensions and w_s is the process noise. The saturation function $\sigma(\cdot) : R^n \rightarrow R^n$ is denoted as follows:

$$\sigma(\tau) \triangleq [\sigma_1(\tau_1) \sigma_2(\tau_2) \dots \sigma_n(\tau_n)] \quad (2)$$

in which

$$\sigma_i(\tau_i) = \text{sign}(\tau_i) \min \{ \zeta_{i,max}, |\tau_i| \} \quad (3)$$

where sign denotes the signum function, $\zeta_{i,max}$ is the i -th element of vector τ_{max} and it shows the saturation level.

The measurement of the i -th sensor can be depicted as follows:

$$z_{i,s} = \xi_{i,s}^1 (C_{i,s} x_s + D_{i,s}^1 v_s) + (1 - \xi_{i,s}^1) \xi_{i,s}^2 (C_{i,s} x_s + D_{i,s}^2 v_s), \quad i = 1, \dots, m \quad (4)$$

$z_{i,s} \in R^p$ is the output of the i -th sensor, $C_{i,s}$, $D_{i,s}^1$ and $D_{i,s}^2$ are known time-variant matrices with appropriate dimensions and v_s is the measurement noise. We assume that w_s and v_s are uncorrelated white noises with zero means and variances that are equal to Q_w and R_v . $\xi_{i,s}^1$ and $\xi_{i,s}^2$ are uncorrelated scalar random variables having Bernoulli distribution with

mean $\bar{\xi}_i^j, \bar{\xi}_i^j \in [0, 1], j = 1, 2$. i.e. $prob \left\{ \xi_{i,s}^j = 1 \right\} = \bar{\xi}_i^j$ and $prob \left\{ \xi_{i,s}^j = 0 \right\} = 1 - \bar{\xi}_i^j$. We assume that $\xi_{i,s}^j$ is uncorrelated with w_s and v_s .

Remark 1: In Eq. (4), It is assumed that the data of each sensor can be sent to its estimator through two redundant communication channels. If the transmitted state information is delivered by the first channel, other channel will be inactive, otherwise the second channel will be automatically enabled to send the state data. Redundant communication channels can help to alleviate the negative effects of packet dropouts and disconnection of partial channels in performance of networked systems [26]. It is worth noting, to avoid text clutter, we only consider two channels and adding more channels does not make a difference in the structure of the proposed method.

For the sake of brevity, lets define $\phi_{i,s} = (1 - \xi_{i,s}^1)\xi_{i,s}^2$, therefore the measurement output in Eq. (4) can be rewritten as follows:

$$z_{i,s} = \xi_{i,s}^1(C_{i,s}x_s + D_{i,s}^1v_s) + \phi_{i,s}(C_{i,s}x_s + D_{i,s}^2v_s). \quad (5)$$

In the networked systems, when each sensor exchanges data with its estimators/neighbors, too much energy will inevitably be consumed. To reduce the consumed energy and enhance bandwidth efficiently, the data is quantized before being transmitted. In this paper, we employ a logarithmic quantizer which is described for the i -th sensor as follows:

$$q_i(u_i) = \left[q_{i,1}(u_i^1) \quad q_{i,2}(u_i^2) \quad \dots \quad q_{i,p_i}(u_i^{p_i}) \right],$$

where $u_i^j, (j = 1, \dots, p_i)$ represents the j -th element of u_i and the set of quantization levels for each $q_{i,j}(\cdot)$ are defined as follows:

$$\Psi_{i,j} = \left\{ \pm \mu_r^{i,j}, \mu_r^{i,j} = \sigma_{i,j}^r \mu_0^{i,j}, r = \pm 1, \pm 2, \dots \right\} \cup \{0\} \\ 0 < \sigma_{i,j} < 1, \quad \mu_0^{i,j} > 0, \quad (6)$$

in which $\sigma_{i,j}$ describes the quantization density. The logarithmic quantizer connected with the quantization level (6) is defined as the following representation:

$$q_{i,j}(u_i^j) = \begin{cases} \mu_r^{i,j}, & \frac{1}{1 + \delta_{i,j}} \mu_r^{i,j} < u_i^j < \frac{1}{1 - \delta_{i,j}} \mu_r^{i,j} \\ 0 & u_i^j = 0 \\ -q_{i,j}(-u_i^j) & u_i^j < 0 \end{cases} \quad (7)$$

where $\delta_{i,j} = \frac{1 - \sigma_{i,j}}{1 + \sigma_{i,j}}$.

It is straightforward to derive $q_{i,j}(u_{i,s}^j) = (1 + \Theta_{i,s}^j)u_{i,s}^j$ with $|\Theta_{i,s}^j| < \delta_{i,j}$, by defining $\Theta_{i,s} = diag \left\{ \Theta_{i,s}^1, \dots, \Theta_{i,s}^{p_i} \right\}$, $\delta_i = diag \left\{ \delta_{i,1}, \dots, \delta_{i,p_i} \right\}$ and $F_{i,s} = \Theta_{i,s} \delta_i^{-1}$ which satisfy $F_{i,s} F_{i,s}^T \leq I$.

Before ending this section, some necessary lemmas are recalled from literature.

Lemma 1 [16]: suppose that there exists a real number $\varepsilon_i \in [0, 1]$ for all $y, x \in R$, such that:

$$\sigma_i(x) - \sigma_i(y) = \varepsilon_i(x - y), \quad i = 1, 2, \dots, m$$

in which the saturation function $(\sigma_i(\cdot), i = 1, \dots, n)$ is described in Eq. (3).

Lemma 2 [27]: For given matrices B, C, D and M satisfying $MM^T \leq I, X$ is considered as a symmetric positive definite matrix, and having a scalar $a > 0$ such that $a^{-1}I - DXD^T > 0$, then we can derive an inequality in the following form:

$$(B + CMD)X(B + CMD)^T \leq B(X^{-1} - aD^T D)^{-1}B^T + a^{-1}CC^T.$$

III. MAIN RESULTS

In this section, first a novel scalable distributed state estimation is presented for a linear system with state-saturated, then we obtain upper bounds for common, prediction and filter error covariances and finally the parameters of proposed filter are calculated such that the upper bounds of the estimation error covariances are minimized. The proposed distributed state estimation equations are defined as follows:

$$\hat{x}_{i,s|s}^- = \hat{x}_{i,s|s-1} + G_{i,s} \sum_{j \in \Lambda_i} (\hat{x}_{j,s|s-1} - \hat{x}_{i,s|s-1}) \quad (8a)$$

$$\hat{x}_{i,s+1|s} = \sigma(A_s \hat{x}_{i,s|s}^-) + L_{i,s} q(\tilde{z}_{i,s}) \quad (8b)$$

$$\hat{x}_{i,s|s} = \hat{x}_{i,s|s}^- + K_{i,s} q(\tilde{z}_{i,s}) \quad (8c)$$

$\hat{x}_{i,s|s}^-$ is the common estimation, $\hat{x}_{i,s|s}$ and $\hat{x}_{i,s+1|s}$ are the current estimation of state, and its one-step ahead prediction at i -th sensor, respectively. $\tilde{z}_{i,s} = z_{i,s} - (\bar{\xi}_i^1 + \bar{\phi}_i)C_{i,s}\hat{x}_{i,s|s}^-$ denotes the innovation sequence before being quantized, $G_{i,s}, L_{i,s}$ and $K_{i,s}$ are the time-varying parameters of proposed filter which will be determined later by minimizing the error covariance matrices.

Remark 2: In the proposed method, differently from existing consensus on estimation algorithms, the consensus is first done on the prediction estimations based on the given topology called common estimation in Eq. (8a). Then the common estimation is used with the prediction estimation in Eq. (8b) and the current estimation in Eq. (8c) to improve the accuracy of computed estimation. As can be seen similarly to [28]–[31], differently from traditional Kalman filter, term $L_{i,s}q(\tilde{z}_{i,s})$ is added to the prediction estimation equation to obtain more accurate estimation.

Based on Eqs. (6)-(7) and (8a), the quantized signal $q(\tilde{z}_{i,s})$ is described as follows:

$$q(\tilde{z}_{i,s}) = (I + \Theta_{i,s})(z_{i,s} - (\bar{\xi}_i^1 + \bar{\phi}_i)C_{i,s}\hat{x}_{i,s|s}^-). \quad (9)$$

By merging Eqs. (1)-(9), the common error $e_{i,s|s}^- = x_s - \hat{x}_{i,s|s}^-$, the prediction error $e_{i,s+1|s} = x_s - \hat{x}_{i,s+1|s}$ and the filter error $e_{i,s|s} = x_s - \hat{x}_{i,s|s}$ are defined as follows:

$$e_{i,s|s}^- = (I - N_i G_{i,s})e_{i,s|s-1} + G_{i,s} \sum_{j \in \Lambda_i} e_{j,s|s-1} \quad (10)$$

$$e_{i,s+1|s} = \bar{\varepsilon}_s(A_s e_{i,s|s}^- + B_s w_s) + (A_s - (\bar{\xi}_i^1 + \bar{\phi}_i)L_{i,s}C_{i,s}) \times e_{i,s|s}^- - A_s e_{i,s|s}^- - L_{i,s}(I + \Theta_{i,s})((\bar{\xi}_i^1 + \bar{\phi}_i) \times C_{i,s}x_s - (\bar{\xi}_i^1 D_{i,s}^1 + \phi_{i,s}D_{i,s}^2)v_s) - (\bar{\xi}_i^1 + \bar{\phi}_i)L_{i,s}\Theta_{i,s}C_{i,s}e_{i,s|s}^- \quad (11)$$

$$e_{i,s|s} = (I - (\bar{\xi}_i^1 + \bar{\phi}_i)K_{i,s}C_{i,s})e_{i,s|s}^- - (\bar{\xi}_i^1 + \bar{\phi}_i) \times K_{i,s}\Theta_{i,s}C_{i,s}e_{i,s|s}^- - K_{i,s}(I + \Theta_{i,s})((\bar{\xi}_i^1 + \bar{\phi}_i) \times C_{i,s}x_s - (\bar{\xi}_i^1 D_{i,s}^1 + \phi_{i,s}D_{i,s}^2)v_s), \quad (12)$$

where $\bar{\xi}_{i,s}^1 = \xi_{i,s}^1 - \bar{\xi}_i^1$, $\bar{\phi}_{i,s} = \phi_{i,s} - \bar{\phi}_i$ and $\bar{\varepsilon}_s = \text{diag}\{\varepsilon_{1,s}, \dots, \varepsilon_{n,s}\}$, $\varepsilon_{i,s} \in [0, 1], (i = 1, \dots, n)$. The covariances of the common, prediction and filtering errors are defined as follows:

$$p_{i,s|s}^- = E \left\{ e_{i,s|s}^- e_{i,s|s}^{-T} \right\} \quad (13)$$

$$p_{i,s+1|s} = E \left\{ e_{i,s+1|s} e_{i,s+1|s}^T \right\} \quad (14)$$

$$p_{i,s|s} = E \left\{ e_{i,s|s} e_{i,s|s}^T \right\} \quad (15)$$

based on Eqs. (10)-(15), the common, prediction and filter error covariances are obtained as follows:

$$p_{i,s|s}^- = (I - N_i G_{i,s})p_{i,s|s-1} (I - N_i G_{i,s})^T + \sum_{j \in \Lambda_i} \sum_{l \in \Lambda_i} G_{i,s} E \left\{ e_{j,s|s-1} e_{l,s+1|s}^T \right\} G_{i,s}^T + \sum_{j \in \Lambda_i} ((I - N_i G_{i,s}) E \left\{ e_{i,s|s} e_{j,s+1|s}^T \right\} G_{i,s}^T + G_{i,s} E \left\{ e_{j,s|s-1} e_{i,s+1|s}^T \right\} (I - N_i G_{i,s})^T) \quad (16)$$

$$p_{i,s+1|s} = (A_s - (\bar{\xi}_i^1 + \bar{\phi}_i)L_{i,s}C_{i,s})p_{i,s|s}^- (A_s - (\bar{\xi}_i^1 + \bar{\phi}_i) \times L_{i,s}C_{i,s})^T + A_s p_{i,s|s}^- A_s^T + E \left\{ \bar{\varepsilon}_s(A_s e_{i,s|s}^- + B_s w_s)(A_s e_{i,s|s}^- + B_s w_s)^T \bar{\varepsilon}_s^T \right\} + \aleph_1 + \aleph_1^T - \aleph_2 - \aleph_2^T + E\{(\bar{\xi}_i^1 + \bar{\phi}_i)^2 L_{i,s}\Theta_{i,s}C_{i,s}e_{i,s|s}^- e_{i,s|s}^{-T} \times (L_{i,s}\Theta_{i,s}C_{i,s})^T\} - \aleph_3 - \aleph_3^T + E\{(I + \Theta_{i,s}) \times (\bar{\xi}_i^1 D_{i,s}^1 + \phi_{i,s}D_{i,s}^2)v_s(L_{i,s}(I + \Theta_{i,s}) \times (\bar{\xi}_i^1 D_{i,s}^1 + \phi_{i,s}D_{i,s}^2)v_s)^T\} + E\{(\bar{\xi}_i^1 + \bar{\phi}_i)^2 \times L_{i,s}(I + \Theta_{i,s})C_{i,s}x_s(L_{i,s}(I + \Theta_{i,s})C_{i,s}x_s)^T\} \quad (17)$$

$$p_{i,s|s} = (I - (\bar{\xi}_i^1 + \bar{\phi}_i)K_{i,s}C_{i,s})p_{i,s|s}^- (I - (\bar{\xi}_i^1 + \bar{\phi}_i) \times K_{i,s}C_{i,s})^T - \tilde{\aleph}_1 - \tilde{\aleph}_1^T + E\{(\bar{\xi}_i^1 + \bar{\phi}_i)^2 K_{i,s}(I + \Theta_{i,s})C_{i,s}x_s(K_{i,s} \times (I + \Theta_{i,s})C_{i,s}x_s)^T\} + E\{K_{i,s}(I + \Theta_{i,s}) \times (\bar{\xi}_i^1 D_{i,s}^1 + \phi_{i,s}D_{i,s}^2)v_s(K_{i,s}(I + \Theta_{i,s})(\bar{\xi}_i^1 D_{i,s}^1 + \phi_{i,s}D_{i,s}^2)v_s)^T\} + E\{(\bar{\xi}_i^1 + \bar{\phi}_i)K_{i,s}\Theta_{i,s} \times C_{i,s}e_{i,s|s}^- e_{i,s|s}^{-T} (K_{i,s}\Theta_{i,s}C_{i,s})^T\} \quad (18)$$

in which

$$\aleph_1 = E\{(\bar{\varepsilon}_s - I)A_s e_{i,s|s}^- e_{i,s|s}^{-T} (A_s - (\bar{\xi}_i^1 + \bar{\phi}_i)L_{i,s}C_{i,s})^T\}$$

$$\aleph_2 = E\{(\bar{\varepsilon}_s - I)A_s e_{i,s|s}^- e_{i,s|s}^{-T} ((\bar{\xi}_i^1 + \bar{\phi}_i)L_{i,s}\Theta_{i,s}C_{i,s})^T\}$$

$$\aleph_3 = E\{(A_s - (\bar{\xi}_i^1 + \bar{\phi}_i)L_{i,s}C_{i,s})e_{i,s|s}^- e_{i,s|s}^{-T} ((\bar{\xi}_i^1 + \bar{\phi}_i) \times L_{i,s}\Theta_{i,s}C_{i,s})^T\}$$

$$\tilde{\aleph}_1 = E\{(I - (\bar{\xi}_i^1 + \bar{\phi}_i)K_{i,s}C_{i,s})e_{i,s|s}^- e_{i,s|s}^{-T} ((\bar{\xi}_i^1 + \bar{\phi}_i) \times K_{i,s}\Theta_{i,s}C_{i,s})^T\}.$$

According to Eqs. (16)-(18), due to the existence of uncertainty matrix $\Theta_{i,s}$ and unknown terms $\tilde{\aleph}_l, \aleph_l, l = 1, \dots, 3$ the common, prediction and filter error covariances cannot be obtained in an explicit way. Therefore, we will obtain upper bounds for them in Theorem 1. Before proceeding further, we introduce a useful lemma that will be used for computing the parameters of proposed filter.

Lemma 3: The state covariance matrix $\bar{q}_{i,s} = E \{x_s x_s^T\}$ for Eq. (1) can be calculated as follows:

$$\bar{q}_{i,s} = \min \left\{ 2\Sigma_{i,s}^- + 2\hat{x}_{i,s|s}^- \hat{x}_{i,s|s}^{-T}, dI \right\} \quad (19)$$

where $d = \sum_{i=1}^n \zeta_i^2$ and $\Sigma_{i,s}^-$ is an upper bound for the common error covariance that will be introduced later.

Proof: Based on Eq. (1) the state covariance matrix is obtained as follows:

$$E \left\{ x_s x_s^T \right\} = E \left\{ \sigma(A_{s-1}x_{s-1} + B_{s-1}w_{s-1}) \times \sigma^T(A_{s-1}x_{s-1} + B_{s-1}w_{s-1}) \right\} \leq E \left\{ \text{tr} \left\{ \sigma(A_{s-1}x_{s-1} + B_{s-1}w_{s-1})\sigma^T \times (A_{s-1}x_{s-1} + B_{s-1}w_{s-1}) \right\} \right\} I = dI \quad (20)$$

On the other hand, according to the definition of common error $x_s = e_{i,s|s}^- - \hat{x}_{i,s|s}^-$, we have:

$$E \left\{ x_s x_s^T \right\} = E \left\{ (e_{i,s|s}^- - \hat{x}_{i,s|s}^-)(e_{i,s|s}^- - \hat{x}_{i,s|s}^-)^T \right\} = \Sigma_{i,s}^- + \hat{x}_{i,s|s}^- \hat{x}_{i,s|s}^{-T} + e_{i,s|s}^- \hat{x}_{i,s|s}^{-T} + \hat{x}_{i,s|s}^- e_{i,s|s}^{-T} \quad (21)$$

using elementary inequality $xy^T + yx^T \leq xx^T + yy^T$ and selecting the third and fourth terms on the right-hand side of Eq. (21), one can conclude that:

$$e_{i,s|s}^- \hat{x}_{i,s|s}^{-T} + \hat{x}_{i,s|s}^- e_{i,s|s}^{-T} \leq \Sigma_{i,s}^- + \hat{x}_{i,s|s}^- \hat{x}_{i,s|s}^{-T} \quad (22)$$

substituting Eqs. (22) into (21), we have:

$$E \left\{ x_s x_s^T \right\} = 2\Sigma_{i,s}^- + 2\hat{x}_{i,s|s}^- \hat{x}_{i,s|s}^{-T}, \quad (23)$$

merging Eqs. (20)-(23), we can obtain Eq. (19).

Theorem 1: If there exists a positive scalar $\alpha_{i,s}$ such that $((C_{i,s}\bar{q}_{i,s}C_{i,s}^T)^{-1} - a_{i,s}\delta_i^T \delta_i) > 0$ and $(\bar{R}_{i,s}^{-1} - a_{i,s}\delta_i^T \delta_i) > 0$,

then $p_{i,s|s}^- \leq \Sigma_{i,s|s}^-$, $p_{i,s+1|s} \leq \Sigma_{i,s+1|s}$ and $p_{i,s|s} \leq \Sigma_{i,s|s}$, where $p_{i,s|s}^-$, $p_{i,s+1|s}$ and $p_{i,s|s}$ are defined in Eqs. (16)-(18), respectively. Moreover, $\Sigma_{i,s|s}^-$ and $\Sigma_{i,s+1|s}$ are the solutions of the following recursive equations:

$$\Sigma_{i,s|s}^- = (1 + N_i)((I - N_i G_{i,s})\Sigma_{i,s|s-1}(I - N_i G_{i,s})^T \times \sum_{j \in \Lambda_i} G_{i,s} \Sigma_{j,s|s-1} G_{i,s}^T) \quad (24)$$

$$\begin{aligned} \Sigma_{i,s+1|s} &= 3(A_s - (\bar{\xi}_i^1 + \bar{\phi}_i)L_{i,s}C_{i,s})\Sigma_{i,s|s}^-(A_s - (\bar{\xi}_i^1 + \bar{\phi}_i) \\ &\times L_{i,s}C_{i,s})^T + 3A_s \Sigma_{i,s|s}^- A_s^T + \bar{\lambda}_i L_{i,s} [((C_{i,s} \bar{q}_{i,s} \\ &\times C_{i,s}^T)^{-1} - a_{i,s} \delta_i^T \delta_i)^{-1} + a_{i,s}^{-1} I] L_{i,s}^T \\ &+ 3(\bar{\xi}_i^1 + \bar{\phi}_i)^2 L_{i,s} tr(\delta_i C_{i,s} \Sigma_{i,s|s}^- C_{i,s}^T \delta_i^T) L_{i,s}^T \\ &+ L_{i,s} [(\bar{R}_{i,s}^{-1} - a_{i,s} \delta_i^T \delta_i)^{-1} + a_{i,s}^{-1} I] L_{i,s}^T \\ &+ tr(3A_s \Sigma_{i,s|s}^- A_s^T + B_s Q_w B_s^T) I. \end{aligned} \quad (25)$$

The parameters of proposed filter in Eqs. (8a)-(8c) are obtained as follows:

$$L_{i,s} = 3(\bar{\xi}_i^1 + \bar{\phi}_i)A_s \Sigma_{i,s|s}^- C_{i,s}^T \tilde{\pi}_{i,s}^{-1} \quad (26)$$

$$K_{i,s} = 2(\bar{\xi}_i^1 + \bar{\phi}_i)\Sigma_{i,s|s}^- C_{i,s}^T \pi_{i,s}^{-1} \quad (27)$$

$$G_{i,s} = N_i(1 + N_i)\Sigma_{i,s|s-1}((1 + N_i)(N_i^2 \Sigma_{i,s|s-1} + (1 + N_i) \sum_{j \in \Lambda_i} \Sigma_{j,s|s-1}))^{-1} \quad (28)$$

where

$$\begin{aligned} \bar{R}_{i,s} &= \bar{\xi}_i^1 D_{i,s}^1 R_v D_{i,s}^{1T} + \bar{\phi}_i D_{i,s}^2 R_v D_{i,s}^{2T} \\ \tilde{\pi}_{i,s} &= 3(\bar{\xi}_i^1 + \bar{\phi}_i)^2 C_{i,s} \Sigma_{i,s|s}^- C_{i,s}^T + \bar{\lambda}_i [((C_{i,s} \bar{q}_{i,s} C_{i,s}^T)^{-1} \\ &- a_{i,s} \delta_i^T \delta_i)^{-1} + a_{i,s}^{-1} I] + 3(\bar{\xi}_i^1 + \bar{\phi}_i)^2 tr(\delta_i C_{i,s} \\ &\times \Sigma_{i,s|s}^- C_{i,s}^T \delta_i^T) + [(\bar{R}_{i,s}^{-1} - a_{i,s} \delta_i^T \delta_i)^{-1} + a_{i,s}^{-1} I] \\ \pi_{i,s} &= (\bar{\xi}_i^1 + \bar{\phi}_i)^2 C_{i,s} \Sigma_{i,s|s}^- C_{i,s}^T + \bar{\lambda}_i [((C_{i,s} \bar{q}_{i,s} C_{i,s}^T)^{-1} \\ &- a_{i,s} \delta_i^T \delta_i)^{-1} + a_{i,s}^{-1} I] + 2(\bar{\xi}_i^1 + \bar{\phi}_i)^2 tr(\delta_i C_{i,s} \\ &\times \Sigma_{i,s|s}^- C_{i,s}^T \delta_i^T) + [(\bar{R}_{i,s}^{-1} - a_{i,s} \delta_i^T \delta_i)^{-1} + a_{i,s}^{-1} I] \\ \bar{\lambda}_i &= E\{(\bar{\xi}_i^1 + \bar{\phi}_i)^2\} = (\bar{\xi}_i + \bar{\phi}_i)(1 - \bar{\xi}_i - \bar{\phi}_i) \end{aligned}$$

Proof: Using elementary inequality $xy^T + yx^T \leq xx^T + yy^T$ and selecting the second and third terms on the right-hand side of Eq. (16), we have:

$$\begin{aligned} &\sum_{j \in \Lambda_i} \sum_{l \in \Lambda_i} G_{i,s} E \left\{ e_{j,s|s-1} e_{l,s|s-1}^T \right\} G_{i,s}^T \\ &\leq \frac{1}{2} \sum_{j \in \Lambda_i} \sum_{l \in \Lambda_i} G_{i,s} (p_{j,s|s-1} + p_{l,s|s-1}) G_{i,s}^T \\ &= N_i \sum_{j \in \Lambda_i} G_{i,s} p_{j,s|s-1} G_{i,s}^T \quad (29) \\ &\sum_{j \in \Lambda_i} ((I - N_i G_{i,s}) e_{i,s|s-1} e_{j,s|s-1}^T G_{i,s}^T + G_{i,s} e_{j,s|s-1} \end{aligned}$$

$$\begin{aligned} &\times e_{i,s|s-1}^T (I - N_i G_{i,s})^T) \leq \sum_{j \in \Lambda_i} ((I - N_i G_{i,s}) e_{i,s|s-1} \\ &\times e_{i,s|s-1}^T (I - N_i G_{i,s})^T + G_{i,s} e_{j,s|s-1} e_{j,s|s-1}^T G_{i,s}^T) \\ &= N_i (I - N_i G_{i,s}) p_{i,s|s-1} (I - N_i G_{i,s})^T \\ &+ \sum_{j \in \Lambda_i} G_{i,s} p_{j,s|s-1} G_{i,s}^T \quad (30) \end{aligned}$$

Now, substituting Eqs. (29)-(30) into Eq. (16), we can obtain Eq. (24). The common gain $G_{i,s}$ is attained by setting the first variation of Eq. (24) to zero as follows:

$$\frac{\partial \Sigma_{i,s|s}^-}{\partial G_{i,s}} = (1 + N_i)((I - N_i G_{i,s})\Sigma_{i,s|s-1}(-N_i I)^T + \sum_{j \in \Lambda_i} G_{i,s} \Sigma_{j,s|s-1}) = 0 \quad (31)$$

$G_{i,s}$ in Eq. (28) is achieved by straightforward manipulation of Eq. (31).

For computing $\Sigma_{i,s+1|s}$, selecting the first term on the right-hand side of Eq. (17) and using lemma 1, we can conclude that:

$$\begin{aligned} &E \left\{ \bar{\epsilon}_s (A_s e_{i,s|s}^- + B_s w_s) (A_s e_{i,s|s}^- + B_s w_s)^T \bar{\epsilon}_s^T \right\} \\ &\leq E \left\{ \left\| \bar{\epsilon}_s (A_s e_{i,s|s}^- + B_s w_s) \right\|^2 \right\} I \\ &\leq E \left\{ \|\bar{\epsilon}_s\|^2 \left\| (A_s e_{i,s|s}^- + B_s w_s) \right\|^2 \right\} I \\ &\leq E \left\{ \left\| (A_s e_{i,s|s}^- + B_s w_s) \right\|^2 \right\} I \\ &= tr(A_s \Sigma_{i,s|s}^- A_s^T + B_s Q_w B_s^T) I \quad (32) \end{aligned}$$

using the aforementioned inequality for unknown terms $\aleph_j, j = 1, \dots, 3$, we have:

$$\begin{aligned} \aleph_1 + \aleph_1^T &\leq (\bar{\epsilon}_s - I) A_s e_{i,s|s}^- e_{i,s|s}^{-T} A_s^T (\bar{\epsilon}_s - I)^T \\ &+ (A_s - (\bar{\xi}_i^1 + \bar{\phi}_i) L_{i,s} C_{i,s}) e_{i,s|s}^- e_{i,s|s}^{-T} \\ &\times (A_s - (\bar{\xi}_i^1 + \bar{\phi}_i) L_{i,s} C_{i,s})^T. \quad (33) \end{aligned}$$

$$\begin{aligned} -\aleph_2 - \aleph_2^T &\leq (\bar{\epsilon}_s - I) A_s e_{i,s|s}^- e_{i,s|s}^{-T} A_s^T (\bar{\epsilon}_s - I)^T \\ &+ ((\bar{\xi}_i^1 + \bar{\phi}_i) L_{i,s} \Theta_{i,s} C_{i,s}) e_{i,s|s}^- e_{i,s|s}^{-T} \\ &\times ((\bar{\xi}_i^1 + \bar{\phi}_i) L_{i,s} \Theta_{i,s} C_{i,s})^T. \quad (34) \end{aligned}$$

$$\begin{aligned} -\aleph_3 - \aleph_3^T &\leq ((\bar{\xi}_i^1 + \bar{\phi}_i) L_{i,s} \Theta_{i,s} C_{i,s}) e_{i,s|s}^- e_{i,s|s}^{-T} ((\bar{\xi}_i^1 + \bar{\phi}_i) \\ &\times L_{i,s} \Theta_{i,s} C_{i,s})^T + (A_s - (\bar{\xi}_i^1 + \bar{\phi}_i) L_{i,s} C_{i,s}) \\ &\times e_{i,s|s}^- e_{i,s|s}^{-T} (A_s - (\bar{\xi}_i^1 + \bar{\phi}_i) L_{i,s} C_{i,s})^T. \quad (35) \end{aligned}$$

Moreover, an upper bound for the third term on the right-hand side of Eq. (17) is obtained as follows:

$$\begin{aligned} &E \left\{ (\bar{\xi}_i^1 + \bar{\phi}_i)^2 L_{i,s} \Theta_{i,s} C_{i,s} e_{i,s|s}^- e_{i,s|s}^{-T} (L_{i,s} \Theta_{i,s} C_{i,s})^T \right\} \\ &\leq (\bar{\xi}_i^1 + \bar{\phi}_i)^2 L_{i,s} E \left\{ tr(\Theta_{i,s} C_{i,s} e_{i,s|s}^- e_{i,s|s}^{-T} (\Theta_{i,s} C_{i,s})^T) \right\} \\ &\times L_{i,s}^T = (\bar{\xi}_i^1 + \bar{\phi}_i)^2 L_{i,s} tr(\delta_i C_{i,s} \Sigma_{i,s|s}^- C_{i,s}^T \delta_i^T) L_{i,s}^T. \end{aligned}$$

And finally, using lemma 2 in the fourth and fifth terms on the right-hand side of Eq. (17), we have:

$$\begin{aligned} E\{(\tilde{\xi}_{i,s}^1 + \tilde{\phi}_{i,s})^2 L_{i,s}(I + \Theta_{i,s})C_{i,s}x_s x_s^T (L_{i,s}(I + \Theta_{i,s}) \\ \times C_{i,s})^T\} = E\{(\tilde{\xi}_{i,s}^1 + \tilde{\phi}_{i,s})^2\}L_{i,s}(I + F_{i,s}\delta_i) \\ \times E\left\{C_{i,s}x_s x_s^T C_{i,s}^T\right\}(I + F_{i,s}\delta_i)^T L_{i,s}^T \\ \leq \bar{\lambda}_i L_{i,s} \left[((C_{i,s}\bar{q}_{i,s}C_{i,s}^T)^{-1} - a_{i,s}\delta_i^T \delta_i)^{-1} + a_{i,s}^{-1}I \right] L_{i,s}^T. \end{aligned} \quad (36)$$

$$\begin{aligned} E\{L_{i,s}(I + \Theta_{i,s})(\xi_{i,s}^1 D_{i,s}^1 + \phi_{i,s} D_{i,s}^2)v_s v_s^T (L_{i,s}(I + \Theta_{i,s}) \\ \times (\xi_{i,s}^1 D_{i,s}^1 + \phi_{i,s} D_{i,s}^2))^T\} = L_{i,s}(I + F_{i,s}\delta_i)E\{(\xi_{i,s}^1 D_{i,s}^1 \\ + \phi_{i,s} D_{i,s}^2)v_s v_s^T (\xi_{i,s}^1 D_{i,s}^1 + \phi_{i,s} D_{i,s}^2)^T\}(I + F_{i,s}\delta_i)^T L_{i,s}^T \\ \leq L_{i,s} \left[(\bar{R}_{i,s}^{-1} - a_{i,s}\delta_i^T \delta_i)^{-1} + a_{i,s}^{-1}I \right] L_{i,s}^T. \end{aligned} \quad (37)$$

Eq. (25) can be attained by substituting Eqs. (32)-(37) into Eq. (17). Similarly, for computing $\Sigma_{i,s|s}$, based on Eq. (21) and utilizing the abovementioned inequality for $\tilde{\mathfrak{N}}_1$, we have:

$$\begin{aligned} -\tilde{\mathfrak{N}}_1 - \tilde{\mathfrak{N}}_1^T \leq ((\bar{\xi}_i^1 + \bar{\phi}_i)K_{i,s}\Theta_{i,s}C_{i,s})e_{i,s|s}^- e_{i,s|s}^{-T} ((\bar{\xi}_i^1 + \bar{\phi}_i) \\ \times K_{i,s}\Theta_{i,s}C_{i,s})^T + (I - (\bar{\xi}_i^1 + \bar{\phi}_i)K_{i,s}C_{i,s}) \\ \times e_{i,s|s}^- e_{i,s|s}^{-T} (I - (\bar{\xi}_i^1 + \bar{\phi}_i)K_{i,s}C_{i,s})^T. \end{aligned} \quad (38)$$

using lemma 2, the upper bounds are calculated for other terms on the right-hand side of Eq. (18) as follows:

$$\begin{aligned} E\left\{(\bar{\xi}_i^1 + \bar{\phi}_i)^2 K_{i,s}\Theta_{i,s}C_{i,s}e_{i,s|s}^- e_{i,s|s}^{-T} (K_{i,s}\Theta_{i,s}C_{i,s})^T\right\} \\ \leq (\bar{\xi}_i^1 + \bar{\phi}_i)^2 K_{i,s}E\left\{tr(\Theta_{i,s}C_{i,s}e_{i,s|s}^- e_{i,s|s}^{-T} (\Theta_{i,s}C_{i,s})^T)\right\} \\ \times K_{i,s}^T = (\bar{\xi}_i^1 + \bar{\phi}_i)^2 K_{i,s}tr(\delta_i C_{i,s}\Sigma_{i,s|s}^- C_{i,s}^T \delta_i^T)K_{i,s}^T \quad (39) \\ E\{(\tilde{\xi}_{i,s}^1 + \tilde{\phi}_{i,s})^2 K_{i,s}(I + \Theta_{i,s})C_{i,s}x_s x_s^T ((I + \Theta_{i,s})C_{i,s})^T \\ \times K_{i,s}^T\} = E\{(\tilde{\xi}_{i,s}^1 + \tilde{\phi}_{i,s})^2\}K_{i,s}(I + F_{i,s}\delta_i)E\{C_{i,s}x_s x_s^T \\ \times C_{i,s}^T\}(I + F_{i,s}\delta_i)^T K_{i,s}^T \\ \leq \bar{\lambda}_i K_{i,s} \left[((C_{i,s}\bar{q}_{i,s}C_{i,s}^T)^{-1} - a_{i,s}\delta_i^T \delta_i)^{-1} + a_{i,s}^{-1}I \right] K_{i,s}^T \end{aligned} \quad (40)$$

$$\begin{aligned} E\{K_{i,s}(I + \Theta_{i,s})(\xi_{i,s}^1 D_{i,s}^1 + \phi_{i,s} D_{i,s}^2) \\ \times v_s v_s^T (K_{i,s}(I + \Theta_{i,s})(\xi_{i,s}^1 D_{i,s}^1 + \phi_{i,s} D_{i,s}^2))^T\} \\ = K_{i,s}(I + F_{i,s}\delta_i)E\{(\xi_{i,s}^1 D_{i,s}^1 + \phi_{i,s} D_{i,s}^2) \\ \times v_s v_s^T (\xi_{i,s}^1 D_{i,s}^1 + \phi_{i,s} D_{i,s}^2)^T\}(I + F_{i,s}\delta_i)^T \\ \times K_{i,s}^T \leq K_{i,s} \left[(\bar{R}_{i,s}^{-1} - a_{i,s}\delta_i^T \delta_i)^{-1} + a_{i,s}^{-1}I \right] K_{i,s}^T. \end{aligned} \quad (41)$$

Therefore, substituting Eqs. (38)-(41) into Eq. (18), the upper bound of filter error covariance is obtained as follows:

$$\begin{aligned} \Sigma_{i,s|s} = 2(I - (\bar{\xi}_i^1 + \bar{\phi}_i)K_{i,s}C_{i,s})\Sigma_{i,s|s}^- (I - (\bar{\xi}_i^1 + \bar{\phi}_i) \\ \times K_{i,s}C_{i,s})^T + B_s Q_w B_s^T + \bar{\lambda}_i K_{i,s} \\ \times \left[((C_{i,s}\bar{q}_{i,s}C_{i,s}^T)^{-1} - a_{i,s}\delta_i^T \delta_i)^{-1} + a_{i,s}^{-1}I \right] K_{i,s}^T \end{aligned}$$

$$\begin{aligned} + 2(\bar{\xi}_i^1 + \bar{\phi}_i)^2 K_{i,s}tr(\delta_i C_{i,s}\Sigma_{i,s|s}^- C_{i,s}^T \delta_i^T)K_{i,s}^T \\ + K_{i,s} \left[(\bar{R}_{i,s}^{-1} - a_{i,s}\delta_i^T \delta_i)^{-1} + a_{i,s}^{-1}I \right] K_{i,s}^T. \end{aligned} \quad (42)$$

The prediction and filter gains are calculated by setting the first variation of Eqs.(24) and (42) to zero as follows:

$$\begin{aligned} \frac{\partial \Sigma_{i,s+1|s}}{\partial L_{i,s}} \\ = 3(A_s - (\bar{\xi}_i^1 + \bar{\phi}_i)L_{i,s}C_{i,s})\Sigma_{i,s|s}^- (-\bar{\xi}_i^1 + \bar{\phi}_i)C_{i,s})^T \\ + \bar{\lambda}_i L_{i,s} [((C_{i,s}\bar{q}_{i,s}C_{i,s}^T)^{-1} - a_{i,s}\delta_i^T \delta_i)^{-1} + a_{i,s}^{-1}I] \\ + 3(\bar{\xi}_i^1 + \bar{\phi}_i)^2 L_{i,s}tr(\delta_i C_{i,s}\Sigma_{i,s|s}^- C_{i,s}^T \delta_i^T) \\ + L_{i,s} \left[(\bar{R}_{i,s}^{-1} - a_{i,s}\delta_i^T \delta_i)^{-1} + a_{i,s}^{-1}I \right] = 0 \end{aligned} \quad (43)$$

$$\begin{aligned} \frac{\partial \Sigma_{i,s|s}}{\partial K_{i,s}} \\ = 2(I - (\bar{\xi}_i^1 + \bar{\phi}_i)K_{i,s}C_{i,s})\Sigma_{i,s|s}^- (-\bar{\xi}_i^1 + \bar{\phi}_i)C_{i,s})^T \\ + \bar{\lambda}_i K_{i,s} [((C_{i,s}\bar{q}_{i,s}C_{i,s}^T)^{-1} - a_{i,s}\delta_i^T \delta_i)^{-1} + a_{i,s}^{-1}I] \\ + 2(\bar{\xi}_i^1 + \bar{\phi}_i)^2 K_{i,s}tr(\delta_i C_{i,s}\Sigma_{i,s|s}^- C_{i,s}^T \delta_i^T) \\ + K_{i,s} [(\bar{R}_{i,s}^{-1} - a_{i,s}\delta_i^T \delta_i)^{-1} + a_{i,s}^{-1}I] = 0 \end{aligned} \quad (44)$$

$L_{i,s}$ and $K_{i,s}$ in Eqs. (26)-(27) are obtained by straightforward manipulation of Eqs. (43) and (44).

Remark 3: The main issue of this paper is also examined in [25]. In [25], a distributed filter with consensus on measurements is presented and an augment vector is used to calculate the parameters of proposed filter. In the mentioned method the cross-covariance matrices between coupling nodes are calculated recursively and used to obtain the gain matrices of nodes simultaneously, but in our method an upper bound is implemented for cross-covariance to compute gain matrix. Therefore, computational burden of our method is remarkably less than [25]. Moreover, other challenge in the method proposed in [25] is that the gain matrix can be non-singular in the case of a link failure or the loss of information from the neighbours, but our method overcomes this challenge. Also, the augmented approach in [25] cannot be applied for a graph with a lot of nodes.

Remark 4: For computing the parameters of the proposed filter, the common, prediction and filter error covariances are computed in Eqs. (16)-(18). According to Eqs. (16)-(18), due to the existence of the uncertainty matrix $\Theta_{i,s}$ and unknown terms $\tilde{\mathfrak{N}}_1, \mathfrak{N}_l, l = 1, \dots, 3$ the common, prediction and filter error covariances cannot be obtained in an explicit way. To overcome this challenge, in Theorem 1 we obtained upper bounds for the common, prediction and filter error covariances. On the other hand, the cross-covariance matrices appear in the second and the third terms on the right-hand side of Eq. (16). If the cross-covariance matrices are directly computed, then the proposed distributed filter is not scalable. Therefore, we attain upper bounds for them in Eqs. (21)-(22) using the inequality $xy^T + yx^T \leq xx^T + yy^T$. Therefore, the parameters of the proposed filter are obtained by minimizing Eqs. (24)-(25) and (42).

IV. SIMULATION RESULTS

In this section, numerical simulation results are employed to indicate the merits of proposed method in Eqs. (8a)-(8c).

Example 1: Consider the following discrete time system:

$$A_s = \begin{bmatrix} 0.96 + 0.05 \sin(0.12s) & 0.4 \\ 0.15 & -0.75 \end{bmatrix}, B_s = \begin{bmatrix} 0.16 \\ 0.18 \end{bmatrix}$$

$$C_{1,s} = [0.9 \quad 0.62 + 0.05 \cos(0.12s)]$$

$$C_{2,s} = [0.74 + 0.04 \sin(0.1s) \quad 0.8]$$

$$C_{3,s} = [0.75 + 0.04 \sin(0.1s) \quad 0.74 + 0.04 \cos(0.1s)]$$

$$C_{4,s} = [0.75 \quad 0.65]$$

The uncorrelated noises w_s and v_s have zero-mean and unity covariances. The initial values of the states and the estimators are considered as follows:

$$x(0) = [1 \ 1]^T, \hat{x}_{i,0|0} = [0 \ 0]^T, \quad i = 1, \dots, 4$$

The saturation levels are $\zeta_{1,max} = 10$ and $\zeta_{2,max} = 2$. $\xi_{i,s}^1$ and $\xi_{i,s}^2$ are chosen 0.85 and 0, respectively. Also, the parameters of quantization are specified as $\mu_0^i = 0.1$, $\sigma_i = 0.15$. The network graph, $\Omega = (\nu, \vartheta)$ is built from the set of nodes $\nu = \{1, 2, 3, 4\}$ and the set of edges $\vartheta = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 3), (3, 1), (4, 4), (4, 1)\}$ with the adjacency elements $a_{ij} = 1$.

Figure.1 illustrates the trajectories of the states and their corresponding estimation versus time samples, obtained from 100 Monte Carlo simulation. As seen, the proposed filter has acceptable performance for four different sensor nodes. In the following to demonstrate the scalability of our method, we consider a new sensor network topology with 50 nodes. As shown in Figure. 2, all 50 sensors are organized in a network with 273 edges. Figure. 3 illustrates the averaged root mean square errors (RMSE) of estimations obtained by our method and a rival method [25]. It is worth noting our method is implemented by two different topologies while the algorithm proposed in [25] is not scalable for the network graph of 50 nodes. The averaged root mean square

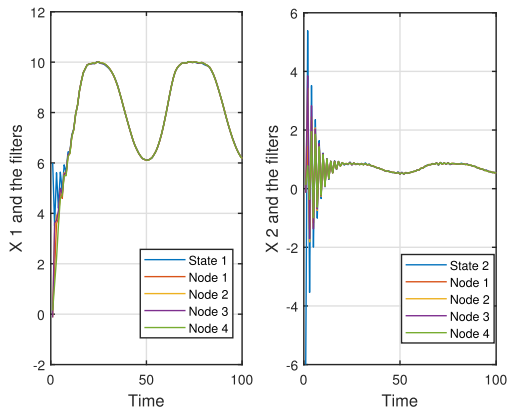


FIGURE 1. Trajectories of states and their estimations.

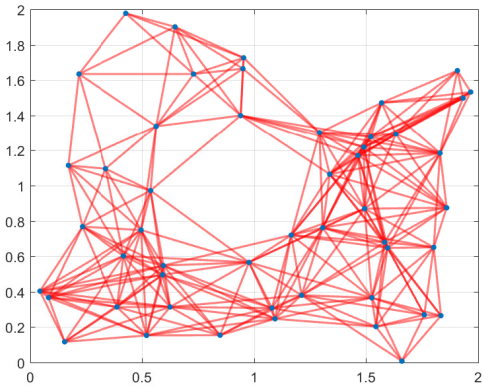


FIGURE 2. Simulated sensor network with 50 nodes and 273 edges.

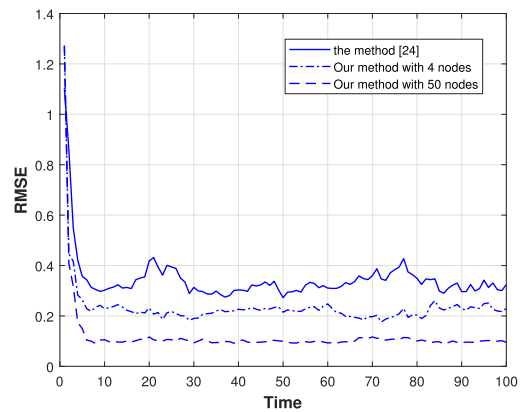


FIGURE 3. RMSE of estimation for two different filters.

TABLE 1. The averages of MSEs for different sensors.

Sensor node	1	2	3	4
MSE	0.2507	0.2319	0.2280	0.2512

error (RMSE) is calculated by:

$$RMSE : \frac{1}{4} \sum_{i=1}^4 \sqrt{\frac{1}{200} \sum_{j=1}^{200} (x_{i,s}^1 - \hat{x}_{i,s|s}^{1j})^2 + (x_{i,s}^2 - \hat{x}_{i,s|s}^{2j})^2}$$

where x_s^1 and x_s^2 denote the first and second states, and $(\hat{x}_{i,s|s}^{1j}, \hat{x}_{i,s|s}^{2j})$ denotes the first and second estimates of the i -th sensor at the j -th Monte Carlo run. The mean square errors (MSEs) for different sensor nodes are reported in Table 1 to demonstrate estimation accuracy for each sensor node in the given topology. As it is obvious, each sensor node has a different accuracy depending on the number of its neighbours and its local information such as measurement of matrix and noise. It should be noted that if we consider a topology with more nodes, the MSEs of sensor nodes become more accurate.

Example 2: Consider the wheeled mobile robot (WMR) in Figure. 4 which consists of two rear driving wheels (A_1, A_2) [32].

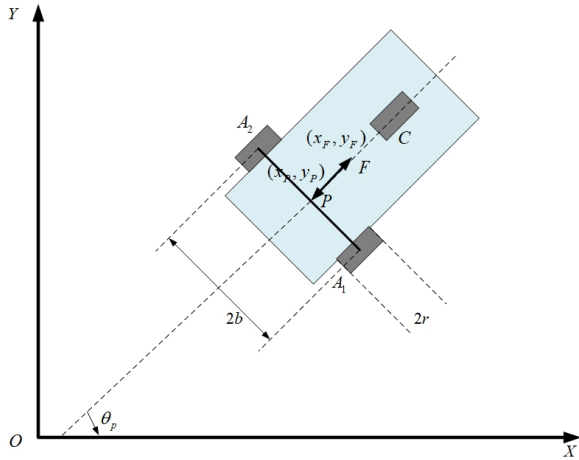


FIGURE 4. Diagram of the wheeled mobile robot.

The parameters of the WMR are reported as follows: F and P are the projection of the mass center and the center of two front wheels of the robot, respectively. Also, (x_p, y_p) and (x_f, y_f) symbolize the coordinates of P and the coordinates of F , respectively. l_f stands for the distance between the points F and P . θ_p denotes the heading angle of the robot and v_p is the transitional speed at the point P . ω shows the angular speed of WMR. If the state variables of the system are taken as $x = [\theta_p, \dot{x}_f, \dot{y}_f, \dot{\theta}_f]^T$, then, the dynamical equations of a typical WMR can be obtained. Discretization of the WMR model with sampling interval $T = 0.1$ yields to:

$$x_{s+1} = \begin{bmatrix} 1 & 0 & 0 & 0.1 \\ 0 & 1 & -0.00175 & -0.001 \\ 0 & 0.00175 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x_s + \begin{bmatrix} 0.01771 \\ 0.0381 \\ -0.0319 \\ 0.3542 \end{bmatrix} w_s,$$

$$y_{i,s} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x_s + \begin{bmatrix} 0.2 \sin(ik) \\ 0.3 \cos(ik) \\ 0.9 \sin(2ik) \\ 1.2 \cos(2ik) \end{bmatrix} v_s,$$

$i = 1, \dots, 5$

The uncorrelated noises, w_s and v_s have zero-mean and unity covariances. Filter parameters are as the following:

$$x(0) = [4 \ 1 \ 0.5 \ -0.5]^T,$$

$$\hat{x}_{1,0|-1} = \hat{x}_{2,0|-1} = \hat{x}_{3,0|-1} = \hat{x}_{4,0|-1} = \hat{x}_{5,0|-1}$$

$$\hat{x}_{5,0|-1} = [4 \ 1 \ 0.5 \ -0.5]^T$$

$$\varsigma_{1,\max} = 3, \quad \varsigma_{r,\max} = 0.5, \quad r = 2, 3, 4$$

The topology of the sensor network with five nodes is $\vartheta = \{(1, 4), (2, 1), (3, 2), (4, 3), (5, 4), (5, 1), (5, 3)\}$. Figure. 5 and Figure. 6 illustrate the trajectories for the actual states and the corresponding estimates for three randomly selected nodes from 500 Monte Carlo simulation.

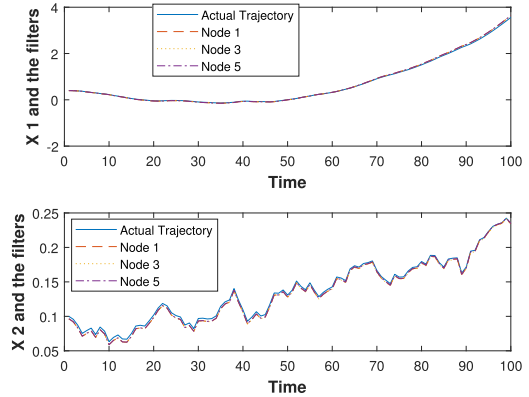


FIGURE 5. Trajectories of states for the first, second and their estimations.

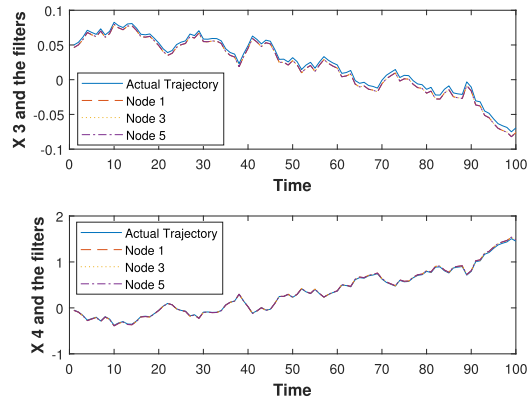


FIGURE 6. Trajectories of states for the third, fourth and their estimations.

V. CONCLUSION

In this paper, a novel scalable distributed state estimation has been presented for a class of state-saturated systems with quantization effects and two redundant channels. Both state-saturation and quantization phenomena have been considered in the proposed estimator. In our approach, the consensus is first achieved on the prediction estimate which is subsequently improved by two successive relations. Finally, by solving three Riccati-like difference equations, the parameters of filter have been determined at each sensor node by utilizing an upper bound for the common error covariance. Comparative simulation results have been reported to show that the averaged root mean square errors (RMSE) of estimation gets decreased by our method compared to a rival one in literature. The main results of this paper can be developed to the following issues in order to improve the proposed method in the future works: (1) consideration of the round-robin protocol for reducing the data exchange between nodes [19], [33], (2) attachment of the gain variations that usually occur in the practical system because of computational or implementation uncertainties in the hardware [34], [35], (3) consideration of the issue that when data is exchanged among the sensors, due to the vulnerability of communication networks, the information can be overheard and modified by the adversary [36], [37], (4) consideration of the Fault Estimation [38].

Thus, based on the view of the author, considering the above topics can improve the performance of the proposed method in practical applications.

REFERENCES

- [1] I. F. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, "A survey on sensor networks," *IEEE Commun. Mag.*, vol. 40, no. 8, pp. 102–114, Aug. 2002.
- [2] J. Yick, B. Mukherjee, and D. Ghosal, "Wireless sensor network survey," *Comput. Netw.*, vol. 52, no. 12, pp. 2292–2330, Aug. 2008.
- [3] F. Wang and J. Liu, "Networked wireless sensor data collection: Issues, challenges, and approaches," *IEEE Commun. Surveys Tuts.*, vol. 13, no. 4, pp. 673–687, 4th Quart., 2011.
- [4] R. Olfati-Saber and J. S. Shamma, "Consensus filters for sensor networks and distributed sensor fusion," in *Proc. 44th IEEE Conf. Decis. Control*, Dec. 2005, pp. 6698–6703.
- [5] R. Olfati-Saber, "Kalman-consensus filter: Optimality, stability, and performance," in *Proc. 48th IEEE Conf. Decis. Control (CDC) Held Jointly With 28th Chin. Control Conf.*, Dec. 2009, pp. 7036–7042.
- [6] W. Li, Y. Jia, and J. Du, "Distributed consensus extended Kalman filter: A variance-constrained approach," *IET Control Theory Appl.*, vol. 11, no. 3, pp. 382–389, Feb. 2017.
- [7] W. Song, J. Wang, C. Wang, and J. Shan, "A variance-constrained approach to event-triggered distributed extended Kalman filtering with multiple fading measurements," *Int. J. Robust Nonlinear Control*, vol. 29, no. 5, pp. 1558–1576, Mar. 2019.
- [8] R. Olfati-Saber, "Distributed Kalman filtering for sensor networks," in *Proc. 46th IEEE Conf. Decis. Control*, Dec. 2007, pp. 5492–5498.
- [9] H. Rezaei, R. M. Esfanjani, A. Akbari, and M. H. Sedaaghi, "Scalable event-triggered distributed extended Kalman filter for nonlinear systems subject to randomly delayed and lost measurements," *Digit. Signal Process.*, vol. 111, Apr. 2021, Art. no. 102957.
- [10] G. Battistelli and L. Chisci, "Kullback–Leibler average, consensus on probability densities, and distributed state estimation with guaranteed stability," *Automatica*, vol. 50, no. 3, pp. 707–718, 2014.
- [11] D. Liu and A. N. Michel, "Asymptotic stability of discrete-time systems with saturation nonlinearities with applications to digital filters," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 39, no. 10, pp. 798–807, Oct. 1992.
- [12] D. Liu and A. N. Michel, "Stability analysis of systems with partial state saturation nonlinearities," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 43, no. 3, pp. 230–232, Mar. 1996.
- [13] D. Ding, Z. Wang, B. Shen, and H. Shu, "State-saturated H_∞ filtering with randomly occurring nonlinearities and packet dropouts: The finite-horizon case," *Int. J. Robust Nonlinear Control*, vol. 23, no. 16, pp. 1803–1821, Nov. 2013.
- [14] C. Wen, Z. Wang, G. Tao, and F. E. Alsaadi, "Event-based distributed recursive filtering for state-saturated systems with redundant channels," *Inf. Fusion*, vol. 39, pp. 96–107, Jan. 2017.
- [15] D. Wang, Z. Wang, B. Shen, and Q. Li, " H_∞ finite-horizon filtering for complex networks with state saturations: The weighted try-once-discard protocol," *Int. J. Robust Nonlinear Control*, vol. 29, no. 7, pp. 2096–2111, May 2019.
- [16] C. Wen, Z. Wang, J. Hu, Q. Liu, and F. Alsaadi, "Recursive filtering for state-saturated systems with randomly occurring nonlinearities and missing measurements," *Int. J. Robust Nonlinearity Control*, vol. 28, pp. 1715–1727, Mar. 2018.
- [17] L. Ma, Z. Wang, H.-K. Lam, and N. Kyriakoulis, "Distributed event-based set-membership filtering for a class of nonlinear systems with sensor saturations over sensor networks," *IEEE Trans. Cybern.*, vol. 47, no. 11, pp. 3772–3783, Nov. 2017.
- [18] H. Rezaei, R. M. Esfanjani, A. Akbari, and M. H. Sedaaghi, "Event-triggered distributed Kalman filter with consensus on estimation for state-saturated systems," *Int. J. Robust Nonlinear Control*, vol. 30, no. 18, pp. 8327–8339, Dec. 2020.
- [19] B. Shen, Z. Wang, D. Wang, and H. Liu, "Distributed state-saturated recursive filtering over sensor networks under round-robin protocol," *IEEE Trans. Cybern.*, vol. 50, no. 8, pp. 3605–3615, Aug. 2020.
- [20] M. Fu and L. Xie, "The sector bound approach to quantized feedback control," *IEEE Trans. Autom. Control*, vol. 50, no. 11, pp. 1698–1711, Nov. 2005.
- [21] M. Fu and C. E. de Souza, "State estimation for linear discrete-time systems using quantized measurements," *Automatica*, vol. 45, no. 12, pp. 2937–2945, 2009.
- [22] J. Hu, Z. Wang, G.-P. Liu, and H. Zhang, "Variance-constrained recursive state estimation for time-varying complex networks with quantized measurements and uncertain inner coupling," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 31, no. 6, pp. 1955–1967, Jun. 2020.
- [23] J. Hu, Z. Wang, B. Shen, and H. Gao, "Quantised recursive filtering for a class of nonlinear systems with multiplicative noises and missing measurements," *Int. J. Control*, vol. 86, no. 4, pp. 650–663, 2013.
- [24] F. Rahimi and H. Rezaei, "An event-triggered recursive state estimation approach for time-varying nonlinear complex networks with quantization effects," *Neurocomputing*, vol. 426, pp. 104–113, Feb. 2021.
- [25] C. Wen, Z. Wang, Q. Liu, and F. E. Alsaadi, "Recursive distributed filtering for a class of state-saturated systems with fading measurements and quantization effects," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 48, no. 6, pp. 930–941, Jun. 2018.
- [26] L. Zhang, Z. Ning, and Z. Wang, "Distributed filtering for fuzzy time-delay systems with packet dropouts and redundant channels," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 46, no. 4, pp. 559–572, Apr. 2016.
- [27] T. Wang, L. Xie, and C. E. de Souza, "Robust control of a class of uncertain nonlinear systems," *Syst. Control Lett.*, vol. 19, no. 2, pp. 139–149, 1992.
- [28] H. Rezaei, R. M. Esfanjani, and M. H. Sedaaghi, "Improved robust finite-horizon Kalman filtering for uncertain networked time-varying systems," *Inf. Sci.*, vol. 293, pp. 263–274, Feb. 2015.
- [29] H. Rezaei, R. M. Esfanjani, and M. Farsi, "Robust filtering for uncertain networked systems with randomly delayed and lost measurements," *IET Signal Process.*, vol. 9, no. 4, pp. 320–327, Jun. 2015.
- [30] H. Rezaei, R. M. Esfanjani, and M. H. Sedaaghi, "Improved Kalman filtering for systems with randomly delayed and lost measurements," *Circuits Syst. Signal Process.*, vol. 33, no. 7, pp. 2217–2236, 2014.
- [31] H. Rezaei, S. Mohamed, R. M. Esfanjani, and S. Nahavandi, "Improved robust Kalman filtering for uncertain systems with missing measurements," in *Proc. Int. Conf. Neural Inf. Process.* Springer, 2014, pp. 509–518.
- [32] A. Farnam, M. M. Kaleybar, and R. M. Esfanjani, "Networked control of wheeled mobile robots," in *Proc. 1st RSJ/ISM Int. Conf. Robot. Mechatronics (ICRoM)*, Feb. 2013, pp. 55–61.
- [33] K. Liu, H. Guo, Q. Zhang, and Y. Xia, "Distributed secure filtering for discrete-time systems under round-robin protocol and deception attacks," *IEEE Trans. Cybern.*, vol. 50, no. 8, pp. 3571–3580, Aug. 2020.
- [34] Q. Liu, Z. Wang, X. He, G. Ghinea, and F. E. Alsaadi, "A resilient approach to distributed filter design for time-varying systems under stochastic nonlinearities and sensor degradation," *IEEE Trans. Signal Process.*, vol. 65, no. 5, pp. 1300–1309, Mar. 2017.
- [35] W. Li, Y. Jia, and J. Du, "Resilient filtering for nonlinear complex networks with multiplicative noise," *IEEE Trans. Autom. Control*, vol. 64, no. 6, pp. 2522–2528, Jun. 2019.
- [36] D. Ding, Q.-L. Han, Z. Wang, and X. Ge, "Recursive filtering of distributed cyber-physical systems with attack detection," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 51, no. 10, pp. 6466–6476, Oct. 2021.
- [37] W. Yang, Y. Zhang, G. Chen, C. Yang, and L. Shi, "Distributed filtering under false data injection attacks," *Automatica*, vol. 102, pp. 34–44, Apr. 2019.
- [38] F. Rahimi and H. Rezaei, "A distributed fault estimation approach for a class of continuous-time nonlinear networked systems subject to communication delays," *IEEE Control Syst. Lett.*, vol. 6, pp. 295–300, 2022.



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